



A Moving Discontinuous Galerkin Method with Interface Condition Enforcement for Reacting Hypersonic Flows

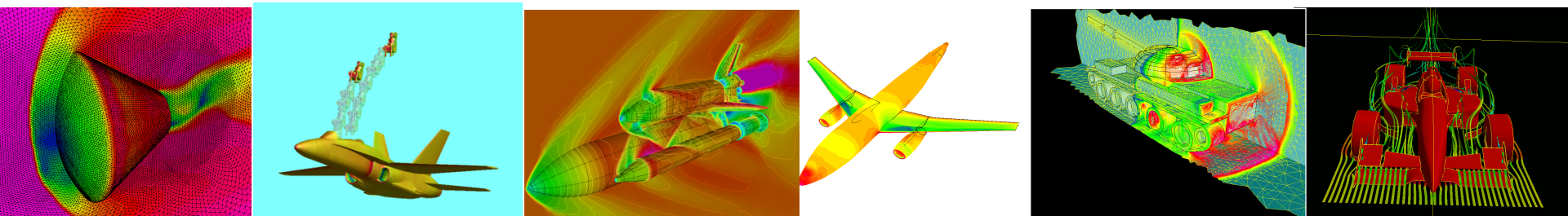
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Outline

- Background, Observation, and Motivation
- Moving Discontinuous Galerkin Method with Interface Conservation Enforcement (MDG-ICE) for Conservation Laws
 - Space-time DG Formulation
 - Variational formulation for Interface Conservation
- Extension to Reacting Hypersonic Flows
- Numerical Examples
- Concluding Remarks and Future Work



Background

- One of the most important topics for solving conservation laws is **how to avoid and suppress spurious oscillations** in the vicinity of discontinuities for high-order methods ($>1^{\text{st}}$).
- Continuous efforts (Limiters, ENO/WENO, BAP, WBAP, Artificial viscosity, Hyper viscosity, etc.) in the last few decades for addressing this issue manifest that a ‘perfect’ or ‘right’ answer to this question remains evasive.
- **The root cause of the problem** is that the **interface conservation** is not enforced. $\Rightarrow$ Formally grid-converged solutions can **NEVER** be achieved for discontinuous solutions.



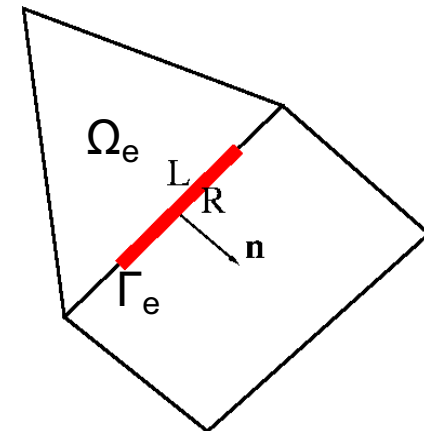
Interface Conservation

- Traditionally, the conservation laws are only solved on elements Ω_e (control volumes).

$$\frac{\partial \mathbf{U}(x,t)}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{U}(x,t))}{\partial x} = 0$$

- Similarly, the conservation laws **should** be enforced on element interfaces Γ_e , as required by the **physics** of the conservation laws.
- Applying the conservation laws on a zero-thickness control volume along an interface Γ_e leads to the following interface condition (jump condition) for the flux function across the interface

$$[\mathbf{F} \cdot \mathbf{n}] = [\mathbf{f}n_x + \mathbf{U}n_t] = 0$$





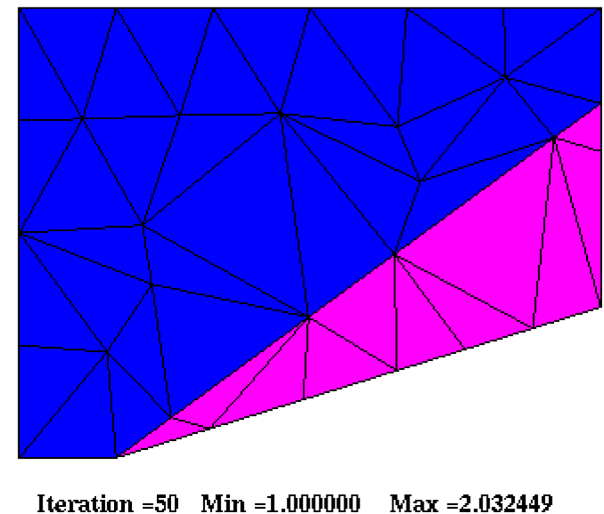
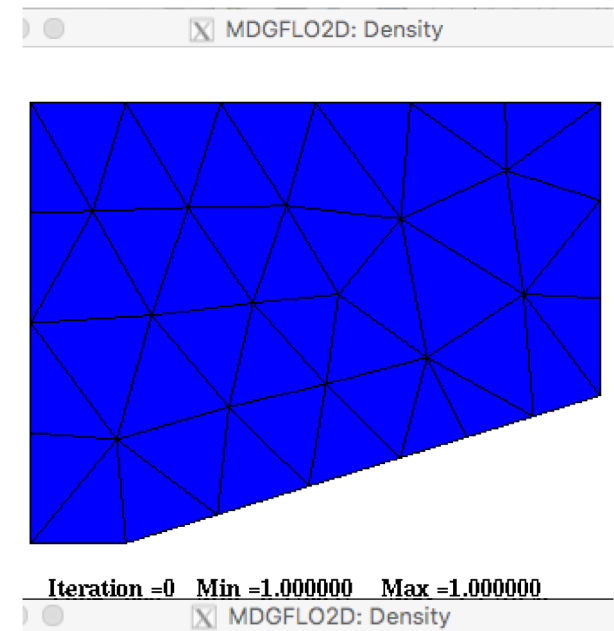
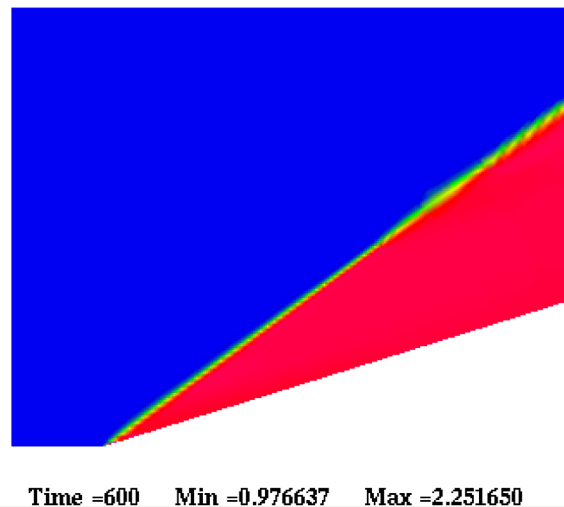
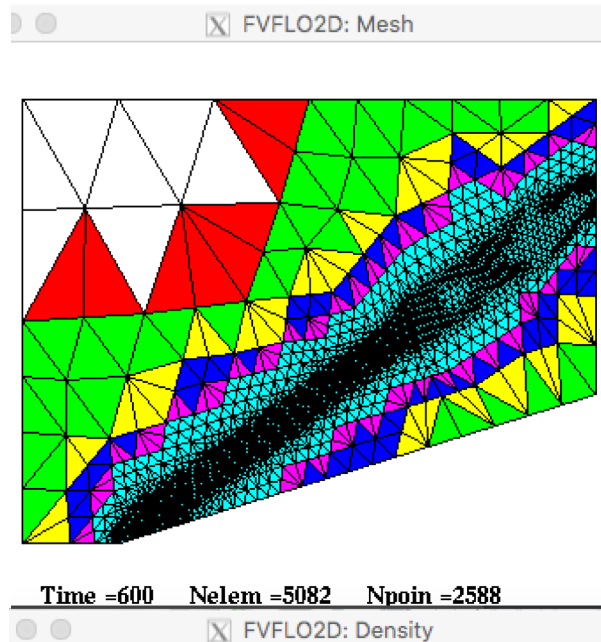
Observation

- Note that application of interface conservation to a shock wave leads to the Rankine-Hugoniot condition, which relates the flow properties before and after a shock wave.
- The interface conservation is never considered in all shock-capturing methods, because
 - 1. **it is automatically satisfied** for smooth flows, as long as the flows are fully resolved, which can always be achieved by using an excessive mesh resolution;
 - 2. **it can NEVER be satisfied** for flows with discontinuities by simply using excessively refined meshes, as long as the discontinuities are not aligned with mesh interfaces. **=>Discontinuities can never be resolved and computed exactly** by any shock-capturing methods, leading to a zero-order rate of convergence for solutions with discontinuities.



Illustrative example: Importance of enforcing IC

Supersonic flow past a wedge at a Mach number of 3





Motivation

- Explore new and innovative approaches to resolve and compute all types of discontinuities exactly.
- Avoid shock anomalies such as carbuncle phenomenon and shock instabilities.
- Reduce grid sensitivities seen in CFD results by shock-capturing methods



Background

- A moving DG finite element method with interface condition enforcement (MDG-ICE)¹ was originally formulated for flows with discontinuous interfaces by Corrigan et al.
 - Enforce the interface condition in addition to the conservation laws;
 - Treat the discrete grid geometry as a variable.
- A variant of MDG-ICE² was introduced by Luo et al., where a different variational formulation is used to enforce the interface conservation.

¹A. Corrigan, A.D. Kercher, and D.A. Kessler, A Moving Discontinuous Galerkin Finite Element Method for Flows with Interfaces, International Journal for Numerical Methods in Fluids, 2019.

²H. Luo, G. Absillis, and R. Nourgaliev, A Moving Discontinuous Galerkin Finite Element Method with Interface Condition Enforcement for Compressible Flows, Journal of Computational Physics, 2021.



Objective

- Further develop the MDG-ICE method for thermal-chemical non-equilibrium reactive hypersonic flows
 - Avoid shock anomalies such as carbuncle phenomenon and shock instabilities.
 - Reduce grid sensitivities seen in CFD results by shock-capturing methods



Example 1. Application to thermal and chemical non-equilibrium hypersonic reacting flows

Set-up of the problem:

- Hypersonic flow over a half-cylinder geometry (radius=1m)
- The free stream conditions: $p=25$, $T=300\text{K}$, $M_\infty=10$
- 5-species (O_2 , O , NO , N , N_2) $\text{O}_2=0.233$, $\text{N}_2=0.767$, $\text{O}=\text{N}=\text{NO}=0$
- RRHO for the 2-temperature model

Mesh information:

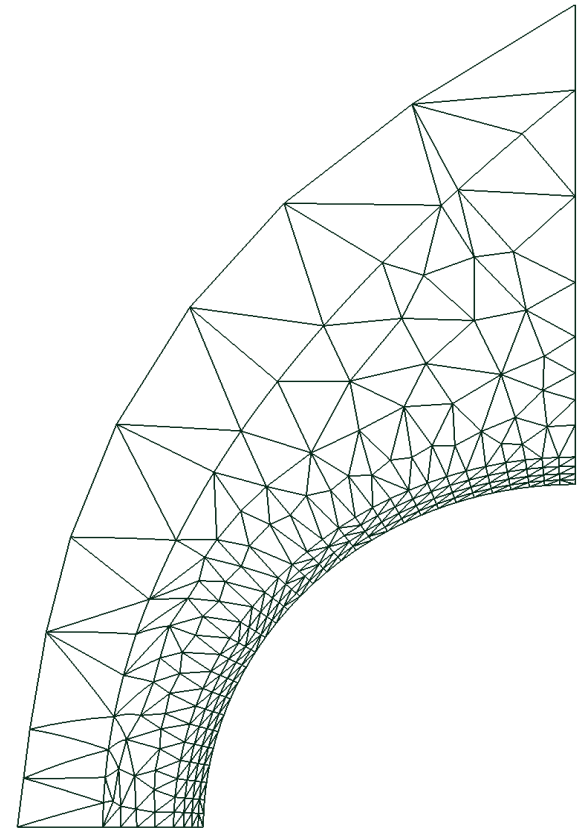
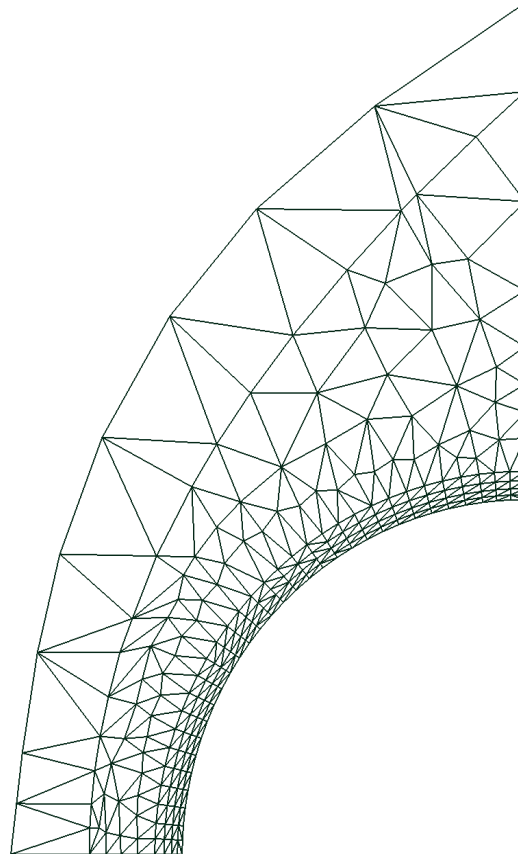
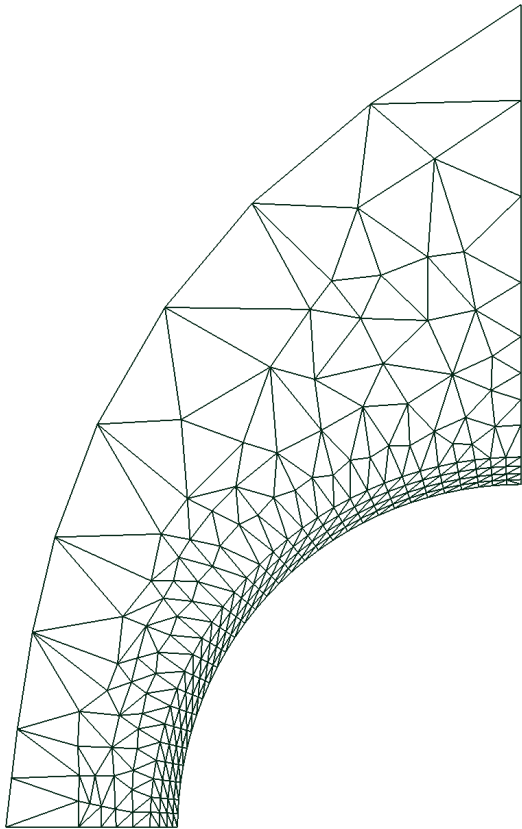
- $\text{nelem}=311$, $\text{nvert}=184$, $\text{npoin}=184$, $\text{nface}=55$
- DOFs: 933(P1), 1,866(P2), 20,000(DPLR)
- Distance of the first point to the SP: $5\text{E-}5$ (DPLR), $2.19\text{E-}2$ (MDG)

Solution process:

- DG(P0) solution is used to initialize MDG(P1) solution
- DG(P1) solution is used to initialize MDG(P2) solution



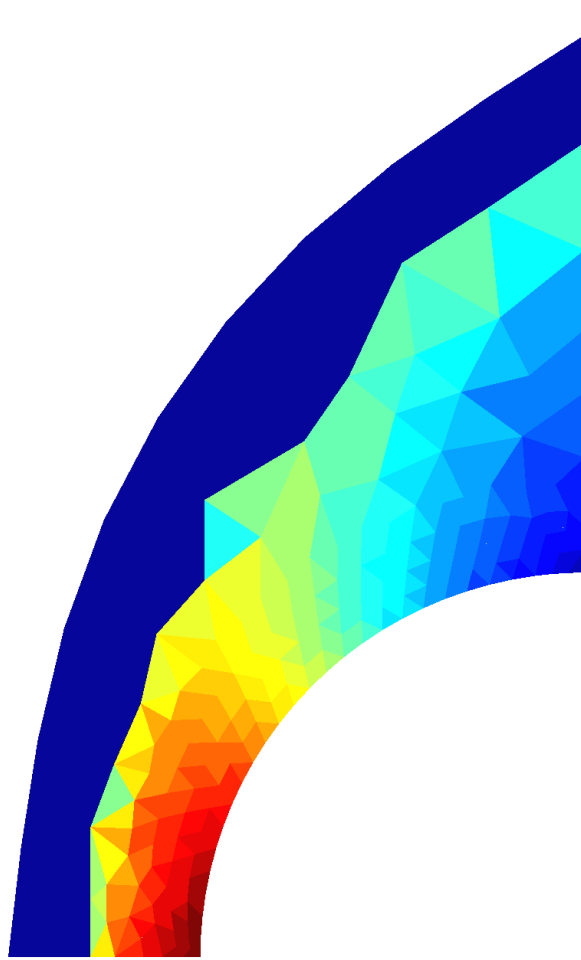
Example: Hypersonic flow past a half-cylinder ($M=10$)



Initial MDG(P1) mesh Converged MDG(P1) mesh MDG(P2) curved mesh

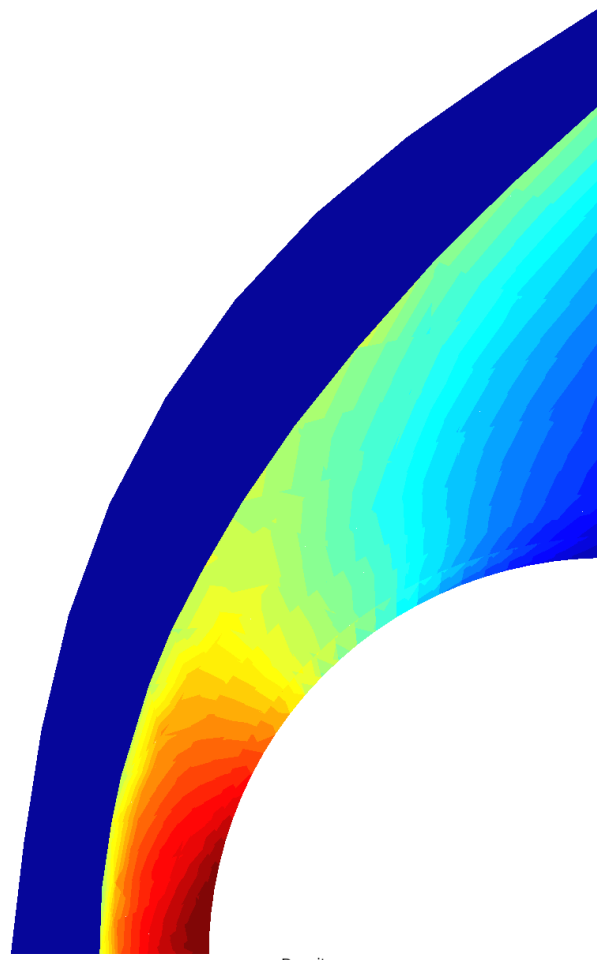


Example: Hypersonic flow past a half-cylinder ($M=10$)



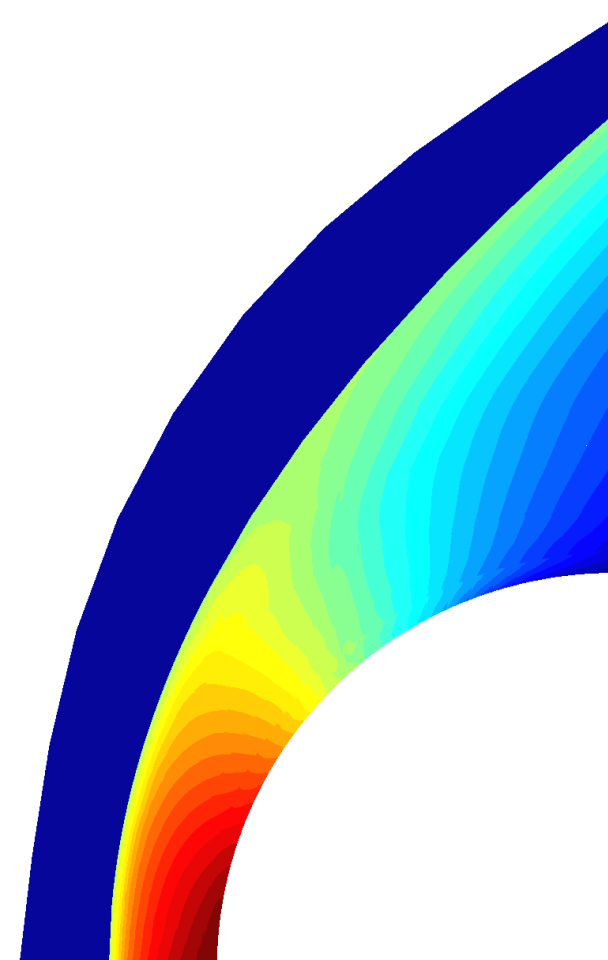
Density
0.000289 0.00156 0.00284

DG(P0)



Density
0.000289 0.00159 0.00289

MDG(P1)

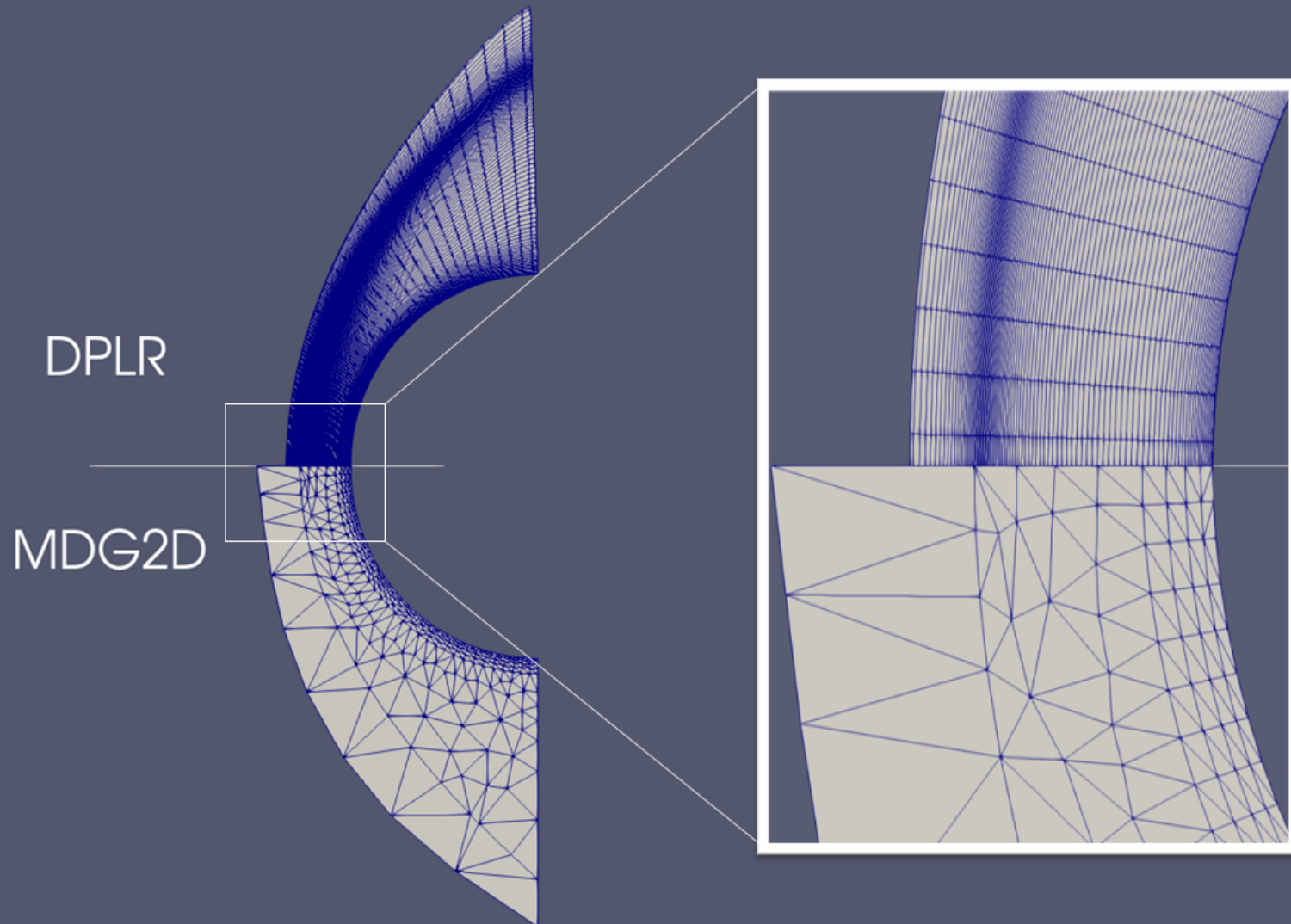


Density
0.000289 0.00159 0.0029

MDG(P2)



Comparison of the LAURA-adapted grid used in DPLR and converged mdgp2 mesh

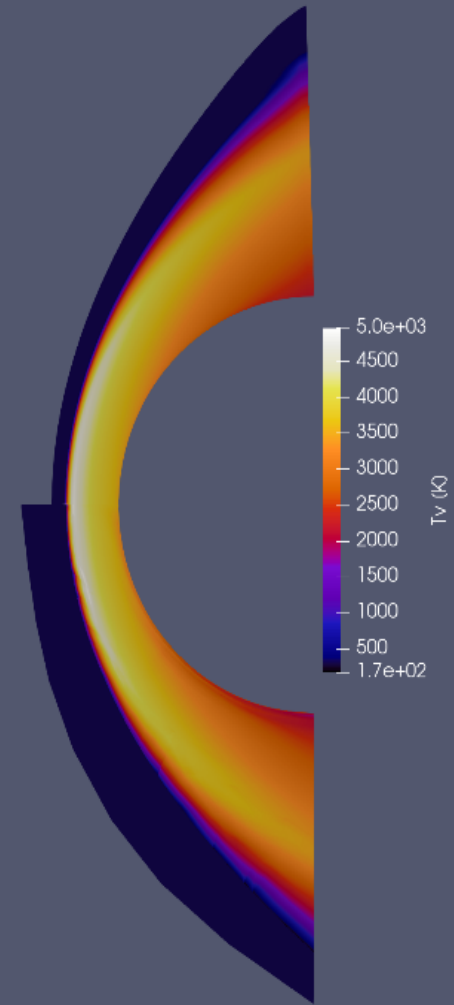
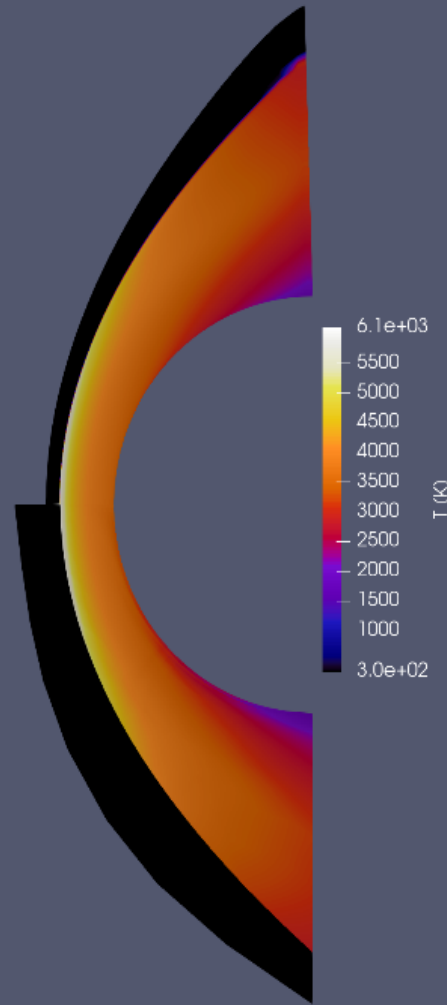
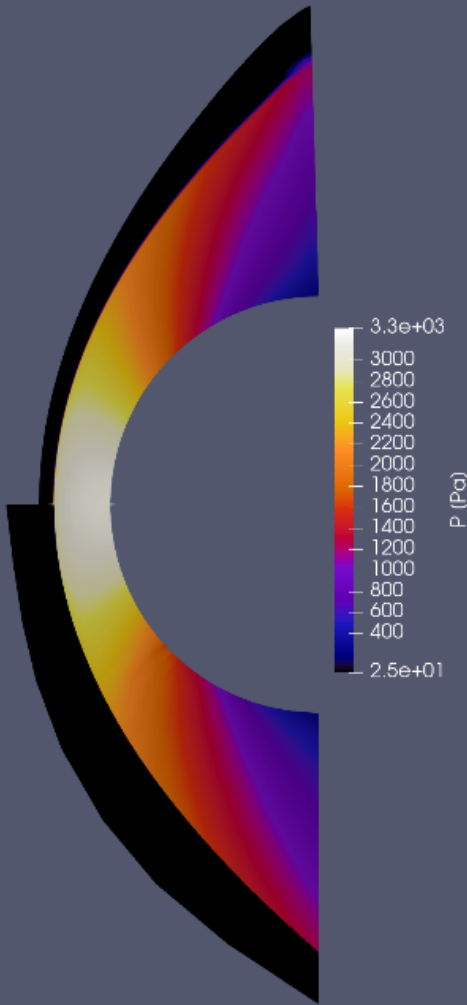




Contours of the pressure, temperature, vibrational temperature (from left to right) for DPLR and mdgp2 solutions

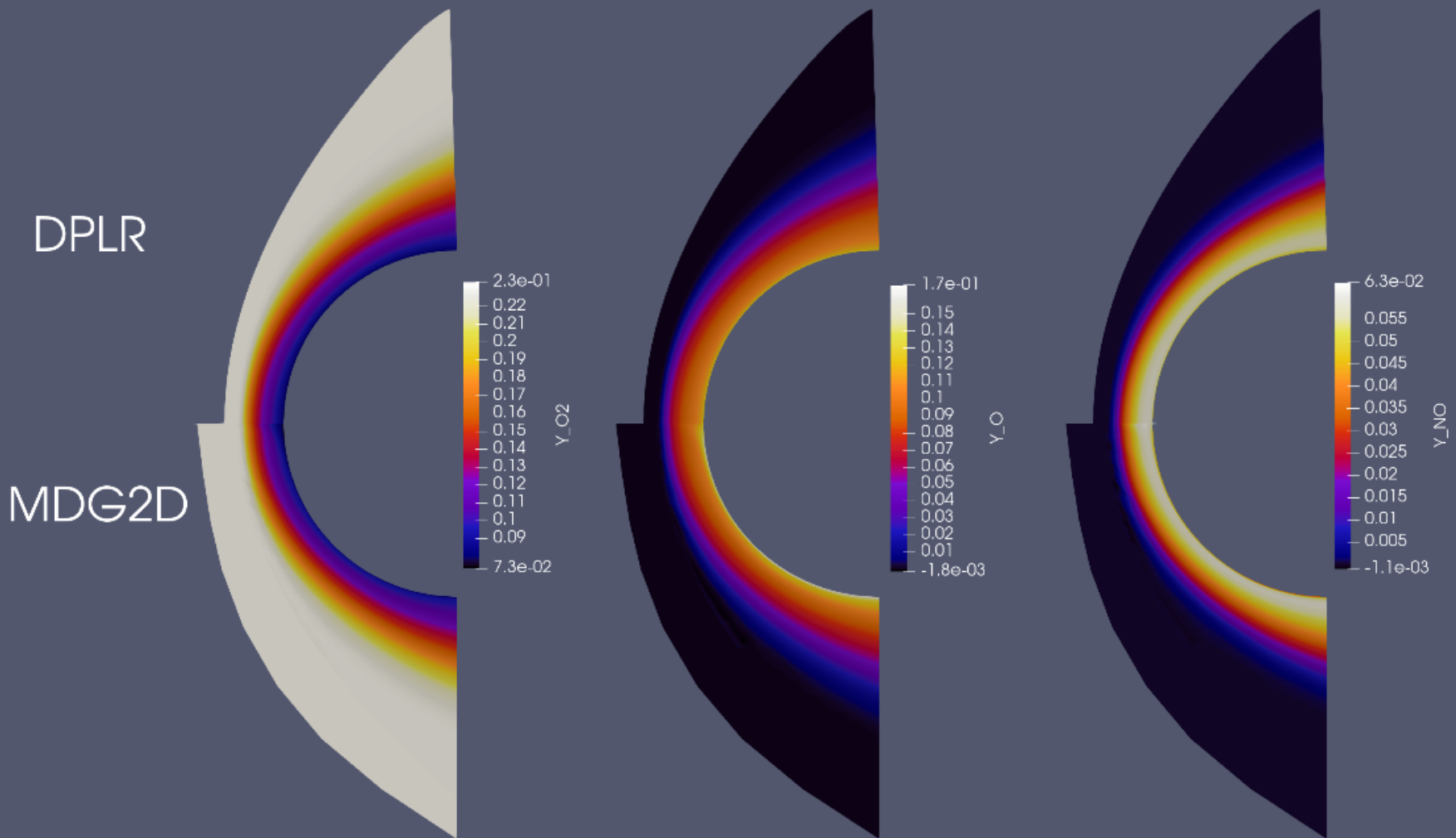
DPLR

MDG2D



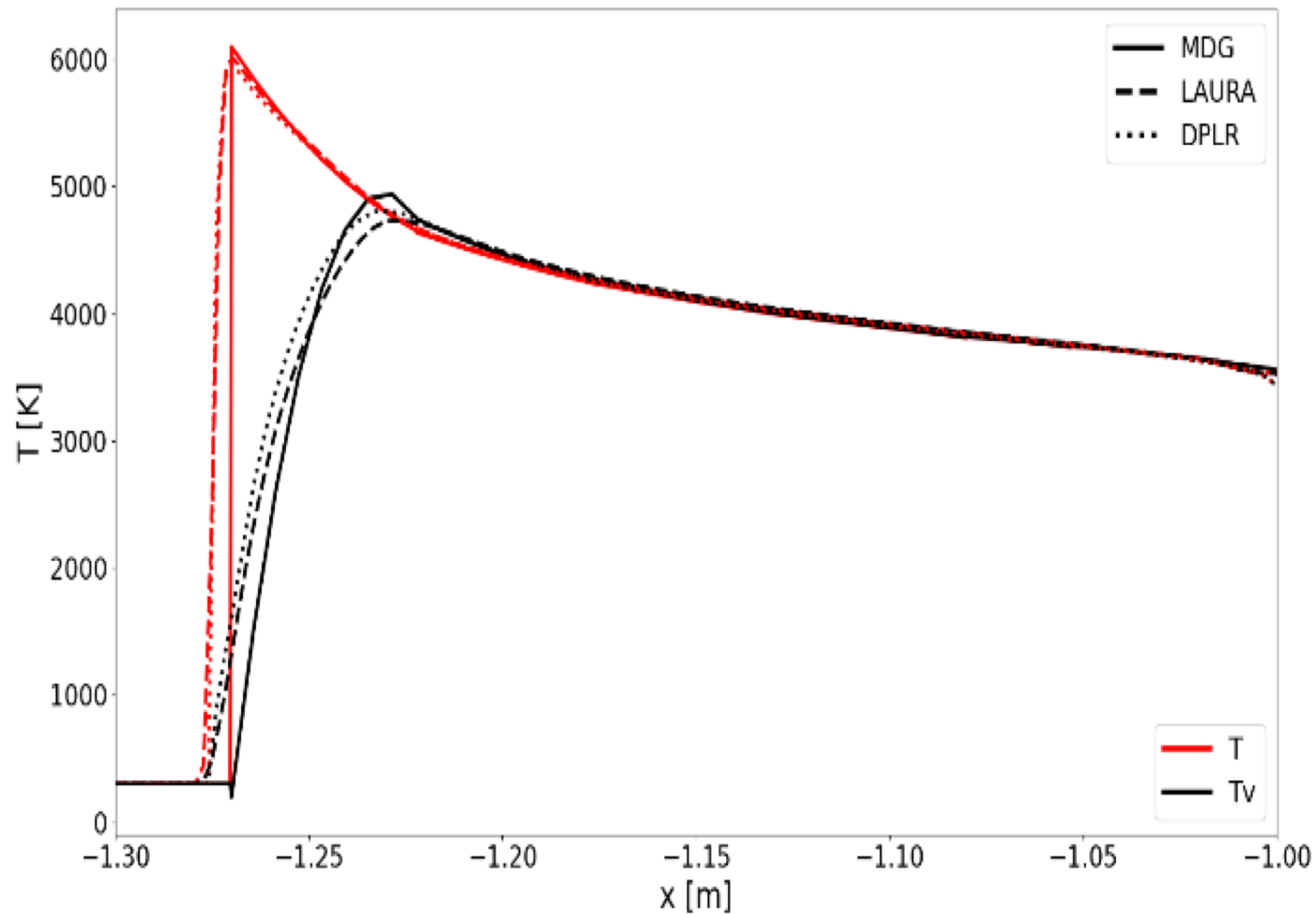


Contours of the mass fractions for O₂, O, and NO (from left to right) for DPLR and mdgp2 solutions



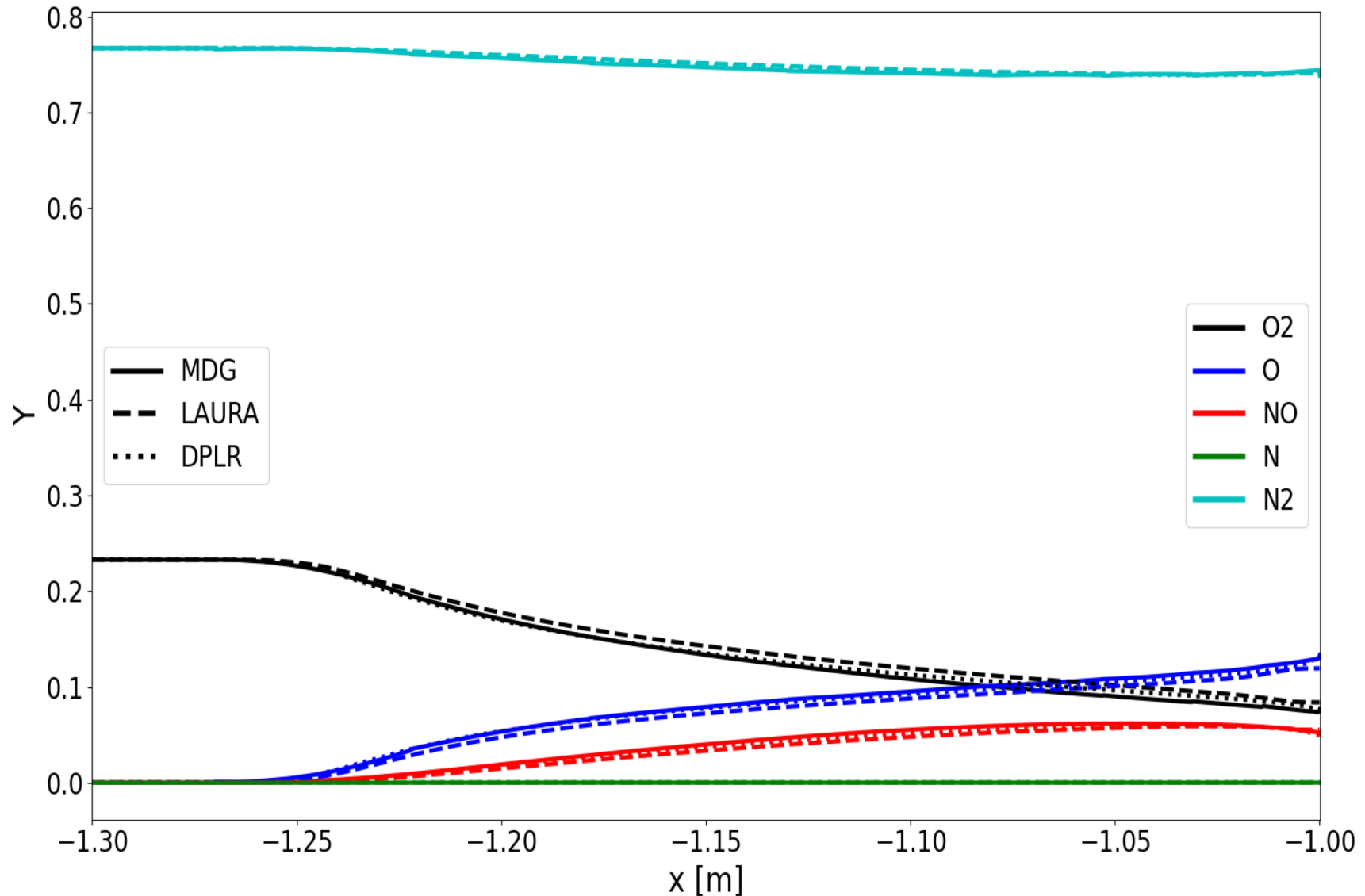


Comparison of translational/rotational temperature (T) and vibrational temperature (T_v) along the stagnation line





Comparison of species mass fractions along the stagnation line between MDG2D P2 (solid), LAURA (dashed), and DPLR (dotted)





Example 2. Application to thermal and chemical non-equilibrium hypersonic reacting flows

Set-up of the problem:

- Hypersonic flow over a blunt body geometry
- The free stream conditions: $p=25$, $T=300\text{K}$, $M_\infty=10$
- 5-species (O_2 , O , NO , N , N_2) $\text{O}_2=0.233$, $\text{N}_2=0.767$, $\text{O}=\text{N}=\text{NO}=0$
- RRHO for the 2-temperature model

Mesh information:

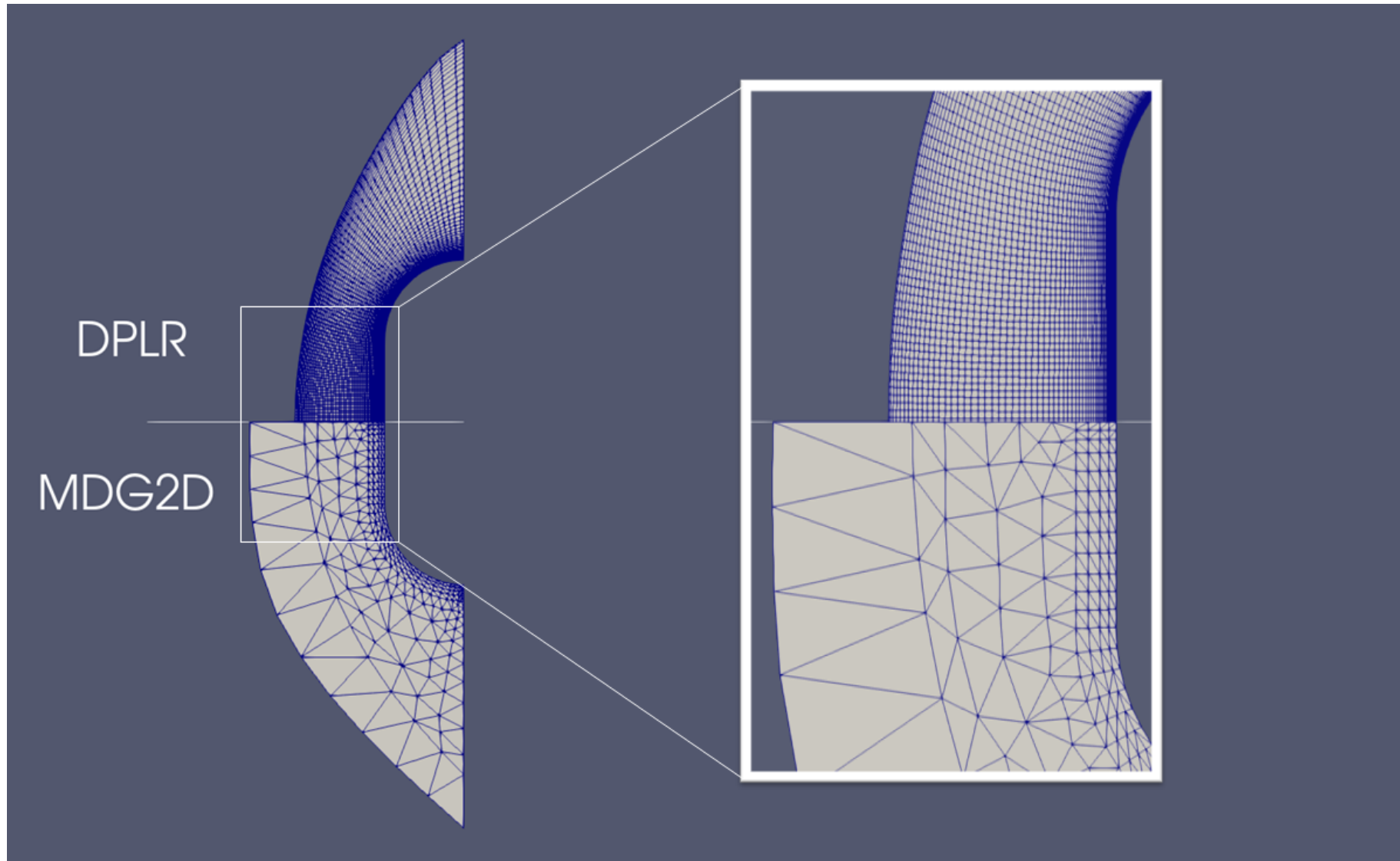
- $\text{nelem}=447$, $\text{nvert}=258$, $\text{npoin}=258$, $\text{nface}=67$
- DOFs: 1,341(P1), 2,682(P2), 28,800(DPLR)
- Distance of the first point to the SP: $5\text{E-}5$ (DPLR), $2.19\text{E-}2$ (MDG)

Solution process:

- DG(P0) solution is used to initialize MDG(P1) solution
- DG(P1) solution is used to initialize MDG(P2) solution

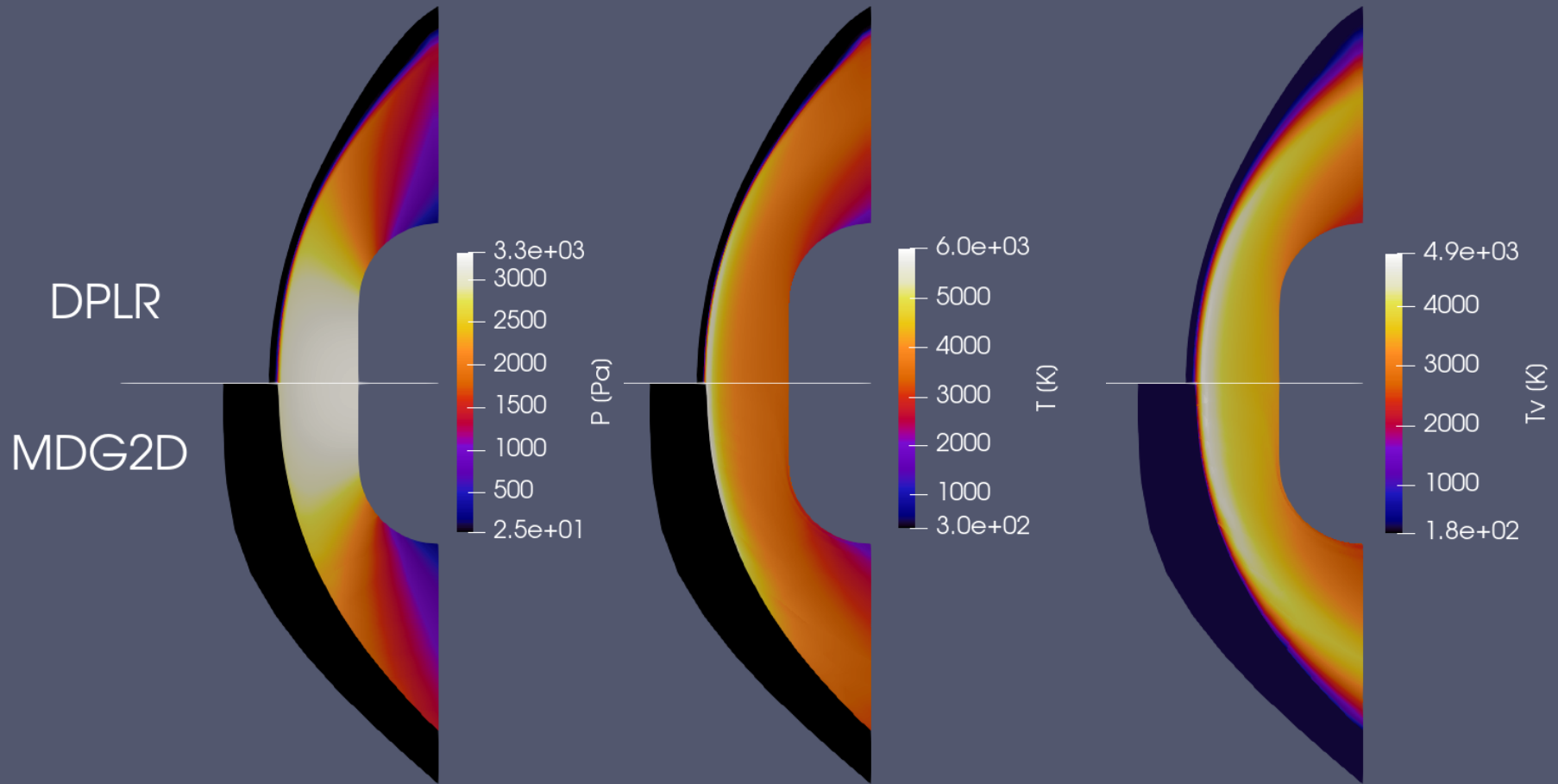


Comparison of the adapted grid used in DPLR and converged mdg-ice(P2) mesh



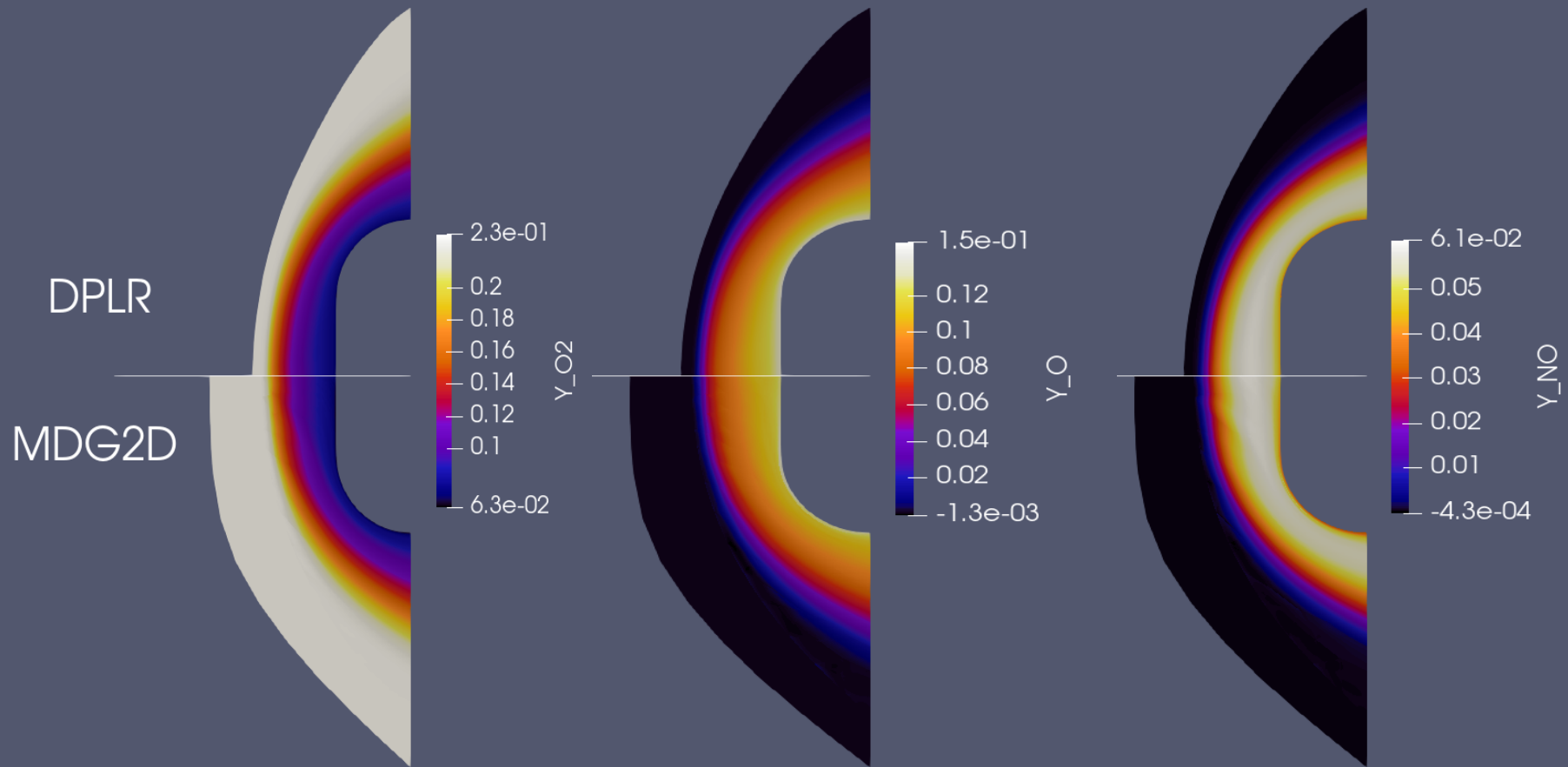


Contours of the pressure, temperature, vibrational temperature (from left to right) for DPLR and mdgp2 solutions



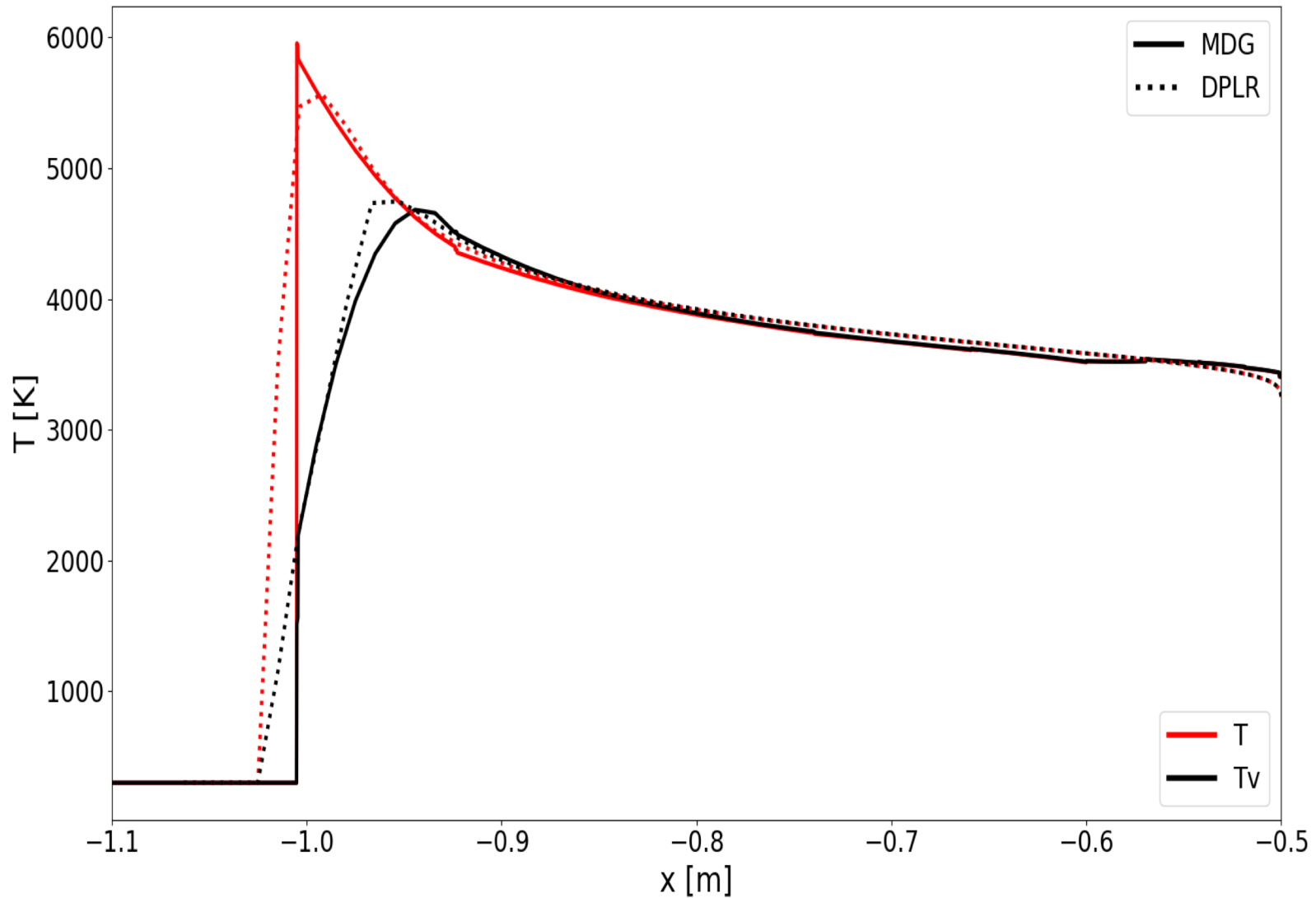


Contours of the mass fractions for O₂, O, and NO (from left to right) for DPLR and mdgp2 solutions



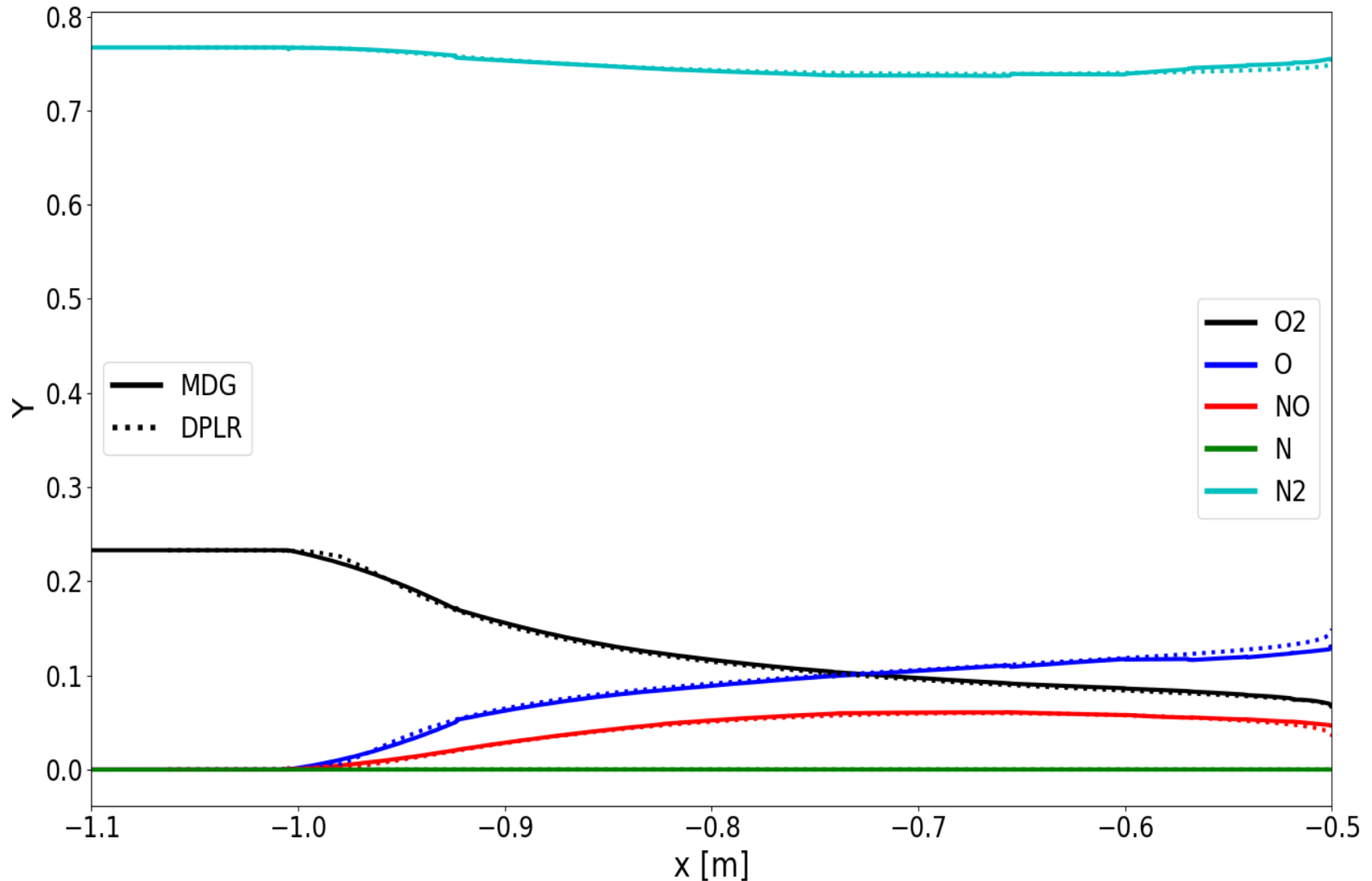


Comparison of translational/rotational temperature (T) and vibrational temperature (T_v) along the stagnation line





Comparison of species mass fractions along the stagnation line between MDG2D P2 (solid) and DPLR (dotted)





Concluding Remarks and future work

- The MDG-ICE method has been successfully extended for reacting hypersonic flows using a mixture model.
- The numerical experiments for a number of chemical and thermal-chemical non-equilibrium reactive hypersonic flows demonstrate that the MDG-ICE method is able to capture shocks (discontinuities) and the associated jumps in thermodynamic properties exactly, while using a fraction of the grid cells typically used in DPLR and LAURA.
- Ongoing work is to further develop the MDG-ICE method for thermal and chemical non-equilibrium viscous hypersonic flows.



Thank you
for attending my presentation