# On a stabilization of the Ingard-Myers impedance boundary condition

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# Presentation Outline

- **Instability issue of the Ingard-Myers Condition**
- **Proposed stabilization of the Ingard-Myers condition**
- **Accuracy assessments**
- **Numerical examples**

# A stabilization strategy

**Ingard-Myers condition written in pressure and its normal derivative:**

$$\rho_0 \left(-i\omega + U\frac{\partial}{\partial x}\right)^2 \hat{p}(\boldsymbol{r},\omega) = i\omega Z(\omega)\frac{\partial \hat{p}}{\partial n}(\boldsymbol{r},\omega)$$

**Proposed *Truncated Ingard-Myers Impedance Boundary Condition*:**

$$\rho_0 \left((-i\omega)^2 \hat{p} + 2(-i\omega)U\frac{\partial \hat{p}}{\partial x} + U^2\frac{\partial^2 \hat{p}}{\partial x^2}\right) = i\omega Z(\omega)\frac{\partial \hat{p}}{\partial n}$$

**TIMIBC:** ⇒ $$-\rho_0 \left((-i\omega)\hat{p} + 2U\frac{\partial \hat{p}}{\partial x}\right) = Z(\omega)\frac{\partial \hat{p}}{\partial n}$$

1. **Free of instability waves**
2. **Good approximate for flows of low to mid subsonic Mach numbers**

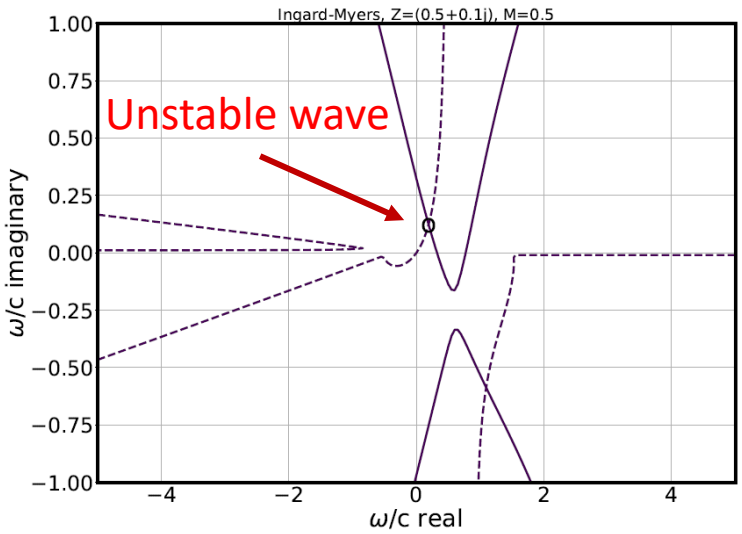
## Dispersion relation for plane waves of the form $\hat{p}(\boldsymbol{r},\omega) = Ae^{ik_x x + ik_y y + i\gamma z - i\omega t}$

For the Ingard-Myers Condition:

$$\rho_0 \left(-i\omega + U\frac{\partial}{\partial x}\right)^2 \hat{p}(\boldsymbol{r},\omega) = i\omega Z(\omega)\frac{\partial \hat{p}}{\partial n}(\boldsymbol{r},\omega)$$

Dispersion relation is:

$$(\omega/c)\gamma Z + \rho_0 c\,(\omega/c - Mk_x)^2 = 0$$



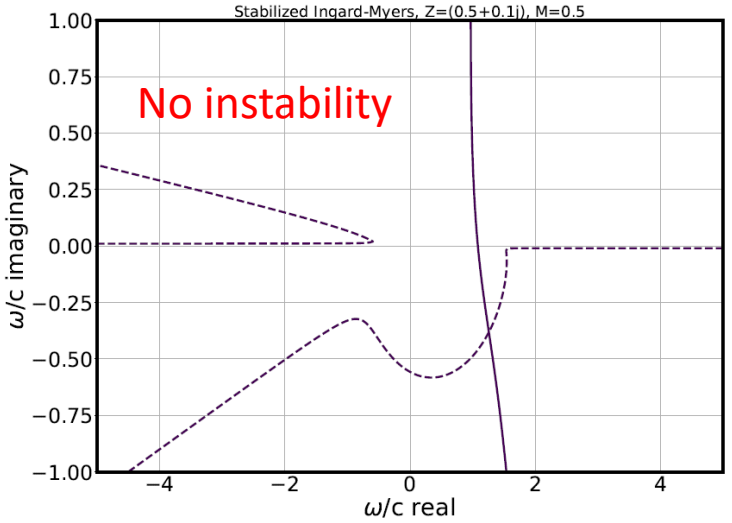

(a)

For the Truncated Ingard-Myers Condition:

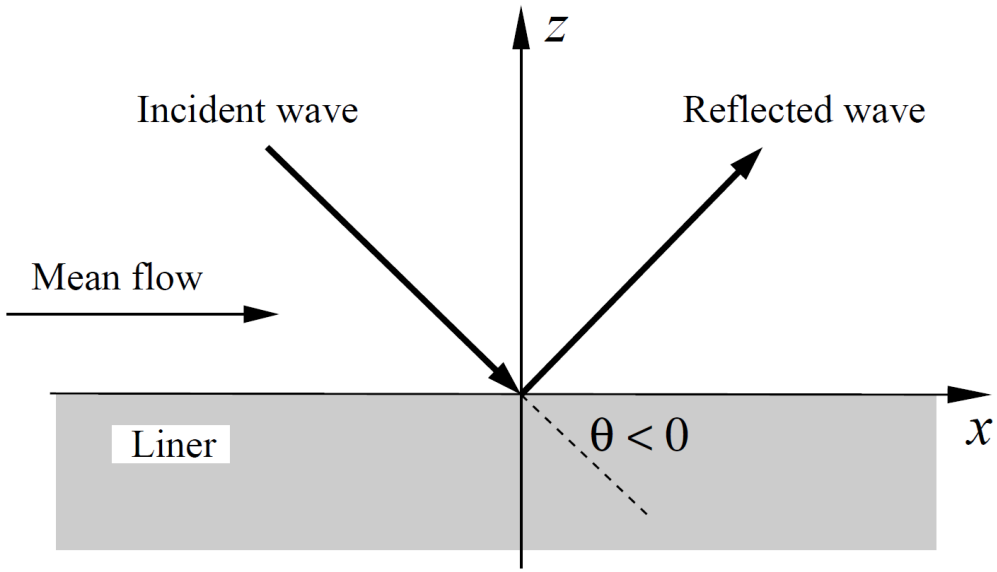
$$-\rho_0 \left((-i\omega)\,\hat{p} + 2U\frac{\partial \hat{p}}{\partial x}\right) = Z(\omega)\frac{\partial \hat{p}}{\partial n}$$

Dispersion relation is:

$$\gamma Z + \rho_0 c\,(\omega/c - 2Mk_x) = 0$$




(b)




**Reflection Coefficient by the Ingard-Myers Condition:**

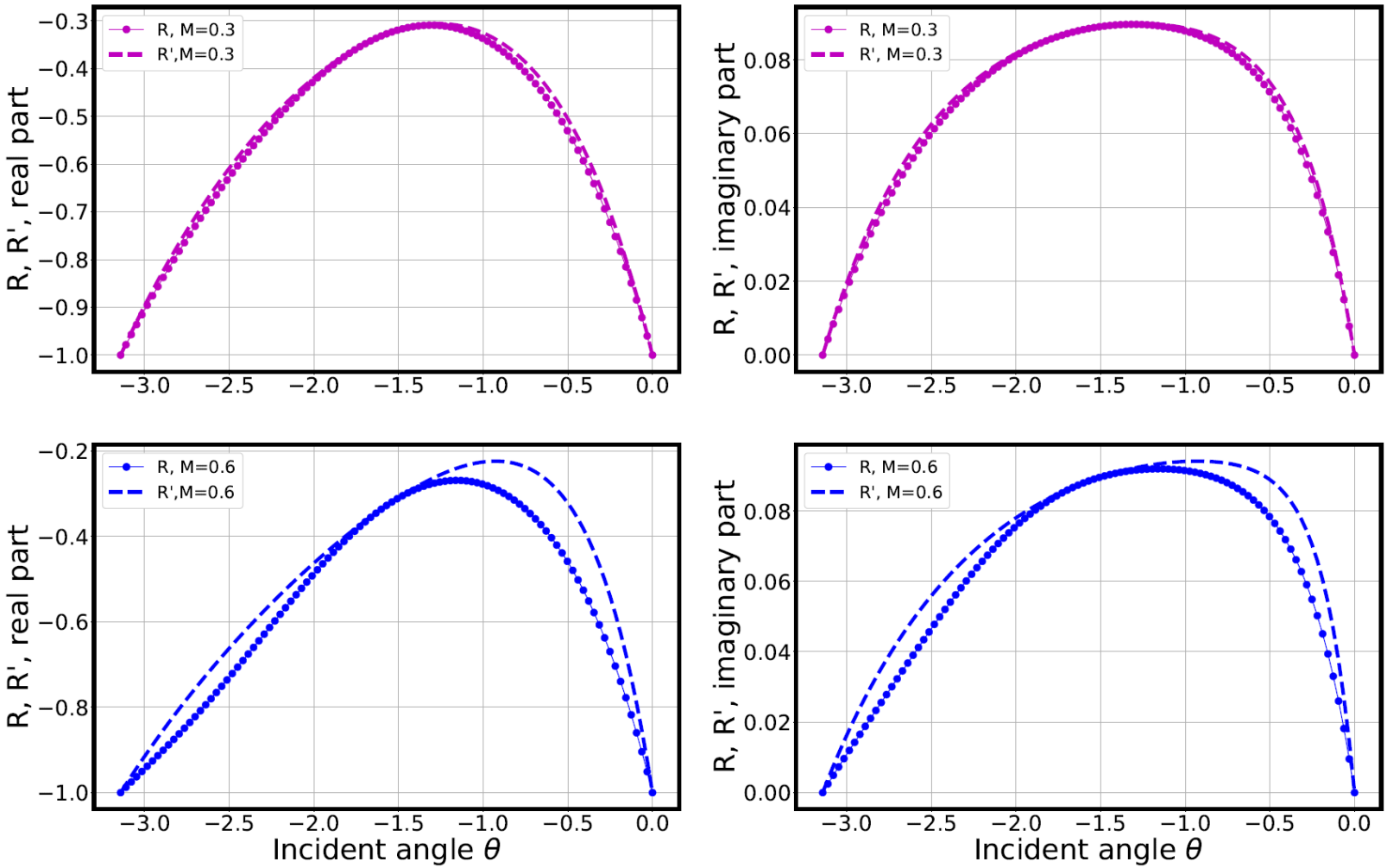
$$R = \frac{Z\,(1 + M\cos\theta)\sin\theta + \rho_0 c}{Z\,(1 + M\cos\theta)\sin\theta - \rho_0 c}$$

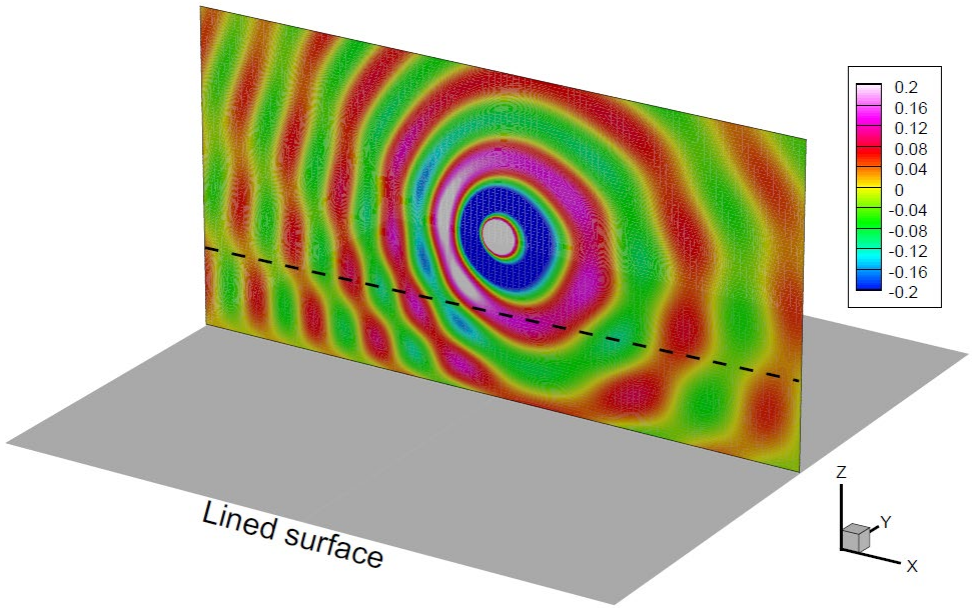
**Reflection Coefficient by the Truncated Condition:**

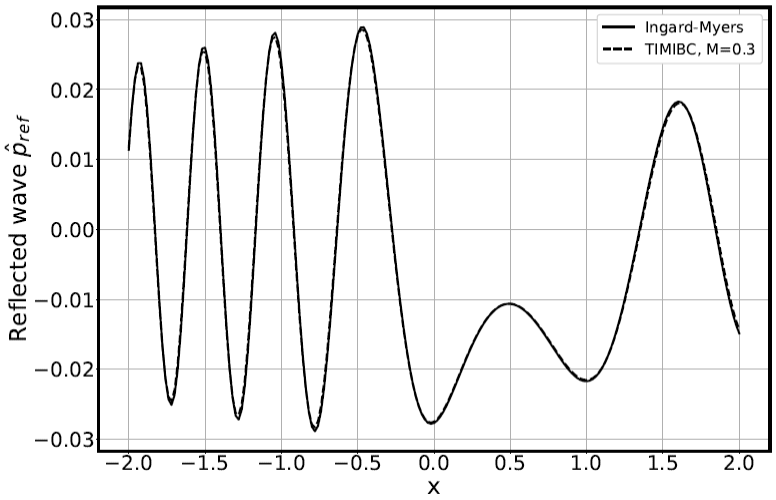
$$R' = \frac{Z\,(1 + M\cos\theta)\sin\theta + \rho_0 c(1 - M^2\cos^2\theta)}{Z\,(1 + M\cos\theta)\sin\theta - \rho_0 c(1 - M^2\cos^2\theta)}$$

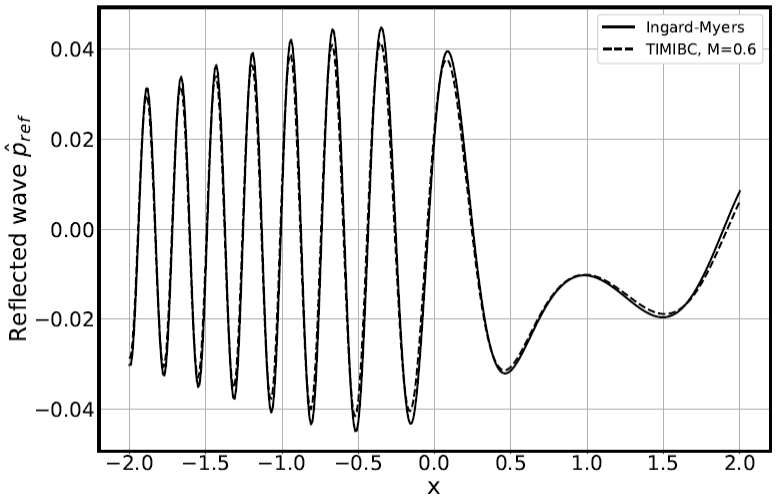
# A comparison of reflection coefficients

$$Z=0.5+0.1i$$




0.2
0.16
0.12
0.08
0.04
0
-0.04
-0.08
-0.12
-0.16
-0.2
Lined surface
Z
Y
X


Ingard-Myers
TIMIBC, M=0.3
Reflected wave $\hat{p}_{ref}$
x


Ingard-Myers
TIMIBC, M=0.6
Reflected wave $\hat{p}_{ref}$
x

# Summary

- A strategy for stabilization of the Ingard-Myers impedance boundary condition has been proposed with numerical examples

- The Truncated Ingard-Myers condition has been found to be stable for any subsonic flow and showed to be a good approximation to the original Ingard-Myers condition at low to mid Mach numbers

- Proposed truncation can be a practical solution for avoiding the instability waves associated with the original formulation in the time domain which many studies have shown to be the correct limit of boundary layer thickness going to zero