



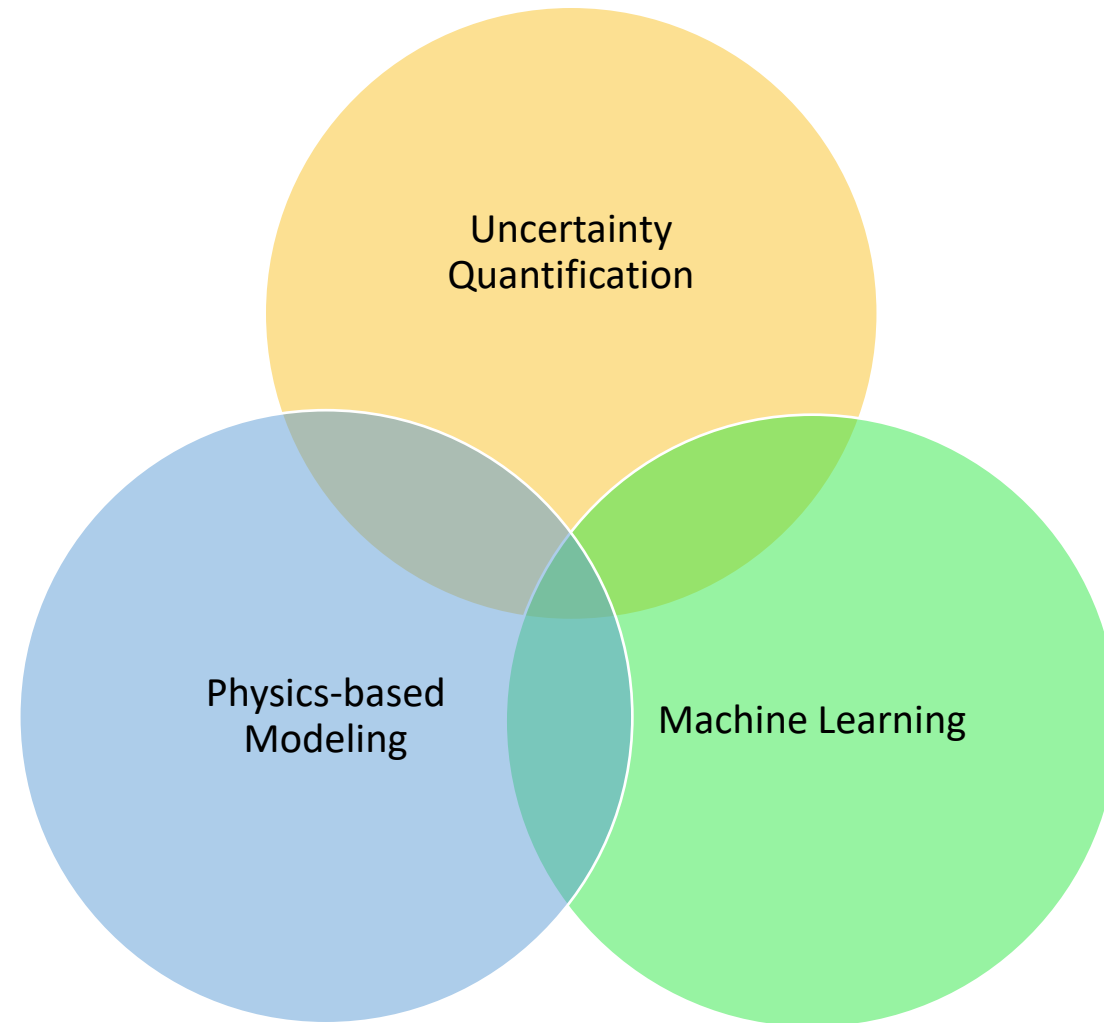
Combining Data with Physical Knowledge for Uncertainty Quantification in Certification and Reliability Analysis

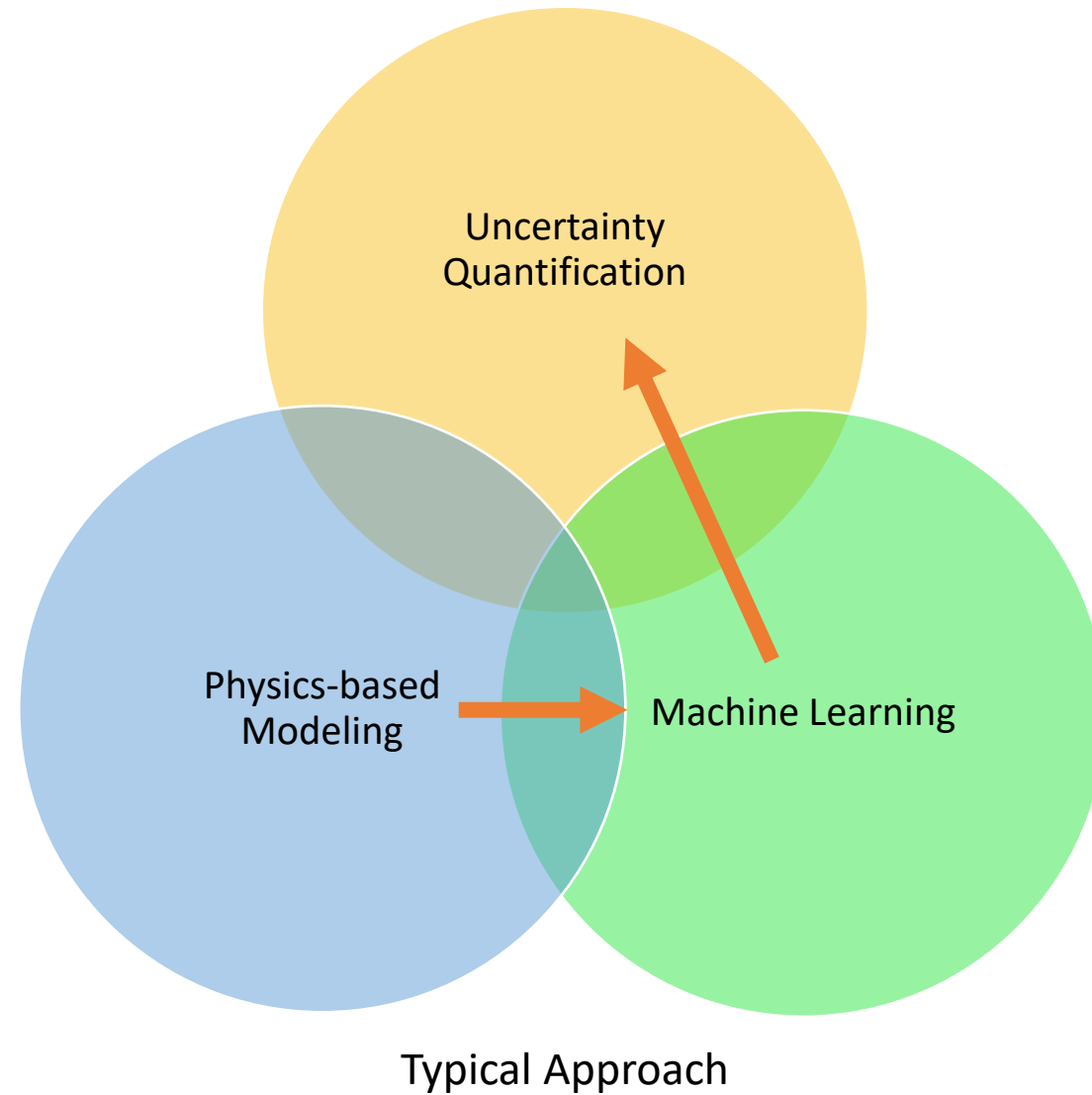
Patrick Leser
NASA Langley Research Center

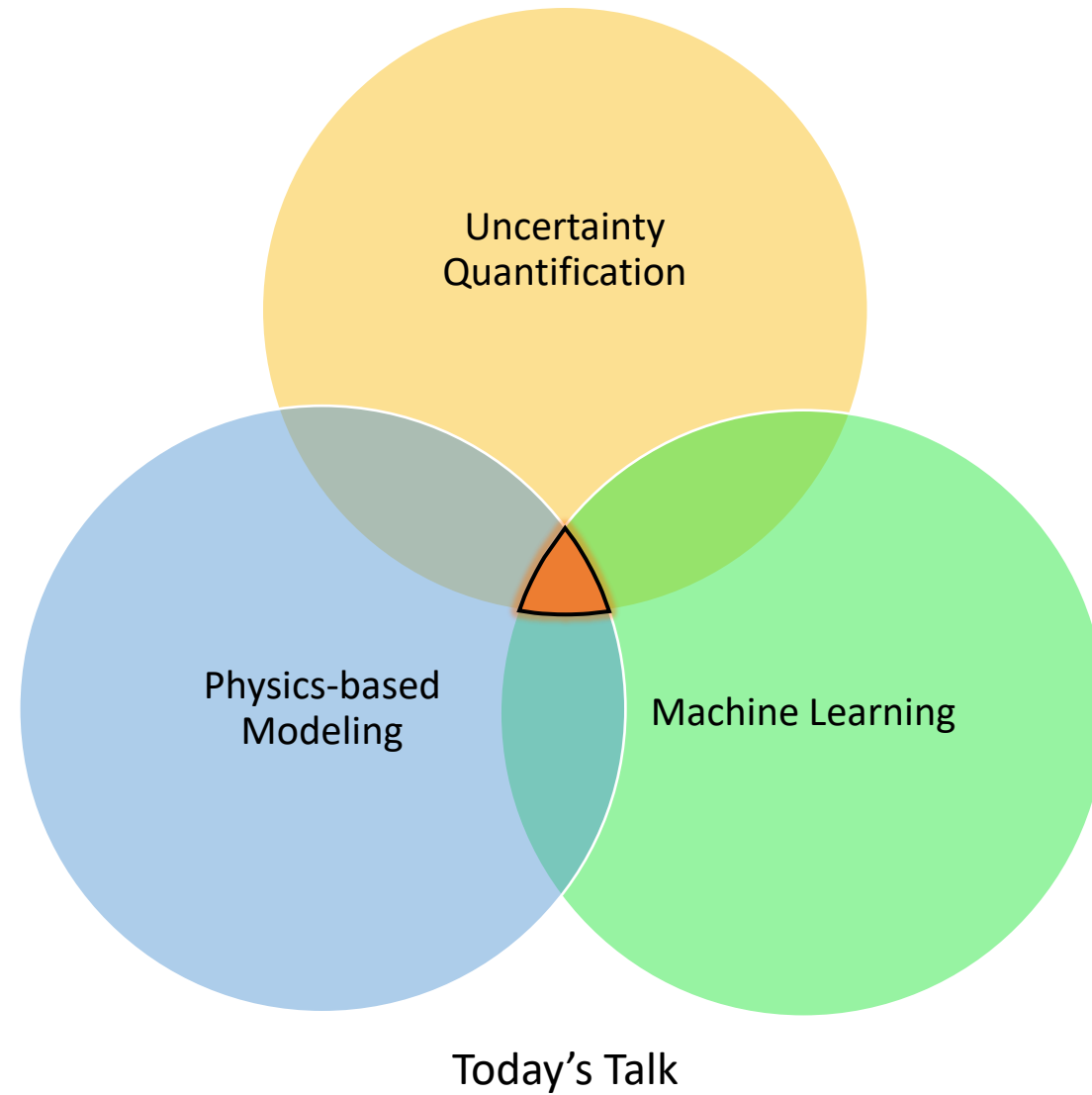
Will Jenkins, Jacob Hochhalter (University of Utah)

Sean Current (Ohio State University), Mohannad Elhamod (Virginia Tech),
Marten Thompson (University of Minnesota)

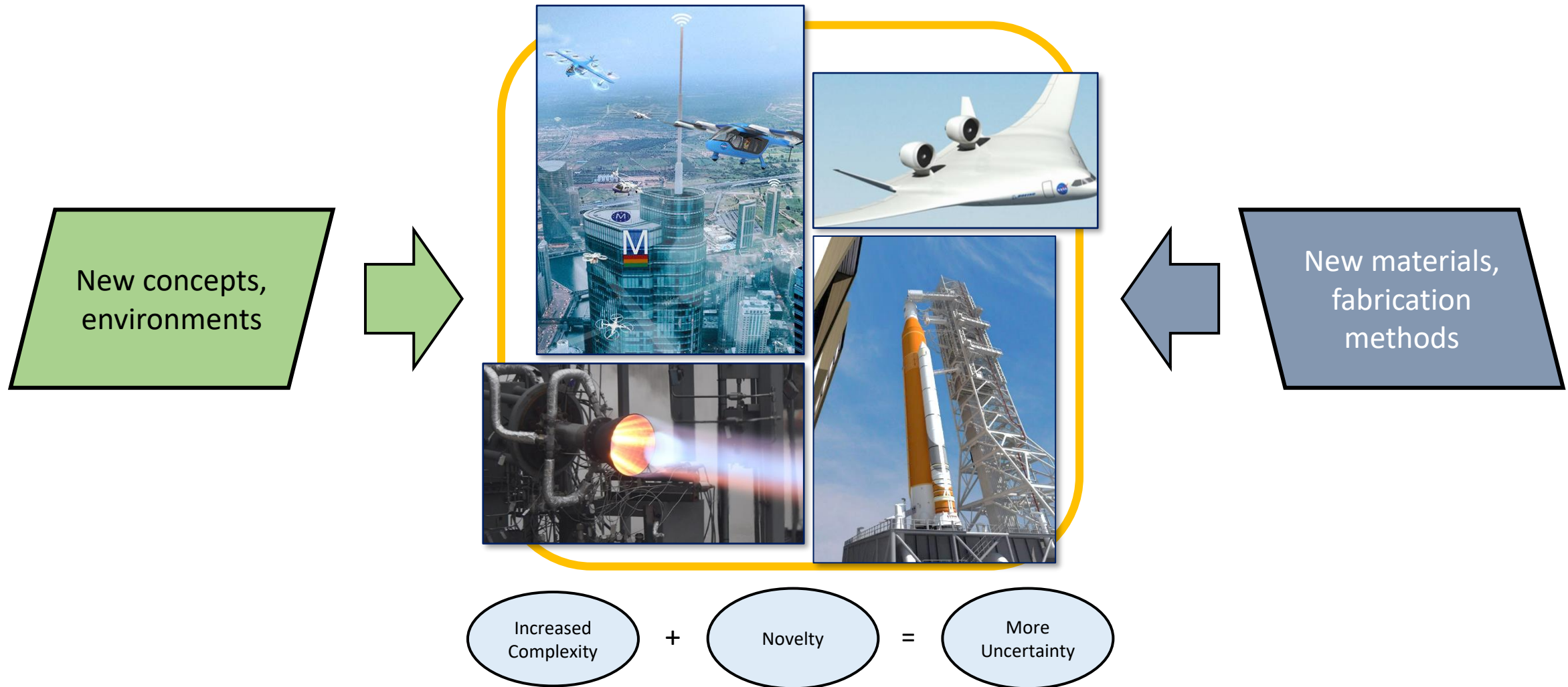
Geoffrey Bomarito, Paul Leser, Jim Warner (NASA Langley Research Center)



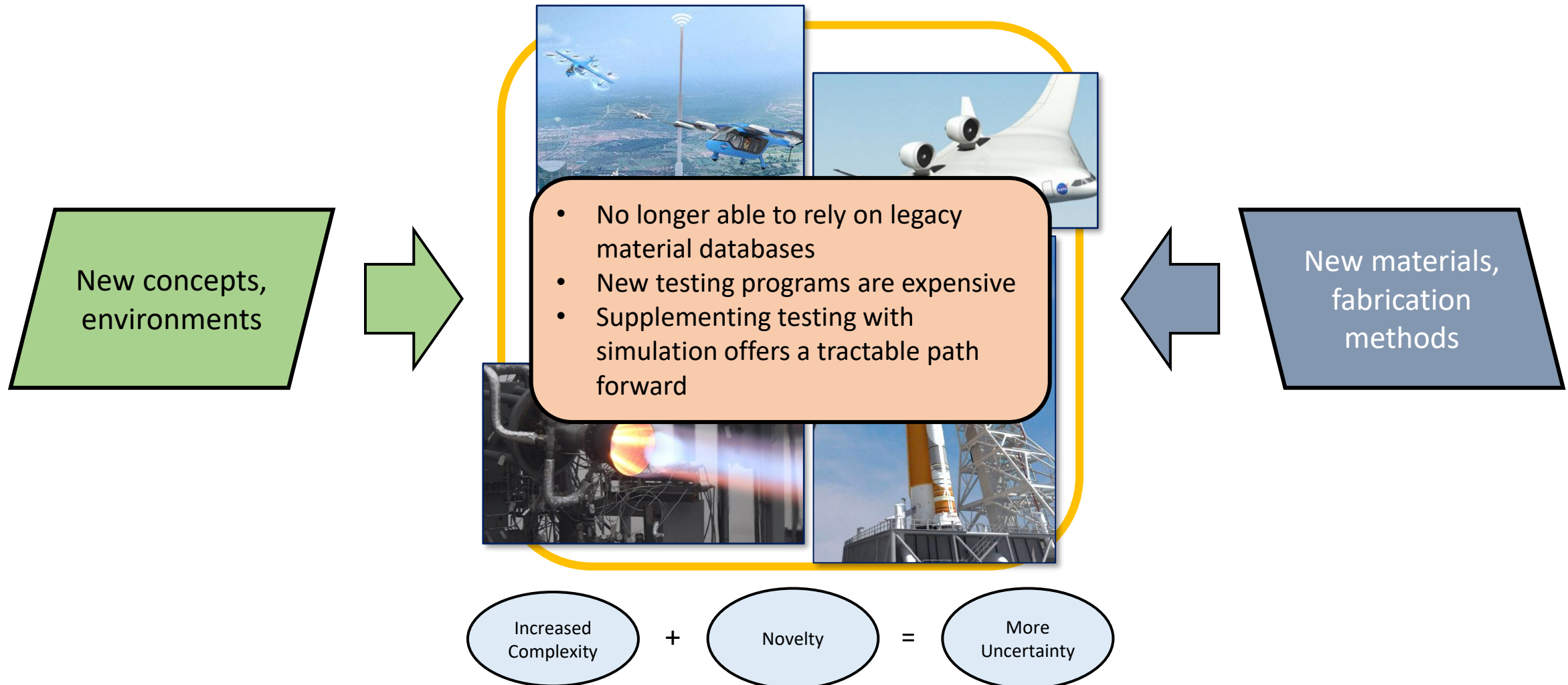




The Role of Simulation in Certification

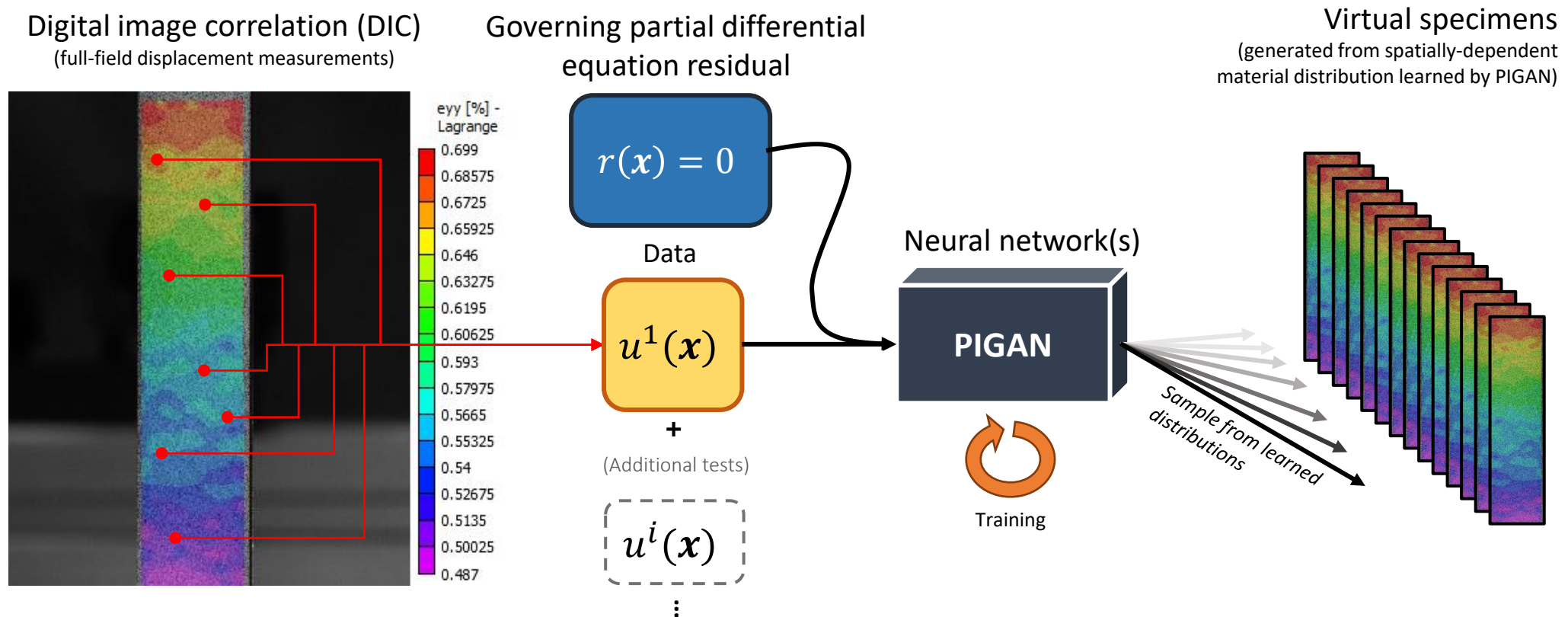


The Role of Simulation in Certification



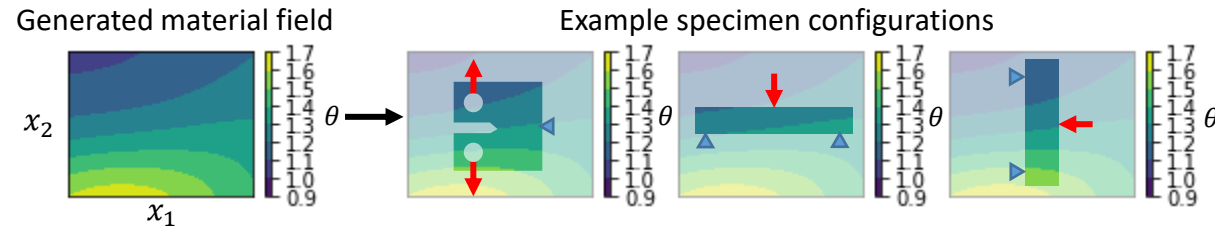
Application: Material Identification

- Learn random material property fields from empirical data using a **Physics-Informed Generative Adversarial Network (PIGAN)**, enabling generation of an infinite number of virtual test specimens and simulation-based uncertainty quantification



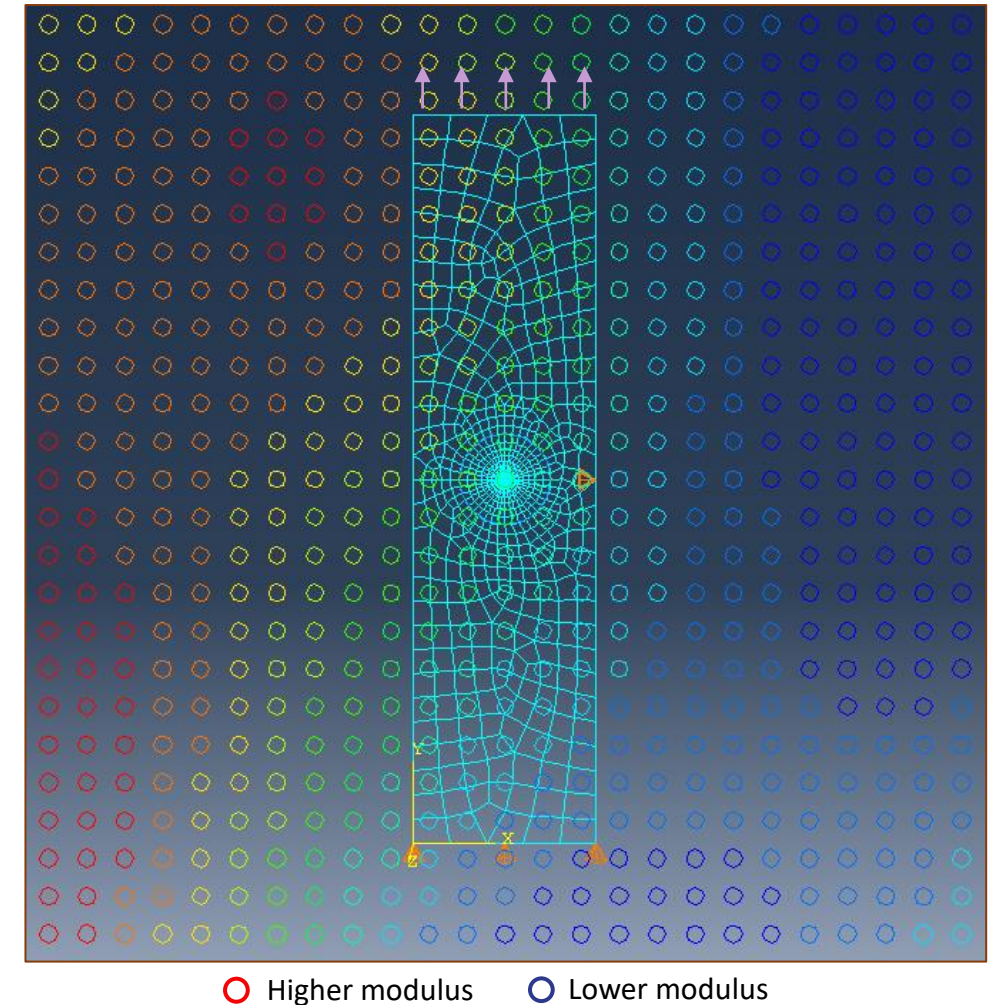
Virtual Test Specimens

- Learned material fields can be mapped to different scenarios (boundary conditions, geometries, etc.)



- Example:** Use finite element analysis (FEA) and Monte Carlo simulation to estimate probability distributions for crack mouth opening displacement (CMOD) due to spatial material variability

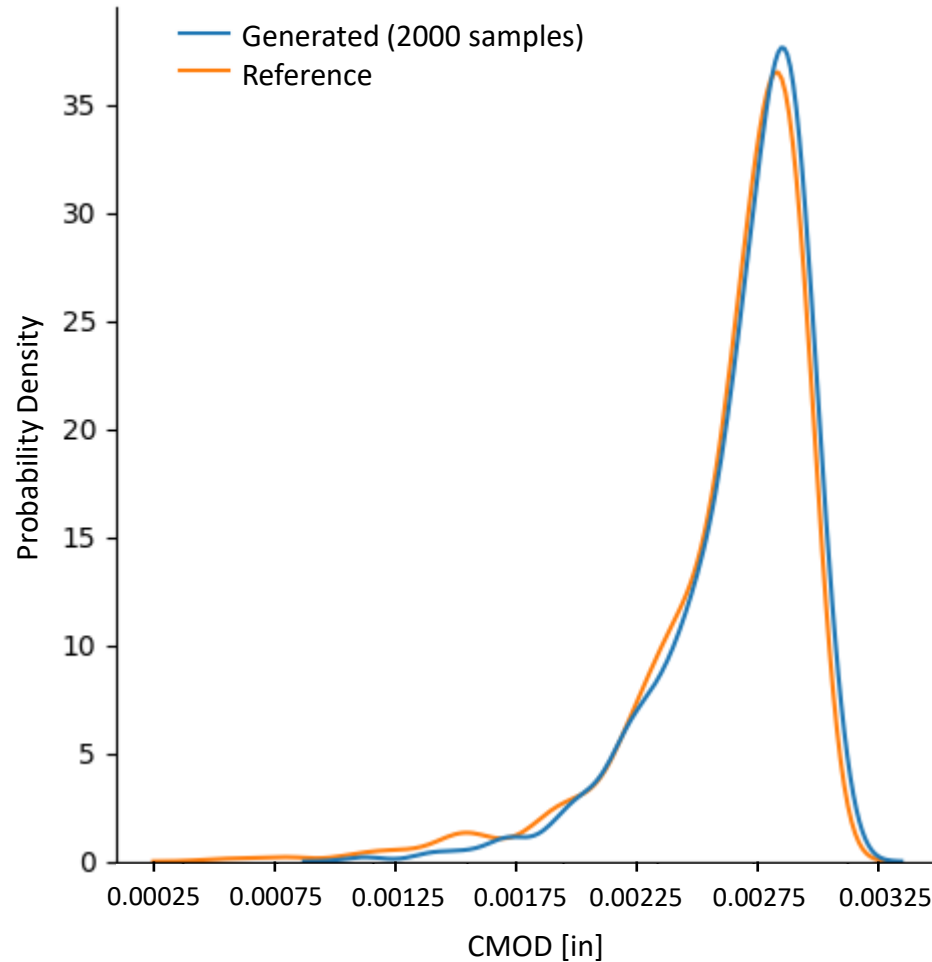
Cracked mesh overlaid on a sampled elastic modulus field
(Abaqus* screenshot)



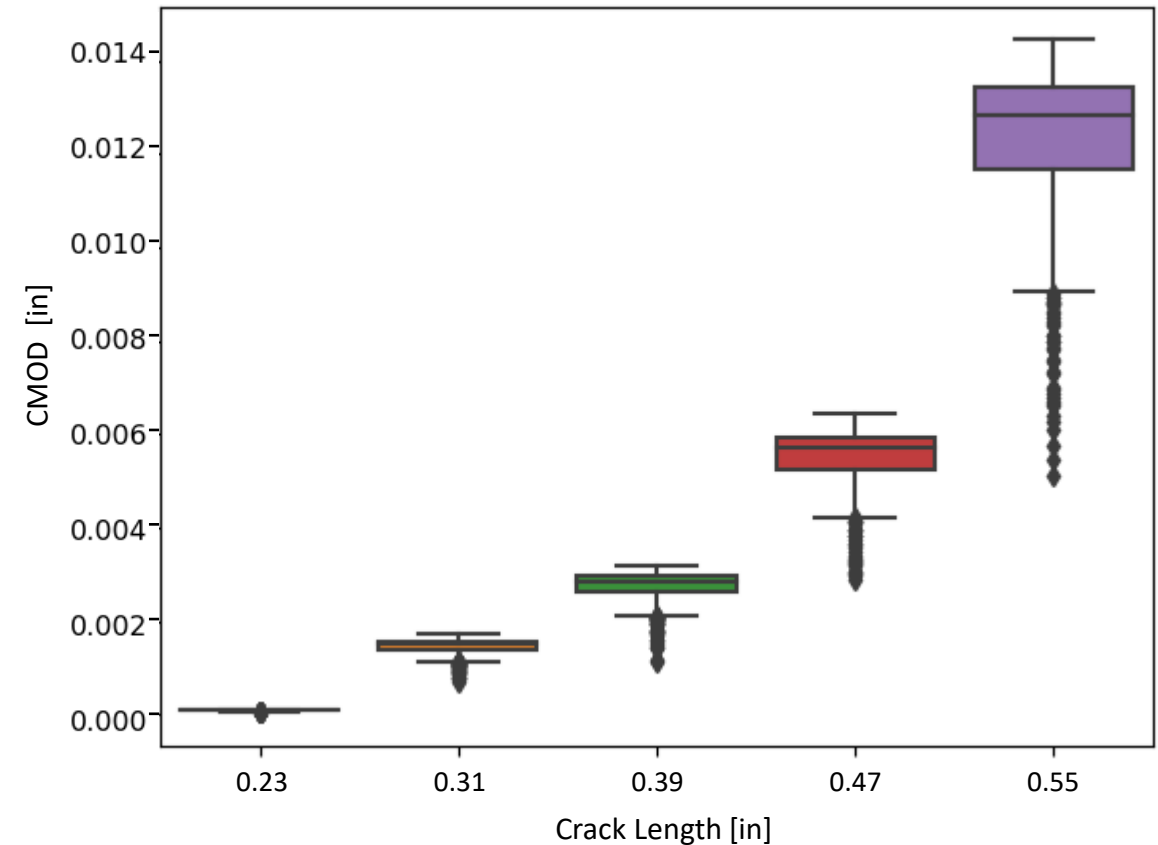
*The use of vendor and manufacturer names does not imply an endorsement by the U.S. Government, nor does it imply that the specified software is the best available

Example: Effect of Material Variability on CMOD

CMOD at fixed crack length of 0.4 in

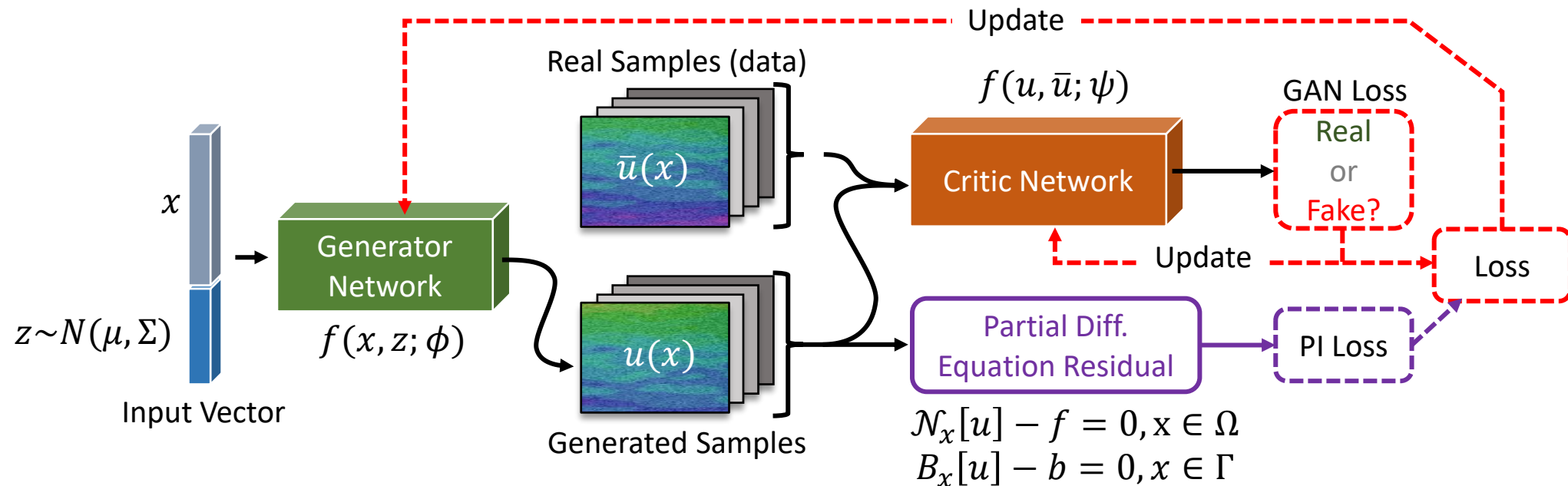


Variability in CMOD as a function of crack length



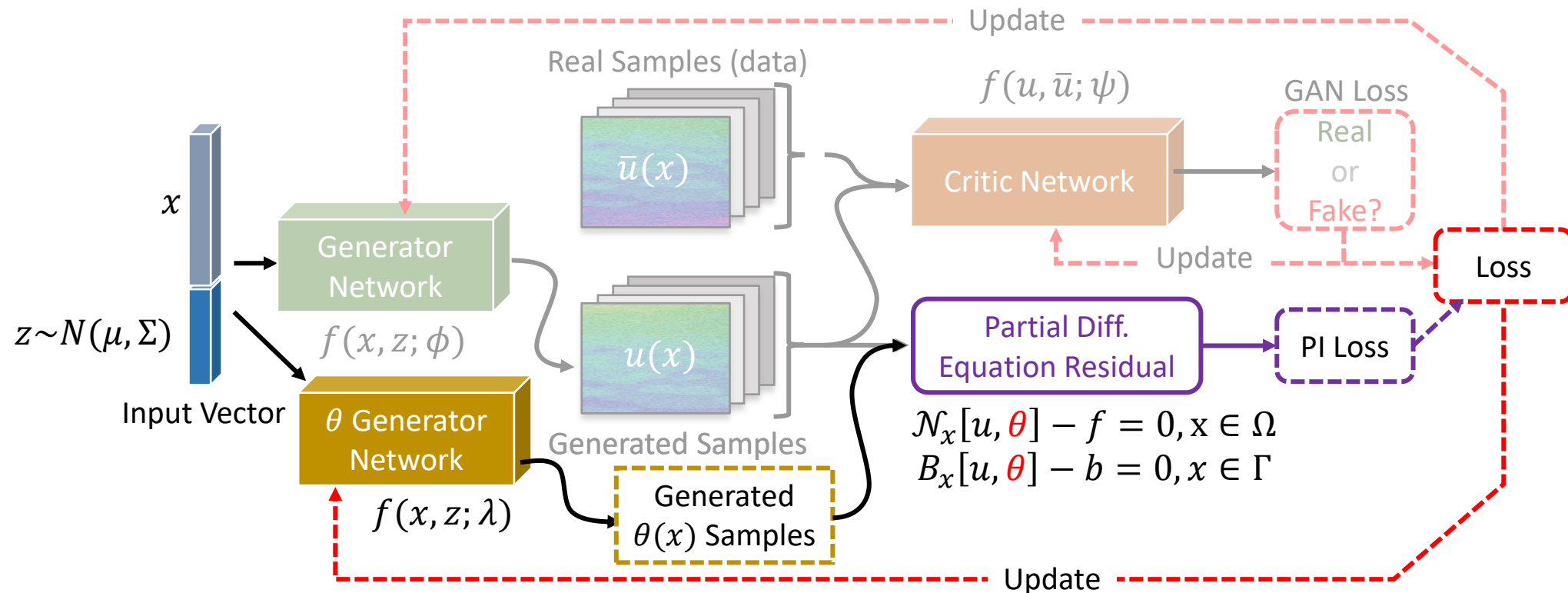
How PIGANs Work

- Combination of physics-informed neural nets (PINNs) and generative adversarial networks (GANs)
- PIGANs enable learning physically admissible, *probabilistic* solution, $\mathbf{u}(x)$



How PIGANs Work

- Additional parameters can be incorporated by introducing additional generators
- Training feedback to the new generator(s) runs through the physics equations



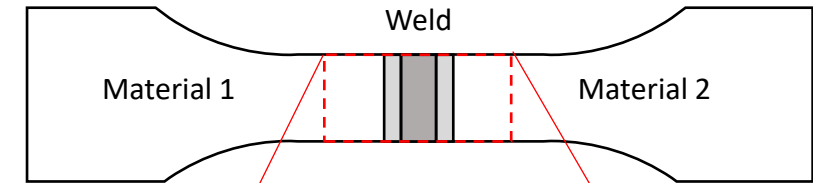
PIGAN Demonstration Overview

- **Objective:** Quantify uncertainty in the elastic modulus field across a bi-material weld given surface displacement measurements from 85 tensile tests
- **Physics:**
 - Measured quantity:
 - $\mathbf{u}(\mathbf{x}, \omega)$ for $\mathbf{x} \in \mathcal{D}$, $\omega \in \Omega$
 - Governing partial differential equation (assuming two-dimensional plane stress, isotropic elasticity, small strains):
 - $\nabla \cdot \boldsymbol{\sigma}(\mathbf{u}, E) = 0$
 - Boundary conditions:
 - $u_1 = 0$, $\mathbf{x} \in \Gamma_1$
 - $u_2 = 0$, $\mathbf{x} = (a, b)$
 - $\boldsymbol{\sigma}(\mathbf{u}, E) \cdot \mathbf{n} = [\tau, 0]$, $\mathbf{x} \in \Gamma_4$ (traction free Γ_2 and Γ_3)

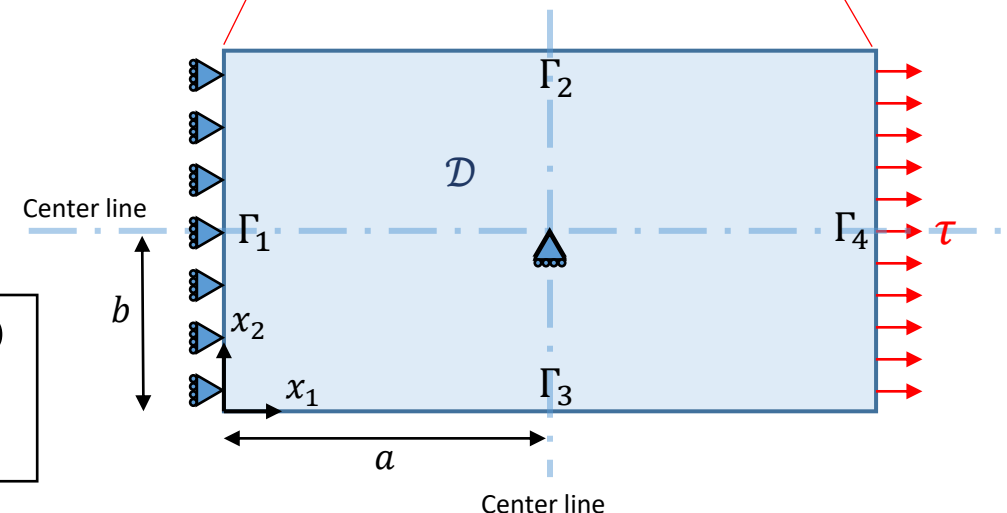
Definitions

$\mathbf{x}$ – position vector	E – elastic modulus field; i.e., $E(\mathbf{x}, \omega)$
$\mathbf{u}$ – displacement vector	$\mathbf{n}$ – surface outward normal vector
ω – random event in sample space, Ω	τ – applied traction
$\boldsymbol{\sigma}$ – second order stress tensor	

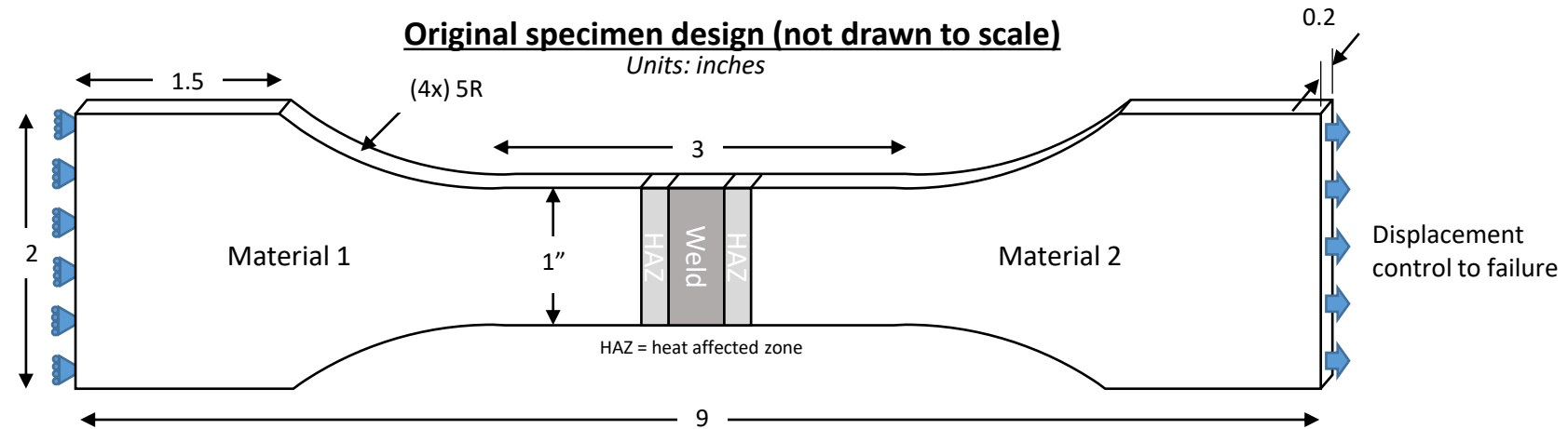
Test specimen schematic (not to scale)



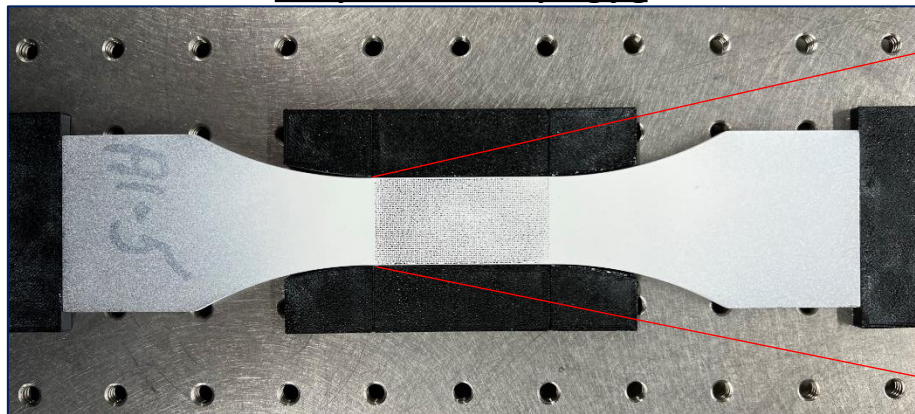
Material region of interest



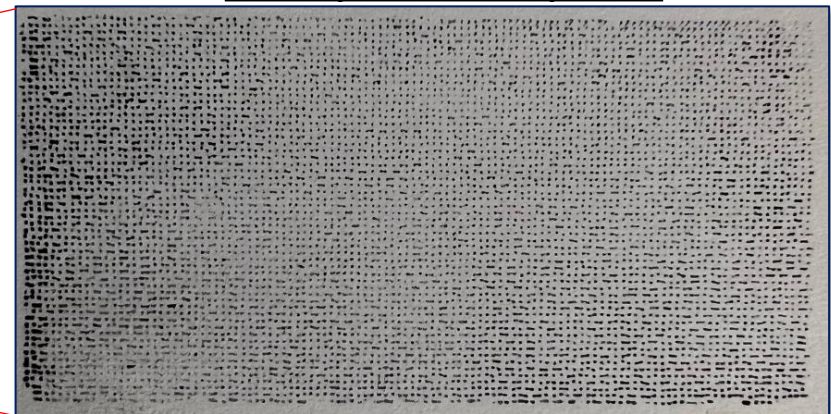
Experimental Setup



DIC pattern stamping jig

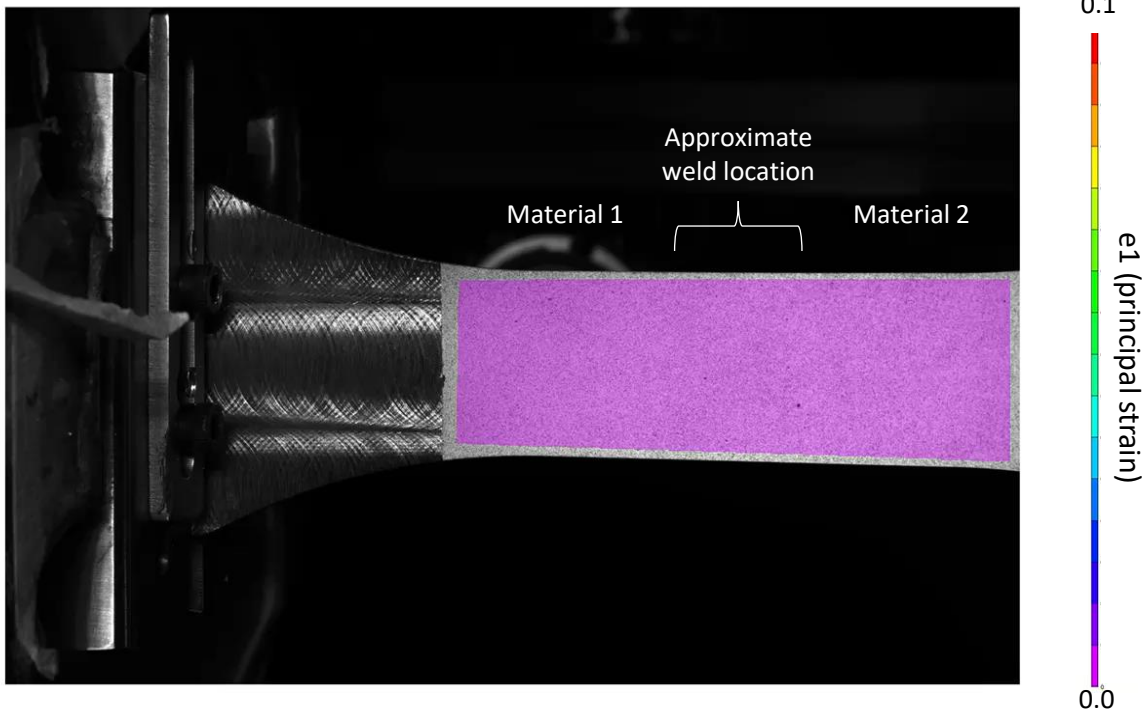


Close up view of DIC pattern

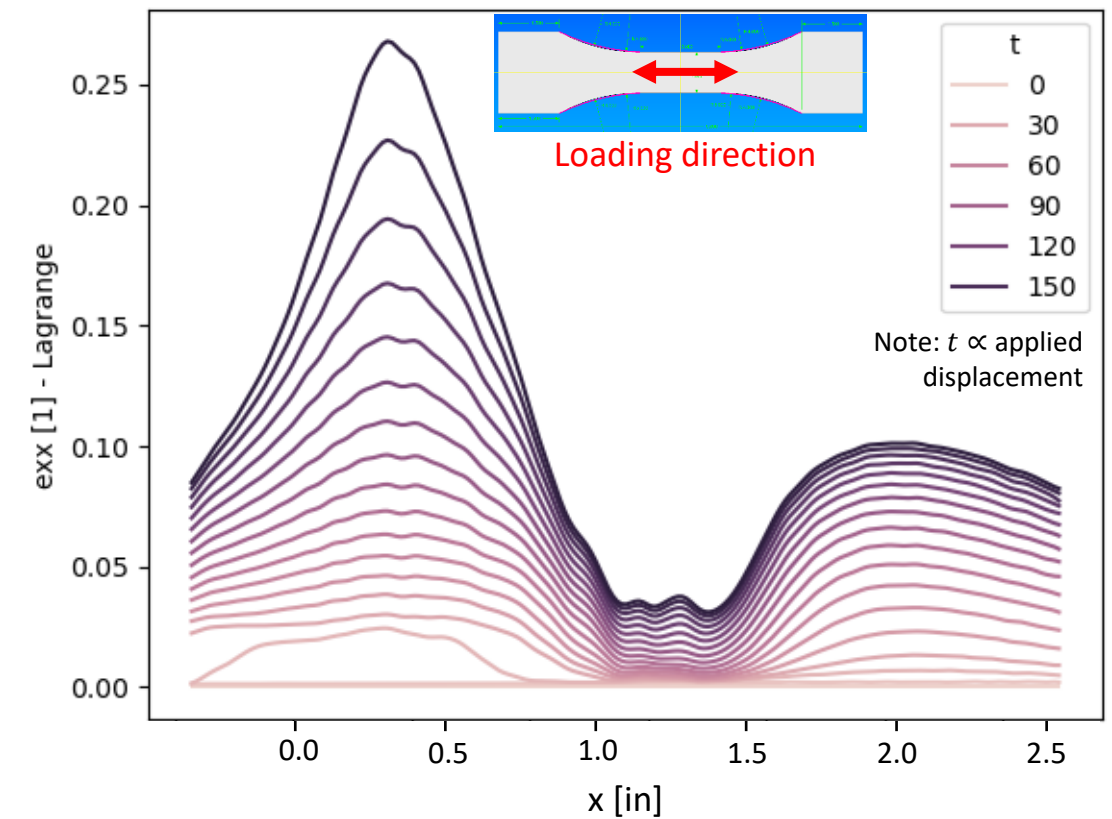


Digital Image Correlation

1/8 inch thick coupon, strain in loading direction

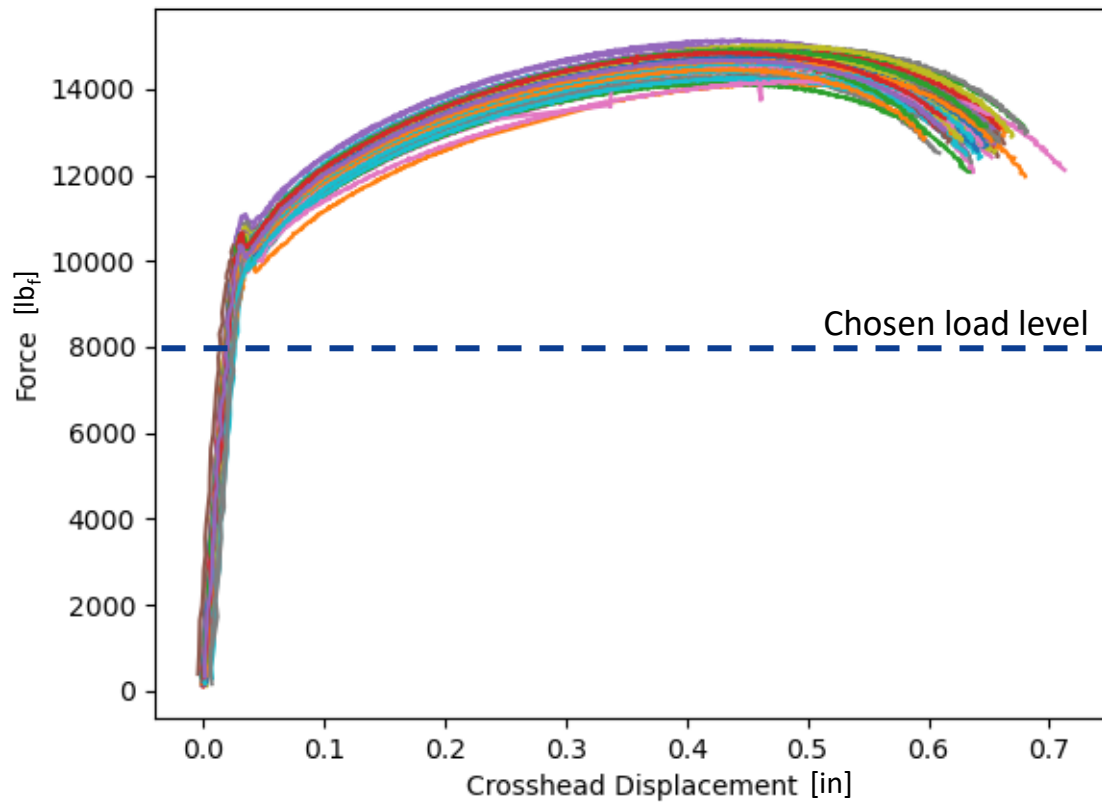


Strain along center of coupon in loading direction

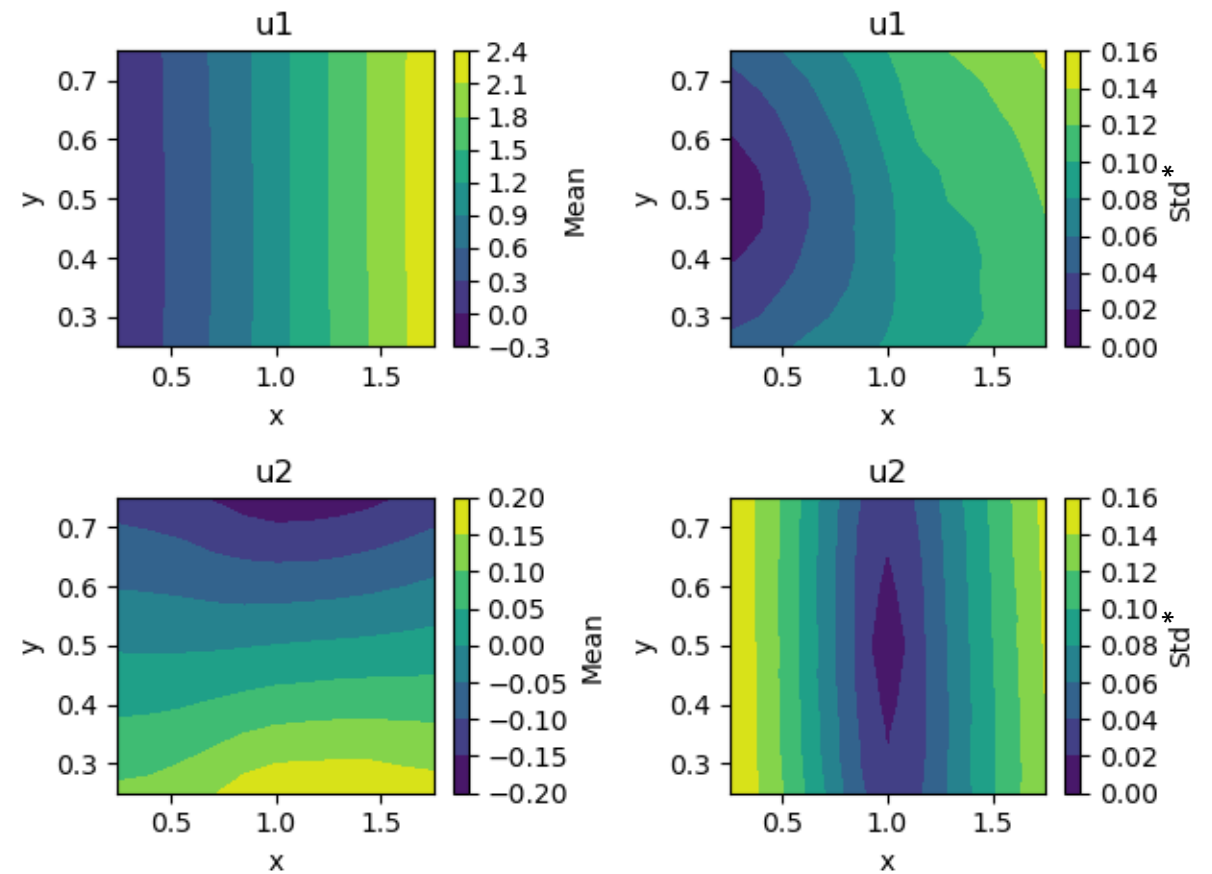


Summary of Test Data

Global force versus displacement response for all 85 tests



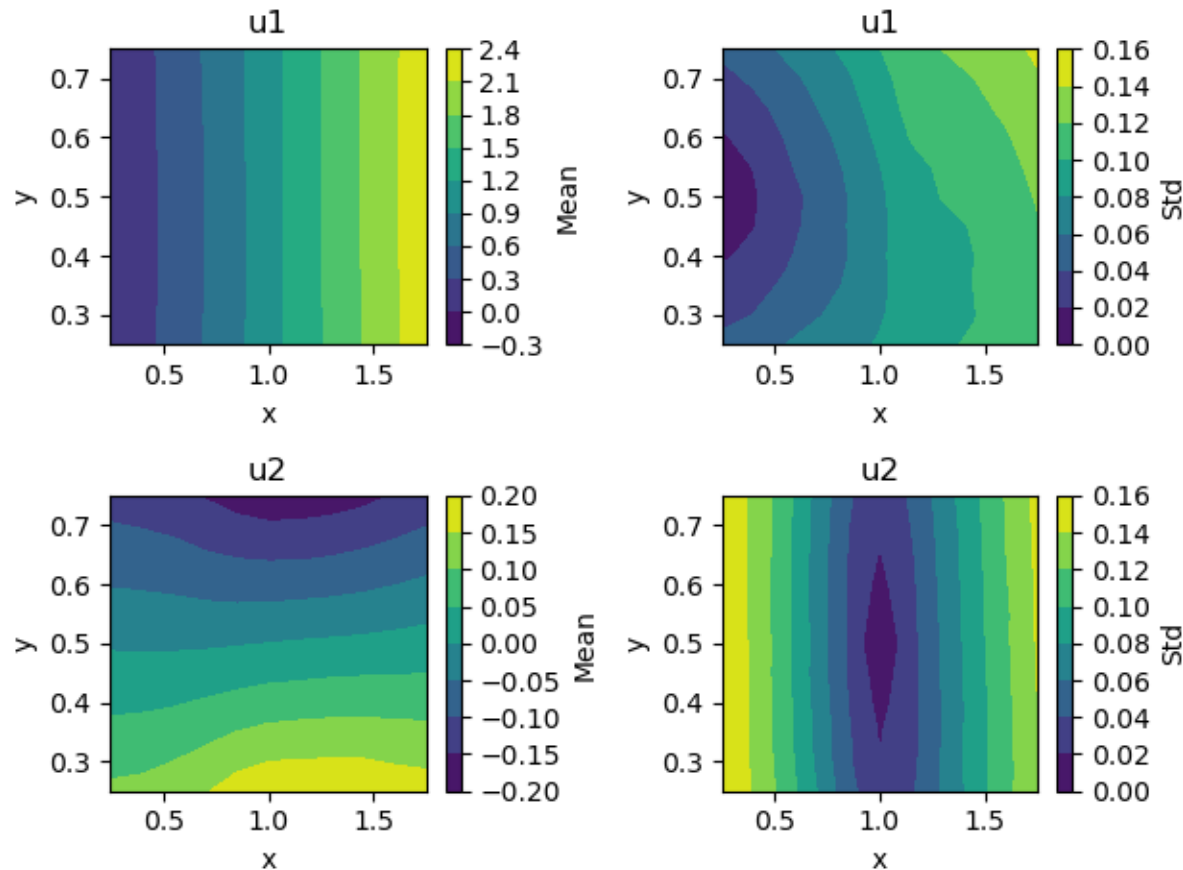
Normalized displacement statistics for all 85 tests



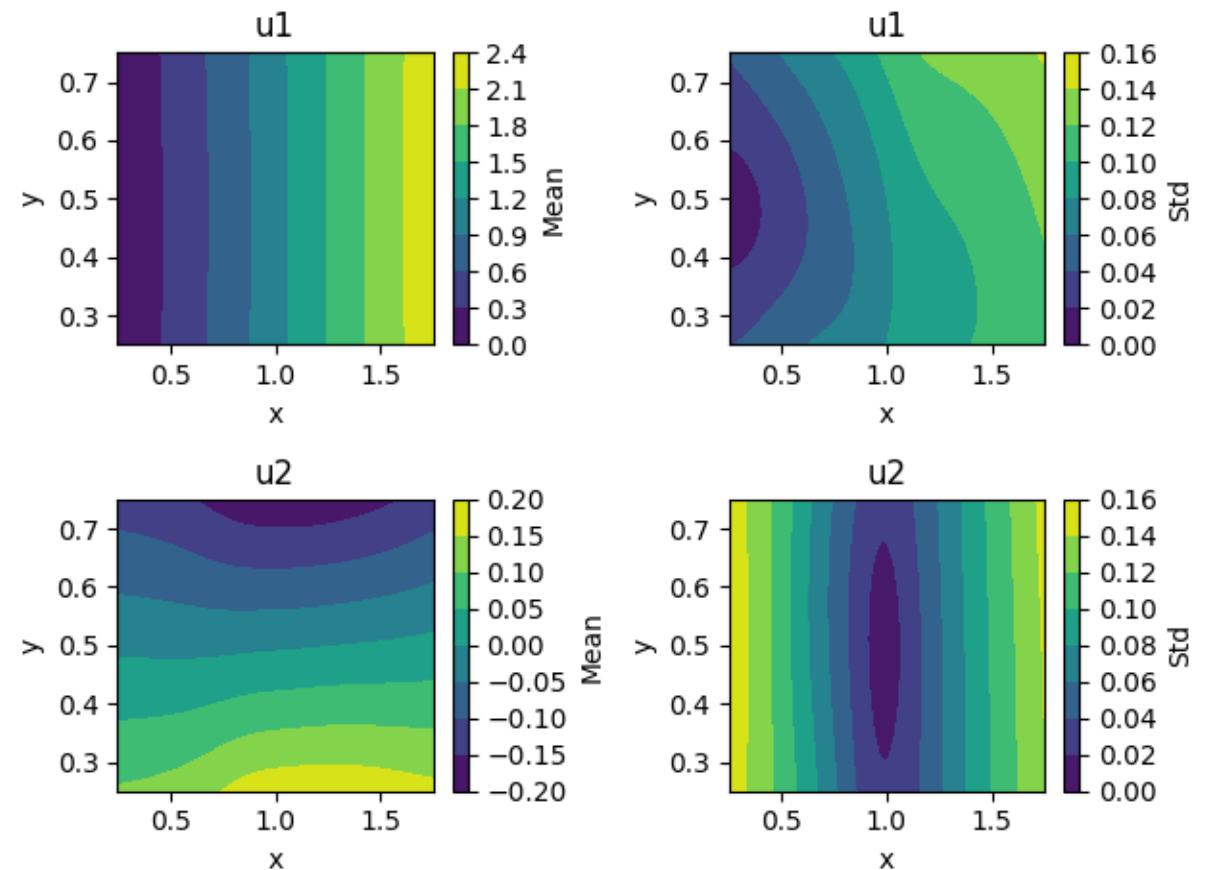
*Std = Standard deviation

Results: Displacement Statistics

Normalized displacement statistics from all 85 tests

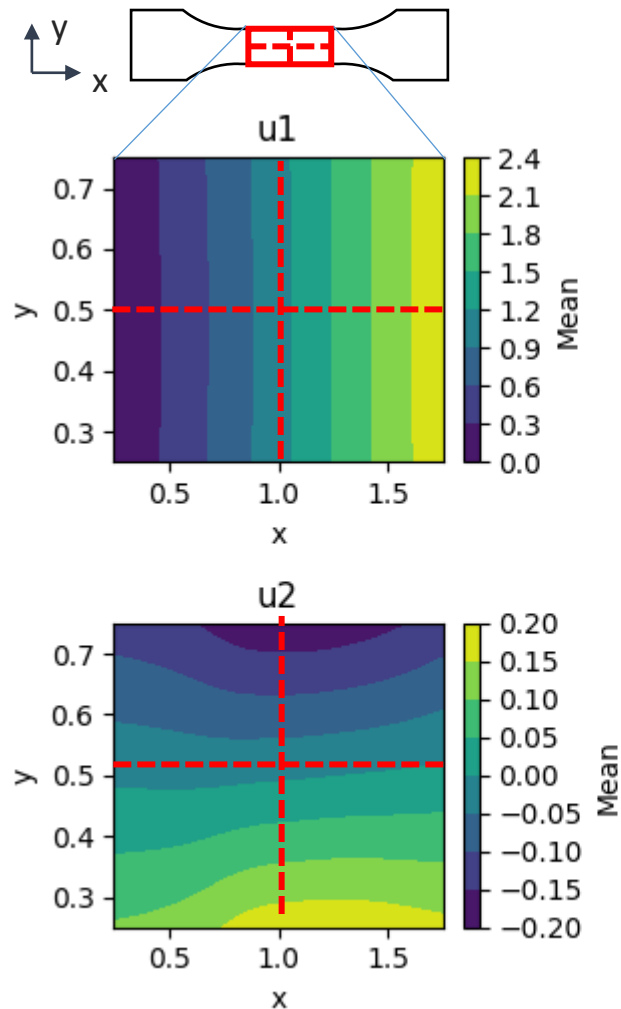


Predicted normalized displacement statistics
(1000 PIGAN-generated samples)

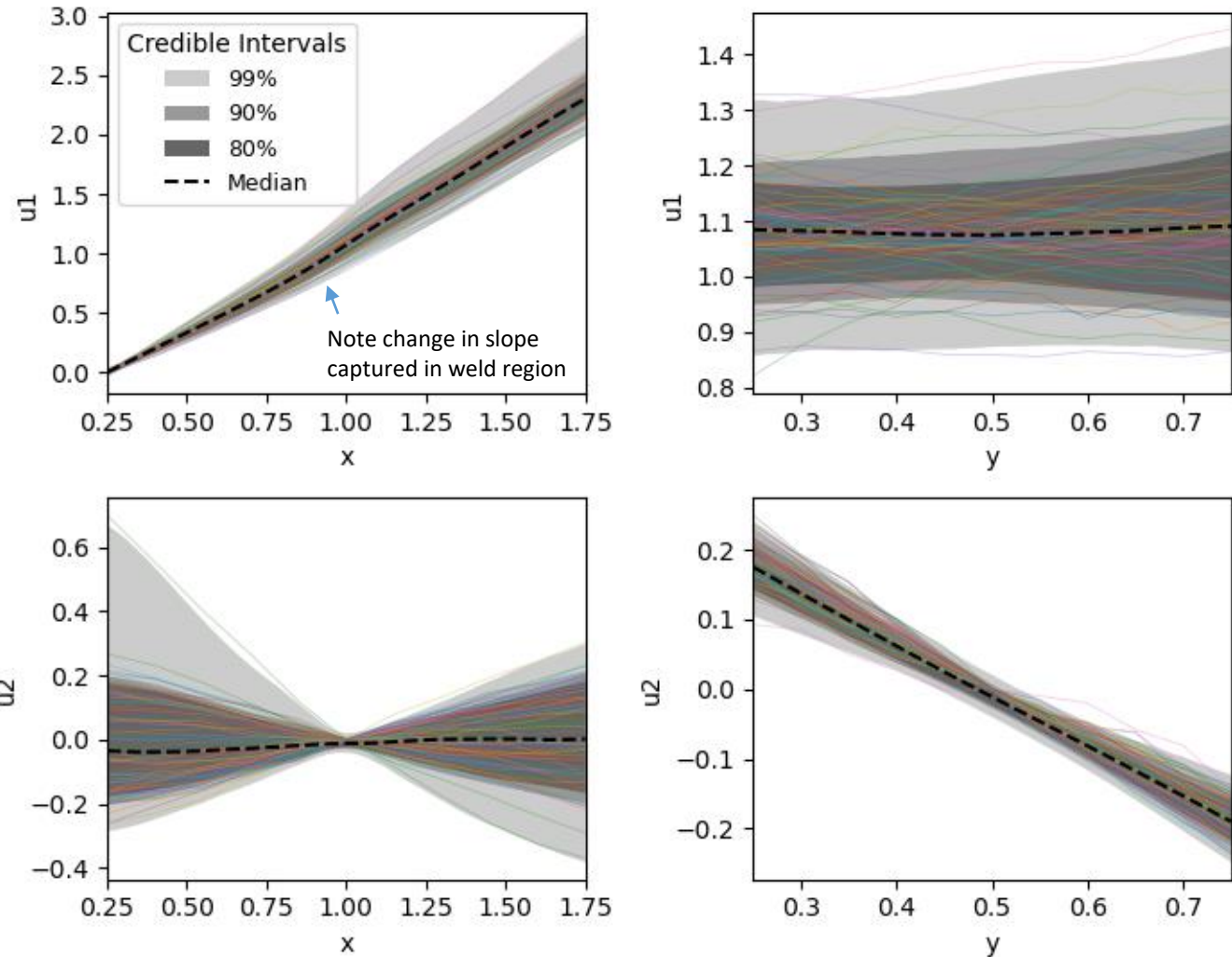


Results: Displacement Distributions

Mean displacement fields for reference

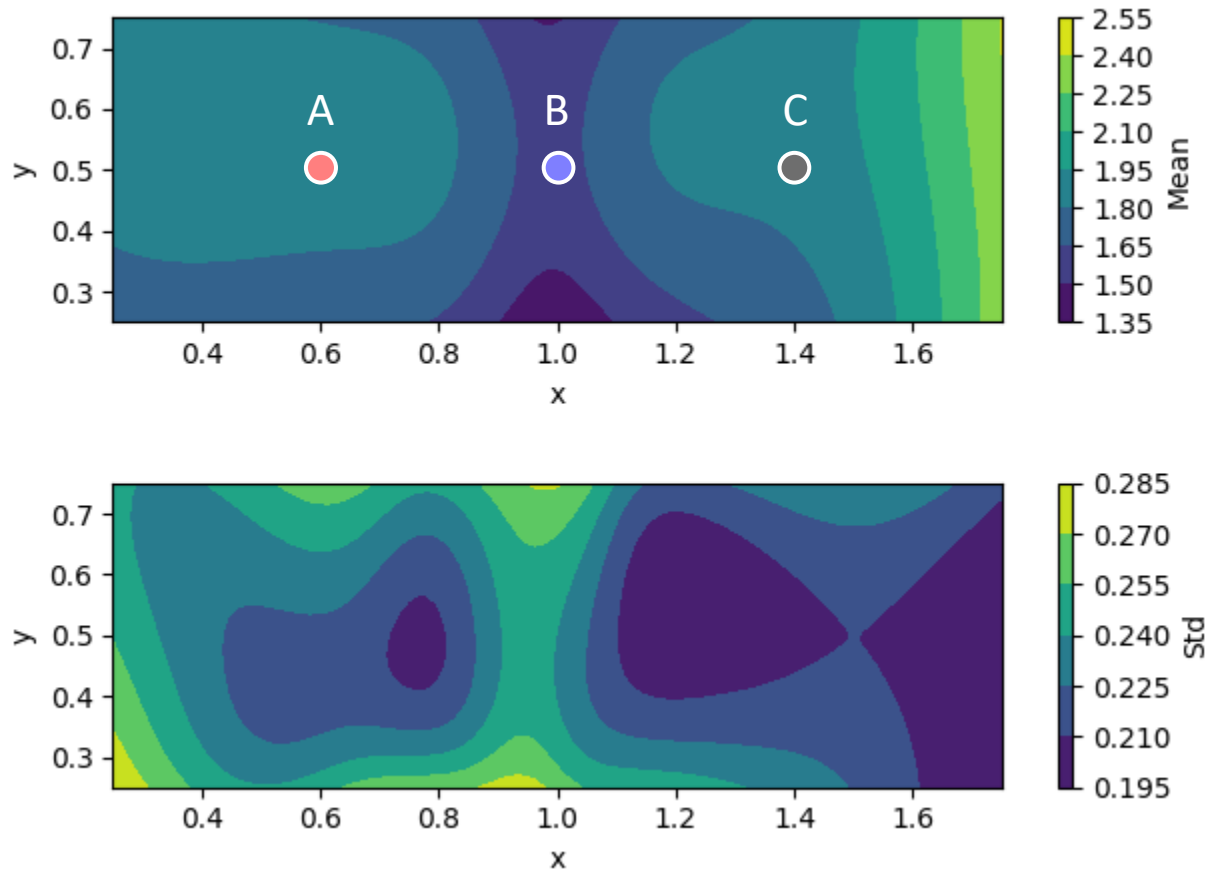


Cross-sectional displacement distributions compared to test data
(10,000 PIGAN-generated samples, colored lines indicate DIC data)

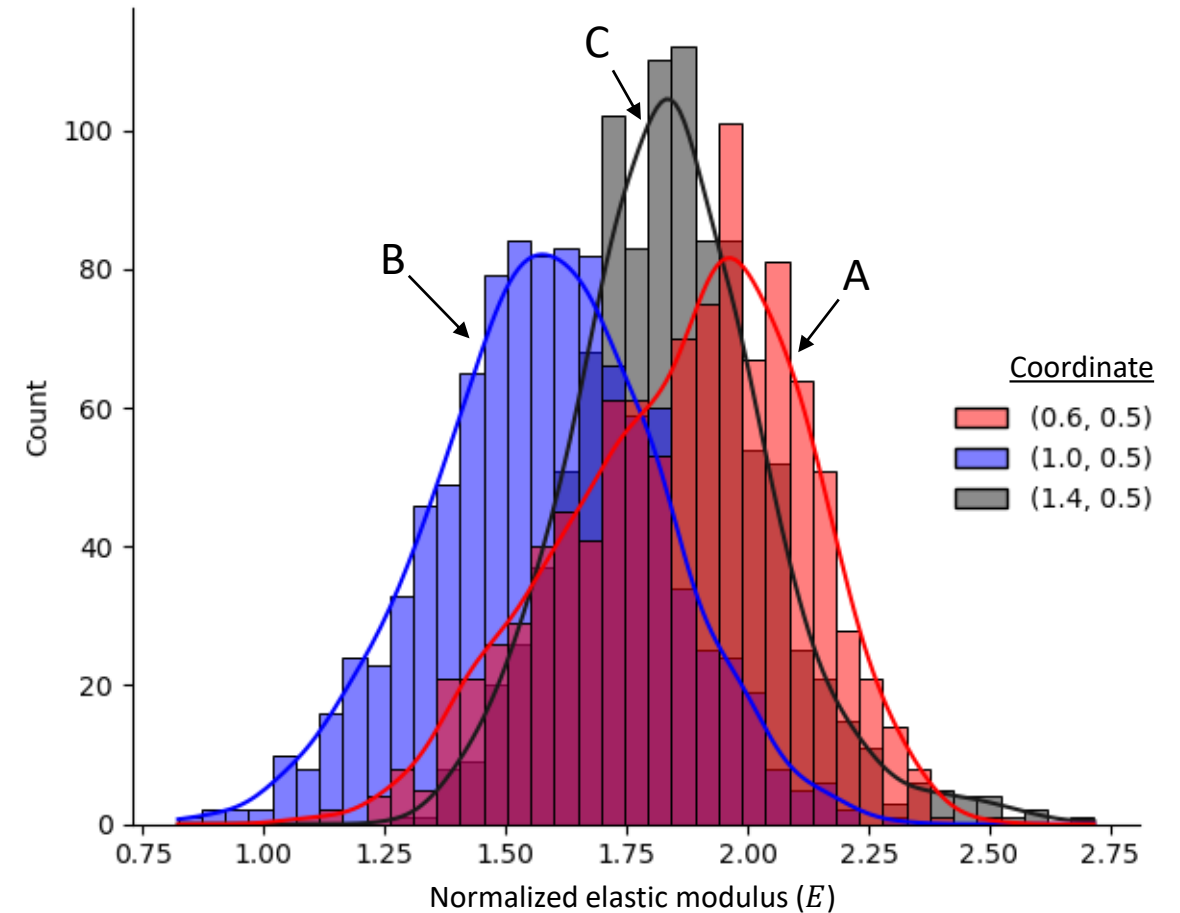


Results: Predicting Elastic Modulus

Predicted normalized elastic modulus statistics
(1000 PIGAN-generated samples)



Predicted normalized elastic modulus at three points
(1000 PIGAN-generated samples)



Concluding Remarks

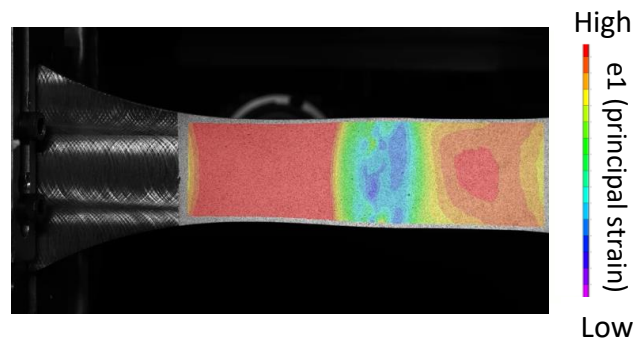
1. Short term goals

- Verification, validation, and publication
- For some existing documentation, see:

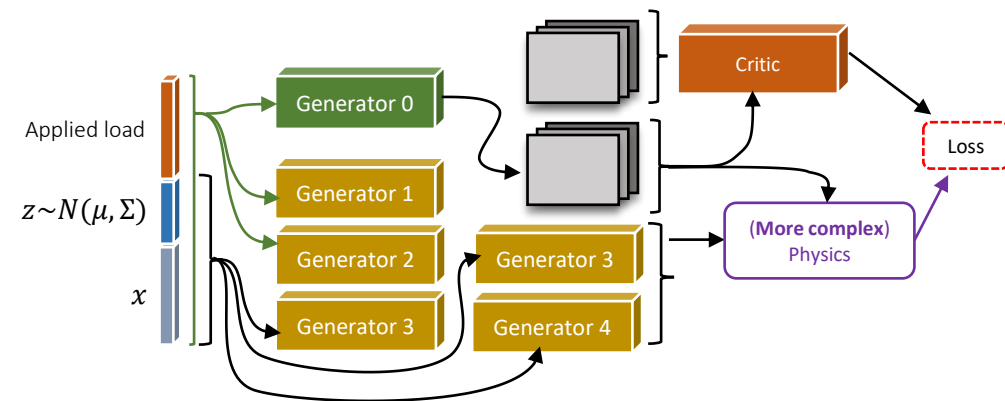
Warner, James E., et al. "Inverse estimation of elastic modulus using physics-informed generative adversarial networks." NASA-TM-2020-5001300 (2020)

2. Estimating properties governing plasticity:

Large variations in response in the plastic regime



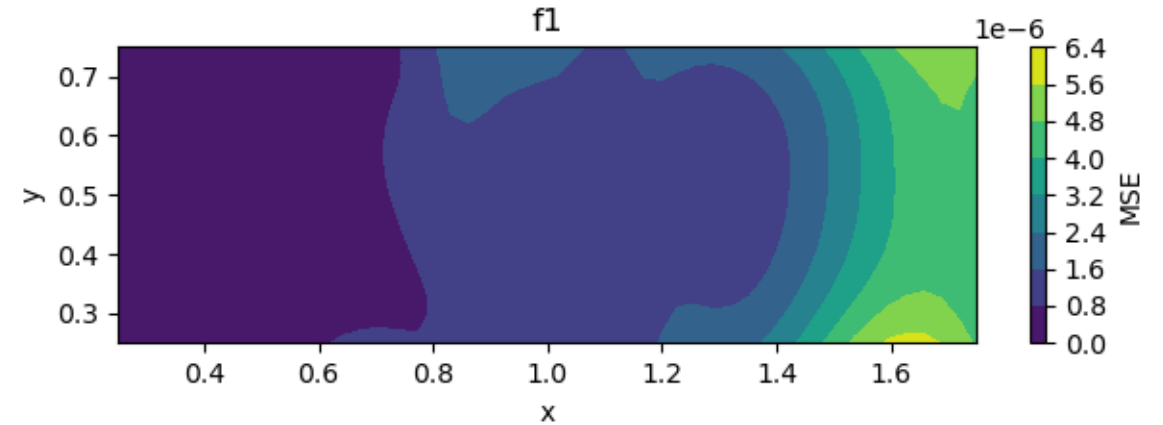
Significant increase in PIGAN architecture complexity



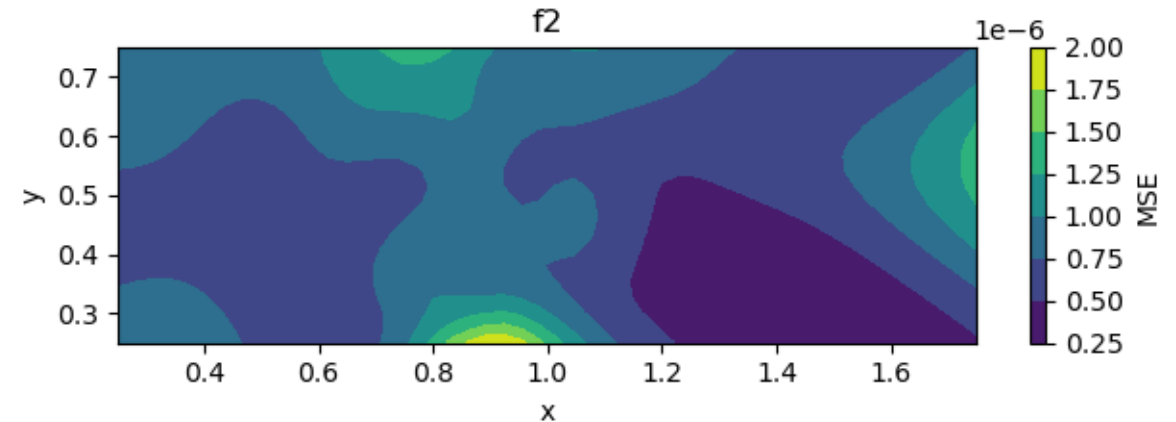
3. Upgrade to more modern generative models

Results: Physics Residual

$$\frac{(1-\nu)}{2} [E_{,2}(u_{1,2} + u_{2,1}) + E(u_{1,22} + u_{2,12})] + E_{,1}(u_{1,1} + \nu u_{2,2}) + E(u_{1,11} + \nu u_{2,21}) = 0 \quad (26)$$



$$\frac{(1-\nu)}{2} [E_{,1}(u_{1,2} + u_{2,1}) + E(u_{1,21} + u_{2,11})] + E_{,2}(\nu u_{1,1} + u_{2,2}) + E(\nu u_{1,12} + u_{2,22}) = 0, \quad (27)$$



See Warner, James E., et al. "Inverse estimation of elastic modulus using physics-informed generative adversarial networks." NASA-TM-2020-5001300 (2020) for additional information on (26) and (27)