Linear Interaction Approximation for the Prediction of Noise Generated by Turbulence/Shock Interactions

S. J. Leib
HX5, LLC
NASA Glenn Research Center
Cleveland, Ohio 44135, USA

AIAA Aviation Forum, San Diego, CA
June 12 - 16, 2023

Work supported by:
NASA Advanced Air Vehicles Program, Commercial Supersonic Technology Project



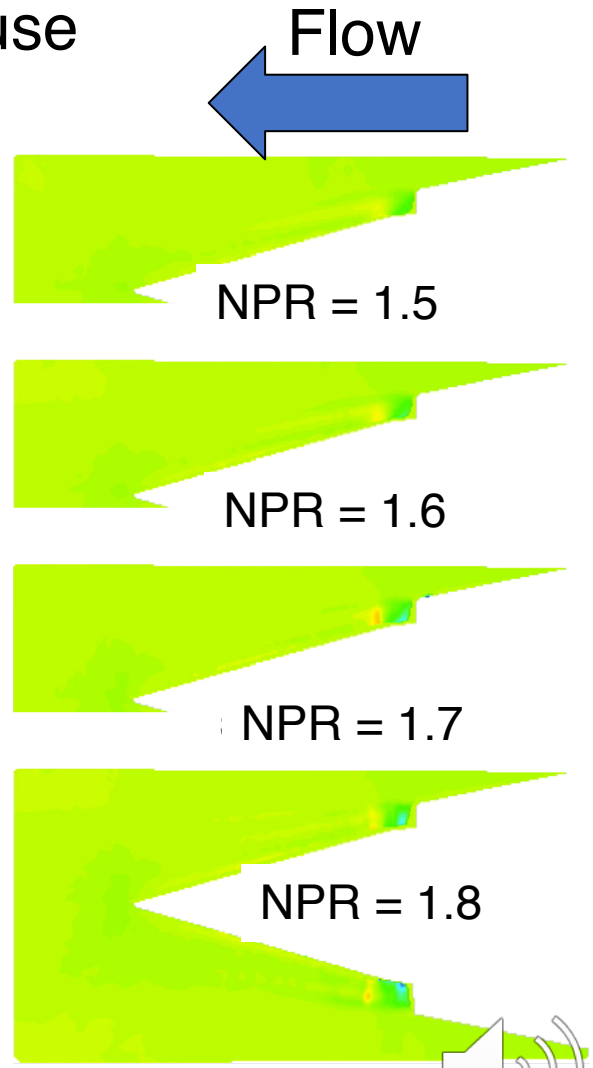
This material is a work of the U.S. Government and is not subject to copyright protection in the United States.

INTRODUCTION

- Exhaust system designs for commercial supersonic aircraft will use internally mixed / externally plugged nozzles

- Additional noise concerns beyond turbulent jet mixing noise:
  - Internally generated noise
  - Impact of external plug
  - Shock wave near the near nozzle exit due to local flow acceleration

- Potential noise impacts of shock:
  - Direct generation of noise due to interaction with turbulence
  - Modification of mixing noise sources passing through
  - Scattering of upstream-generated noise



BOS images – Bridges & Werne, 2021

Purpose and Scope of Work

- Implement a linear approach to evaluate the sound field generated by the interaction of unsteady flows (turbulence, high-temperature spots) with a shock near the exit of internally mixed/ externally plugged nozzles

- Demonstrate on canonical incident flows

Ultimate Goals:

- *Develop tools and scalings to quantify noise impacts of shock and separate it from other sources in mixed/plugged nozzles*



Technical Approach

- Linear Interaction Approximation (LIA) to model turbulence/shock interactions

  (Moore (1953), Ribner (1954, 1955, 1987), Sagaut & Cambon (2018))

- Parallel, uniform mean flow with small-amplitude perturbations incident on a planar shock

- Impose: (Independent) Vorticity, Entropy, Acoustic modes upstream

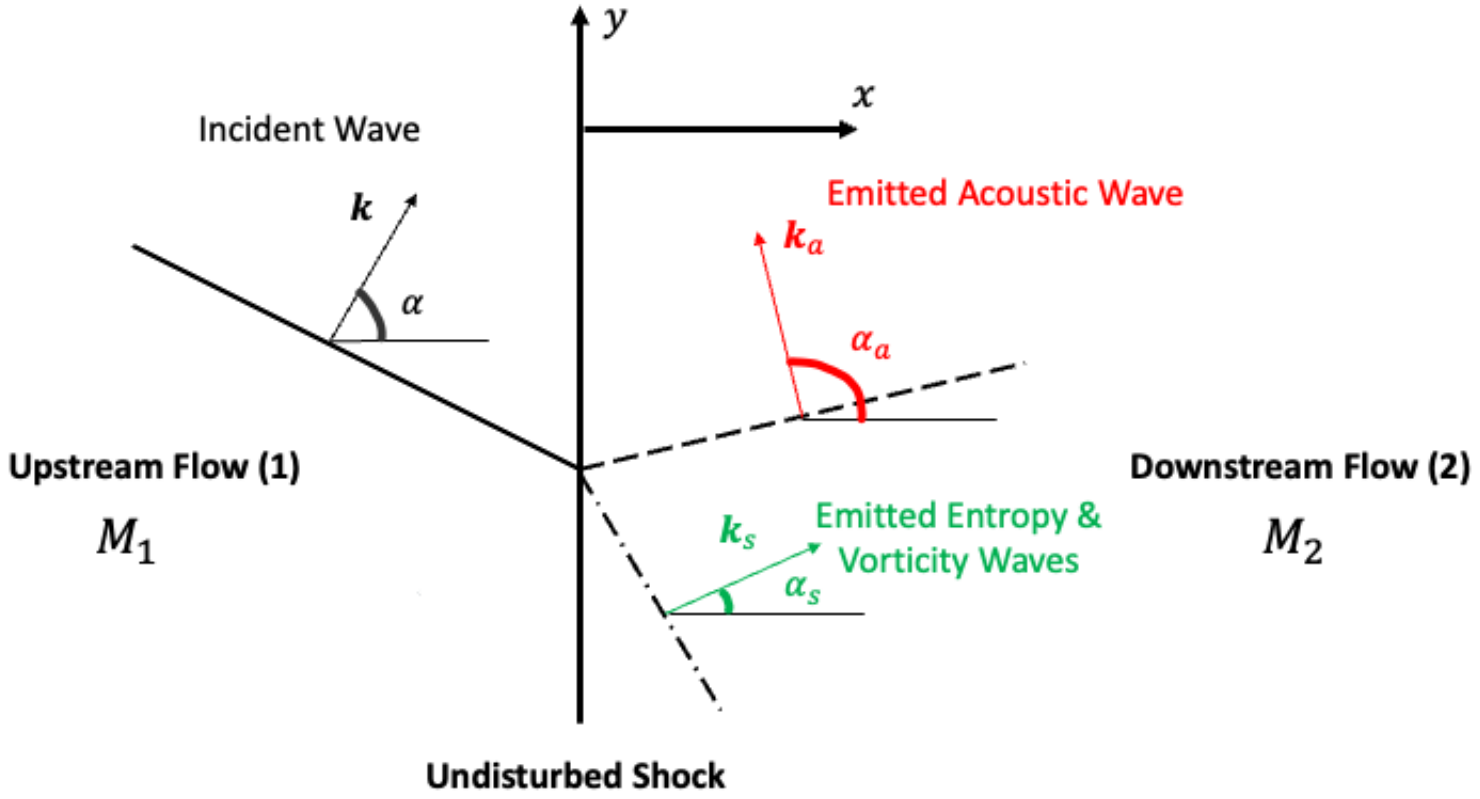
- Generate: (all three) Vorticity, Entropy, Acoustic modes downstream

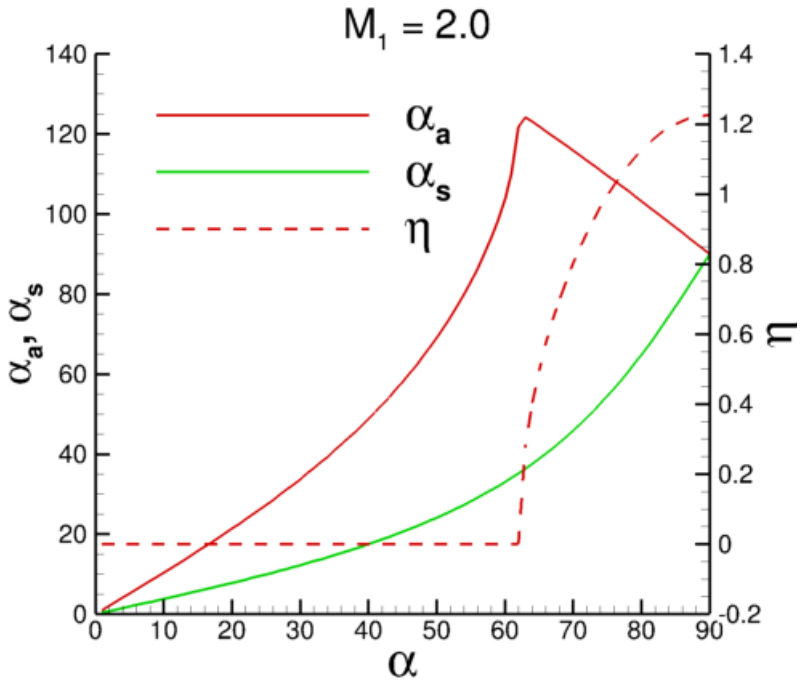
- Relate flow downstream of shock (emitted) to that upstream (incident)
  - Combine solutions for individual Fourier components to obtain results for specified incident field

Current Work

Incident Vorticity or Entropy Mode

Emitted Vorticity, Entropy and Acoustic Modes





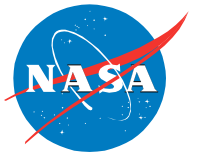
Emission vs. Incident Angles

Incident Vorticity Mode

$$\begin{Bmatrix} \frac{u_1'}{U_1} \\ \frac{v_1'}{U_1} \\ \frac{\rho_1'}{\bar{\rho}_1} \\ \frac{p_1'}{\bar{p}_1} \\ \frac{T_1'}{\bar{T}_1} \\ \frac{s_1'}{c_p} \end{Bmatrix} = A_v(\boldsymbol{k}) \begin{Bmatrix} \sin\alpha \\ -\cos\alpha \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} e^{i(k[x\cos\alpha + y\sin\alpha] - \omega t)}$$

- Incompressible
- Described by velocity components

Emitted Field – Incident Vorticity



Vorticity and Entropy

Acoustic

$$\begin{Bmatrix} \frac{u_2'}{U_2} \\ \frac{v_2'}{U_2} \\ \frac{\rho_2'}{\bar{\rho}_2} \\ \frac{p_2'}{\bar{p}_2} \\ \frac{T_2'}{\bar{T}2} \\ \frac{s_2'}{c_p} \end{Bmatrix} = A_v \begin{Bmatrix} Z_{vv}\sin\alpha_s \\ -Z_{vv}\cos\alpha_s \\ -Z_{vs} \\ 0 \\ Z_{vs} \\ Z_{vs} \end{Bmatrix} e^{i(k_s[x\cos\alpha_s + y\sin\alpha_s]-\omega t)} + A_v Z_{va} \begin{Bmatrix} \frac{\cos\alpha_a + i\eta}{M_2\zeta} \\ \frac{\sin\alpha_a}{M_2\zeta} \\ 1 \\ 1 \\ (\gamma-1) \\ 0 \end{Bmatrix} e^{-k_a\eta x} e^{i(k_a[x\cos\alpha_a + y\sin\alpha_a]-\omega t)}$$

Shock Displacement

$$\xi(y,t) = Z_{vx} e^{i(k\sin\alpha y - \omega t)}$$

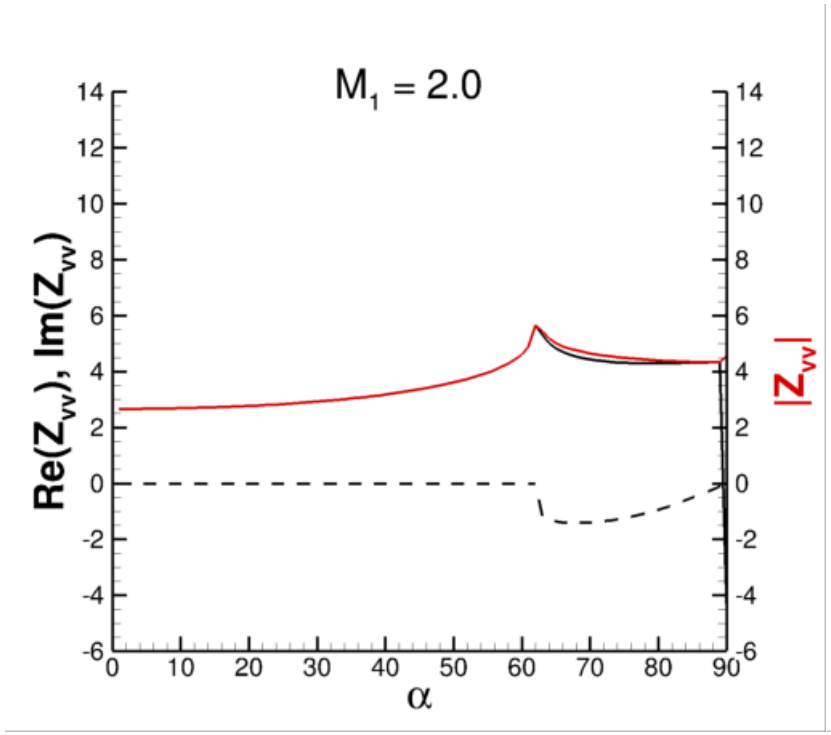
$$Z_{vv}, Z_{vs}, Z_{va}, Z_{vx}$$

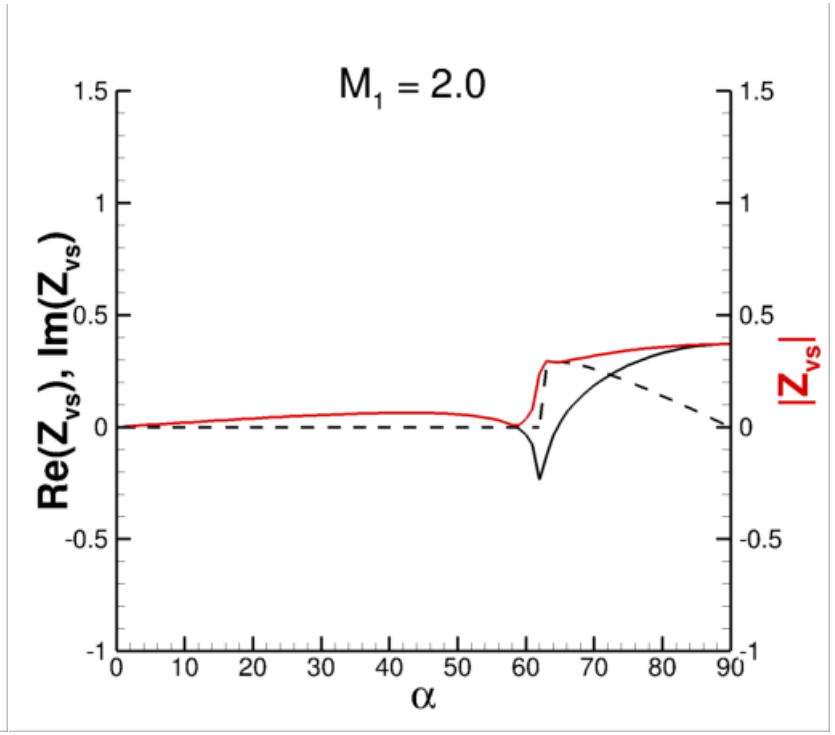
Transfer Coefficients for emitted vorticity, entropy modes and displacement
From linearized jump conditions across the shock

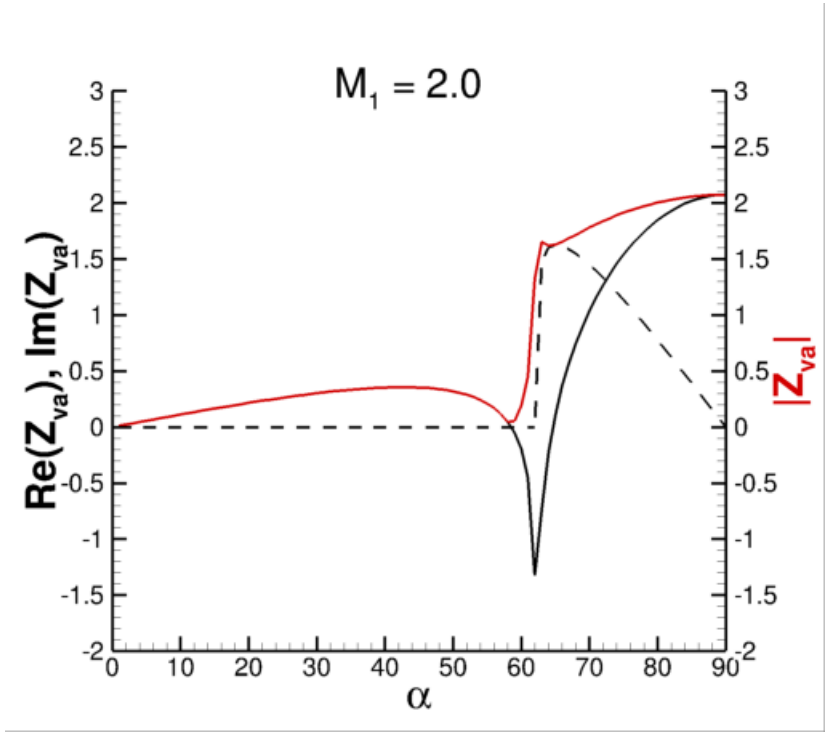


Incident Vorticity Mode

Transfer Coefficients







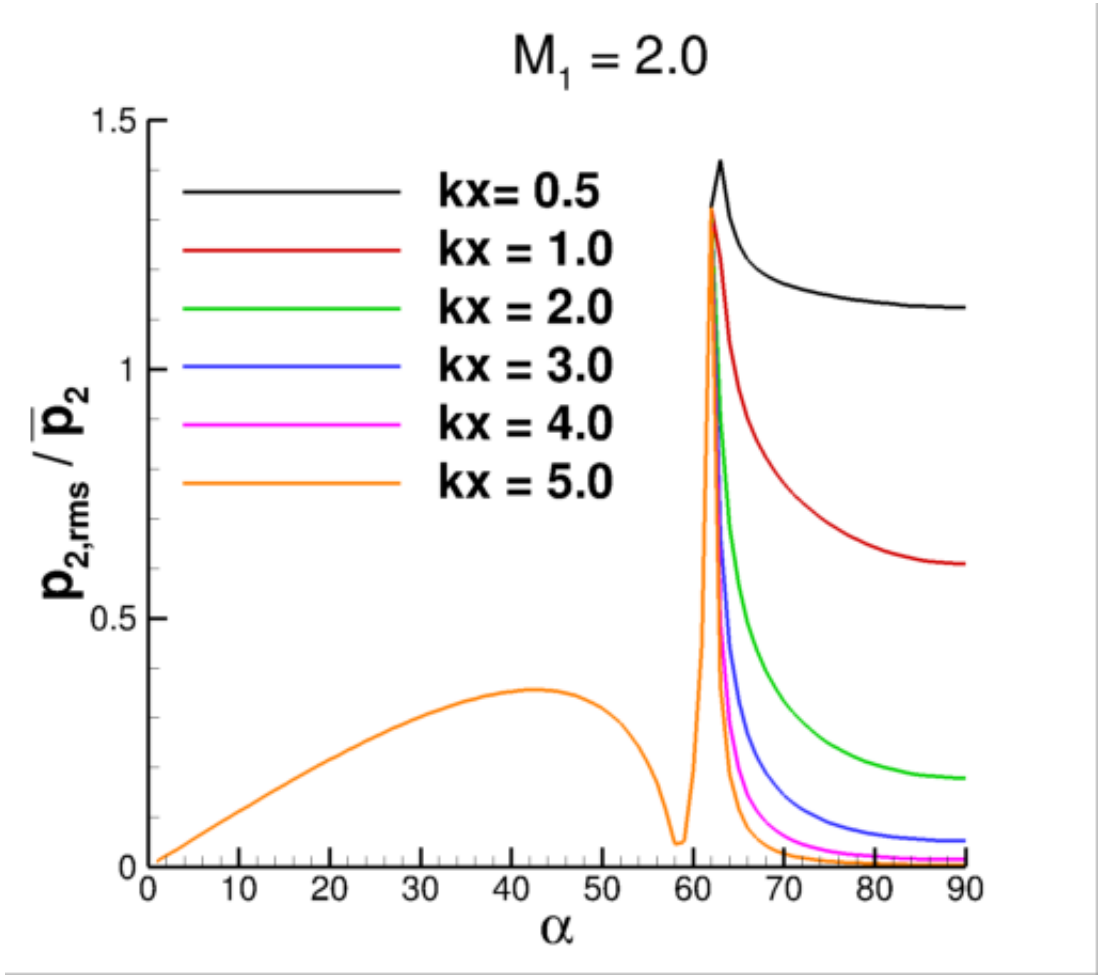
Vorticity Mode Amplified

Entropy Mode Triggered, Small Amplitudes

Acoustic Mode Triggered, Large Amplitudes $\alpha > \alpha_c$

Incident Vorticity Mode

Emitted Acoustic Mode – Single Fourier Component



Incident Vorticity Mode

Emitted Acoustic Field – Incident Isotropic Turbulence

Energy Spectrum

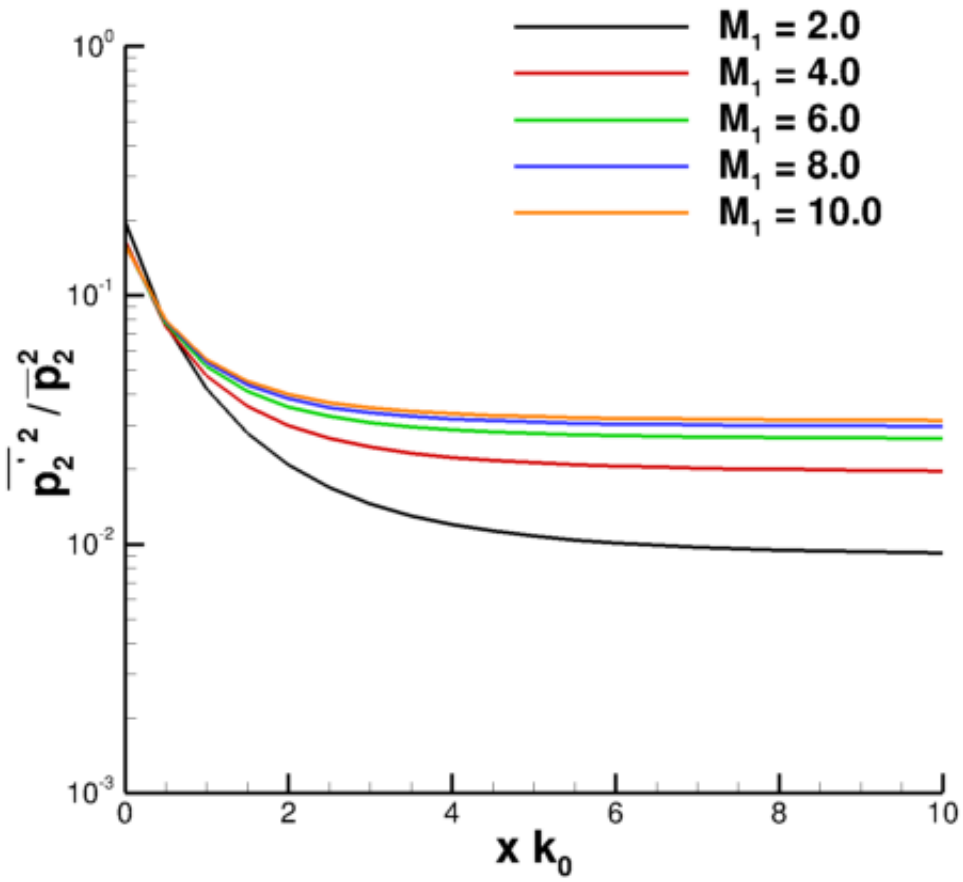
$$E_{ij}(\boldsymbol{k}) = \frac{E(k)}{4\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)$$

Mode Amplitudes

$$|A_v(k)|^2 = \frac{E(k)}{4\pi k^2}$$

3D Spectrum Model

$$E(k) = \frac{(k/k_0)^4}{\left[1 + (k/k_0)^2\right]^{17/6}}$$



$$\frac{\overline{p_2'^2}(x)}{\bar{p}_2^2} = \frac{1}{2}\int_0^{\infty}\int_0^{\pi/2} E(k)|Z_{va}|^2 e^{-2k_a\eta x}\sin\alpha\, d\alpha\, dk$$

Incident Entropy Mode

- Irrotational
- Described by density, temperature and entropy

$$\begin{Bmatrix} \frac{u_1'}{U_1} \\ \frac{v_1'}{U_1} \\ \frac{\rho_1'}{\bar{\rho}_1} \\ \frac{p_1'}{\bar{p}_1} \\ \frac{T_1'}{\bar{T}_1} \\ \frac{s_1'}{c_p} \end{Bmatrix} = A_s(\boldsymbol{k}) \begin{Bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \\ 1 \end{Bmatrix} e^{i(k[x\cos\alpha + y\sin\alpha] - \omega t)}$$

Incident Entropy Mode

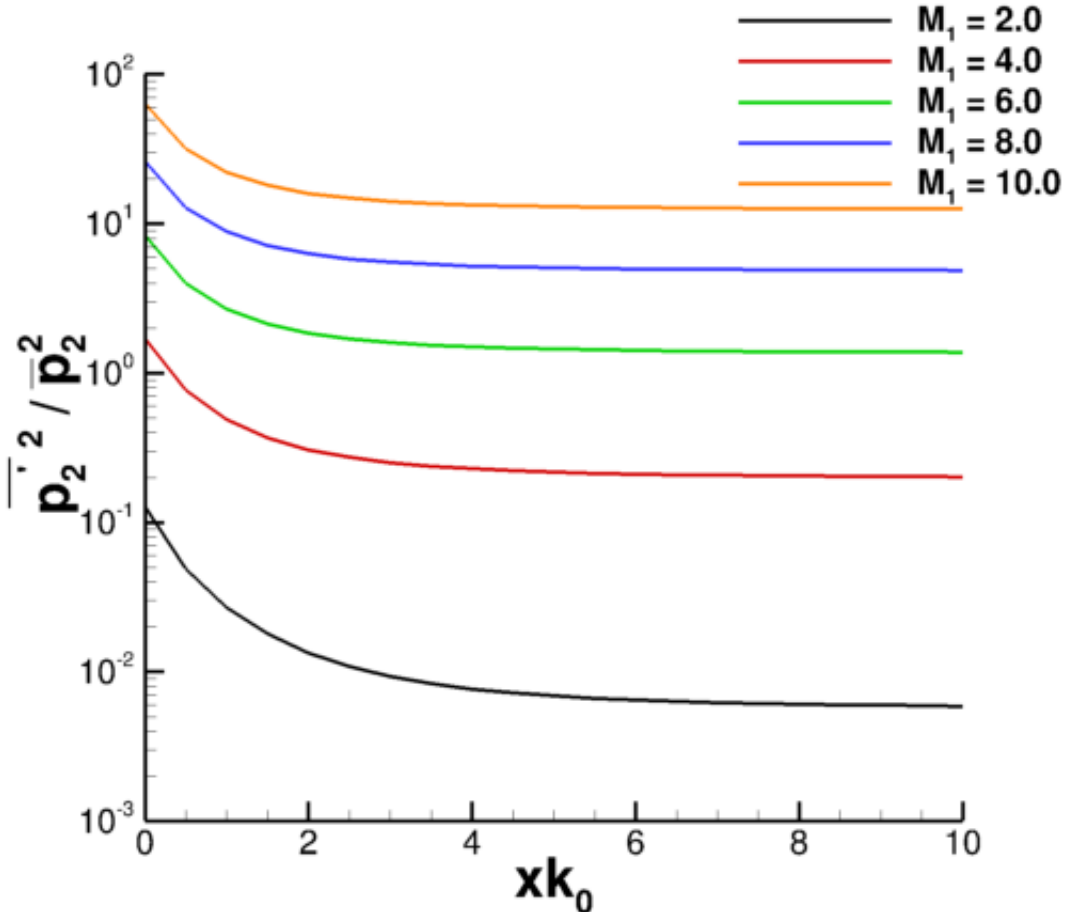
Emitted Acoustic Field – Incident Isotropic Entropy

Amplitude Scaling
Morkovin's hypothesis (1960)

$$\frac{s_1'}{c_p} = -(\gamma - 1) M_1^2 \frac{u_1'}{U_1}$$

Mode Amplitudes

$$|A_s(\boldsymbol{k})|^2 = (\gamma - 1)^2 M_1^4 \sin^2 \alpha \frac{E(k)}{4\pi k^2}$$



$$\frac{\overline{p_2'^2}(x)}{\bar{p}_2^2} = (\gamma - 1)^2 M_1^4 \frac{1}{2} \int_0^\infty \int_0^{\pi/2} E(k) |Z_{sa}|^2 e^{-2k_a \eta x} \sin^3 \alpha \, d\alpha \, dk$$

Summary and Conclusions

- Reviewed the basic formulation of Linear Interaction Approximation

- Implemented the formulation and demonstrated on canonical incident turbulent fields
  - Linear theory! Breaks down at ‘large’ amplitudes
  - Use as ‘first-cut’ to indicate where to employ higher-fidelity tools

- Next: Use for applications specific to exhaust systems being designed for commercial supersonics
  - Obtain incident turbulence spectrum from LES
  - Modification of mixing noise sources when passing through shock
  - Scattering of internally generated noise

THE END

THANK YOU !

