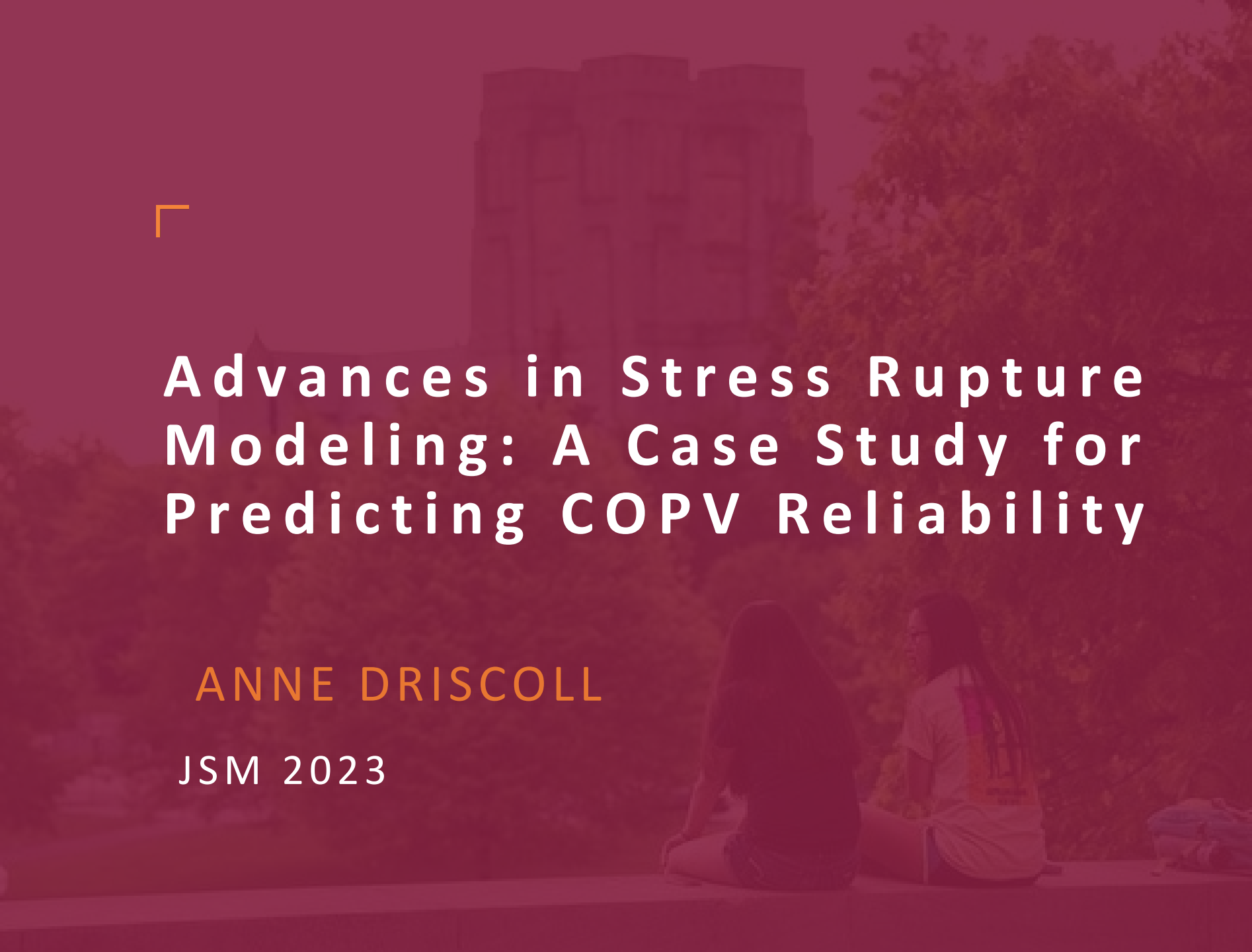




VIRGINIA TECHTM



Advances in Stress Rupture Modeling: A Case Study for Predicting COPV Reliability

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Outline

- Testing Overview
- Details on Stress Rupture Testing
- Weibull Regression Discussion
- Model Building
- Vessels vs. Strands—New Approach
- Next Steps

NASA Strand and Vessel Testing

- NASA's Engineering Safety Center (NESC) project to assess safety of Composite Overwrapped Pressure Vessels (COPVs)
- COPVs
 - Transport gasses under high pressure
 - Metal Liner
 - Wrapped by a Series of Carbon Strands
- Research Question: **Determine Reliability of COPVs at Use Conditions for the Expected Mission Life**
 - Primary Focus on Strands
 - Secondary Focus on Relationship to Vessels
 - Strands Less Expensive to Test
- [https://www.nasa.gov/offices/nesc/home/Feature COPVs Jan-2012.html](https://www.nasa.gov/offices/nesc/home/Feature_COPVs_Jan-2012.html)

NASA Strand and Vessel Testing

- Analyses Use Classic Weibull Model

$$R(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^\rho\right)^\beta}$$

- Observed Life Time: t_i
- SR : Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
 - ρ : Sensitivity to Stress Ratio
 - β : Shape parameter for time to Failure
 - t_{ref} : Reference time to Failure when $SR=1$

NASA Strand Study

- Previous Strand Test
 - Relevant strand study conducted at a national lab
 - 57 strands at high loads for 10 years
 - Net information learned:
 - Strands either fail very early or
 - Last more than 10 years
 - Limited information based on 10 years of study!
- Estimates of Critical Parameters for Planning

NASA Strand Study

- Team's Initial Concept
 - Much larger study than the original 10 year study
 - Sensor very early
 - Reduces time
 - Allows for the larger study in a practical amount of time
- Proceed in phases
- Have detailed data records to track any problems

NASA Strand Study

Experimental Phases

- Phase A – During “shake-out” of tests rigs
- Phase B – “Gold Standard” Experiment for Strands
- Phase C – “Proof” Study
- In Parallel: Vessel Studies (Opportunistic)

Phase A

- Conducted During Shake-Out of Equipment
 - Small study (although bigger than the national lab study!)
 - Statistical goal: Determine if the parameters from the national lab study are valid as the basis for planning the larger study!
 - Note: Phase A gave the team an opportunity to re-plan the larger experiment, if necessary!

Phase B

- “Gold Standard” Experiment
 - Planned time required: 1 year
 - Used 4 “blocks” of almost equal numbers of strands
 - Allowed the team to correct for time effects
 - Allowed the team to mitigate problems, especially early
 - Study assumed the “classic” Weibull model
 - Size of the experiment assured ability to assess model

Block	SR	Number	Proportion
1	0.80	176	0.718
	0.85	50	0.204
	0.90	19	0.078
	Sum	245	1.00
2	0.80	170	0.708
	0.85	50	0.208
	0.90	20	0.083
	Sum	240	1.00
3	0.80	174	0.710
	0.85	51	0.208
	0.90	20	0.082
	Sum	245	1.00
4	0.80	176	0.718
	0.85	49	0.200
	0.90	20	0.082
	Sum	245	1.00

Observations

- Phase A: Surprisingly Similar to Initial Study
- Phase B:
 - Serious problem occurred with the gripping in the first block
 - Serious conversations with possibility of replacing!
 - Other three blocks well behaved and by themselves produced better than the planned precision for the estimates
- Final Decision: Drop the First Block

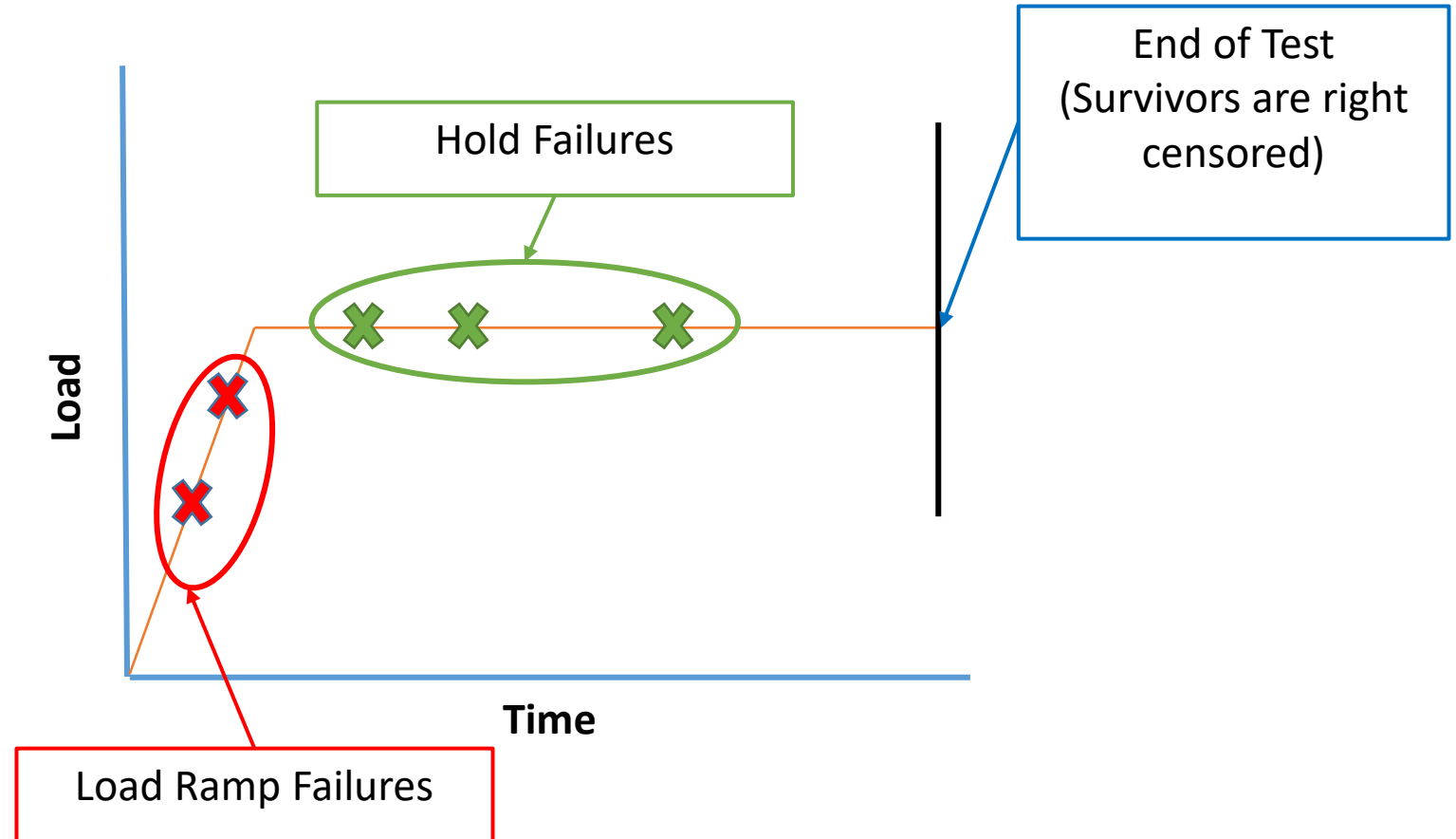
NASA Strand Study: Benefits

- Phase A:
 - Opportunity to Confirm Initial Study Parameter Estimates
 - Allowed opportunity to revise the experimental protocol if the estimates were significantly different
- Phase B:
 - Allowed opportunity to model changes in time over the year.
 - Mitigated the problem with the first block!
 - Provided simple mechanism for replacing the first block if needed!

Analysis: Stress Rupture Data

Description of Stress Rupture Test

- Stress Rupture
 - Failures occur after a period of time where there is no increase in load
- Failures are needed to determine reliability
- Must extrapolate from where test is performed versus where reliability predictions are made
- Test strands at higher loads and then extrapolate
- Need a model to make predictions



Basic Weibull Distribution

- Probability Density Function

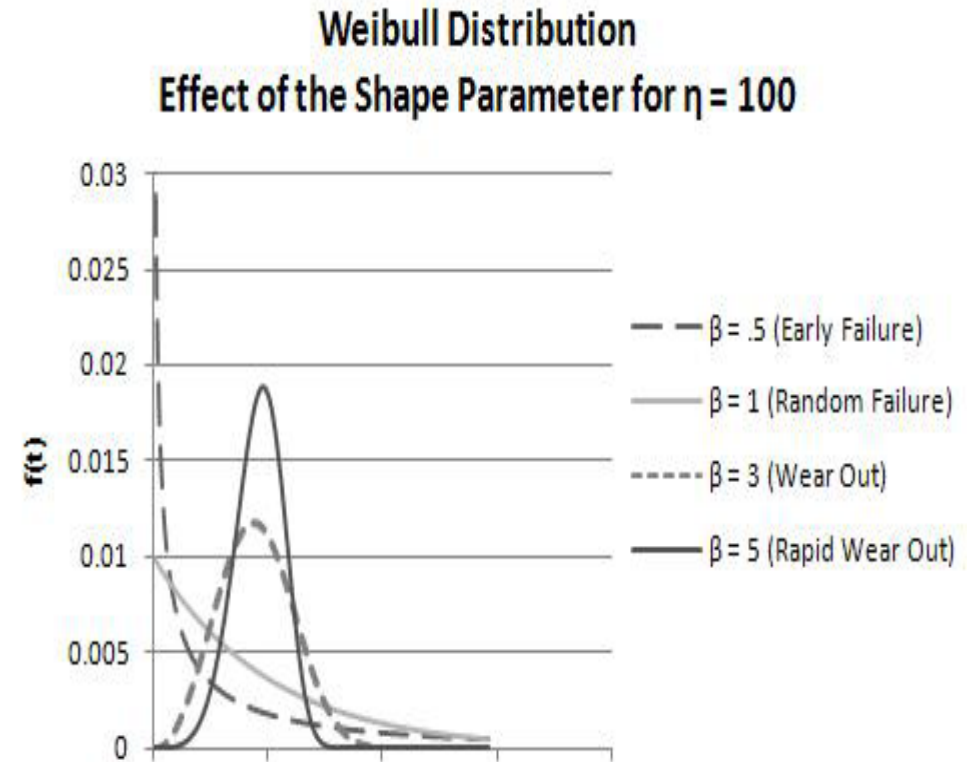
$$f(t, \beta, \eta) = \frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta-1} e^{-\left(\frac{t}{\eta} \right)^{\beta}}$$

- Survivor Function = $1 - F(t)$

$$S(t) = e^{-\left(\frac{t}{\eta} \right)^{\beta}}$$

β : shape parameter

η : scale parameter (characteristic life, time where 63.2% of units will fail)



Smallest Extreme Value Distribution

- Smallest extreme value (SEV) distribution is an alternate parameterization of the Weibull distribution
- SEV represents Weibull as a log-location-scale distribution
 - If t_i is Weibull, then $\log(t_i)$ is SEV
- Weibull/SEV relationship mimics the Normal/Lognormal relationship
- Parameters
 - Log-location: $\mu = \log(\eta)$ Scale: $\sigma = \frac{1}{\beta}$
- Residuals
 - $e_i = \log t_i - \mu$
 - Scaled: $z_i = \beta e_i = \beta(\log t_i - \mu) = \frac{\log t_i - \mu}{\sigma}$
- Survivor Function
 - $S(t_i) = P(T > t_i) = e^{-e^{z_i}}$

Classic Stress Rupture Model: Weibull

- Classic Weibull Survival Function

$$S(t_i) = P(T > t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^\rho\right)^\beta} \quad \text{Note: } \eta = t_{ref} SR^{-\rho}$$

- Observed Life Time: t_i
- SR : Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
 - ρ : controls the relationship between the failure time and stress ratio (SR)
 - β : Shape parameter for time to Failure
 - t_{ref} : Reference time to Failure

Classic Stress Rupture Model: SEV

- SEV Survival Function

$$S(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^\rho\right)^\beta} = e^{-e^{\beta(\log t_i - \theta + \rho \ln(SR))}}$$

where $\theta = \log(t_{ref})$ and $\mu = \log(\eta) = \theta - \rho \ln(SR)$

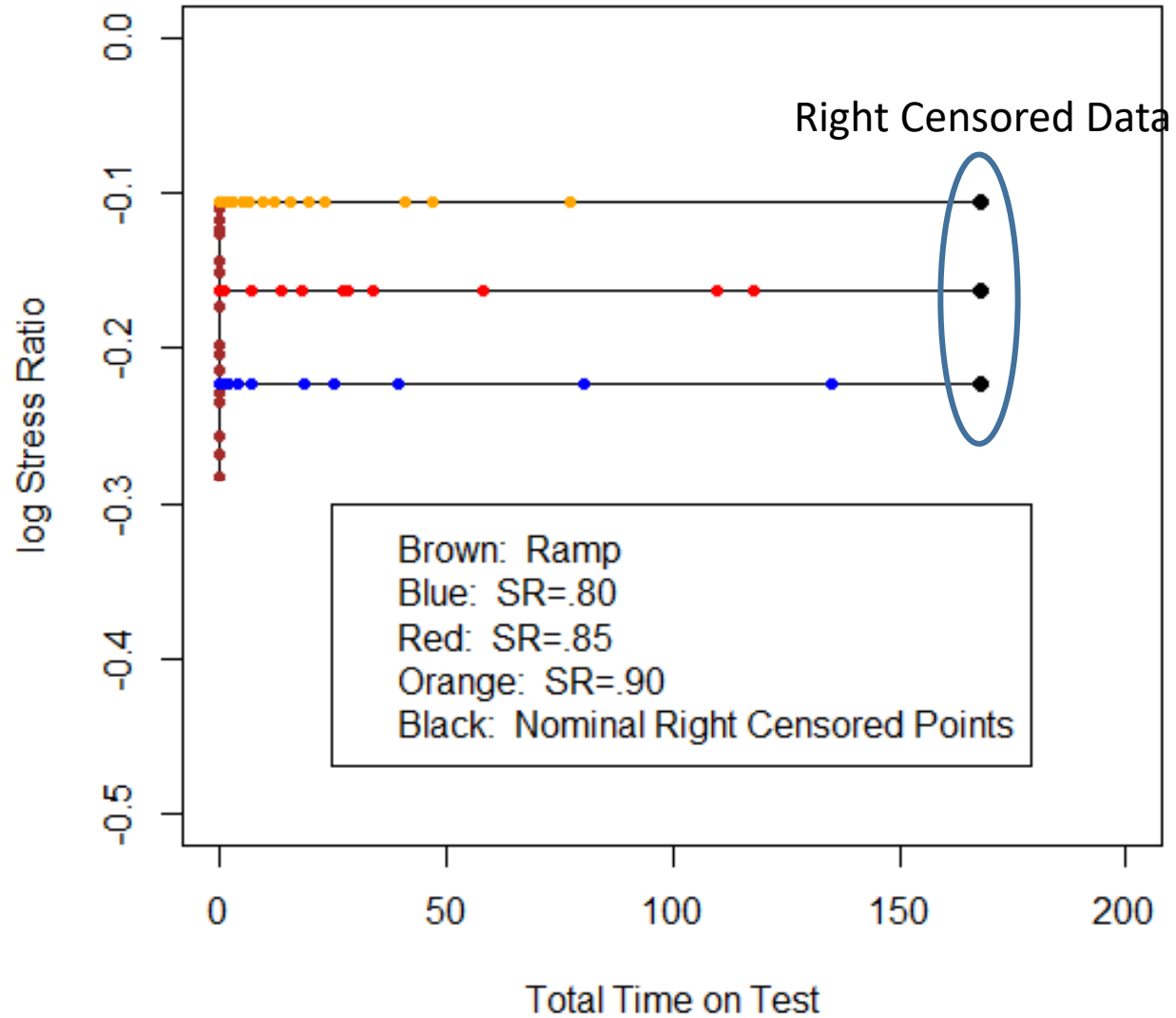
Now working with a linear model, similar to simple linear regression

- Scaled Residuals

- $z_i = \beta e_i = \beta(\log t_i - \mu) = \beta(\log t_i - \theta + \rho \ln(SR))$
- Used for predictions of the log probability for specific observations

Weibull Regression: How do we fit this model?

Data Structure for the Ramp and Hold Phase B Data



**Estimate the Stress
Rupture Parameters**
 ρ, t_{ref}, β

Use the ramp data

Don't use the ramp data

Effective Time

Conditional Weibull
Analysis

What is Effective Time?

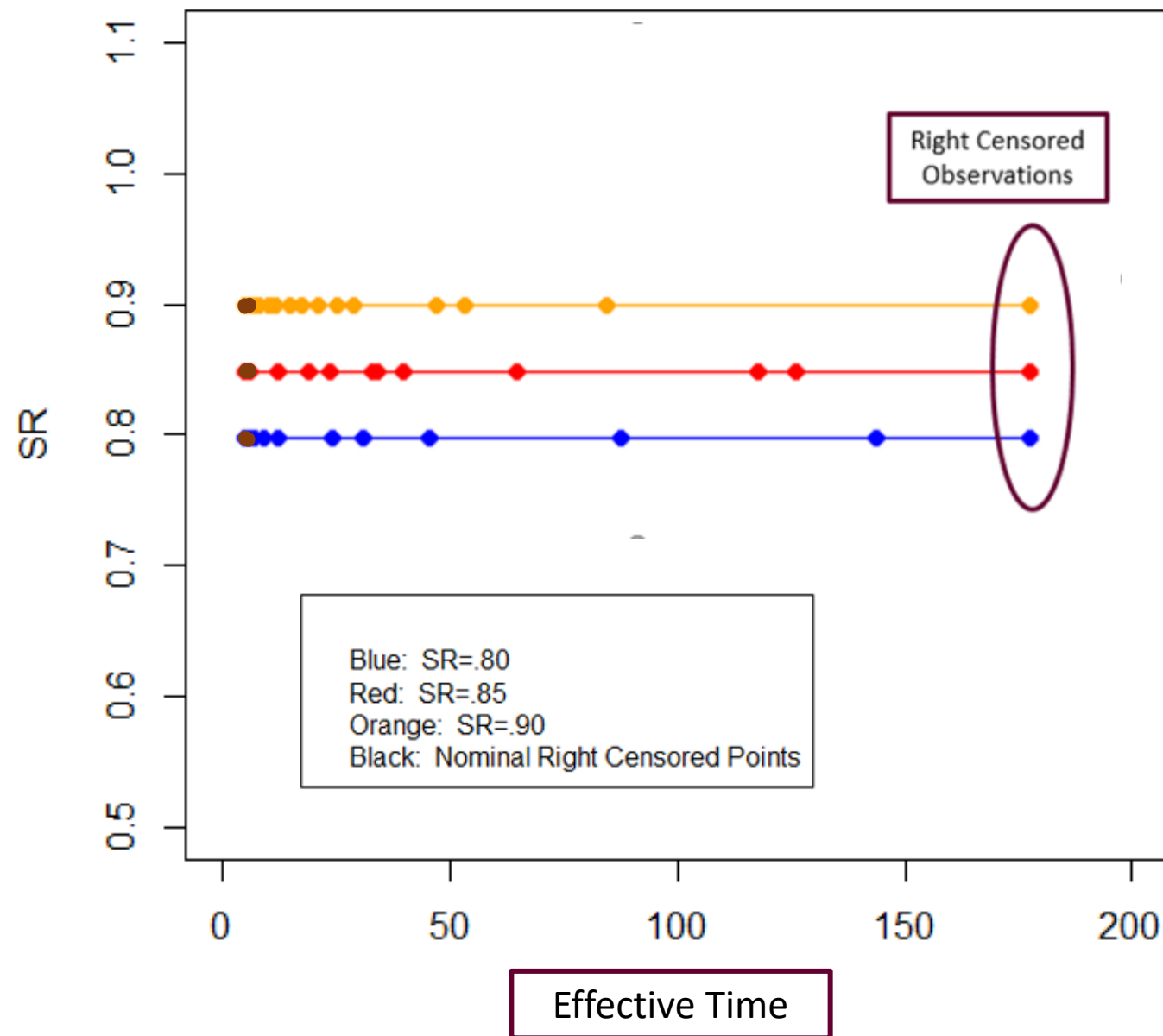
- Basic Idea:
 - “Correct” the actually observed ramp time for what the time would have been at a specified target load
- Effective Time on Load
 - Replace the observed ramp failure time with
$$t_{eff} = \frac{t}{\rho + 1}$$
- Yin and Sheng (1987)
 - Theoretical development of the concept
 - Parameters (constraint)
 - $\beta_{ramp} = \beta(\rho + 1)$
 - $\eta_{ramp} = \left[\frac{(\rho+1)}{R\rho+1} \right]^{1/\rho+1}$ (no SR here!)
- Ramp behavior is a two-dimensional problem and stress rupture is really a three-dimensional problem.
- Rapid pull to failure on the ramp is a different acceleration equation than the classic stress rupture that assumes constant stress load

Both functions of
 ρ

“Different products have different acceleration equations. The inverse power law is only one of the possible models. The acceleration equation we have used may be wrong. In this situation, further investigation to understand the relationship between life and stress is necessary.”

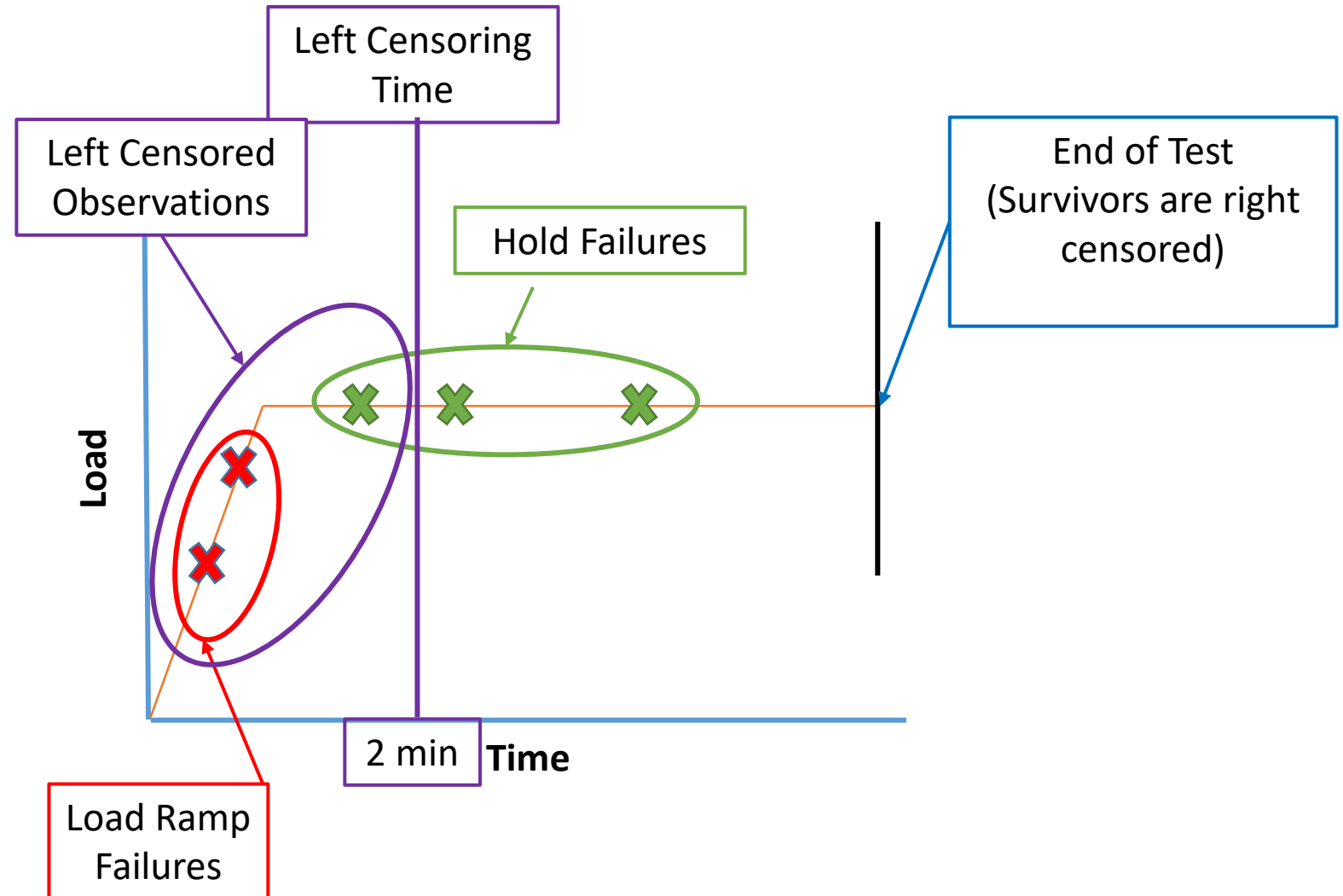
- Yin and Sheng (1987)

Stress Ratio versus Effective Time



Effective Time → Left Censored

- Left Censors specimen that fail on load and those that fail within the first two minutes of stress rupture testing
- Left censoring assumes the data from the ramp can be modeled by the same model parameters as the hold data



Deeper Dive into Left Censoring

- Left censor when failure times are missing
 - Only have information that specimen failed before a specified time
- It is not appropriate to left censor outliers
- Left censoring assumes that the failure time is missing at random
 - Left censoring ramp failures and failures within a certain window of time violates this assumption
- Left censoring assumes the missing failure times follow the exactly the same distribution as the other data being estimated
 - Does this make sense for the ramp and hold?

“Left-censored observations occur in life test applications when a unit has failed at the time of its first inspection; all that is known is that the unit failed before the inspection time. In other situations, left-censored observations arise when the exact value of a response has not been observed and we have, instead, an upper bound on that response.”

- Meeker and Escobar
(p.38)

**Estimate the Stress
Rupture Parameters**
 ρ, t_{ref}, β

Use the ramp data

Don't use the ramp data

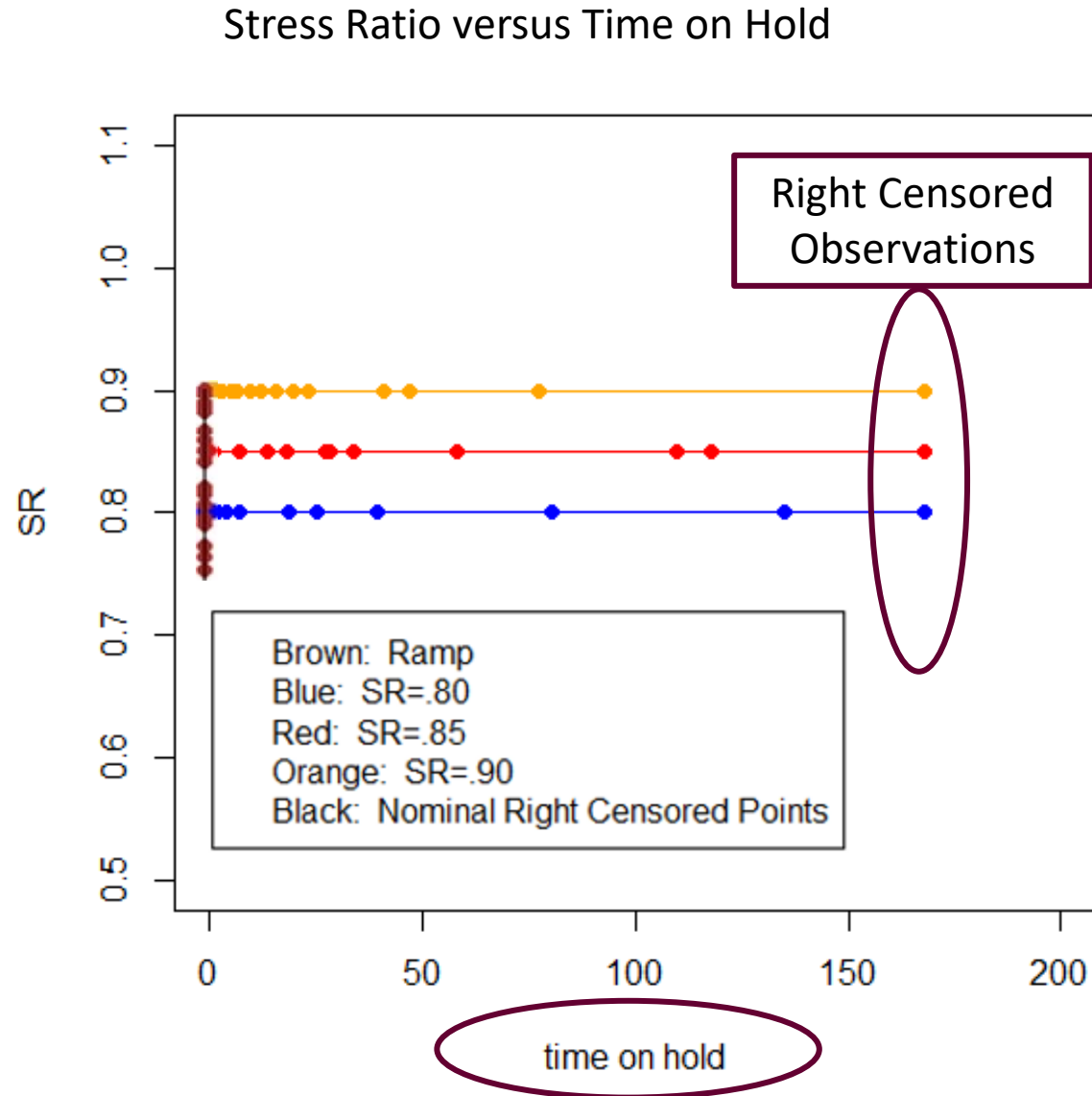
Effective Time

Conditional Weibull
Analysis

Left Censored analysis

Fit-to-Hold Analysis

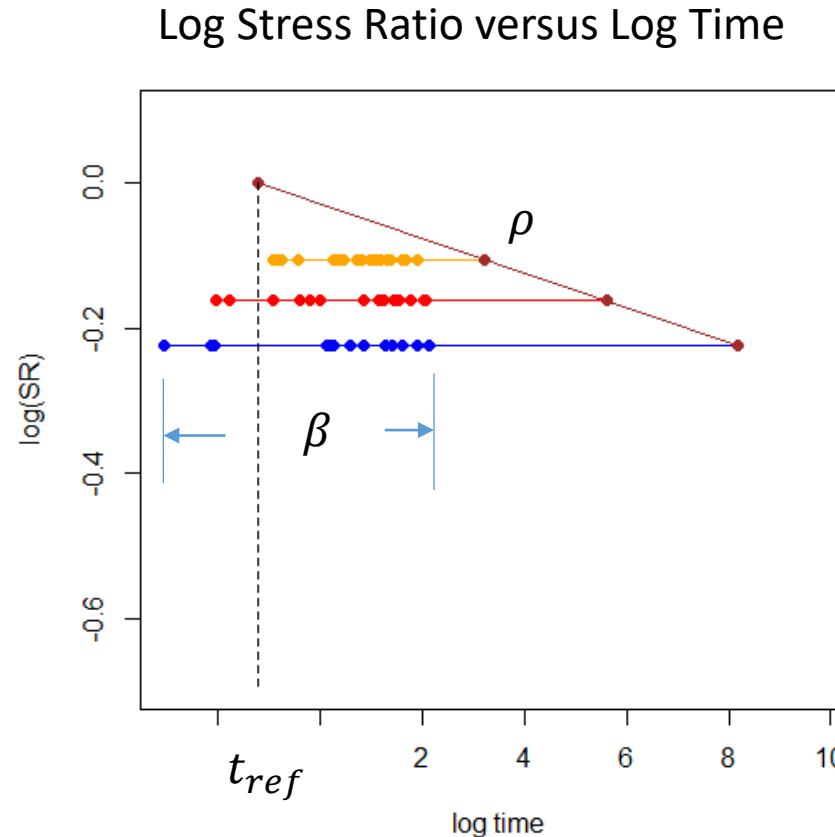
- Focus on Time on Hold not Time on Test.
- Fit the Classic Weibull Model using Maximum Likelihood Estimation with Right Censoring.
- Failures on load are not used because they have no time on hold!



True Structure of the Stress Rupture Model

$$\mu = \log(\eta) = \theta - \rho \ln(SR)$$

- ρ : controls the relationship between the failure time and stress ratio (SR)
- β : Shape parameter for time to Failure
- t_{ref} : Reference time to Failure



Stress Rupture model explains the behavior of the items *on hold*.

- Weibull regression gives us estimates for ρ , β and t_{ref}

**Estimate the Stress
Rupture Parameters**
 ρ, t_{ref}, β

Use the ramp data

Don't use the ramp data

Effective Time

Conditional Weibull
Analysis

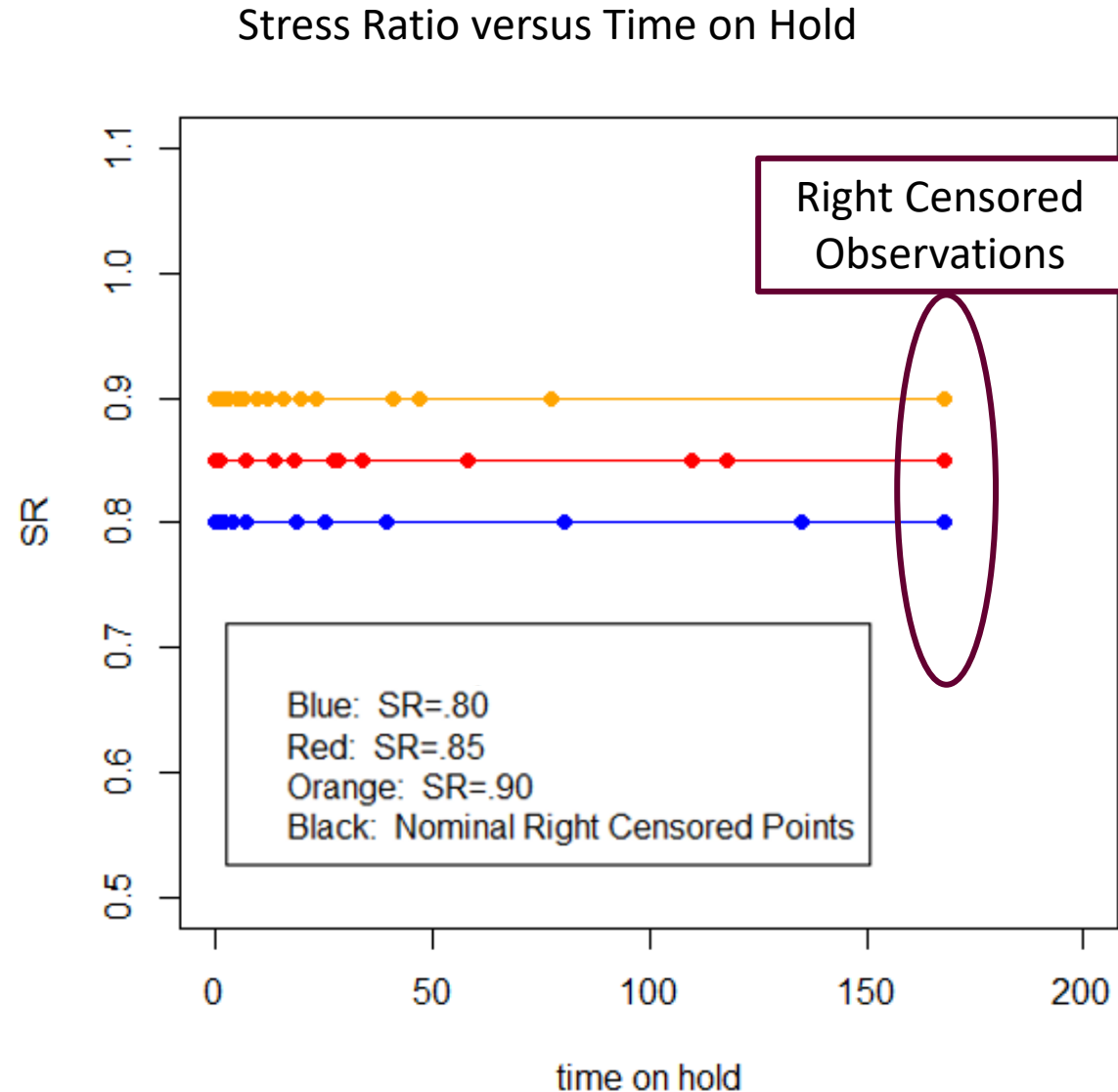
Left Censored analysis

Fit-to-Hold Analysis

Weibull Regression: Model Building

Full Model

- Separate individual models to each stress ratio
 - Two parameters for the SR=1 data: $\eta_{.80}$ and $\alpha_{.80}$
 - Two parameters for the SR=2 data: $\eta_{.85}$ and $\alpha_{.85}$
 - Two parameters for the SR=3 data: $\eta_{.90}$ and $\alpha_{.90}$
- Largest possible Weibull model for the data
 - Has the largest log-likelihood
- Will compare to the Full Model to subset models to determine whether the improvement in log-likelihood justifies the extra parameters



Fit-to-Hold (subset model)

- Model the data that have achieved the target load as defined by the experimental protocol (no ramp data)
- Defines that the time at the sustained constant load begins the moment the test item achieves the target load
- Assumes a Weibull distribution to describe the time to failure under the sustained constant load
- Experimental protocol uses right-censoring at a nominal time

Fit-to-Hold (subset model)

- Ramp and Hold data are modeled separately
 - Three parameters to explain the hold data: ρ , β , and $\theta = \log t_{ref}$
- Model assumes

$$\alpha_{.80} = \alpha_{.85} = \alpha_{.90} = \beta$$

Comparisons

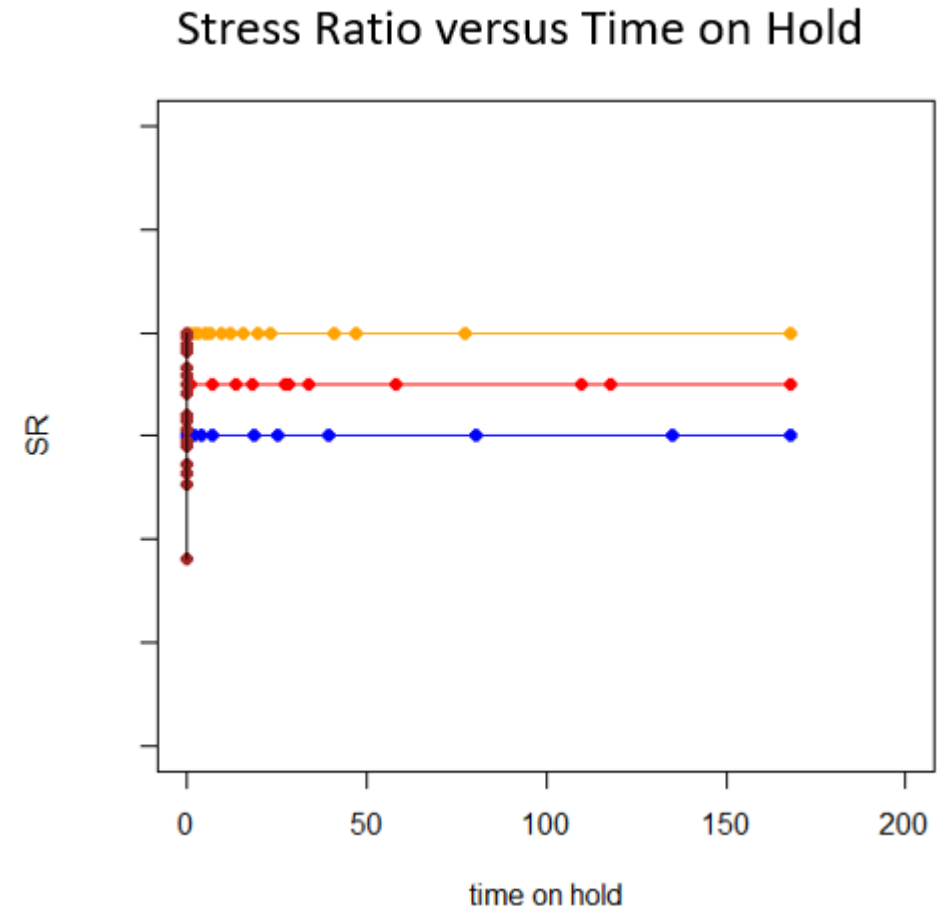
Model:	Fit-to-Hold	Full Model
Number of Observations:	708	708
True Log-Likelihood:	−306.411	−305.900
Log-Likelihood Statistic:	612.822	611.800
<i>AIC</i> :	618.822	623.800

- The p -value associated with the χ^2 based on the difference in the log-likelihood statistics is 0.7959
 - The three extra parameters in the full model are not significant
- Smallest AIC value for Fit-to-Hold (adjustment for parameters)

Weibull Regression: Including Ramp Fit-to-Hold versus Effective Time

Adaptations to Include Ramp Failures

- Rigorous Approach
 - Add two additional parameters for a Weibull Distribution fit to only the ramp data along with the Fit-to-Hold Analysis
 - Two parameters for the ramp data: η and α
 - Three parameters to explain the hold data: ρ , β , and $\theta = \log t_{ref}$
- Left Censored Analysis
 - Assume that ALL data follow the same failure mechanism
 - Left censor all ramp failures and some early stress rupture failures
 - Three parameters to explain the ramp and hold data: ρ , β , and $\theta = \log t_{ref}$

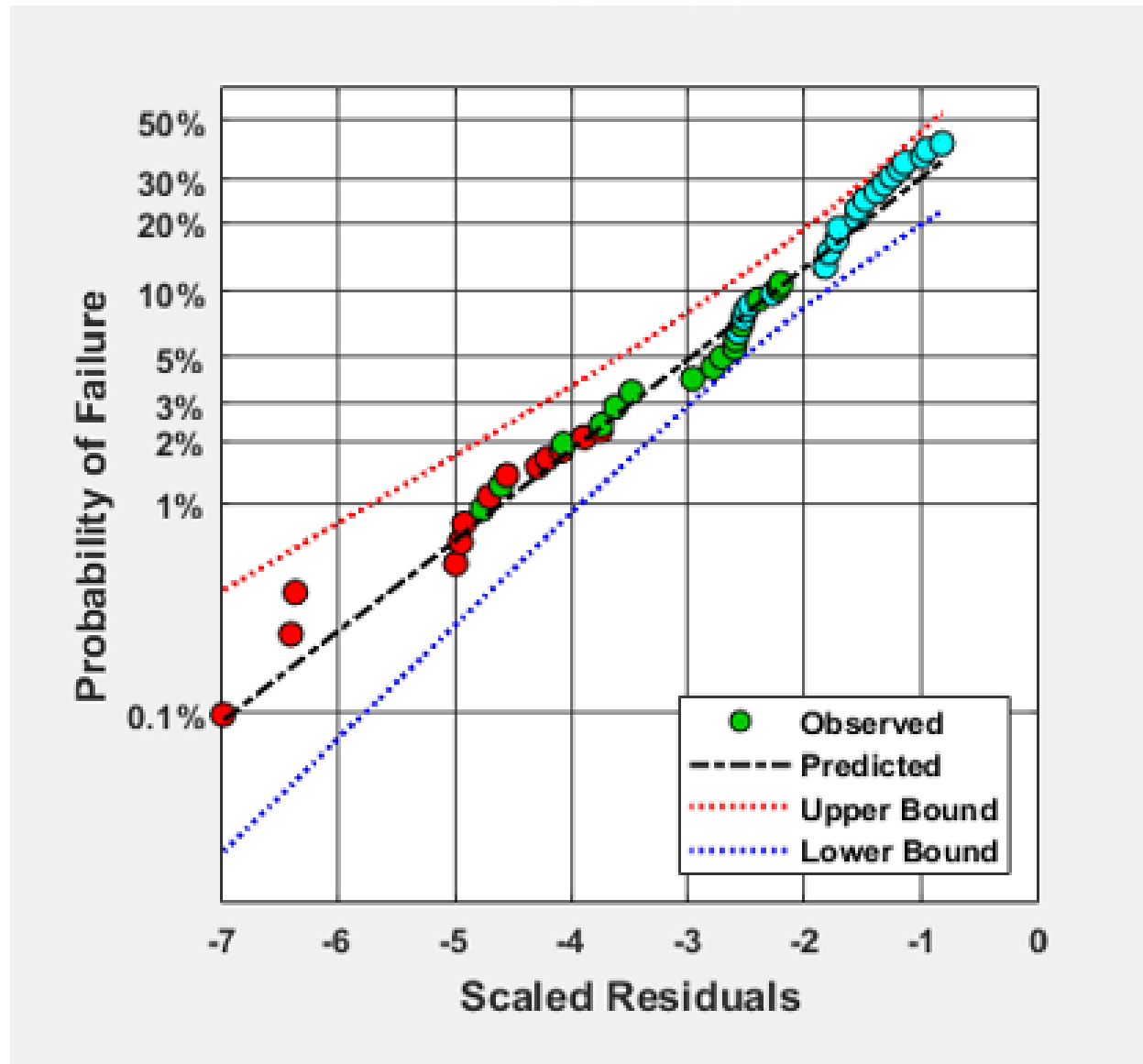


Comparisons

	Rigorous	Left-Censored	Full Model
Overall log $\mathcal{L}$:	-251.103	-390.779	-250.592
Log-Like Stat:	502.206	781.558	501.184
AIC:	512.206	789.558	517.184
Ramp log $\mathcal{L}$:	55.308		55.308
Hold log $\mathcal{L}$:	-306.4114		-305.900

- All fit statistics indicate that the rigorous model is the superior fit to the data compared to the Left-Censored approach
 - Smallest AIC value (512.206) and log-likelihood statistic (502.206) and the largest overall log-likelihood value (-251.103) The three extra parameters in the full model are not significant
- The probability that the left censored analysis explains the data at least as well as the rigorous model is 1.03946 E-62
- Counter-intuitive to penalize the maximum likelihood fit to the data with left censoring especially when we know the precise time these items failed on the ramp

Weibull Probability Plot

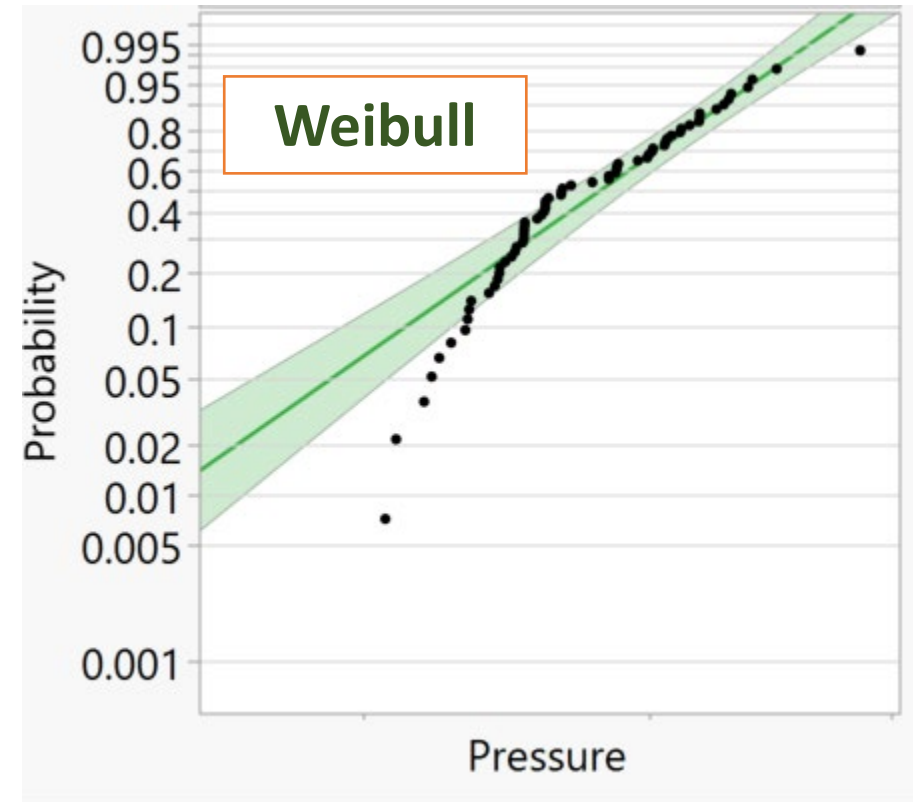
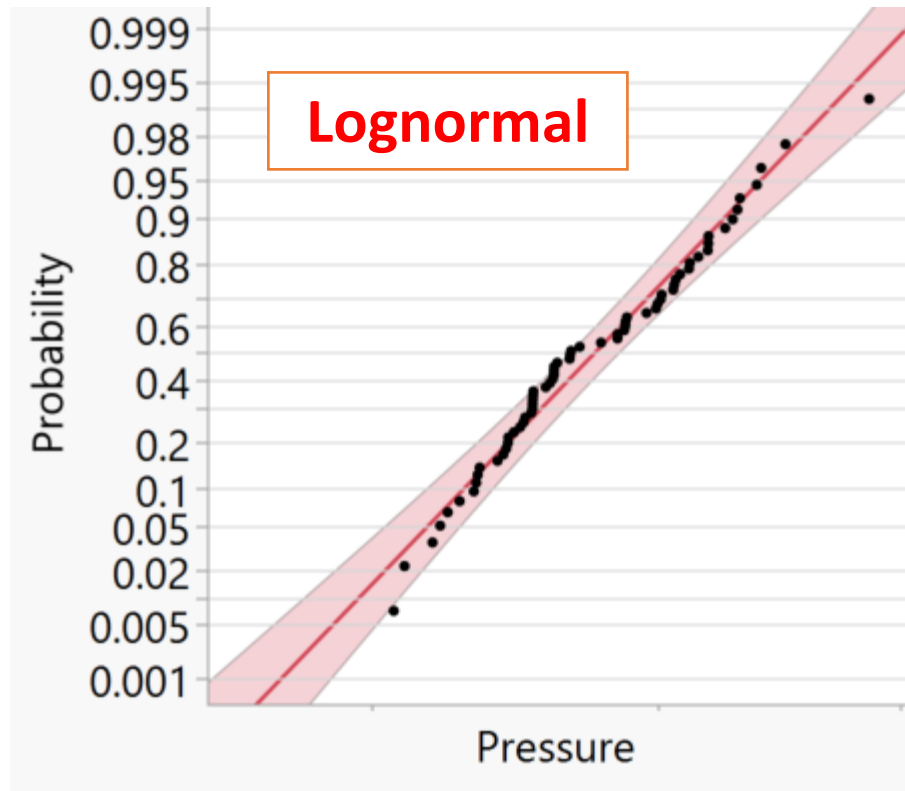


Vessels vs. Strands

Recall our Overall Testing Goal

- **Determine Reliability of COPVs at Use Conditions for the Expected Mission Life**
 - Primary Focus on Strands
 - Secondary Focus on Relationship to Vessels
 - Strands Less Expensive to Test
- Is there a link between strands and vessels?
 - Does the Weibull distribution fit both the strands and the vessels?
- Joint work with Pete Parker and Bill Prosser from NASA have led to new insights

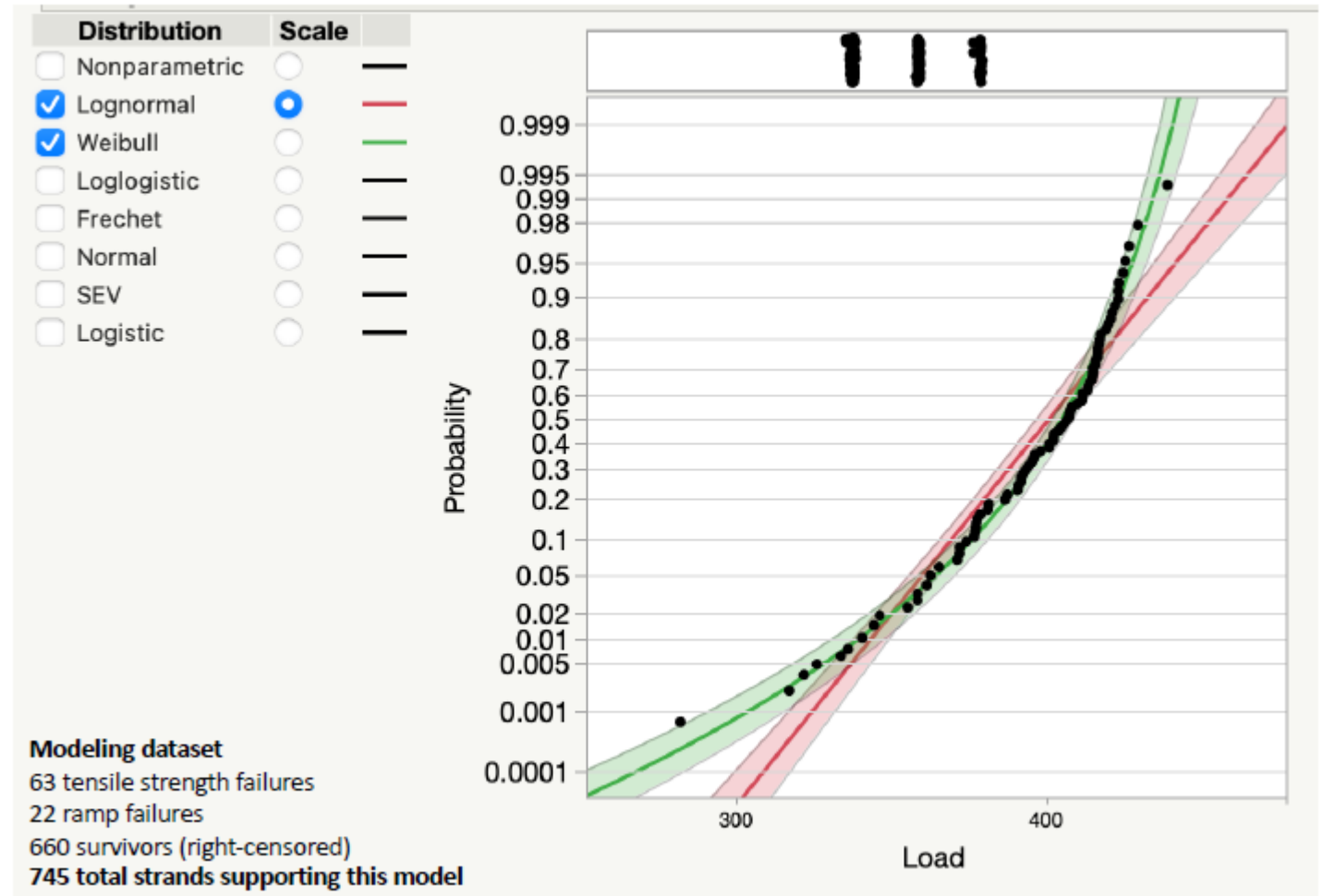
Vessel Strength Modeling of Burst Data



- Would you prefer the red distribution fit to the data of the green?
- Lognormal is a better model fit to the burst data.
 - Results in high reliability estimates!

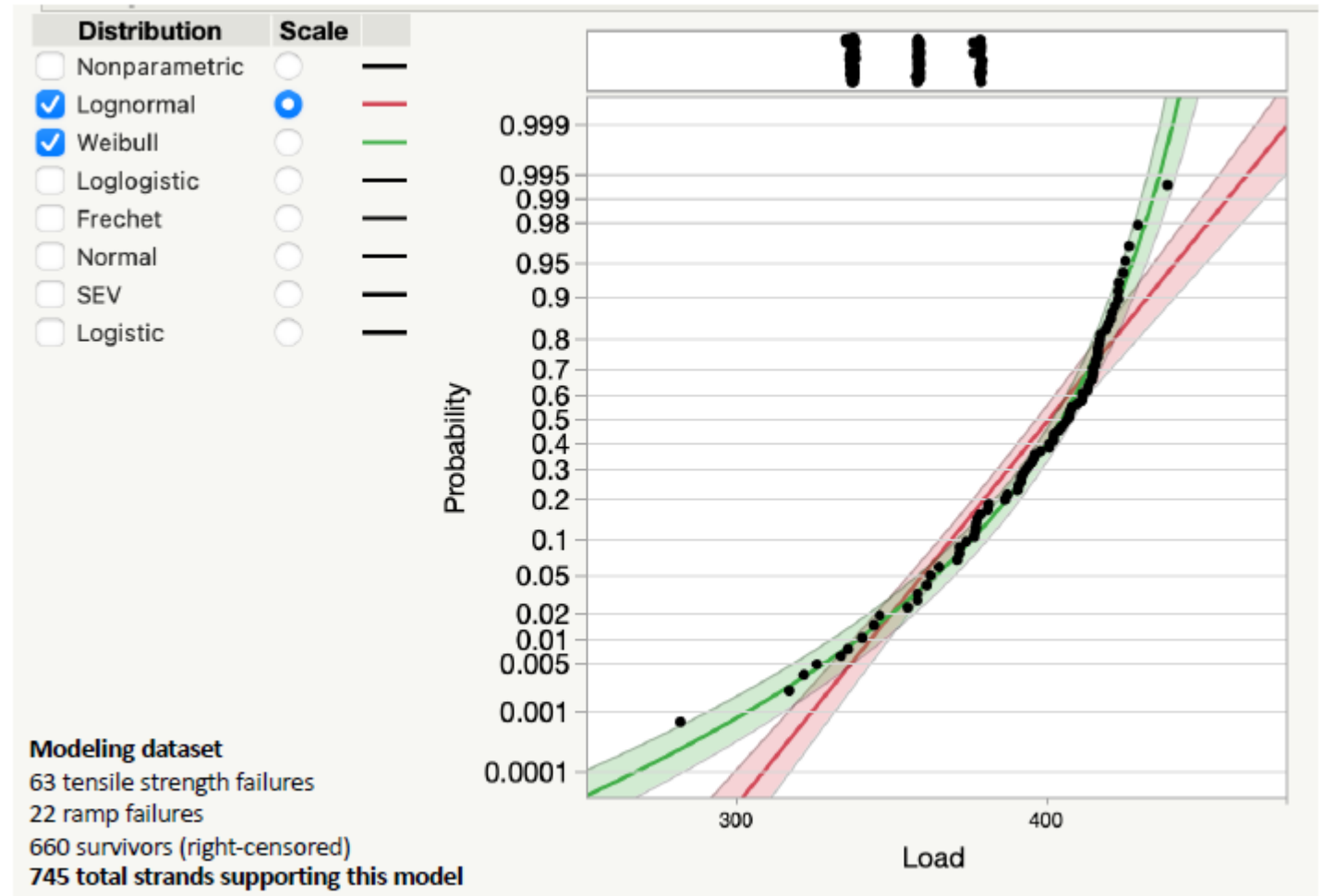
Strand Tensile Strength Modeling

- Weibull model is a better fit for the strand strength tests
- There appears to be different distributional models for strength of strands and burst for vessels



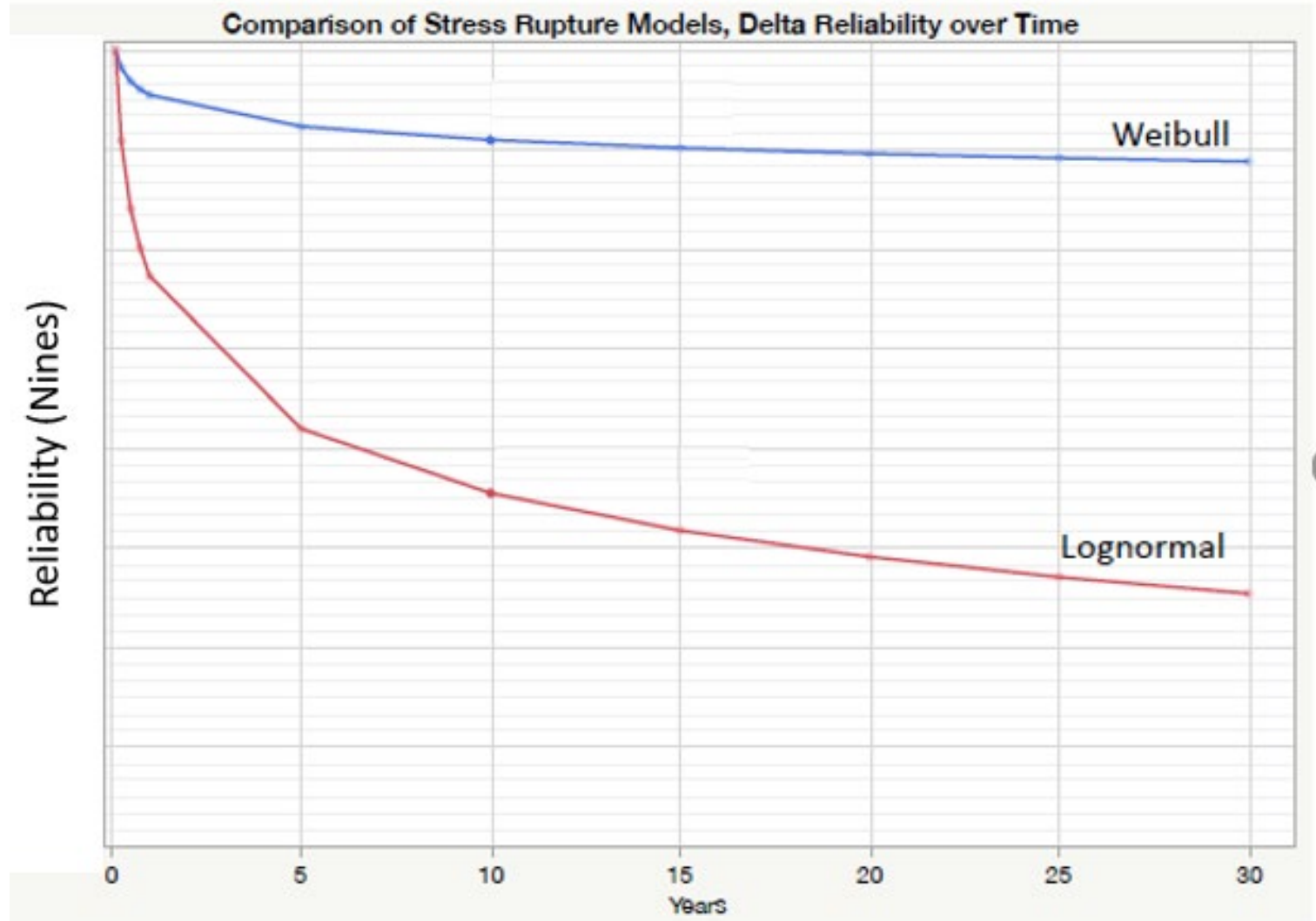
Strand Tensile Strength Modeling

- Weibull model is a better fit for the strand strength tests
- There appears to be different distributional models for strength of strands and burst for vessels



Vessel Stress Rupture Modeling

- Shows the shape of the Reliability over time with the Weibull model and the lognormal model fits
- We can use this drop in reliability to inform our model about the change in reliability over time



Uncertainty in Initial Reliability

- **Stress rupture test data alone provides a poor estimate of initial reliability**
 - Much uncertainty around the estimate of t_{ref}
 - Much debate on how to treat ramp failures
- **New Statistical Modeling Strategy**
 - Anchor initial reliability with strength burst/tensile testing, not stress rupture testing
 - Only use time-based stress rupture data to estimate drop in reliability over hold time
 - Leverages strength and time-based rupture testing to their maximum advantage

Strength-Informed Rupture Reliability (SIRR)

- **Strength/Burst model**

- Used to estimate the probability of failures with a lower bound at the operational stress ratio to anchor any rupture model reliability estimate

- **Stress rupture model**

- Used only to estimate the drop in reliability over mission time, from t_1 to t_2 , where t_1 is based on the operational stress ratio

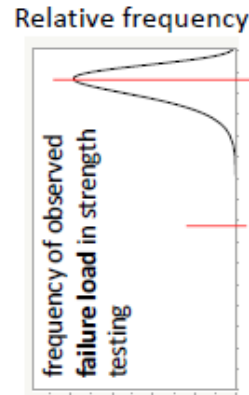
- **Reliability at Operation Conditions**

$$R_{use}(SR) = R_{strength_{LB}}(SR) - [R_{rupture}(t_1, SR) - R_{rupture}(t_2, SR)]$$

R is the calculated reliability in nines

Modeling Failure in Strength and Stress Rupture Testing

Strength Model
Function of Failure Load



Stress Rupture Model
Function of Failure Time

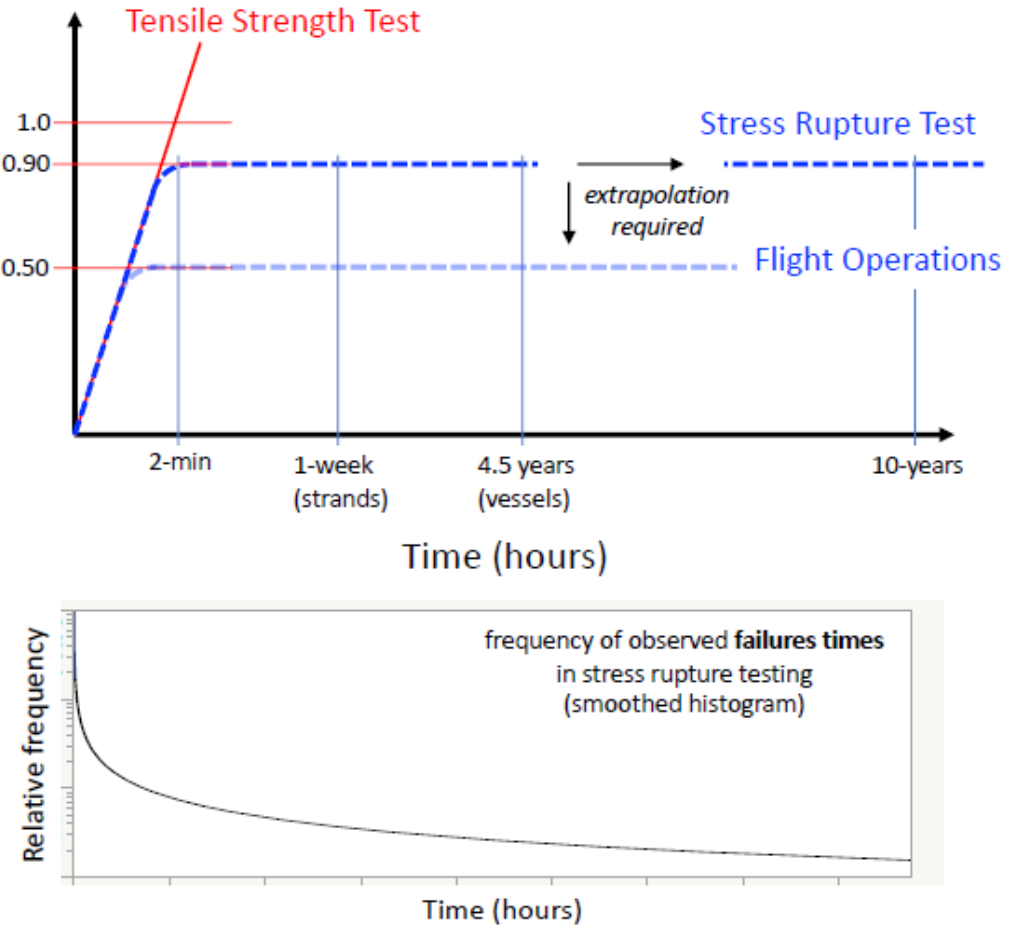


*Figure provided by Pete Parker, NASA

Where to Next...

- Working on case study manuscript to explain project
- Advocating for the use of the Strength-Informed Rupture Reliability for COPV reliability estimates
- Study new ways to perform stress rupture testing

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References

- Meeker, W.Q. and Escobar, L.A. (1998). Statistical Methods for Reliability Data. John Wiley & Sons: Hoboken, NJ.
- Yin, X-K and Sheng, B-Z (1987). “Some Aspects of Accelerated Life Testing by Progressive Stress,” *IEEE Transactions of Reliability*, R-36, pp. 150-155.