Asteroseismic Inversions of Mixed Acoustic-Gravity Modes to Probe the Stellar Core Structure

Alexander Kosovichev (New Jersey Institute of Technology) and Irina Kitiashvili (NASA Ames Research Center)

The discovery of mixed acoustic-gravity modes of oscillations of moderate mass stars opens a unique opportunity to infer the structure of the inner energy-generating cores and thus test the stellar evolution theory. The mixed modes have properties of internal gravity waves (g-modes) in the convectively stable helium core and properties of acoustic modes outside the core. We select several sets of the oscillation mode frequencies in the mass range from about 1.3 to 1.6 solar masses from the Kepler Legacy database, and apply the optimally localized averaging inversion technique previously developed for low-degree helioseismology. The inversion technique takes into account the uncertainties in the determination of the mass and radius of the stars, as well as the surface effects. The methodology provides sensitivity kernels for various structure properties, including the sound speed, density, and Ledoux parameter of convective stability, and, thus, the direct relationship between the stellar properties and the deviation of observed frequencies from the reference models. The inversion results reveal significant deviations in the core structure from the reference models calculated using the MESA evolutionary code for the stellar parameters obtained by the asteroseismic model grid fitting. Our analysis shows that the best resolution of the inner helium core and surrounding shell is achieved in inversions for the Ledoux parameter.

Motivation

Our goal is to investigate how the detection of mixed modes in the oscillation spectra of F-type stars helps to resolve the structure of the inner stellar cores. The mixed modes have properties of internal gravity waves (g-modes) in the convectively stable helium core and properties of acoustic modes outside the core. The oscillation frequencies of these modes may be quite sensitive to the properties of the core. Including them in the inversion procedure allows us to localize the averaging kernels in the core region. This opens a unique opportunity for testing the stellar evolution theory for subgiant stars where the energy release is in a hydrogen shell surrounding a helium core.

Target selection

From the Kepler Asteroseismic Legacy Sample (Silva Aguirre et al. 2017) we selected 6 stars in the mass range from about 1.3 to 1.6 solar masses, and, using the MESA stellar evolutionary code, we calculated models of the internal structure, matching as close as possible the stellar parameters determined by grid fitting methods. Specifically, we chose the stellar mass, chemical composition, and mixing-length parameter from the grid pipeline YMCM (Silva Aguirre et al.2015) and evolved starting from a pre-main sequence phase to the age estimated from the grid-fitted models. The basic parameters of the calculated models are shown in Table.

KIC	$M/M_\odot$	$\lg(R/R_\odot)$	Age(Gyr)	$\lg(L/L_\odot)$	Ysurf	Zsurf	Teff	alpha
7206837	1.298	0.1980	2.90	0.5523	0.2800	0.0220	6320	1.791
1435467	1.382	0.2436	2.56	0.6694	0.2637	0.0197	6414	1.692
10162436	1.461	0.3082	2.51	0.8110	0.2460	0.0173	6494	1.684
9353712	1.516	0.3261	2.03	0.9008	0.2603	0.0180	6665	1.713
5773345	1.579	0.3074	2.07	0.7931	0.2593	0.0306	6400	1.715
12069127	1.588	0.3403	1.76	0.9385	0.2669	0.0216	6700	1.672

Reference Evolutionary Models

In this poster, we present the results for two stars: KIC 10162436 and KIC 5773345 with similar structures.

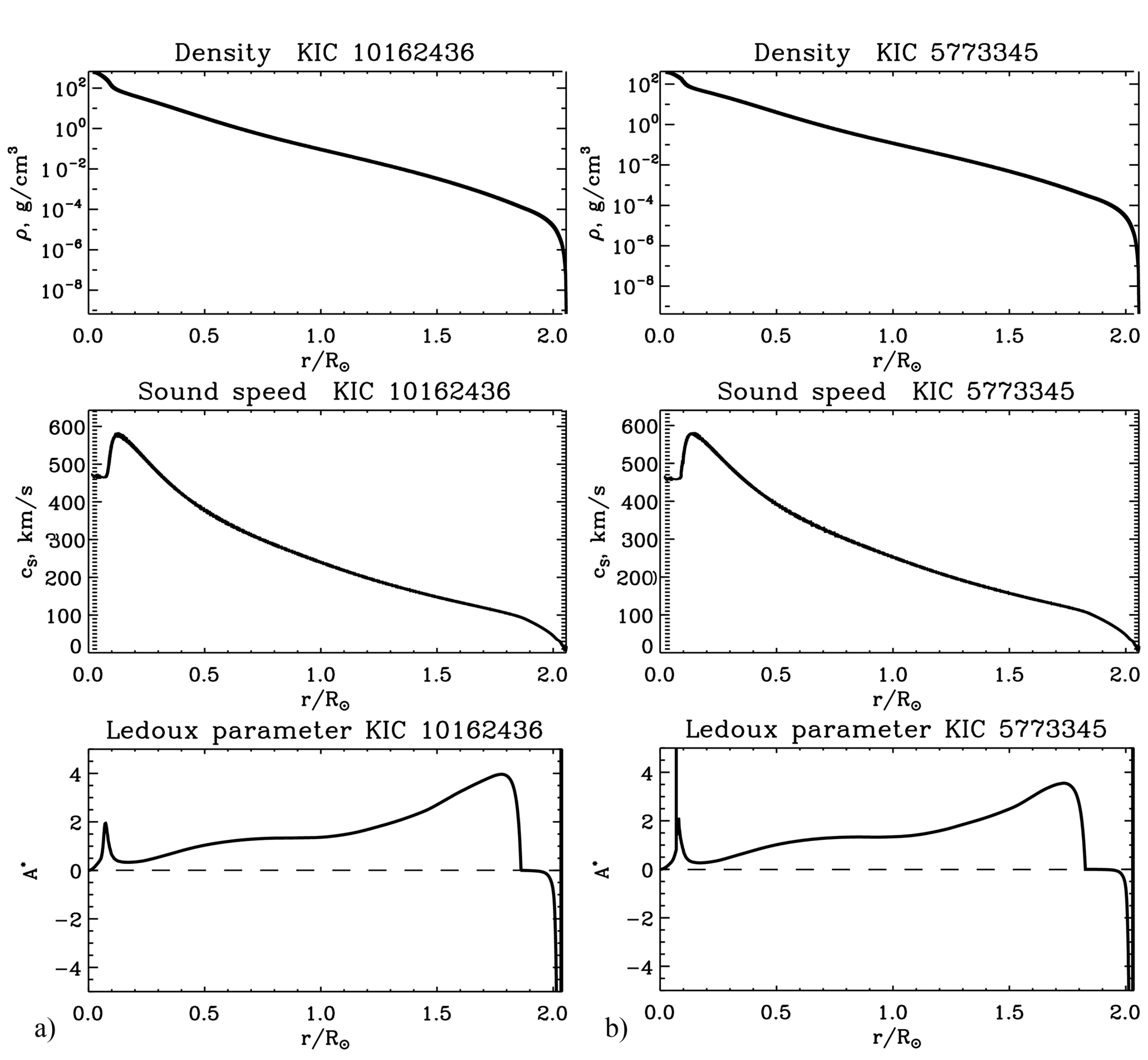

Figure 1. The density, sound speed, and the Ledoux parameter

$$A^* = 1/\gamma(d\log P/d\log r) - (d\log\rho/d\log r)$$

as a function of radius in the reference MESA models of two Kepler target stars. Sharp peaks in A^* at about $0.05-0.07\,R_\odot$ correspond to the helium core boundary.

Difference between observed and model frequencies

The oscillations frequencies for the reference models are calculated in the adiabatic approximation.

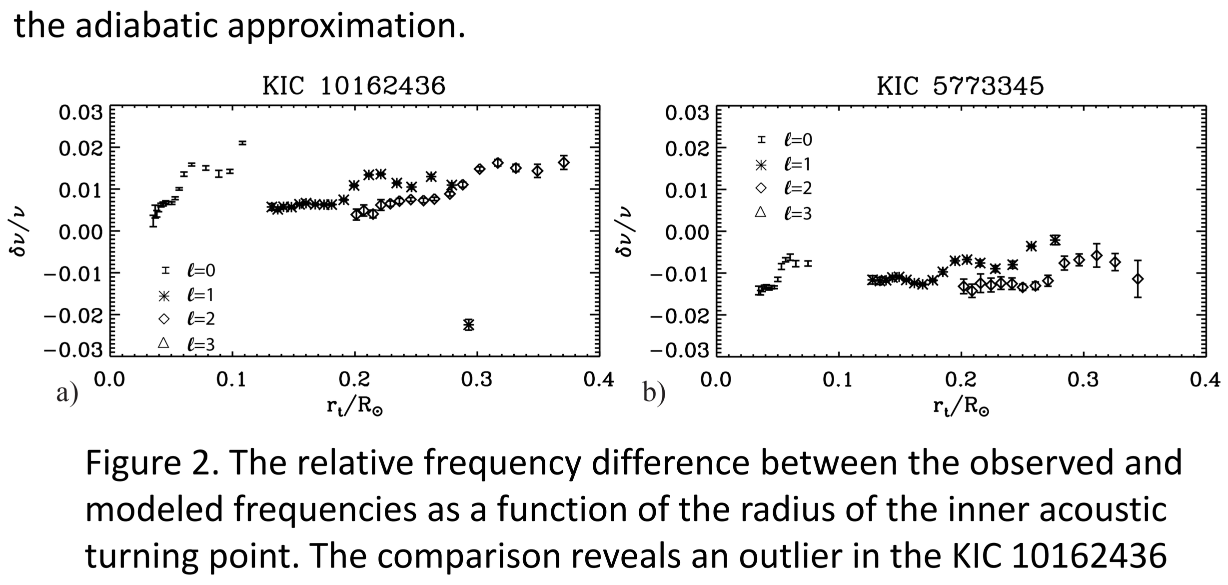

Figure 2. The relative frequency difference between the observed and modeled frequencies as a function of the radius of the inner acoustic turning point. The comparison reveals an outlier in the KIC 10162436 mode set.

The acoustic turning points of some $\ell=0$ modes are located in the stellar core. Nevertheless, the sensitivity of these modes to the core structure is low. Some of the observed low-frequency non-radial modes of $\ell=1$ and 2 represent mixed acoustic-gravity modes which have properties of internal gravity (g) modes in the stellar cores and acoustic (p) modes outside the core. The oscillation frequencies of these modes are very sensitive to the core properties.

References

1. Silva Aguirre, V., et al. 2015, MNRAS, 2015, **452**, 2127
2. Silva Aguirre, V., et al. 2017, ApJ, **835**, 173
3. Gough, D. O. and Kosovichev, A. G.: 1988, in E. J. Rolfe (ed.), *Seismology of the Sun and Sun-Like Stars*, Vol. 286 of *ESA Special Publication*, pp 195–201
4. Gough, D. O. and Kosovichev, A. G.: 1993, in W. W. Weiss and A. Baglin (eds.), *IAU Colloq. 137: Inside the Stars*, Vol. 40 of *Astronomical Society of the Pacific Conference Series*, p. 541
5. Kosovichev, A. G.: 1999, *Journal of Computational and Applied Mathematics* **109**, 1

Primary Sensitivity Kernels

Using explicit formulations for the variational principle, frequency perturbations can be reduced to a system of integral equations for a chosen pair of independent hydrostatic variables, e.g. (ρ,γ):

$$\frac{\delta\omega^{(n,l)}}{\omega^{(n,l)}} = \int_0^R K_{\rho,\gamma}^{(n,l)}\frac{\delta\rho}{\rho}dr + \int_0^R K_{\gamma,\rho}^{(n,l)}\frac{\delta\gamma}{\gamma}dr,$$

where $K_{\rho,\gamma}^{(n,l)}(r)$ and $K_{\gamma,\rho}^{(n,l)}(r)$ are sensitivity (or 'seismic') kernels. These are calculated using the initial solar model parameters, ρ_0, P_0, γ, and the oscillation eigenfunctions for these model, $\vec{\xi}$.

Secondary Sensitivity Kernels

The sensitivity for various pairs of solar parameters, such the sound speed, Ledoux parameter, Brunt-Väisälä frequency, temperature, and chemical abundances, can be obtained by using the relations among these parameters, which follow from the equations of solar structure ('stellar evolution theory').

These 'secondary' kernels are then used for direct inversion of the various parameters (Gough and Kosovichev, 1988; Kosovichev, 1999). A general procedure for calculating the sensitivity kernels can be illustrated in operator form. Consider two pairs of solar variables, $\vec{X}$ and $\vec{Z}$, e.g. $\vec{X} = \left(\frac{\delta\rho}{\rho},\frac{\delta\gamma}{\gamma}\right)$; $\vec{Z} = \left(A^*,\frac{\delta Y}{Y}\right)$,

where A^* is the Ledoux parameter and Y is the helium abundance. The linearized structure equations (the hydrostatic equilibrium equation and the equation of state) that relate these variables can be written symbolically as $A\vec{X} = \vec{Z}$.

Let $\vec{K}_X$ and $\vec{K}_Z$ be the sensitivity kernels for the pairs of variables $\vec{X}$ and $\vec{Z}$; then the frequency perturbation is:

$$\frac{\delta\omega}{\omega} = \int_0^R \vec{K}_X\cdot\vec{X}dr \equiv \langle\vec{K}_X\cdot\vec{X}\rangle,$$

where $\langle\cdot\rangle$ denotes the integrated inner product.

Similarly, $\frac{\delta\omega}{\omega} = \langle\vec{K}_Z\cdot\vec{Z}\rangle$.

Then, from the stellar structure equation $A\vec{X} = \vec{Z}$:

$$\langle\vec{K}_Z\cdot\vec{Z}\rangle = \langle\vec{K}_Z\cdot A\vec{X}\rangle = \langle A^*\vec{K}_Z\cdot\vec{X}\rangle,$$

where A^* is an adjoint operator. Thus: $\langle A^*\vec{K}_Z\cdot\vec{X}\rangle = \langle\vec{K}_X\cdot\vec{X}\rangle$. This is valid for any $\vec{X}$ if $A^*\vec{K}_Z = \vec{K}_X$. This means that the equation for the sensitivity kernels is adjoint to the stellar structure equations.

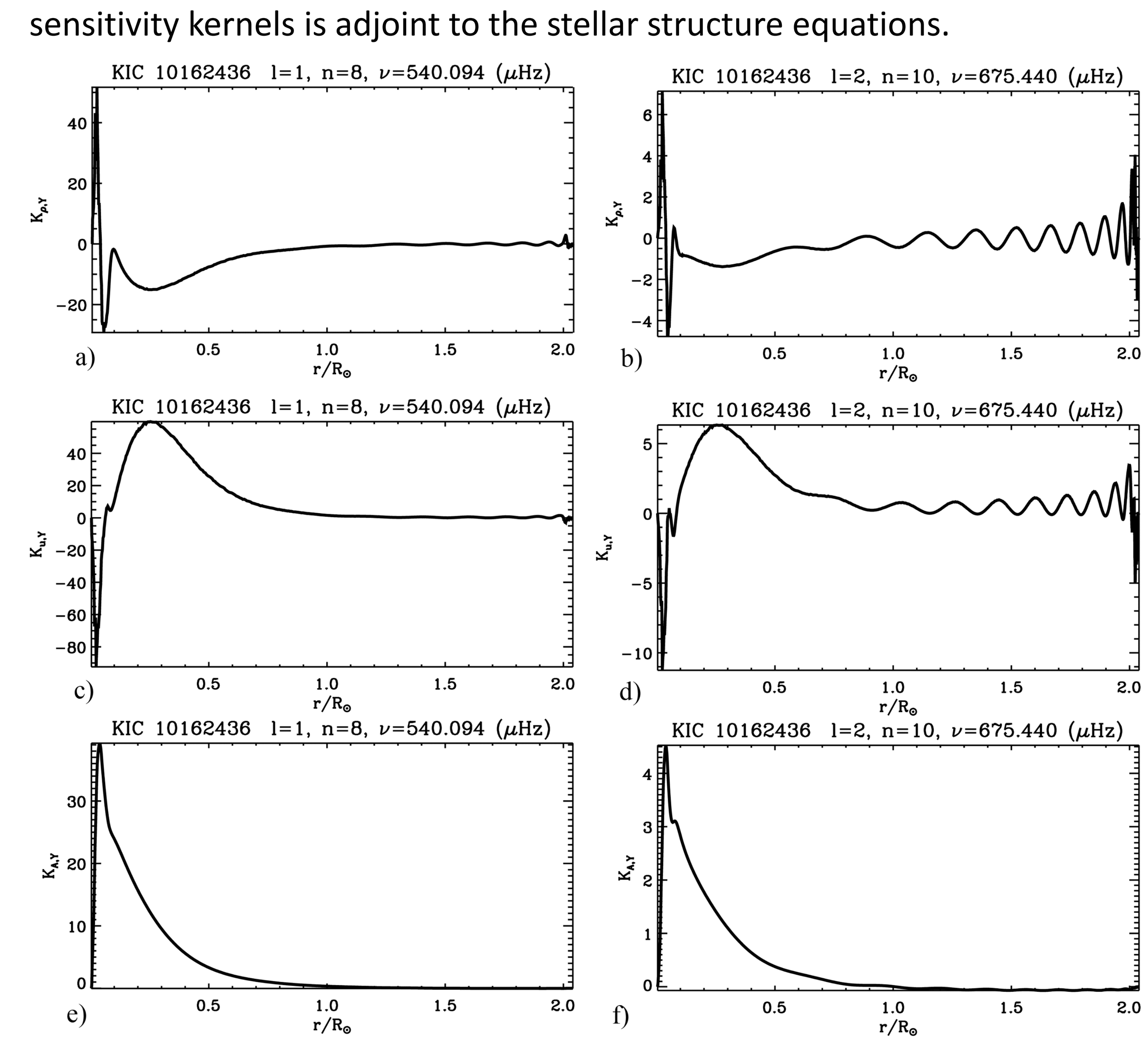

Figure 3. Examples of the sensitivity kernels of two mixed modes observed in KIC 10162436 for density, and parameters $u = \frac{P}{\rho}$, and A^*

Inversion Procedure

It is important to take into account potential systematic uncertainties in the stellar mass and radius. Because the oscillation frequencies are scaled linearly with the factor $q = M/R^3$, then, the mode frequencies ω_i can be expressed in terms of their relative difference $\delta\omega_i^2/\omega_i^2$ from those of a standard reference model of similar mass and radius according to the linearized expression (Gough and Kosovichev, 1993):

$$\delta\omega_i^2/\omega_i^2 = \int_0^1\left(K_{f,Y}^i\frac{\delta f}{f} + K_{Y,f}^i\delta Y\right)dx - I_q^i\delta q,$$

where $x = r/R$, $q = M/R^3$, Y is the helium abundance, and f can be any property defined by the sensitivity kernels $K_{f,Y}$ and $K_{Y,f}$; M and R are stellar mass and radius, in solar units, and I_q is an integral over the reference model. In this poster, we consider three cases: $f=\rho$, $f = u \equiv P/\rho$, and $f = A^*$.

The following constraints can provide localized averages of $\delta\ln f$ and estimates of δY and δq of the kind

$$\overline{\delta\ln f} \equiv \int_0^1\sum_i a_i(x_0)K_{f,Y}^i\delta\ln f\,dx \equiv \int_0^1 A_{f,Y}(x,x_0)\delta\ln f\,dx = \sum_i a_i(x_0)\frac{\delta\omega_i^2}{\omega_i^2}$$

by minimizing over the coefficients $a_i(x_0)$ the functional :

$$\int_0^1 A_{f,Y}^2(x,x_0)J_f dx + \lambda_1\int_0^1\left(\sum_i a_iK_{Y,f}^i\right)^2 J_Y dx + \lambda_2\left(\sum_i a_iI_q^i\right)^2 + \alpha\sum_i a_i^2\epsilon_i^2$$

for tradeoff parameters λ_1, λ_2 and α, where $J_f = (x-x_0)^2$, and ϵ_i are standard relative errors in the data. The tradeoff parameters are chosen empirically, by selecting a sufficiently smooth solution and using the L-curve criterion. This inversion procedure is called Optimally Localized Averaging (OLA). Its advantage is that the shape of the averaging kernels is not prescribed.

Inversion results

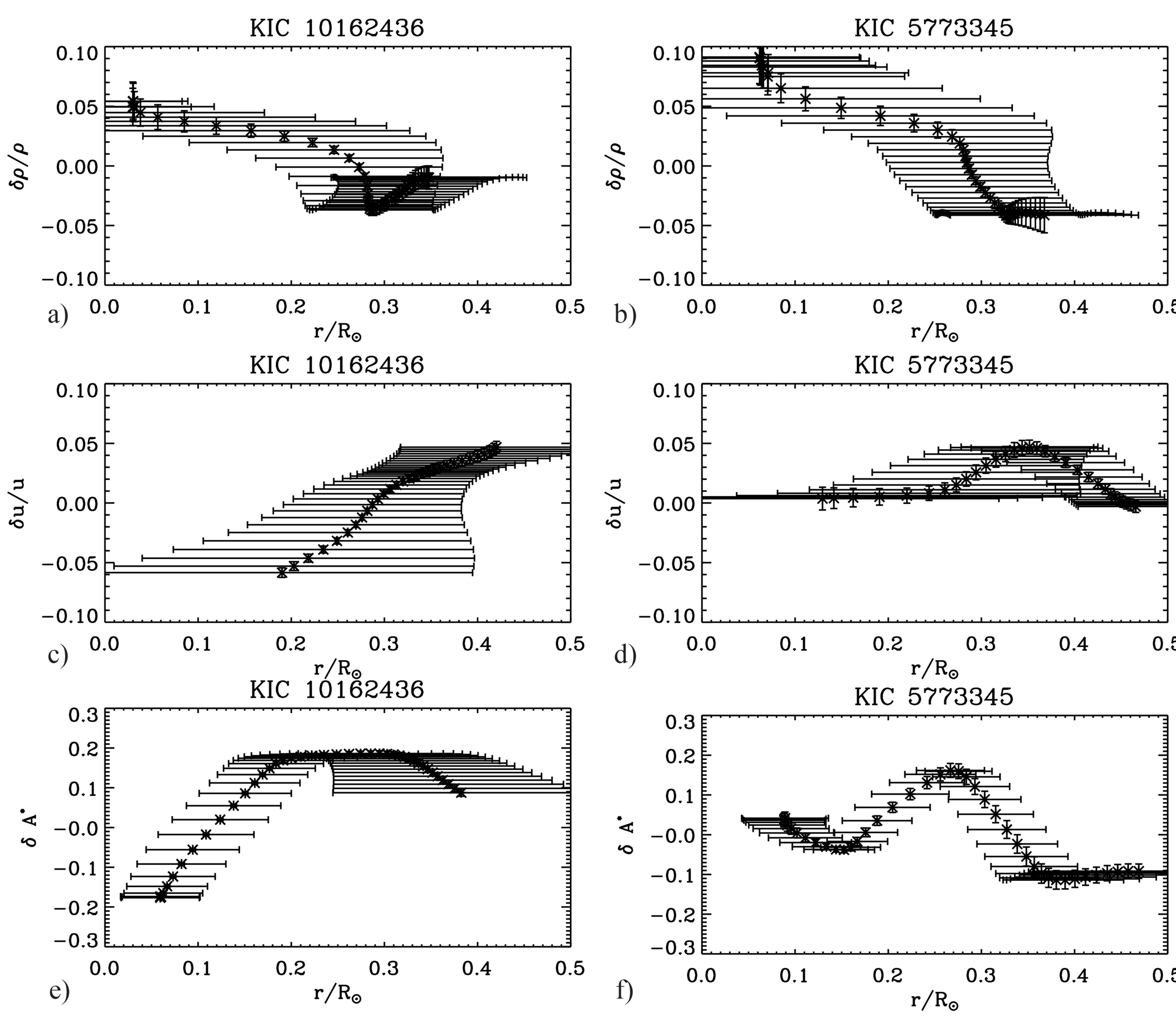

Figure 4. The relative deviations in stellar structure from the evolutionary models of density, $u = P/\rho$ and the Ledoux parameter as a function of radius for KIC 10162436 and KIC 5773345. Crosses show the weighted center locations of the localized averaging kernels, the horizontal bars show the spread of the averaging kernels, and the vertical bars show the uncertainties calculated using the observational error estimates.

Averaging Kernels

The resolving power of the astreroseismic inversions can be assessed from the corresponding averaging kernels.

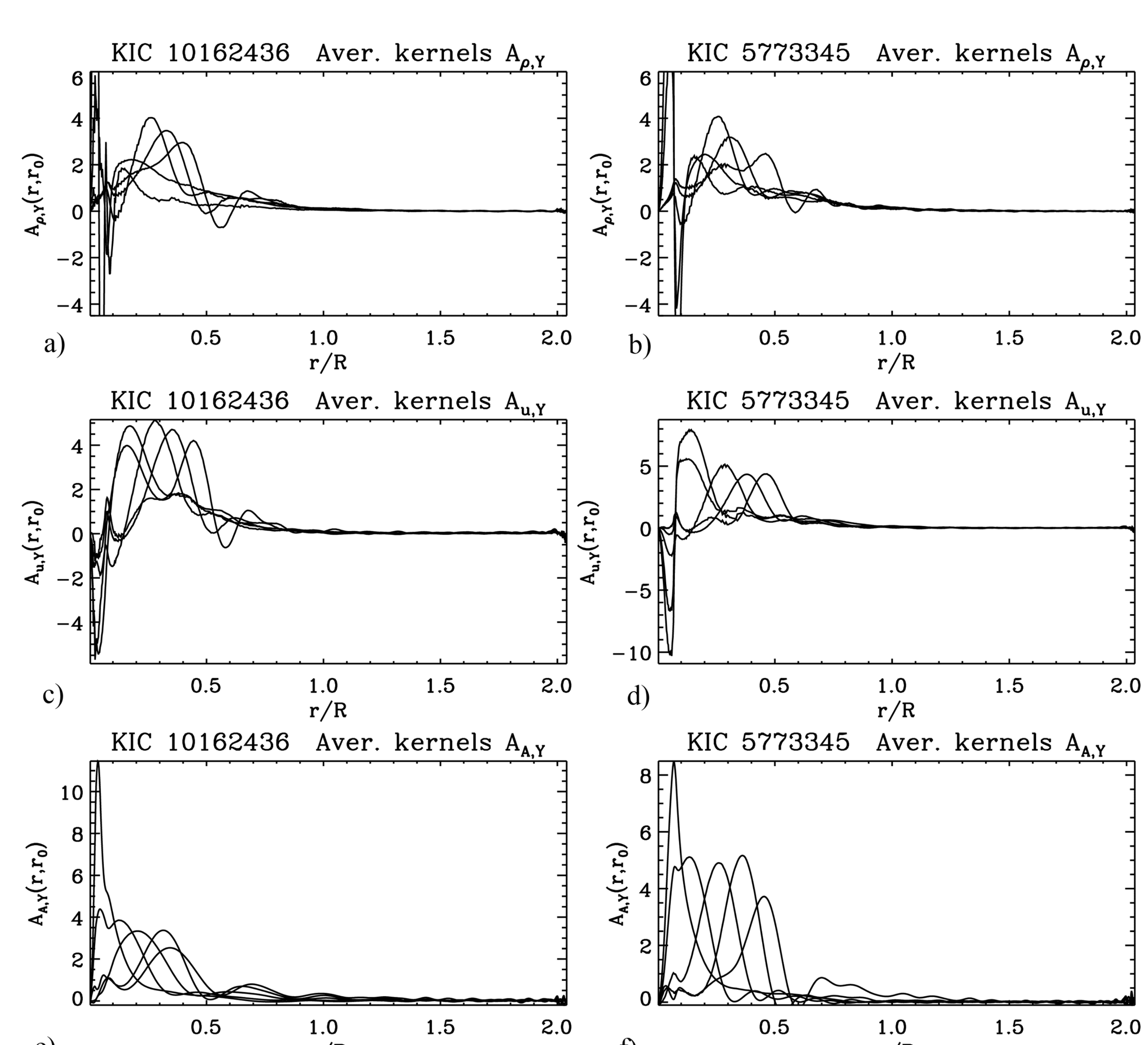

Figure 5. The averaging kernels density, $u = P/\rho$ and the Ledoux parameter as a function of radius for KIC 10162436 and KIC 5773345, for five target positions: r_0/R=0.0125, 0.125, 0.25, 0.375, and 0.5.

Evidently, the mixed modes observed in the oscillation spectra of these stars allow us to obtain constraints on the structural properties of the central helium core and the hydrogen-burning shell.

The averaging kernels for parameter u (or, equivalently, the sound speed) are localized outside the core. The density kernels centered in the core have significant negative sidelobes, which complicate the interpretation of the inversion results. The best localization without negative sidelobes is achieved for the Ledoux parameter.

Therefore, it is very important to choose the pairs of inversion variables that provide the best resolution. These variables can be different for different regions of a star. In our cases, only inversions for the Ledoux parameter provide the optimally localized averaging kernels centered in the stellar cores.

Discussion

For both stars, the inversion results show that the density of the core and the surrounding shell are about 5% higher than in the stellar models, but lower outside the energy-release shell. The Ledoux parameter reveals significant deviations from the models at the helium core boundary.

The boundary of the helium core is located at $0.05\,R_\odot$ in KIC 10162436, and at $0.07\,R_\odot$ in KIC 5773345. Outside the helium cores, the nuclear energy production shells extend $0.3\,R_\odot$, with the peak rate at $\sim 0.08\,R_\odot$ in both models. Perhaps these stellar regions involve physical processes that are not described by the evolutionary models. For understanding these deviations. it will be beneficial to perform more detailed structure inversion studies, including inversions for the chemical abundances, for a large sample of post-main sequence stars with hydrogen-burning shells.