



Anisotropic Mach Cone Aligned Mesh Adaptation for Low Boom Simulations

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➤ Introduction

- Motivation and Objectives
- Low-Boom Simulations

➤ Methodology

- Overset Grid System
- Adaptation Equations
- Error Indicator Formulations

➤ Results

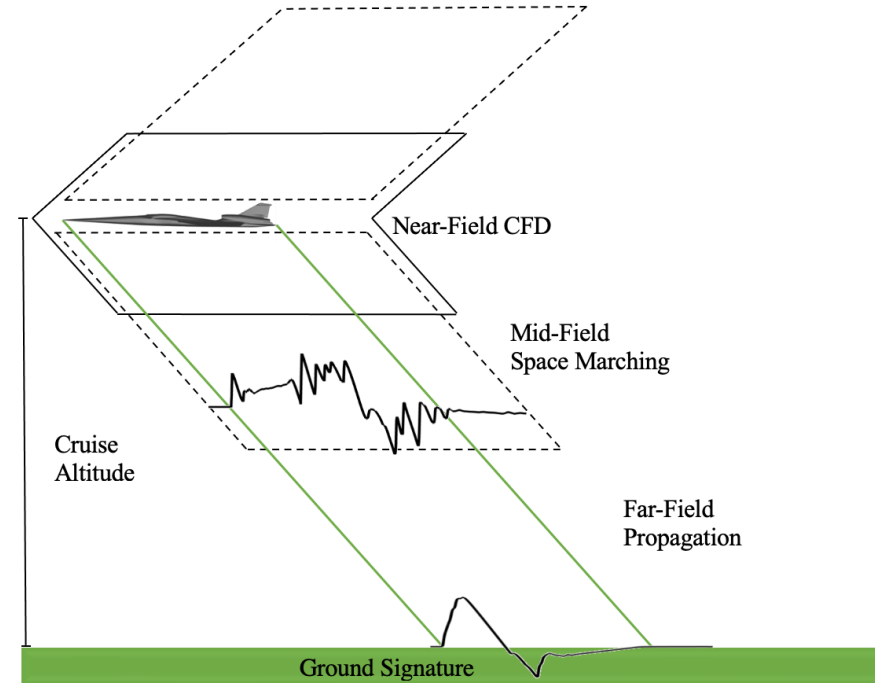
- X-59 C608 Demonstrator Model

➤ Summary

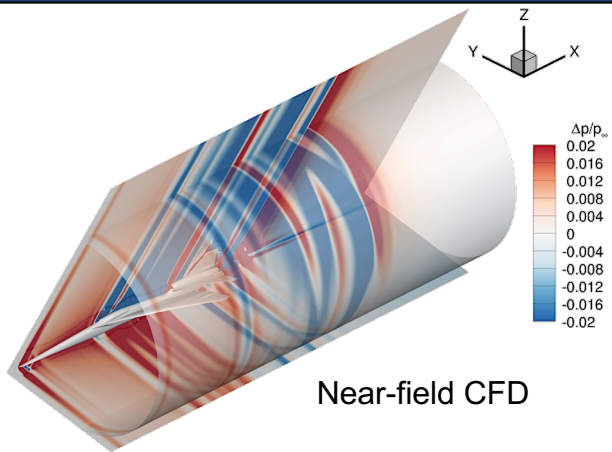
Motivation



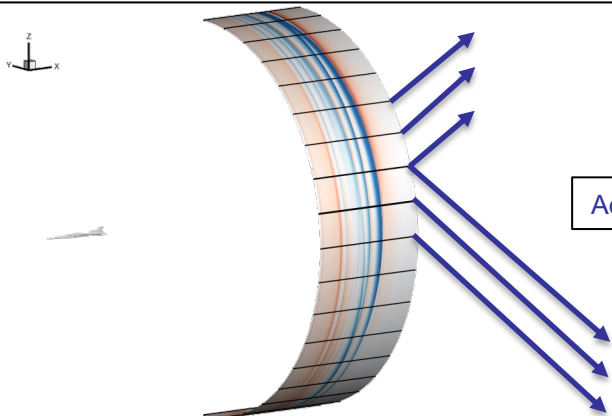
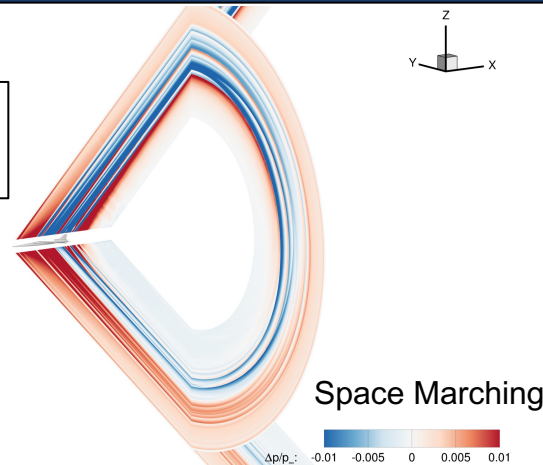
- Evaluation of low boom aircraft designs rely on accurate predictions of ground-level noise
- The Launch, Ascent, and Vehicle Aerodynamics (LAVA) framework uses a three-step prediction procedure
- Near-field CFD simulations demand the majority of the required computational resources



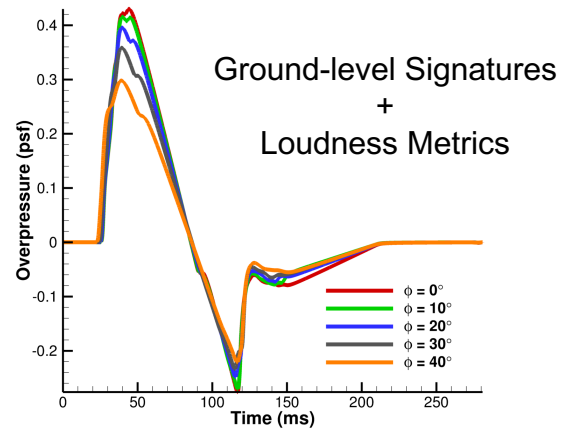
Low Boom Simulation Procedure



Interpolation
+
Space Marching



Pressure Cylinder Extraction



Objectives

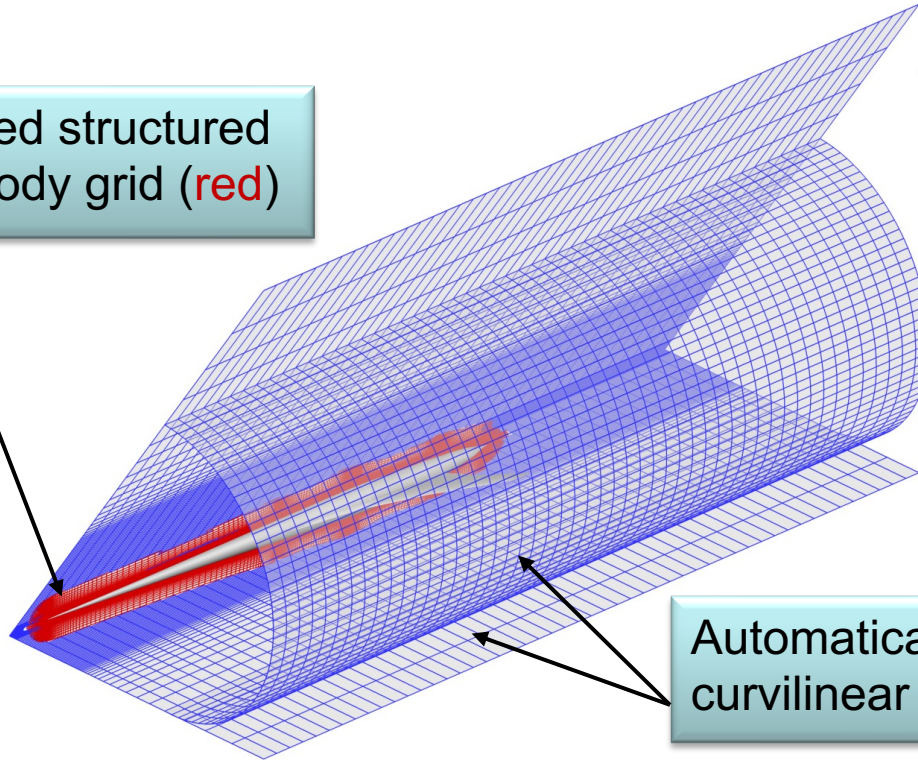
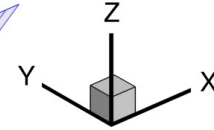


1. Automate the off-body grid generation procedure for near-field simulations
 - Increase robustness to flight plan changes
2. Generate improved CFD grid system with fixed degrees of freedom
 - Reduce resource requirements for achievable accuracy levels
3. Construct error indicators to facilitate rapid anisotropic mesh adaptation
 - Capture error for non-symmetric complex supersonic flows

Methodology: Overset Grid System

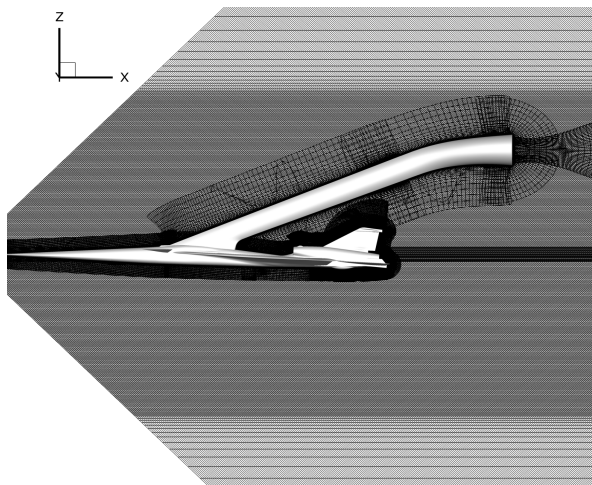
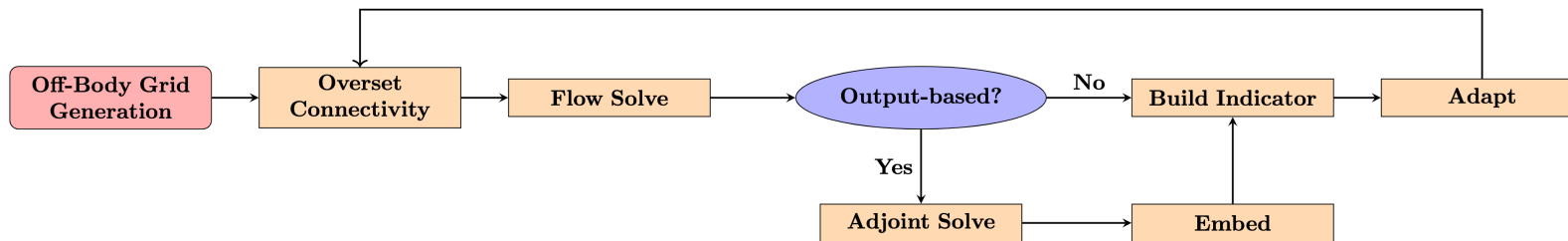


Manually generated structured curvilinear near-body grid (red)

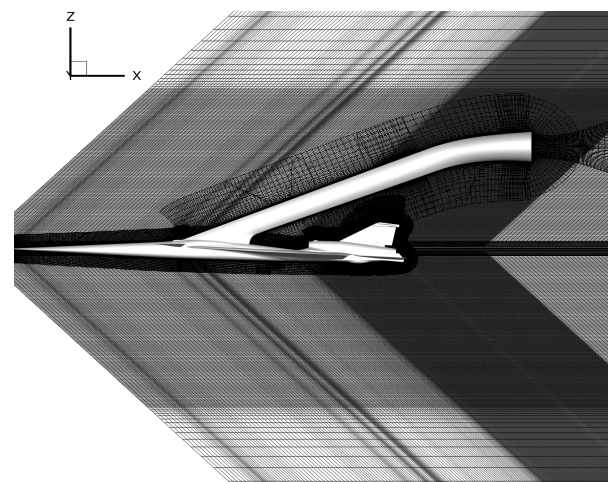


Automatically generated structured curvilinear off-body grid (blue)

Methodology



Initial Off-body



Adapted Off-body

Methodology: Adaptation Equations



- Direction-based mesh redistribution is governed by the equation

$$w_i \underbrace{\Delta s_i}_{\text{cell-size}} = C$$

Discrete equidistribution principle

- Adaptation weights control the location of local refinement

$$w_i = 1 + A \underbrace{f_i}_{\text{error indicator}}$$

- Clustering constant limits the maximum and minimum cell-size

$$A = \underbrace{c_f r_f}_{\text{coarsening and refinement factors}} \frac{\Delta s_{\max}^0}{\Delta s_{\min}^0} - 1$$

Methodology: Error Indicators



- The construction of direction-based indicators relies on the initial computation of standard error indicators
- **Feature-based** indicators target errors in a specific flow quantity
 - Mach, vorticity, pressure disturbance, etc.
- **Output-based** indicators target errors in a single or multiple scalar output quantities of interest
 - Lift, drag, pitching moment, line integral of pressure, etc.

Methodology: Error Indicators



- Feature-based indicators are formed from the second undivided difference

$$e_i = |q_{i+1} - 2q_i + q_{i-1}|$$

- This is proportional to the remaining error term of linear interpolation across cell i

$$e_i \propto \frac{(\Delta\xi)^2}{8} \max_{\xi_i \leq \xi \leq \xi_{i+1}} |q''(\xi)|$$

- Normalize by a reference quantity and compute the maximum over each computational space direction

$$e(\xi_j, \eta_k, \zeta_l) = \max_{i=j,k,l} \frac{|q_{i+1} - 2q_i + q_{i-1}|}{q_{\text{ref}}}$$

Methodology: Error Indicators



- Near-field CFD governed by RANS in strong conservative form

$$\mathbf{R}(\mathbf{Q}) = 0 \text{ in } \Omega$$

- Output-based indicators target the coarse grid functional discretization error

$$\varepsilon = |J(\mathbf{Q}) - J_H(\mathbf{Q}_H)|$$



- Discrete adjoint derivation gives the estimate

$$J(\mathbf{Q}) - J_H(\mathbf{Q}_H) \approx \frac{r_f^p}{r_f^p - 1} (J_h(\mathbf{Q}_h^{LO}) - J_H(\mathbf{Q}_H)) - \frac{r_f^p}{r_f^p - 1} \left[(\boldsymbol{\psi}_h^{LO})^T \mathbf{R}_h(\mathbf{Q}_h^{LO}) + (\boldsymbol{\psi}_h^{HI} - \boldsymbol{\psi}_h^{LO})^T \mathbf{R}_h(\mathbf{Q}_h^{LO}) \right]$$

- High- and low-order prolongation operators are defined

$$\begin{aligned}\mathbf{Q}_h^{LO} &= \mathbf{P}_{LO} \mathbf{Q}_H \\ \boldsymbol{\psi}_h^{LO} &= \mathbf{P}_{LO} \boldsymbol{\psi}_H\end{aligned}$$

Methodology: Error Indicators

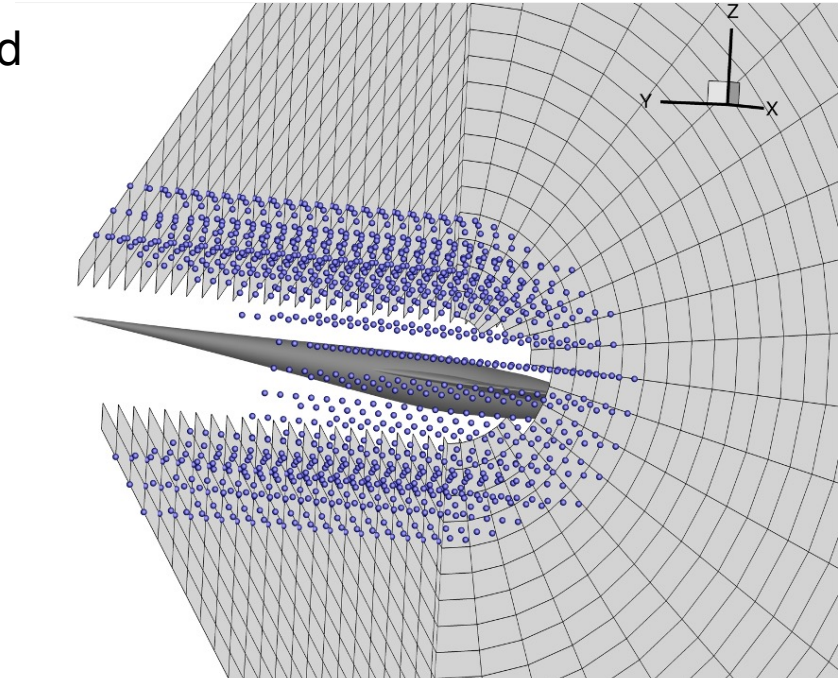


- Output-based indicator on the embedded grid

$$e_h = (\psi_h^{\text{HI}} - \psi_h^{\text{LO}})^T \mathbf{R}_h(\mathbf{Q}_h^{\text{LO}})$$

- Output indicator is defined

$$J(\mathbf{Q}) := \sum_{i=1}^{N_{\text{fringe}}} \left(\frac{\Delta p_i}{p_\infty} \right)^2$$



Space Marching Fringe Points

Methodology: Error Indicators



- For a fixed radial surface ζ^* a streamwise indicator is computed

$$g(\xi) = \max_{\eta} |e_0(\xi, \eta, \zeta^*)|$$

- Apply a low number of elliptic smoothing iterations

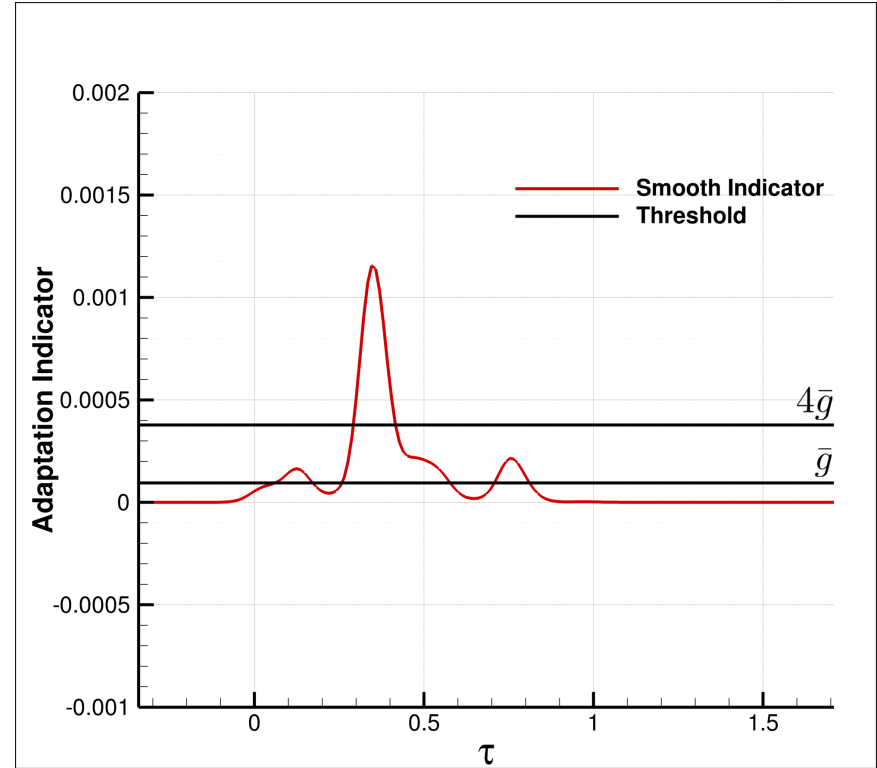
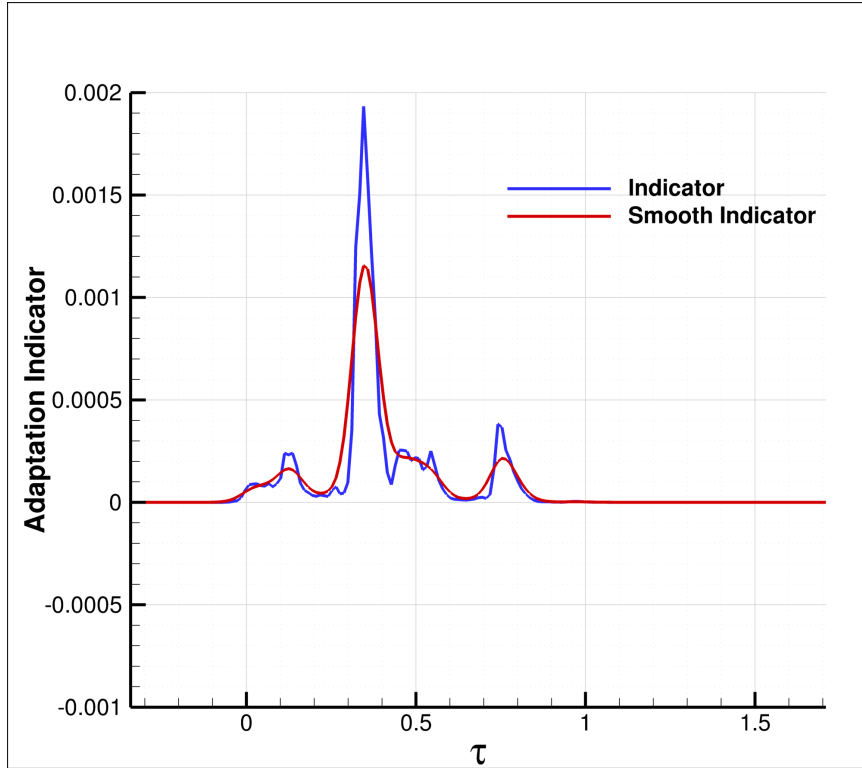
$$g_i^s = \frac{g_{i-1} + 4g_i + g_{i+1}}{6}$$

- Smoothed indicator is mapped between $[0, 1)$

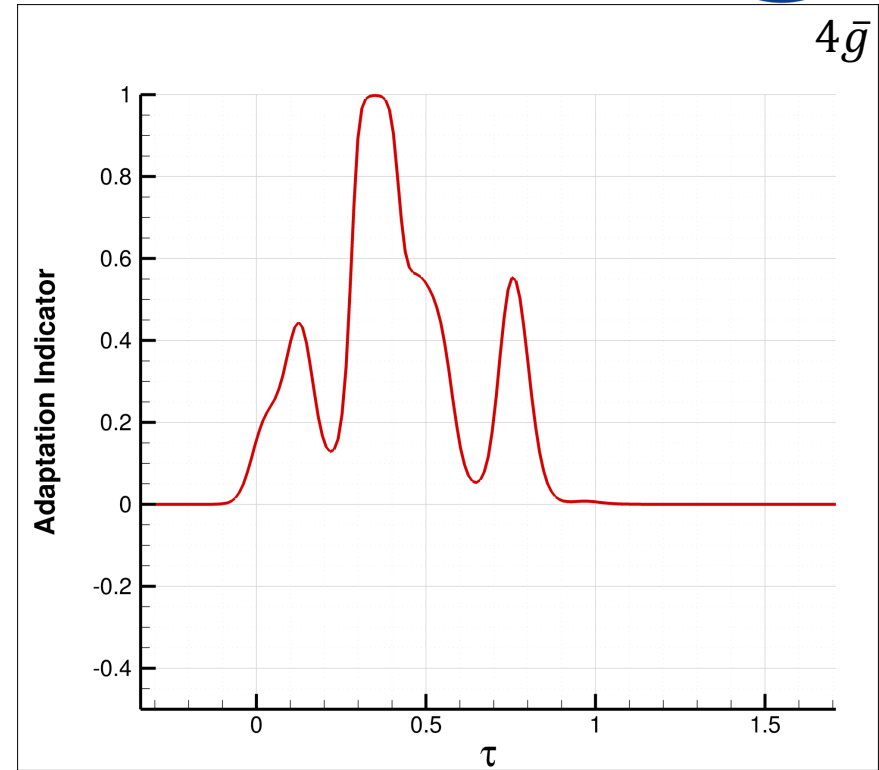
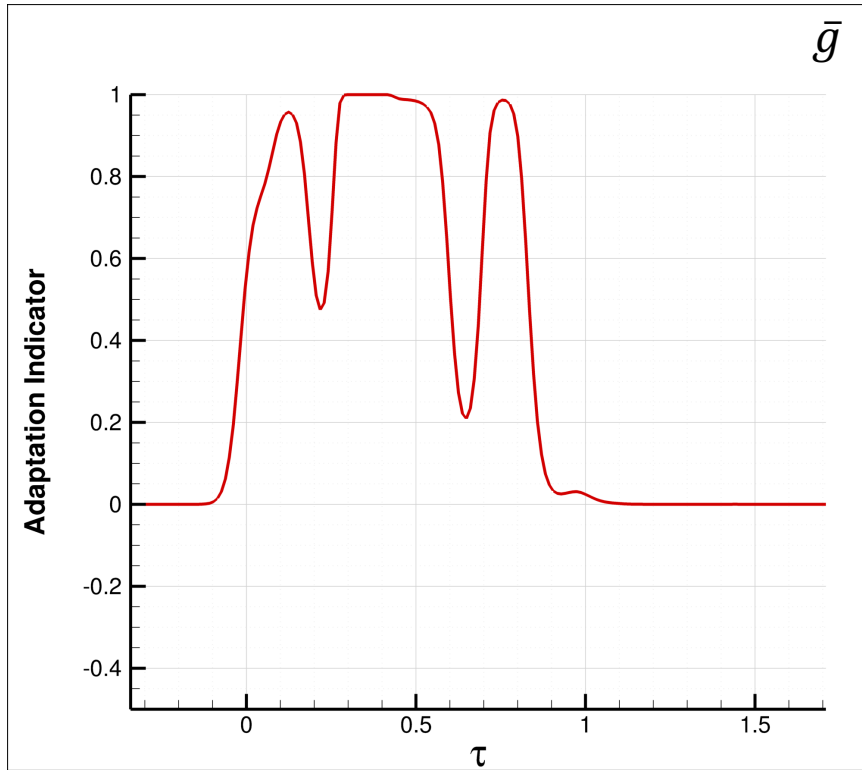
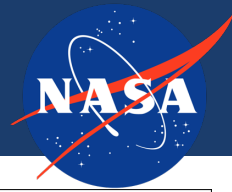
$$f(\xi, \eta; \zeta^*) = \tanh(\beta g^s(\xi)) \in [0, 1)$$

$$\beta = \tanh^{-1}(0.8)/(C \bar{g})$$

Methodology: Error Indicators



Methodology: Error Indicators

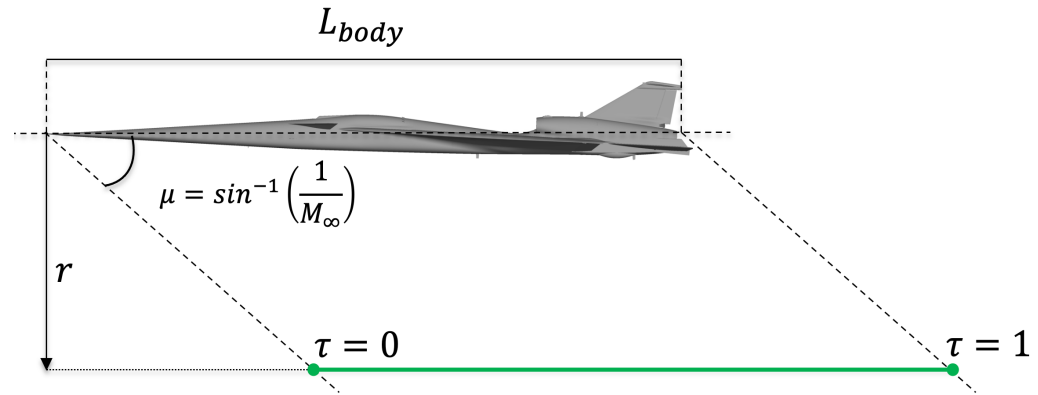


Methodology: Error Indicators

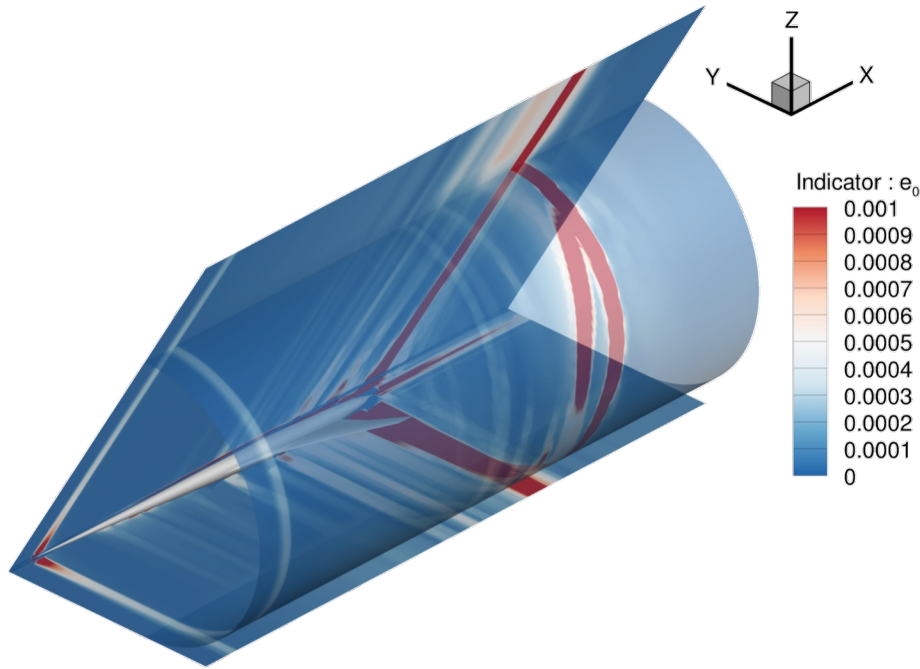


- Direction-based surface indicator is interpolated to off-body grid using a normalized distance metric

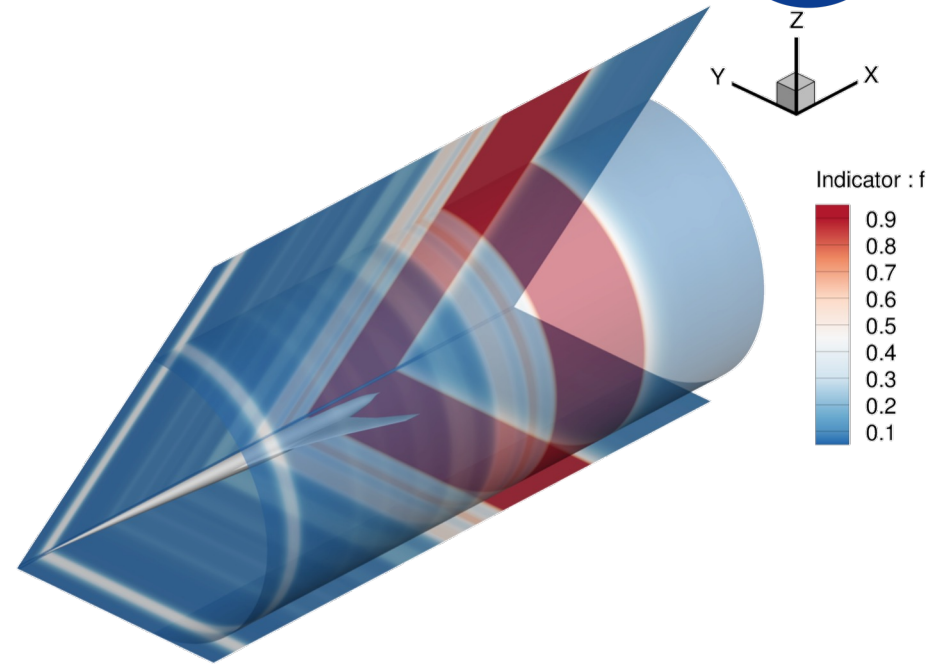
$$\tau = \frac{x - r \sqrt{M_\infty^2 - 1}}{L_{body}}$$



Methodology: Error Indicators



Initial Feature-Based Error Indicator



Final Direction-based Error Indicator

Methodology: Error Indicators



- Direction-based indicator construction procedure is generalizable

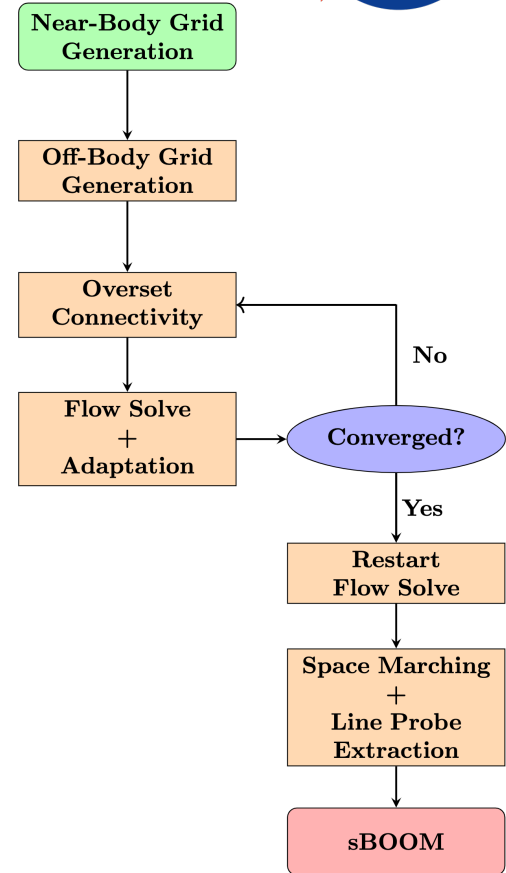
- Adaptation equations are solved along **ONE** coordinate line
 - Resulting coordinate displacements are used to update the rest of the off-body grid

- Sequential multi-directional adaptation
 - Studied combined circumferential and streamwise adaptation
 - No benefit of circumferential adaptation has been observed

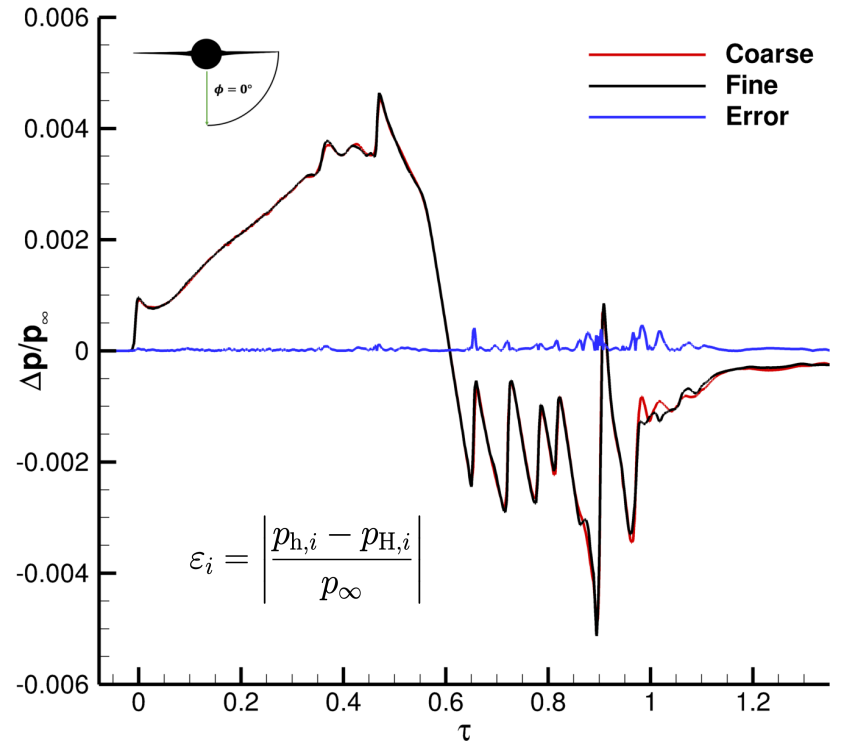
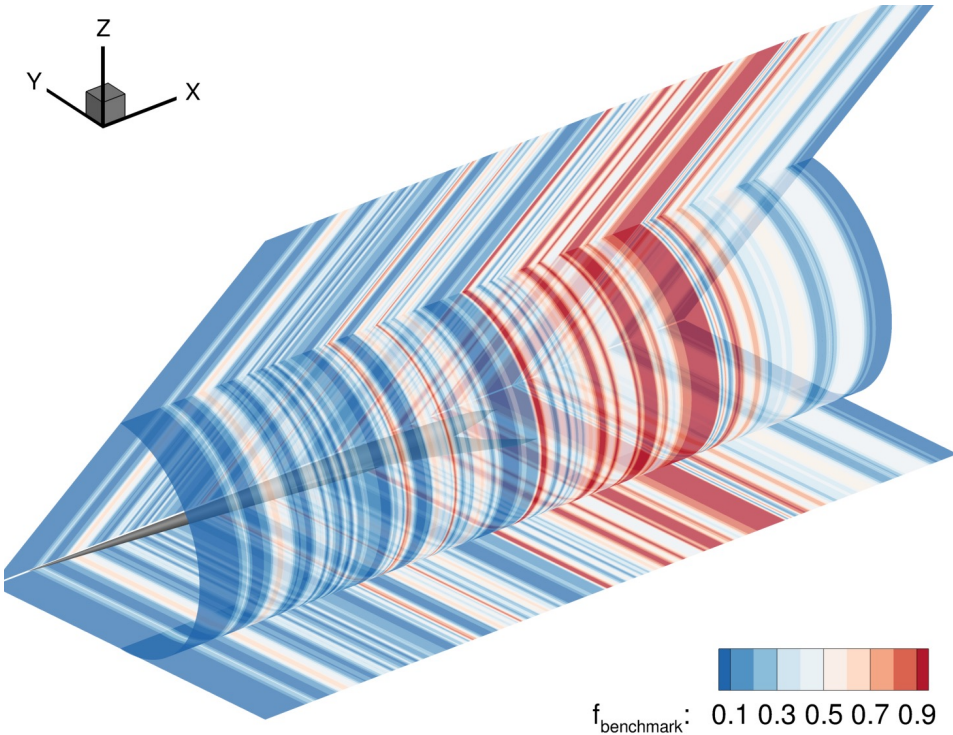
Results



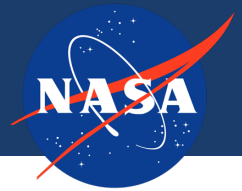
- Streamwise adapted off-body grids are compared to automatically generated uniform grids
 - Constant streamwise and circumferential spacing
- Near-body grids are held fixed
 - Grid refinement studies performed during AIAA SBPW
- Comparison of off-body grid-levels for feature-based and output-based indicators
 - 1st order adjoint solver used for presented results



Results: Benchmark



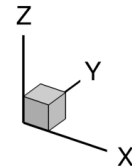
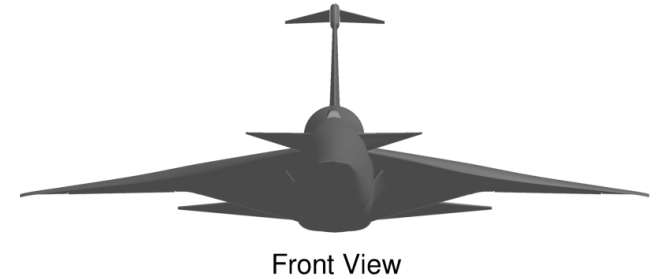
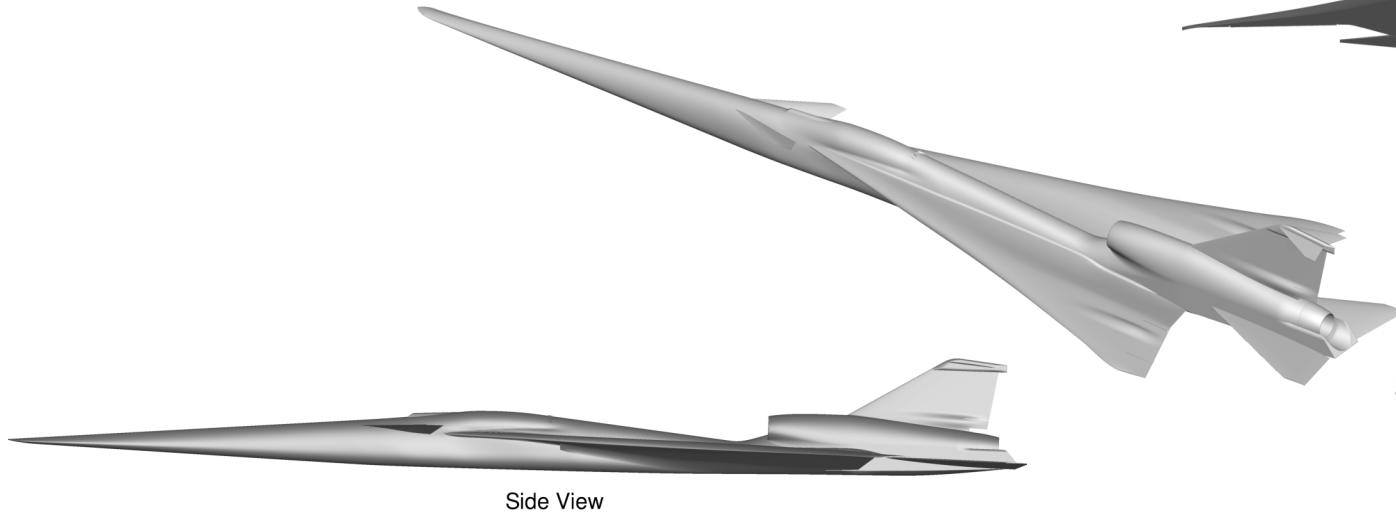
Results: X-59 C608



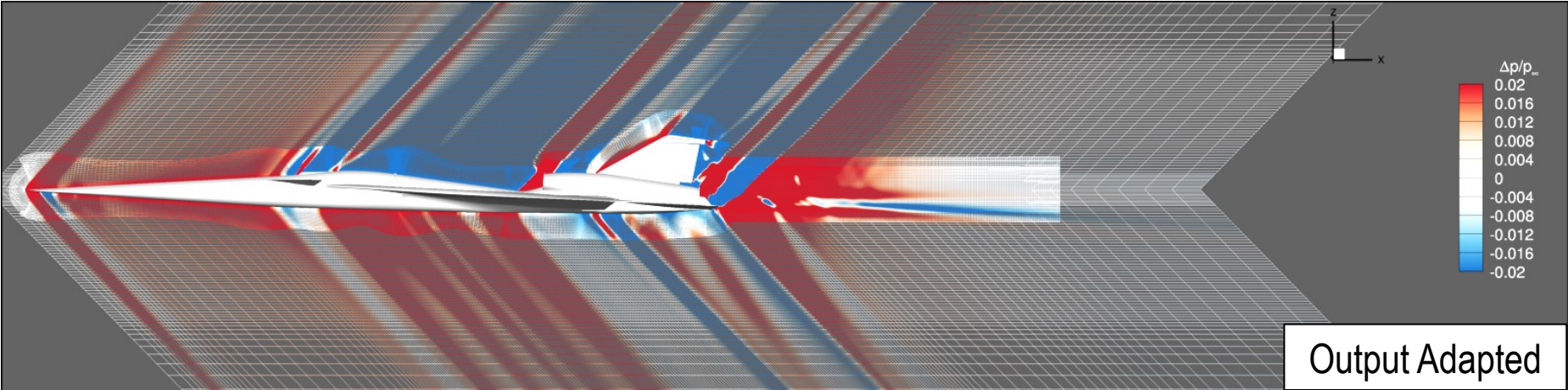
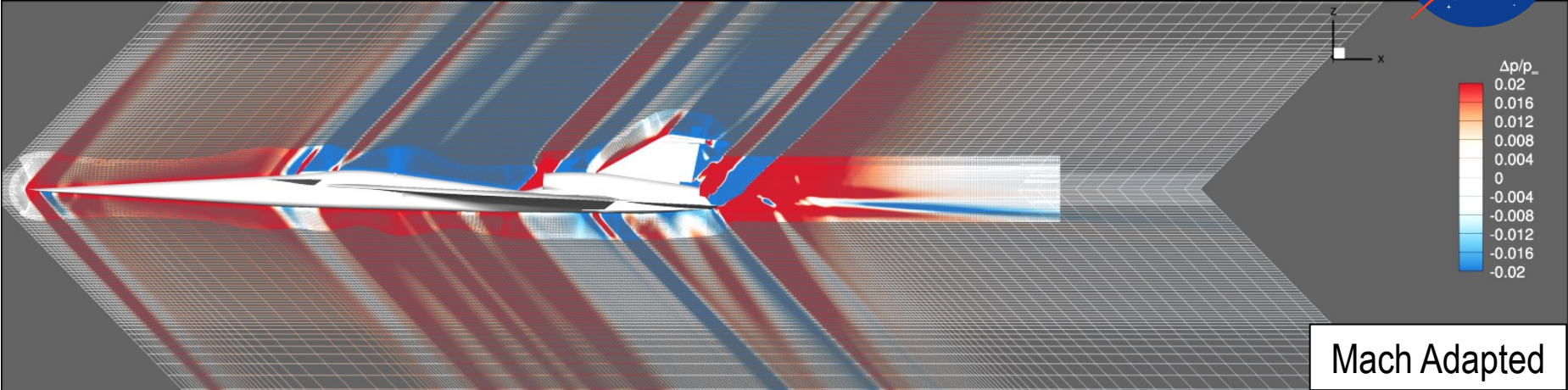
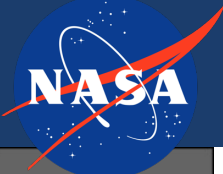
Reference length : $L_{ref} = 27.432 \text{ m}$

Cruise Altitude : 16,215 m

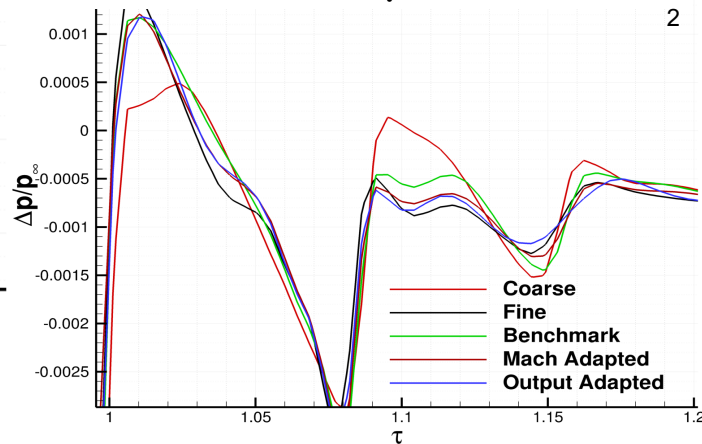
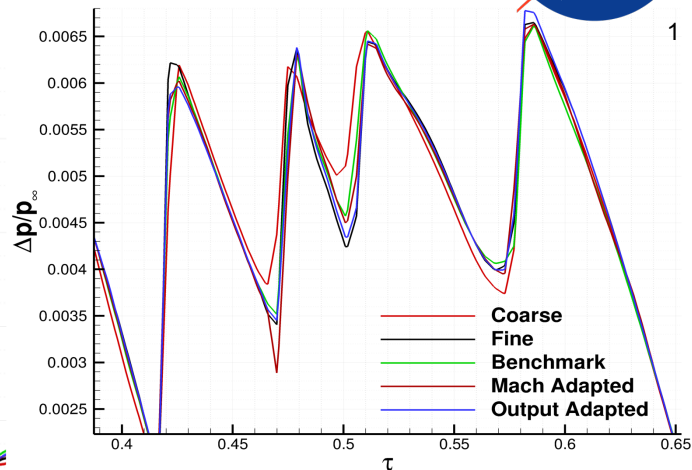
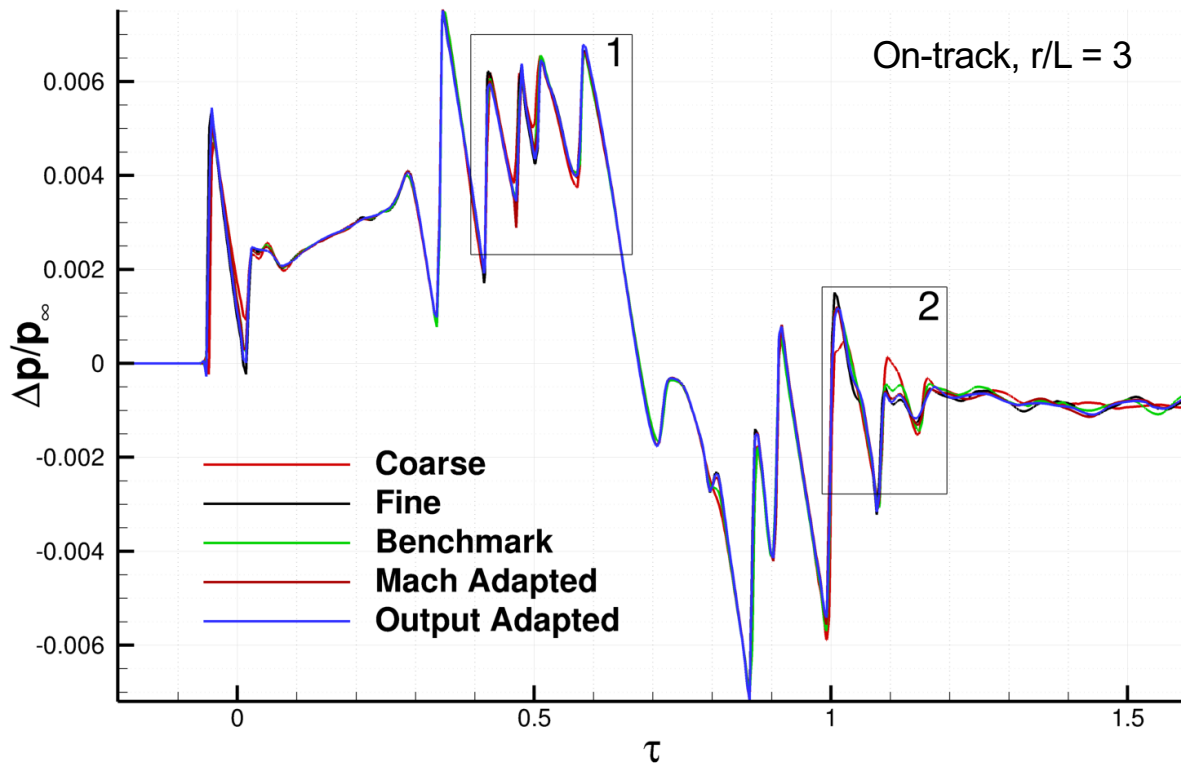
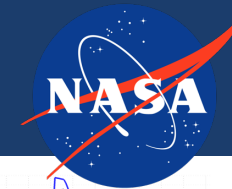
Mach = 1.4, $Re/m = 4.3$ million, and $\alpha = 2.15^\circ$



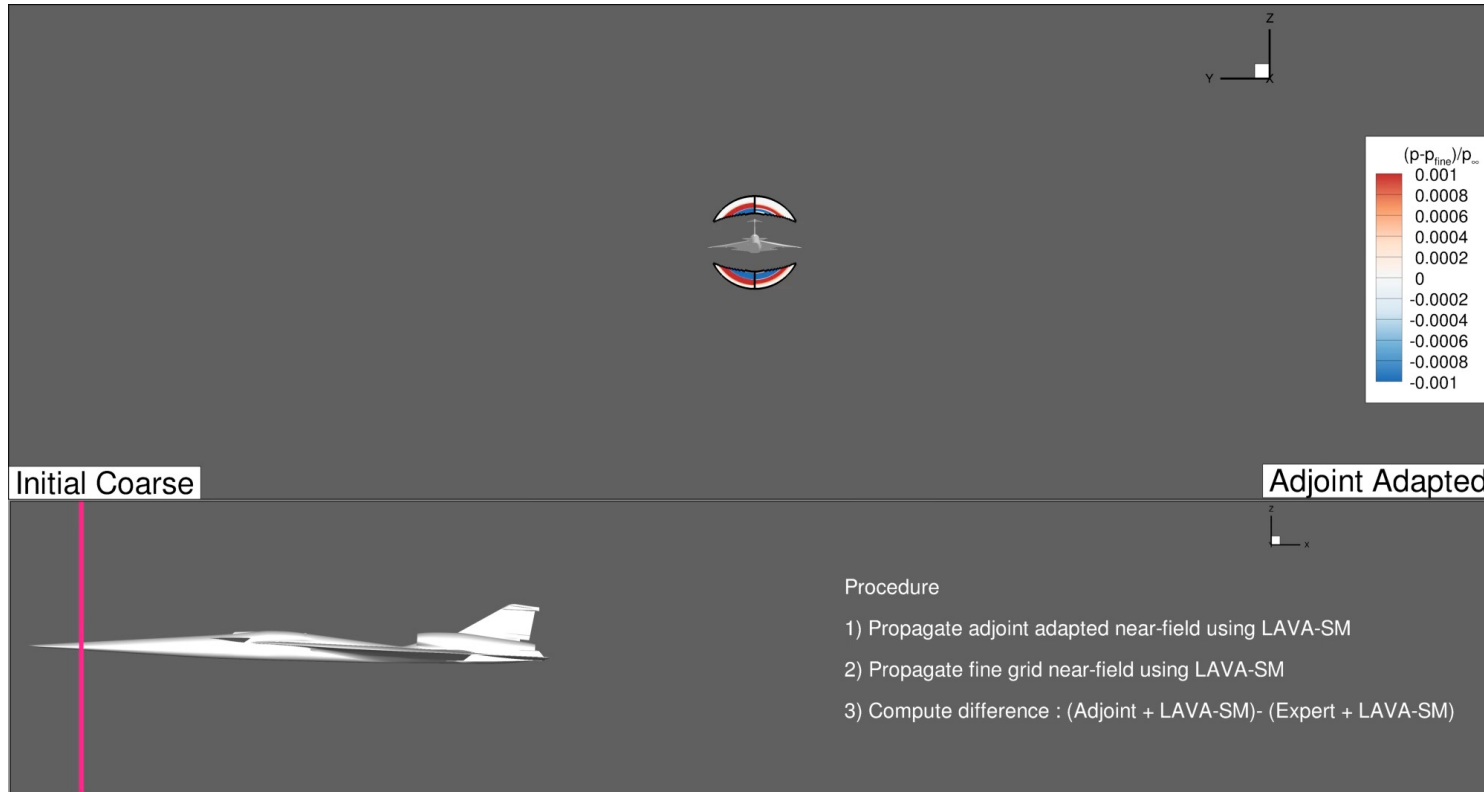
Results: X-59 C608



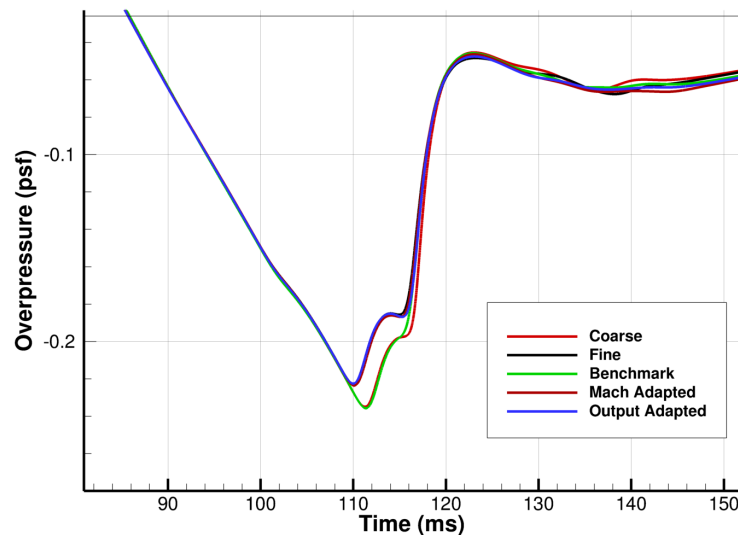
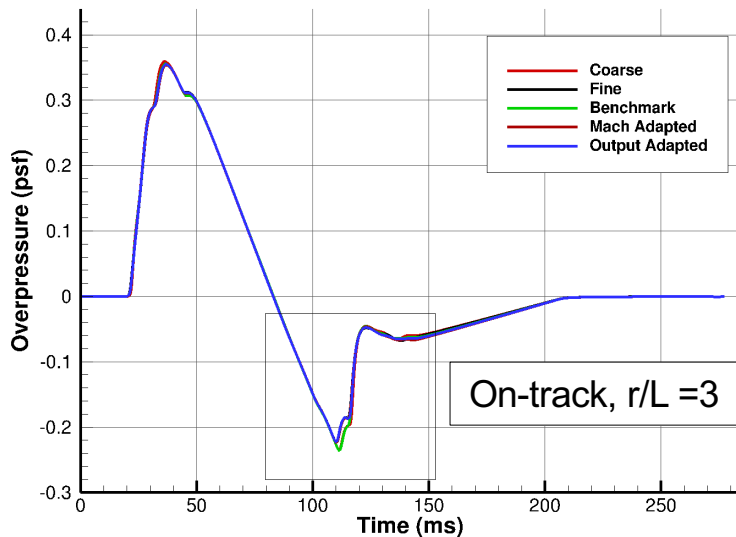
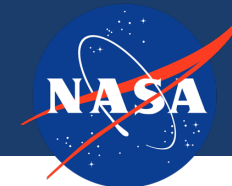
Near-field Accuracy Improvement



Near-Field Propagation Difference



Ground Level Accuracy Improvement



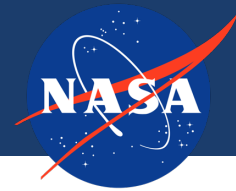
Grid	ASEL [dB(A)]	BSEL [dB]	CSEL [dB(C)]	PL [dB]	ASEL Rel. Error	BSEL Rel. Error	CSEL Rel. Error	PL Rel. Error
Fine	62.21	76.49	91.27	76.53				
Coarse	63.40	77.78	91.72	77.80	1.917%	1.692%	0.497%	1.660%
Benchmark	62.80	77.22	91.53	77.41	0.946%	0.952%	0.285%	1.149%
Mach Adapted	62.63	76.73	91.29	76.94	0.664%	0.312%	0.025%	0.544%
Output Adapted	62.11	76.53	91.30	76.45	0.160%	0.056%	0.032%	0.103%

Cost Analysis



- Off-body grids may not account for the majority of the the total degrees of freedom
 - C608 off-body grid accounts for only 23.59% of total coarse grid nodes
- Off-body grid adaptation procedure cost is compared to LAVA's baseline best practice grid from the 3rd AIAA Sonic Boom Prediction Workshop
- Reference grid has manually generated off-body refinement regions

Cost Analysis



Grid	Total Nodes*	Degrees of Freedom*	Nodes per Core
Reference	161.990	127.379	318,448
Fine	36.329	23.613	295,166
Coarse	29.754	20.874	260,925

*In millions.

***3.7x reduction in
computational resources***

Grid	Connectivity	Primal Solve	Adjoint Solve	Embedding	Adaptation	Restart	Total	SBU
Reference	-	1.4	-	-	-	-	1.4	18.3
Coarse	0.02	1.59	-	-	-	-	1.77	4.6
Fine	0.02	2.60	-	-	-	-	2.62	6.7
Benchmark Adapted	0.04	1.59	-	-	-	0.14	1.77	4.6
Mach Adapted	0.04	1.59	-	-	-	0.14	1.77	4.6
Output Adapted	0.04	1.59	0.10	0.01	0.01	0.14	1.89	4.9

Cost Analysis

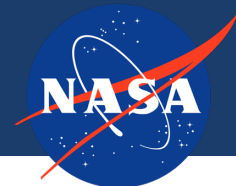


Grid	ASEL [dB(A)]	BSEL [dB]	CSEL [dB(C)]	PL [dB]	Δ ASEL	Δ BSEL	Δ CSEL	Δ PL
Reference	61.59	76.73	91.20	76.81	-	-	-	-
Fine	62.21	76.49	91.27	76.53	1.017%	0.309%	0.076%	0.368%
Coarse	63.40	77.78	91.72	77.80	2.953%	1.378%	0.574%	1.286%
Benchmark	62.80	77.22	91.53	77.41	1.972%	0.640%	0.361%	0.777%
Mach Adapted	62.63	76.73	91.29	76.94	1.687%	0.002%	0.101%	0.174%
Output Adapted	62.11	76.53	91.30	76.45	0.855%	0.254%	0.108%	0.470%

On-track loudness metrics propagated from r/L = 3

Grid	ASEL [dB(A)]	BSEL [dB]	CSEL [dB(C)]	PL [dB]	Δ ASEL	Δ BSEL	Δ CSEL	Δ PL
Reference	59.09	75.64	89.98	74.42	-	-	-	-
Fine	60.14	75.51	89.90	74.51	1.782%	0.186%	0.086%	0.122%
Coarse	61.80	76.34	90.29	76.24	4.584%	0.928%	0.346%	2.447%
Benchmark	60.91	76.18	90.22	75.58	3.092%	0.718%	0.277%	1.570%
Mach Adapted	60.85	75.90	89.99	75.25	2.982%	0.341%	0.017%	1.117%
Output Adapted	60.73	75.77	89.95	75.09	2.772%	0.173%	0.029%	0.906%

Off-track loudness metrics propagated from r/L = 3

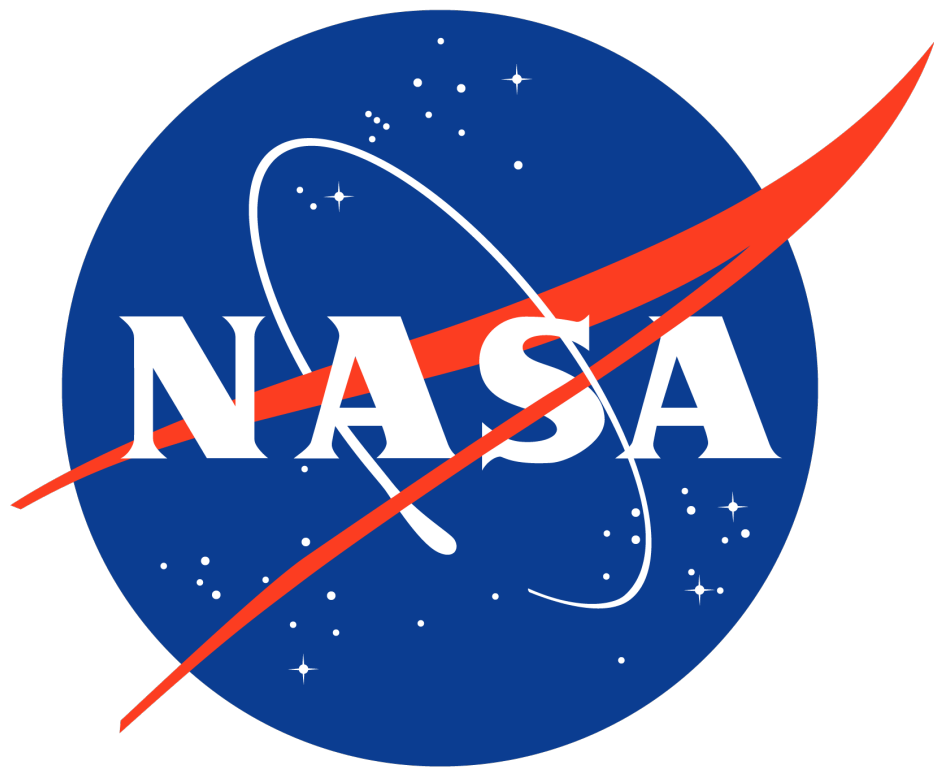


- Introduced a new anisotropic mesh adaptation algorithm for structured curvilinear Mach cone aligned grids
 - Eliminates need for manual off-body grid generation
- Novel direction-based error indicators
 - Enables error capturing for complex supersonic flows
 - Allows for one small nonlinear system solve per adaptation direction
- Adaptation methodology enables factor of two reduction in computational resources for the same level of sonic boom prediction accuracy

Acknowledgments



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- Gratitude is expressed to Brandon Lowe for the development of the LAVA adjoint solver.
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- Special thanks are due to Marian Nemec from NASA Ames Research Center for his technical guidance.
- A special thank you to Gaetan Kenway for his supervision of the code development and Cetin Kiris for his exceptional leadership and technical guidance.
- Results in this presentation are from the doctoral dissertation of the first author to be submitted in partial fulfillment of the requirements for a PhD at the University of Kentucky. Special thanks to Peter Hislop and Christoph Brehm for their academic advisory roles.





- A benchmark indicator is formed by propagating uniform coarse and fine CFD solutions to the mid-field
- Pointwise error is computed along the propagated waveform

$$\varepsilon_i = \left| \frac{p_{h,i} - p_{H,i}}{p_\infty} \right|$$

- Mapped to the off-body CFD grid using normalized distance metric

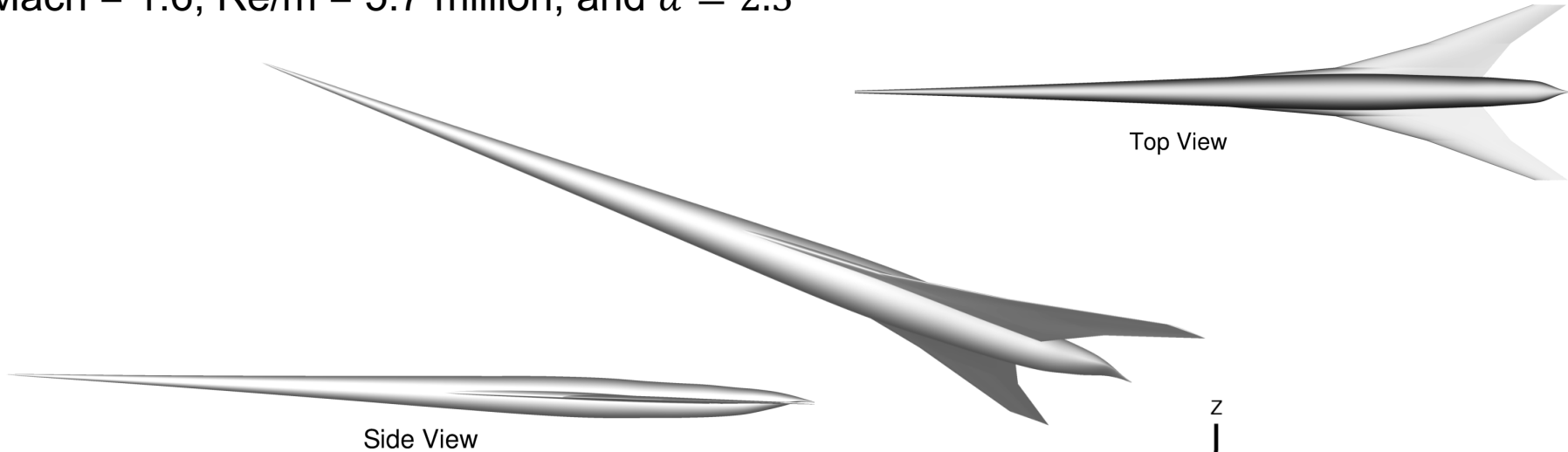
Results: JAXA Wing Body



Reference length : $L_{ref} = 38.7 \text{ m}$

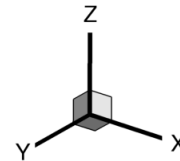
Cruise Altitude : 15,276 m

Mach = 1.6, $Re/m = 5.7$ million, and $\alpha = 2.3^\circ$

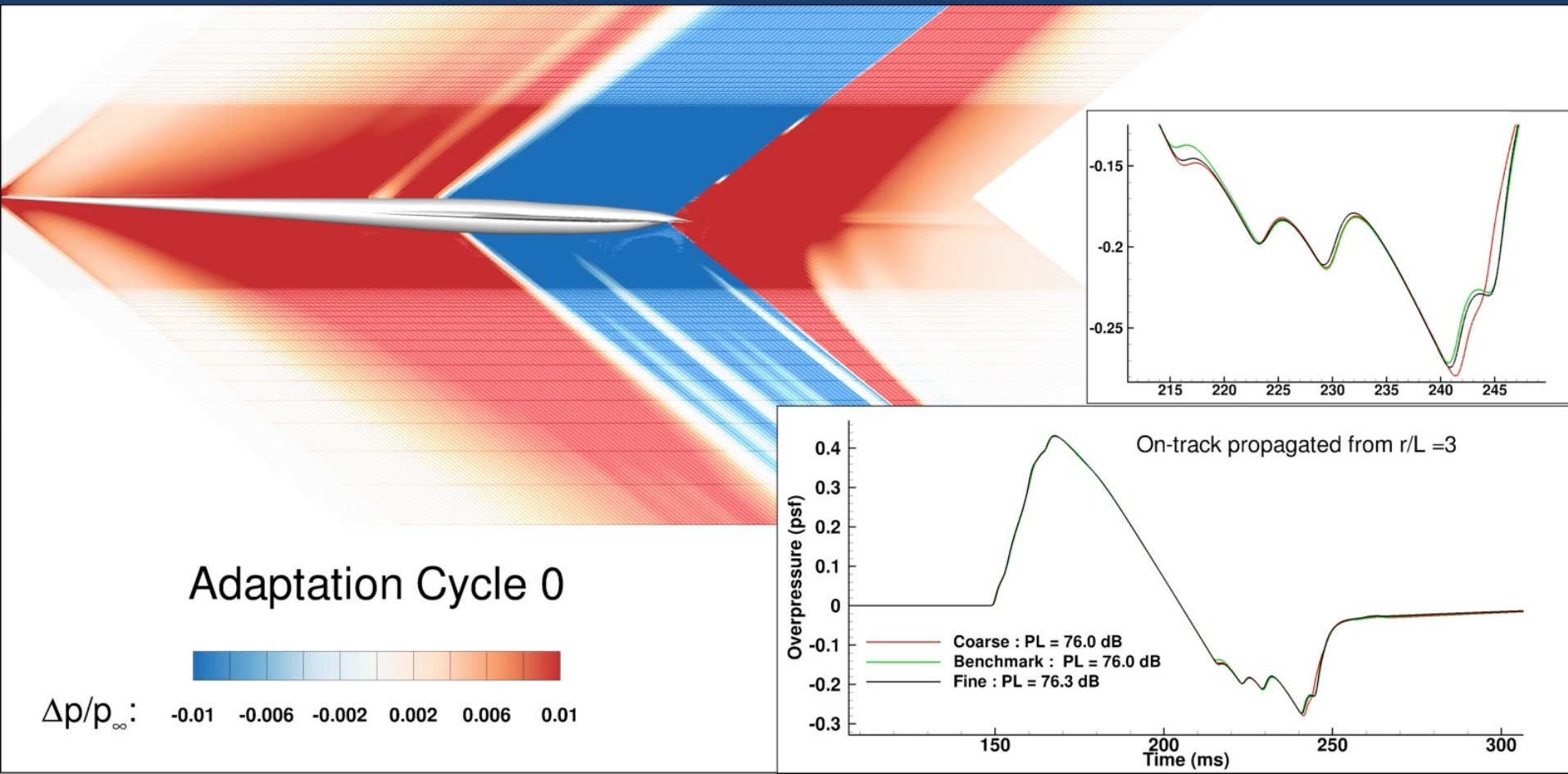


Side View

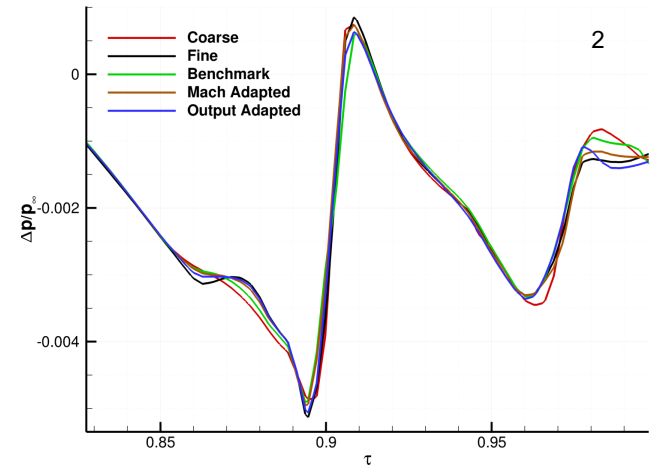
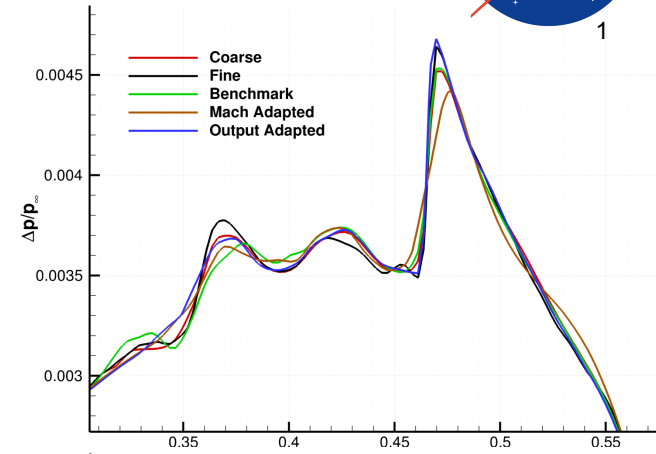
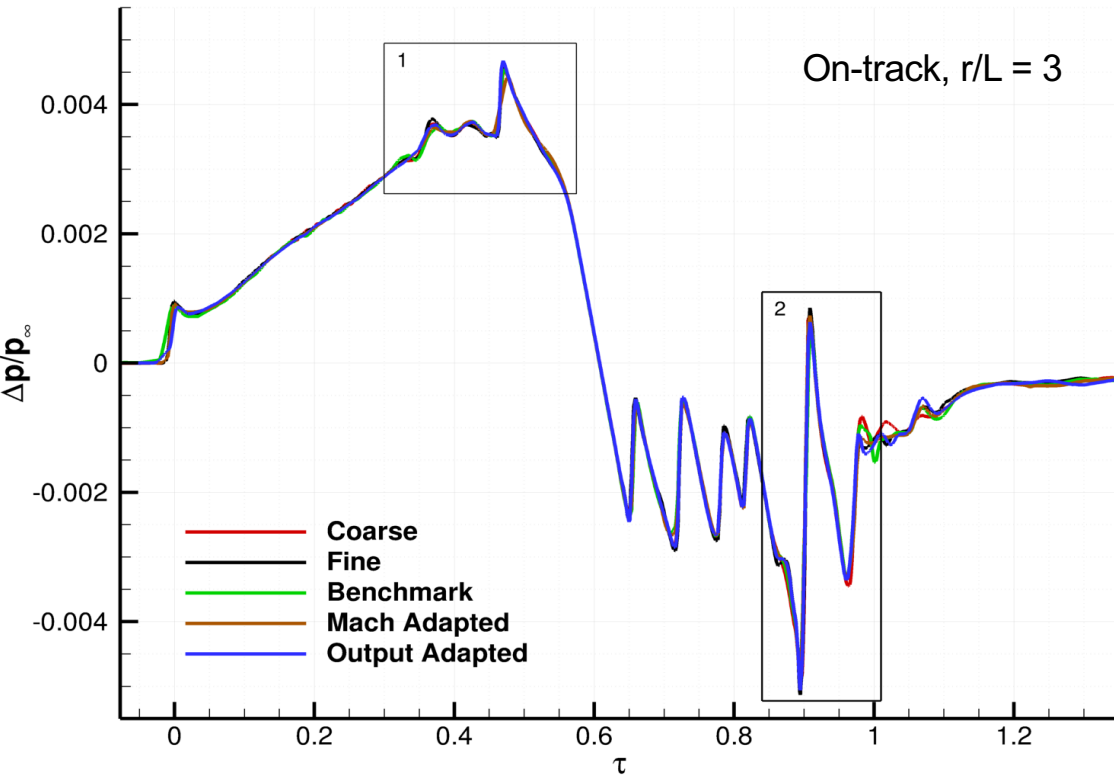
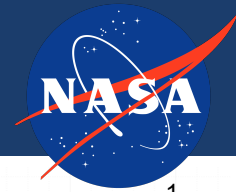
Top View



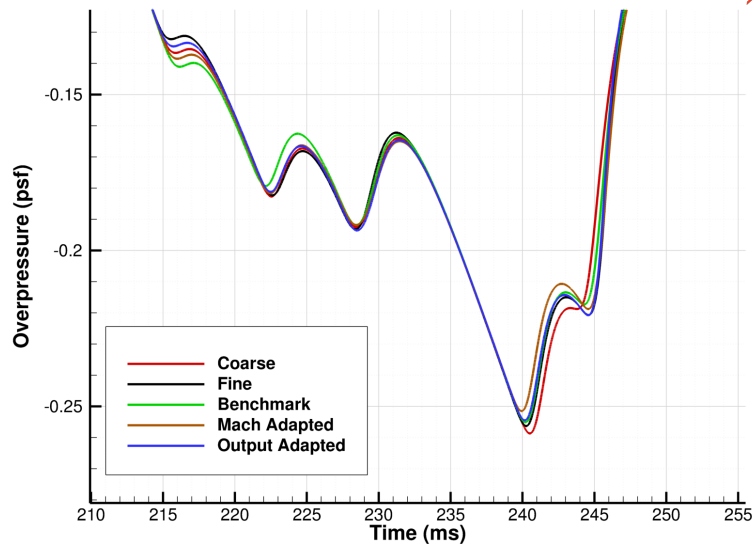
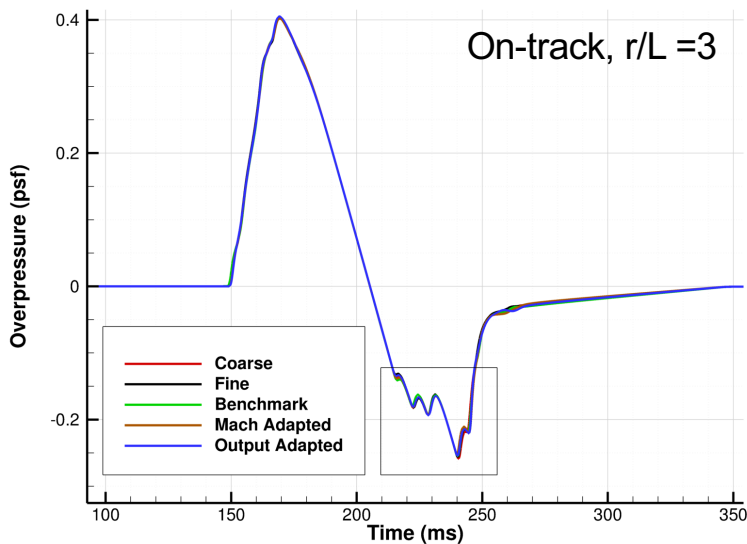
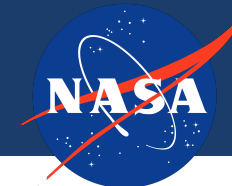
Adjoint Grid Adaptation: Ground-Level Signatures



Near-field Accuracy Improvement

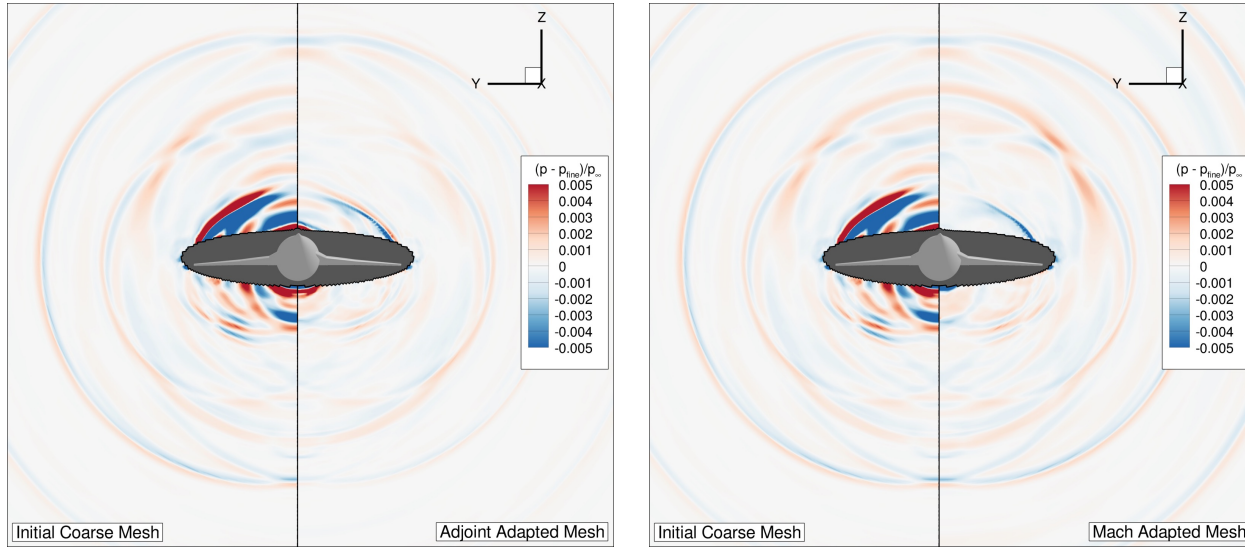


Ground Level Accuracy Improvement



Grid	ASEL [dB(A)]	BSEL [dB]	CSEL [dB(C)]	PL [dB]	ASEL Rel. Error	BSEL Rel. Error	CSEL Rel. Error	PL Rel. Error
Fine	62.15	75.96	90.78	75.98	-	-	-	-
Coarse	60.86	75.43	90.71	74.83	2.064%	0.692%	0.072%	1.514%
Benchmark	61.51	75.45	90.66	75.18	1.026%	0.673%	0.131%	1.045%
Mach Adapted	61.73	75.61	90.66	75.45	0.661%	0.457%	0.134%	0.699%
Output Adapted	62.37	75.86	90.74	76.10	0.359%	0.132%	0.048%	0.161%

Cost Analysis



Grid	Connectivity	Primal Solve	Adjoint Solve	Embedding	Adaptation	Restart	Total
Coarse	0.14	25.46	-	-	-	-	25.60
Benchmark Adapted	0.28	25.46	-	-	0.02	4.70	30.46
Mach Adapted	0.29	25.46	-	-	0.01	5.66	31.42
Output Adapted	0.28	25.46	0.87	0.17	0.03	5.03	31.56
Fine	0.13	55.99	-	-	-	-	56.12

Resource usage in CPU-hours for each JWB simulation performed on NASA's Electra supercomputer using 5 Skylake nodes each containing two 20-core Xeon Gold 6148 sockets (2.4 GHz).