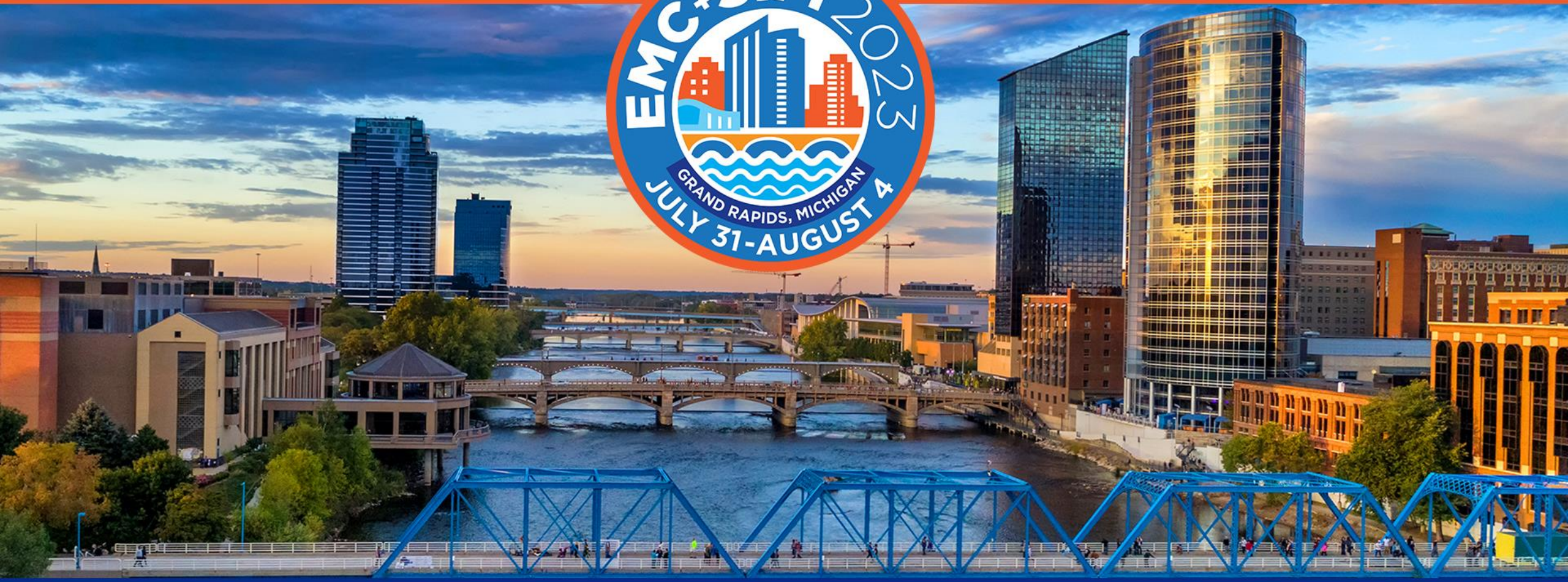
2023 IEEE INTERNATIONAL SYMPOSIUM ON ELECTROMAGNETIC COMPATIBILITY, SIGNAL & POWER INTEGRITY

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The Yin/Yang Relationship Between Conducted and Radiated Coupling

John McCloskey, NASA Goddard Space Flight Center

John McCloskey

John McCloskey is the Chief EMC Engineer at NASA's Goddard Space Flight Center in Greenbelt, Maryland, where he has worked since May 1988

John provides EMC support to many space flight projects, including the James Webb Space Telescope (JWST) which launched on Christmas Day of 2021

John really enjoys teaching and training, and he has provided many EMC training and education opportunities for the Goddard workforce

Contact: John.C.McCloskey@nasa.gov





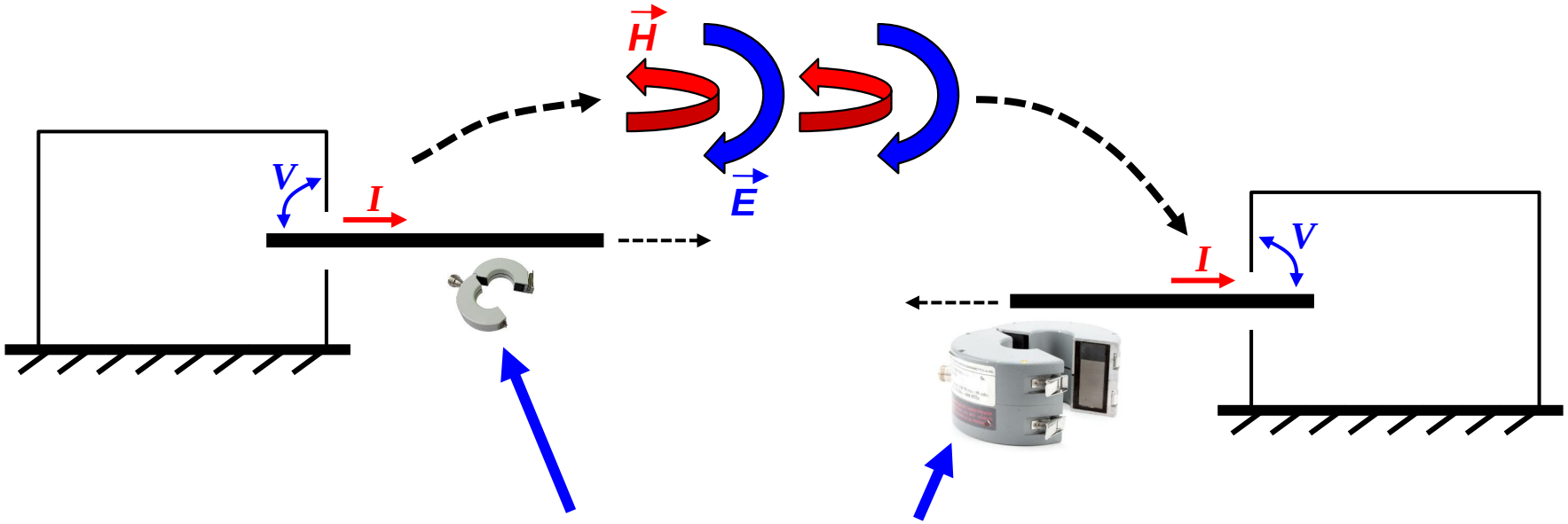
Introduction

- One of the biggest challenges an EMC engineer faces on a nearly daily basis is the need to convince electronics developers that EMI actually exists as something to which they should pay attention
- To an electronics designer who is accustomed to thinking in terms of voltages and currents, the concept of radiated energy is a mere abstraction and a distraction from their work. (“I can’t see it, I can’t smell it, I can’t feel it, so why should I care?”)
- Purposes of this presentation:
  - Provide basis for understanding “yin/yang” relationship between conducted & radiated coupling
  - Show how to correlate radiated energy to voltages and currents that electronics designers understand and provides tangible guidance that electronics designers can implement in order to better control radiated emissions and susceptibility
  - Provide tools for EMC engineers to explain these concepts to electronics designers to get them to finally “see it, smell it, feel it, taste it, etc.”
- ***NOTE: This presentation is most applicable to platforms with conductive structures that serve as metallic ground planes with interconnecting cables routed over them***

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Executive Summary

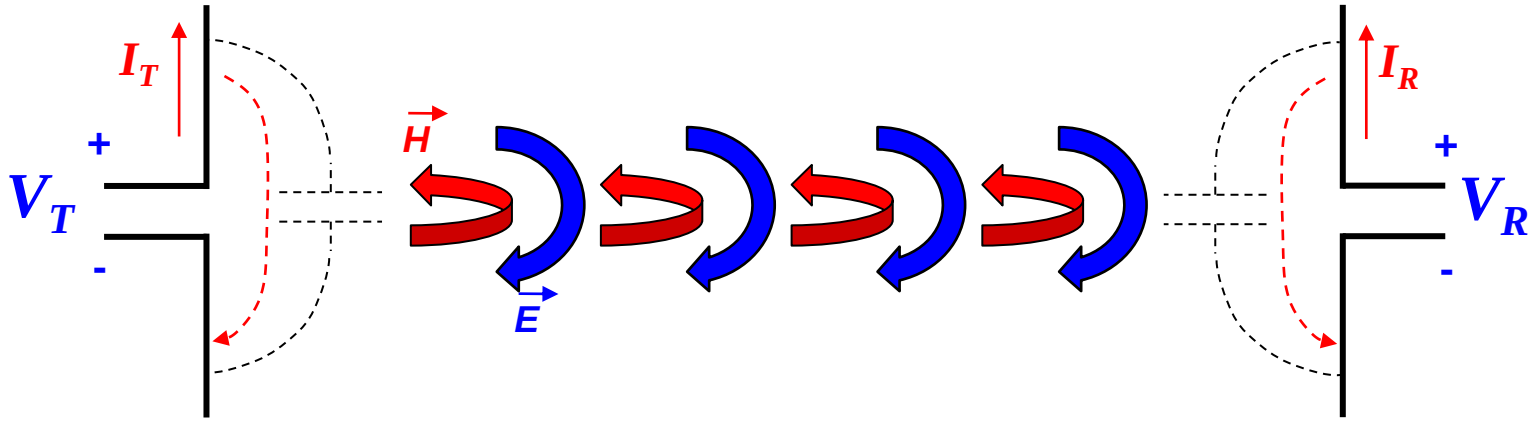
Voltages and currents in culprit equipment... → *...produce electric and magnetic fields...* → *...that produce voltages and currents in victim equipment*



- Current emissions and susceptibility measurements possible
- Provide indirect assessement of radiated emissions and susceptibility
- May be peformed in hardware development laboratory before bringing equipment to dedicated EMI test facility

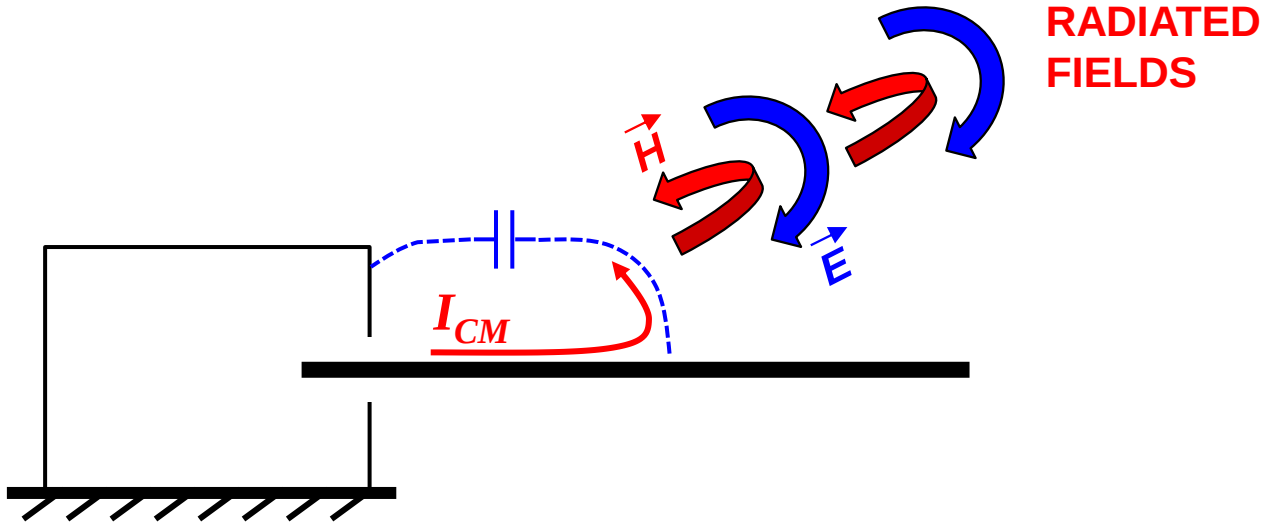
Dipole Antenna Basics

- Inspired by (the late, great) Henry Ott's "Dipoles for Dummies"
- Antenna is transducer between conducted and radiated energy
- Antenna is formed by establishing an RF potential between two conductors
- Capacitance between two conductors provides current path
- Dipole provides simple analytical model for upper bound of radiated efficiency of unintentional antenna



Unintentional Radiators, a.k.a. Antennas

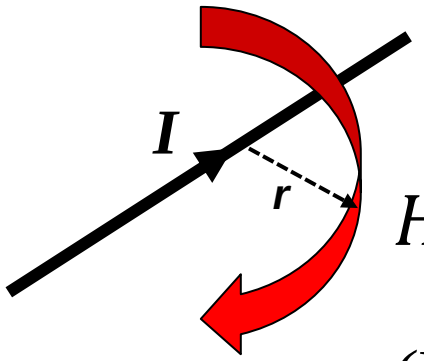
- If any part of your equipment radiates or is susceptible to radio frequency (RF) energy, you have created an antenna whether or not you call it one
- Unintentional antenna can be formed between any two conductors, e.g.:
  - Cable and chassis
  - Cable and ground plane (structure)
  - PC board trace and chassis
  - Poorly bonded connector and chassis (seams on chassis)
  - Etc.



Radiated Fields from Current-Carrying Wire (Bare Wire in Space - No Ground Plane Cancellation)

CURRENT PRODUCES MAGNETIC FIELD- AMPÈRE'S LAW

(accurate close to long wire)



$$H = \frac{I}{2\pi r}$$

$$(B = \mu_0 H)$$

B = magnetic flux density

H = magnetic field intensity

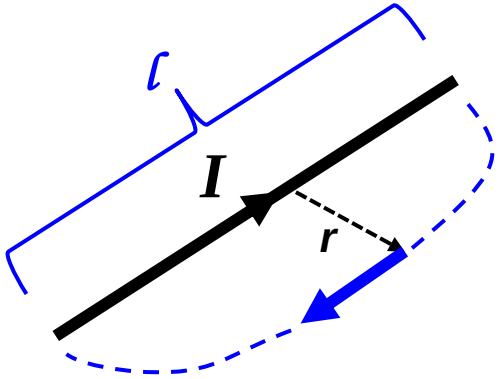
μ_0 = *permeability of free space*

$= 4\pi \times 10^{-7}\ H/m$

(~ 1.3 μH/m)

CURRENT PRODUCES ELECTRIC FIELD

(worst-case upper bound approximations; details in backup slides)



Electrically short wire (l < λ/10):

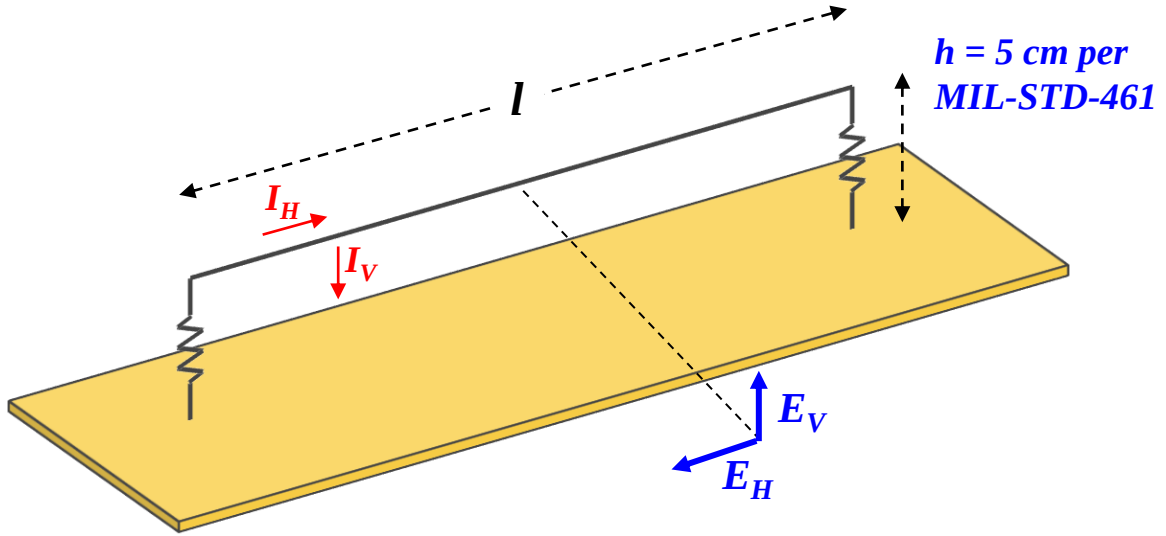
$$E_{short} \approx \frac{\mu_0 l f I}{2r}$$

Maximum for tuned half-wave dipole:

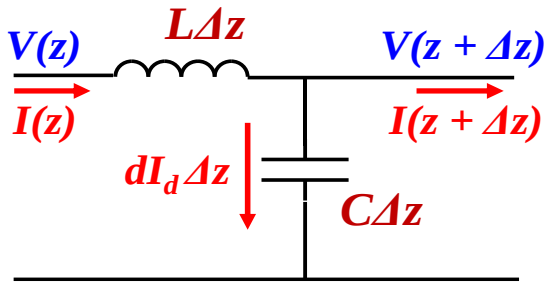
$$E_{MAX} = \frac{60\ I}{r}$$

Wire-Above Ground Transmission Line

- Cable above ground plane behaves as wire-above-ground transmission line
- Transmission line current has horizontal and vertical components
  - Horizontal component is current flowing on wire itself
  - Vertical component is displacement current flowing in dielectric of transmission line
- Resulting electric field will have horizontally and vertically polarized components with magnitudes corresponding to horizontal and vertical current components
- Electric field will be function of average current integrated along length of wire
- Details in backup slides

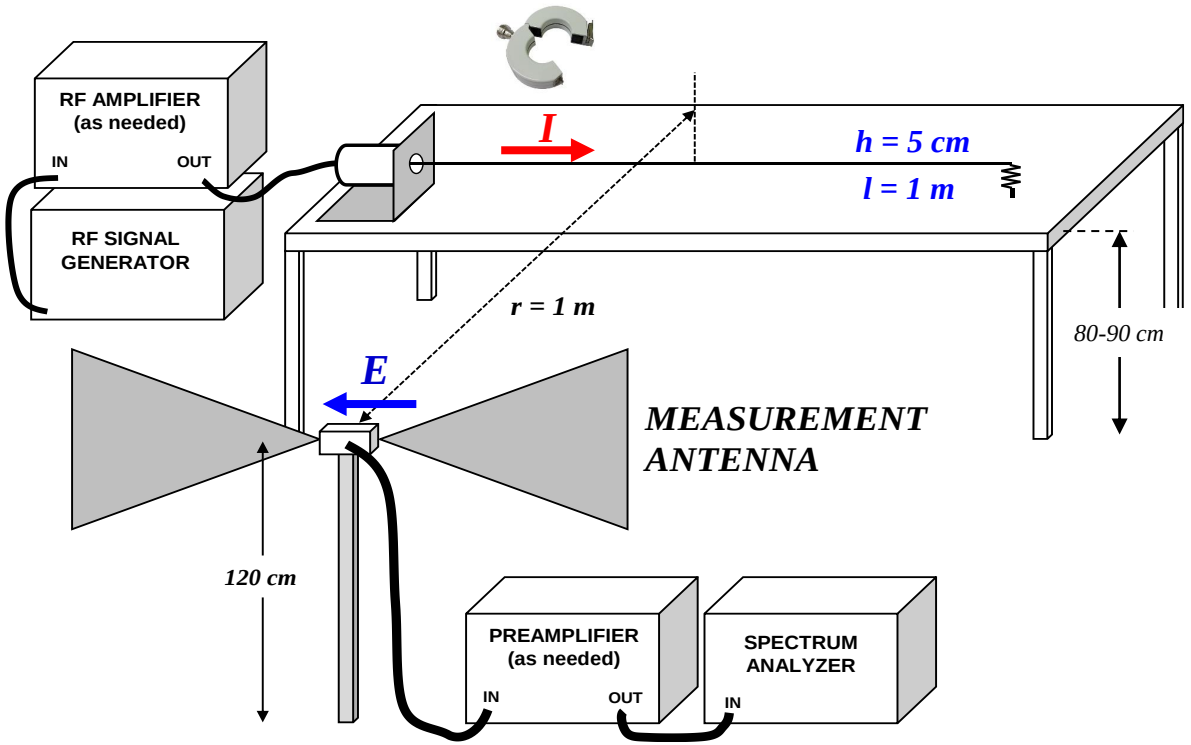


Lossless transmission line model



|E/I| Measured vs. Predicted for 1-Meter Wire

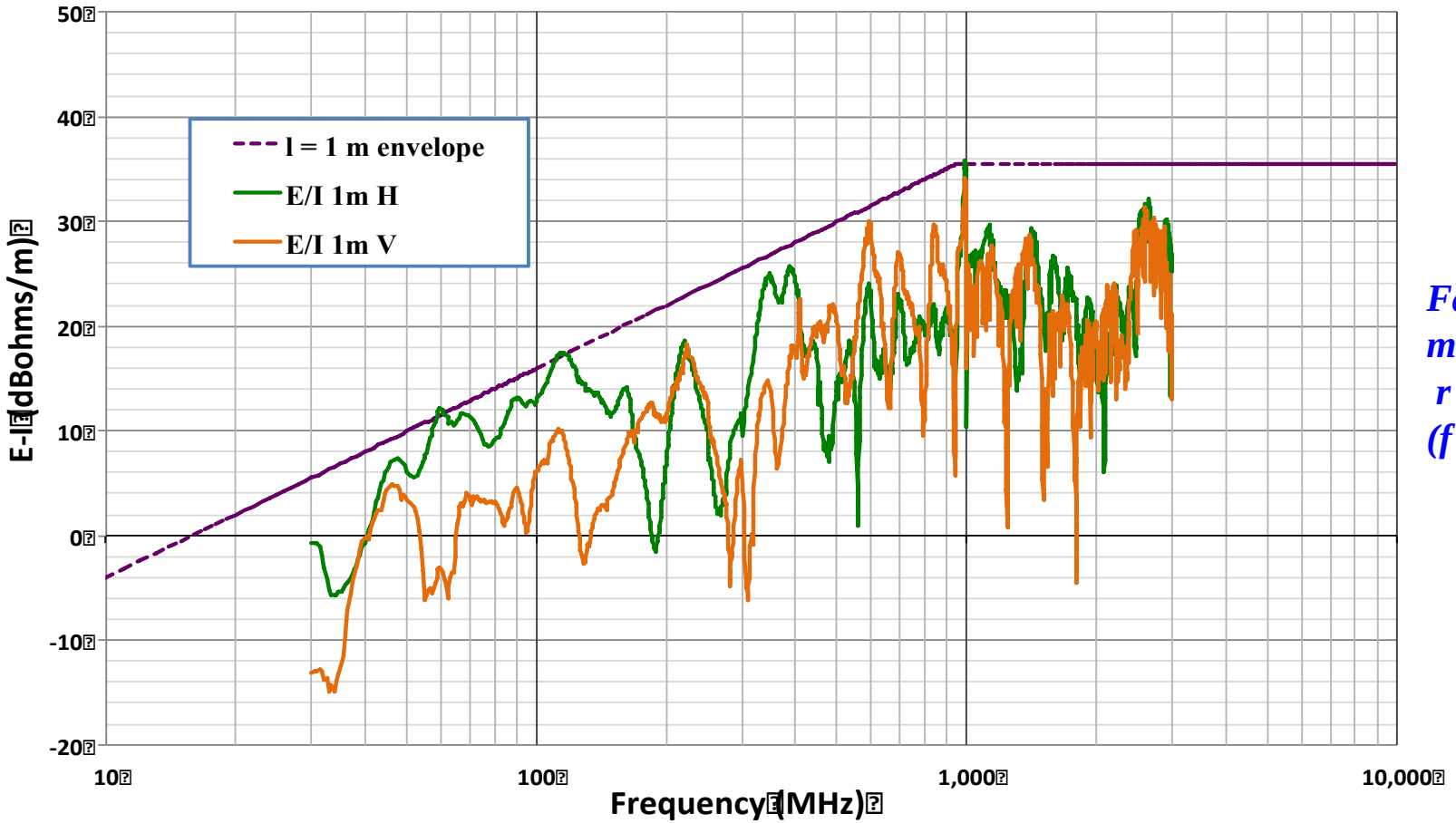
- Predicted envelope derived in backup slides
- Additional info provided in “Radiated Emissions as a Function of Common Mode Current,” demo recorded for 2016 IEEE EMC Symposium by John McCloskey and Jen Dimov (link on “References” page)



|E/I| Measured vs. Predicted for 1-Meter Wire (cont.)

- Predicted |E/I| envelope provides reasonable upper bound for measured values

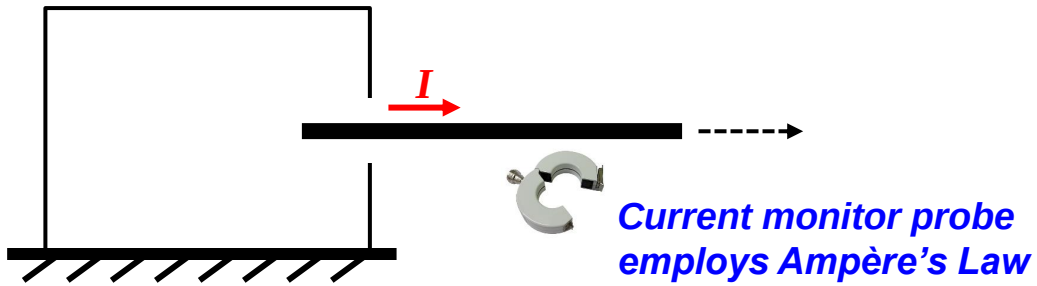
THEORY EXPERIMENT



Far field approximation most accurate for:
$r > \lambda/2\pi$
(f > 48 MHz)

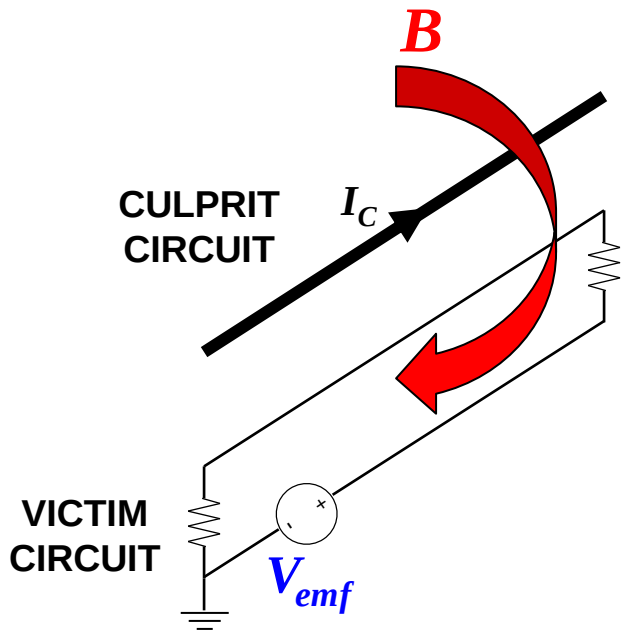
Current and Fields – Things to Consider

- Measured current is excellent predictor of magnetic field in close proximity to long wire
  - Per Ampère's Law (current probe directly employs that very principle)
  - Useful for assessing coupling to nearby conductors, a.k.a. "near field" (e.g. crosstalk)
- Measured current is useful for predicting UPPER BOUND for the electric field (horizontal and vertical components) measured in far field
  - Most likely common mode current on bulk cable (can be measured directly with current probe; no disconnections or power cycles needed)
- These analytical techniques are NOT intended nor useful for estimating a current on a wire/cable from a measured electric field in the near field
  - VERY large error bar if attempted in that direction



Magnetic Coupling: Faraday's Law

- Time-varying magnetic flux through loop induces potential (a.k.a. electromotive force, emf)



Total magnetic flux through loop:

$$\Phi = \int \vec{B} \cdot \overrightarrow{ds}$$

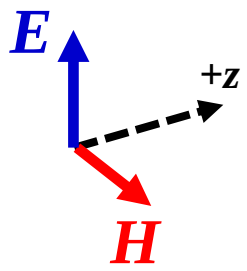
Induced potential:

$$V_{emf} = -\frac{d\Phi}{dt}$$

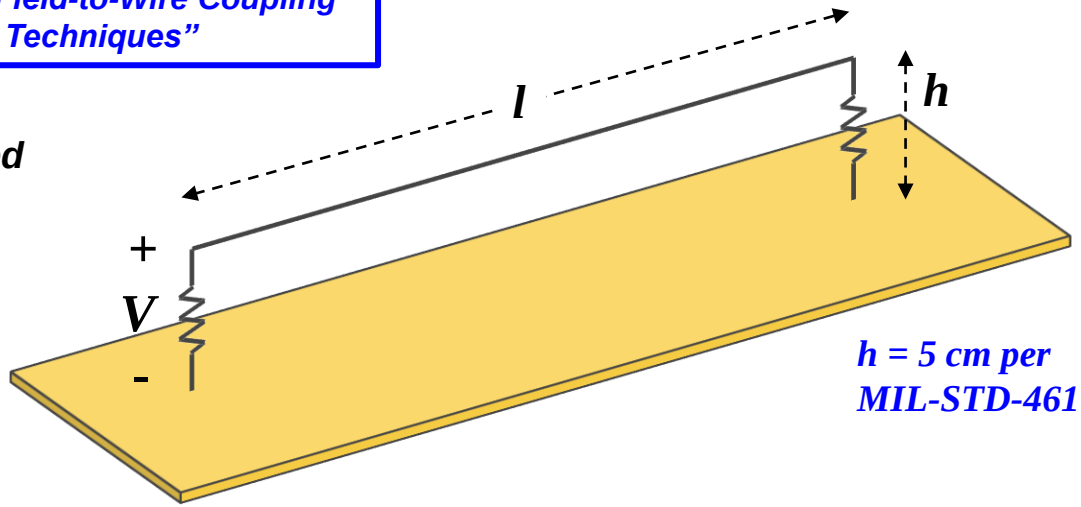
Field-to-Wire Coupling – Transmission Line Model

Inspired by Ken Javor, "On Field-to-Wire Coupling Versus Conducted Injection Techniques"

Incident plane wave oriented for worst-case coupling (H-field normal to loop)



$$E(t,z) = E_0 e^{j(\omega t - \beta z)}$$
$$H(t,z) = H_0 e^{j(\omega t - \beta z)}$$
$$B(t,z) = B_0 e^{j(\omega t - \beta z)}$$



h = 5 cm per MIL-STD-461

Faraday's Law:

$$V = \left|\frac{d\Phi}{dt}\right| = \frac{d}{dt}\int \vec{B}\cdot\overrightarrow{ds} = \frac{d}{dt}\int \vec{B}\cdot\overrightarrow{ds}$$

Wave impedance: $\frac{E}{H} = \eta = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi \approx 377\ \Omega$ *(air/vacuum)*

Velocity of propagation: $c = \frac{1}{\sqrt{\mu\varepsilon}} = 3\times10^8\ m/s$ *(air/vacuum)*

B in terms of E: $B_0 = \mu H_0 = \frac{\mu E_0}{\sqrt{\mu/\varepsilon}} = \sqrt{\mu\varepsilon}\cdot E_0 = \frac{E_0}{c}$

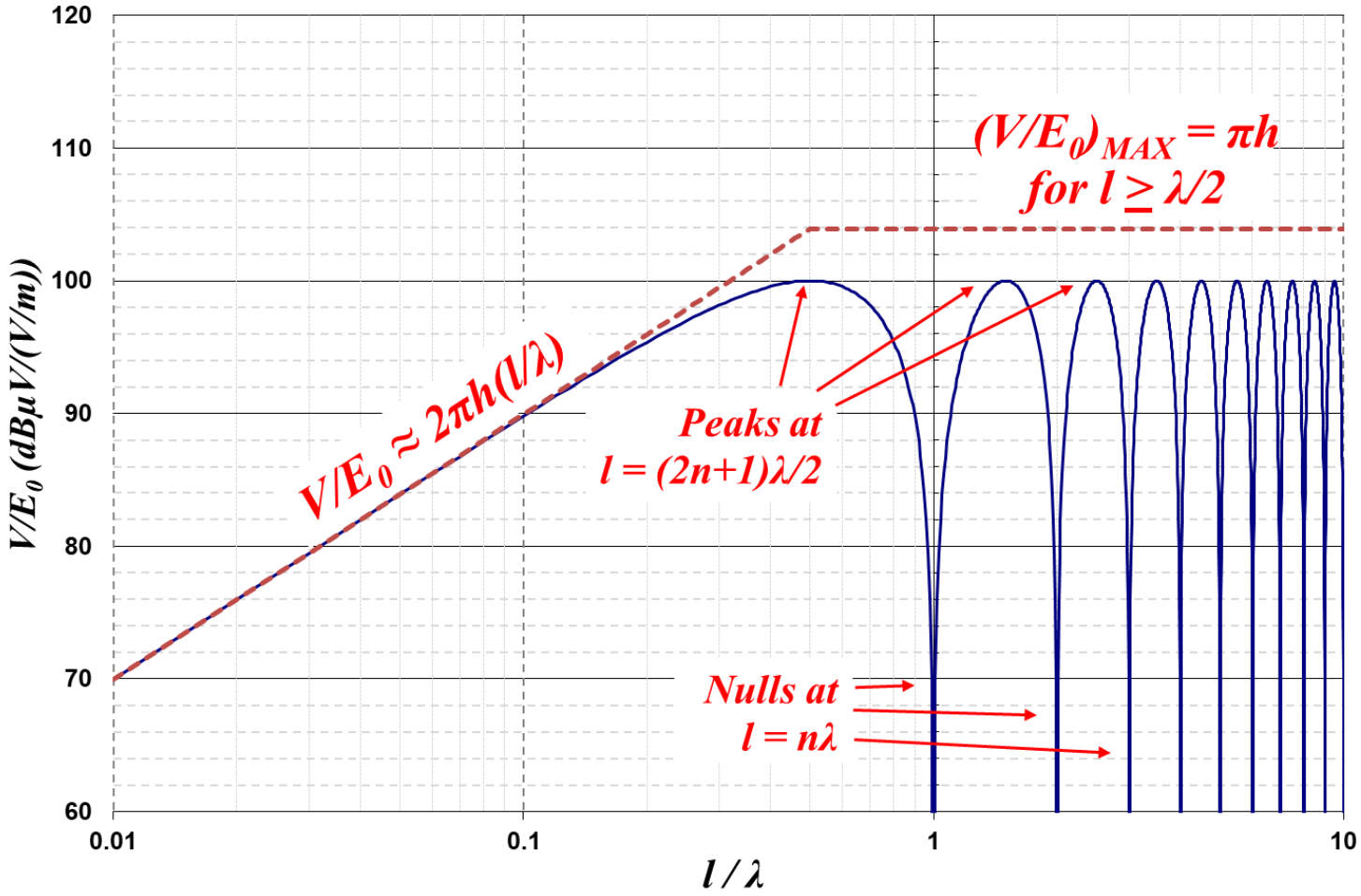
Wavenumber: $\beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

$$= hB_0 \cdot \frac{d}{dt}\int_0^l e^{j(\omega t-\beta z)}dz$$
$$= \frac{hE_0}{c}\cdot\frac{d}{dt}\int_0^l e^{j(\omega t-\beta z)}dz$$

Derivation in backup slides

$$\frac{V}{E_0} = 2h\sin\left(\frac{\pi l}{\lambda}\right)$$

Field-to-Wire Coupling – Transmission Line Model (cont.)

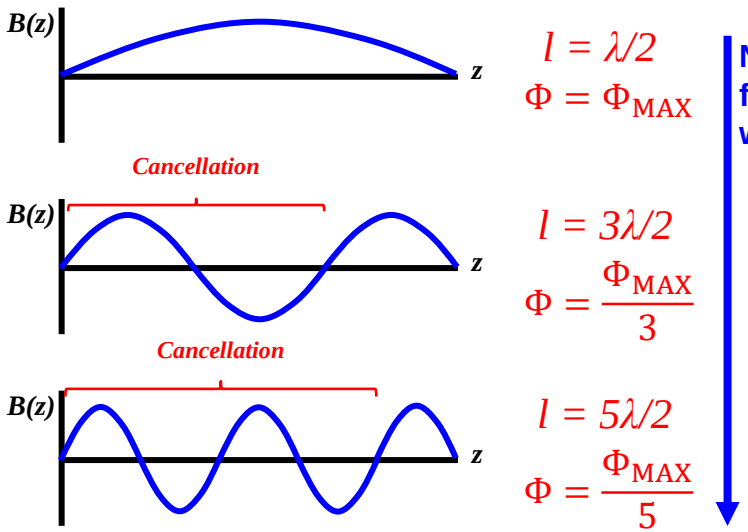


$$\frac{V}{E_0} = 2h \sin\left(\frac{\pi l}{\lambda}\right)$$

$(V/E)_{MAX}$ = 150 mV/(V/m) for h = 5 cm

= 104 dBμV/(V/m)

Field-to-Wire Coupling – Transmission Line Model (cont.)

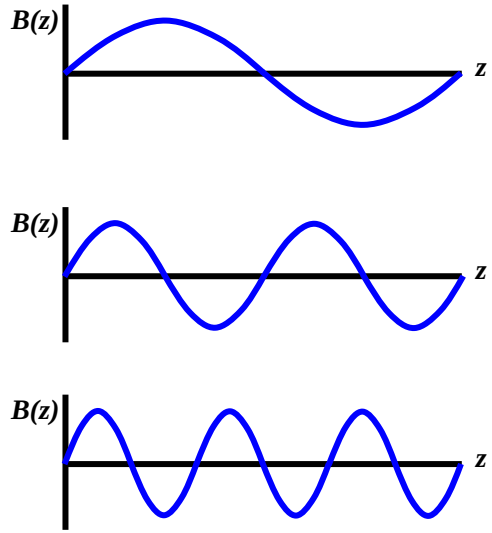


$l = \lambda/2$
$\Phi = \Phi_{MAX}$

$l = 3\lambda/2$
$\Phi = \frac{\Phi_{MAX}}{3}$

$l = 5\lambda/2$
$\Phi = \frac{\Phi_{MAX}}{5}$

Net coupled flux decreases with frequency



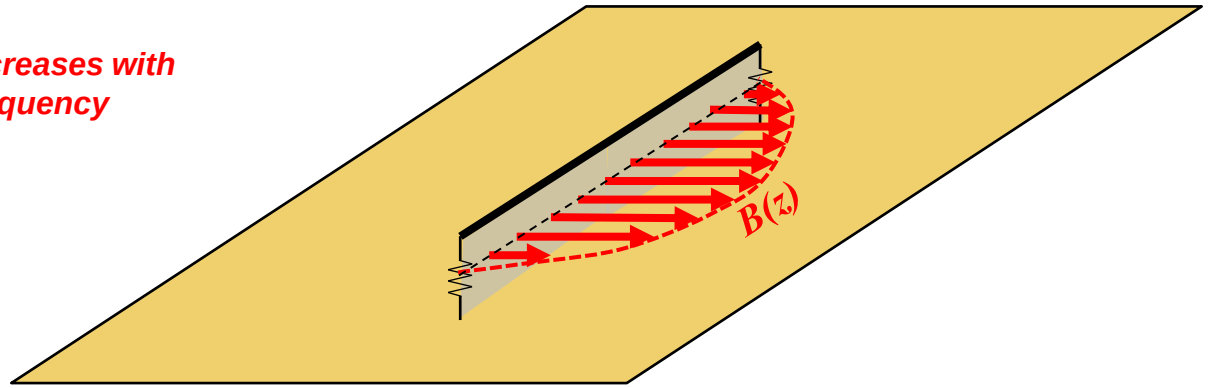
$l = \lambda$
$\Phi = 0$

$l = 2\lambda$
$\Phi = 0$

$l = 3\lambda$
$\Phi = 0$

But: $|V| = \left|\frac{d\Phi}{dt}\right| = 2\pi f \Phi$ *Increases with frequency*

Constant B (vs. f)
→ Constant V_{peak}

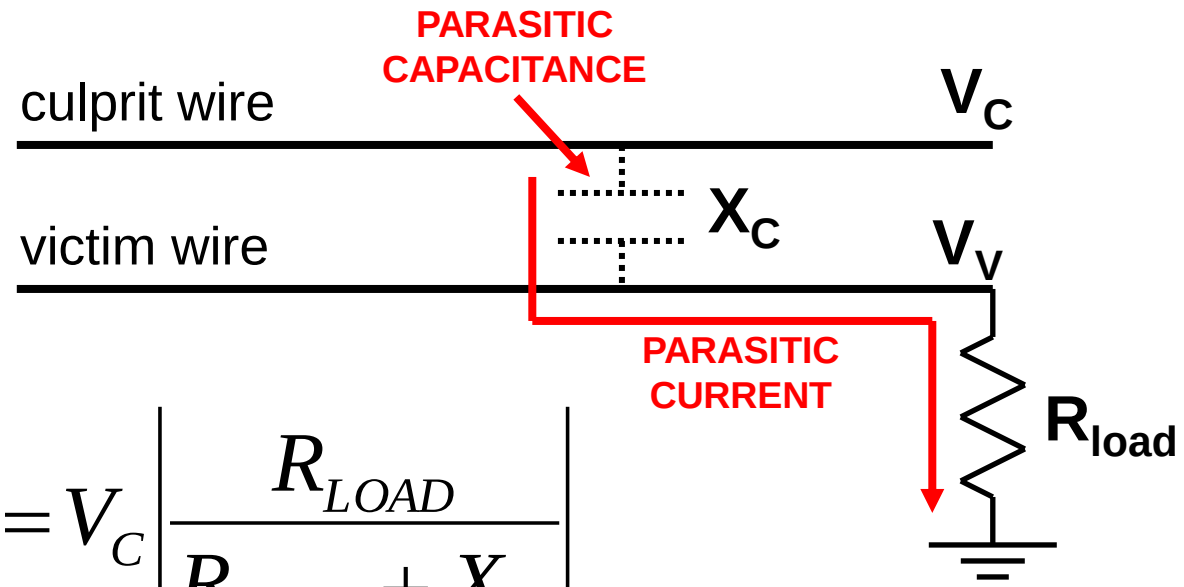


Capactive Crosstalk

- Parasitic capacitance forms impedance divider between culprit and victim circuits
- Most common between unshielded conductors in close proximity (e.g. printed circuit board traces)

$$C = \frac{\varepsilon A}{d}$$

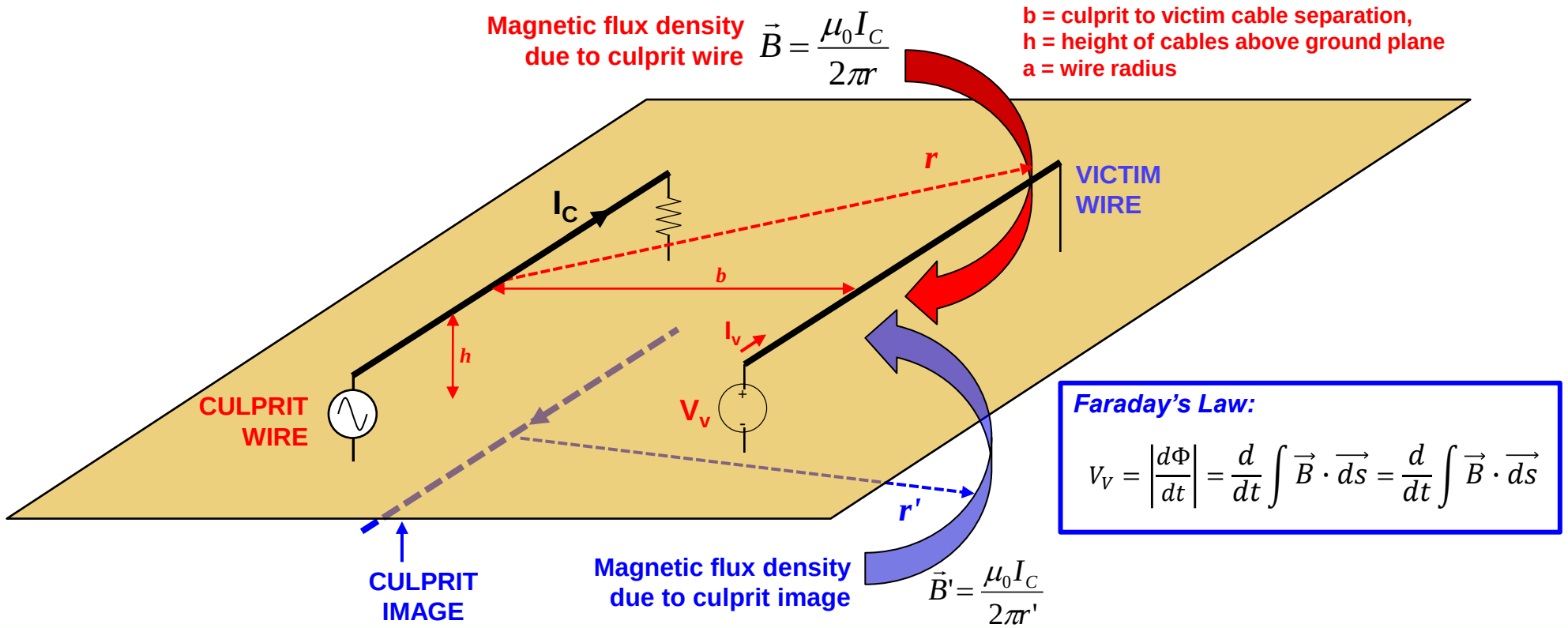
$$X_C = \frac{1}{j\omega C}$$



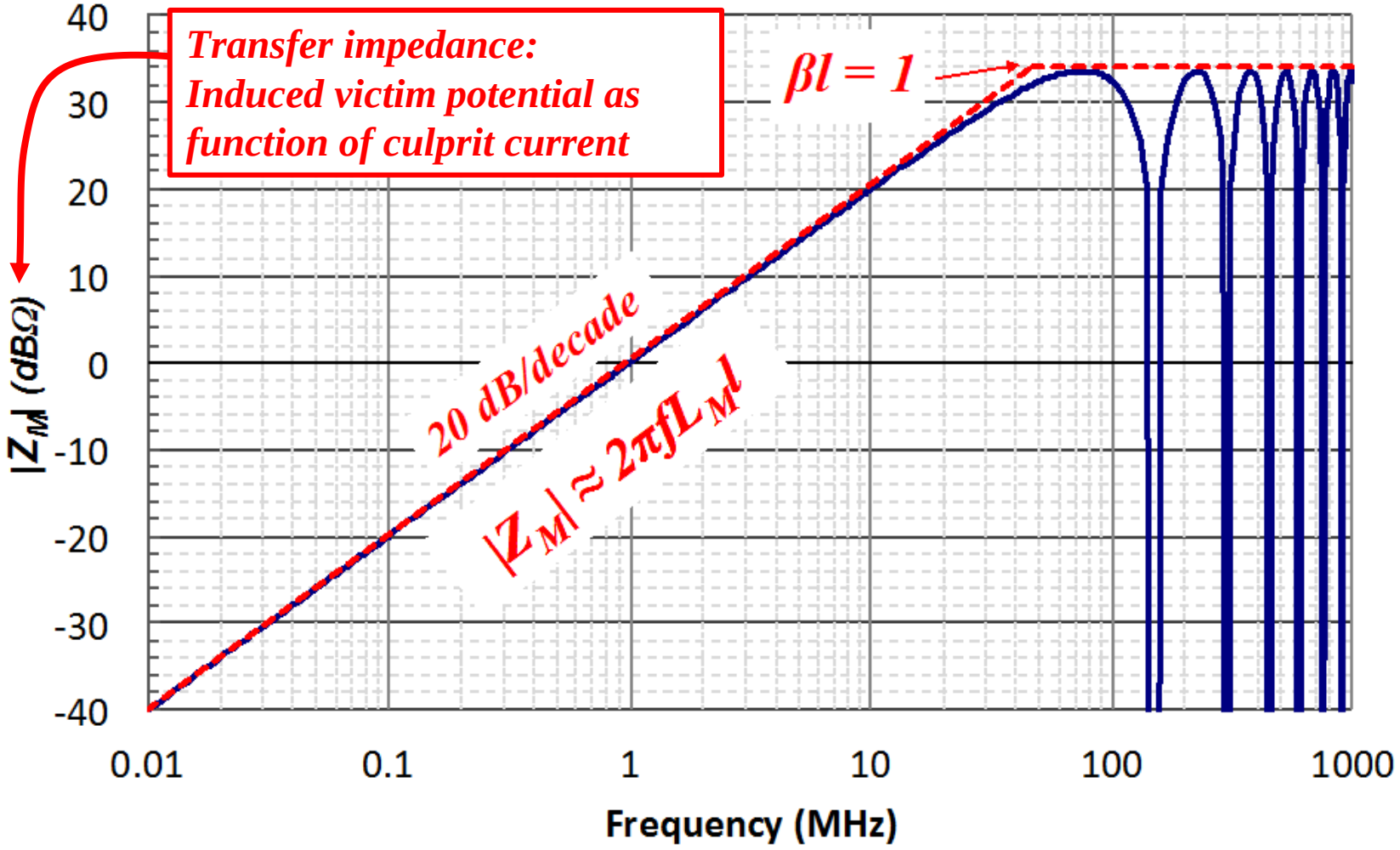
$$V_V = V_C \left| \frac{R_{LOAD}}{R_{LOAD} + X_C} \right|$$

Cable-to-Cable Inductive Crosstalk

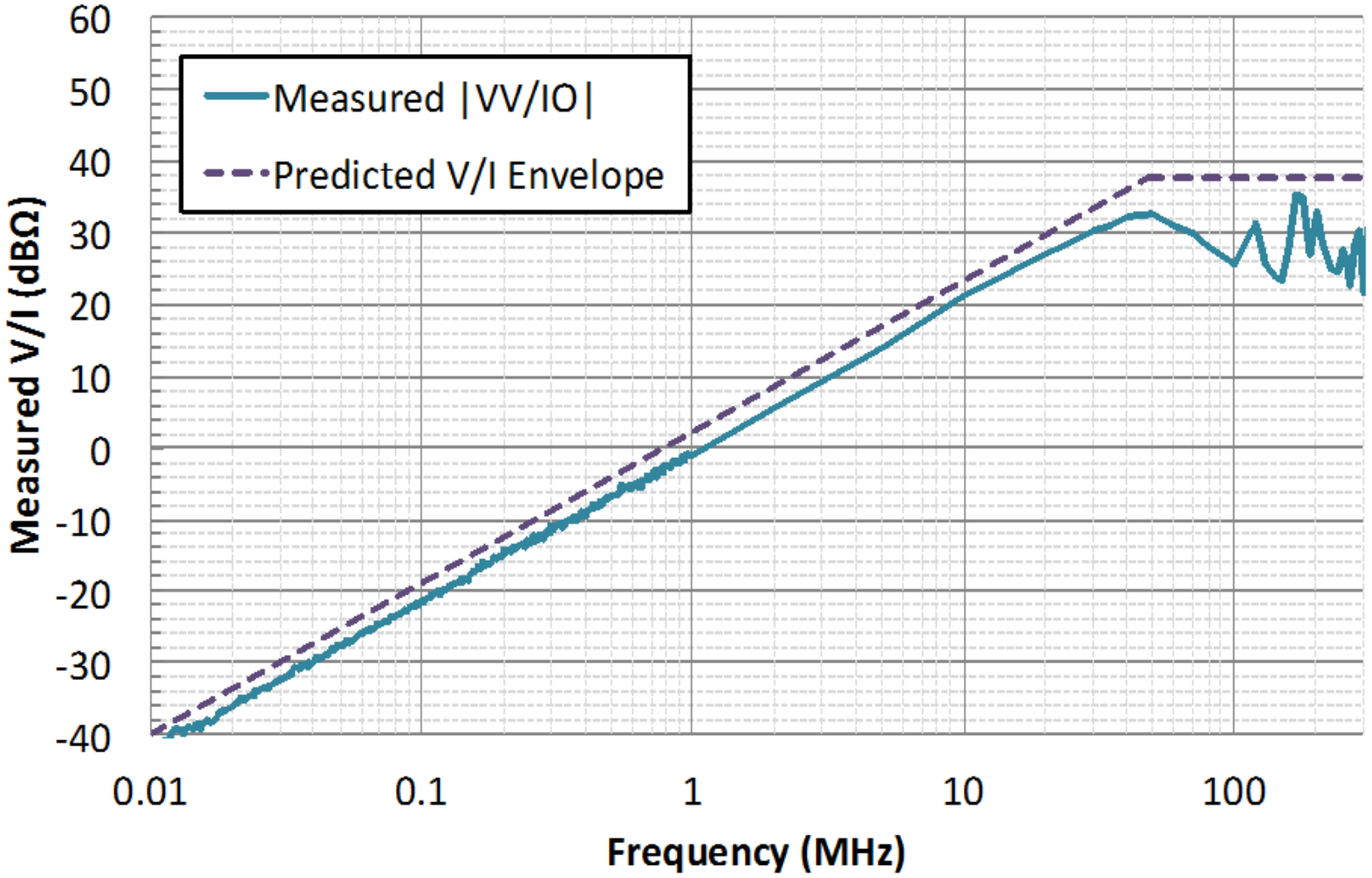
- Similar analysis applies to cable-to-cable crosstalk
  - Coupled potential in terms of culprit current instead of incident electric field
- Once again, it comes down to Faraday's Law (details in backup)



Predicted Coupled Voltage Due to Average Current
Example: *b = h = 5 cm, l = 1 m*

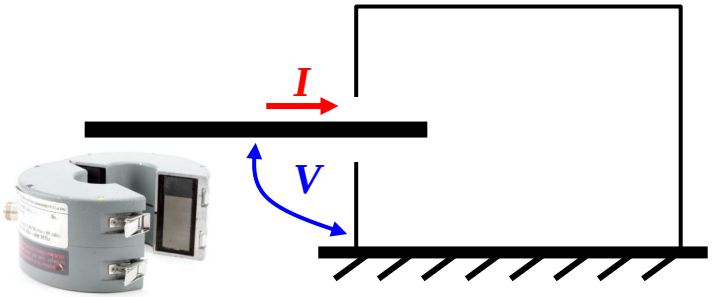


Measured Coupled Voltage Due to Average Culprit Current
Example: *b = h = 5 cm, l = 1 m*



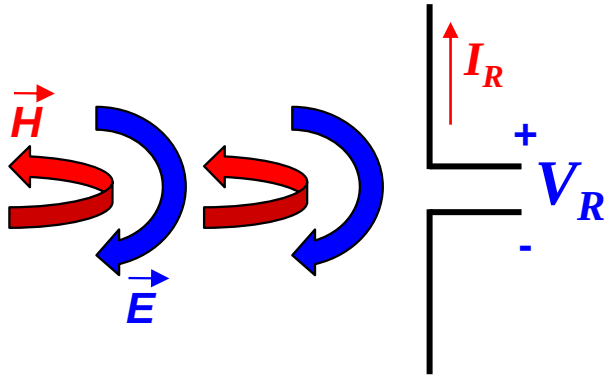
Field-to-Wire Coupling / Crosstalk Summary

- Incident field induces potential in wire-above-ground loop (Faraday's Law)
- Induced current is determined by impedance of configuration
- For shielded cable with shields terminated to chassis at both ends:
  - Impedance is dominated by loop inductance for $l < \lambda/2$
  - ~1 μH/m for h = 5 cm
  - Impedance plateaus at characteristic impedance of wire-above-ground transmission line for $l > \lambda/2$ (~300 Ω for h = 5 cm); wire-above-ground transmission line parameters in backup slides
- MIL-STD-461G CS114 test method for bulk cable injection
  - Developed specifically to simulate coupled currents from incident fields (injection probe employs Faraday's Law)
  - Includes precalibration step into 100 Ω fixture, giving "transfer function" in high frequency plateau of 1.5 mA per V/m; actual injected current determined by loop impedance
  - Injects common mode current on bulk cable; no disconnections or power cycles needed



Current injection probe employs Faraday's Law

Tuned Half-Wave Dipole Parameters (Worst-Case Coupling)

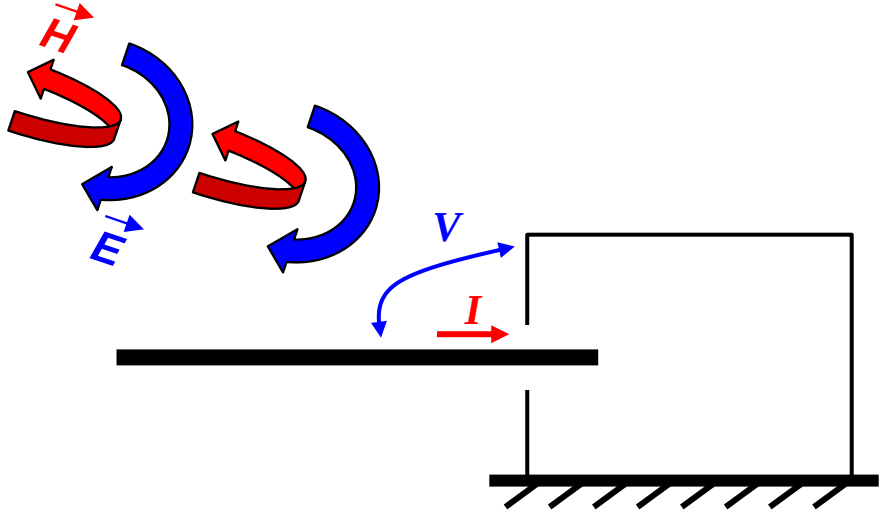


Tuned half-wave dipole antenna factor effective height h_e (ratio of open-circuit received potential to incident electric field):

$$h_e = \frac{V_R}{E} = \frac{\lambda}{\pi}$$

Tuned half-wave dipole impedance (details in backup slides):

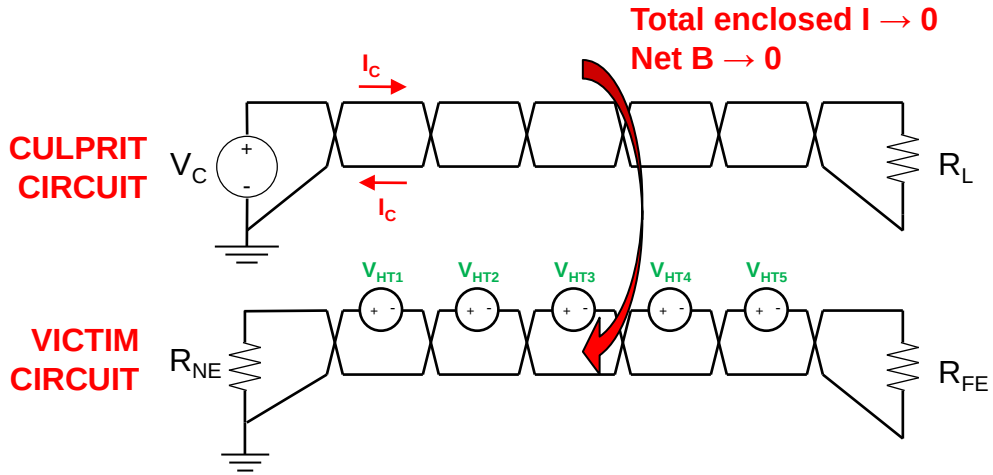
$$73\ \Omega$$



Useful analytical tools for calculating worst-case coupling to victim equipment

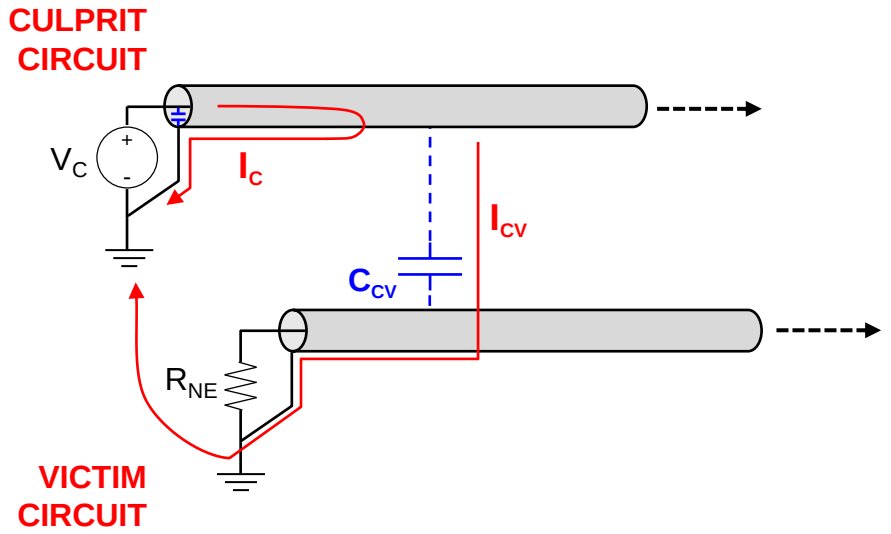
Mitigation Strategies: Twisted Pairs

- Provide dedicated return path for differential mode currents
  - Twist (+) and (-) wires together
- Emissions
  - Provides return current path to cancel culprit current
  - Reduced net current reduces net magnetic field
- Susceptibility
  - Reduced loop area
  - Voltages induced on adjacent half-twist loops cancel
  - Worst case induced voltage is that induced on loop area of single half twist

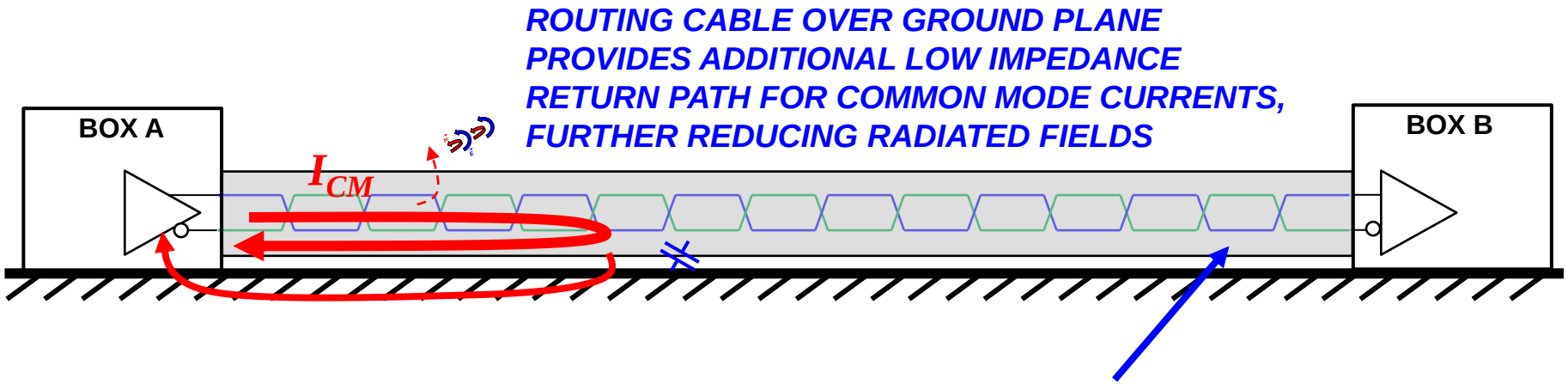


Mitigation Strategies: Shielding

- Emissions
  - Culprit common mode noise current couples to its own shield and returns to its source
  - Minimizes emissions
- Susceptibility
  - Any remaining current that makes its way to victim couples to victim's shield and gets shunted back to source (via ground), protecting victim signal wire(s)
  - Incident field couples to shield, which shunts current away from signal-carrying conductors
- Shields must be properly terminated to provide low impedance path back to source



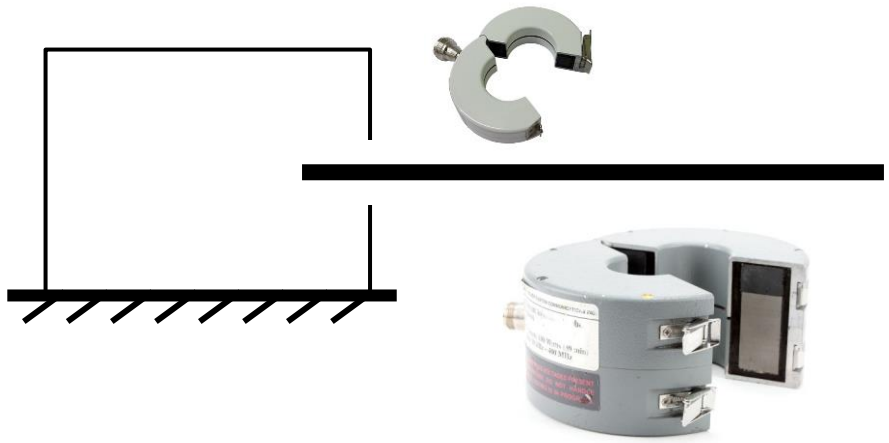
Mitigation Strategies: Route Cable Over Ground Plane



Common Mode Current Measurements

- Measure common mode conducted emissions (CMCE) and common mode conducted susceptibility (CMCS) at component level
- Measure on all cables (power and signal) to provide:
  - Direct assessment of cable-to-cable crosstalk risk
  - Indirect assessment of radiated emissions and radiated susceptibility
- Use clamp-on current probes
  - Easily move from cable-to-cable
  - No disconnections or power cycles needed
  - May be performed in hardware development lab before taking product to EMI facility
  - Don't have to fight RF background, room resonances, etc.
  - Identify specific sources of potential problems as early as possible

Current monitor probe for CMCE



Current injection probe for CMCS

Summary

- Radiated and conducted EMI issues are "joined at the hip”
  - You can’t have one without the other
  - Well-established analytical techniques facilitate correlation between the two, giving EMC engineers useful tools for explaining issues to electronics engineers in terms they will best understand (voltages and currents)
- Current probe is your friend
  - Measure currents early and often; anechoic chamber not needed
  - Measure in development lab long before going to EMI lab
- Control current return paths
  - Provide desired low impedance path (e.g. dedicated return wire, twisted pair, shield, ground plane, etc.) and make them flow where you want them to
  - Let nature do the work
- When you control current return paths, you go long way toward controlling radiated EMI problems

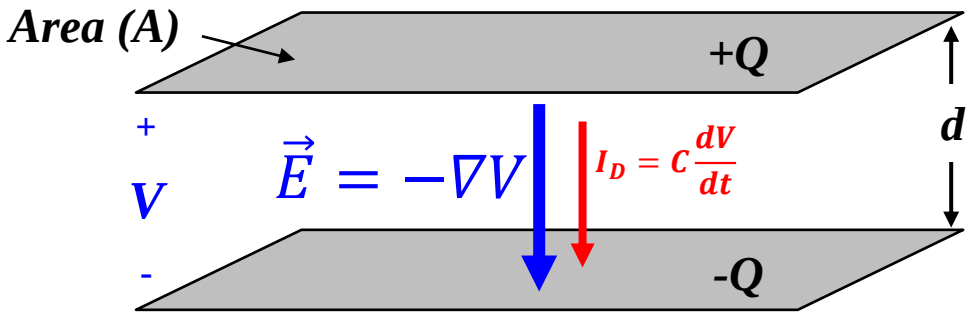
References

- Textbooks
  - Ott, Henry: Electromagnetic Compatibility Engineering
  - Paul, Clayton: Introduction to Electromagnetic Compatibility
  - Cheng, David: Field and Wave Electromagnetics
- Papers, articles, etc.
  - Henry Ott, “Dipoles for Dummies”
    - http://www.hottconsultants.com/pdf_files/dipoles-1.pdf
    - http://www.hottconsultants.com/pdf_files/dipoles-2.pdf
  - Ken Javor, “On Field-to-Wire Coupling Versus Conducted Injection Techniques”, 1997 IEEE EMC Symposium record
- Video of hardware demonstration
  - John McCloskey and Jen (Roberts) Dimov, “Radiated Emissions as a Function of Common Mode Current,” recorded for 2017 IEEE EMC Symposium
  - https://ieeetv.ieee.org/radiated-emissions-as-function-of-common-mode-current-emc-society-demo

BACKUP

Electric Field, Potential, and Capacitance

- Electric field is negative gradient of potential (units of V/m)
- Capacitance is the ratio of charge to potential between 2 conductors
- Capacitive reactance decreases with frequency
- Time-varying electric field induces displacement current (I_D) through dielectric



Capacitance:

$$C = \frac{Q}{V}$$

Parallel plate:

$$C = \frac{\varepsilon_0 A}{d}$$

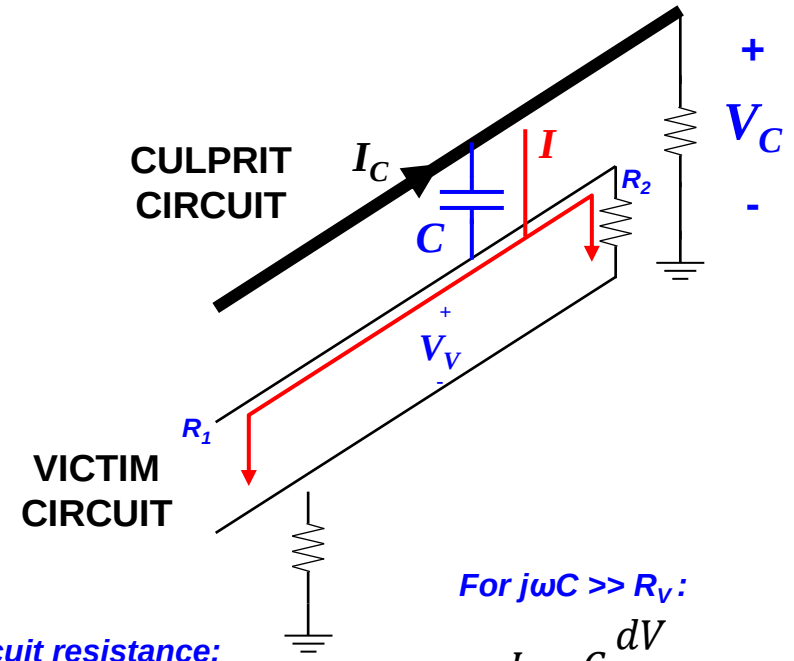
Capacitive reactance (decreases with frequency):

$$X_C = \frac{1}{j\omega C}$$

$$\varepsilon_0 = \text{permittivity of free space} \approx \frac{1}{36\pi} \times 10^{-9}\, F/m \quad \textbf{(\sim 8.84 pF/m)}$$

Electric Field (Capacitive) Coupling

- AC potential in culprit circuit couples AC potential in victim circuit according to impedance divider
- More likely between unshielded conductors in close proximity (e.g. adjacent PC board traces, unshielded wires, etc.)



CAPACITIVE COUPLING EXAMPLE



Victim circuit resistance:

$$R_V = R_1 || R_2$$

$$V_V = V_C \frac{R_V}{R_V + j\omega C}$$

For $j\omega C >> R_V$:

$$I = C\frac{dV}{dt}$$

Coupled potential into victim circuit:

$$V_V \approx IR_V$$

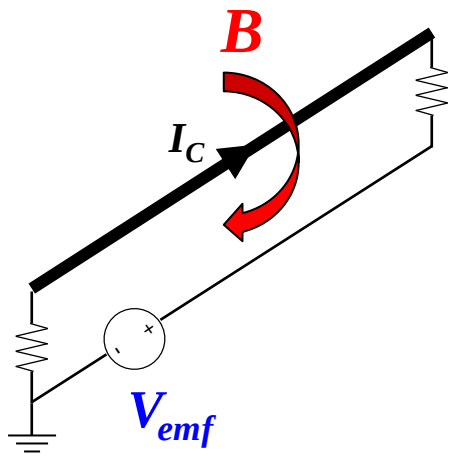
$$I = C\frac{dV}{dt} \approx (200\ pF)\left(\frac{500\ mV}{50\ nsec}\right) \approx 2\ mA$$

$$V_V \approx (2\ mA)(25\ \Omega) \approx 50\ mV$$

50 Ω // 50 Ω

Magnetic Field (Flux), Current, and Inductance

- **Magnetic flux induces potential (emf) in loop (Faraday's Law)**
  - Magnitude is time derivative of total magnetic flux through loop (Φ)
  - Induced emf opposes (impedes) coupled flux (negative sign)
- **Inductance is the ratio of induced magnetic flux to the current causing it**
- **Inductive reactance increases with frequency**



Total magnetic flux through loop:

$$\Phi = \int \vec{B} \cdot \overrightarrow{ds}$$

induced potential (emf):

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B} \cdot \overrightarrow{ds}$$

Inductance:

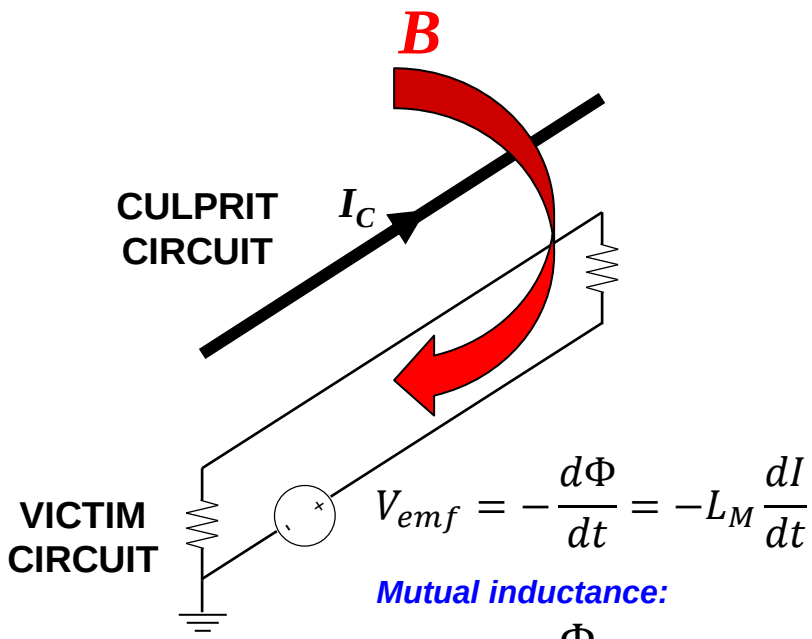
$$L = \frac{\Phi}{I}$$

Inductive reactance (increases with frequency):

$$X_L = j\omega L$$

Magnetic Field (Inductive) Coupling

- **Magnetic flux generated by culprit current produces noise potential in victim circuit (Faraday's Law)**

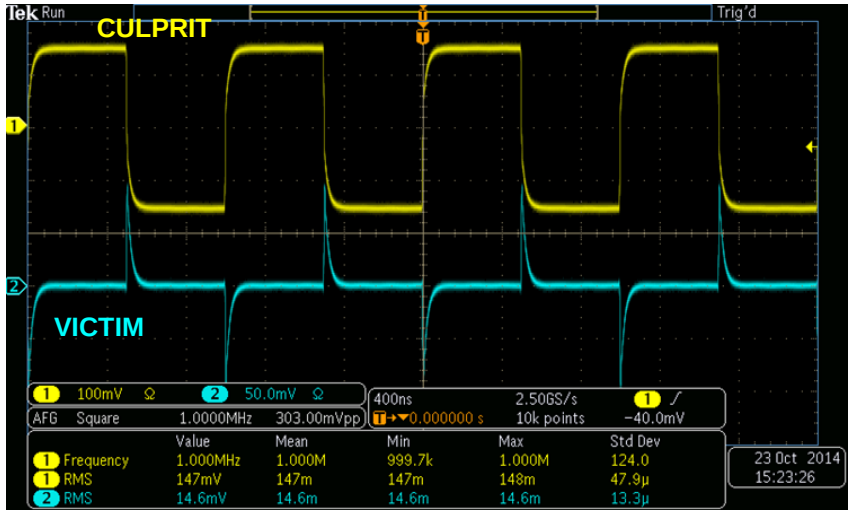


$$V_{emf} = -\frac{d\Phi}{dt} = -L_M \frac{dI}{dt}$$

Mutual inductance:

$$L_M = \frac{\Phi}{I_C}$$

INDUCTIVE COUPLING EXAMPLE

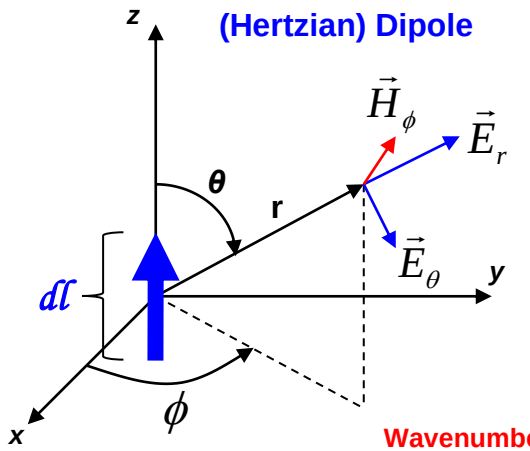


$$|V_{emf}| = L_M \frac{dI}{dt} \approx (1\ \mu H)\left(\frac{6\ mA}{50\ nsec}\right) \approx 120\ mV$$

6 mA = 300 mV / 50 Ω

Radiated Fields from Elemental Dipoles

Electric (Hertzian) Dipole



Wavenumber:

$$\beta_0 = \frac{2\pi}{\lambda}$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = \frac{Idl}{4\pi}\beta_0^2 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} \right) e^{-j\beta_0 r}$$

Complex dependence on 1/r and 1/r² in near field

$$H_{far} = j\frac{Idl}{4\pi}\beta_0 \sin\theta \frac{e^{-j\beta_0 r}}{r}\vec{a}_\phi$$

Only 1/r dependence remains in far field

$$E_r = 2\frac{Idl}{4\pi}\eta_0\beta_0^2 \cos\theta \left(\frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

$$E_\theta = \frac{Idl}{4\pi}\eta_0\beta_0^2 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

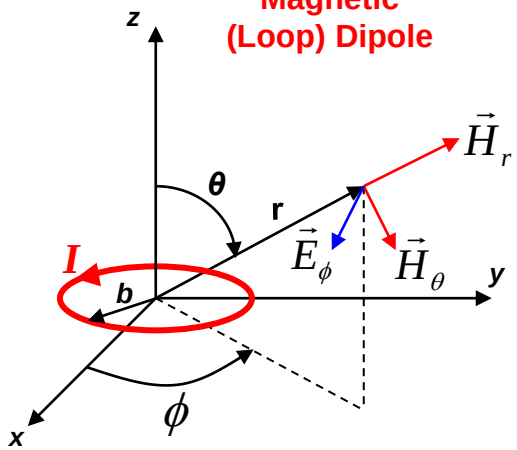
$$E_\phi = 0$$

Complex dependence on 1/r, 1/r², & 1/r³ in near field

$$E_{far} = j\frac{Idl}{4\pi}\eta_0\beta_0 \sin\theta \frac{e^{-j\beta_0 r}}{r}\vec{a}_\theta$$

Only 1/r dependence remains in far field

Magnetic (Loop) Dipole



Magnetic dipole moment:

$$m = I\pi b^2$$

$$E_r = 0$$

$$E_\theta = 0$$

$$E_\phi = -j\frac{\omega\mu_0 m}{4\pi}\beta_0^2 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} \right) e^{-j\beta_0 r}$$

Complex dependence on 1/r and 1/r² in near field

$$E_{far} = \frac{\omega\mu_0 m}{4\pi}\beta_0 \sin\theta \frac{e^{-j\beta_0 r}}{r}\vec{a}_\phi$$

Only 1/r dependence remains in far field

$$H_r = j\frac{2\omega\mu_0 m}{4\pi\eta_0}\beta_0^2 \cos\theta \left(\frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

$$H_\theta = j\frac{\omega\mu_0 m}{4\pi\eta_0}\beta_0^2 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

$$H_\phi = 0$$

Complex dependence on 1/r, 1/r², & 1/r³ in near field

$$H_{far} = -\frac{\omega\mu_0 m}{4\pi\eta_0}\beta_0 \sin\theta \frac{e^{-j\beta_0 r}}{r}\vec{a}_\theta$$

Only 1/r dependence remains in far field

Radiated Field from Electrically Short Wire (Bare Wire in Space - No Ground Plane Cancellation)

Velocity of propagation:

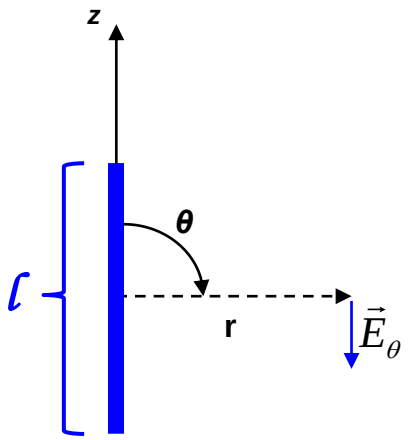
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3 \times 10^8 \; m/s$$ ***(air/vacuum)***

Wave impedance:

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \approx 377 \; \Omega$$ ***(air/vacuum)***

Wavenumber:

$$\beta_0 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 2\pi f \sqrt{\mu_0 \varepsilon_0}$$



A Very Sick Integral:

$$E_\theta = \int_{-l/2}^{l/2} \frac{I(z)}{4\pi} \eta_0 \beta_0^2 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r_0^2} - j\frac{1}{\beta_0^3 r_0^3} \right) e^{-j\beta_0 r} dz$$

Far field assumption for high frequencies ($\beta_0 r > 1$):

$$E_\theta = \int_{-l/2}^{l/2} j\frac{I(z)}{4\pi} \eta_0 \beta_0 \sin\theta \frac{e^{-j\beta_0 r}}{r} dz$$ ***(still a bit sick)***

Additional simplifying assumptions:

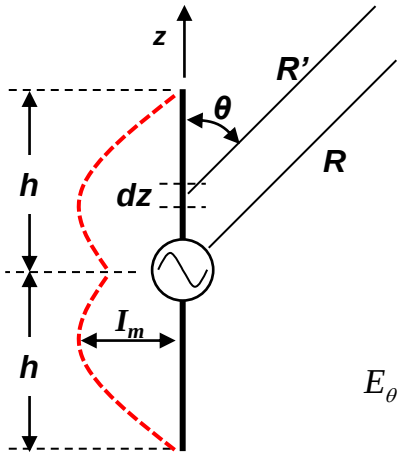
- ***Wire is electrically short ($l < \lambda/10$)***
- ***$\theta = 90^o$ ($\sin\theta = 1$)***
- ***r is constant for all points along wire***
- ***Current is constant along length***
- ***All contributions are in phase***
- ***(unrealistic, but provides useful worst-case upper bound)***

$$E_{MAX} \approx \frac{Il}{4\pi r} \eta_0 \beta_0 \rightarrow \eta_0 \beta_0 = 2\pi f \mu_0$$

$$\boxed{E_{MAX} \approx \frac{\mu_0 l f I}{2r}}$$

Upper bound for ELECTRICALLY SHORT wire

Radiated Fields from Finite Linear Dipole



$$dE_\theta = j\frac{I(z)dz}{4\pi}\eta_0\beta_0\sin\theta\frac{e^{-j\beta_0 R'}}{R'}$$

"h" used for height of each diplole "leg"

$$I(z) = I_m\sin\left[\beta\left(h-|z|\right)\right]$$

$$R' \cong R - z\cos\theta$$

$$E_\theta = j\frac{I_m\eta_0\beta_0\sin\theta}{4\pi R}e^{-j\beta_0 R}\int_{-h}^{h}\sin\left[\beta_0\left(h-|z|\right)\right]\cdot e^{j\beta_0 z\cos\theta}dz$$

$$E_\theta = j\frac{I_m\eta_0\beta_0\sin\theta}{4\pi R}e^{-j\beta_0 R}\int_{-h}^{h}\left\{\sin\left[\beta_0\left(h-|z|\right)\right]\cdot\cos\left(j\beta_0 z\cos\theta\right)+\underbrace{\sin\left[\beta_0\left(h-|z|\right)\right]\cdot j\sin\left(\beta_0 z\cos\theta\right)}\right\}\cdot dz$$

= 0 with symmetrical integration limits

$$E_\theta = j\frac{I_m\eta_0\beta_0\sin\theta}{2\pi R}e^{-j\beta_0 R}\int_{0}^{h}\sin\left[\beta_0\left(h-z\right)\right]\cdot\cos\left(\beta_0 z\cos\theta\right)dz$$

But: $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$

$$E_\theta = j\frac{I_m\eta_0\beta_0\sin\theta}{4\pi R}e^{-j\beta_0 R}\int_{0}^{h}\left\{\sin\left[\beta_0\left(h-z\right)+\beta_0 z\cos\theta\right]+\sin\left[\beta_0\left(h-z\right)-\beta_0 z\cos\theta\right]\right\}\cdot dz$$

$$E_\theta = j\frac{I_m\eta_0\beta_0\sin\theta}{4\pi R}e^{-j\beta_0 R}\cdot\left\{\frac{1}{\beta_0-\beta_0\cos\theta}\cdot\cos\left[\beta_0\left(h-z\right)+\beta_0 z\cos\theta\right]+\frac{1}{\beta_0+\beta_0\cos\theta}\cos\left[\beta_0\left(h-z\right)-\beta_0 z\cos\theta\right]\right\}\Bigg|_0^h$$

$$E_\theta = j\frac{I_m\eta_0\beta_0\sin\theta}{4\pi R}e^{-j\beta_0 R}\cdot\left\{\frac{1}{\beta_0(1-\cos\theta)}\cdot\left[\cos\left(\beta_0 h\cos\theta\right)-\cos\beta_0 h\right]+\frac{1}{\beta_0(1+\cos\theta)}\left[\cos\left(\beta_0 h\cos\theta\right)-\cos\beta_0 h\right]\right\}$$

CONTINUED…

Radiated Fields from Linear Dipole (cont.)

$$E_\theta = j\frac{I_m \eta_0 \beta_0 \sin\theta}{4\pi R} e^{-j\beta_0 R} \cdot \left\{ \frac{(1+\cos\theta+1-\cos\theta)}{\beta_0(1-\cos^2\theta)} \cdot \left[\cos(\beta_0 h\cos\theta) - \cos\beta_0 h\right] \right\}$$

$$E_\theta = j\frac{I_m \eta_0 \beta_0 \sin\theta}{4\pi R} e^{-j\beta_0 R} \cdot \left\{ \frac{2}{\beta_0 \sin^2\theta} \cdot \left[\cos(\beta_0 h\cos\theta) - \cos\beta_0 h\right] \right\}$$

$$E_\theta = j\frac{I_m \eta_0}{2\pi R} e^{-j\beta_0 R} \cdot \left(\frac{\cos(\beta_0 h\cos\theta) - \cos\beta_0 h}{\sin\theta} \right)$$

$$\eta_0 \cong 120\pi$$

$$E_\theta = j\frac{60 I_m}{R} e^{-j\beta_0 R} \cdot F(\theta)$$

$$H_\phi = \frac{E_\theta}{\eta_0} = j\frac{I_m}{2\pi R} e^{-j\beta_0 R} \cdot F(\theta)$$

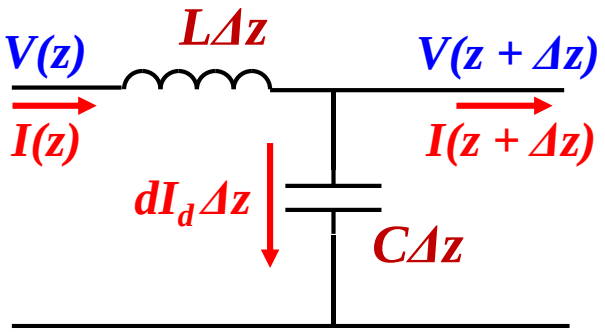
Pattern Function:

$$F(\theta) = \frac{\cos(\beta_0 h\cos\theta) - \cos\beta_0 h}{\sin\theta}$$

Maximum field levels for tuned half-wave dipole (h = λ/4) and θ = 90°:

$$E_{MAX} = \frac{60\ I_M}{R} \qquad H_{MAX} = \frac{I_M}{2\pi R}$$

Transmission Line Currents



Current on wire:

$$I(z) = I_0 e^{-j\beta z}$$

$$Re[I(z)] = I_0 \cos\beta z$$

Integrated average horizontal current:

$$I_H = \frac{1}{l}\int_0^l I_0 \cos\beta z dz = \frac{I_0}{\beta l}\sin\beta z\Big|_0^l = I_0\frac{\sin\beta l}{\beta l}$$

$$\beta l \le 1: \quad \left|\frac{I_H}{I_0}\right|_{ENV} = 1$$

$$\beta l \ge 1: \quad \left|\frac{I_H}{I_0}\right|_{ENV} = \frac{1}{\beta l}$$

$l = \lambda/2\pi$ is breakpoint

Integrated average vertical current:

$$dI_d(z)\Delta z = I(z) - I(z+\Delta z)$$

$$dI_d(z) = \frac{I(z) - I(z+\Delta z)}{\Delta z} = \frac{dI(z)}{dz} = -\beta I_0 \sin\beta z$$

$$I_V = -\beta I_0 \int_0^l \sin\beta z dz = I_0 \cos\beta z\Big|_0^l = I_0(\cos\beta l - 1)$$

$$\left|\frac{I_V}{I_0}\right|_{ENV} = 2$$

Response and Envelope of sin(x)/x

Response of |sin(x)/x|:

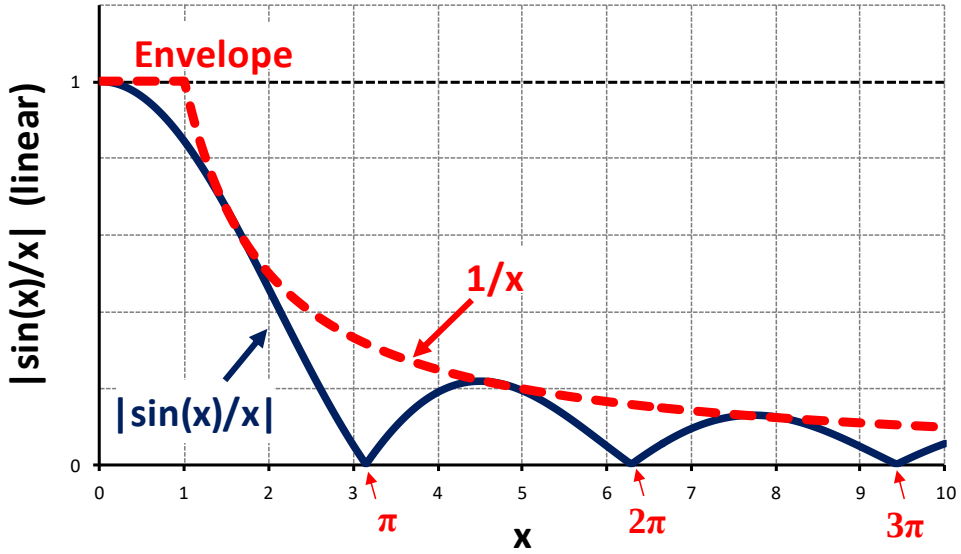
0 for x = nπ

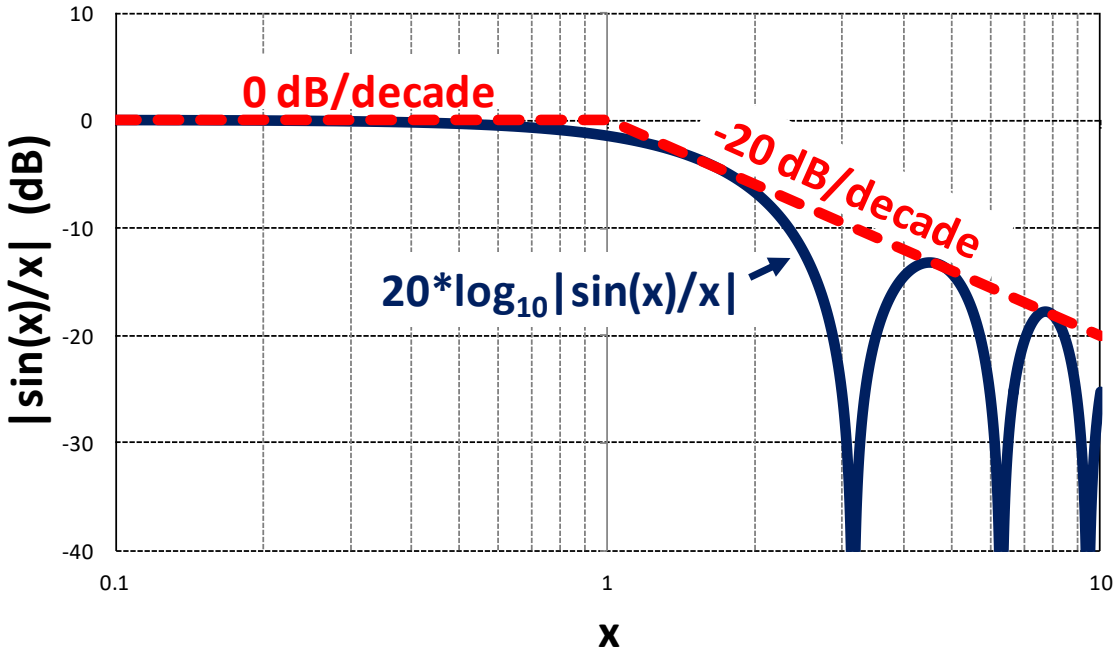
1/x for x = (n+1)π/2

Envelope of |sin(x)/x|:

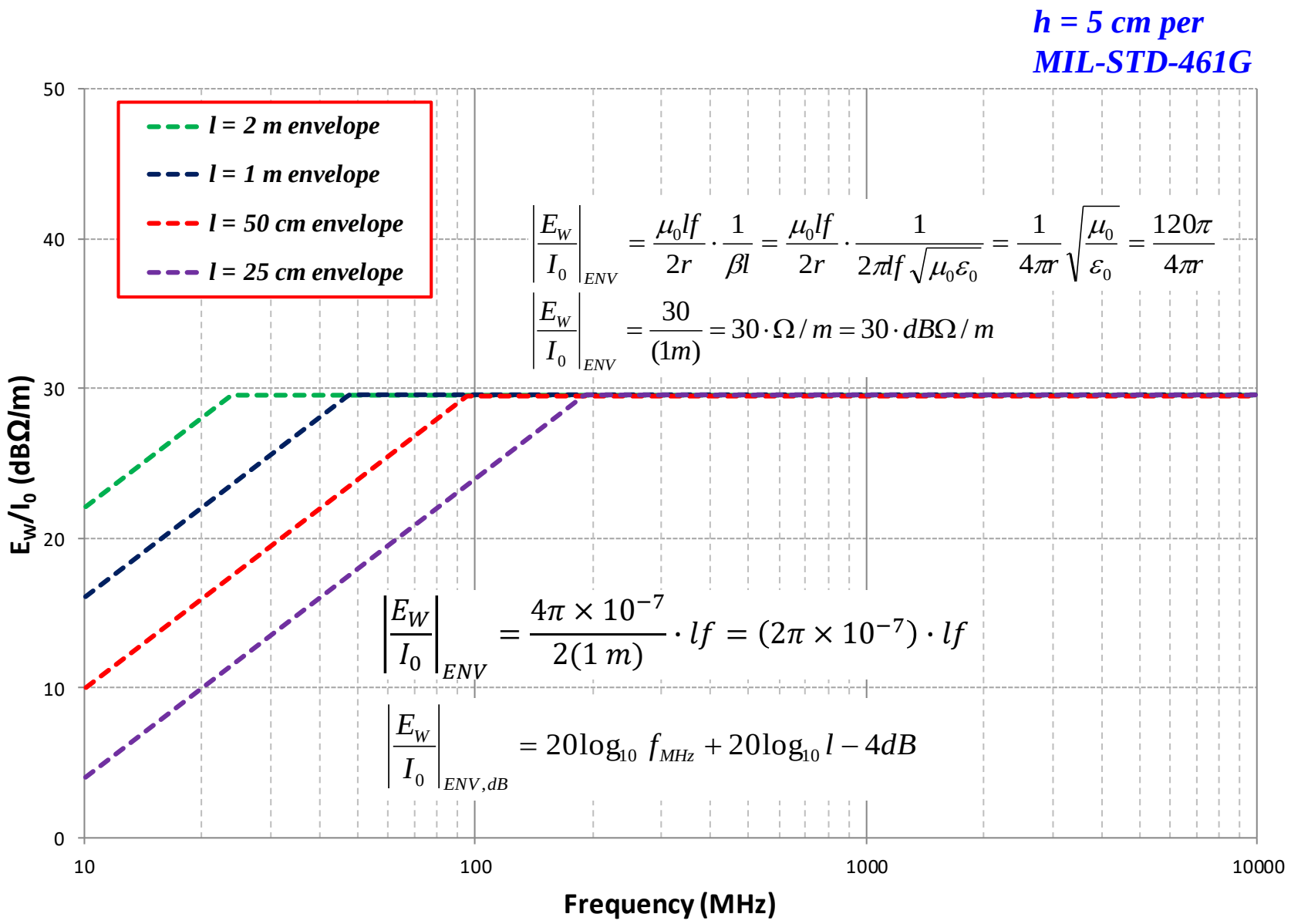
1 for x ≤ 1

1/x for x ≥ 1





Horizontal E-Field (No Ground Plane Attenuation)

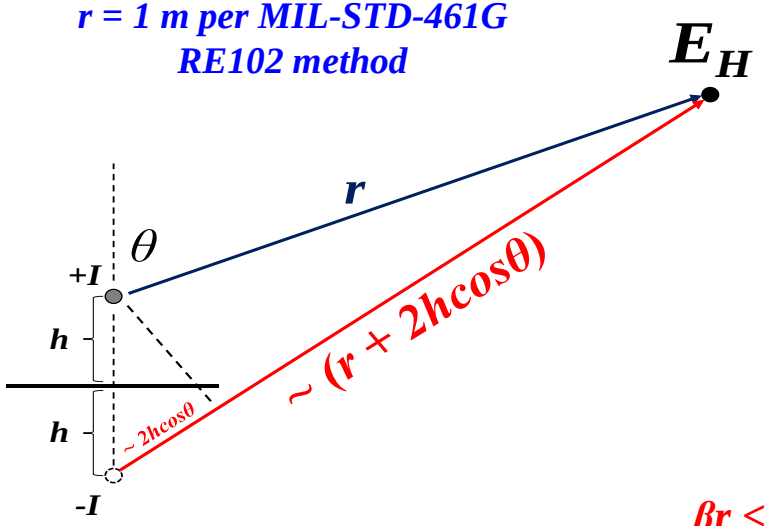


Ground Plane Attenuation (Horizontal Polarization Only)

h = 5 cm per MIL-STD-461G

r = 1 m per MIL-STD-461G

RE102 method



$$E_H = \frac{\mu_0 lf}{2r} I_H e^{-j\beta r} - \frac{\mu_0 lf}{2(r+2h\cos\theta)} I_H e^{-j\beta(r+2h\cos\theta)}$$

$$E_H = \frac{\mu_0 lf}{2r} I_H \left[e^{-j\beta r} - \left(\frac{r}{r+2h\cos\theta} \right) e^{-j\beta(r+2h\cos\theta)} \right]$$

$$E_H = E_W \left[e^{-j\beta r} - \left(\frac{r}{r+2h\cos\theta} \right) e^{-j\beta(r+2h\cos\theta)} \right]$$

where: $E_W = \frac{\mu_0 lf}{2r} I_H$ *Electric field from current on bare wire*

Setting θ = 0 for worst-case and taking real part:

$$\left| \frac{E_H}{E_W} \right| = \left| \cos\beta r - \frac{r}{(r+2h)} \cos[\beta(r+2h)] \right|$$

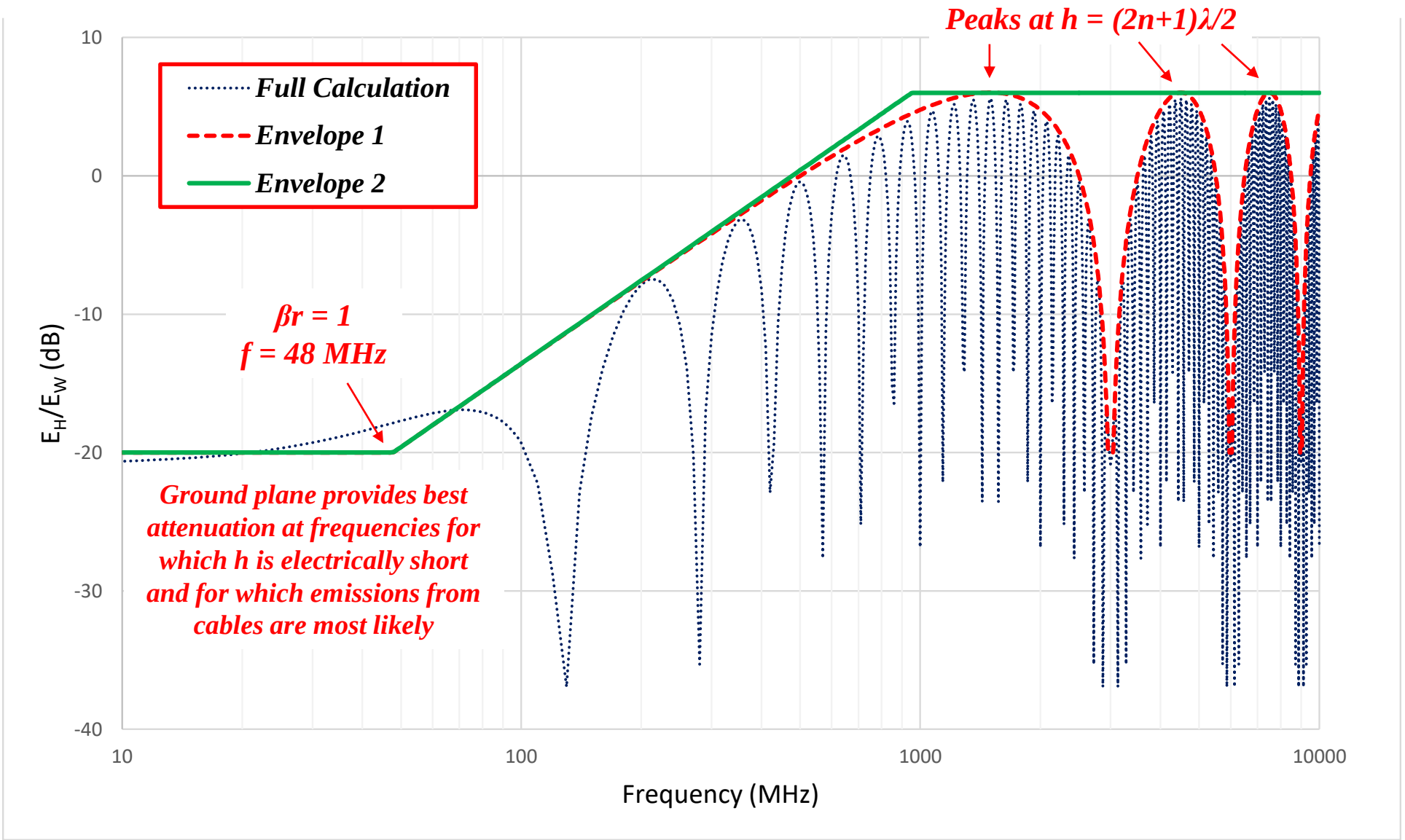
βr ≤ 1: $\left| \frac{E_H}{E_W} \right|_{ENV} \approx 1 - \frac{r}{(r+2h)} = \frac{2h}{(r+2h)} \approx 0.1 \approx -20dB$

βr ≥ 1: $\left| \frac{E_H}{E_W} \right| \approx \left| \cos\beta r - \cos[\beta(r+2h)] \right|$

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ → $\left| \frac{E_H}{E_W} \right| \approx \left| 2\sin[\beta(r+h)]\sin(\beta h) \right|$

$$\left| \frac{E_H}{E_W} \right|_{ENV} \approx \left| 2\sin(\beta h) \right| = +6\ dB\ max$$

Ground Plane Attenuation (Gain, Really)

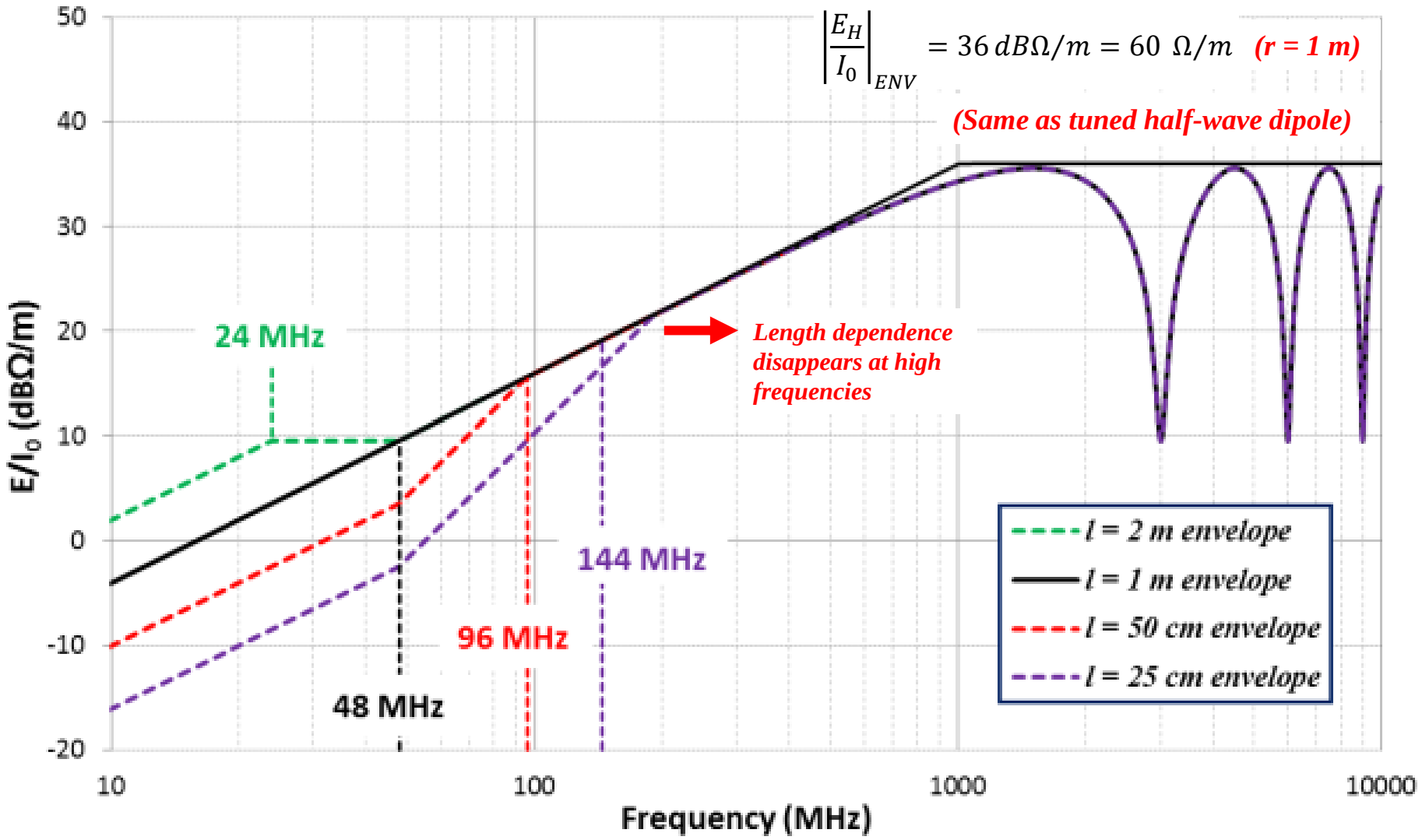


$|E_H/I_0|$ with Ground Plane Attenuation

High frequency asymptote:

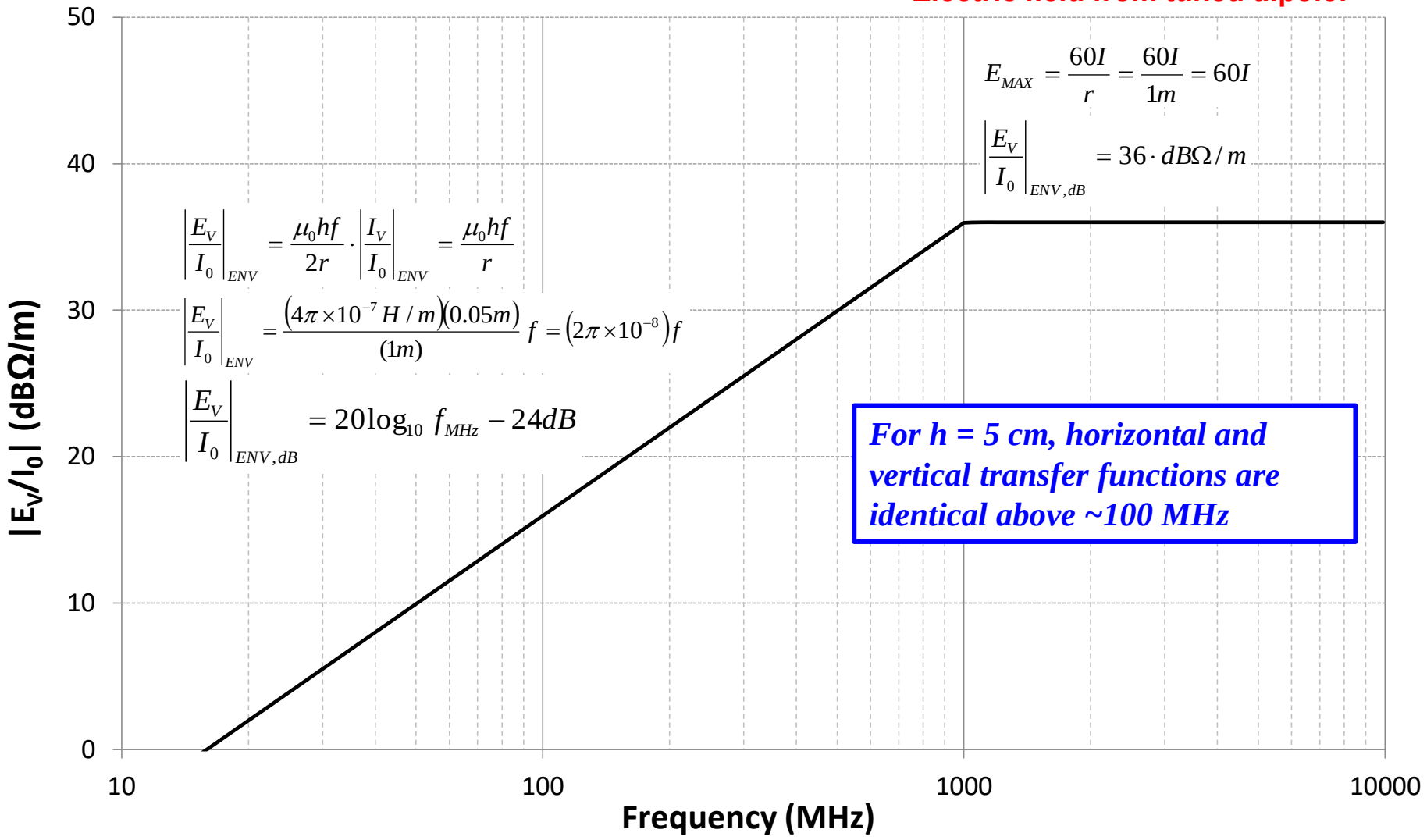
$$\left|\frac{E_H}{I_0}\right|_{ENV} = 36\ dB\Omega/m = 60\ \Omega/m$$ ***(r = 1 m)***

(Same as tuned half-wave dipole)



Vertical: $|E_V/I_0|$

Electric field from tuned dipole:



Linear Dipole Impedance

$$P_{av} = \frac{1}{2} E_\theta H_\phi^* = \frac{15 I_m^2}{\pi \cdot R^2} \cdot F^2(\theta)$$

$$P_r = \int_0^{2\pi} \int_0^{\pi} P_{av} R^2 \sin\theta \cdot d\theta \cdot d\phi$$

$$P_r = 30 I_m^2 \int_0^{\pi} \sin\theta \cdot F^2(\theta) d\theta$$

$$P_r = 30 I_m^2 \underbrace{\int_0^{\pi} \frac{\left[\cos(\beta_0 h \cos\theta) - \cos\beta_0 h\right]^2}{\sin\theta} \cdot d\theta}$$

Evaluated numerically

For half-wave dipole, *2h = λ/2, $\beta_0 h = \pi/2$:*

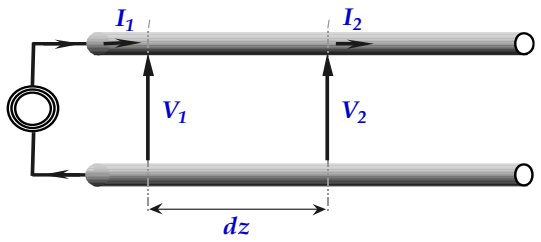
$$P_r \cong 30 I_m^2 (1.218)$$

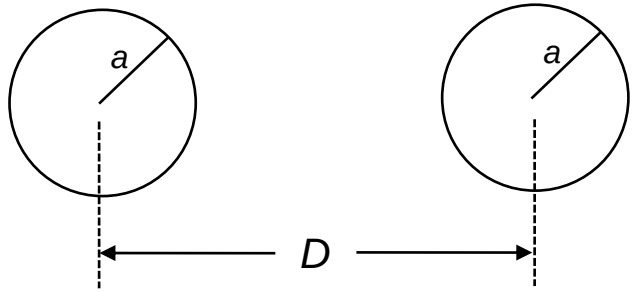
$$P_r \cong 36.54 I_m^2$$

$$P_r = \frac{1}{2} I_m^2 R_r$$

$$R_r \cong 73.1\Omega$$

2-Wire Transmission Line Parameters





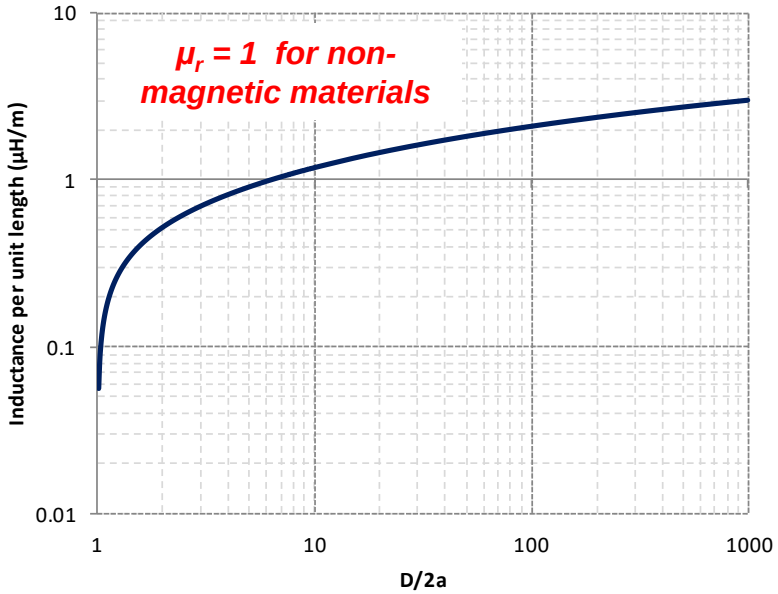
Inductance per unit length:

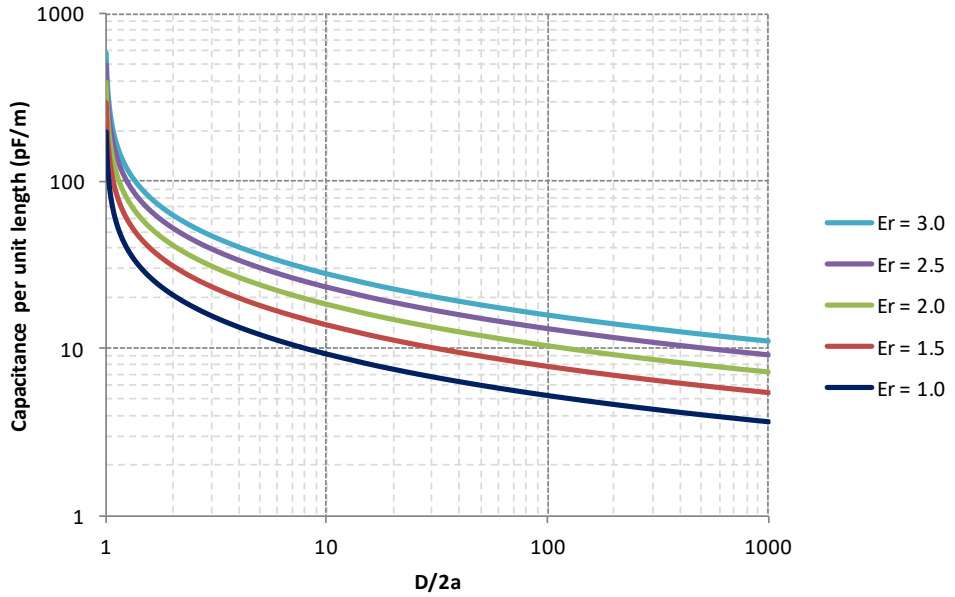
$$\frac{L}{l} = \frac{\mu_r \mu_0}{\pi} \cdot \ln\left[(D/2a) + \sqrt{(D/2a)^2 - 1}\right]$$

D/2a dependence

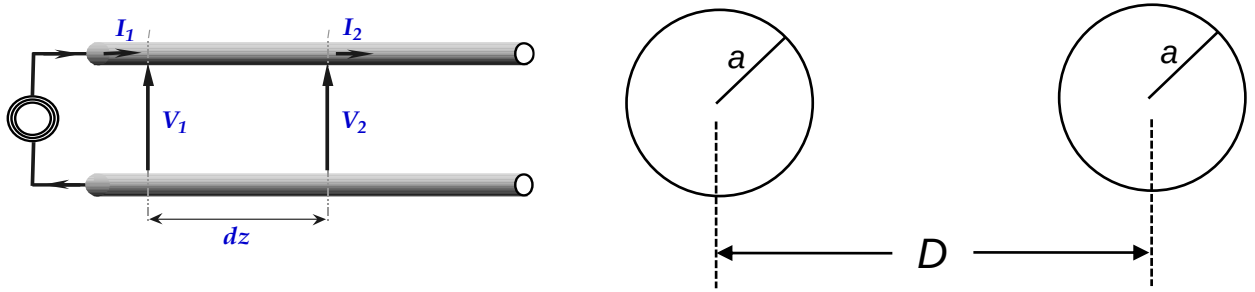
Capacitance per unit length:

$$\frac{C}{l} = \frac{\pi \varepsilon_r \varepsilon_0}{\ln\left[(D/2a) + \sqrt{(D/2a)^2 - 1}\right]}$$





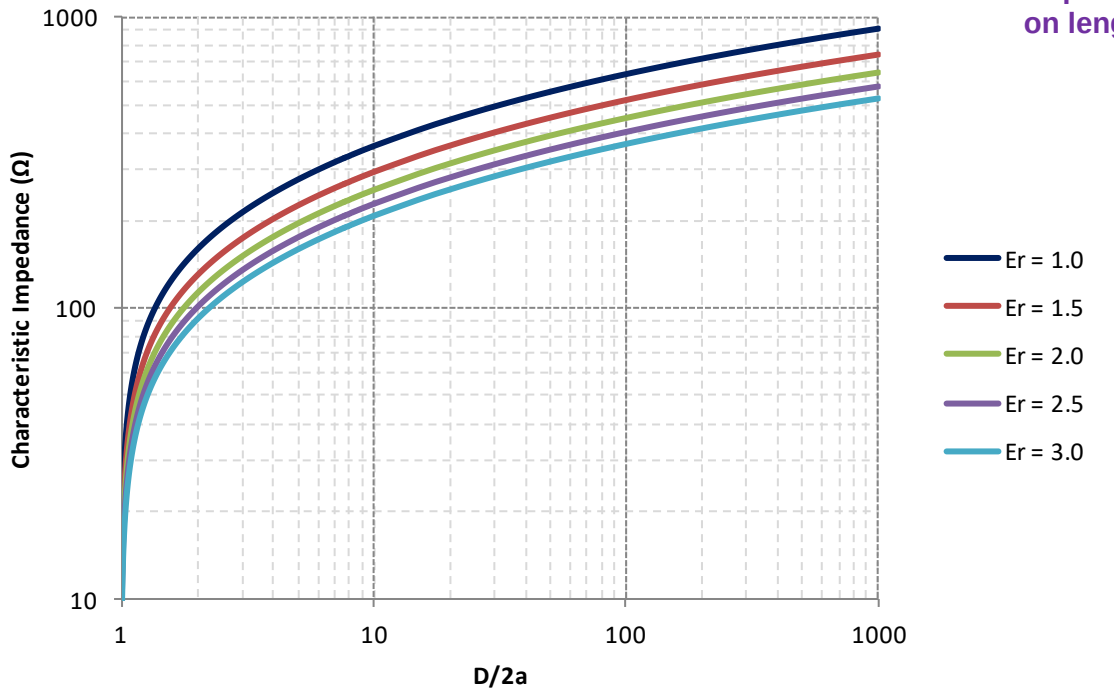
2-Wire Transmission Line Parameters (cont.)



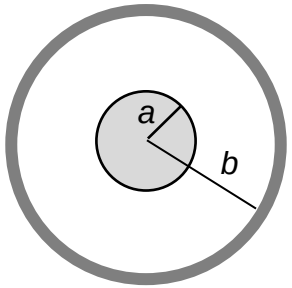
$$Z_0 = \sqrt{\frac{L}{C}} = 120 \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot \ln\left[(D/2a) + \sqrt{(D/2a)^2 - 1}\right]$$

D/2a dependence

No dependence on length



Coaxial Transmission Line Parameters



Inductance per unit length (H/m):

$$\frac{L}{l} = \frac{\mu_r \mu_0}{2\pi} \cdot \ln\left(\frac{b}{a}\right)$$

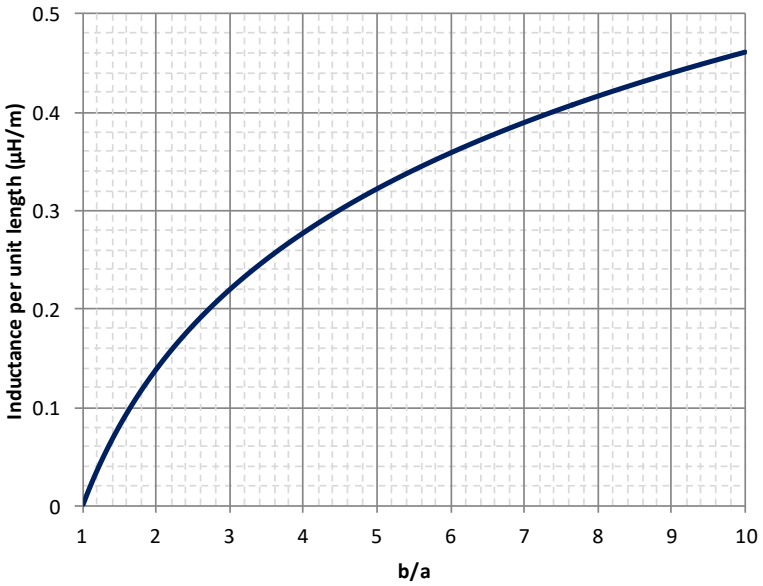
$\mu_r = 1$

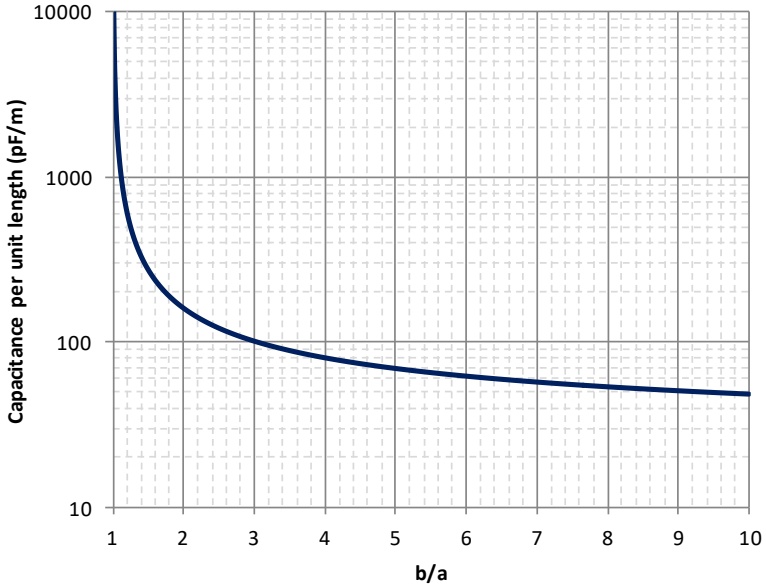
$\varepsilon_r = 2$

(typ.)

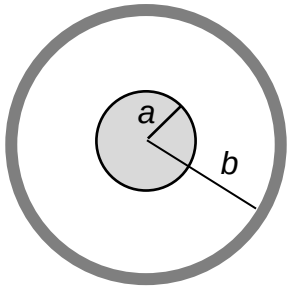
Capacitance per unit length (F/m):

$$\frac{C}{l} = \frac{2\pi\varepsilon_r \varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$



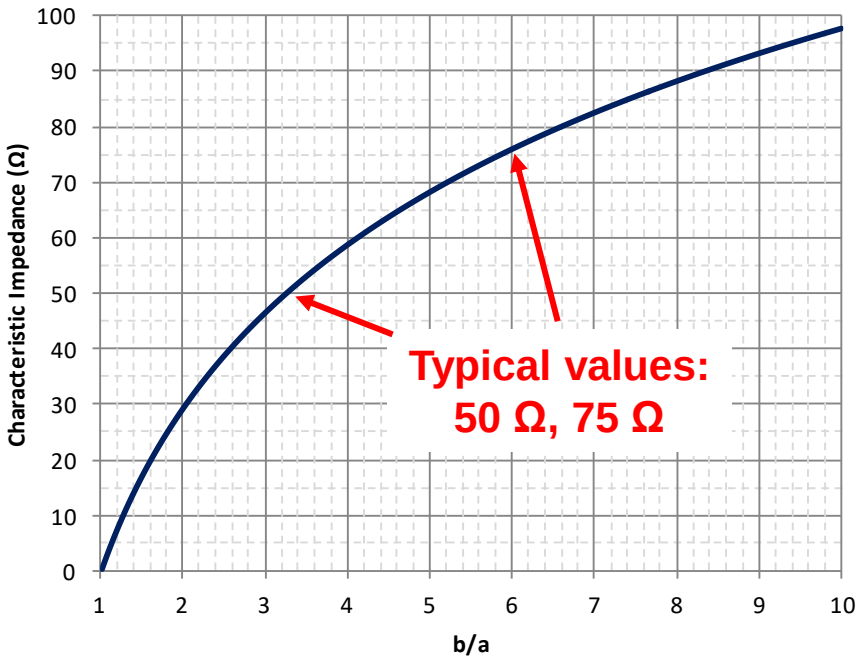


Coaxial Transmission Line Parameters (cont.)



Characteristic impedance (Ω):

$$Z_0 = \sqrt{\frac{L}{C}} = 60 \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot \ln\left(\frac{b}{a}\right)$$



$\mu_r = 1$

$\varepsilon_r = 2$

(typ.)

Wire-Above-Ground Transmission Line Parameters

Inductance per unit length:

$$L = \frac{\mu_0}{2\pi} ln\left[\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right]$$

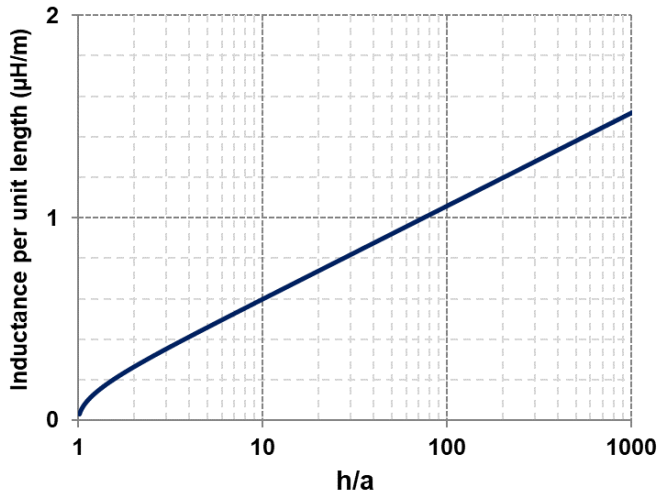
Capacitance per unit length:

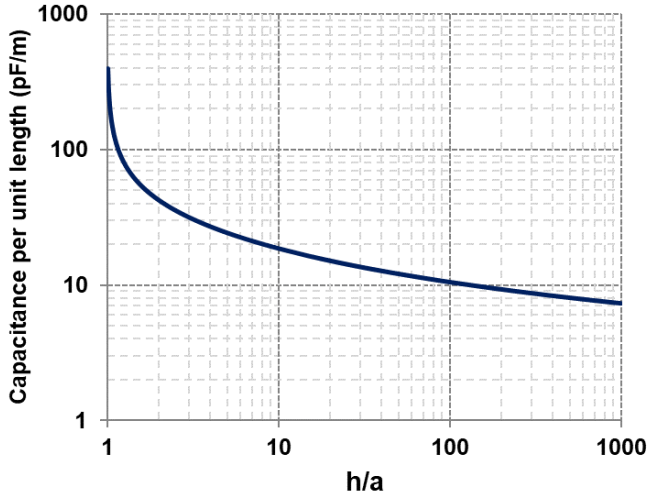
$$C = \frac{2\pi\varepsilon_0}{ln\left[\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right]}$$

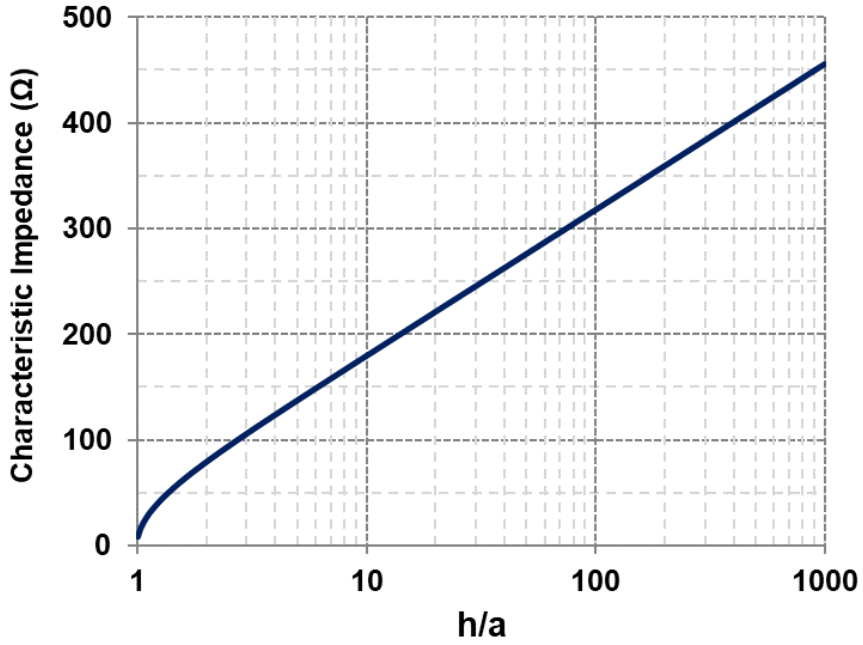
Characteristic impedance:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}} ln\left[\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right]$$

μ_0 = permeability of free space = 4π x 10^{-7} H/m
ε_0 = permittivity of free space = 1/(36π) x 10^{-9} F/m
h = height of wire (cable) above ground plane
a = wire (cable) radius







Field-to-Wire Coupling: The Math

Faraday's Law: $|V| = \left|\frac{d\Phi}{dt}\right|$

"h" used for height of wire above ground plane

$$B = B_0 e^{j(\omega t - \beta z)} = \mu H_0 e^{j(\omega t - \beta z)} \qquad H_0 = \frac{E_0}{\eta} = \frac{E_0}{\sqrt{\mu/\varepsilon}}$$

$$B = \frac{\mu E_0}{\sqrt{\mu/\varepsilon}} e^{j(\omega t - \beta z)} = \sqrt{\mu\varepsilon}\cdot E_0 e^{j(\omega t - \beta z)} = \frac{E_0}{c} e^{j(\omega t - \beta z)}$$

B put in terms of E ***(In terms of culprit current for crosstalk)***

Total integrated magnetic flux: $\Phi = \frac{hE_0}{c}\int_0^l e^{j(\omega t - \beta z)} dz = -\frac{hE_0}{\beta c}\left[e^{j(\omega t - \beta l)} - e^{j\omega t}\right]$

$$V = \frac{d\Phi}{dt} = -\frac{j\omega h E_0}{\beta c}\left[e^{j(\omega t - \beta l)} - e^{j\omega t}\right]$$

$$V = -jhE_0\left[e^{j(\omega t - \beta l)} - e^{j\omega t}\right]$$

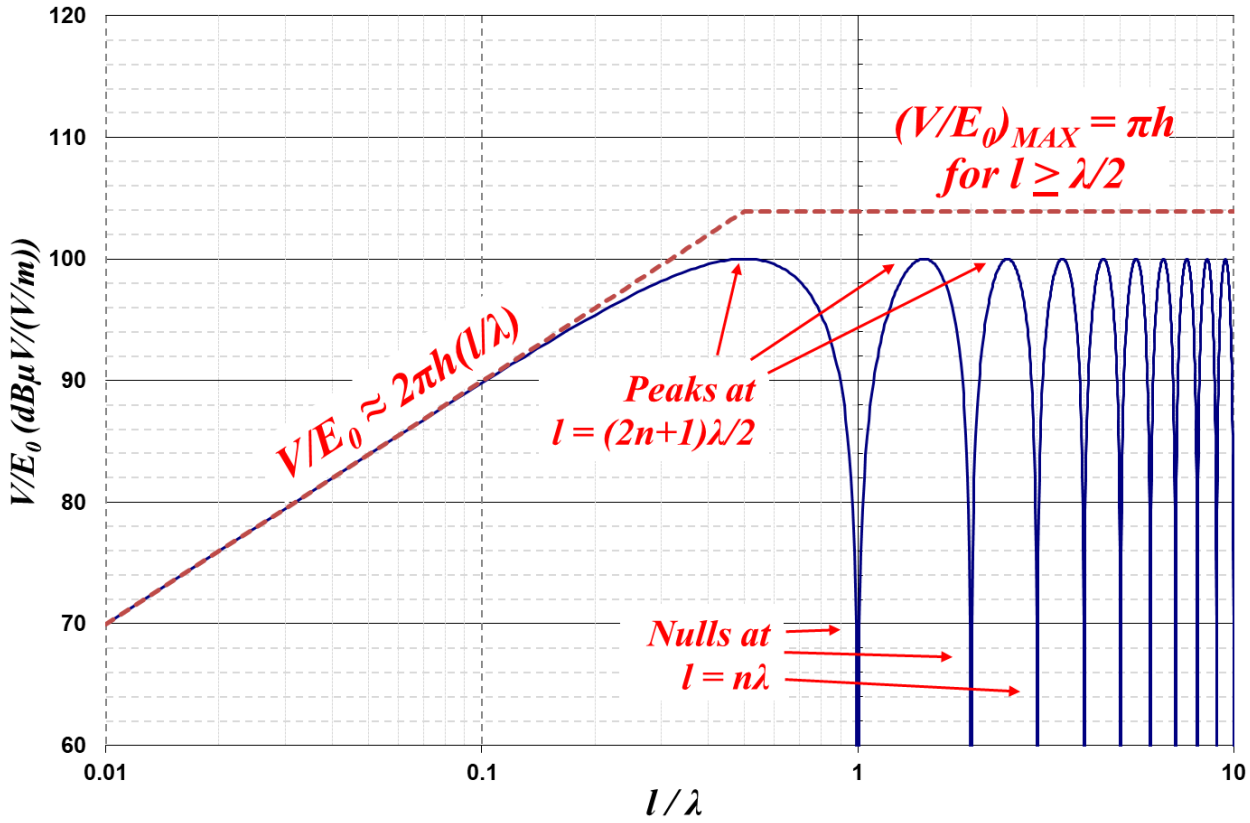
$$V = hE_0[-\sin(\omega t - \beta l) + \sin(\omega t)] \Rightarrow \sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$V = 2hE_0 \sin\left(\frac{\beta l}{2}\right)\cos(2\omega t - \beta l)$$

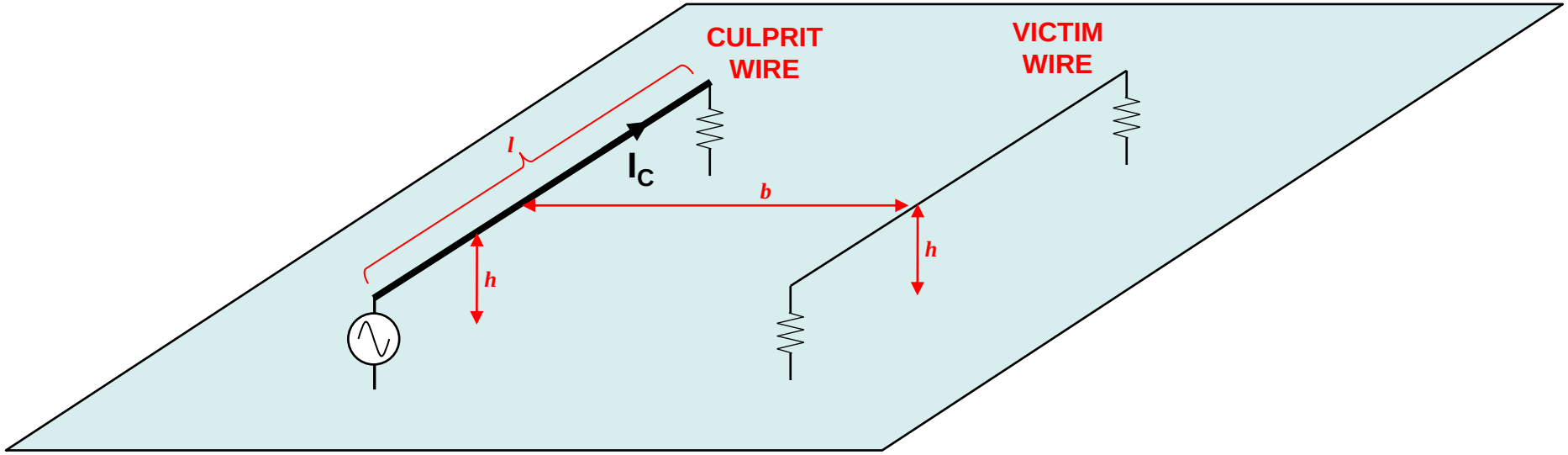
Maximum occurs when time dependent term = 1

$$\frac{V}{E_0} = 2h\sin\left(\frac{\beta l}{2}\right) \Rightarrow \beta = \frac{2\pi}{\lambda}$$

$$\boxed{\frac{V}{E_0} = 2h\sin\left(\frac{\pi l}{\lambda}\right)}$$

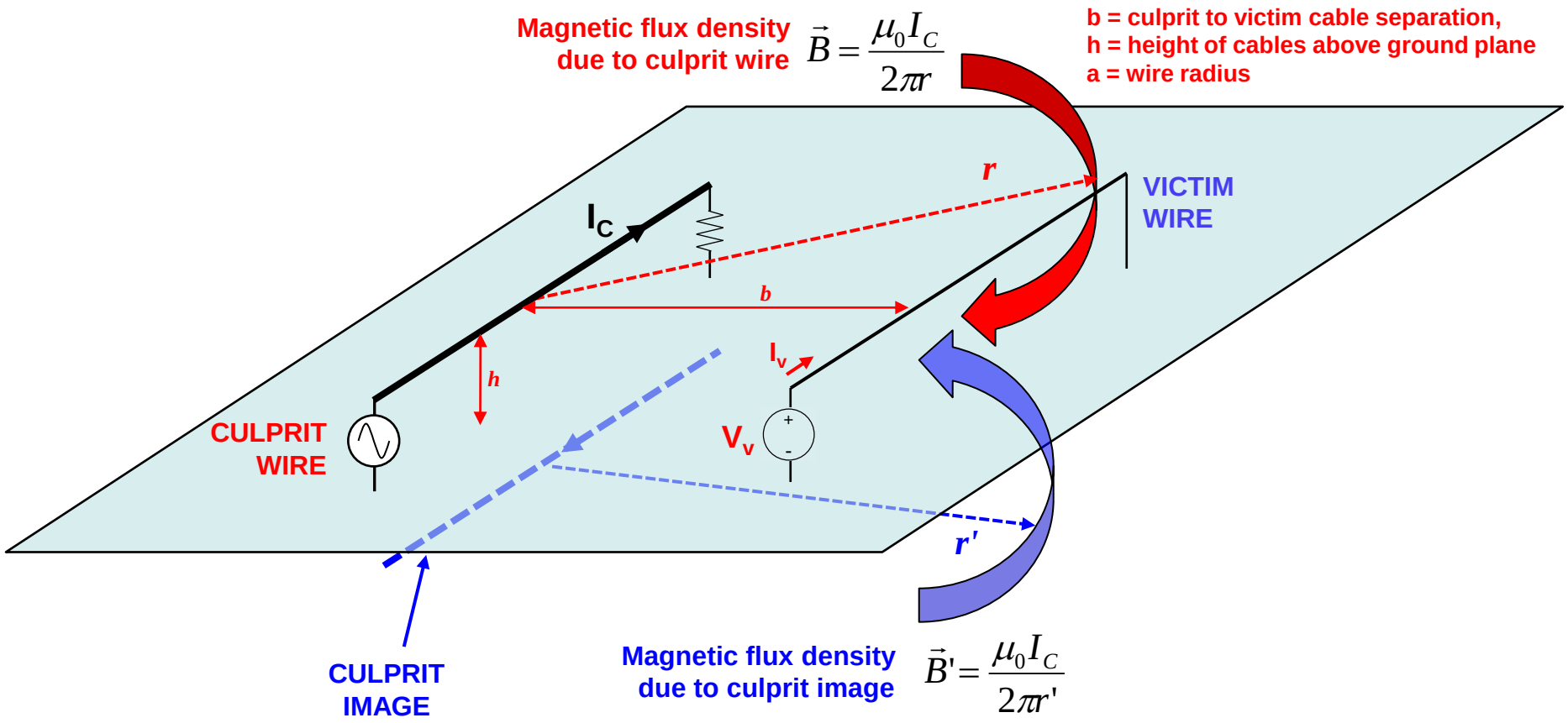


Inductive (Magnetic Field) Crosstalk



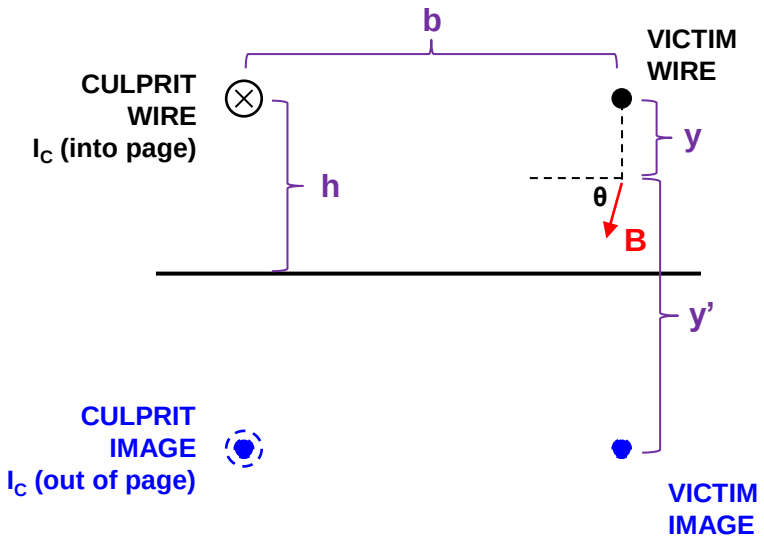
I_C = current on culprit wire
b = culprit to victim wire separation
h = height of wires above ground plane
a = wire radius
l = length of parallel run of culprit and victim wires

Inductive (Magnetic Field) Crosstalk (cont.)



Inductive Crosstalk (cont.)

b = culprit to victim cable separation
h = height of culprit & victim cables above ground plane
a = wire radius



Magnetic flux density at distance y from victim wire:

$$\vec{B}(y) = \frac{\mu_0 I_C}{2\pi\sqrt{b^2 + y^2}}$$

Differential magnetic flux per unit length through victim circuit:

$$\frac{d\Phi}{l} = \vec{B}\cdot d\vec{s} = \frac{\mu_0 I_C}{2\pi}\cdot\frac{dy\cdot\cos\theta}{\sqrt{b^2+y^2}} = \frac{\mu_0 I_C}{2\pi}\cdot\frac{y\cdot dy}{b^2+y^2}$$

Magnetic flux per unit length through victim circuit:

$$\frac{\Phi}{l} = \frac{\mu_0 I_C}{2\pi}\left[\underbrace{\int_0^h \frac{y}{b^2+y^2}\cdot dy}_{\text{CULPRIT WIRE CONTRIBUTION}} + \underbrace{\int_h^{2h} \frac{y'}{b^2+y'^2}\cdot dy'}_{\text{IMAGE CONTRIBUTION}}\right]$$

$$= \frac{\mu_0 I_C}{4\pi}\cdot\left[\ln\left(b^2+y^2\right)\Big|_a^h + \ln\left(b^2+y'^2\right)\Big|_h^{2h-a}\right]$$

$$= \frac{\mu_0 I_C}{4\pi}\left[\ln\left(\frac{b^2+h^2}{b^2+a^2}\right) + \ln\left(\frac{b^2+(2h-a)^2}{b^2+h^2}\right)\right]$$

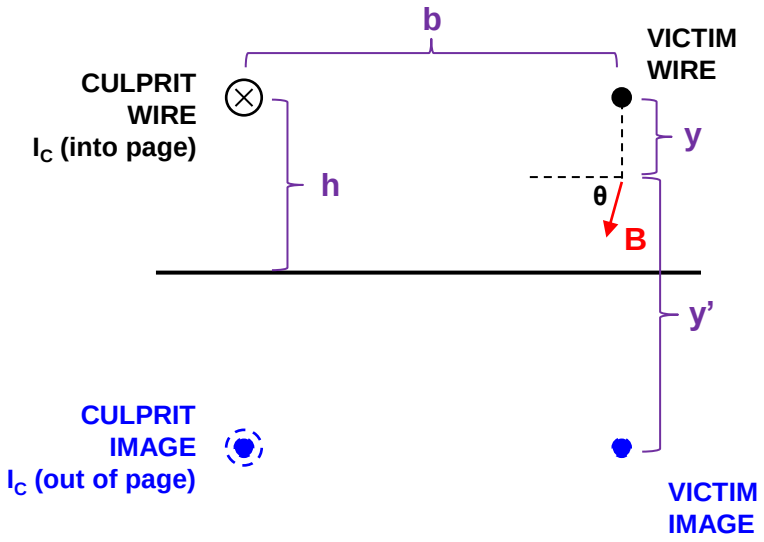
$$= \frac{\mu_0 I_C}{4\pi}\ln\left(\frac{b^2+(2h-a)^2}{b^2+a^2}\right)$$

Mutual inductance per unit length:

$$L_M = \frac{\Phi}{I_C l} = \frac{\mu_0}{4\pi}\ln\left(\frac{b^2+(2h-a)^2}{b^2+a^2}\right)$$

Inductive Crosstalk (cont.)

b = culprit to victim cable separation
h = height of culprit & victim cables above ground plane
a = wire radius



Mutual inductance per unit length:

$$L_M = \frac{\Phi}{I_C l} = \frac{\mu_0}{4\pi} \ln\left(\frac{b^2 + (2h-a)^2}{b^2 + a^2}\right)$$

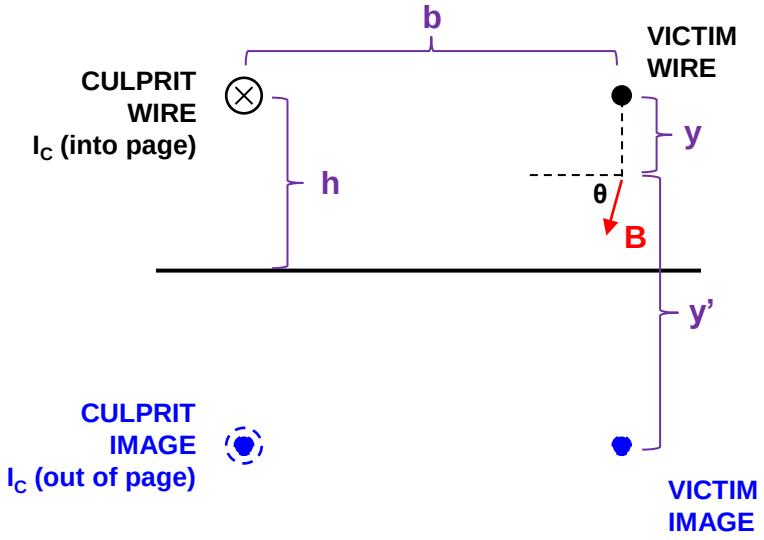
Potential coupled into victim circuit:

$$V_V = -\frac{d\Phi}{dt} = -L_M \frac{dI_C}{dt} l = -j\omega L_M I_C l = -j2\pi f L_M I_C l$$

$$\boxed{V_V = -j2\pi f L_M I_C l}$$

Inductive Crosstalk – Electrically Short Model

b = culprit to victim cable separation
h = height of culprit & victim cables above ground plane
a = wire radius



For electrically short ($l << \lambda$) shielded bulk cable:
R_{NE}, R_{FE} = 0
Impedance of victim circuit is purely inductive
Self inductance per unit length of victim circuit (a = cable radius, h = height above ground plane):

$$\frac{L_V}{l} = \frac{\mu_0}{2\pi} \cdot \ln\left[(h/a) + \sqrt{(h/a)^2 - 1}\right]$$

Current coupled into victim circuit:

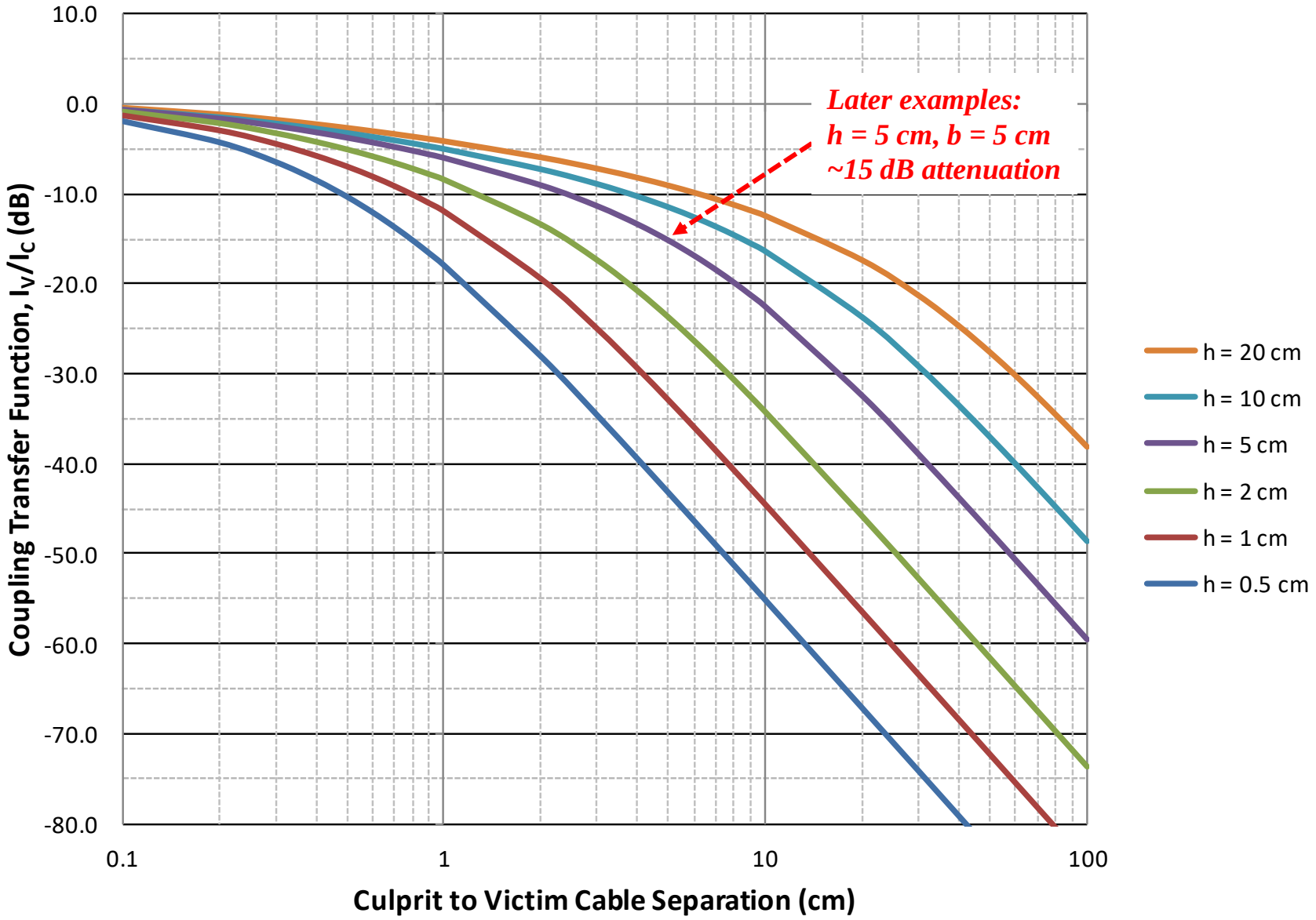
$$|I_V| = \frac{V_V}{j2\pi f L_V} = \frac{j2\pi f L_M I_C}{j2\pi f L_V} = \frac{L_M I_C}{L_V}$$

Frequency dependence cancels out

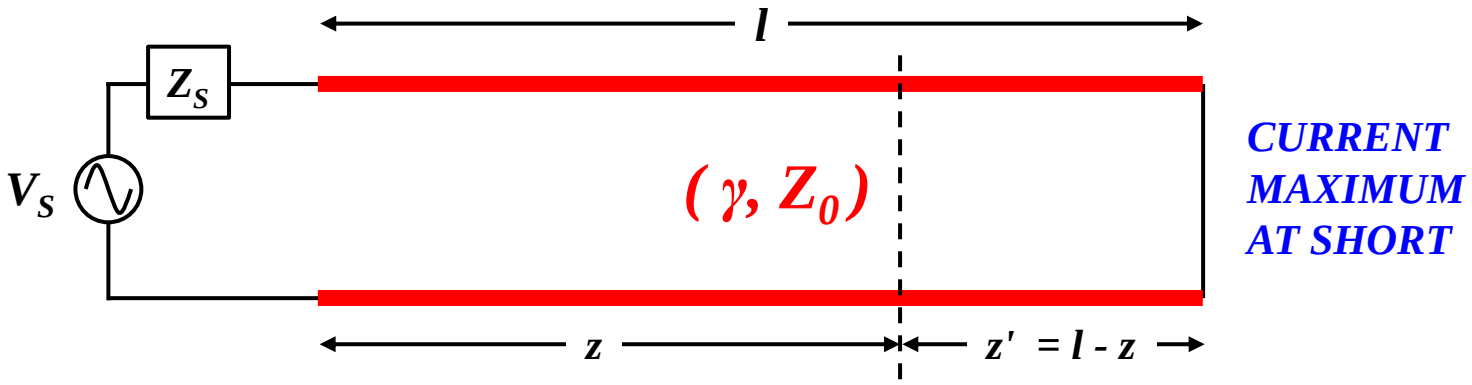
Coupling Transfer Function:

$$\left|\frac{I_V}{I_C}\right| = \frac{L_M}{L_V} = \frac{\ln\left(\frac{b^2 + (2h-a)^2}{b^2 + a^2}\right)}{2 \cdot \ln\left[(h/a) + \sqrt{(h/a)^2 - 1}\right]}$$

Inductive Crosstalk – Electrically Short Model (cont.)



Transmission Line Current Distribution (cont.)



CURRENT MAXIMUM AT SHORT

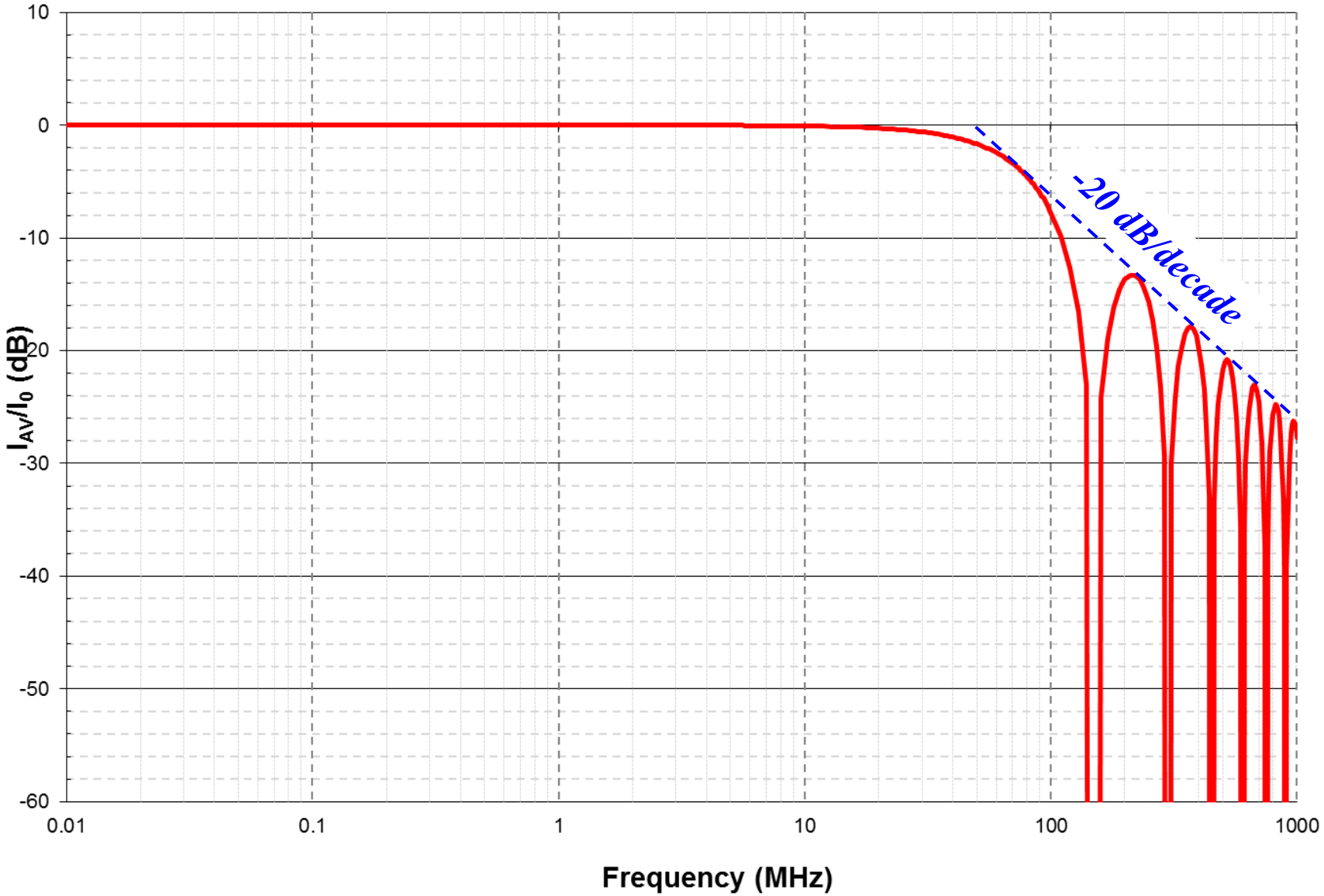
Average current:

$$I_{AV} = \frac{1}{l}\int_0^l I_0 \cos\beta z' dz' = \frac{I_0}{\beta l}\sin\beta z'\Big|_0^l = I_0\frac{\sin\beta l}{\beta l}$$

$$\beta l \le 1: \quad \left|\frac{I_H}{I_0}\right|_{ENV} = 1$$

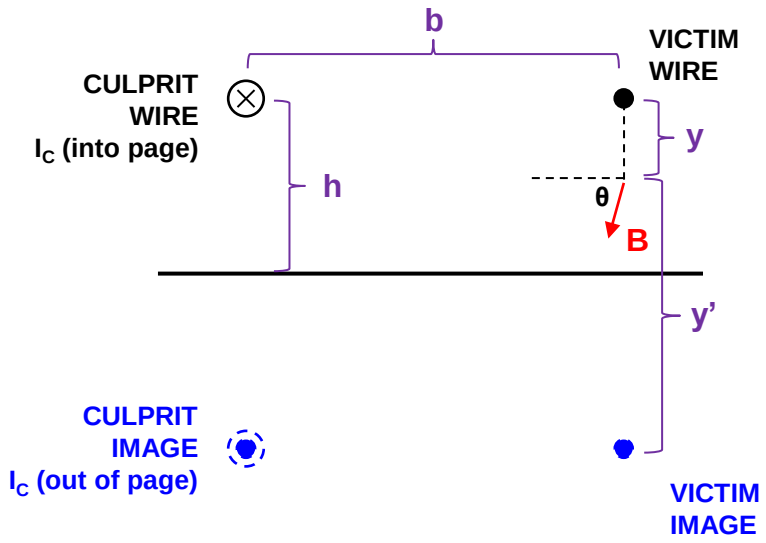
$$\beta l \ge 1: \quad \left|\frac{I_H}{I_0}\right|_{ENV} = \frac{1}{\beta l}$$

Average Current Example: h = 5 cm, l = 1 m



Coupled Voltage Due to Average Current

b = culprit to victim cable separation
h = height of culprit & victim cables above ground plane
a = wire radius



Mutual inductance per unit length:

$$L_M = \frac{\Phi}{I_C l} = \frac{\mu_0}{4\pi} \ln\left(\frac{b^2 + (2h-a)^2}{b^2 + a^2}\right)$$

Potential coupled into victim circuit:

$$V_V = -\frac{d\Phi}{dt} = -L_M \frac{dI_C}{dt} l = -j\omega L_M I_C l = -j2\pi f L_M I_C l$$

$$\boxed{V_V = -j2\pi f L_M I_C l}$$

For $l > 0.1\lambda$,
$I_C = \sim I_{AV}$

I_V/I_C for various geometries, $\beta l < 1$

$$\frac{I_V}{I_C} \approx \frac{L_M}{L_V} = \frac{ln\left[\frac{b^2+(2h-a)^2}{b^2+a^2}\right]}{2 \cdot ln\left[\frac{h}{a}+\sqrt{\left(\frac{h}{a}\right)^2-1}\right]}$$

