The ROTATING RAKE MODE MEASUREMENT SYSTEM

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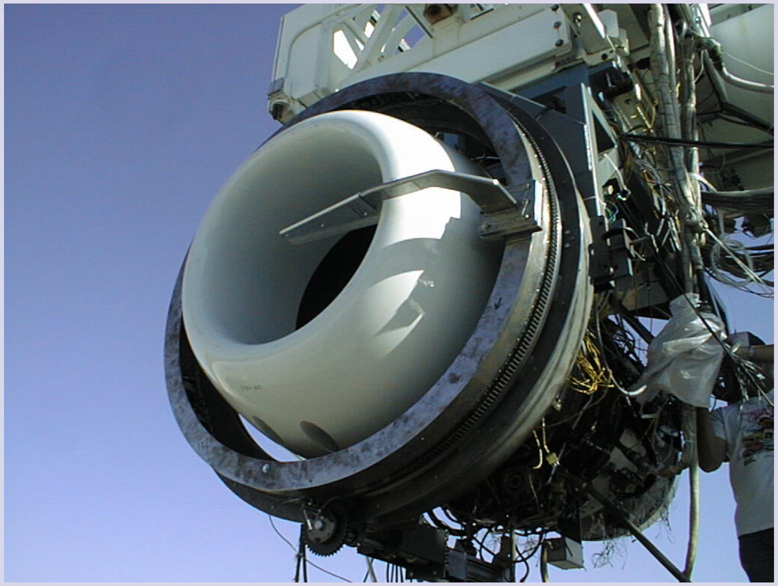
OVERVIEW

ROTATING RAKE: A radial rake inserted in to a turbofan inlet or exhaust duct. It has radially distributed pressure transducers. The rake continuously and synchronously rotates about the duct centerline to obtain a time history of the dynamic pressure. A complete acoustic mode map at the fan harmonics is obtained from the signal processing.

- Concept based on idea by T. Sofrin NASA CR-168040 Nov 1982.
- First Implemented by L. Heidelberg on 17-inch ADP model in the NASA Glenn 9x15 Wind Tunnel in 1991 described in AIAA Journal of Aircraft Vol 32, No 4, July-Aug 1995.
- Put into practice and continual improvement on 4' Low Speed Fan dedicated test bed at NASA Glenn.
- Continued use in NASA Glenn 9x15 Wind Tunnel.
- Multiple Engine tests.



ROTATING RAKE CONCEPTS

Continuously rotating, radially distributed from tip to hub array of pressure transducers installed at inlet/exhaust duct acoustic release point.

- Rake electronically very accurately synchronized to fan shaft
- Signals brought across rotating boundary via telemetry

KEY CONCEPT: Each m-order rotates at a different spin rate.

- The rotating rake imparts a unique Doppler shift depending on spin rate, hence m-order.
- **Radial modes in a given m-order complex curve fit using least-squares fit to the derived duct E-functions (eigen-functions, basis functions).**
  - **To-date only hardwall, plug flow solutions and conditions**
  - **Extending to softwall, plug flow solutions and conditions**

A complete circumferential and radial modal magnitude & phase map is obtained for the 1st three harmonics.

- (only as accurate as the basis function description)

ROTATING RAKE APPLICATIONS

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Put into practice and continual improvement on 4’ Low Speed Fan dedicated test bed at NASA Glenn.

Continued use in NASA Glenn 9x15 Wind Tunnel.

Presentation Outline

(I) Description of Acoustic Field that is Measured
- rotor/stator interaction
- duct modes

(II) Rotating Rake Theory and Implementation
- Doppler shift
- Radial Curve Fitting
- Mechanical Parameters

(III) Examples of Rotating Rake Applications

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(I) Description of Acoustic Field that is Measured

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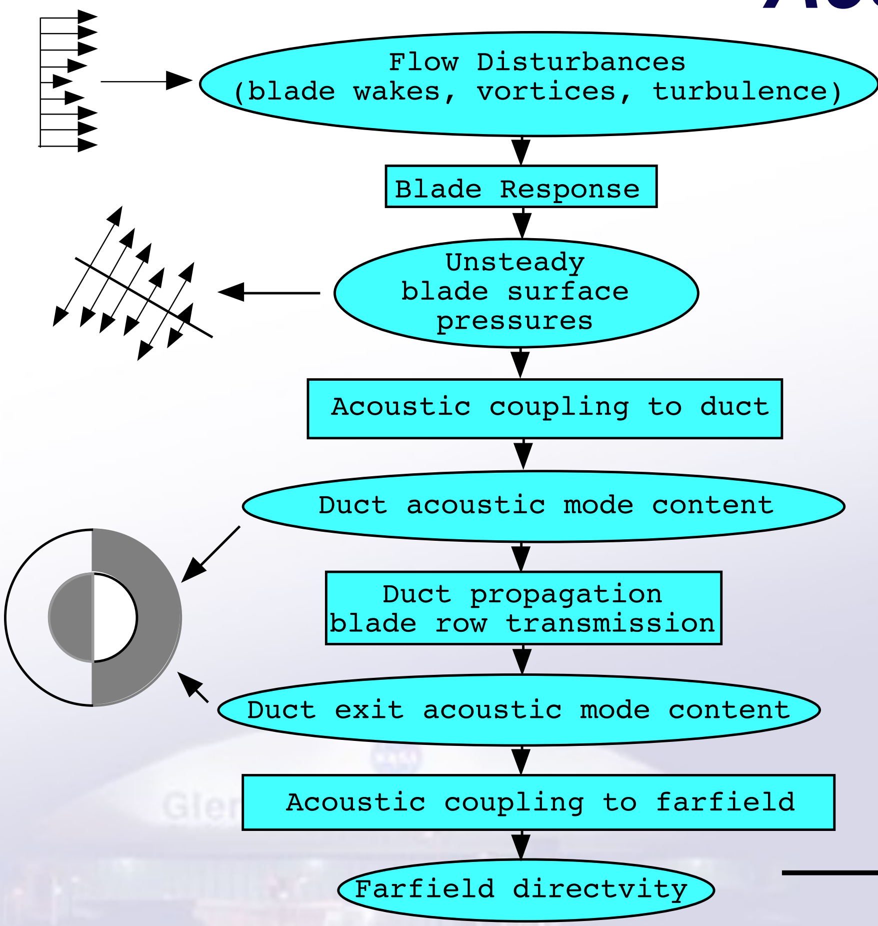
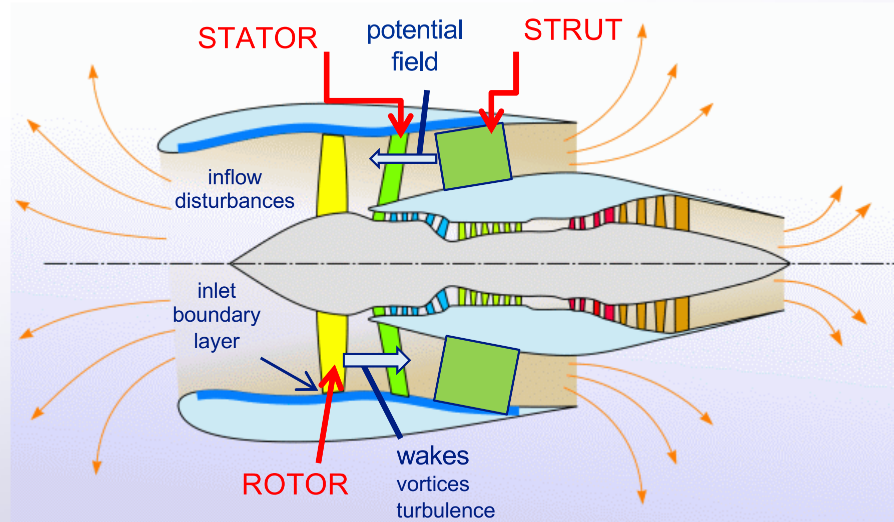
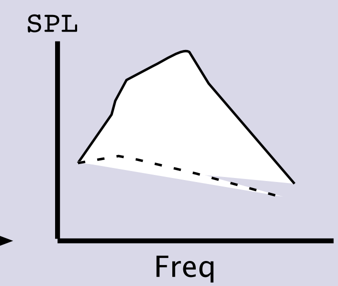
(III) Examples of Rotating Rake Applications

Physics/Measurement of Fan/Duct Aero-Acoustics







OVALS: Physically measurable quantity.

RECTANGLES: transfer function that can be investigated for prediction (noise reduction)

Acoustic Wave Equation in a Duct

Wave Equation:

$$\frac{\partial^2 p}{\partial t^2} + M\frac{\partial^2 p}{\partial x^2} = \nabla^2 p \quad \Longrightarrow \quad P(\theta, r, x, t) = p_{mnf} * E_{mn}(k_{mn} r) e^{i(\eta t + m\theta \pm \gamma x)}$$

Separated solution in the x-direction (wave-number):

$$\frac{k_x}{\eta} = \frac{1}{(1 - M^2)}\left[-M \pm \sqrt{1 - (1 - M^2)(\frac{\kappa}{\eta})^2}\right]$$

Separated solution in the Θ -direction is the Bessel equation:

$$P(\theta) = P(\theta + 2\pi); \quad \sin(m\theta)$$

Separated solution in the r-direction is the Bessel equation:

$$\frac{d^2P}{dr^2} + \frac{1}{r}\frac{dP}{dr} + (\kappa^2 - \frac{m^2}{r^2})P = 0 \quad \Longrightarrow$$

Acoustic Wave Equation in a Duct

Solution to Bessel's equation: $\Psi_m(\kappa,r) = AJ_m(\kappa r) + BY_m(\kappa r)$ A,B arbitrary

These functions are orthogonal:

$$\int_a^b r\psi_m(\kappa_{mn}r)\psi_k(\kappa_{kl}r)dr = \begin{cases} 0, & if\ k \neq m, l \neq m \\ \int_a^b r^2\psi_m(\kappa_{mn}r)dr, & if\ k = m, l = m \end{cases}$$

$$\int_a^b r^2\psi_m(\kappa_{mn}r)dr = \left\{\frac{1}{2}r^2\left[(1-\frac{m^2}{\kappa_{mn}^2 r^2})\psi_m{}^2(\kappa_{mn}r)\right] + \psi_m'{}^2(\kappa_{mn}r)\right\}\Bigg|_a^b$$

$$\int_A p^2 dA = A \longrightarrow \int_0^{2\pi}\int_{rt}^{rh} C_{mn}^2\Psi_{mn}^2(\kappa_{mn}r)d\Theta dr = 2\pi(r_t^2 - r_h^2) \longrightarrow \int_{rt}^{rh} C_{mn}^2\Psi_{mn}^2(\kappa_{mn}r)dr = (r_t^2 - r_h^2)$$

$$\frac{(1-\sigma^2)}{C_{mn}^2} = \frac{1}{2}\left\{\left[(1-\frac{m^2}{\kappa_{mn}^2})\psi_m{}^2(\kappa_{mn})\right] + \psi_m'{}^2(\kappa_{mn}) - \left[(1-\frac{m^2}{\kappa_{mn}^2\sigma}{}^2)\psi_m{}^2(\kappa_{mn}\sigma)\right] + \psi_m'{}^2(\kappa_{mn}\sigma)\right\}$$

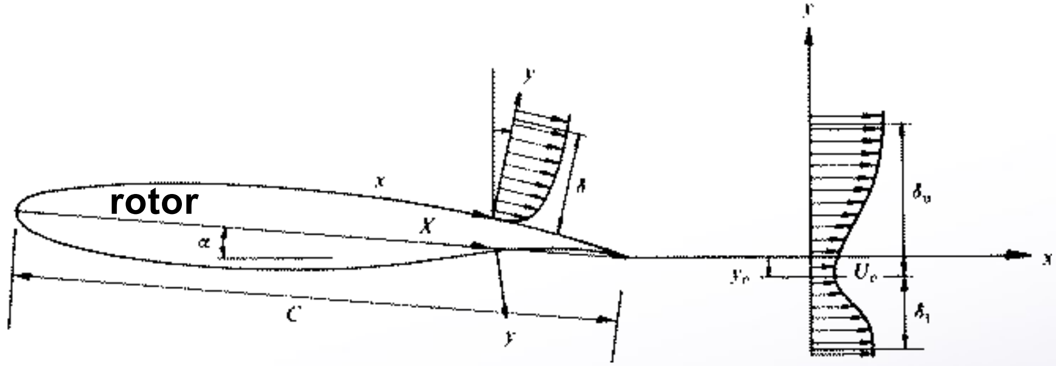
$\sigma = \frac{r_h}{r_r}; \tilde{r} = \frac{r}{r_r}$

hub-to-tip ratio
non-dimensional radius

$$E_{mn} = C_{mn}[J_m(\kappa_{mn}\tilde{r}) + Q_{mn}Y_m(\kappa_{mn}\tilde{r})] \longrightarrow p(r) = P_{mn}E_{mn}(\kappa_{mn}\tilde{r})$$

J & Y are Bessel functions of the 1st kind
k & Q are “eigenvalues” -> modes

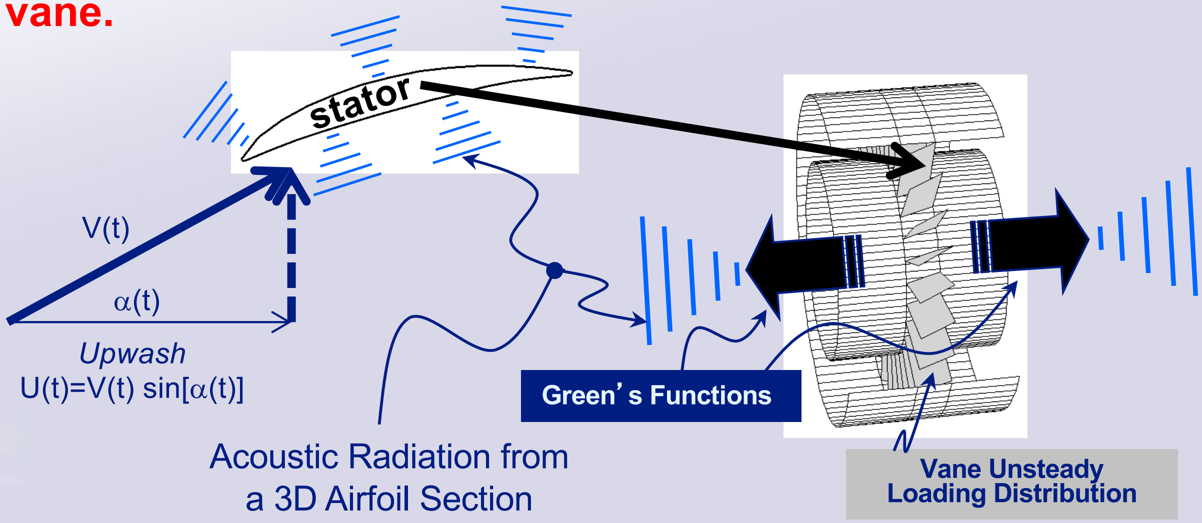
Rotor/Stator Interaction



- Boundary layer growth along blade creates a velocity deficit at the trailing edge.
- This velocity profile impinging on the stator surface generates noise.

Significant source of noise is the interaction of the rotor viscous wake with the stator. The wake itself is not the primary noise generation mechanism, but the interaction of the wake perturbations with the stator vane.

- The fluctuation of the velocity vector normal to the surface of the stator induces a pressure since the solid surface cannot support a normal velocity.
- The fluctuating normal velocity is called the *UPWASH* – combination of the variation in the velocity magnitude and angle.



Solution to Eigenvalue Problem

For illustrative purposes, σ=0

$$p(r) = P_{mn} E_{mn}(\kappa_{mn}\tilde{r}) E_{mn} = C_{mn}[J_m(\kappa_{mn}\tilde{r})]$$

$$\frac{1}{C_{mn}^2} = \frac{1}{2}\left\{\left[(1-\frac{m^2}{\kappa_{mn}^2})J_m{}^2(\kappa_{mn})\right] + J_m'{}^2(\kappa_{mn})\right\}$$

Eigen-value problem based on boundary conditions:

HARDWALL:
acoustic velocity normal to wall is zero, momentum equation gives:

$$\frac{dP}{dr} = 0;\ J_m'(\kappa_{mn}) = 0$$

A set of basis functions for each (m,n) mode.

SOFTWALL: (passive treatment - liner)

$$\frac{p}{v} = \frac{Z}{\rho c};\ A = \frac{\rho c}{Z};\ \eta = \frac{\omega R}{c}$$

impedance, admittance, reduced freq

$$\frac{dP}{dr} = -i\eta AP \longrightarrow \kappa\frac{J_m'(\kappa)}{J_m(\kappa)} = -i\eta A$$

$$\kappa\frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A$$

$$\kappa\frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A(1 - M\frac{k_x}{\eta})^2$$

A set of basis functions for each (m,n) mode - for each frequency, impedance, duct Mach.

Acoustic Duct Modes

Solution to separated wave equation in cylindrical coordinates:

$$P(\theta, r, x, t) = p_{mnf} * E_{mn}(k_{mn} r) e^{-i(2\pi f t + m\theta \pm \gamma x)}$$

Acoustic Duct Modes

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2-Dimensions (duct modes) :

circumferential direction:

$P(\theta) = P(\theta + 2\pi); \quad \sin(m\theta)$

m is defined as the number of cycles
-circumferential mode, m order

radial direction:

$$E_{mn}(k_{mn} r) = C_{mn}[J_m(k_{mn} r) + Q_{mn} * Y_m(k_{mn} r)]$$

n is defined as the number of zero crossings
-radial mode, n order

J & Y are Bessel functions / k & Q are “eigenvalues”

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3rd Dimension (propagation) :

axial direction:

$$\gamma_{mn}(x) = \sqrt{\left(2\pi f r_t/c\right)^2 - k_{mn}^2}$$

γ is defined as axial wavelength

(i) $2\pi f r_t/c < k_{mn};\ \gamma\ real, cyclical\ propagation$

(ii) $2\pi f r_t/c = k_{mn};\ cut-off/on\ frequency$

(iii) $2\pi f r_t/c > k_{mn};\ \gamma\ imag, exponential\ decay$

define the cut-off ratio from (ii)

$\zeta = f/f_{co};\quad > 1\ \ propagation$

$\quad\quad\quad\quad < 1\ \ decay$

Acoustic Duct Modes

Solution to separated wave equation in cylindrical coordinates:

$$P(\theta, r, x, t) = p_{mnf} * E_{mn}(k_{mn}r)e^{-i(2\pi ft+m\theta\pm\gamma x)}$$

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define the cut-off ratio from (ii)

$$\zeta = f/f_{co}; \quad \begin{array}{ll} >1 & propagation \\ <1 & decay \end{array}$$

Rotor-Stator Interaction

Tyler, J.M., & Sofrin, T.G., “Axial Flow Compressor Studies”, *SAE Transactions*, Vol 70, 1962.

The kinematic relationship between the rotor and stator of varying counts causes a strong interaction pressure ‘lobe’ related to the relative positions.

This ‘lobe’ rotates locked to the fan.

The number of lobes (i.e. circumferential mode) is related to the blade/vane count:

$$m = nB - kV$$

(n harmonic, k= any integer)

To preserve frequency a low count mode must rotate faster:

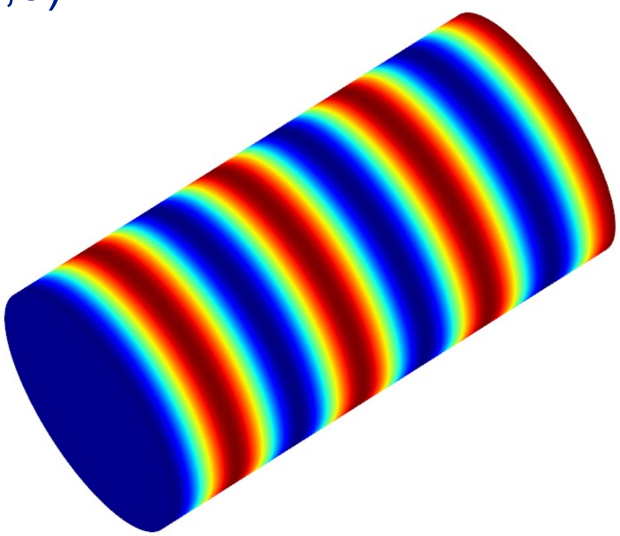
$$m^*\Omega_{mode} = B^*\Omega_{rotor}$$

$$\Omega_{mode} = (B/m)^*\ \Omega_{rotor}$$

If $^*\Omega_{mode}$ > 1.0M modes propagates

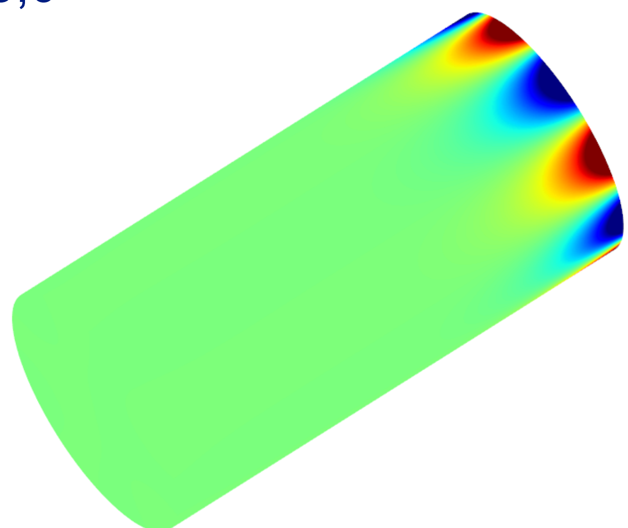
Propagating / Decaying Modes

(0,0)



$\zeta = {}^{f}/_{f_{co}} > 1 \quad propagation$

(5,0
)



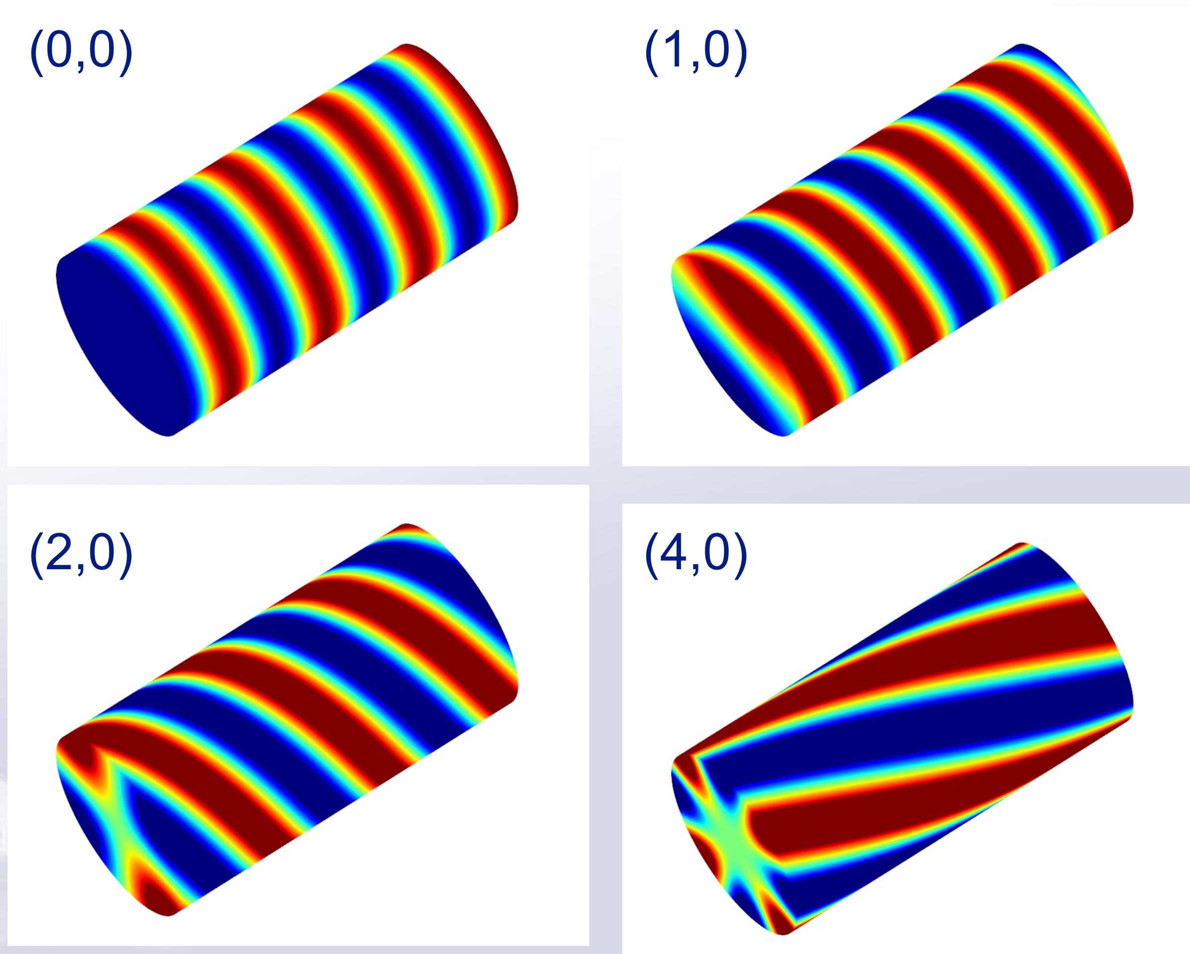
$\zeta = {}^{f}/_{f_{co}} < 1 \quad decay$

Circumferential Modes (m-order)

circumferential direction:

$P(\theta) = P(\theta + 2\pi);\quad \sin(m\theta)$

m is defined as the number of cycles
-circumferential mode, m order



Radial Modes (n-order)

radial direction:

$$E_{mn}(k_{mn}r) = C_{mn}[J_m(k_{mn}r) + Q_{mn} * Y_m(k_{mn}r)]$$

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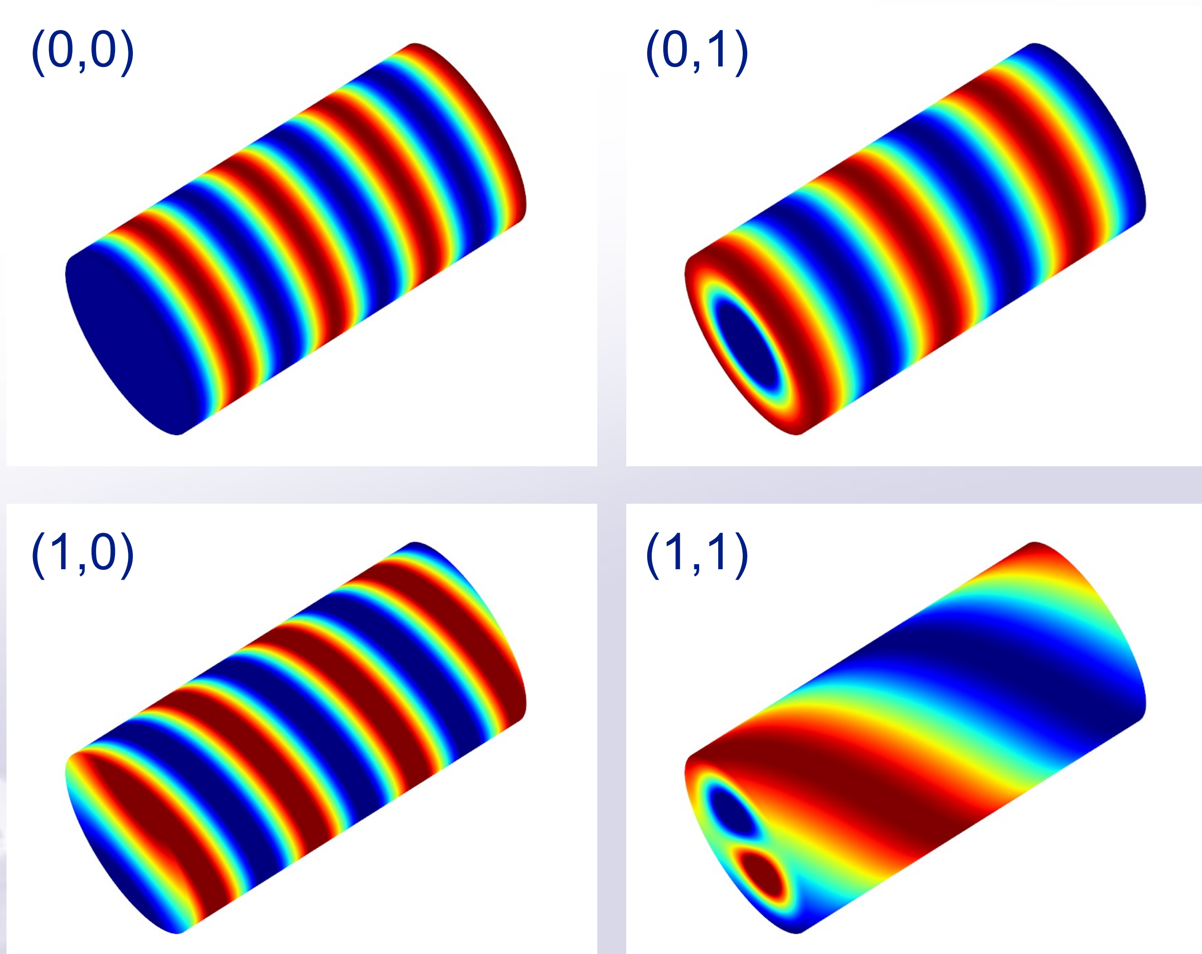
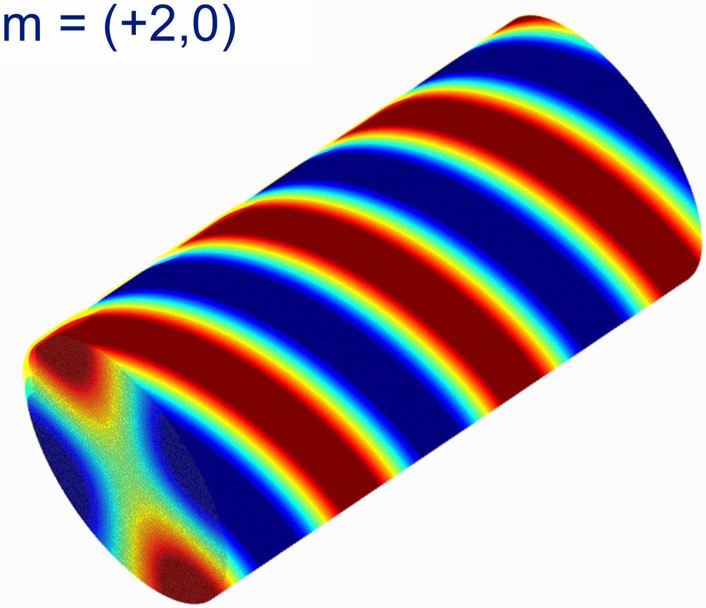


Co / Contra Rotating Modes (+/- m)

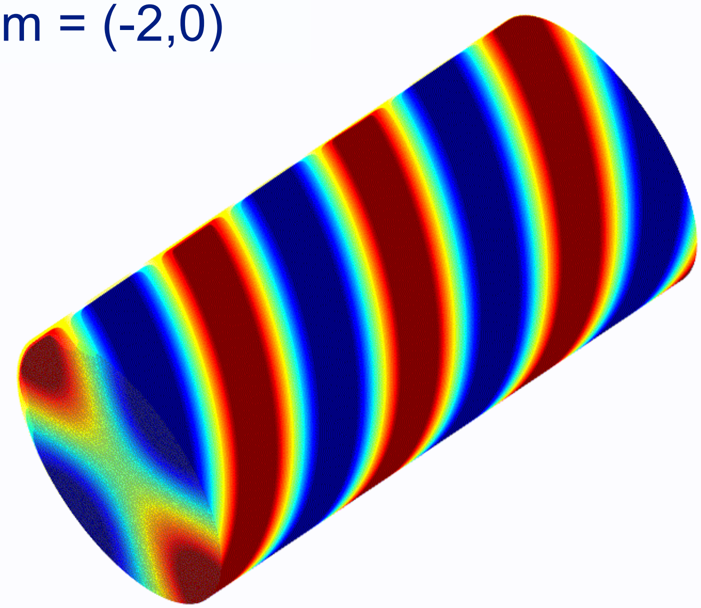
Arbitrarily define +m if rotating in same direction as fan. (co-rotating)

m = (+2,0)



co-rotating

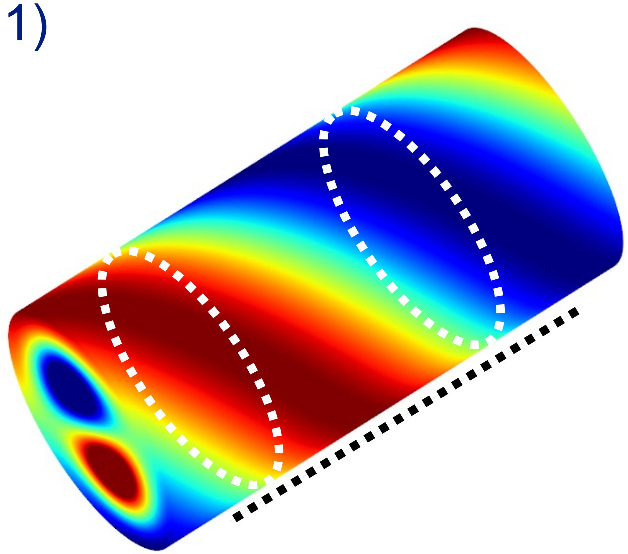
m = (-2,0)



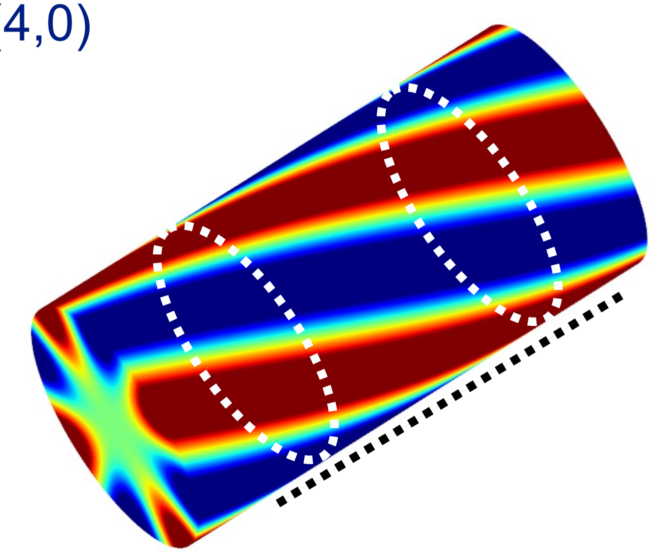
counter-rotating

Mode Measurement

(1,1)



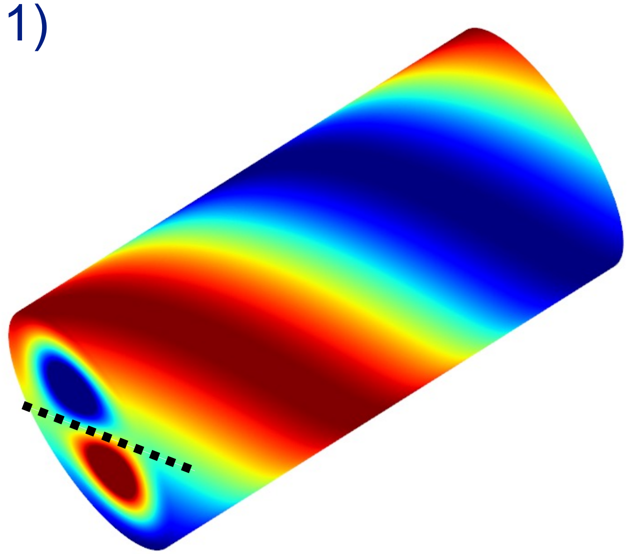
(4,0)



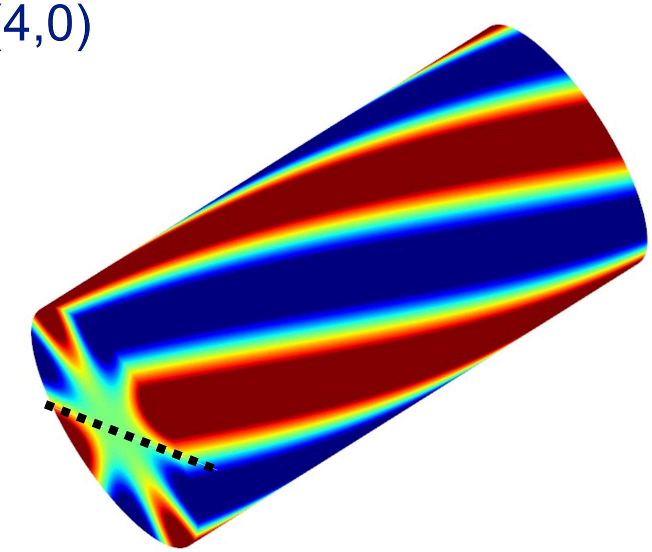
IN-DUCT ARRAY(s)
Axial/Circumferential wall mounted array of microphones
– using complex signal decomposition and cross-correlations

Mode Measurement

(1,1)



(4,0)



ROTATING RAKE

Radial rake array of microphones

– “doppler shift” ; continuously rotating to avoid interference

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(I) Description of Acoustic Field that is Measured
- rotor/stator interaction
- duct modes

(II) Rotating Rake Theory and Implementation
- **Doppler shift**
- **Radial Curve Fitting**
- **Mechanical Parameters**

(III) Examples of Rotating Rake Applications

Doppler Shift of Circumferential Modes

Acoustic pressure at a fixed radial and circumferential location in a plane:

$$p(\Theta,t)=\sum_{s=1}^{\infty}\sum_{m=-M}^{m=+M}C_m^s e^{-i(s\omega t-m\theta)}$$

Acoustic pressure at a fixed radial location in a plane and point is slowly rotating in the circumferential direction:

$$\Theta=\Omega t;\quad p(\Theta,t)=\sum_{s=1}^{\infty}\sum_{m=-M}^{m=+M}C_m^s e^{-i(s\omega-m\Omega)t}$$

$$let\ \Omega=\frac{\omega}{100},\ s=B;\quad freq\rightarrow(s\omega-m\Omega)=\omega(B-\frac{m}{100})$$

Thus each circumferential mode is induced to appear at a unique shifted frequency proportional to the rake/fan speed ratio. Plane wave (m=0) is at fan harmonic. Positive and negative modes are shifted about the the fan harmonic.

p pressure
t time
ω fan shaft speed
m circumferential mode
s shaft harmonic
Ω rake rotational speed
B rotor blade count

What about the rake-wake/rotor interference? (Frame of reference).
Pressure due to a fixed inlet distortion:

$$p_l^s=C_l^s e^{-i\left[(s\pm l)\Theta-s\omega t\right]}$$

Pressure due to a slowly rotating inlet distortion:

$$p_l^s=C_l^s e^{-i\left[(s\pm l)\Theta-s(\omega\pm l\Omega)t\right]}$$

$$\Theta=\Omega t;\quad p_l^s=C_l^s e^{-i\left[(s\pm l)\Omega t-(s\omega\pm l\Omega)t\right]}$$

$$=C_l^s e^{-i\left[s\Omega t\pm l\Omega t-s\omega t\mp l\Omega t\right]}$$

$$=C_l^s e^{-i\left[s(\Omega-\omega)t\right]}$$

Freq of distortion given by B(ω–Ω) for all rake induced modes.

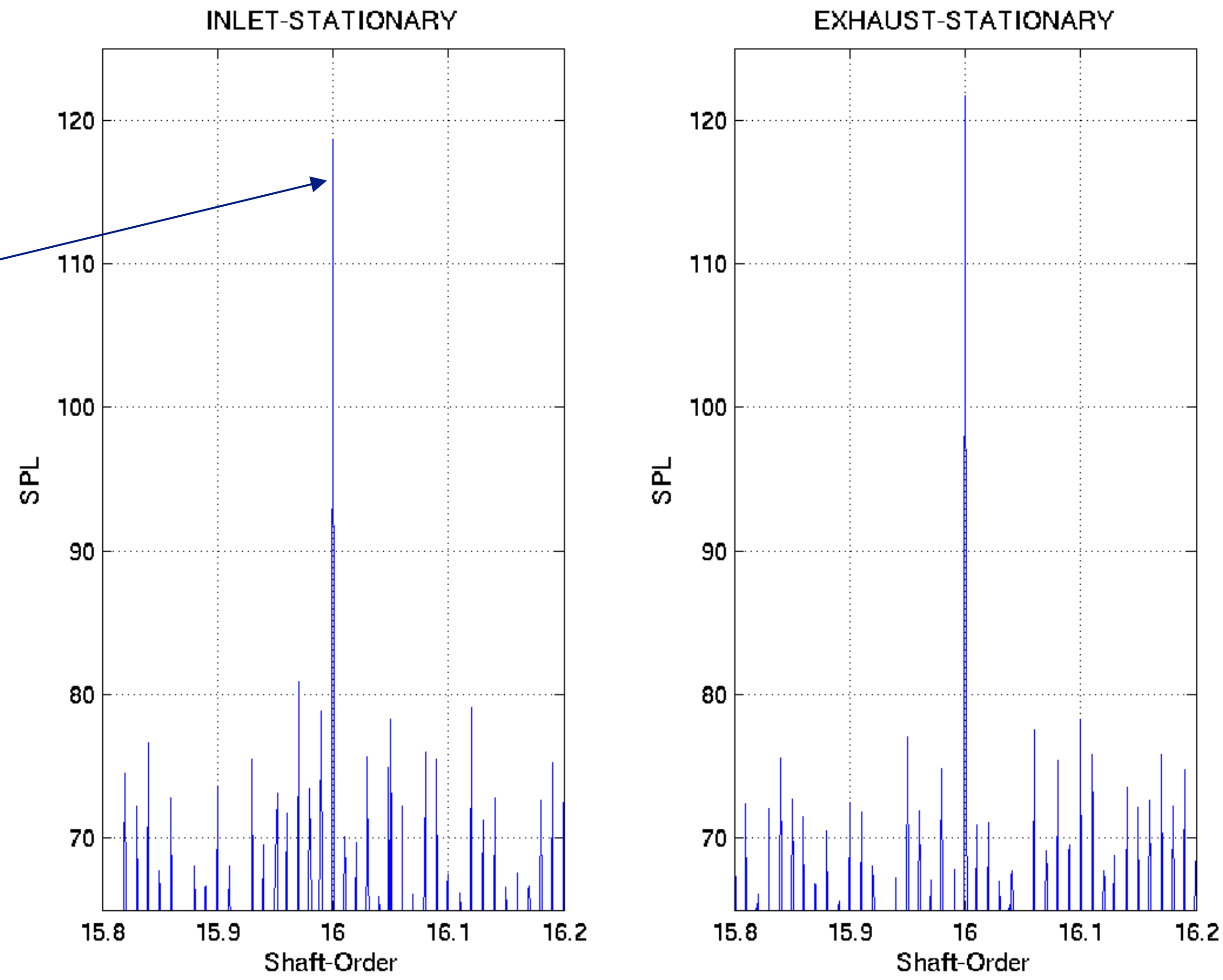
From above let m=B; B(ω–Ω); e.g. all rake induced modes appear at the equivalent 'rotor' count mode m=B.

Rotating Rake Measurements

single microphone

narrow band FFT ~2^{17}
zoom about 1BPF (16)

BPF HARMONIC



Rotating Rake Measurements

single microphone

narrow band FFT ~2^{17}
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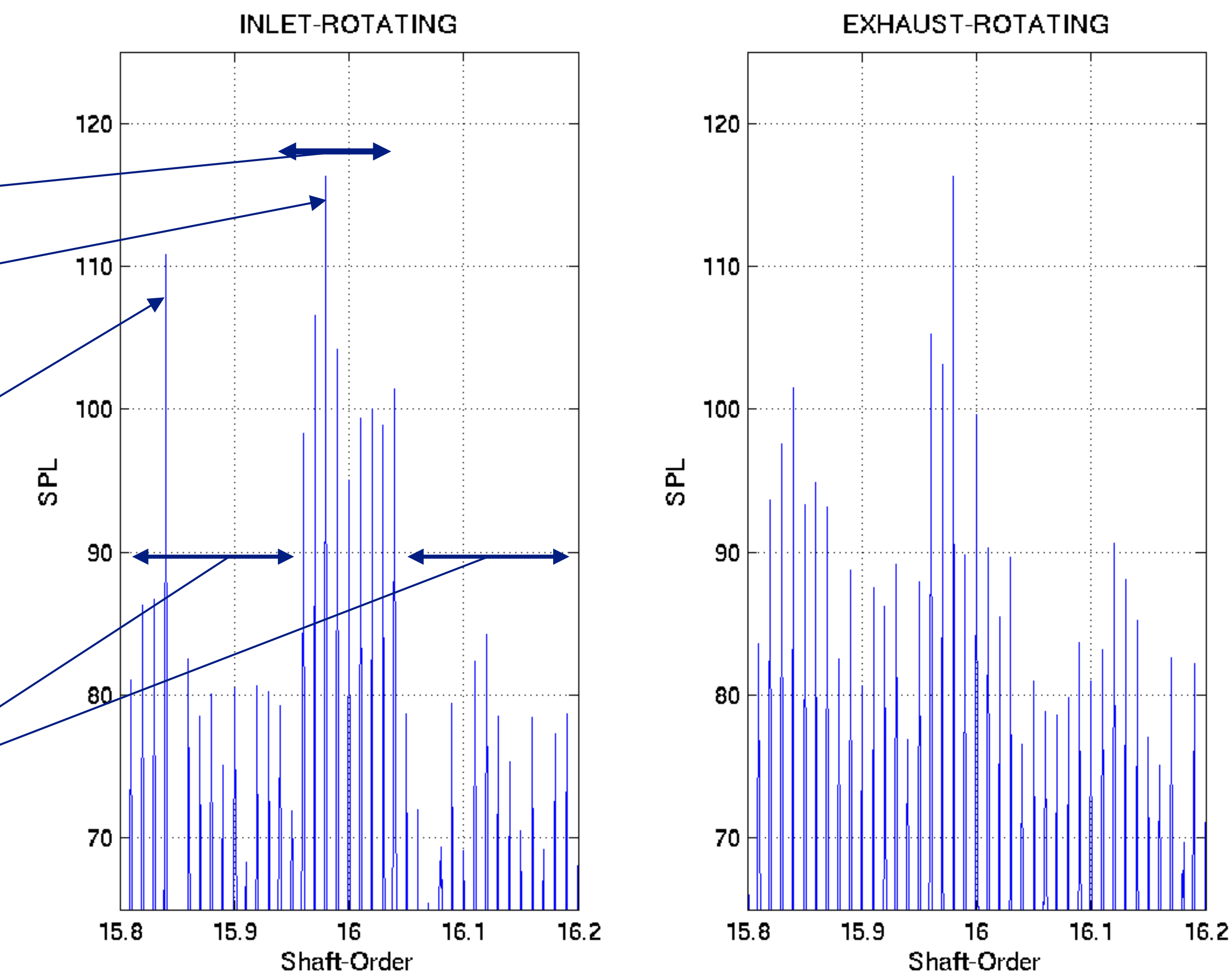
Individual Cut-On Modes

Rotor-Stator Interaction Mode

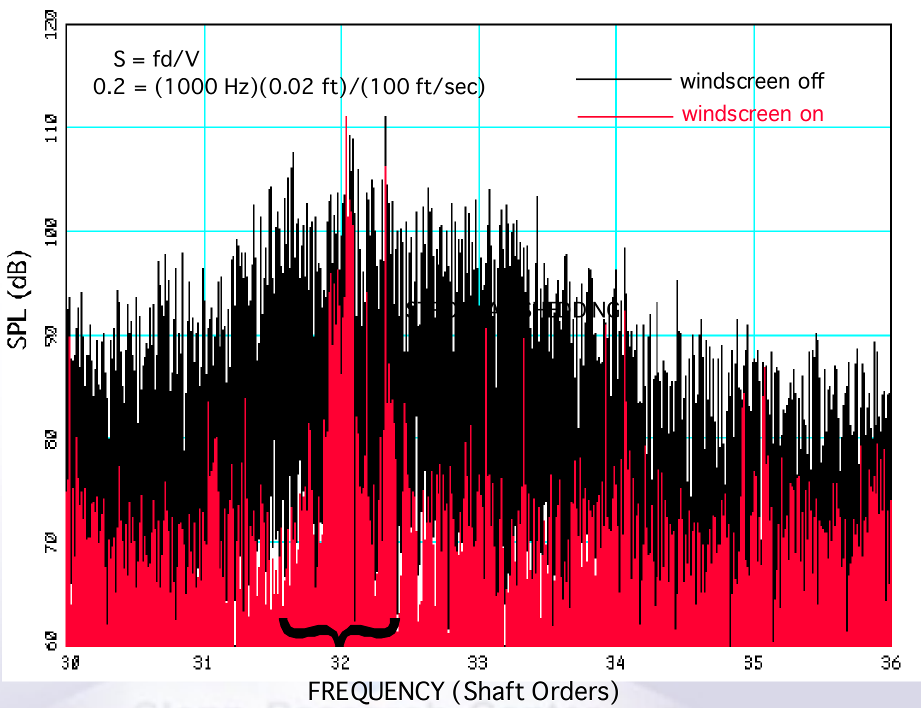
Rake-Wake Rotor Mode

Cut-Off Modes

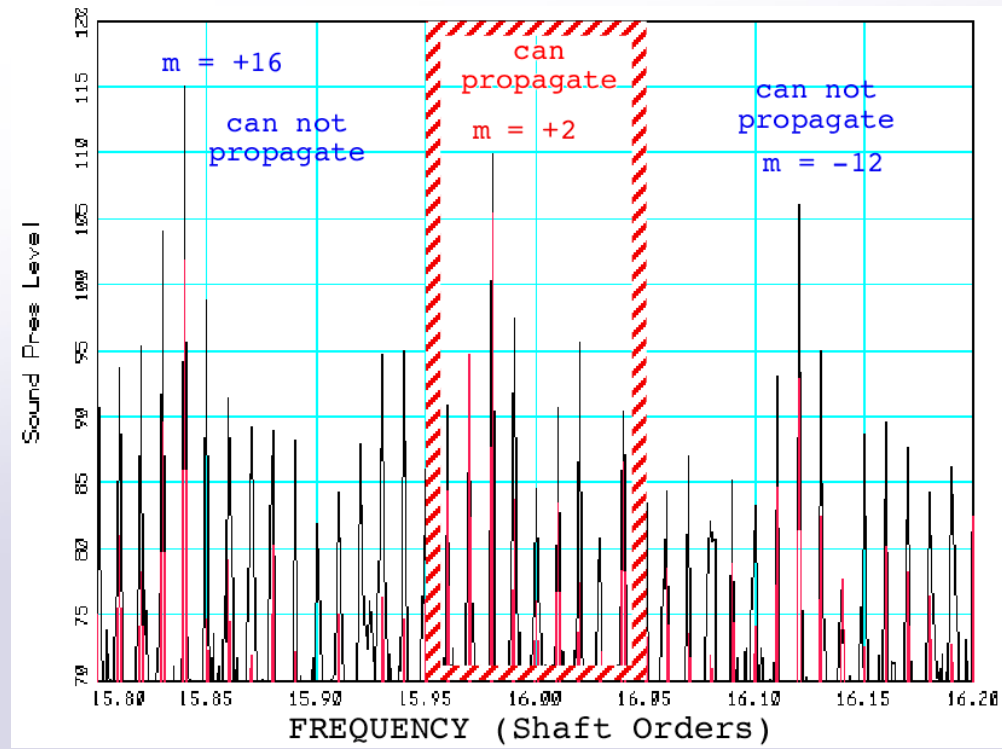


Installation Effects

Inlet



Exhaust



Strouhal shedding eliminated by porous Al windscreen along rake.

Interaction of wake reduced by porous Al windscreen.

Duct Radial Mode Solutions

Determination of radial modal content from measured radial pressure:

$$p_{m,r,f} = \sum_{n=0}^{\infty} p_{m,n,f} E_{mn}(\kappa_{mn} r)$$

measured

unknown

determined

For each m-order & harmonic calculate the least squares fit:

$$\sum_{n=0}^{N_{\max}} p_{m,n,f} E_{mn}(\kappa_{mn} r) - p_{m,r,f} \qquad \delta^2 = \sum_{mic=1}^{N_{mic}} \left[W_{mic} \sum_{n=0}^{N_{\max}} p_{m,n,f} E_{mn}(\kappa_{mn} r) - p_{m,r,f} \right]^2$$

To minimize error differentiate w/r to each mode amplitude to from a set of N_{mic}+1simultaneous equations :

$$\frac{\partial \delta^2}{\partial p_{n1}} = 2 \sum_{i=1}^{N_{mic}} W_i E_{mn1}(\kappa_{mn1} r_i) \left[\sum_{n=0}^{N_{\max}} p_n E_{mn}(\kappa_{mn} r) - p_i \right],$$

$$\frac{\partial \delta^2}{\partial p_{n2}} = 2 \sum_{i=1}^{N_{mic}} W_i E_{mn2}(\kappa_{mn2} r_i) \left[\sum_{n=0}^{N_{\max}} p_n E_{mn}(\kappa_{mn} r) - p_i \right], \ldots$$

Least Squares Solution for Radial Modes

Setting error derivative to zero and re-arranging, each equation (index *v*) becomes:

$$\sum_{n=0}^{N_{\max}} p_n \sum_{i=1}^{Np} E_{mv}(\kappa_{mv} r_i) E_{mn}(\kappa_{mv} r_i) = \sum_{i=1}^{Np} W_i E_{mv}(\kappa_{mv} r_i) p_i$$

$$\left[F_{vn}\right] = \begin{bmatrix} \sum_{i=1}^{N_p} E_{m0}^2(\kappa_{m0} r_i) & \ldots & \sum_{i=1}^{N_p} E_{m0}(\kappa_{m0} r_i) E_{mN_{\max}}(\kappa_{mN_{\max}} r_i) \\ \vdots & & \vdots \\ \sum_{i=1}^{N_p} E_{mN_{\max}}(\kappa_{mN_{\max}} r_i) E_{m0}(\kappa_{m0} r_i) & \ldots & \sum_{i=1}^{N_p} E_{mN_{\max}}^2(\kappa_{mmN_{\max}} r_i) \end{bmatrix}$$

$$\left[E_{vr}\right] = \begin{bmatrix} E_{m0}(\kappa_{m0} r_1) & \ldots & E_{m0}(\kappa_{m0} r_{Np}) \\ \vdots & & \vdots \\ E_{mN_{\max}}(\kappa_{mN_{\max}} r_1) & \ldots & E_{mN_{\max}}(\kappa_{mN_{\max}} r_{Np}) \end{bmatrix}$$

[# of radials X # of radials]

[# of radials X # of mics]

$$\left[F_{vn}\right]\left[P_n\right] = \left[E_{vr}\right]\left[P_r\right]$$

measured sound pressure vector (microphones)

desired radial mode vector

$$\left[P_n\right] = \left[F_{vn}\right]^{-1}\left[E_{vr}\right]\left[P_r\right]$$

Note: # of radials to curve is equal to number of radials present.

-Hardwall away from boundaries equal to max cut-on

-Softwall or near boundary/source consider decay rate

NORMALIZING COEFFICENT

- Since the solution to Bessel's Equation is un-normalized, it is desirable to choose normalization coefficient to make meaningful comparisons between modes, and connection to the physical.

- Normalize to area so that the square of the mode amplitude times area gives mode power.

$$\int_A p^2 dA \equiv A$$

$$\Psi_{mn}(r) = J_m(\kappa_{mn} r) + Q_{mn} Y_m(\kappa_{mn} r); \quad n = 0,1,2,\ldots$$

$$\int_0^{2\pi} \int_\sigma^1 C_{mn}^2 \Psi_{mn}^2 (\kappa_{mn} r) r dr d\Theta = 2\pi(1-\sigma^2)$$

$$C_{mn}^2 \int_\sigma^1 \Psi_{mn}^2 (\kappa_{mn} r) r dr = (1-\sigma^2)$$

$$\frac{(1-\sigma^2)}{C_{mn}^2} = \int_\sigma^1 \Psi_{mn}^2 (\kappa_{mn} r) r dr$$

HARDWALL: $$\frac{1}{C_{mn}^2} = \frac{1}{2}\left(1 - \frac{m^2}{\kappa_{mn}^2}\right) J_m{}^2(\kappa_{mn})$$

SOFTWALL: $$\frac{1}{C_{mn}^2} = \frac{1}{2}\left[\left(1 - \frac{m^2}{\kappa_{mn}^2}\right) J_{mn}^2(\kappa_{mn}) - \left(\frac{\eta \hat{A}_{ow}}{\kappa_{mn}}\right)^2 \left(1 - M\frac{k_x}{\eta}\right)^2 J^2{}_m(\kappa_{mn})\right]$$

POWER CALCULATION

Power from integrated intensity
I_x ~ axial component acoustic energy flux

cut-off ratio: $\zeta = \dfrac{\Omega_{fan} R / c}{\kappa_{mn} \beta}$

$$Power = \int_A \langle I_x \rangle dA; \quad p = c_0^2 \rho; \;\; M = U / c_0$$

$$I_x = (\frac{p}{\rho_0} + Uu)(\rho_0 u + \rho U)$$

$$I_x = (1 + M^2) p\bar{u} + \frac{M}{\rho_0 c_0} p\bar{p} + \rho_0 c_0 M u\bar{u}$$

$$\kappa_{mn} \to real; \; \zeta \to real$$
$$\kappa_{mn} \to complex; \; \zeta \to complex$$

$$Hardwall: \quad \kappa_{mn} \to real$$
$$Softwall: \quad \kappa_{mn} \to complex$$

$$\overline{\mathrm{P}} = \mp \frac{\pi R^2 (1 - \sigma^2)}{\rho_0 c_0} |\beta|^4 \; RE \left\{ \frac{\sqrt{1 - \frac{1}{\zeta^2}}}{\left| 1 \pm M_D \sqrt{1 - \frac{1}{\zeta^2}} \right|^2} \right\} |P|^2$$

Hardwall: (ζ real)

$$(i) \quad \zeta < 1; \quad RE\{Yi\}; \quad \overline{\mathrm{P}} = 0$$
$$(ii) \quad \zeta > 1; \quad RE\{X\}; \quad \overline{\mathrm{P}} \neq 0$$

Softwall: (ζ complex)

$$(i) \quad \zeta < 1; \quad RE\{X + Yi\}; \quad \overline{\mathrm{P}} \neq 0$$
$$(ii) \quad \zeta > 1; \quad RE\{X + Yi\}; \quad \overline{\mathrm{P}} \neq 0$$

ROTATING RAKE – MODAL ANALYSIS (TOMBSTONE PLOTS)

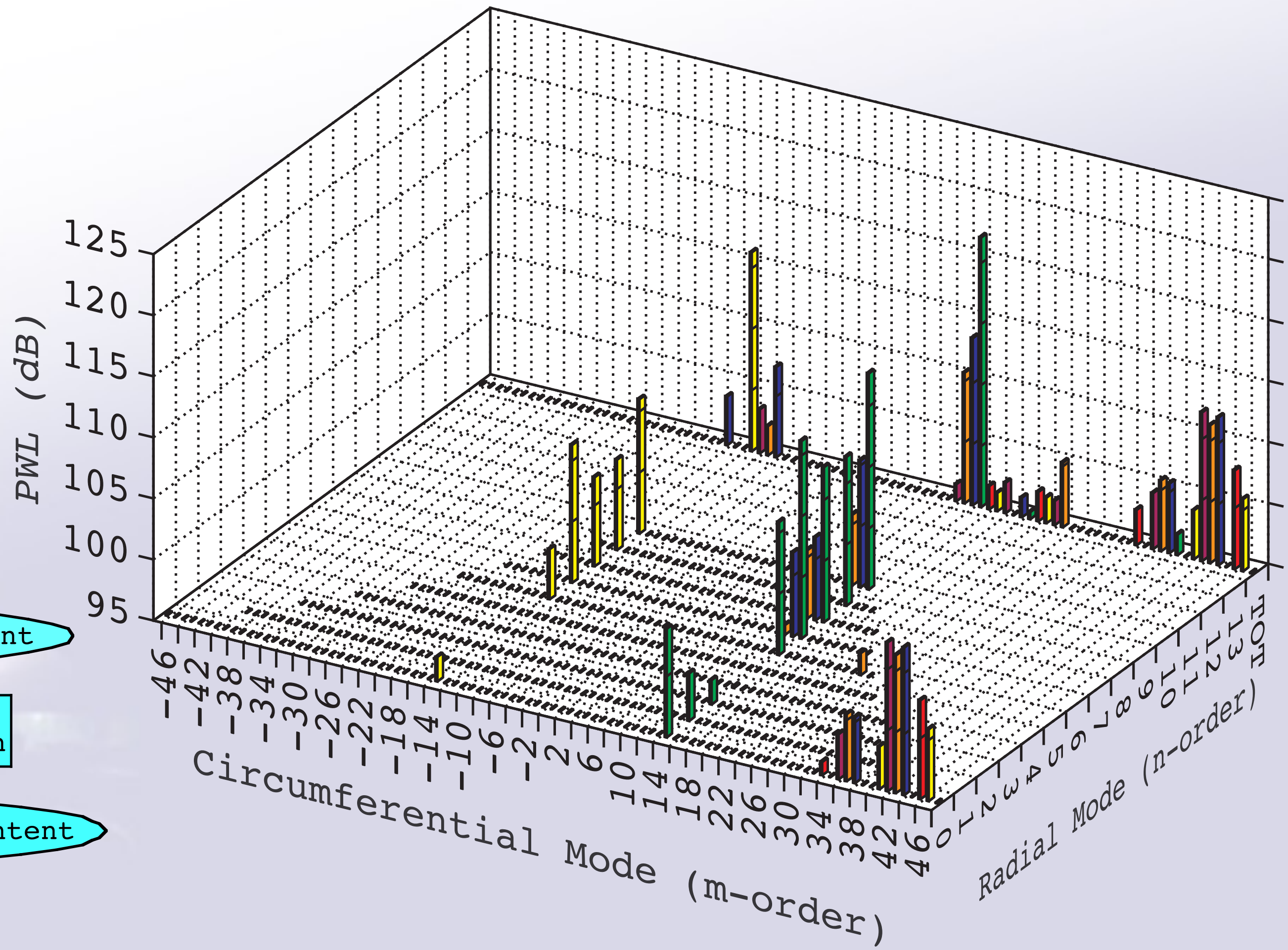
Magnitude & phase of each (m,n) mode.

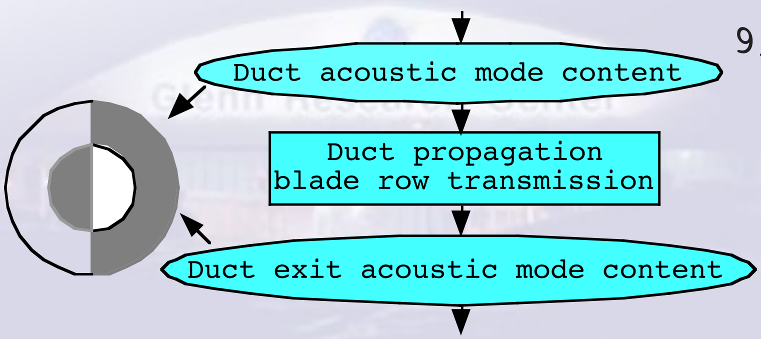
Sum for interaction or tone PWL.

Identifies strong interactions (expected & unexpected).

Note dominance of positive modes (co-rotating).

PWL_{tot} = 120.6 dB
$PWL_{m=+14}$ 117.1 dB



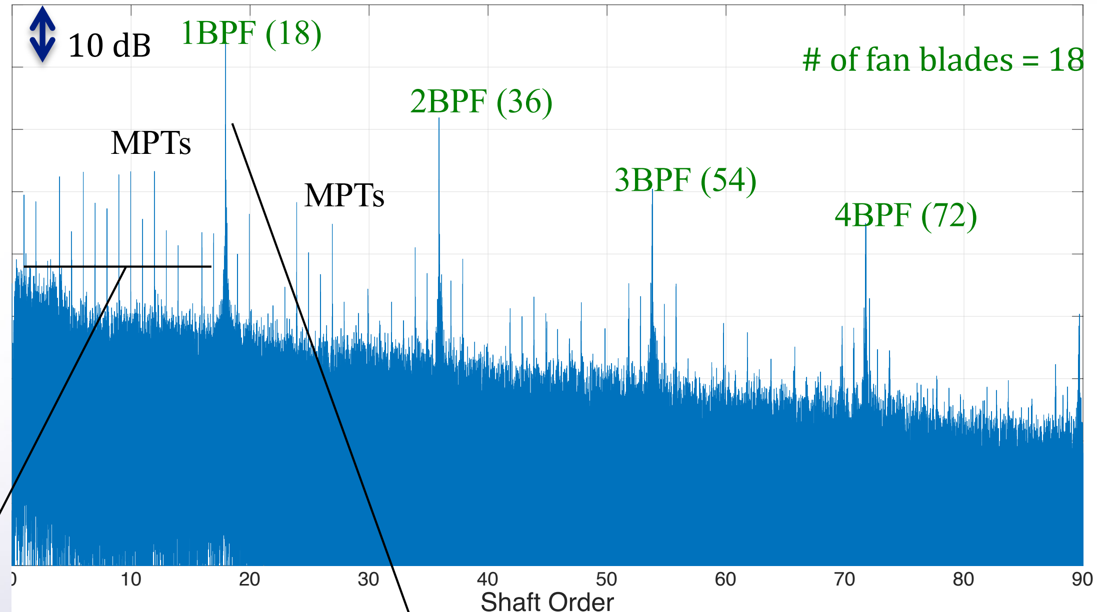


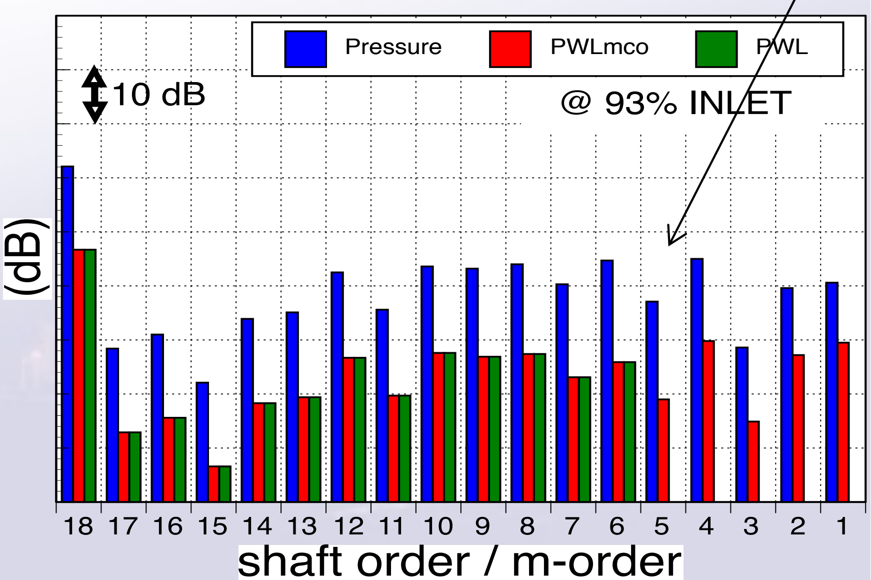
INLET MODE PLOT @ 1xBPF of FAN (RRMS)

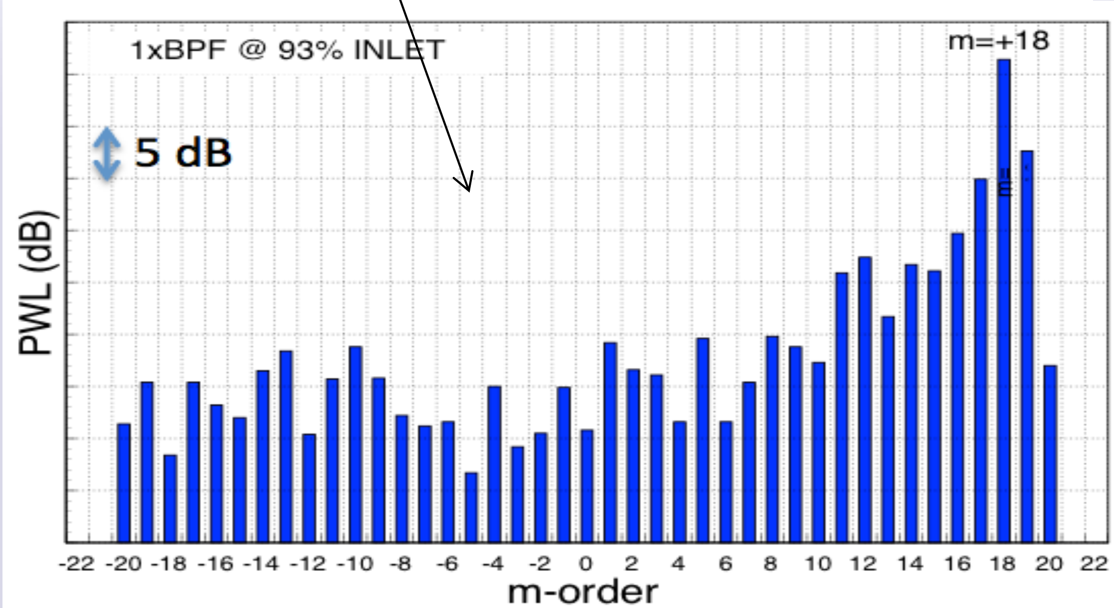
MPTs at multiples of shaft order
m = s.o.

Rotor locked mode: *m*=+18

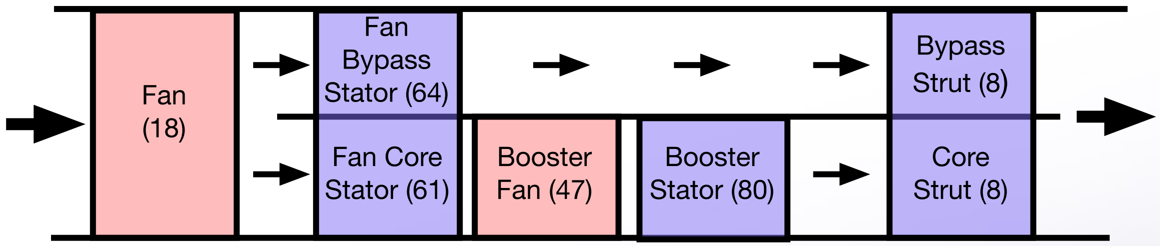
Pressure, PWL in MPTs
In nearly cut-on modes compute instantaneous PWL
(radiates to FF at release point)







ROTATING/STATIONARY COUNTS TYLER-SOFRIN MODAL PREDICTIONS



frequency (shaft order): $\Omega_{s.o} = nB$

generation: $m = nB \pm kV$

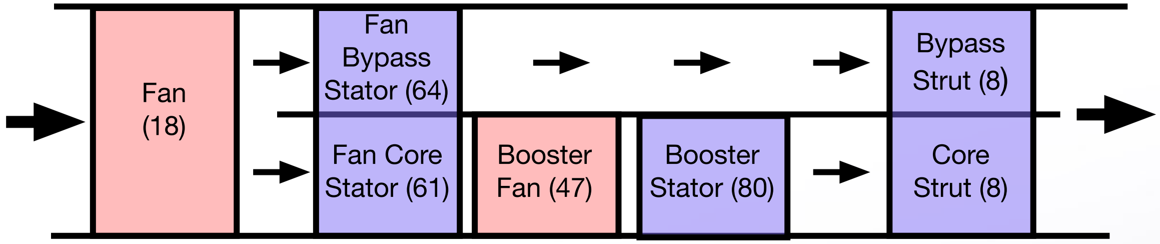
Shaft-Order / Modal Analysis for Fan (B=18)

FAN HARMONIC	SHAFT ORDER	FAN BYPASS STATOR (64)	BYPASS STRUT (8)
1st	18	cut-off	+18,+10,+2,-6,-14,-22
2nd	36	-28	+36,+28,+20,+12,+4, -4,-12,-20,-28,-36
3rd	54	-10	+54,+46,+38,+30,+22,+14,+6, -2,-10,-18,-26,-34,-42,-50

Shaft-Order / Modal Analysis for Booster Fan (B=47)

BOOSTER HARMONIC	SHAFT ORDER	FAN CORE STATOR (61)	BOOSTER STATOR (80)	CORE STRUTS (8)
1st	47	+47,-14	+47,-33	47,-39,-31,-23,-15,-7, +1,+9,+17,+25,+33,+41

SCATTERING PREDICTIONS



frequency scatter: $\Omega_{s.o} = n_1 B_1 \pm n_2 B_2$

mode scatter: $m = n_1 B_1 \pm n_2 B_2 - k_1 V_1 - k_2 V_2$

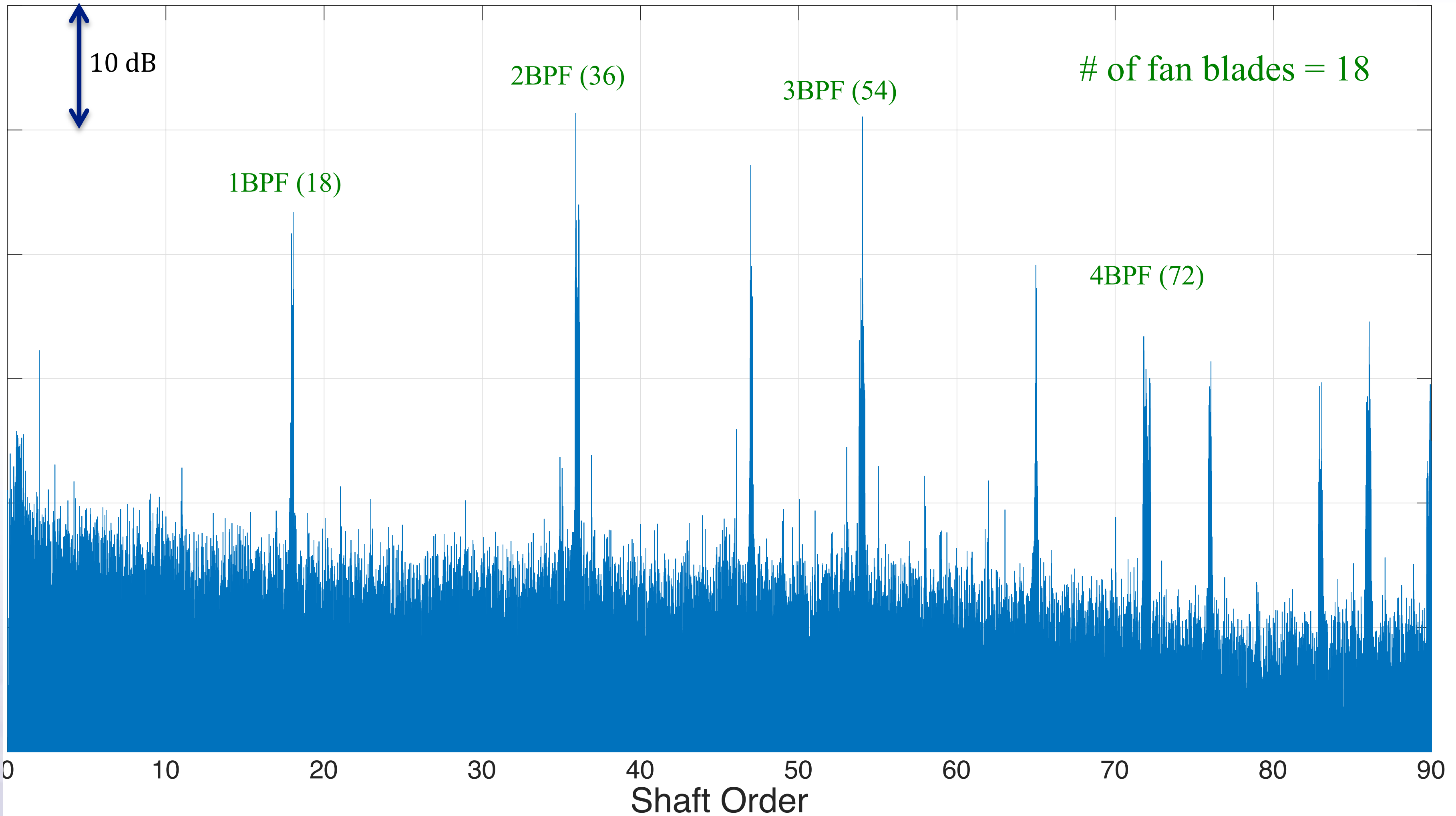
Booster Fan Fundamental (Shaft Order 47) Scattering Due to

	CORE STRUTS (8)	
k_1	FAN CORE STATOR (61)	BOOSTER STATOR (80)
-1	-44,-36,-28,-20,-12,-4, +4,+12,+20,+28,+36,+44	" "
0	-41,-33,-25,-17,-9, -1,+7,+15,+23,+31,+39,+47	-41,-33,-25,-17,-9,-1, +7,+15,+23,-31,+39,+47
+1	-46,-38,-30,**-22,-14,-6,-2, +10,+18,+26,+32,+40**	" "

Fan Fundamental + Booster Fan Fundamental (Shaft Order 65) Scattering Due to

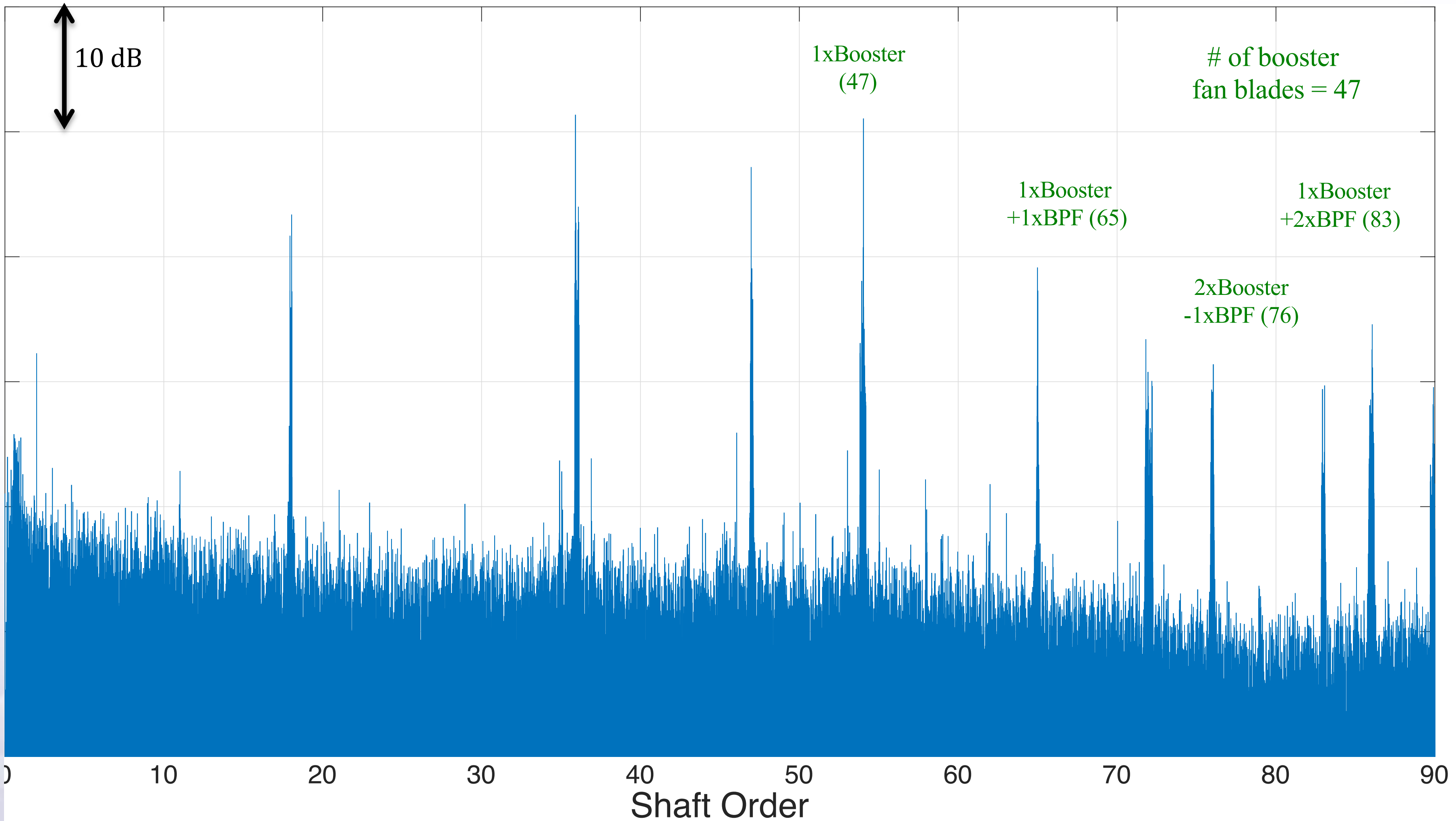
	CORE STRUTS (8)	
k_1	FAN CORE STATOR (61)	BOOSTER STATOR (80)
-1	-42,-34,-26,-18,-10,-2, +6,+14,+22,+30,+38	" "
0	-47,-39,-31,-23,-15,-7, +1,+9,+17,+25,+33,+41	-47,-39,-31,-23,-15,-7, +1,**+9**,+17,+25,+33,+41
+1	-44,-36,-28,-20,-12,**-4, +4,+12**,+20,+28,+36,+44	" "

FFT OF EXHAUST SIGNAL IN ROTATING FRAME OF REFERENCE



Shaft Order Spectral Content from a Single Microphone of Rotating Rake Showing Fan Tone Harmonics

FFT OF EXHAUST SIGNAL IN ROTATING FRAME OF REFERENCE

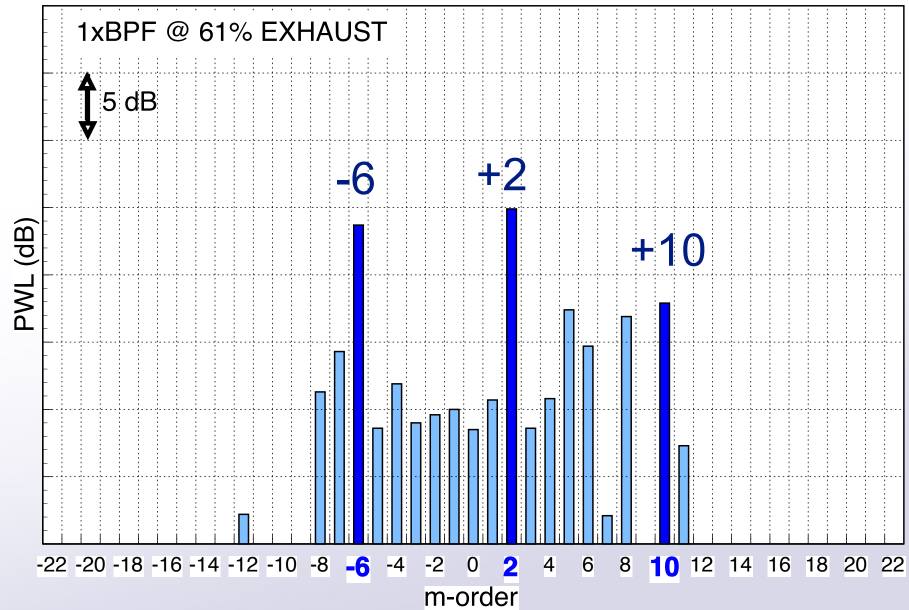


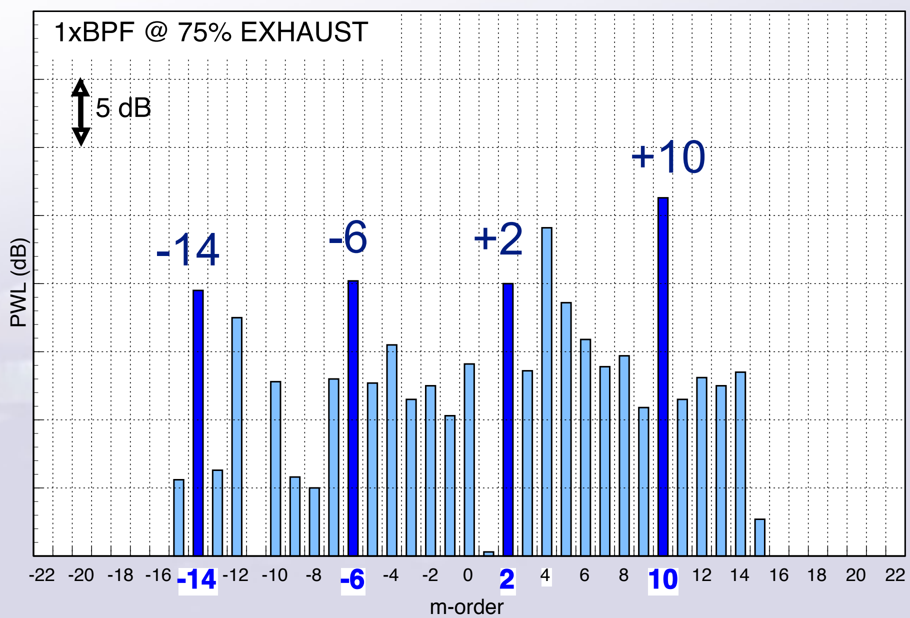
Shaft Order Spectral Content from a Single Microphone of Rotating Rake Showing Booster Tones and Rotor/Booster Interaction Tones.

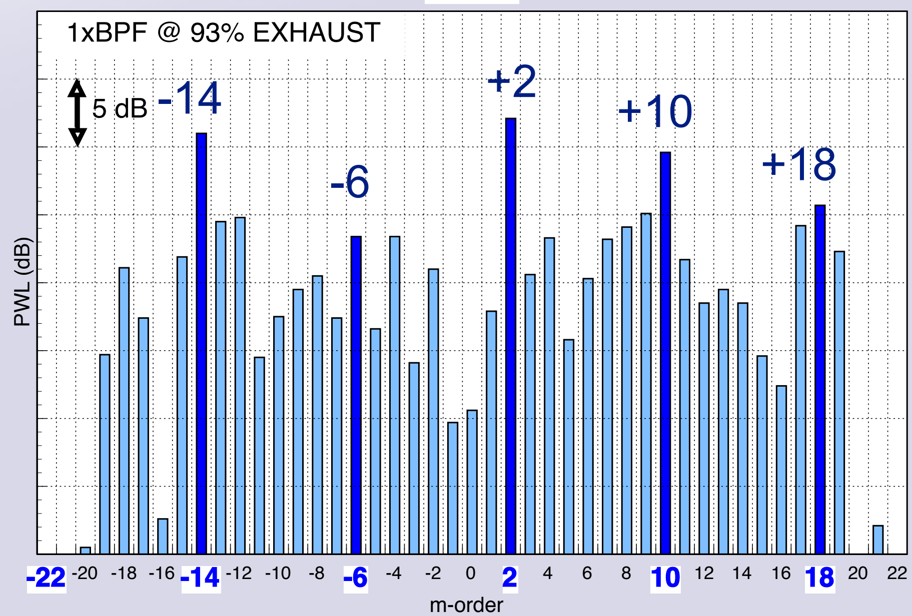
EXHAUST MODE PLOTS @ 1xBPF of FAN

Strut Modes identified & dominant
(m=-14,-6, +2, +10, +18)*
Note δ of 8 between strut interaction modes

Vane Mode cut-off
Rotor locked mode: *m*=+18



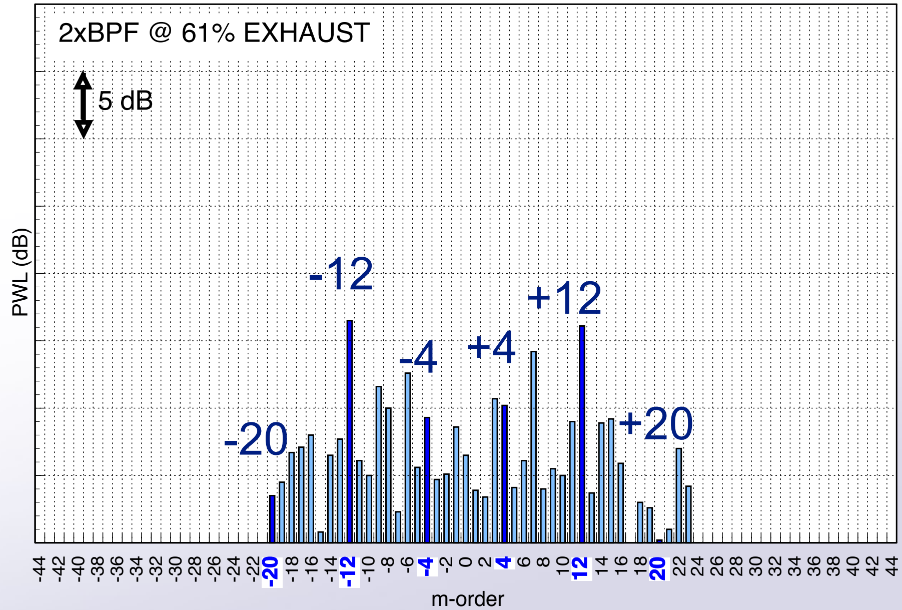


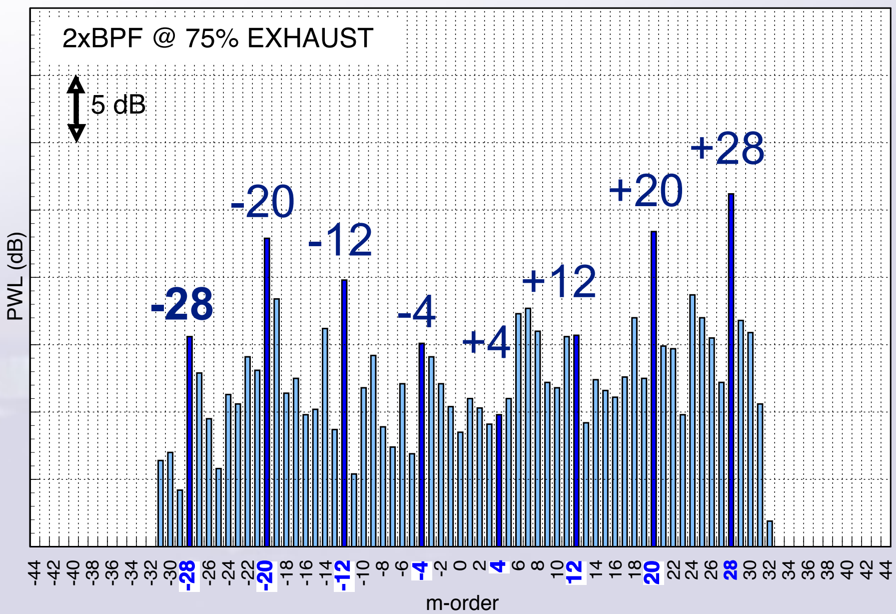


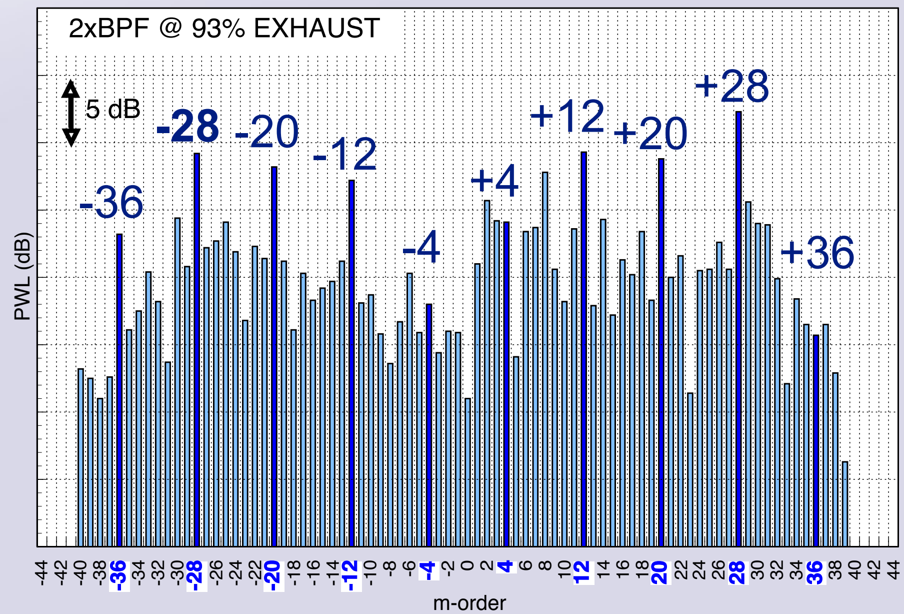
EXHAUST MODE PLOTS @ 2xBPF of FAN

Strut Modes identified & dominant
*(m=-36,**-28**,-20,-12,-4,
+36,+28,+20,+12,+4)*
Note δ of 8 between and symmetry of strut interaction modes

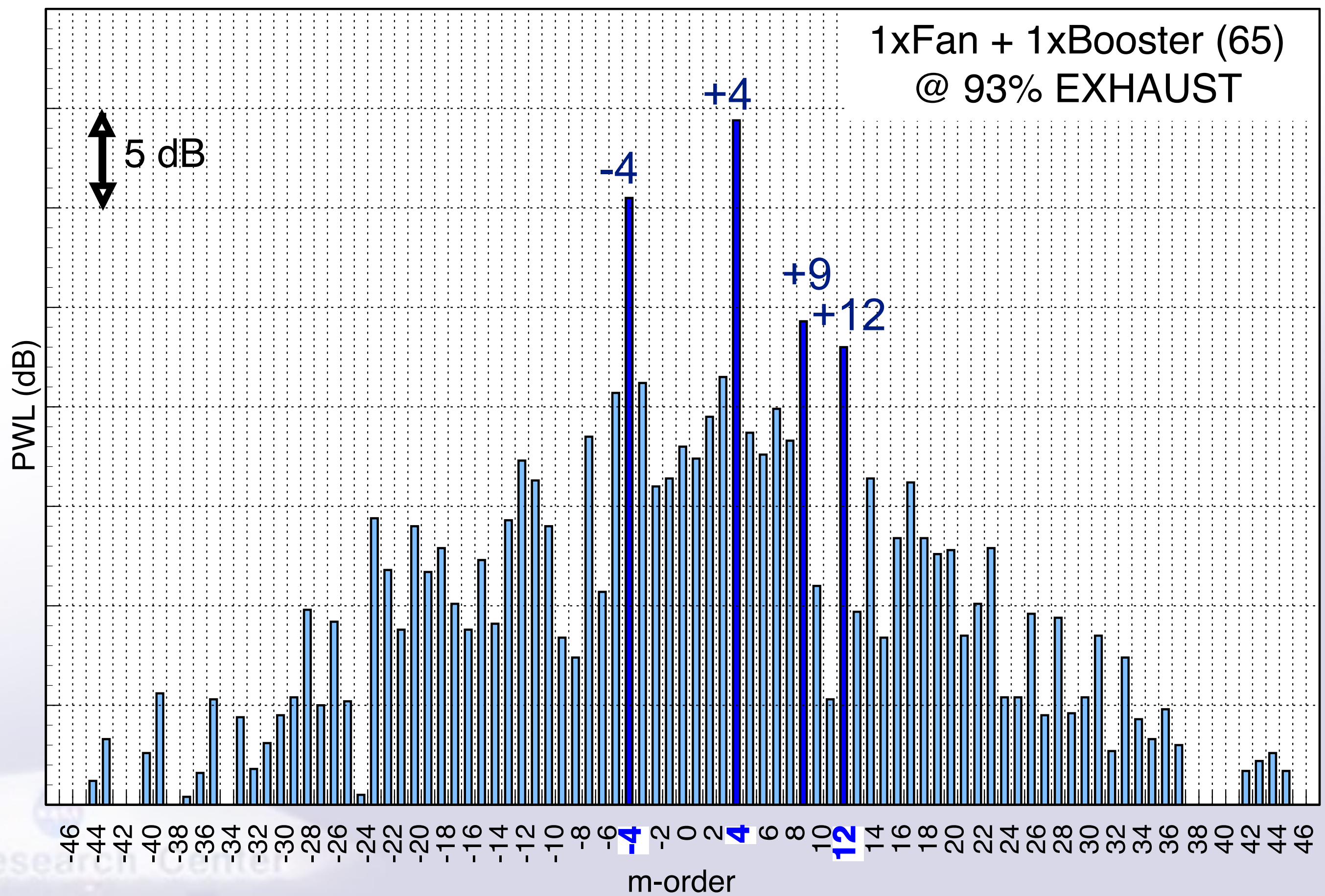
Rotor/Vane mode: *m*=-28
Rotor locked mode: *m*=+36







EXHAUST MODE PLOT @ FAN 1xBPF + BOOSTER 1xBPF INTERACTION (SHAFT ORDER 65)



k₁	CORE STRUTS (8)	
	FAN CORE STATOR (61)	BOOSTER STATOR (80)
+1	-44,-36,-28,-20,-12,**-4**, **+4**,**+12**,+20,+28,+36,+44	-47,-39,-31,-23,-15,-7, +1,**+9**,+17,+25,+33,+41

PHYSICAL SOLUTION to EIGENVALUE PROBLEM

Eigenvalue problem based on boundary conditions (radial direction):

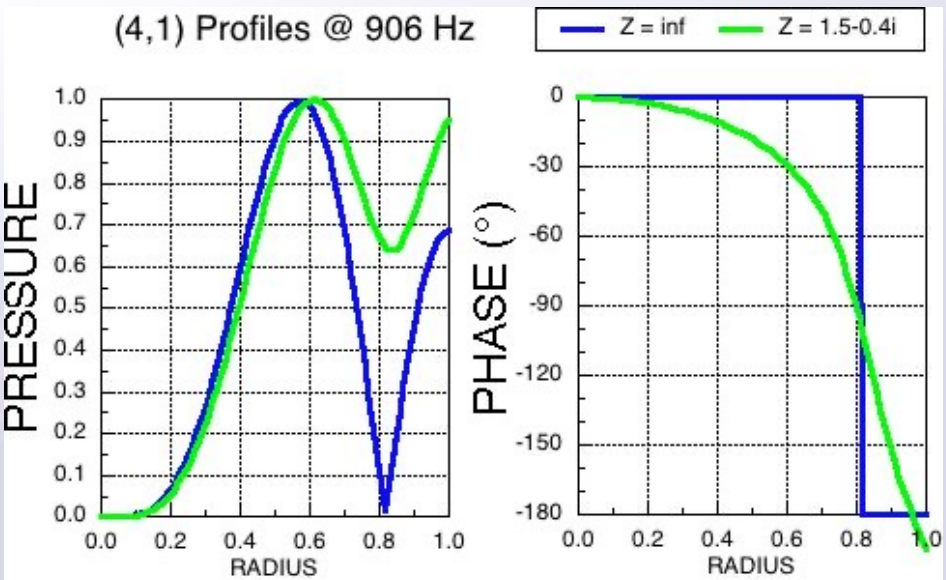
(For illustrative purposes, σ=0)

$$P(r) \propto J_{mn}(\kappa_{mn} r)$$

HARDWALL: acoustic velocity normal to wall is zero, momentum equation gives:

$$\frac{dP}{dr} = 0;\ J'_m(\kappa_{mn}) = 0;\quad \kappa_{mn}\frac{J_{m+1}(\kappa_{mn})}{J_m(\kappa_{mn})} - m = 0$$

A set of basis functions for each (m,n) mode.



SOFTWALL: (passive treatment - liner)

$$\frac{p}{v} = \frac{Z}{\rho c};\ A = \frac{\rho c}{Z};\ \eta = \frac{\omega R}{c}$$

impedance, admittance, reduced freq

$$\frac{dP}{dr} = -i\eta AP$$

$$\kappa\frac{J'_m(\kappa)}{J_m(\kappa)} = -i\eta A \longrightarrow \kappa\frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A$$

$$\kappa\frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A(1 - M\frac{k_x}{\eta})^2$$

A set of basis functions for each (m,n) mode - for each frequency, impedance, duct Mach.

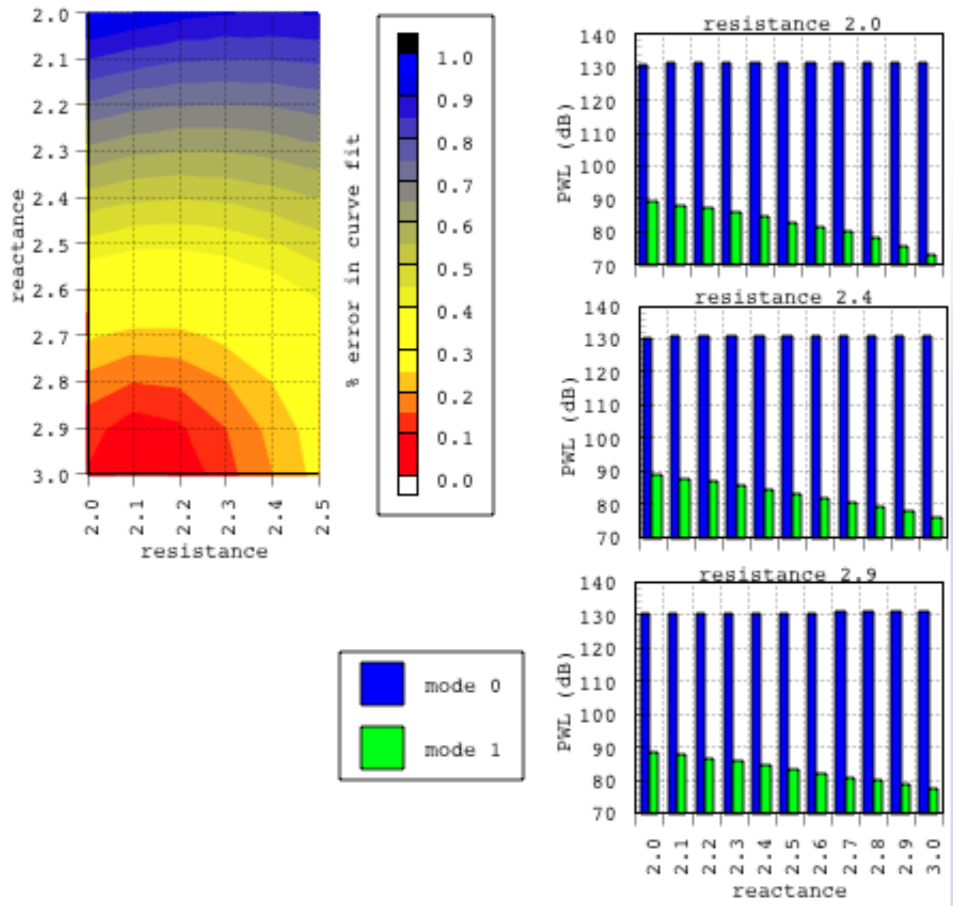
$$Hardwall:\quad \kappa_{mn} \rightarrow real$$

$$Softwall:\quad \kappa_{mn} \rightarrow complex$$

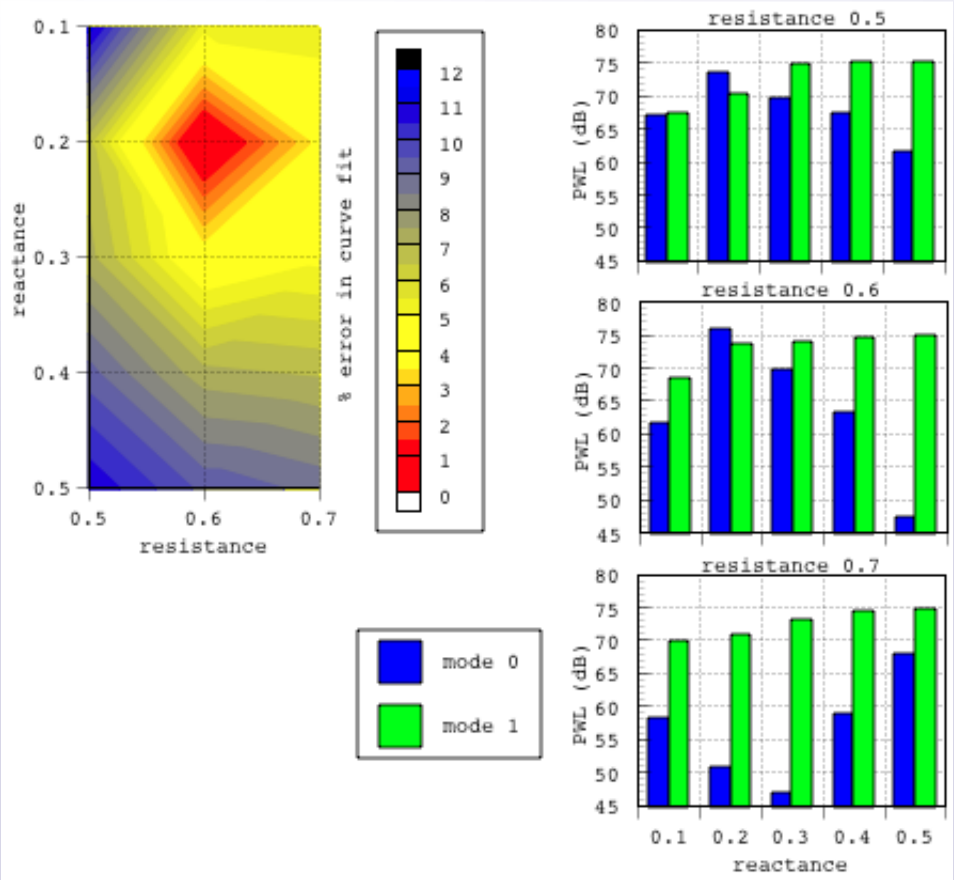
IMPEDANCE SOLUTIONS

Obtained modal solution for matrix of impedances

- Error / sensitivity analysis
- Iterative method for in-situ impedance measurements



Non-Optimum Input Impedance

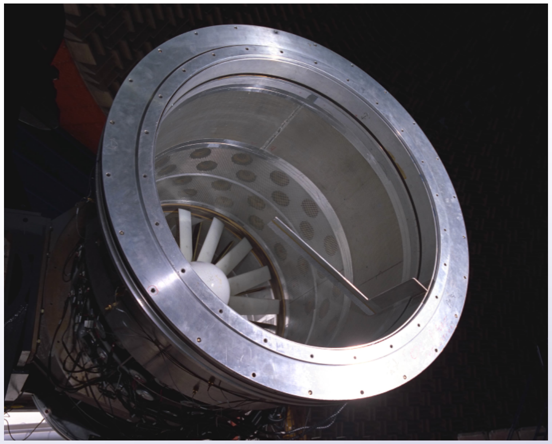


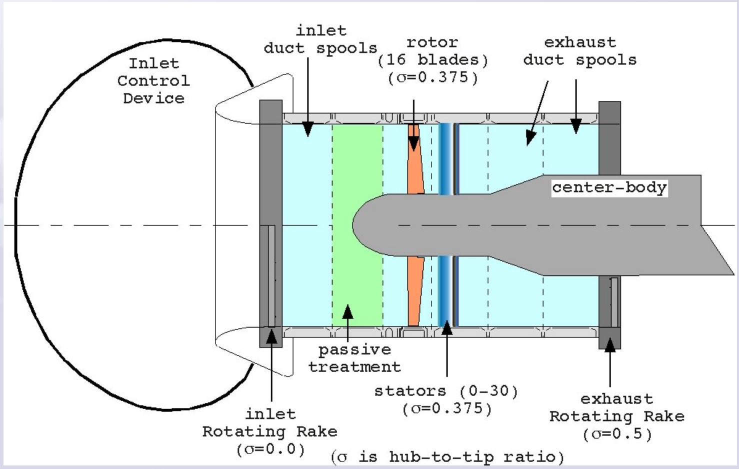
Near Optimum Input Impedance

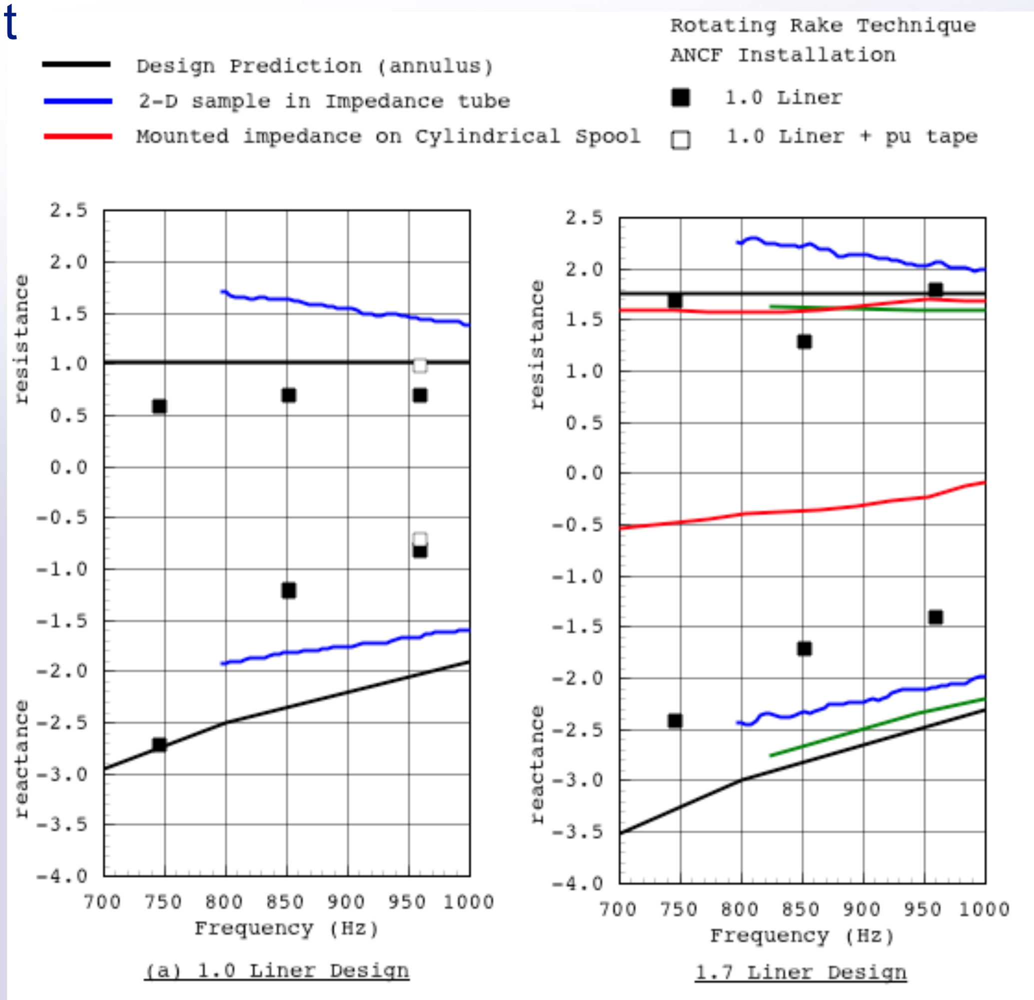
ANCF TEST CASE

4-ft Diameter Ducted Low Speed Fan (Typ 500 Hz BPF : M_{duct} = 0.125)

- Two 16” liner sections tested (res = 1.0, 1.7)
  - Added resistance increasing tape
- Rotating Rake extended over treatment







Presentation Outline

(I) Description of Acoustic Field that is Measured
- rotor/stator interaction
- duct modes

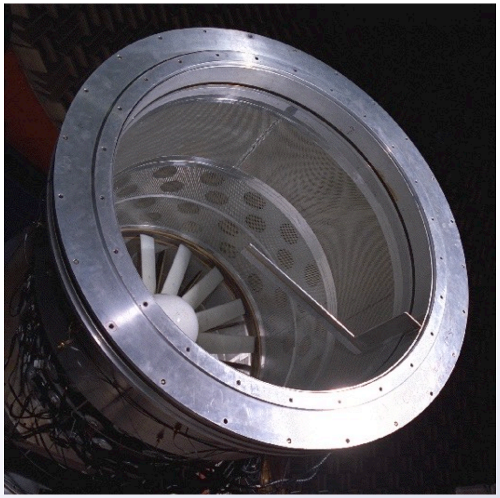
(II) Rotating Rake Theory and Implementation
- Doppler shift
- Radial Curve Fitting
- Mechanical Parameters

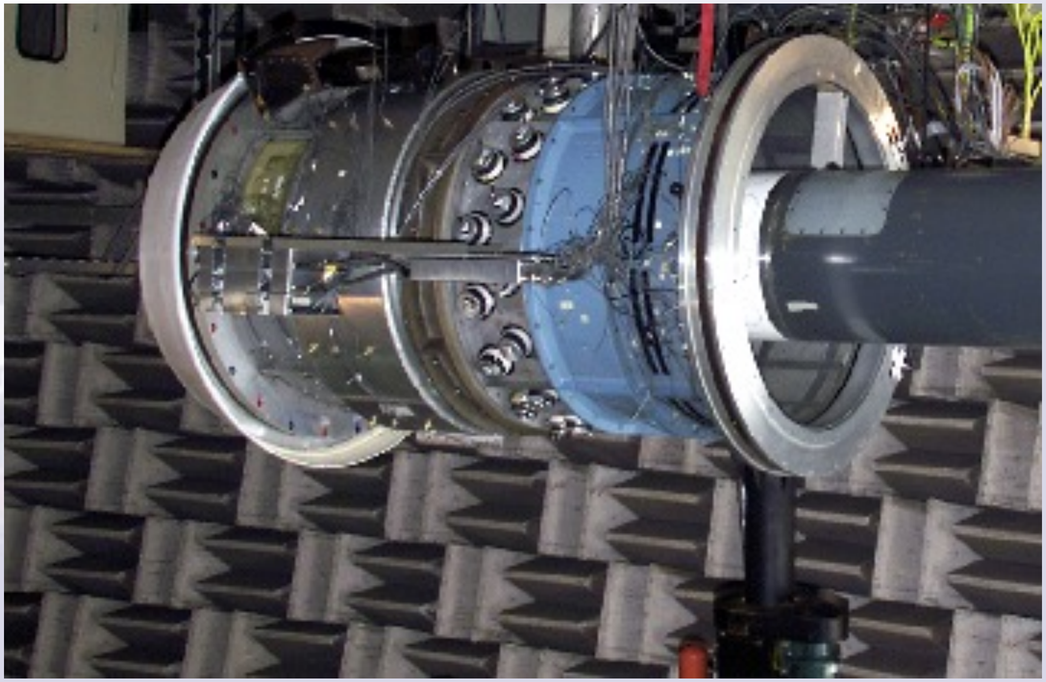
(III) Examples of Rotating Rake Applications

Advanced Noise Control Fan

-rotor tip velocity ~ 400 ft/sec
-16 rotor-blades/variable stator count
BPF ~ 500Hz
-rake rotates 1/100th fan speed
18 rpm
-7/6 radial pressures

Active Noise Control
Trailing Edge Blowing
Code verification
Passive treatment optimization





22” Fans in 9x15 WT

HIGH SPEED FAN
(ADP, SDT, QHSF)

- -rotor tip velocity ~ 800 – 1200 ft/sec
- -22 rotor-blades/variable stator geometry
- BPF ~ 4000 to 5000 Hz
- -rake rotates 1/200th fan speed
- 75 rpm
- -14/8 radial pressures

- Source reduction
- Complex interactions
- Active Noise Control

Full-Scale Engine

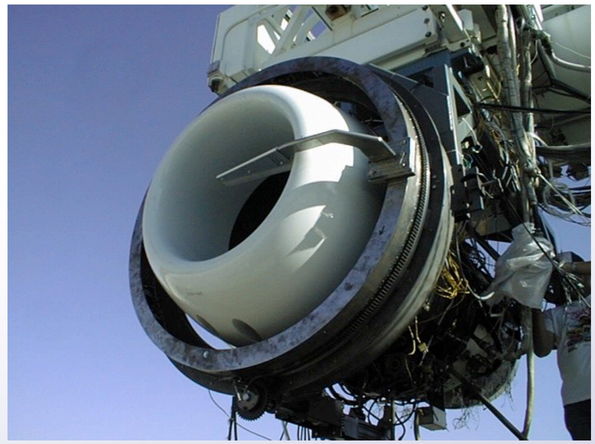
-rotor tip velocity ~ sonic
-22 rotor-blades/52 stators/10 struts
    BPF ~ 3600
-rake rotates 1/200th fan speed
    50 rpm
-14/8 radial pressures

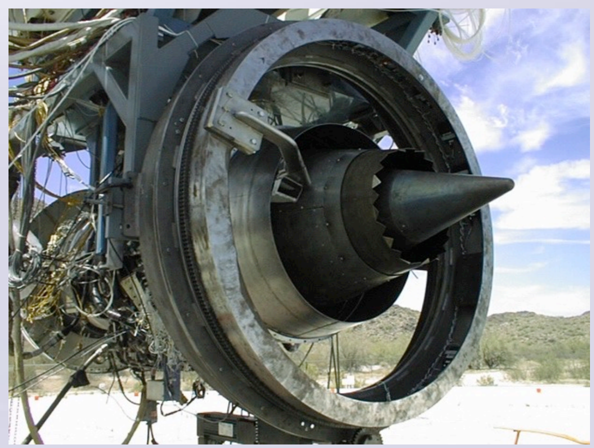
Engine is currently certified on the
    Dassault Falcon 900EX

Part of the TFE731-20/40/60 engine
    family that also powers Lear 45 IAI
    Astra SPX Dassault Falcon 50X.

Basic Engine Weight - 988 lbs
Takeoff Thrust - 5000 lb
Airflow - 187 lb/sec

Honeywell TFE731-60 engine.





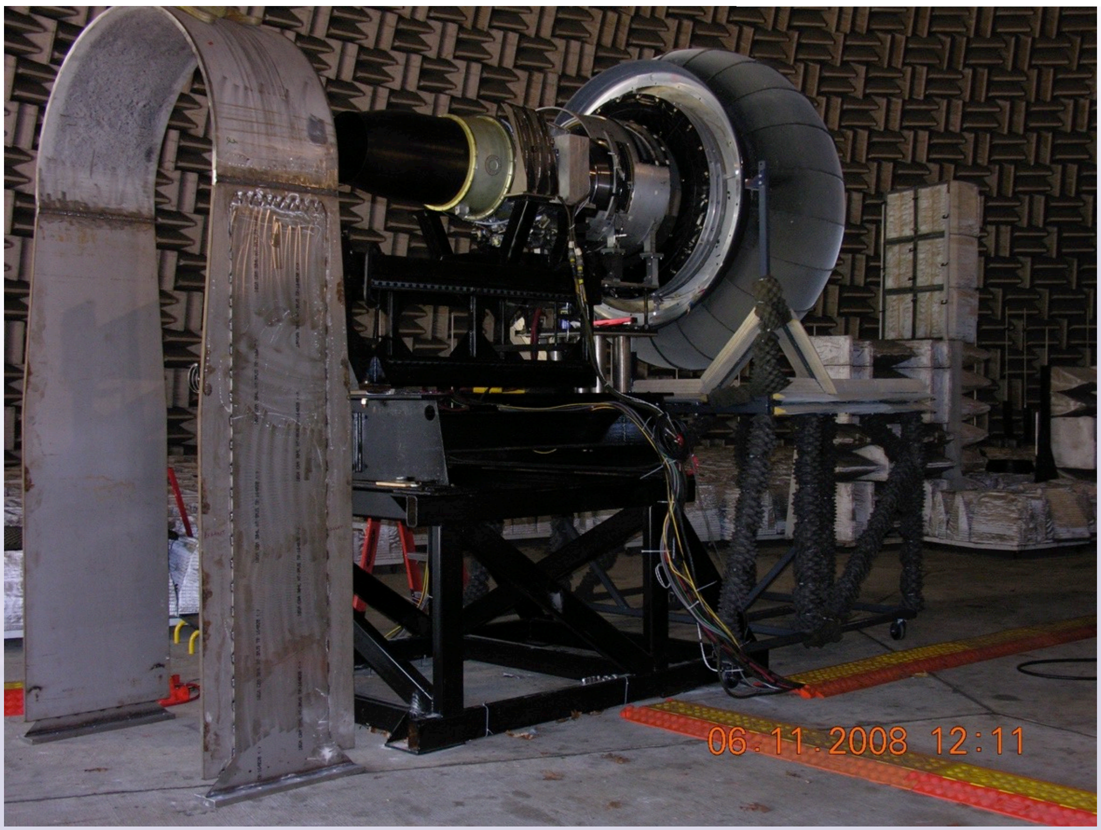
Full-Scale Engine

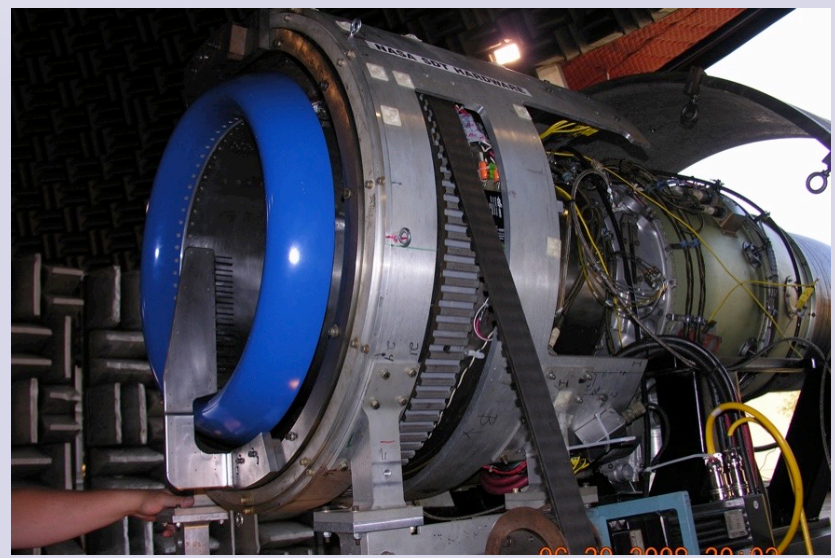
Williams International FJ44 engine

- -rotor tip velocity > sonic
- -16 rotor-blades   BPF ~ 3600
- -rake rotates 1/300th fan speed (50 rpm)
- -14 pressures – inlet only

Table I. FJ44-3A Characteristic Engine Parameters

Base Engine Weight	525 lbs
Base Engine Length	48 inches
Base Engine Diameter	23 inches
Thrust Class	3,000 lbf
Blade Count	16





CONCLUSIONS

The Rotating Rake duct mode measurement system provides a complete modal map – magnitude & phase of circumferential & radial for shaft locked tones.

Full range of TRL applications.

Continuing extensions of the technique:

- Treated annular sections (plug-flow)
- Hard-wall sheared flows
- Soft-wall sheared flows
- Reflections

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- Tyler, J.M., & Sofrin, T.G., "Axial Flow Compressor Studies" *SAE Transactions*, Vol. 70, 1962.
- Moore, C.J., "Measurement of Radial and Circumferential Modes in Annular and Circular Ducts", *JSV*, Vol 62, No 2, 1979.
- Sutliff, D.L., "Rotating Rake Mode Measurement System" NASA TM-2005-213828.