



Rotating Rake Mode Measurement System – Signal Collection and Analysis Techniques

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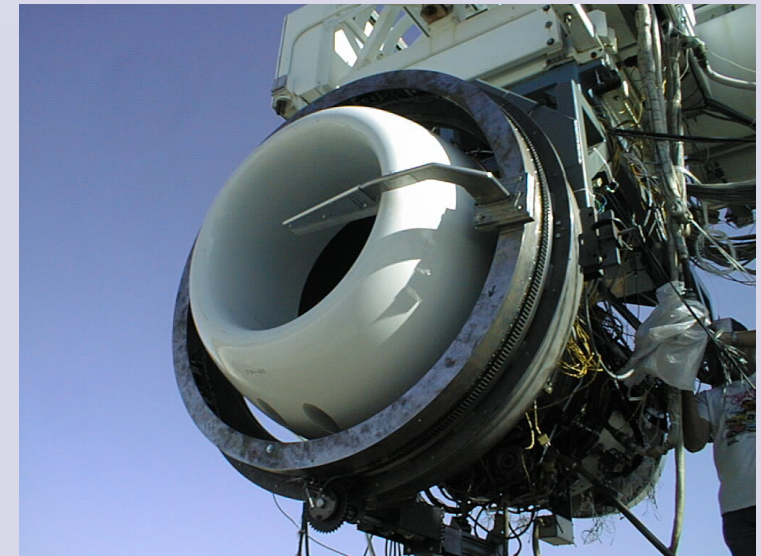


OVERVIEW

ROTATING RAKE: A radial rake inserted into a turbofan inlet or exhaust duct. It has radially distributed pressure transducers. The rake continuously and synchronously rotates about the duct centerline to obtain a time history of the dynamic pressure. A complete acoustic mode map at the fan harmonics is obtained from the signal processing.



- Concept based on idea by T. Sofrin NASA CR-168040 Nov 1982.
- First Implemented by L. Heidelberg on 17-inch ADP model in the NASA Glenn 9x15 Wind Tunnel in 1991 described in AIAA Journal of Aircraft Vol 32, No 4, July-Aug 1995.
- Put into practice and continual improvement on 4' Low Speed Fan dedicated test bed at NASA Glenn.
- Continued use in NASA Glenn 9x15 Wind Tunnel.
- Multiple Engine tests.





ROTATING RAKE CONCEPTS

Continuously rotating, radially distributed from tip to hub array of pressure transducers installed at inlet/exhaust duct acoustic release point.

- Rake electronically very accurately synchronized to fan shaft
- Signals brought across rotating boundary via telemetry

KEY CONCEPT: Each m-order rotates at a different spin rate.

- The rotating rake imparts a unique Doppler shift depending on spin rate, hence m-order.
- **Radial modes in a given m-order complex curve fit using least-squares fit to the derived duct E-functions (eigen-functions, basis functions).**

A complete circumferential and radial modal magnitude & phase map is obtained for the 1st three harmonics.

- (only as accurate as the basis function description)



Presentation Outline

- (I) Description of Acoustic Field that is Measured
 - duct modes
 - rotor/stator interaction

- (II) Rotating Rake Theory and Implementation
 - Doppler shift
 - Radial Curve Fitting
 - Mechanical Parameters

- (III) Examples of Rotating Rake Applications



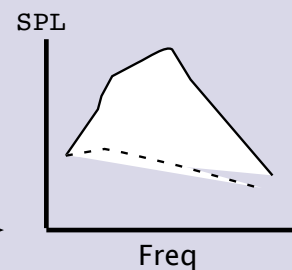
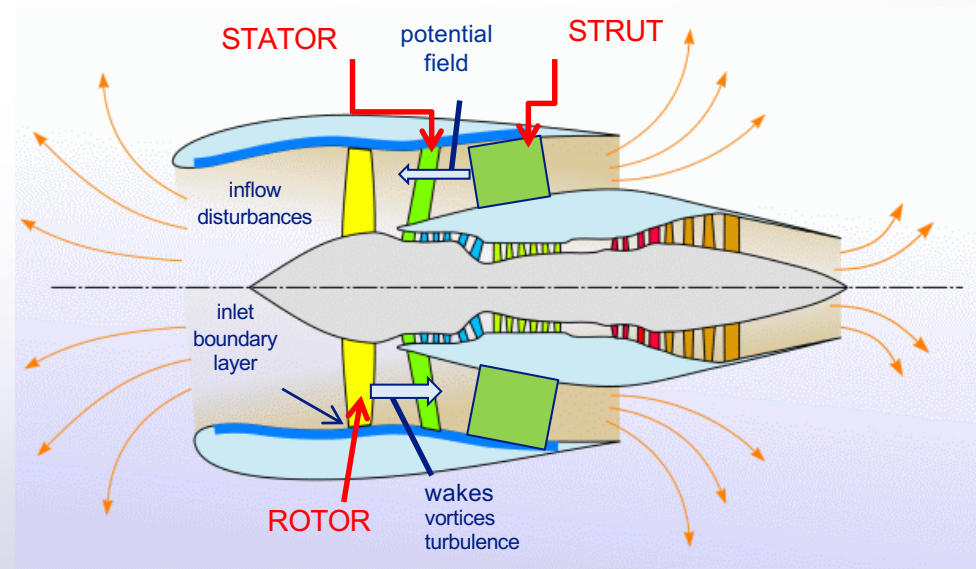
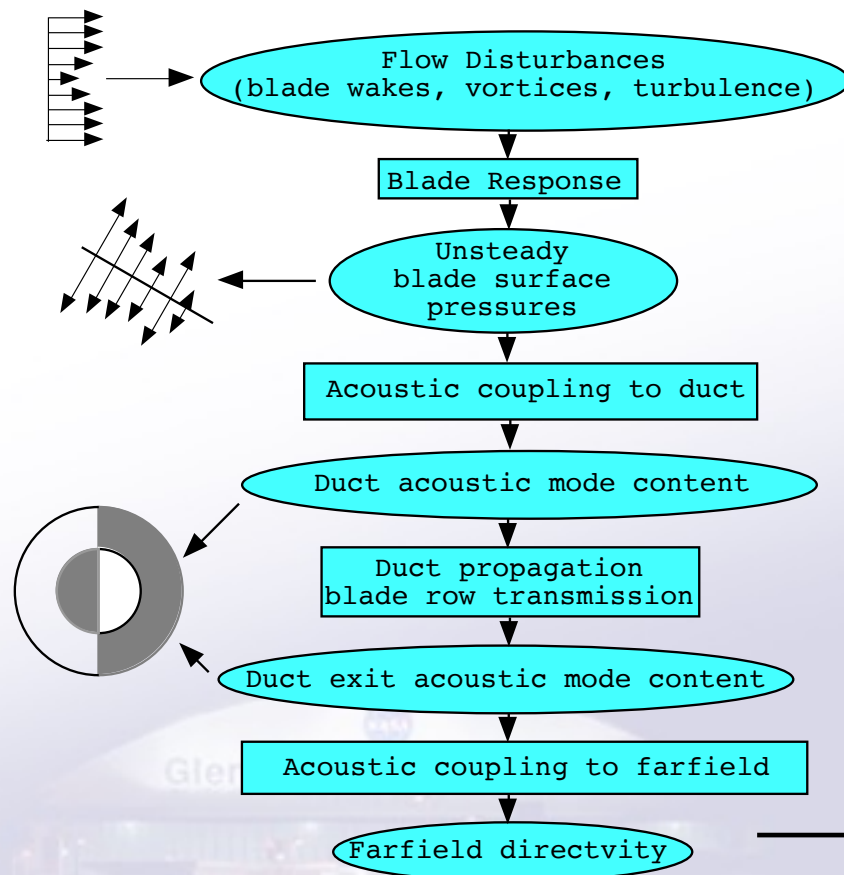
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Physics/M Measurement of Fan/Duct Aero-Acoustics



OVALS: Physically measurable quantity.

RECTANGLES: transfer function that can be investigated for prediction (noise reduction)



Acoustic Wave Equation in a Duct

Wave Equation: $\frac{\partial^2 p}{\partial t^2} + M \frac{\partial^2 p}{\partial x^2} = \nabla^2 p \implies P(\theta, r, x, t) = p_{mnf} * E_{mn}(k_{mn} r) e^{i(\eta t + m\theta \pm \gamma x)}$

Separated solution in the x-direction (wave-number): $\frac{k_x}{\eta} = \frac{1}{(1 - M^2)} \left[-M \pm \sqrt{1 - (1 - M^2) \left(\frac{\kappa}{\eta} \right)^2} \right]$

Separated solution in the θ -direction is the Bessel equation: $P(\theta) = P(\theta + 2\pi); \sin(m\theta)$

Separated solution in the r-direction is the Bessel equation: $\frac{d^2 P}{dr^2} + \frac{1}{r} \frac{dP}{dr} + (\kappa^2 - \frac{m^2}{r^2}) P = 0$

$p(r) = P_{mn} E_{mn}(\kappa_{mn} \tilde{r}) \longrightarrow E_{mn} = C_{mn} [J_m(\kappa_{mn} \tilde{r}) + Q_{mn} Y_m(\kappa_{mn} \tilde{r})]$

J & Y are Bessel functions of the 1st kind
 κ & Q are "eigenvalues" -> modes

C_{mn} coefficient determined by integrating over duct area and normalizing with boundary conditions



Solution to Eigenvalue Problem

For illustrative purposes, $\sigma=0$

hub-to-tip ratio
non-dimensional radius

$$p(r) = P_{mn} E_{mn}(\kappa_{mn} \tilde{r}) E_{mn} = C_{mn} [J_m(\kappa_{mn} \tilde{r})] \quad \frac{1}{C_{mn}^2} = \frac{1}{2} \left\{ \left[\left(1 - \frac{m^2}{\kappa_{mn}^2}\right) J_m^2(\kappa_{mn}) \right] + J_m'^2(\kappa_{mn}) \right\}$$

Eigen-value problem based on boundary conditions:

HARDWALL:

acoustic velocity normal to wall is zero,
momentum equation gives:

$$\frac{dP}{dr} = 0; \quad J_m'(\kappa_{mn}) = 0$$

A set of basis functions for each (m,n) mode.

SOFTWALL: (passive treatment - liner)

$$\frac{p}{v} = \frac{Z}{\rho c}; \quad A = \frac{\rho c}{Z}; \quad \eta = \frac{\omega R}{c}$$

impedance, admittance, reduced freq

$$\frac{dP}{dr} = -i\eta AP \quad \longrightarrow \quad \kappa \frac{J_m'(\kappa)}{J_m(\kappa)} = -i\eta A$$

$$\kappa \frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A$$

$$\kappa \frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A \left(1 - M \frac{k_x}{\eta}\right)^2$$

**A set of basis functions for each (m,n) mode -
for each frequency, impedance, duct Mach.**



Acoustic Duct Modes

Solution to separated wave equation in cylindrical coordinates:

$$P(\theta, r, x, t) = p_{mnf} * E_{mn}(k_{mn}r)e^{-i(2\pi ft + m\theta \pm \gamma x)}$$

2-Dimensions (duct modes) :

circumferential direction:

$$P(\theta) = P(\theta + 2\pi); \quad \sin(m\theta)$$

m is defined as the number of cycles
-circumferential mode, m order

radial direction:

$$E_{mn} = C_{mn}[J_m(\kappa_{mn}\tilde{r}) + Q_{mn}Y_m(\kappa_{mn}\tilde{r})]$$

n is defined as the number of zero crossings
-radial mode, n order

J & Y are Bessel functions / κ & Q are “eigenvalues”

3rd Dimension (propagation) :

axial direction:

$$\gamma_{mn}(x) = \sqrt{\left(2\pi f r_t / c\right)^2 - k_{mn}^2}$$

γ is defined as axial wavelength

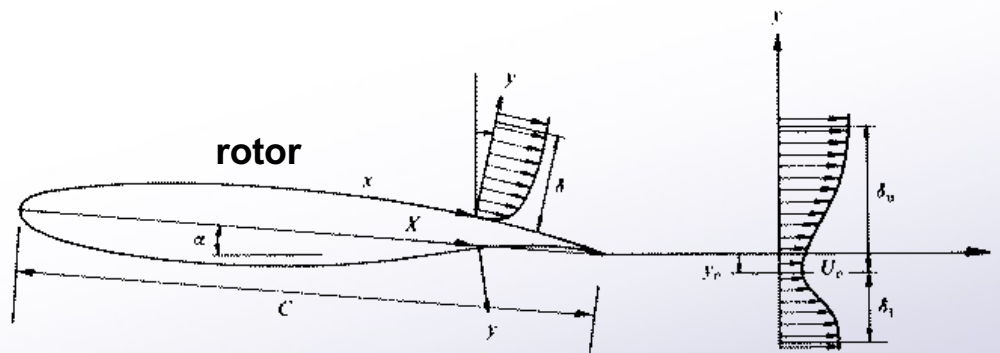
- (i) $2\pi f r_t / c < k_{mn}$; γ real, cyclical propagation
- (ii) $2\pi f r_t / c = k_{mn}$; cut - off / on frequency
- (iii) $2\pi f r_t / c > k_{mn}$; γ imag, exponential decay

define the cut-off ratio from (ii)

$$\zeta = f / f_{co}; \quad \begin{array}{ll} > 1 & \text{propagation} \\ < 1 & \text{decay} \end{array}$$

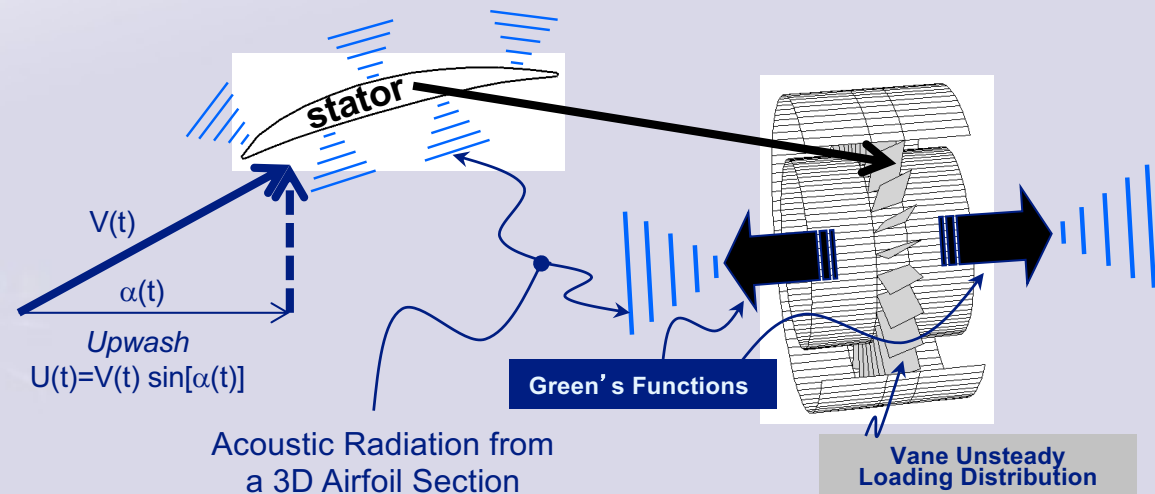
Rotor/Stator Interaction

Significant source of noise is the rotor viscous wake interaction with the stator. The wake itself is not the primary noise generation mechanism, but the interaction of the wake perturbations with the stator vane.



- Boundary layer growth along blade creates a velocity deficit at the trailing edge.
- This velocity profile impinging on the stator surface generates noise.

- The fluctuation of the velocity vector normal to the surface of the stator induces a pressure since the solid surface cannot support a normal velocity.
- The fluctuating normal velocity is called the *UPWASH* – combination of the variation in the velocity magnitude and angle.





Rotor/Stator Interaction

**Tyler, J.M., & Sofrin, T.G., “Axial Flow Compressor Studies”,
SAE Transactions, Vol 70, 1962.**

The kinematic relationship between the rotor and stator of varying counts causes a strong interaction pressure ‘lobe’ related to the relative positions.

This ‘lobe’ rotates locked to the fan.

The number of lobes (i.e. circumferential mode) is related to the blade/vane count:

$$m = nB - kV$$

(n harmonic, k= any integer)

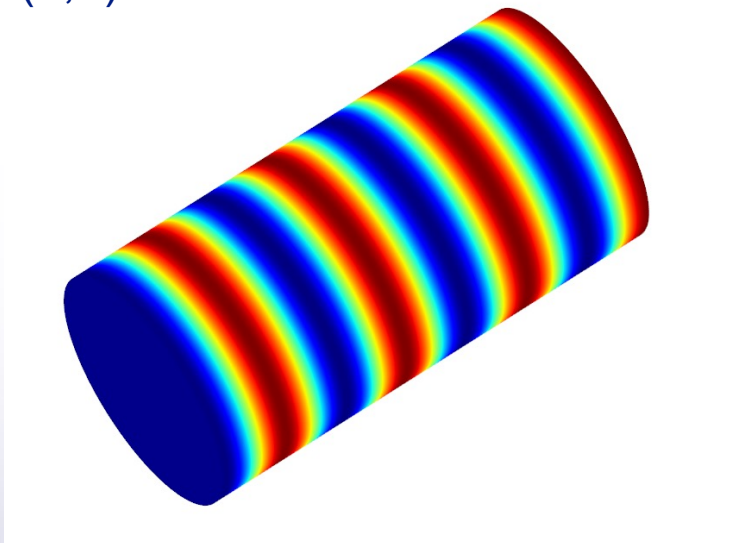
To preserve frequency a low count mode must rotate faster:

$$m \cdot \Omega_{\text{mode}} = B \cdot \Omega_{\text{rotor}}$$
$$\Omega_{\text{mode}} = (B/m) \cdot \Omega_{\text{rotor}}$$

If $\Omega_{\text{mode}} > 1.0M$ modes propagates

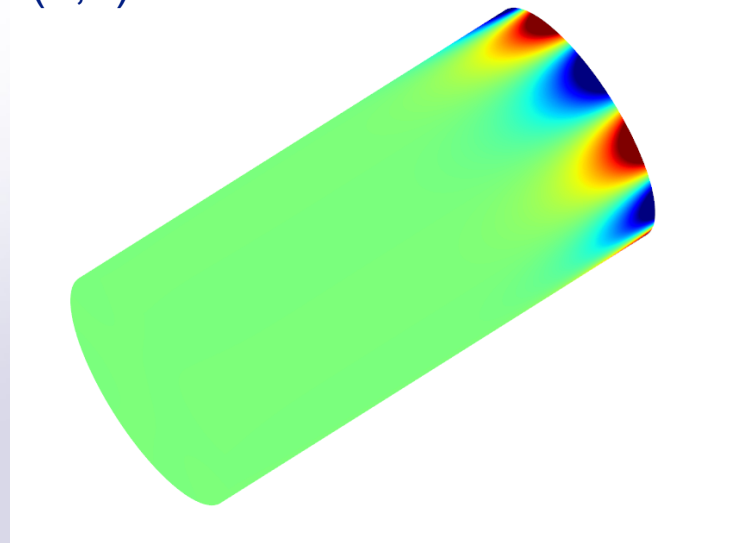
Propagating / Decaying Modes

(0,0)



$$\zeta = f/f_{co} > 1 \text{ propagation}$$

(5,0)



$$\zeta = f/f_{co} < 1 \text{ decay}$$

graphics courtesy of David Stephens GRC/LTV

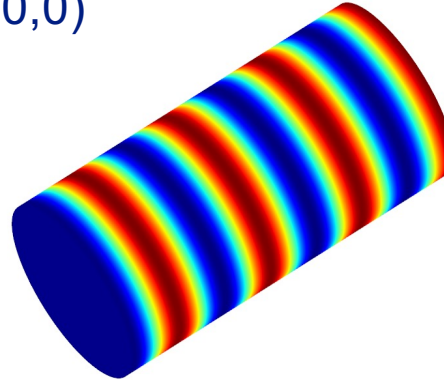
Circumferential Modes (m-order)

circumferential direction:

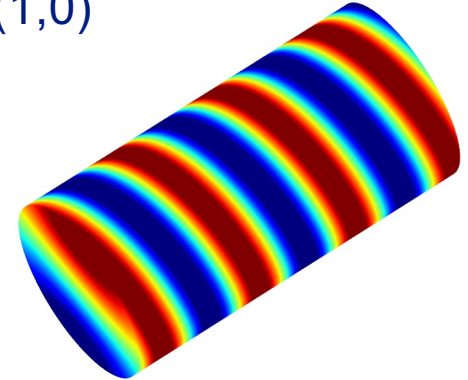
$$P(\theta) = P(\theta + 2\pi); \quad \sin(m\theta)$$

m is defined as the number of cycles
-circumferential mode, m order

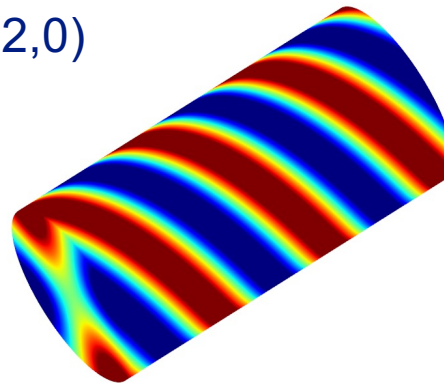
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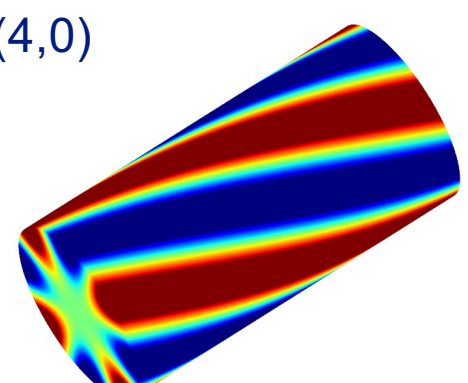
(1,0)



(2,0)



(4,0)



graphics courtesy of David Stephens GRC/LTV

Radial Modes (*n*-order)

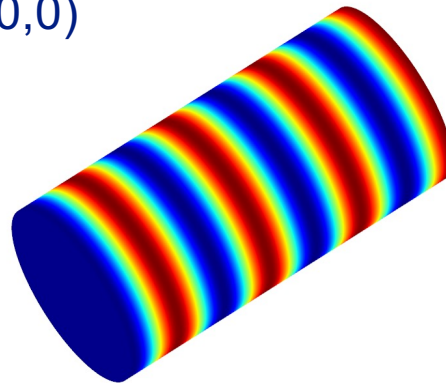
radial direction:

$$E_{mn} = C_{mn} [J_m(\kappa_{mn} \tilde{r}) + Q_{mn} Y_m(\kappa_{mn} \tilde{r})]$$

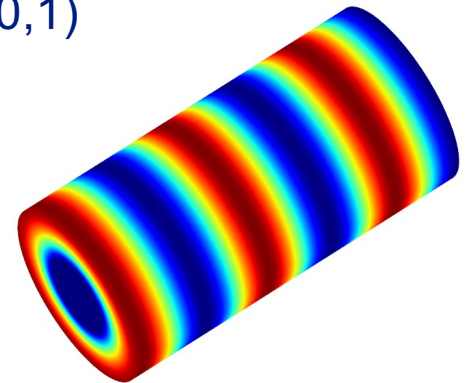
n is defined as the
number of zero crossings
-radial mode, *n* order

J & *Y* are Bessel functions
/*κ* & *Q* are “eigenvalues”

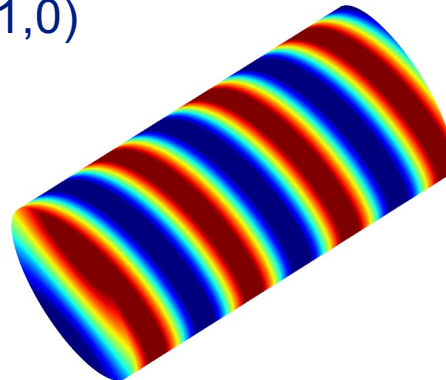
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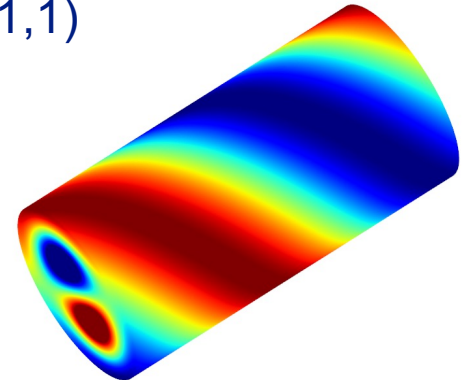
(0,1)



(1,0)



(1,1)



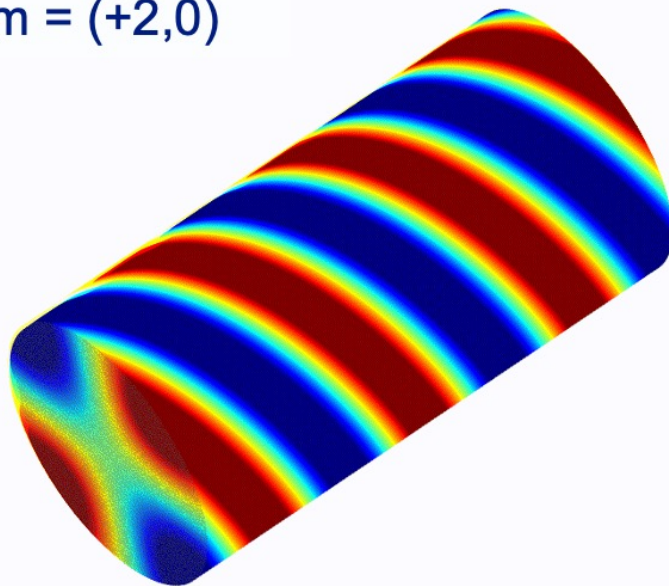
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graphics courtesy of David Stephens GRC/LTV

Co/Contra Rotating Modes (+/- m)

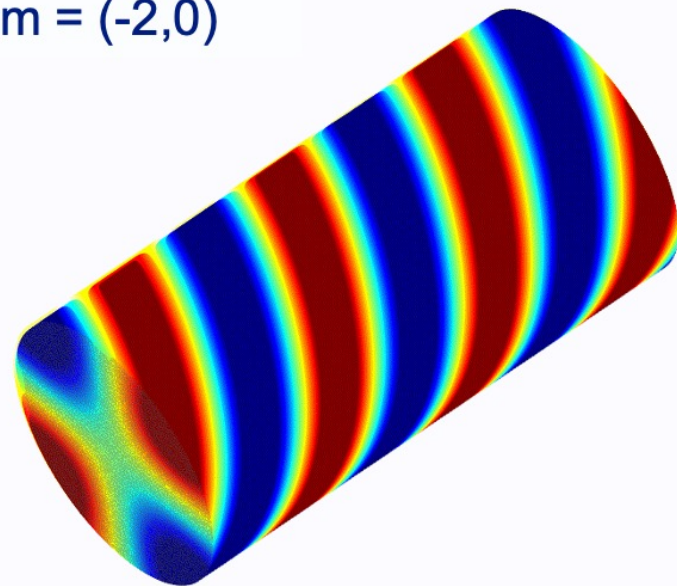
Arbitrarily define +m if rotating in same direction as fan. (co-rotating)

$m = (+2,0)$



co-rotating

$m = (-2,0)$



counter-rotating

graphics courtesy of David Stephens GRC/LTV



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(I) Description of Acoustic Field that is Measured
rotor/stator interaction
duct modes

(II) **Rotating Rake Theory and Implementation**
Doppler shift
Radial Curve Fitting
Mechanical Parameters

(III) Examples of Rotating Rake Applications



Doppler Shift of Circumferential Modes

Acoustic pressure at a fixed radial and circumferential location in a plane:

$$p(\Theta, t) = \sum_{s=1}^{\infty} \sum_{m=-M}^{m=+M} C_m^s e^{-i(s\omega t - m\theta)}$$

Acoustic pressure at a fixed radial location in a plane and point is slowly rotating in the circumferential direction:

$$\Theta = \Omega t; \quad p(\Theta, t) = \sum_{s=1}^{\infty} \sum_{m=-M}^{m=+M} C_m^s e^{-i(s\omega - m\Omega)t}$$

$$\text{let } \Omega = \frac{\omega}{100}, s = B; \quad \text{freq} \rightarrow (s\omega - m\Omega) = \omega(B - \frac{m}{100})$$

Thus, each circumferential mode is induced to appear at a unique shifted frequency proportional to the rake/fan speed ratio. Plane wave (m=0) is at fan harmonic.

Positive and negative modes are shifted about the the fan harmonic.

p pressure

t time

ω fan shaft speed

m circumferential mode

s shaft harmonic

Ω rake rotational speed

B rotor blade count

What about the rake-wake/rotor interference?
(Frame of reference).

Pressure due to a fixed inlet distortion:

$$p_l^s = C_l^s e^{-i[(s\pm l)\Theta - s\omega t]}$$

Pressure due to a slowly rotating inlet distortion:

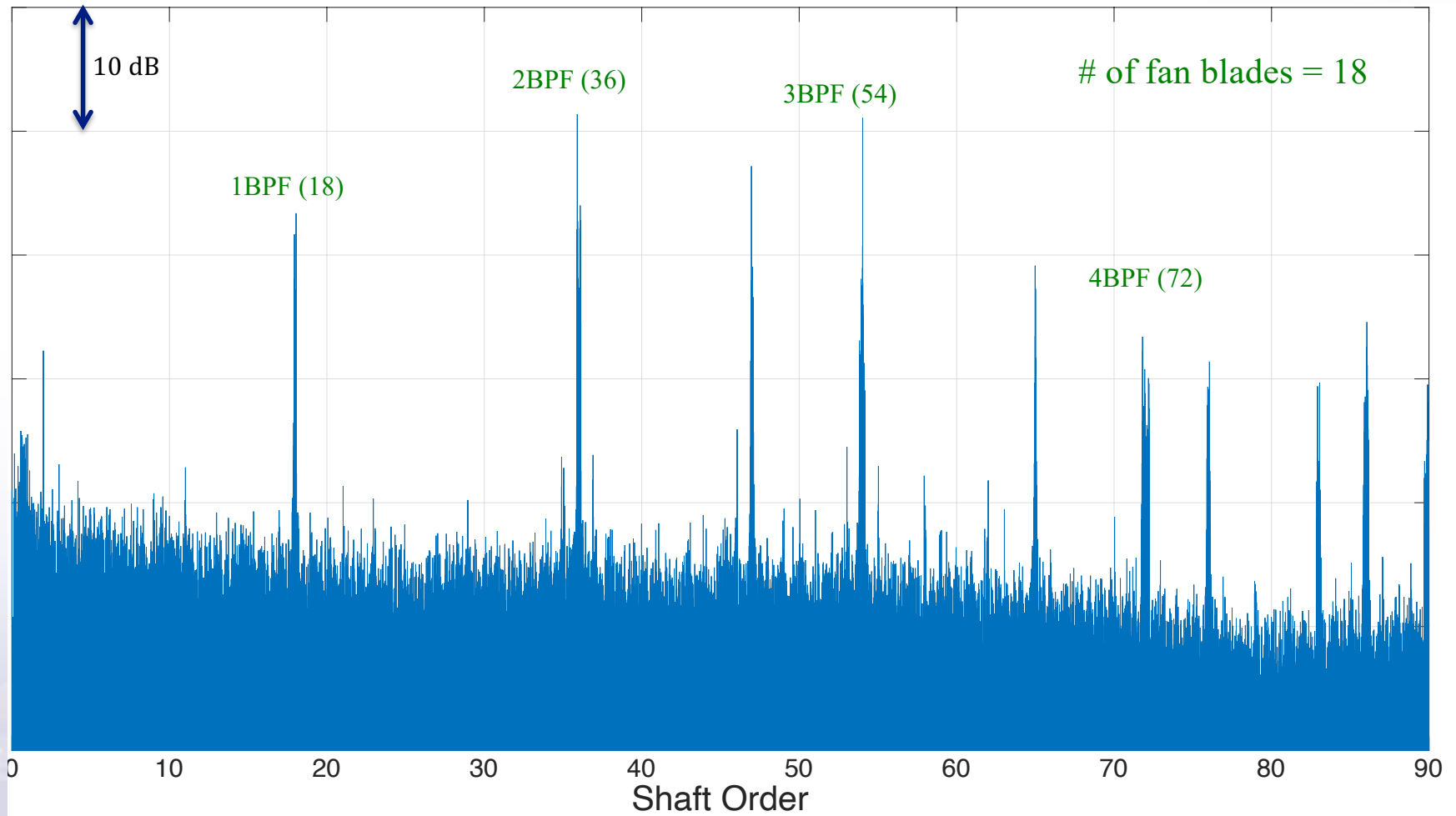
$$\begin{aligned} p_l^s &= C_l^s e^{-i[(s\pm l)\Theta - s(\omega \pm l\Omega)t]} \\ \Theta = \Omega t; \quad p_l^s &= C_l^s e^{-i[(s\pm l)\Omega t - (s\omega \pm l\Omega)t]} \\ &= C_l^s e^{-i[s\Omega t \pm l\Omega t - s\omega t \mp l\Omega t]} \\ &= C_l^s e^{-i[s(\Omega - \omega)t]} \end{aligned}$$

Freq of distortion given by $B(\omega - \Omega)$ for all rake induced modes.

From above let $m=B$; $B(\omega - \Omega)$; e.g. all rake induced modes appear at the equivalent 'rotor' count mode $m=B$.



FFT of Signal in Rotating Frame of Reference



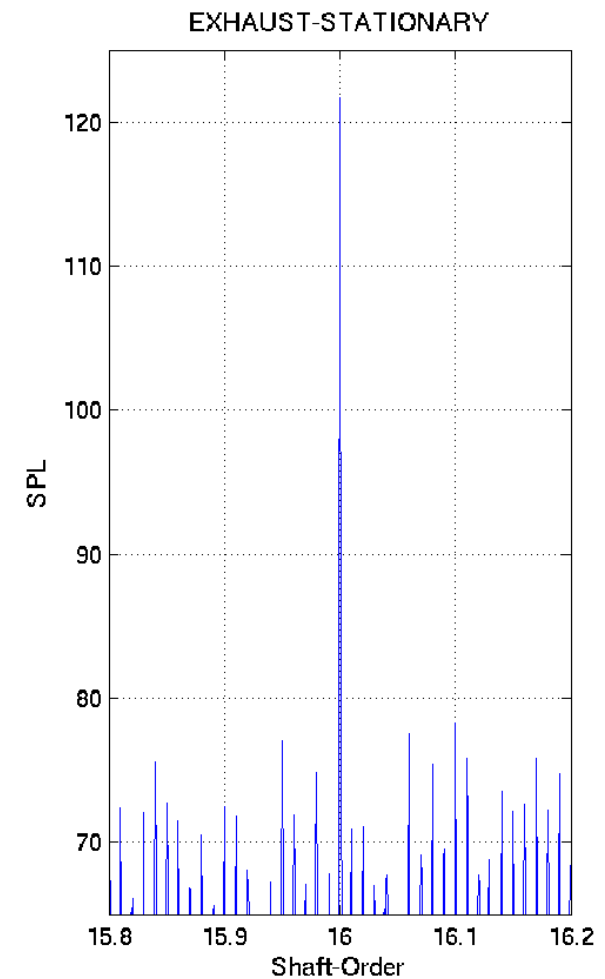
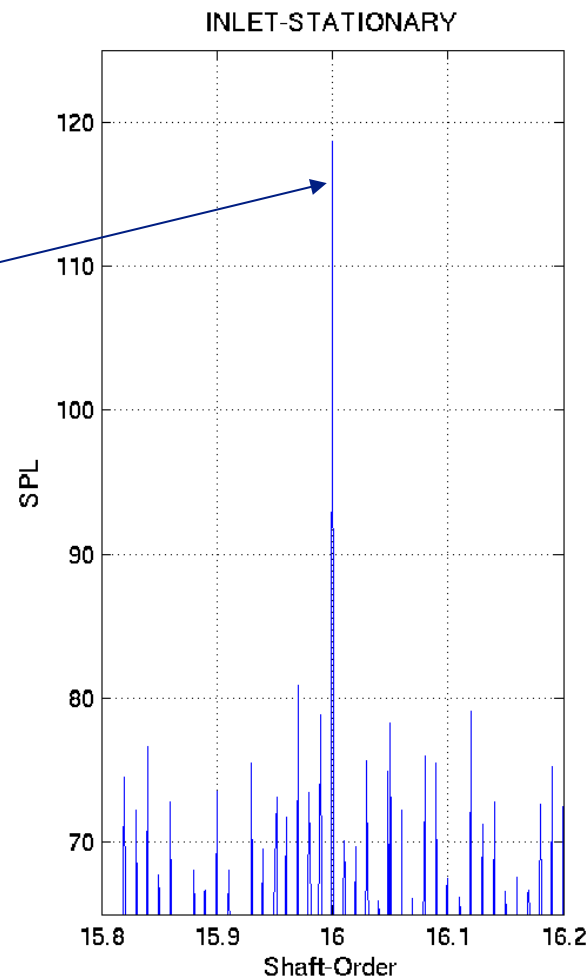
Shaft Order Spectral Content from a Single Microphone of Rotating Rake
Showing Fan Tone Harmonics

Rotating Rake Measurements

single microphone

narrow band FFT $\sim 2^{17}$
zoom about 1BPF (16)

BPF HARMONIC



Rotating Rake Measurements

single microphone

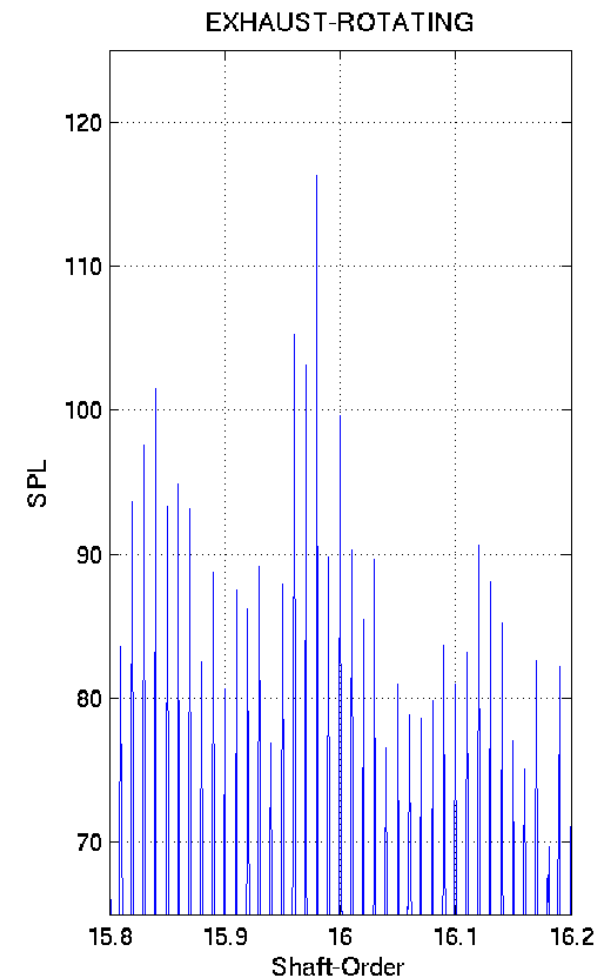
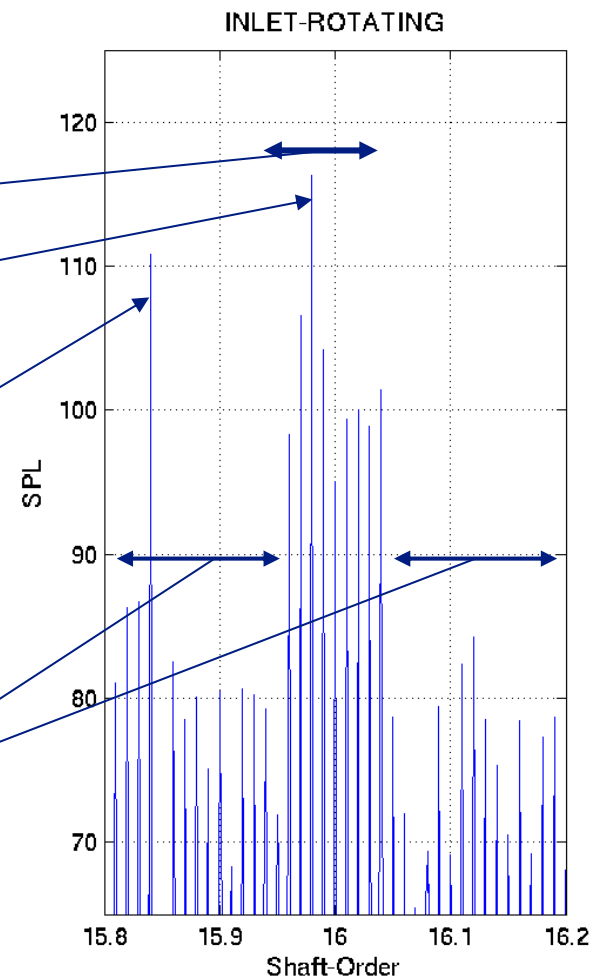
narrow band FFT $\sim 2^{17}$
zoom about 1BPF (16)

Individual Cut-On
Modes

Rotor-Stator Interaction
Mode

Rake-Wake
Rotor Mode

Cut-Off Modes



Duct Radial Mode Solutions

Determination of radial modal content from measured radial pressure:

$$\begin{array}{c}
 \text{measured} \nearrow p_{m,r,f} = \sum_{n=0}^{\infty} \underset{\substack{\uparrow \\ \text{unknown}}}{p_{m,n,f}} \underset{\nwarrow \text{determined}}{E_{mn}(\kappa_{mn}r)}
 \end{array}$$

For each m-order & harmonic calculate the least squares fit:

$$\sum_{n=0}^{N_{\max}} p_{m,n,f} E_{mn}(\kappa_{mn}r) - p_{m,r,f} \quad \delta^2 = \sum_{mic=1}^{N_{mic}} \left[W_{mic} \sum_{n=0}^{N_{\max}} p_{m,n,f} E_{mn}(\kappa_{mn}r) - p_{m,r,f} \right]^2$$

To minimize error, differentiate w/r to each mode amplitude to form a set of $N_{mic}+1$ simultaneous equations :

$$\begin{aligned}
 \frac{\partial \delta^2}{\partial p_{n1}} &= 2 \sum_{i=1}^{N_{mic}} W_i E_{mn1}(\kappa_{mn1}r_i) \left[\sum_{n=0}^{N_{\max}} p_n E_{mn}(\kappa_{mn}r) - p_i \right], \\
 \frac{\partial \delta^2}{\partial p_{n2}} &= 2 \sum_{i=1}^{N_{mic}} W_i E_{mn2}(\kappa_{mn2}r_i) \left[\sum_{n=0}^{N_{\max}} p_n E_{mn}(\kappa_{mn}r) - p_i \right], \dots
 \end{aligned}$$



Least Squares Solution for Radial Modes

Setting error derivative to zero and re-arranging, each equation (index v) becomes:

$$\sum_{n=0}^{N_{\max}} p_n \sum_{i=1}^{N_p} E_{mv}(\kappa_{mv} r_i) E_{mn}(\kappa_{mv} r_i) = \sum_{i=1}^{N_p} W_i E_{mv}(\kappa_{mv} r_i) p_i$$

$$[F_{vn}] = \begin{bmatrix} \sum_{i=1}^{N_p} E_{m0}^2(\kappa_{m0} r_i) & \dots & \sum_{i=1}^{N_p} E_{m0}(\kappa_{m0} r_i) E_{mN_{\max}}(\kappa_{mN_{\max}} r_i) \\ \vdots & & \vdots \\ \sum_{i=1}^{N_p} E_{mN_{\max}}(\kappa_{mN_{\max}} r_i) E_{m0}(\kappa_{m0} r_i) & \dots & \sum_{i=1}^{N_p} E_{mN_{\max}}^2(\kappa_{mN_{\max}} r_i) \end{bmatrix}$$

[# of radials X # of radials]

[# of radials X # of mics]

$$[F_{vn}] [P_n] = [E_{vr}] [P_r]$$

desired radial mode vector

measured sound pressure vector (microphones)

$$[P_n] = [F_{vn}]^{-1} [E_{vr}] [P_r]$$

Note: # of radials to curve is equal to number of radials present.

-Hardwall away from boundaries equal to max cut-on

-Softwall or near boundary/source consider decay rate



Physical Solution to Eigenvalue Problem

Eigenvalue problem based on boundary conditions (radial direction):

(For illustrative purposes, $\sigma=0$)

$$P(r) \propto J_{mn}(\kappa_{mn}r)$$

HARDWALL: acoustic velocity normal to wall is zero, momentum equation gives:

$$\frac{dP}{dr} = 0; \quad J'_m(\kappa_{mn}) = 0; \quad \kappa_{mn} \frac{J_{m+1}(\kappa_{mn})}{J_m(\kappa_{mn})} - m = 0$$

SOFTWALL: (passive treatment - liner)

$$\frac{p}{v} = \frac{Z}{\rho c}; \quad A = \frac{\rho c}{Z}; \quad \eta = \frac{\omega R}{c}$$

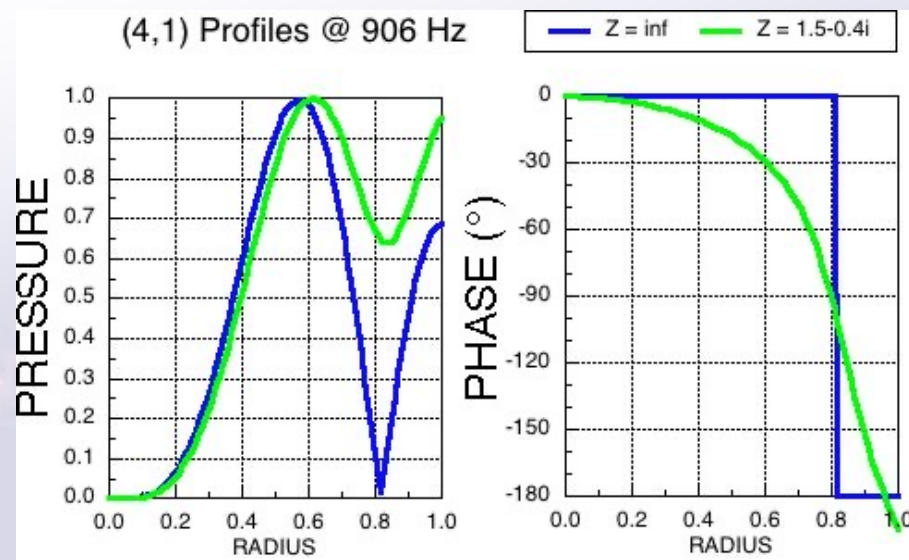
impedance, admittance, reduced freq

$$\frac{dP}{dr} = -i\eta AP$$

$$\kappa \frac{J'_m(\kappa)}{J_m(\kappa)} = -i\eta A \quad \longrightarrow \quad \kappa \frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A$$

$$\kappa \frac{J_{m-1}(\kappa)}{J_m(\kappa)} - m = -i\eta A (1 - M \frac{k_x}{\eta})^2$$

A set of basis functions for each (m,n) mode.



A set of basis functions for each (m,n) mode - for each frequency, impedance, duct Mach.

Hardwall: $\kappa_{mn} \rightarrow \text{real}$

Softwall: $\kappa_{mn} \rightarrow \text{complex}$

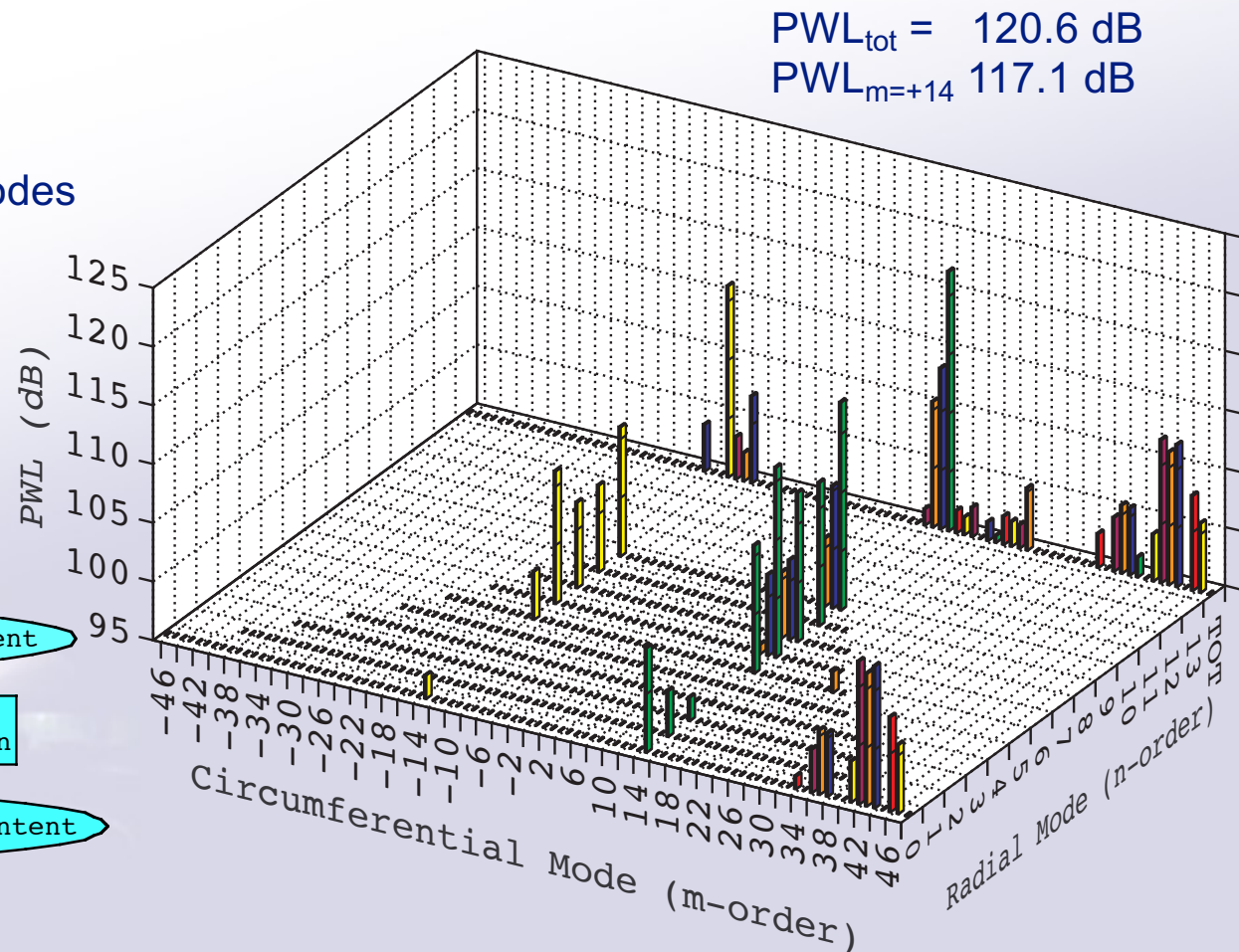
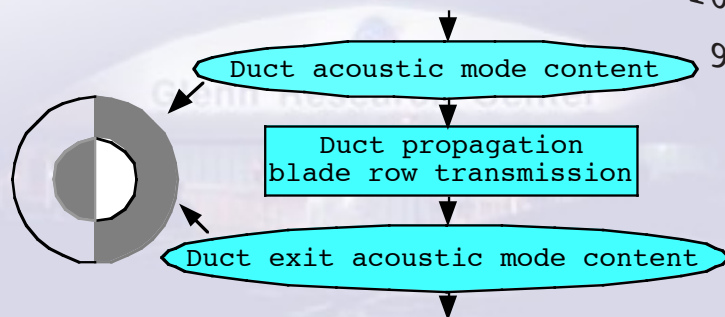
ROTATING RAKE – MODAL ANALYSIS (TOMBSTONE PLOTS)

Magnitude & phase of each (m,n) mode.

Sum for interaction of tone PWL.

Identifies strong interactions
(expected & unexpected).

Note dominance of positive modes
(co-rotating).





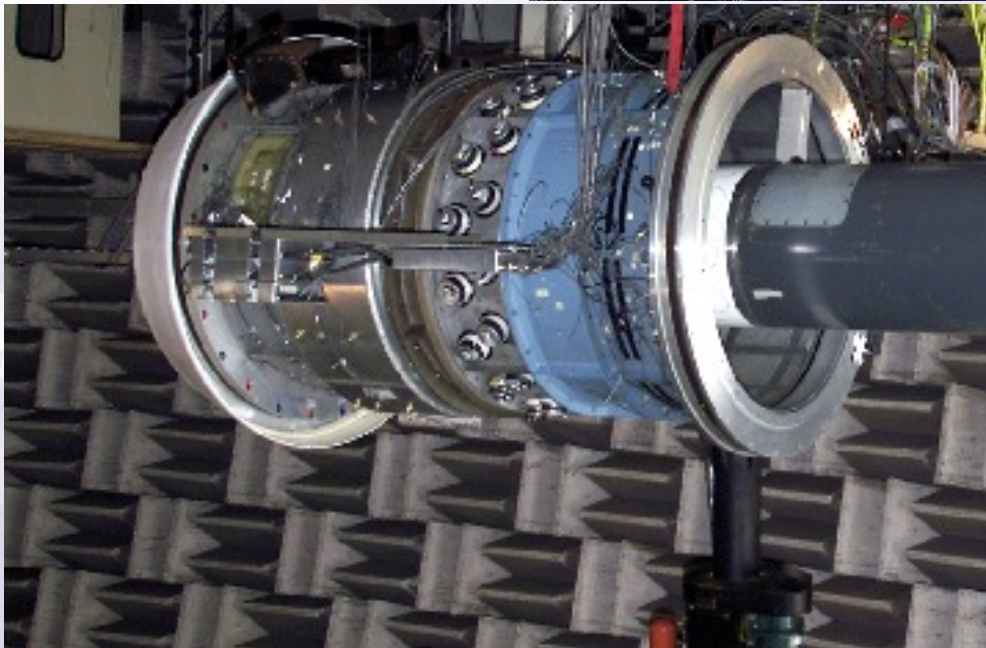
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Advanced Noise Control Fan

- rotor tip velocity ~ 400 ft/sec
- 16 rotor-blades/variable stator count
BPF ~ 500 Hz
- rake rotates 1/100th fan speed
18 rpm
- 7/6 radial pressures

Active Noise Control
Trailing Edge Blowing
Code verification
Passive treatment optimization

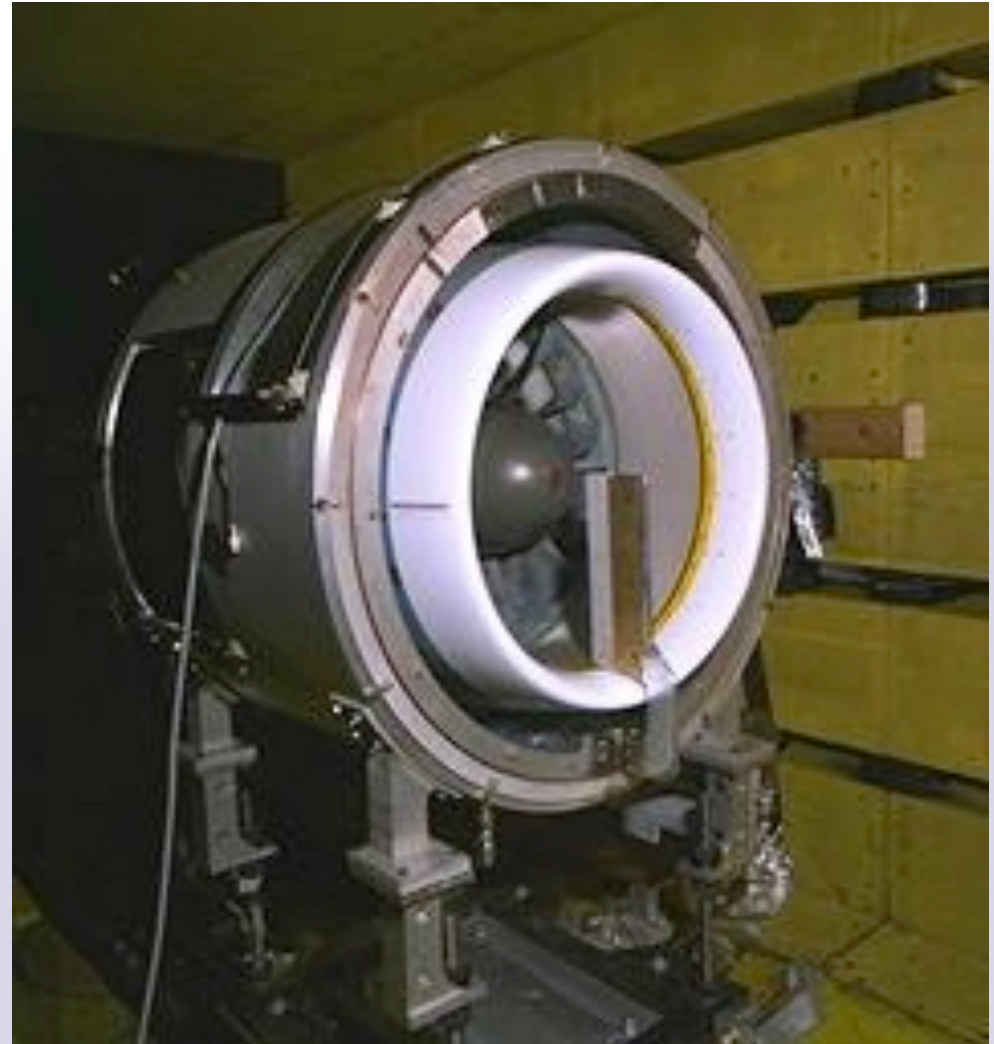


22" Fans in 9x15 WT

HIGH SPEED FAN (ADP, SDT, QHSF)

- rotor tip velocity ~ 800 – 1200 ft/sec
- 22 rotor-blades/variable stator geometry
BPF ~ 4000 to 5000 Hz
- rake rotates 1/200th fan speed
75 rpm
- 14/8 radial pressures

Source reduction
Complex interactions
Active Noise Control



Full-Scale Engine

Honeywell TFE731-60 engine.

- rotor tip velocity ~ sonic
- 22 rotor-blades/52 stators/10 struts
BPF ~ 3600
- rake rotates 1/200th fan speed
50 rpm
- 14/8 radial pressures

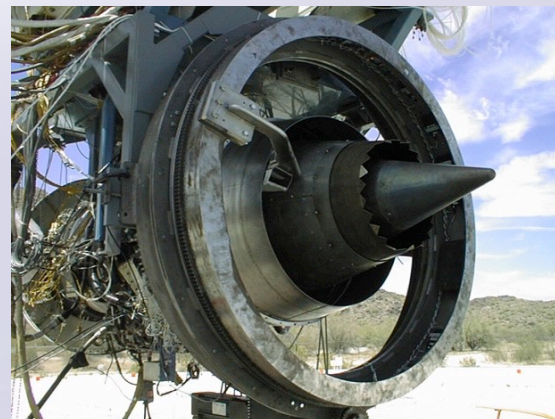
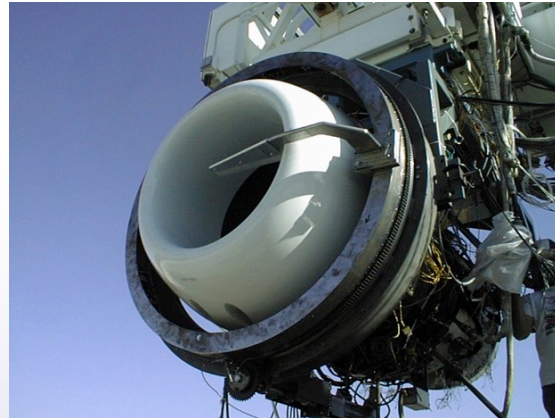
Engine is currently certified on the
Dassault Falcon 900EX

Part of the TFE731-20/40/60 engine
family that also powers Lear 45 IAI
Astra SPX Dassault Falcon 50X.

Basic Engine Weight - 988 lbs

Takeoff Thrust - 5000 lb

Airflow - 187 lb/sec



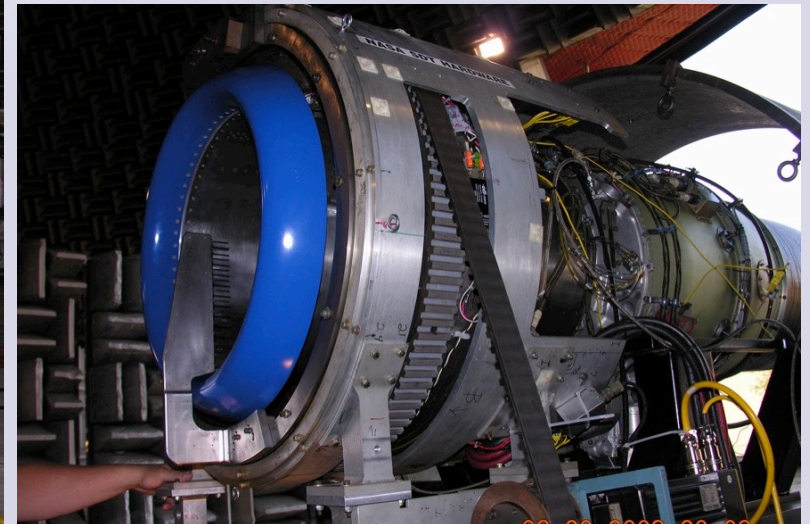
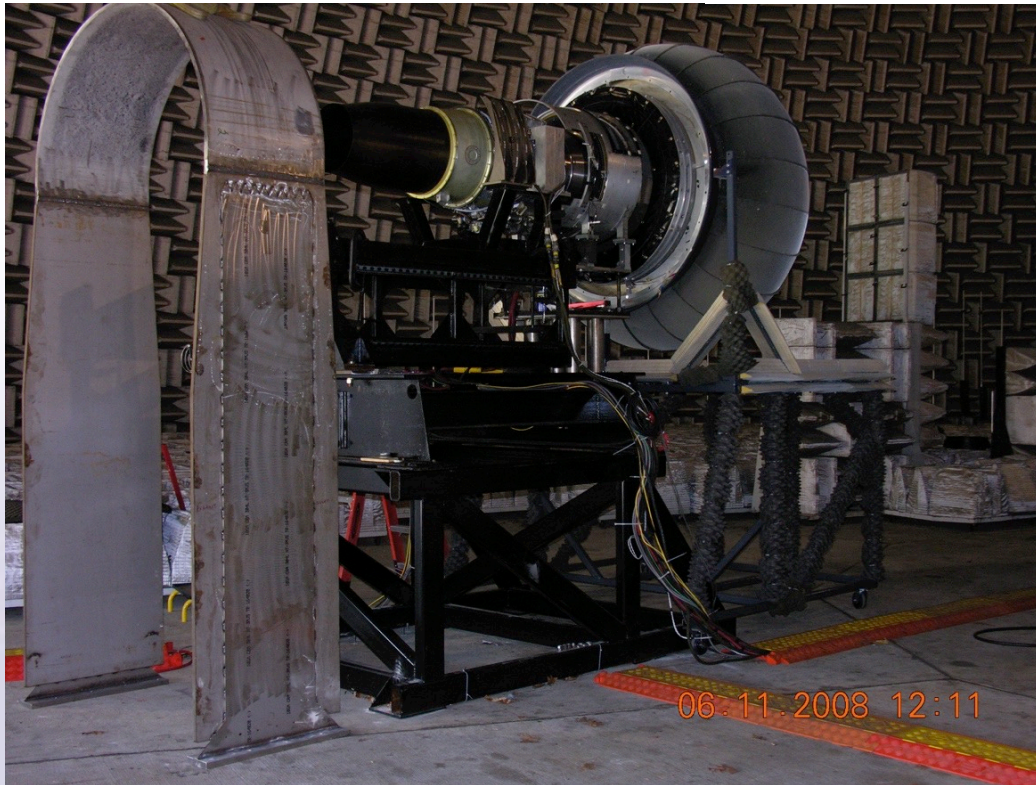
Full-Scale Engine

Williams International FJ44 engine

- rotor tip velocity > sonic
- 16 rotor-blades BPF ~ 3600
- rake rotates 1/300th fan speed (50 rpm)
- 14 pressures – inlet only

Table I. FJ44-3A Characteristic Engine Parameters

Base Engine Weight	525 lbs
Base Engine Length	48 inches
Base Engine Diameter	23 inches
Thrust Class	3,000 lbf
Blade Count	16





CONCLUSIONS

The Rotating Rake duct mode measurement system provides a complete modal map – magnitude & phase of circumferential & radial for shaft locked tones.

Full range of TRL applications.

Continuing extensions of the technique:

- Treated annular sections (plug-flow)
- Hard-wall sheared flows
- Soft-wall sheared flows
- Reflections

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REFERENCES:

- Tyler, J.M., & Sofrin, T.G., “Axial Flow Compressor Studies” *SAE Transactions*, Vol. 70, 1962.
- Moore, C.J., “Measurement of Radial and Circumferential Modes in Annular and Circular Ducts”, *JSV*, Vol 62, No 2, 1979.
- Sutliff, D.L., “Rotating Rake Mode Measurement System” NASA TM-2005-213828.