Analog Quantum Algorithms

Lucas T. Brady

NASA Quantum Artificial Intelligence Lab (QuAIL)/ KBR Inc.

March 27, 2023

Classical Optimization Problems

Classical Optimization

Given some constraints, find a configuration that minimizes your function: Find x such that $f(x)$ is minimized

- Often we just want a better heuristic for NP-complete (i.e. exponentially hard) problems

- Quantum will probably not solve NP-complete problems efficiently

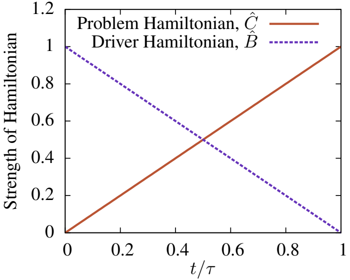
- Often an approximate solution is good enough
- Given a fixed amount of time, how good of a solution can you get

Quantum Adiabatic Optimization

$$\frac{d}{dt}|x(t)\rangle = -i\hat{H}(t)|x(t)\rangle$$

$$\hat{H}(t) = \left(1 - \frac{t}{\tau}\right)\hat{B} + \frac{t}{\tau}\hat{C}$$

- $\hat{B}$: Simple (Driver) Hamiltonian
- $\hat{C}$: Complicated (Problem) Hamiltonian
- τ: Total Runtime



Algorithm

1. Start in the ground state of $\hat{B}$
2. Slowly change the system in total time τ
3. At $t = \tau$, measure to get ground state of $\hat{C}$.

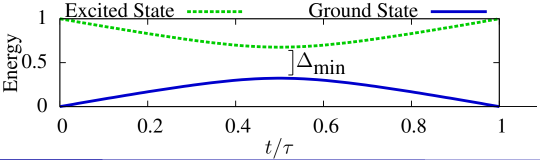
Quantum Adiabatic Theorem

Adiabatic Theorem

If a system starts in the ground state and evolves slowly enough, it will remain in the instantaneous ground state

$$\tau_{AC} \gg \frac{\left|\left|\frac{d\,\hat{H}}{d(t/\tau)}\right|\right|}{\Delta_{min}^2}$$

- $\Delta_{min} = \min_{t\in[0,\tau]} \Delta(t)$ is minimum of the energy gap from the ground state to the first excited state
- If this condition is met, adiabaticity is guaranteed*



Quantum Annealing

- What happens if you go faster than adiabatic or have a lot of noise?

- Quantum Annealing is non-ideal QAO
- Often there are no/fewer guarantees of success
- Often works partially and justifies the tradeoff of quality for speed
- Leads to weird, complicated dynamics

More General Problem

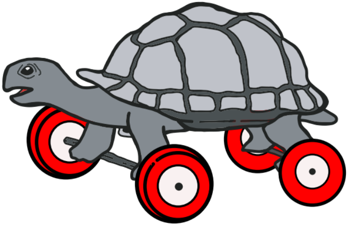
What if we allow more general evolutions

$$\hat{H}(t) = u(t)\hat{B} + (1 - u(t))\hat{C}$$

Can we find a $u(t)$ that works well

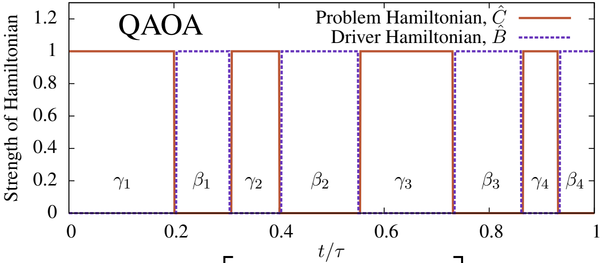
Previously $u(t) = t/\tau$

- Analytically optimized annealing paths
- Variational Optimization
- Shortcuts to Adiabaticity
- Quantum speed-limits



arXiv:1904.08448

Quantum Approximate Optimization Algorithm



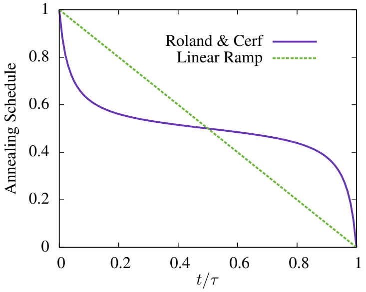
$$|x(\tau)\rangle = \left[\prod_{j=1}^{p} e^{-i\beta_j\hat{B}} e^{-i\gamma_j\hat{C}}\right]|\varphi\rangle$$

- Treat the quantum computer as a black-box
  $E(\vec{\gamma},\vec{\beta}) = \left\langle x(\tau)|\hat{C}|x(\tau)\right\rangle$
- Use a classical optimizer to search for the lowest energy by varying γs and βs

Analytic Optimization

Roland & Cerf Schedule

- Quantum Advantage is linked to the annealing schedule

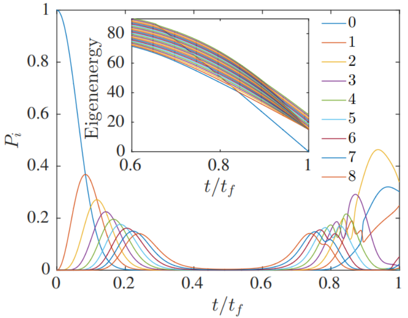


J. Roland, N. J. Cerf, arXiv:quant-ph/0107015

- R& C focuses on solving the unstructured search problem
- With a linear schedule, this takes $\mathcal{O}(N)$ time recovering the classical scaling
- In order to get the square root Grover speed-up in an analog setting, you need a schedule optimized based off the local adiabatic condition.

Non-Adiabatic Optimization

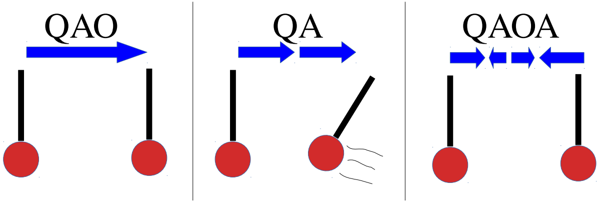
- Your schedule doesn't need to be adiabatic
- Diabatic quantum annealing permits evolution in excited states
- Diabatic annealing is quantum universal even with simple Hamiltonians (adiabatic annealing has more restrictions)



S. Muthukrishnan, T. Albash, D. A. Lidar, arXiv:1505.01249

Variational Optimization

What is Best

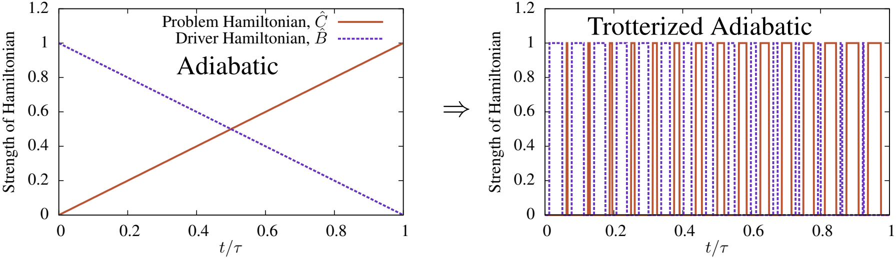


The core question we will ask is which of these is best

- What form of $u(t)$ is optimal for a given τ
- For now, we ignore the difficulty in finding this procedure
- ”Best” is the state with the lowest energy at the end (Alternative would be highest overlap with ground state)

Trotterization of QAO

Trotterization makes smooth adiabatic look like bang-bang



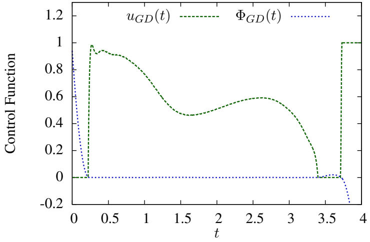
- The bang-bang form of QAOA can approximate Adiabatic
- Since Adiabatic is Quantum Universal, QAOA is as well

Bang-Anneal-Bang

$$\hat{H}(t) = u(t)\hat{B} + (1 - u(t))\hat{C}$$

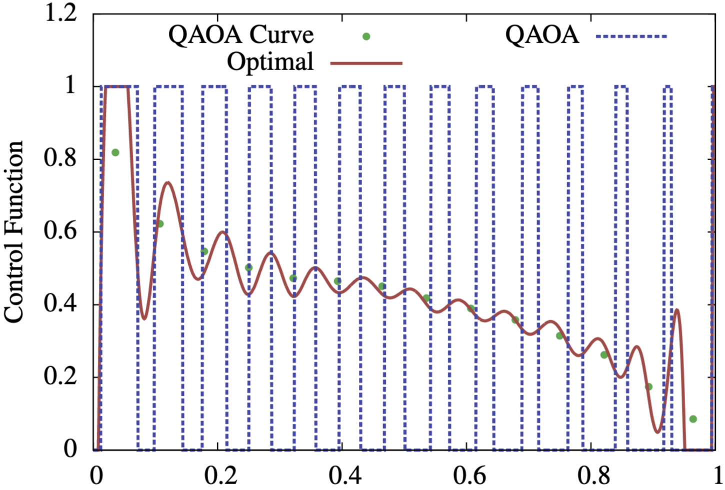
- We can also ask for the optimal form of $u(t) \in [0, 1]$[1]

- Optimal procedure has bangs at the beginning and end
- In the middle, there is a smooth annealing region
- The initial and final bangs decrease in length as time increases



This is Diabatic Annealing

Connections Between QAOA and Optimal

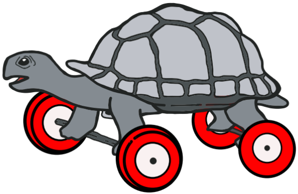


L. T. B., Lucas Kocia, Przemyslaw Bienias, Aniruddha Bapat, Yaroslav Kharkov, Alexey V. Gorshkov, arXiv:2107.01218

Shortcuts to Adiabaticity

Goals of Shortcuts

- Take an adiabatic evolution and run it faster
- Use an additional Hamiltonian
- Exactly follows adiabatic frame



arXiv:1904.08448

$$\hat{H}(t) = u(t)\hat{B} + (1 - u(t))\hat{C} + \hat{H}_{CD}(t)$$

- If $\hat{H}_{CD}$ is unbounded, this works for any t_f
- Mimics Adiabaticity, not Annealing

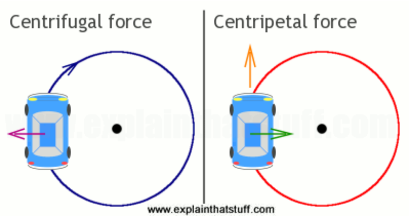
Counter Diabatic Formulation

$$\hat{H}(t) = \sum_{j=1}^{N} E_j(t) \left|j(t)\right\rangle \left\langle j(t)\right|$$

- The CD Hamiltonian is derived to keep us in the adiabatic frame

$$\hat{H}_{CD}(t) = i\hbar \sum_{j} \left|\partial_t j(t)\right\rangle \left\langle j(t)\right|$$

- $\hat{H}_{CD}(t)$ can act alone
- The original Hamiltonian determines phase
- Eigenstate phases determined by adiabatic frame.



Open Questions

Areas of Active Research

- How do we engineer diabatic evolution
- Do QAOA or the optimal curve mimic counter-diabaticity (in the limit of short QAOA steps, the answer is yes)
- How can we scale up QAOA to larger systems and more variational parameters (This is Quantum Machine Learning)
- Can we consistently recover the speed-ups of QAOA without relying on variational approaches