Conjunction Assessment Risk Analysis

A Multistep Probability of Collision Computational Algorithm

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Outline

- **Motivation and objectives**
- **Probability of collision (Pc) estimation methods**
  - Simplest method: Approximates trajectories using rectilinear motion
  - Advanced methods: Account for curvilinear motion and multi-encounter interactions
- **Multistep Pc computation algorithm**
  - Uses several increasingly advanced methods, but only as required
  - Evaluates “usage violations” for each method, indicating potential inaccuracies
- **Examples of satellite conjunctions**
  - Illustrating conjunctions strongly affected by curvilinear trajectory effects
  - Also showing both high and low relative velocity effects
- **Conclusions**

Motivation and Objectives

- **Motivation**
  - Monte Carlo studies indicate that some low Earth orbit satellite conjunctions require advanced methods to ensure accurate Pc estimation
  - These include interactions affected by curvilinear trajectories, low velocities, and multi-encounter or repeating conjunctions
- **Objectives**
  - Combine multiple Pc estimation methods into one omnibus algorithm
  - Implement a multistep approach that uses increasingly advanced methods, but only as required
  - At each step evaluate “usage violations” for each method, which indicate potentially inaccurate Pc estimates
  - Test and validate using a large set of actual archived conjunctions

Collision Probability Estimation Methods

Name	Description	Details
2D-Pc	Collision Plane Integration	• For single conjunctions, with high relative velocities • Rectilinear motion from the time of closest approach (TCA) • Constant position covariance and zero velocity covariance
2D-Nc*	Collision Sphere Integration	• For single or multiple conjunctions, with medium-to-high relative velocities • Curvilinear motion from TCA, using two-body equations of motion • Time dependent equinoctial orbital element state covariance
3D-Nc*	Series of Collision Sphere Integrations	• For single or multiple conjunctions, with low-to-high relative velocities • Curvilinear motion from TCA, using two-body equations of motion • Time dependent equinoctial orbital element state covariance
SDMC	Simplified Dynamics Monte Carlo	• For single conjunctions, with low-to-high relative velocities • Curvilinear motion from TCA, using two-body equations of motion • Samples and propagates multiple equinoctial element orbital states
BFMC	Brute Force Monte Carlo	• For single or multiple conjunctions, and for all relative velocities • High-fidelity curvilinear motion from OD epoch, over multiple encounters • Samples and propagates multiple special perturbations orbital states

*Derived using the statistically expected number of collisions (Nc) method

Multistep Pc Computation Algorithm

Input processing (100%)

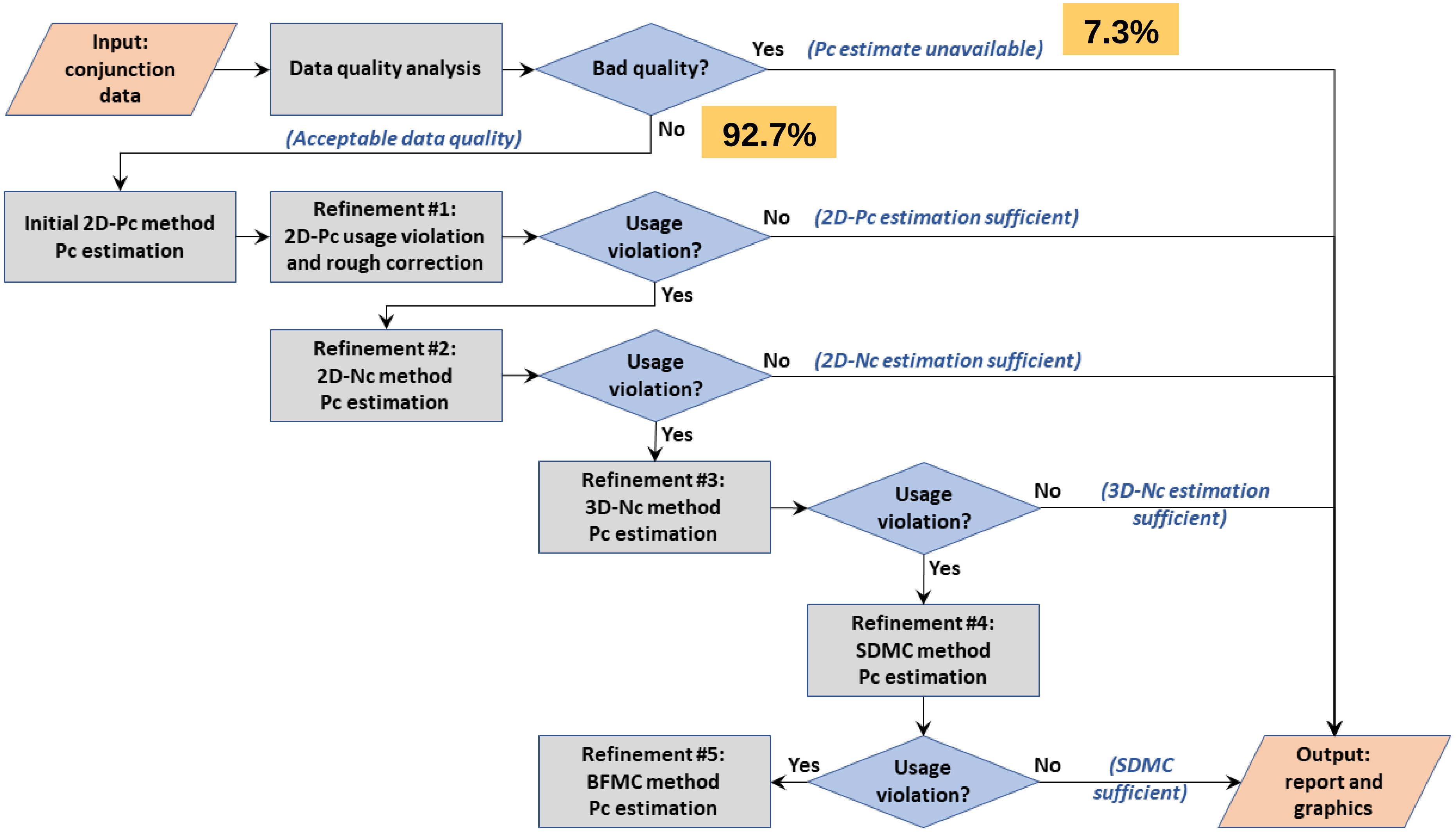
2D-Pc processing (~94.7%)

2D-Nc refinements (~5.0% + overrides)

3D-Nc refinements (~0.3% + overrides)

SDMC refinements (~0.006% + overrides)

BFMC refinements (~0.002% + overrides)

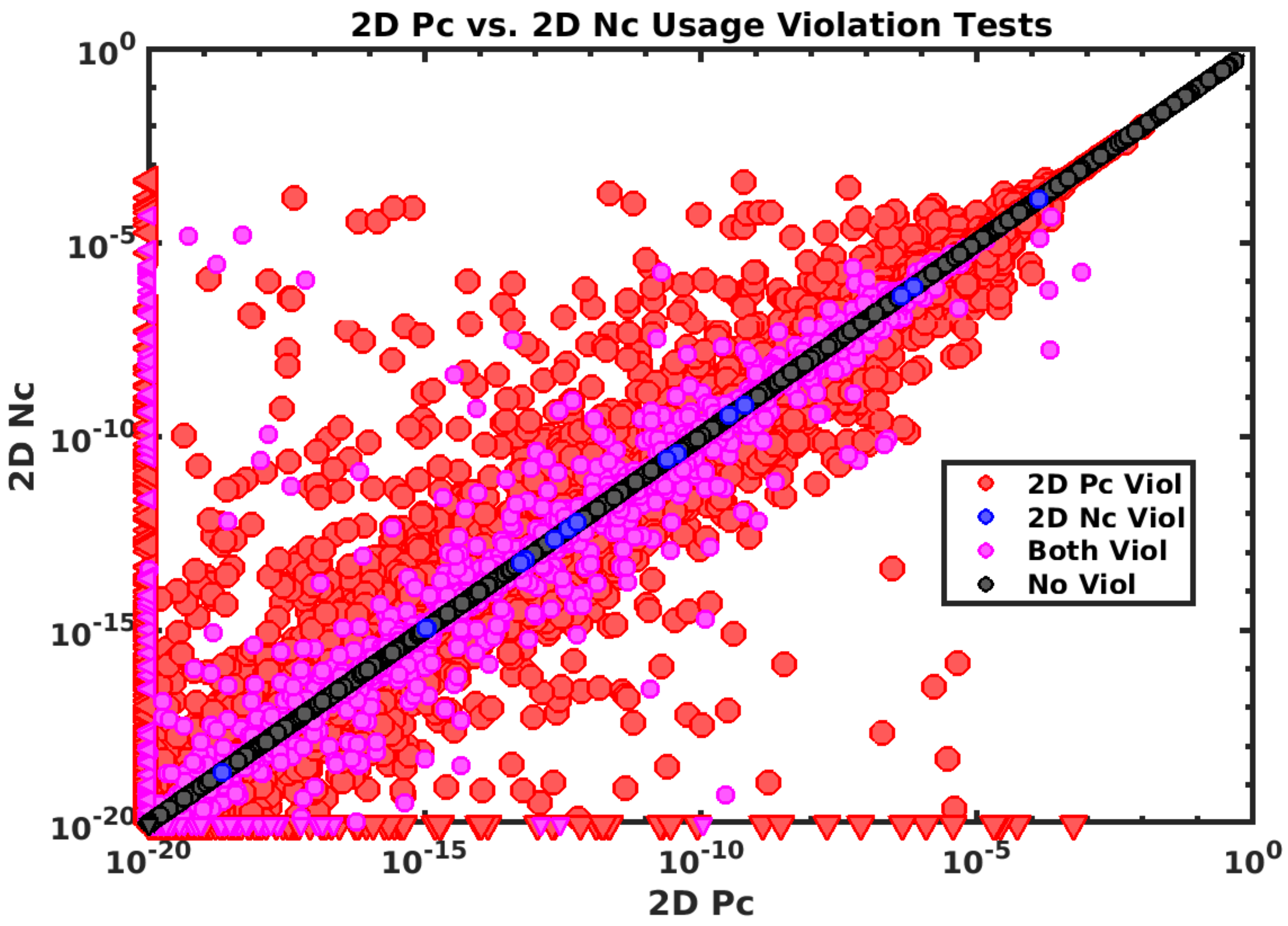


Collision Probability Method Usage Violations

Name	Description	Usage Violation Criteria
2D-Pc	Collision Plane Integration	• "Non-positive definite" (NPD) covariance matrix usage violations • "Extended" or "offset" in time violations for low relative velocities • "Inaccuracy" violations for conjunctions affected by curvilinear trajectories
2D-Nc	Collision Sphere Integration	• "Convergence" violations indicate iterative process failed to converge • "Extended" or "offset" in time violations for very low relative velocities • "Inaccuracy" violations also for very low relative velocities
3D-Nc	Series of Collision Sphere Integrations	• "Convergence" violations indicate iterative processes failed to converge • "Extended" or "offset" violations for extremely low relative velocities • "Inaccuracy" violations also for extremely low relative velocities
SDMC	Simplified Dynamics Monte Carlo	• "Extended" and "offset" violations for extremely low relative velocities that indicate potentially blended multi-encounter interactions
BFMC	Brute Force Monte Carlo	• No usage violations (provides "gold standard" Pc estimates)

Comparison of 2D-Nc and 2D-Pc Method Collision Probabilities

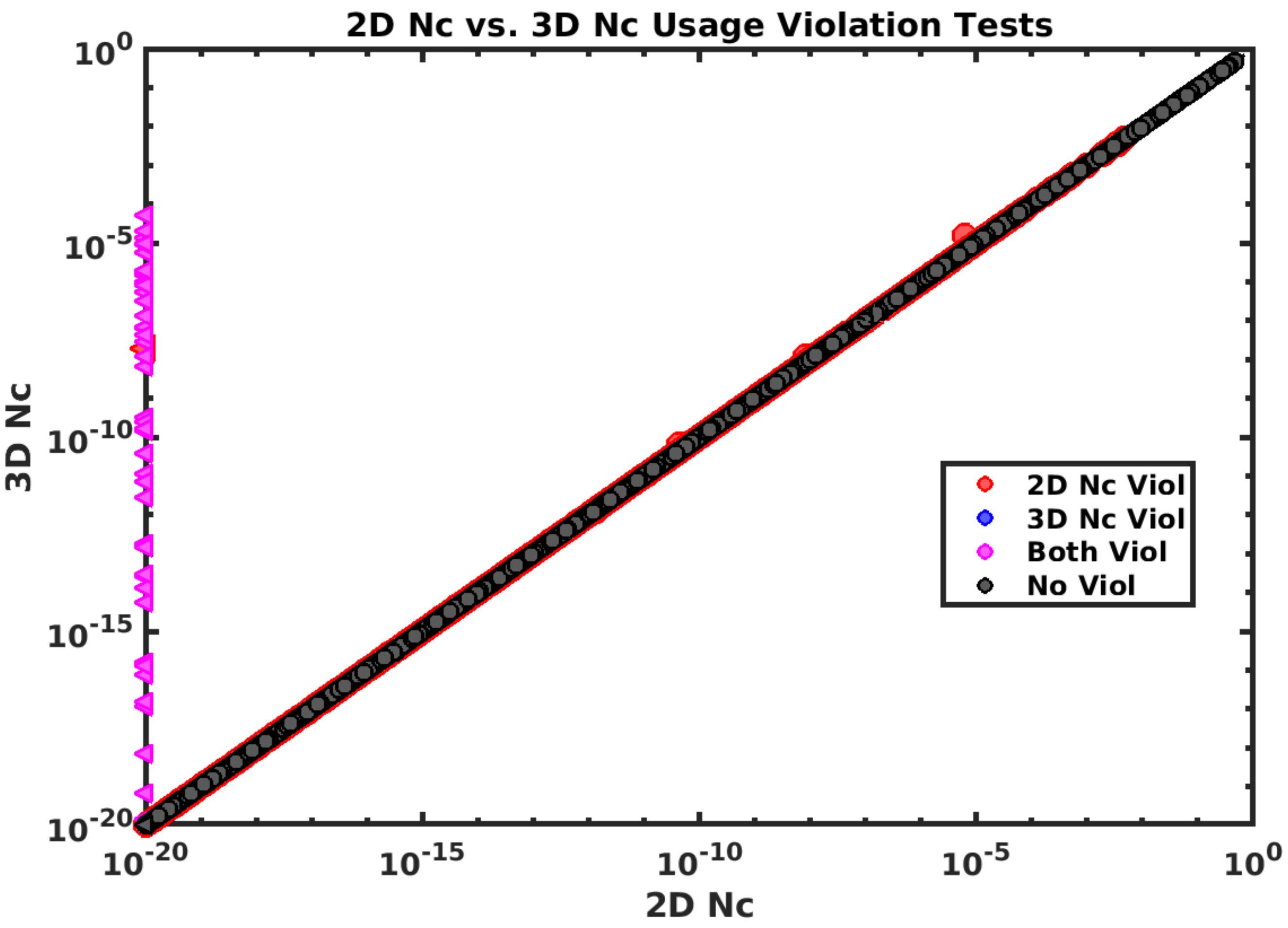
- **Testing based on ~2 million archived LEO conjunctions***
- **Gray points indicate conjunctions with no detected usage violations**
  - No off-diagonal gray points!
  - This means the algorithm has <u>zero</u> missed detections for this test data set
- **Colored points indicate conjunctions with detected usage violations**
  - The far off-diagonal colored points indicate properly detected 2D-Pc method failures
  - The algorithm detects all such failures, along with a somewhat larger number of false alarms



*Low Earth orbit conjunctions archived by CARA during 2021 and 2022

Comparison of 3D-Nc and 2D-Nc Method Collision Probabilities

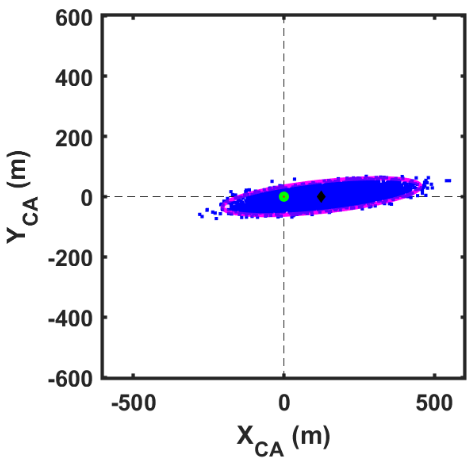
- **The 2D-Nc method has significantly fewer usage violations than 2D-Pc**
- **Gray points indicate conjunctions with no detected usage violations**
  - Again, no off-diagonal gray points!
  - This means the algorithm has zero missed detections for the test data set
- **Colored points indicate conjunctions with detected usage violations**
  - The far off-diagonal colored points indicate properly detected 2D-Nc method failures, mostly due to very low relative velocities
  - Algorithm detects all such failures, along with a somewhat larger number of false alarms



*Low Earth orbit conjunctions archived by CARA during 2021 and 2022

Example High Relative Velocity Conjunction with no Detected Usage Violations

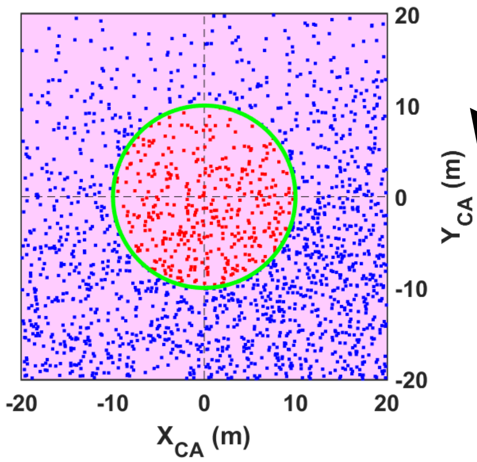
Fully zoomed-out view of close approach (CA) distribution



Conjunction Close Approach Distribution Plot
HBR = 10m CA = 125m
RelV = 13.4km/s Angle = 124°
Pri Last Obs Age = 0.679 days
Sec Last Obs Age = 0.65 days

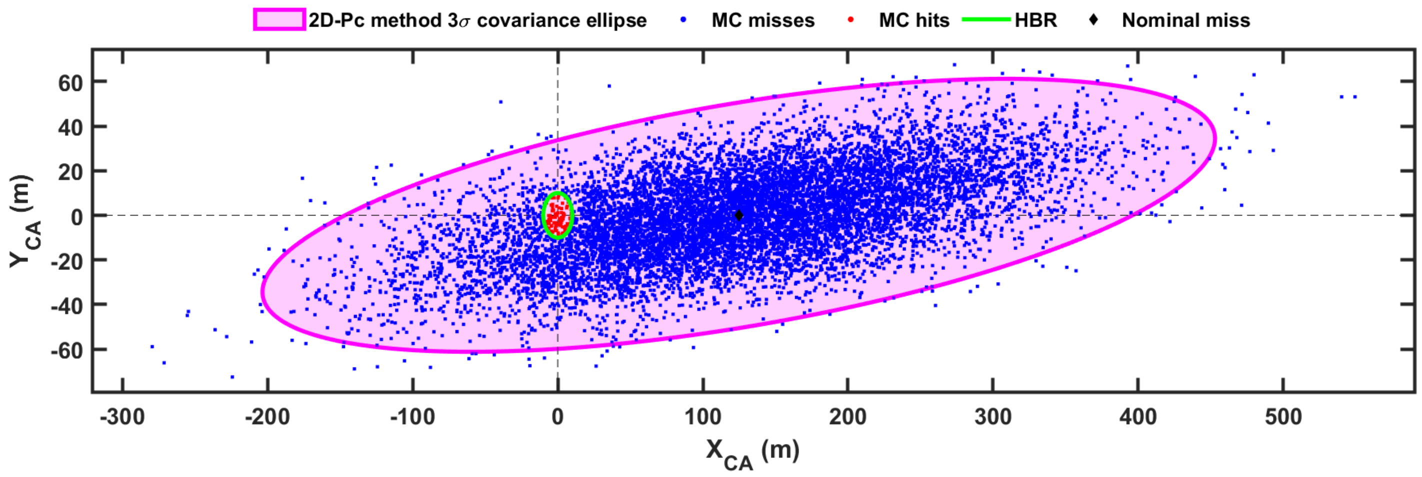
2D-Pc Method Pc = 0.0103
2D-Nc Method Pc = 0.0103
3D-Nc Method Pc = 0.01031
SDMC Method Pc = 0.0099
(396 hits / 4e4 trials)
SDMC 95% Confidence:
9e-3 $\leq$ Pc $\leq$ 0.0109

2D-Pc Method Pc Violations: None
2D-Nc Method Pc Violations: None
3D-Nc Method Pc Violations: None
SDMC Method Pc Violations: None



Fully zoomed-in view of CA distribution with hard-body circle

View of CA distribution zoomed in by unequal amounts on x and y axes



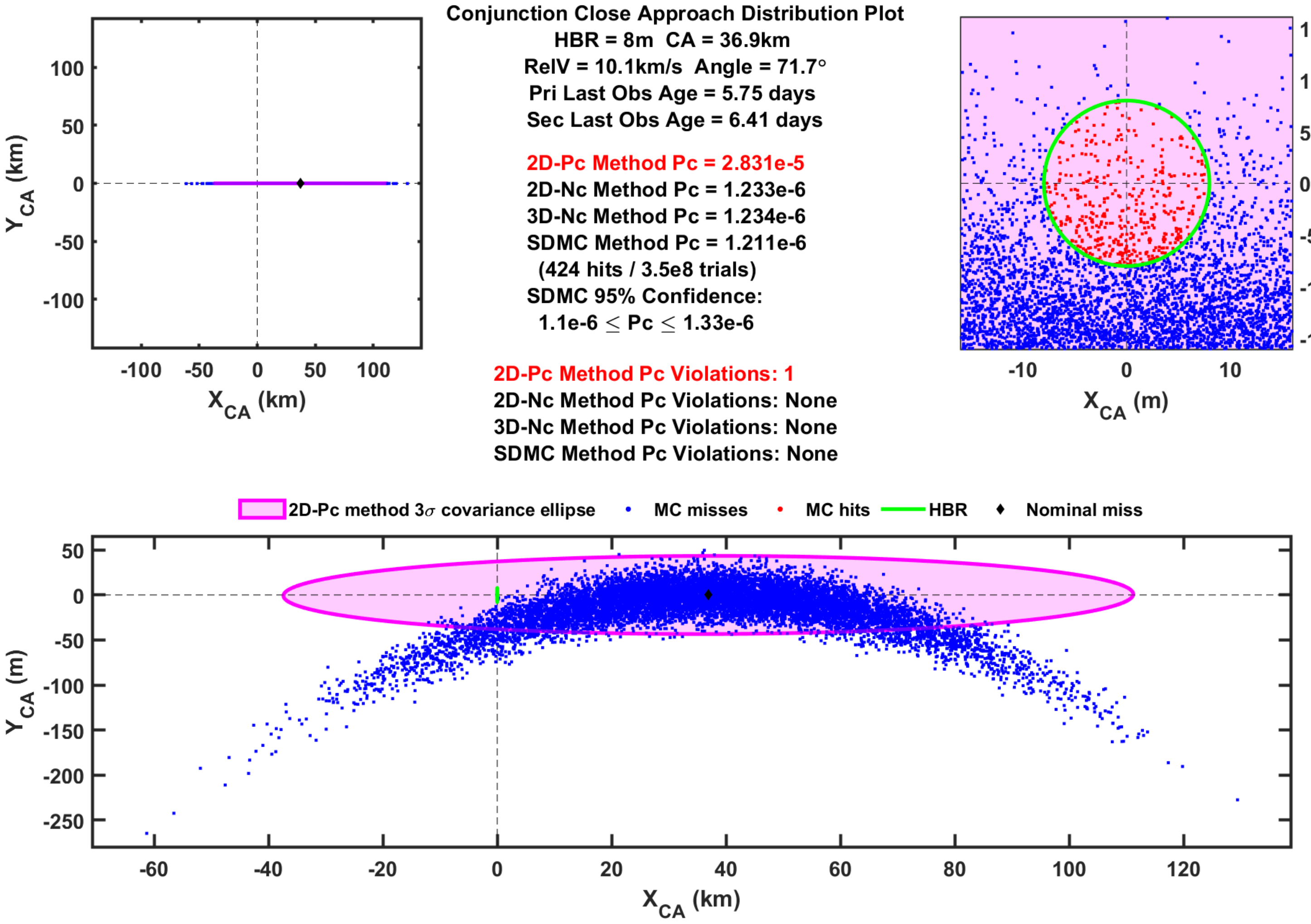
Monte Carlo distribution of dots aligns well with 2D-Pc covariance 3σ ellipse

All estimation methods indicate a high collision probability of Pc ≈ 0.01

Example High Relative Velocity Conjunction with a 2D-Pc Method Usage Violation

Fully zoomed-out view of close approach (CA) distribution

Fully zoomed-in view of CA distribution with hard-body circle



View of CA distribution zoomed in by unequal amounts on x and y axes

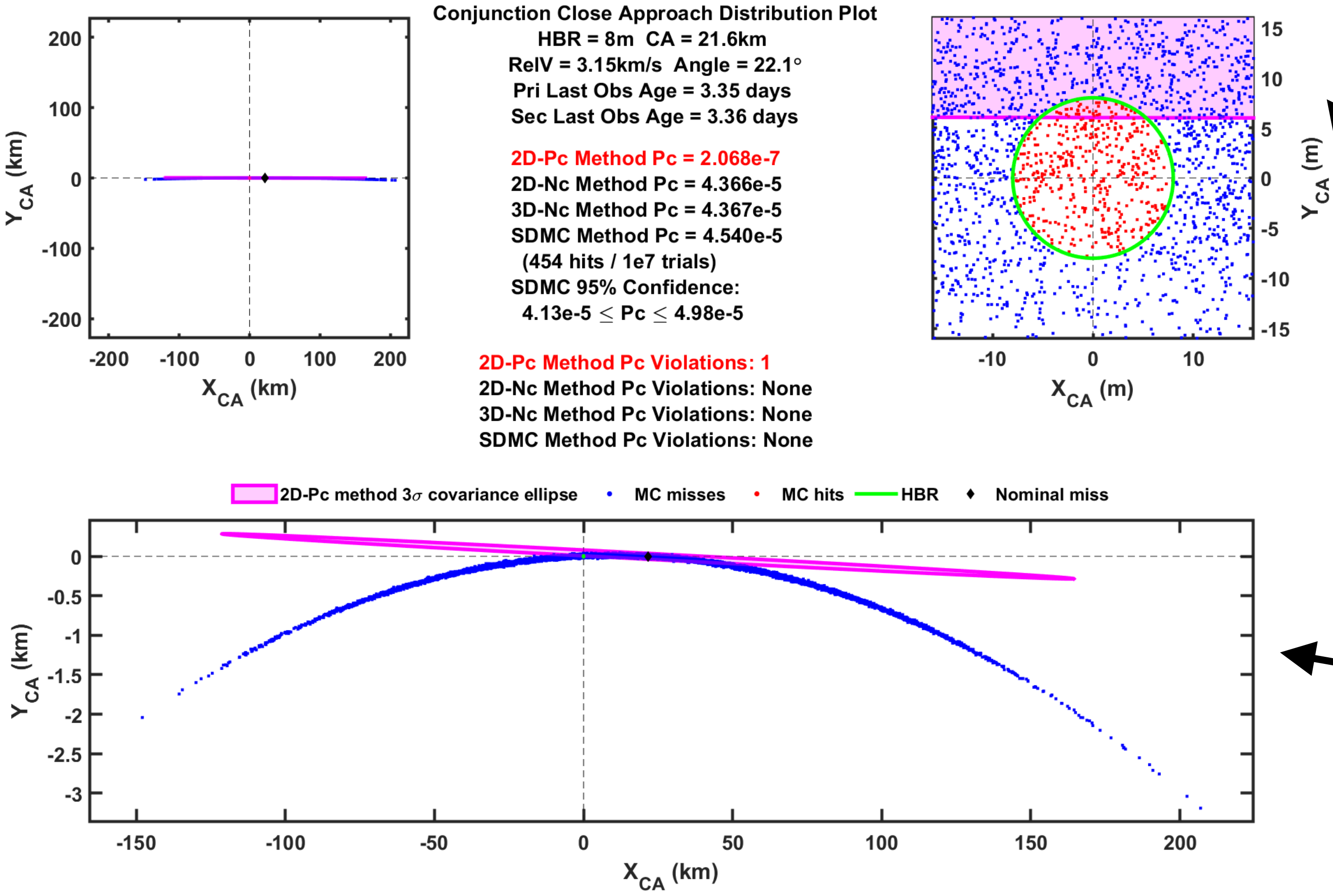
Monte Carlo distribution does not align well with 2D-Pc covariance 3σ ellipse

2D-Pc is erroneously high by a factor of ~23 due to curvilinear effects

Example High Relative Velocity Conjunction with a 2D-Pc Method Usage Violation

Fully zoomed-out view of close approach (CA) distribution

Fully zoomed-in view of CA distribution with hard-body circle



View of CA distribution zoomed in by unequal amounts on x and y axes

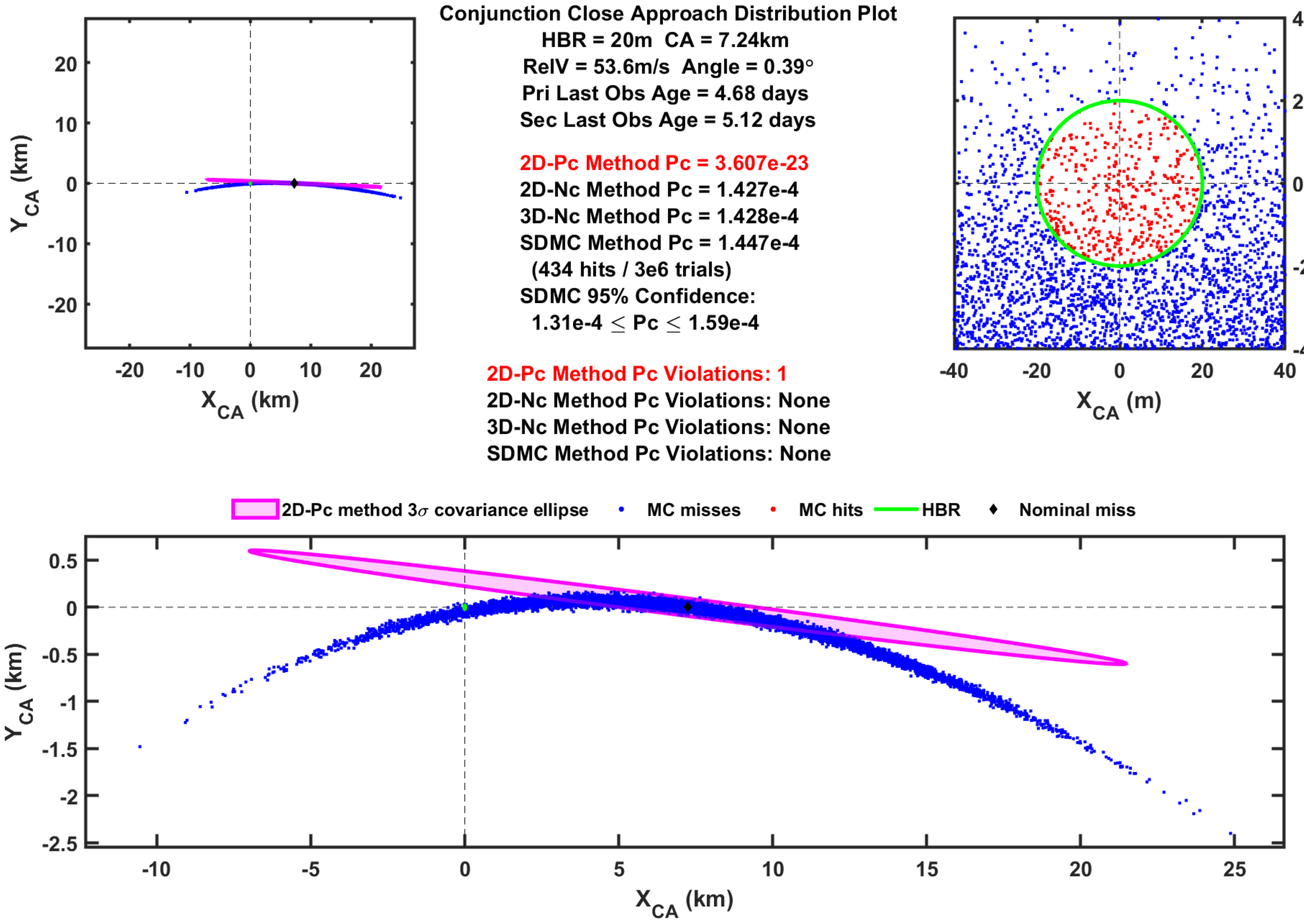
Monte Carlo distribution does not align well with 2D-Pc covariance 3σ ellipse

2D-Pc is erroneously low by a factor of ~210 due to curvilinear effects

Example Low Relative Velocity Conjunction with a 2D-Pc Method Usage Violation

Fully zoomed-out view of close approach (CA) distribution

Fully zoomed-in view of CA distribution with hard-body circle



View of CA distribution zoomed in by unequal amounts on x and y axes

Monte Carlo distribution does not align well with 2D-Pc covariance 3σ ellipse

2D-Pc is erroneously low by a factor of ~10^{18} due to low velocity effects

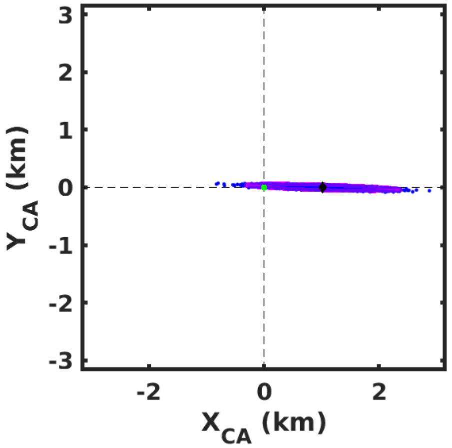
Conclusions

- **The multistep algorithm provides an efficient and accurate means to estimate collision probabilities for satellites in low Earth orbit**
  - Accuracy achieved by using several increasingly advanced estimation methods
  - Efficiency achieved by using advanced methods only when required
- **Usage violation algorithms detect Pc inaccuracies for each method**
  - Based on multiple criteria, developed and tested using ~2 million LEO conjunctions
  - Usage violation algorithms tuned to produce no missed detections for test data set
  - Resulting usage violation tests also produce a moderate number of false alarms
  - Curvilinear trajectory effects cause 2D-Pc method estimation inaccuracies for both low and high relative velocity conjunctions
- **Semi-analytical methods accurately process >99.99% of conjunctions**
  - Monte Carlo methods required for <0.01%, which all have very low relative velocities

Backup Charts

Example Low Relative Velocity Conjunction with no Detected Usage Violations

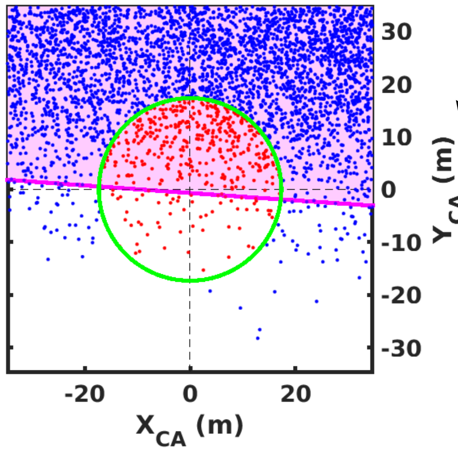
Fully zoomed-out view of close approach (CA) distribution



CA Distribution Plot
HBR = 17.3m CA = 1.02e+03m
RelV = 128m/s Angle = 0.974°
Pri Last Obs Age = 2.61 days
Sec Last Obs Age = 2.62 days

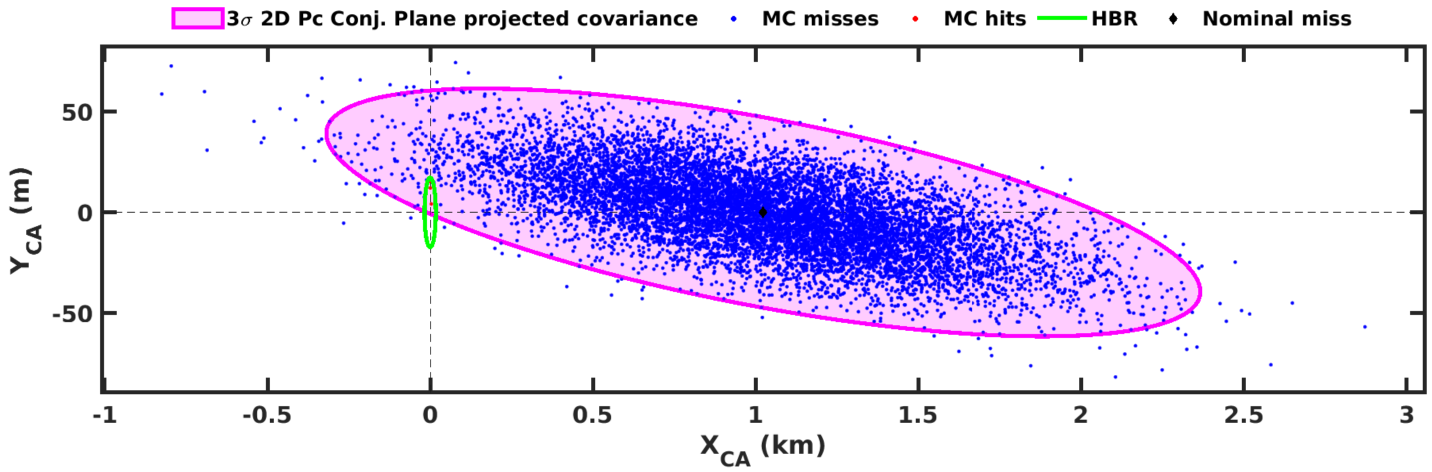
2D Pc Conj. Plane = 3.460e-4
2D Pc Unit Sphere = 3.455e-4
3D Pc Unit Sphere = 3.458e-4
SDMC Pc = 3.346e-4
(435 hits / 1.3e6 trials)
SDMC 95% confidence:
3.04e-4 ≤ Pc ≤ 3.68e-4

2D Pc Conj. Plane Violations: None
2D Pc Unit Sphere Violations: None
3D Pc Unit Sphere Violations: None
SDMC Pc Violations: None



Fully zoomed-in view of CA distribution with hard-body circle

View of CA distribution zoomed in by unequal amounts on x and y axes



Monte Carlo distribution of dots aligns well with 2D-Pc covariance 3σ ellipse

All estimation methods indicate a collision probability of Pc ≈ 3×10^{-4}