Error mitigation protocol for multi-photon noise in quantum repeater chains

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Outline

Part I - Entanglement distribution and multi-photon noise

'Practical' repeater architecture

Multi-photon noise

Part II - Alternating Bell state measurement (ABSM) Protocol

Part III - Analysis of ABSM repeater chain

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'Practical' repeater architecture

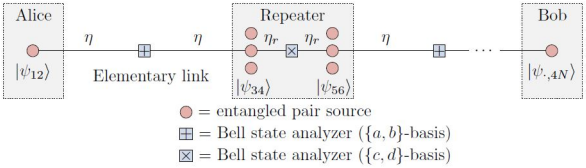
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Repeater architectures with minimal quantum processing

We consider quantum repeater architectures with **minimal quantum processing:**



1. Photon sources produce **dual-rail Bell pairs**
2. Repeaters with **unheralded quantum memory** (e.g. delay-line)
3. Repeaters are **highly multiplexed** with active switching
4. Entanglement swapping via **probabilistic Bell state measurement** (BSM)

Represents a "practical" repeater architecture, as studied by Sinclair *et al.* (2014), Guha *et al.* (2015), Krovi *et al.* (2016), and others.

Target is high fidelity F and mode efficiency η_{AB} (ebits per receiver mode-pair).

Dual-rail photonic qubits

A dual-rail qubit consists of a single photon in one of two modes $\{a_i, b_i\}$ with $[a_i, b_i] = 0$

$$\alpha_{10}|\mathbf{0}\rangle + \alpha_{01}|\mathbf{1}\rangle = (\alpha_{10} a_i^\dagger + \alpha_{01} b_i^\dagger)|0\rangle_i$$

- **Vacuum** $1|0\rangle_i$ and **multi-photon** states $a_i^2, b_i^2, a_i b_i, ...$, outside qubit space

$$|\psi\rangle = (... + \bar{\alpha}_{20} a_i^2 + \bar{\alpha}_{11} a_i b_i + \bar{\alpha}_{02} b_i^2 + \bar{\alpha}_{10} a_i + \bar{\alpha}_{01} b_i + \bar{\alpha}_{00} 1)^\dagger |0\rangle_i$$

- Optical losses/vacuum convert multi-photon noise into qubit errors

Entangled two-mode squeezed vacuum (TMSV)

Photonic state from parametric down-conversion modeled as a product of two TMSV states:

$$|\psi_{ij}\rangle = (1-|\lambda|^2)\sum_{n=0}^{\infty}\lambda^n(a_i^\dagger b_j^\dagger - b_i^\dagger a_j^\dagger)^n|0\rangle$$
$$= (1-|\lambda|^2)\Big(\ldots + \bar{\lambda}^2(a_i^2 b_j^2 + b_i^2 a_j^2 - 2a_i b_i a_j b_j) + \bar{\lambda}(a_i b_j + b_i a_j) + 1\Big)^\dagger|0\rangle$$

- Isolating single-pair emission yields Bell state

$$|\Psi_{ij}^-\rangle = \frac{1}{\sqrt{2}}(a_i^\dagger b_j^\dagger - b_i^\dagger a_j^\dagger)|0\rangle$$

- If secondary photons are lost, double-pair emission reduces to a statistical mixture of a **Bell state** and a completely **mixed 2-photon state**:

$$\frac{2}{3}|\Psi_{ij}^-\rangle\langle\Psi_{ij}^-| + \frac{1}{3}\hat{\rho}_{\text{mix}}$$

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Entanglement swapping with PDC sources

Balanced entanglement swap with PDC sources with pair generation probability $p \ll 1$:

- BSM triggered by 2-photon state with opposite polarizations $a^\dagger b^\dagger |0\rangle$
- Success requires that one photon was received from each adjacent source

$$\text{BSM success probability} \equiv \beta^{(1,1)} = \frac{1}{2} p^2 \eta^2$$

- Equally likely that a single source produced both photons ($p \ll 1$):

$$\text{BSM error probability} \simeq \beta^{(2,0)} + \beta^{(0,2)} = \frac{1}{2} p^2 \eta^2$$

- With vacuum filtering the remaining error arises mainly from 3-photon states

$$\text{Vacuum-filtered BSM error} \simeq \beta^{(2,1)} + \beta^{(1,2)} = O(p^3)$$

An **imbalanced entanglement swap** with PDC sources is dominated by double-pair errors unless pair generation parameter λ is reduced to effectively rebalance the link

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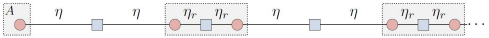
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Entanglement swapping chains with PDC sources

Multi-photon noise in entanglement swapping chains analyzed through sequences $\vec{\nu} = (\nu_1, \nu_2, ..., \nu_{2N})$ describing the number of pairs ν_i emitted by the i-th source.



- Enumerate $(2N+1)$-pair sequences with n_2 double-pairs from $2N$ independent sources:

$\underline{n_2 = 1}$	$\underline{n_2 = 2}$	$\underline{n_2 = 3}$	$\underline{n_2 > 3}$
$(2,1,1,1,...,1)$	$(2,0,2,1,1,1,...,1)$	$(2,0,2,0,2,1,1,...,1)$	
$(1,2,1,1,...,1)$	$(1,2,0,2,1,1,...,1)$	$(1,2,0,2,0,2,1,...,1)$	...
$\vdots$	$\vdots$	$\vdots$	
$(1,1,1,1,...,2)$	$(1,...,1,1,1,2,0,2)$	$(1,...,1,1,2,0,2,0,2)$	

- Photon number resolving (PNR) detectors at repeater nodes can identify errors except boundary sequences $\nu_1 = 2$ or $\nu_{2N} = 2$. Demands high efficiency repeaters $\eta_r \simeq 1$.
- Number of error sequences grows **quadratically** with number of elementary links N
- If all secondary photons are lost, yields relatively high fidelity $F(\vec{\nu}) = \frac{1}{4} + \frac{3}{4}\left(\frac{2}{3}\right)^{n_2}$.

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Double-pairs produced by PDC contain correlations due to stimulated emission

$$\lambda^2(a_1^\dagger b_2^\dagger + b_1^\dagger a_2^\dagger)^2|0\rangle = \sqrt{\frac{p(3)}{3}}\left(|2,0;0,2\rangle + |0,2;2,0\rangle - |1,1;1,1\rangle\right).$$

Expand idler modes in diagonal basis

$$c_2 = \sqrt{1/2}(a_2 + b_2)$$
$$d_2 = \sqrt{1/2}(a_2 - b_2).$$

Measurement of opposite polarizations $a_1^\dagger b_1^\dagger$ projects the idler onto a 2-photon N00N state

$$(b_1^2)^\dagger(c_2^2 + d_2^2 + 2c_2d_2)^\dagger|0\rangle + (a_1^2)^\dagger(c_2^2 + d_2^2 - 2c_2d_2)^\dagger|0\rangle - 2a_1^\dagger b_1^\dagger(c_2^2 - d_2^2)^\dagger|0\rangle.$$

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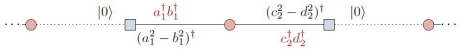
Alternating Bell state measurement (ABSM) protocol

Idea: Exploit stimulated double-pair correlations to reduce errors

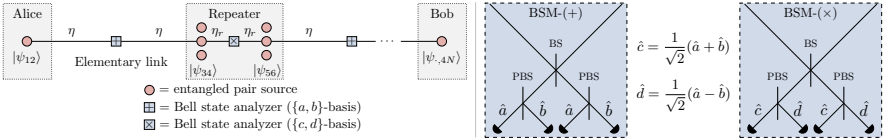
1. BSM can be performed in either basis

$$|\Psi^+\rangle_{ab} = |\Phi^-\rangle_{cd}, \qquad |\Phi^+\rangle_{ab} = |\Phi^+\rangle_{cd}, \qquad |\Psi^-\rangle_{ab} = -|\Psi^-\rangle_{cd}, \qquad |\Phi^-\rangle_{ab} = |\Psi^+\rangle_{cd},$$

2. Probabilistic BSM succeeds only if opposite polarizations are detected ($a_i^\dagger b_i^\dagger$ or $c_i^\dagger d_i^\dagger$).



3. By alternating BSM basis, a single double-pair cannot trigger adjacent BSMs.



Remarks

- Does not prevent errors when one of the photons in the double-pair is lost
- If secondary pair emitted in adjacent spatio-temporal mode, correlations do not exist

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Assumptions

Assumptions for repeater chain with N elementary links:

- Probabilistic sources (based on PDC)
- Probabilistic linear optical BSMs
- Repeaters have PNR detection with efficiency η_r
- Neglect repeater infidelities introduced by quantum memory and mode switching
- Neglect errors due to dark counts
- Bell state fidelity assumes receivers are equipped with vacuum filtering (no PNR)

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Bell state fidelity for balanced chain

The Bell state fidelity produced by a balanced chain is given by

$$F = \frac{1 + p\left(\epsilon_+ + \frac{11+\sigma}{5+\sigma}\epsilon_0\right) + O(p^2)}{1 + 4p(\epsilon_0 + \epsilon_0' + \epsilon_+ + \epsilon_+') + O(p^2)},$$

where p is the pair emission probability and

$$\epsilon_0' = (1-\eta), \qquad \epsilon_+' = \frac{1}{4}(1+\sigma)[1+\sigma(N-2)](1-\eta)$$

$$\epsilon_0 = \frac{N-1}{2}(5+\sigma)(1-\eta)(1-\eta_r), \qquad \epsilon_+ = \frac{N-2}{4}(1+\sigma)^2[2+\sigma(N-3)](1-\eta)(1-\eta_r)$$

with $\sigma = 1$ for the standard BSM protocol and $\sigma = 0$ for the ABSM protocol.

- ABSM protocol eliminates quadratic growth of errors
- Eliminates all errors associated to pair sequences $\vec{\nu}$ with subsequence $(0, 2, 0)$

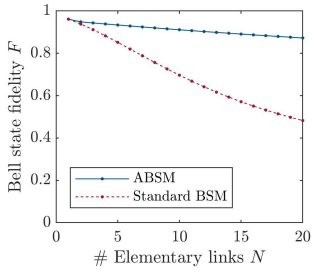
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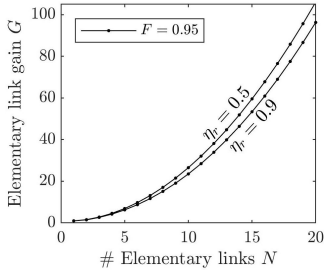
$$F = \frac{1 + p\left(\epsilon_+ + \frac{11+\sigma}{5+\sigma}\epsilon_0\right) + O(p^2)}{1 + 4p(\epsilon_0 + \epsilon_0' + \epsilon_+ + \epsilon_+') + O(p^2)},$$

where p is the pair emission probability.

- Fidelity assuming $p = 0.01$



- Gain for fixed $F = 0.95$



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