

Transformational Tools and Technologies (T³) Project

Stiffness And Fatigue Life Estimator For Polymer Matrix Composite Laminates Using Machine Learning

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*Innovative solutions through
foundational research and
cross-cutting tools*

Outline

- Background
 - MAC/GMC Computer Code
 - Stiffness and Strength Fatigue Degradation Models
- Objective
- Artificial Neural Nets (ANN)
- Data Generation
- Results (MLP and RNN)
- Summary

Background

- Machine learning (ML) models are increasingly being used in many engineering fields due to the advancements in ML algorithms and availability of high-speed computers.
- One of the most popular ML class of models is artificial neural networks (ANN).
- ML is increasingly being used in the design and analysis of composite materials and structures
 - specifically in the constitutive modeling of composite materials with the focus on greatly accelerating multiscale analyses of composite materials and structures through surrogate modeling
- ML models typically require large amounts of data for training and validation and this quantity of measured data is usually not readily available
 - That's why in this study we generated synthetic / virtual data from physics-based models to train ML surrogate model
- Composite Fatigue Life is function of (fiber/matrix properties, V_f , fiber packing, ply architecture (e.g., number of plies, thicknesses, orientation), Loading (mechanical, thermal))



NASA GRC's Multiscale Analysis Approach Links the Behavior Of A Multiphase Material/Structure To The Behavior Of Its Constituents



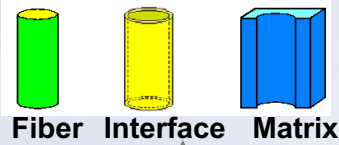
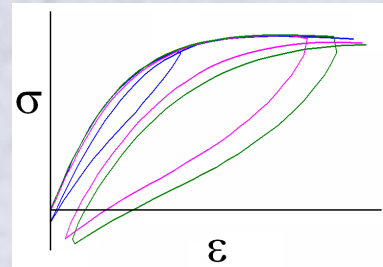
MAC/GMC

Composite Micromechanics RUC

Structural Laminate

Constituent Response

Micromechanics Provides the Link Between Structures and Materials



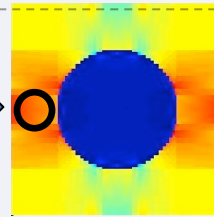
Continuum Deformation and damage models

Micro-level Field Equations (subcell)

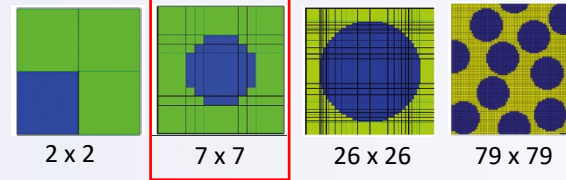
$$\epsilon^{(\alpha\beta\gamma)} = A^{(\alpha\beta\gamma)} \bar{\epsilon} + D^{(\alpha\beta\gamma)} (\epsilon_s^I + \epsilon_s^T)$$

$$\sigma^{(\alpha\beta\gamma)} = C^{(\alpha\beta\gamma)} \left[A^{(\alpha\beta\gamma)} \bar{\epsilon} + D^{(\alpha\beta\gamma)} (\epsilon_s^I + \epsilon_s^T) - (\epsilon^{I(\alpha\beta\gamma)} + \epsilon^{T(\alpha\beta\gamma)}) \right]$$

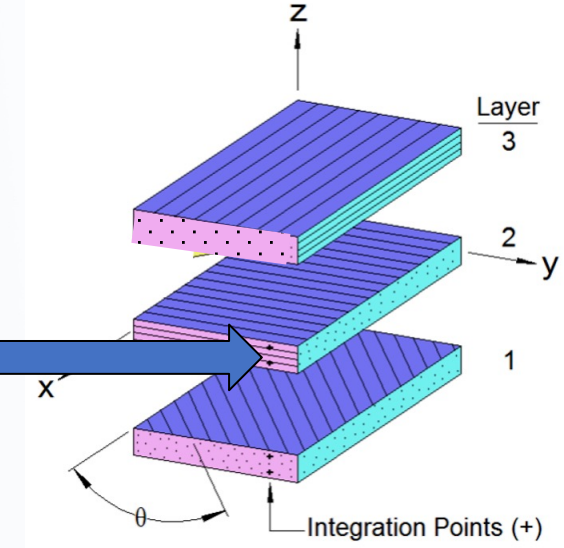
Atomistic Informed



Speed



Accuracy



Macro-level Constitutive Equations

$$\bar{\sigma} = B^* (\bar{\epsilon} - \bar{\epsilon}^I - \bar{\epsilon}^T)$$

$$B^* = \frac{1}{dhl} \sum_{\alpha=1}^{N_a} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_\alpha h_\beta l_\gamma C^{(\alpha\beta\gamma)} A^{(\alpha\beta\gamma)}$$

$$\bar{\epsilon}^I = \frac{-[B^*]^{-1}}{dhl} \sum_{\alpha=1}^{N_a} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_\alpha h_\beta l_\gamma C^{(\alpha\beta\gamma)} (D^{(\alpha\beta\gamma)} \epsilon_s^I - \epsilon^{I(\alpha\beta\gamma)})$$

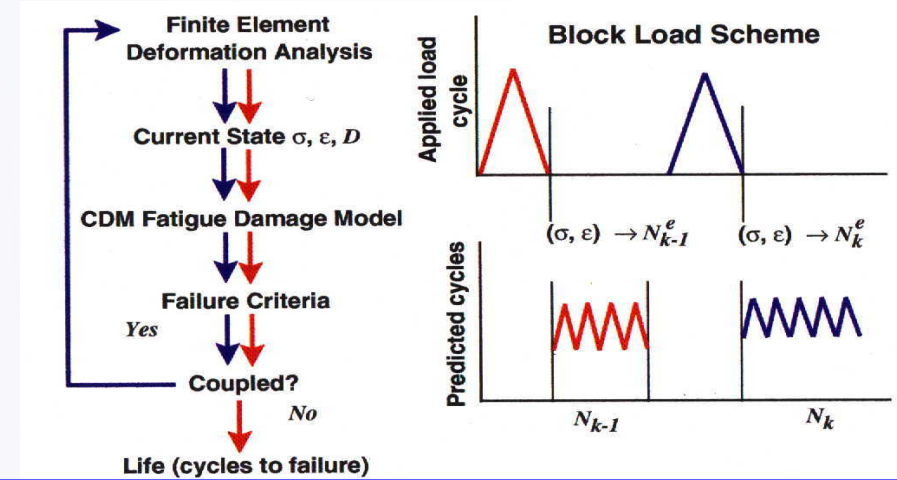
$$\bar{\epsilon}^T = \frac{-[B^*]^{-1}}{dhl} \sum_{\alpha=1}^{N_a} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_\alpha h_\beta l_\gamma C^{(\alpha\beta\gamma)} (D^{(\alpha\beta\gamma)} \epsilon_s^T - \epsilon^{T(\alpha\beta\gamma)})$$

Coupled Deformation and Damage Methodology Utilized For Fatigue Life Prediction

Life Prediction Scheme

Continuum Deformation: Elastic

Carbon Fiber	Epoxy Matrix
$E_{f11}=388.2$ GPa	$E_m=3.45$ GPa
$E_{f22}=7.6$ GPa	$\nu_m=0.35$
$\nu_{f12}=0.41$	$G_{m23}=1.278$ GPa
$\nu_{f23}=0.45$	
$G_{f12}=14.9$ GPa	



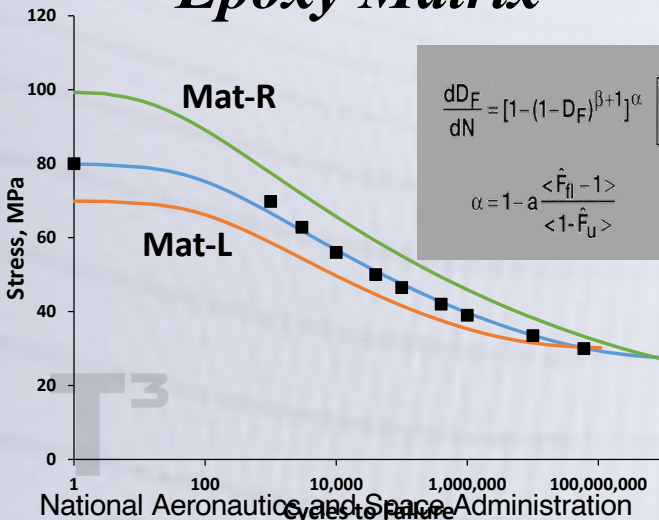
Characterization of the stiffness reduction fatigue damage model

$$\sigma = (1 - D)C\epsilon$$

Baseline Parameters

Property	Value	Units
β	9	
M	150	MPa
σ_{ul}	80	MPa
σ_{fl}	27	MPa
a	0.05	
ω_u	1	
ω_m	1	
ω_{fl}	1	
η_u	1	
η_m	1	
η_{fl}	1	

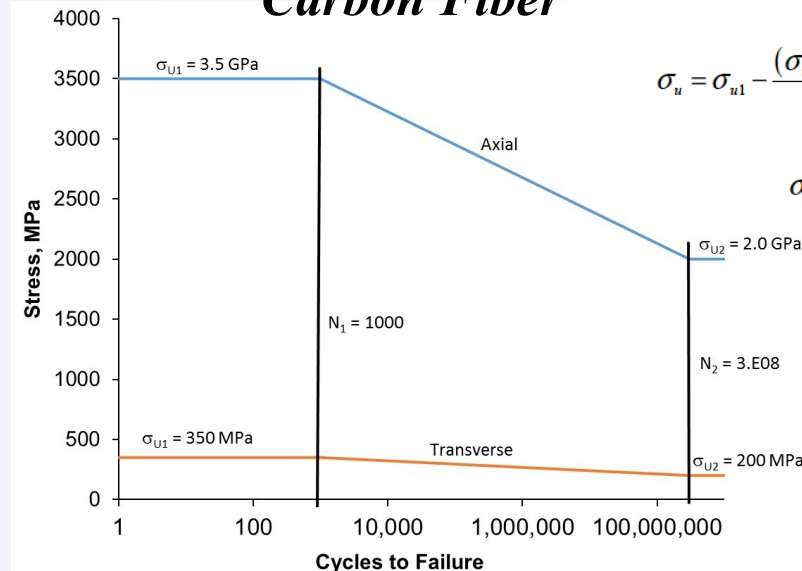
Epoxy Matrix



$$\frac{dD_F}{dN} = [1 - (1 - D_F)^{\beta+1}]^{\alpha} \left[\frac{\hat{F}_m}{(1 - D_F)} \right]^{\beta}$$

$$\alpha = 1 - a \frac{\langle \hat{F}_{fl} - 1 \rangle}{\langle 1 - \hat{F}_u \rangle}$$

Carbon Fiber



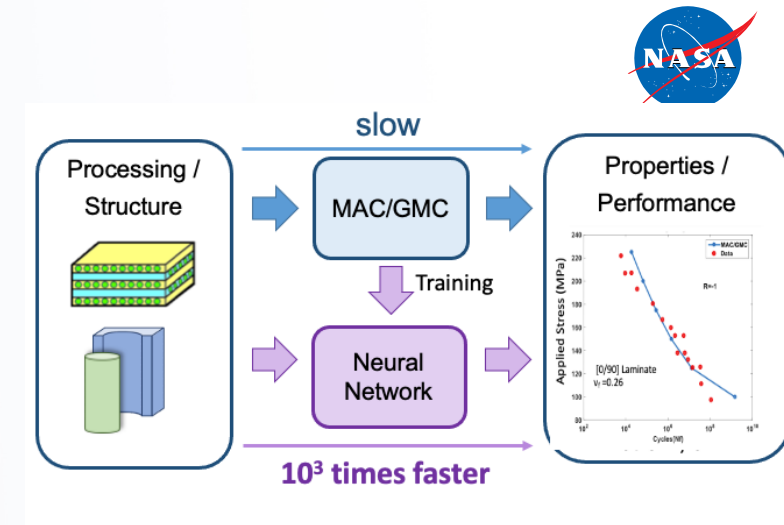
Failure defined:
 $D \sim 1$ (subcell)
Fiber failure (RUC)

$$F_{(I)} = \sqrt{\frac{1}{C_L^2} \left\{ \left(4\omega_{(I)}^2 - 1 \right) I_1 + \frac{\left(4\omega_{(I)}^2 - 1 \right)}{\eta_{(I)}^2} I_2 + \frac{9}{4} I_3 \right\}}$$

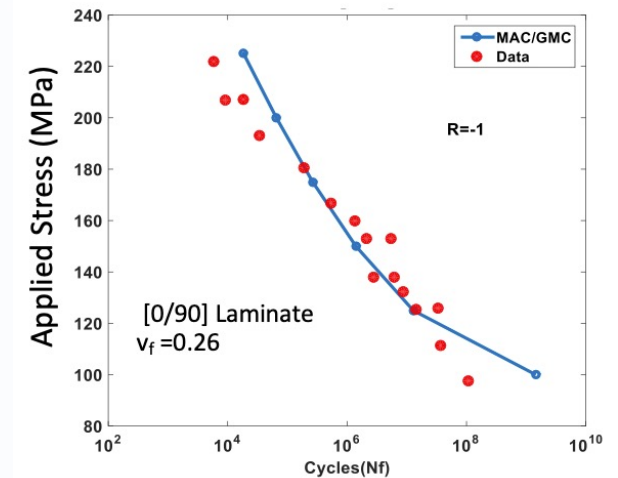
Objective

- Develop a MATLAB or Python-based Machine learning (ML) tool that can rapidly predict the initial laminate stiffness (A, B, D) matrices and fatigue life (S-N curve) of a polymer matrix composite (PMC) laminate

- ✓ Resulting tool can enable **efficient** system level studies by providing reasonably accurate estimates of desired properties and life of PMC composite laminates.
- ✓ The associated surrogate models can be used in composite multiscale analyses to replace the actual physics-based calculations at lower scales and thereby significantly increase the computational efficiency of such analyses (industrial required speed with research level accuracy)
 - ✓ Embedded ML surrogate was able to calculate the composite response **145 times faster** while maintaining an **accuracy of 98%** compared to the original physics-based model**



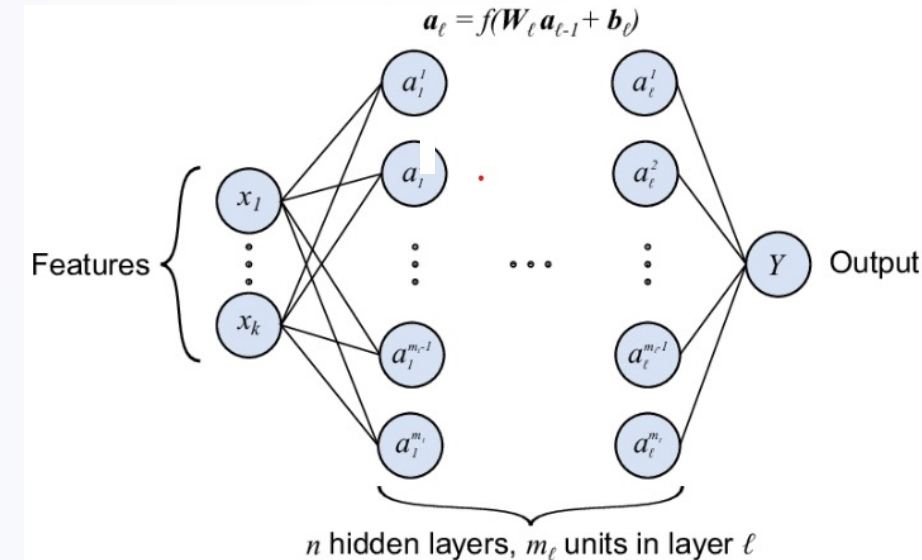
Output: **ABD & Life Curve**



Anatomy of a Deep Learning ANN



- The elements of a network are nodes or neurons (units), where weighted signals (W_{ij}) are combined, and biases added.
- In a single layer, the inputs are multiplied by weights and then added together at the end, after passing through a threshold function.
- In a multi-layer or “deep learning” network, the inputs are combined in the second layer before being output. There are more weights, and the added connections allow the network to learn and solve more complex problems.
- There are many other details – loss function, activation function, optimizer, drop-out nodes, learning rate etc.

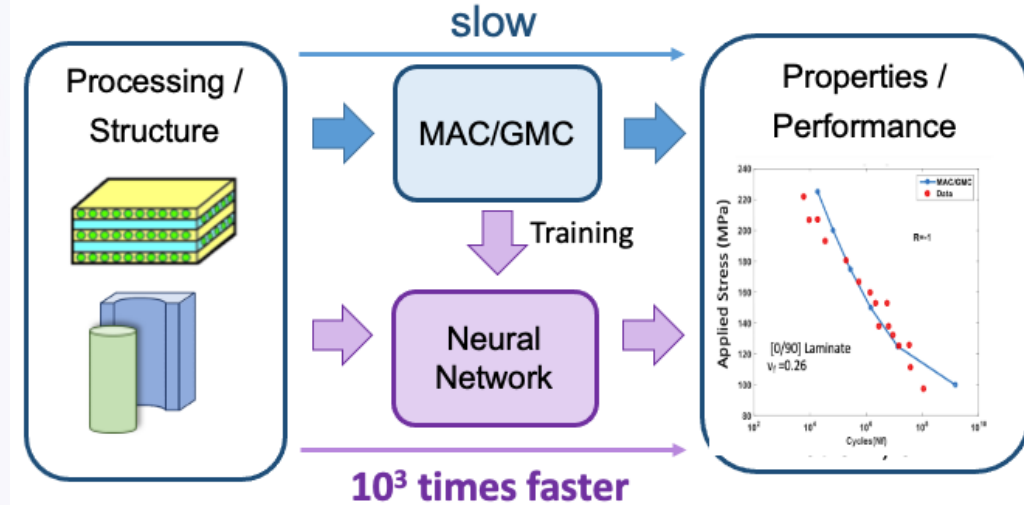


Most Common Types of Deep Learning ANN

- **Multi-layer Perceptron (MLP)** - most basic. Sometimes referred to as multi-layer neural network or simply artificial neural network. Also called feedforward networks. They have no memory and entire sequence is shown all at once.
- **Convolutional Neural Networks (CNN)** – Used for finding patterns or image analysis. It convolves a feature with input matrix so that output emphasizes that feature. Human vision system is a CNN of sorts.
- **Recurrent Neural Networks (RNN)** – Generally, used for time-dependent problems. They combine the last time step's data with the data from hidden layers to represent current time step. They are widely used in speech recognition, language translation and many other applications. They also often use long short-term memory (LSTM) layers to avoid dependency on old information.

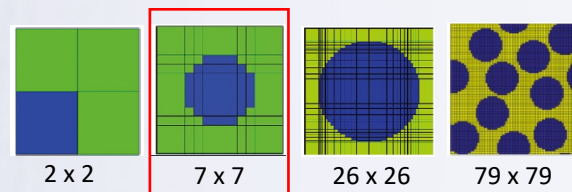
Data Generation - Architecture

- Typically, in material science the amount of experimental data doesn't exist in sufficient quantity required by neural nets due to the expense involved in generating this data.
- Therefore, turned to physics-based GMC micromechanics method to generate the large amount of ***synthetic*** training data



Physics-Based Virtual Data

- Stiffness matrices **[A]** and **[D]** (**[B]** = 0 for symmetric laminates) and fatigue lives (S-N curve) were generated for eight-ply, symmetric, PMC laminates with varying fiber volume fractions as well as varying fiber and matrix material properties for an applied load in global X-direction.
 - 10,000 Laminates with approx. 10 load levels per laminates. (~15,000 hrs of computations on single CPU)
 - Utilized physical linearity knowledge of S-N curve to generate additional 100,000 data points without additional computational cost
- 7X7 RUC** with ordered square packing --used to represent the microstructure of the composite ply
- Each ply has **same thickness** and fatigue load **is fully-reversible (R=-1)** and applied in the global X direction.



Data Generation: 19 Input Parameters Ranges



Constituent Properties

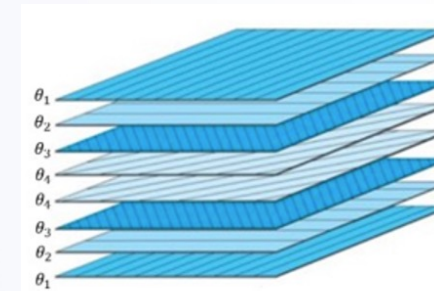
Property	Lower	Upper
Fiber Properties		
E_{fa} , GPa	70	700
E_{ft} , GPa	70	200
ν_{fa}	0.2	0.4
ν_{ft}	0.2	0.4
G_{fa} , GPa	25	200
Matrix Properties		
E_m	2.5	4.5
ν_m	0.2	0.45

Fatigue Parameters

Property	Lower	Upper
Fiber Parameters		
SU_{11} , MPa	2500	9500
SU_{21} , MPa	300	2500
Matrix Parameters		
E_{psm_1}	0.015	0.03
E_{psm_2}	0.015	0.03
Beta	4	12
SFL, MPa	15	30
XML, MPa	80	160

RUC/Laminate Parameters

Parameter	Lower	Upper
FVR	0.4	0.7
θ_1	-90°	90°
θ_2	-90°	90°
θ_3	-90°	90°
θ_4	-90°	90°



- Constituent property limits, fatigue parameters and RUC/Laminate parameter limits were chosen in a manner to cover commonly used constituents of typical PMCs (e.g., IM-7 and T300 fibers, epoxy matrices)
- Input parameters were randomly generated between upper and lower bounds following all applicable laws of material behavior
- Data divided into training data (~80%), validation data (~10%) and test data (~10%)

Importance of Capturing Embedded Assumptions in Data Generation



Fiber properties:

- ☐ $E_t \leq E_a$
- ☐ $E_t/(2(1+\nu_t)) \leq G_a \leq E_a/(2(1+\nu_a))$
- ☐ $SU_{12} = SU_{13} = (SU_{11}/E_a) * E_t$
- ☐ $SU_{14} = SU_{12}/\text{sqrt}(3)$
- ☐ $SU_{15} = SU_{16} = SU_{11}/\text{sqrt}(3)$
- ☐ $SU_{21} < SU_{11}$
- ☐ $SU_{22} = SU_{23} = (SU_{21}/E_a) * E_t$
- ☐ $SU_{24} = SU_{22}/\text{sqrt}(3)$
- ☐ $SU_{25} = SU_{26} = SU_{21}/\text{sqrt}(3)$
- ☐ $X_{11}=SU_{11}, X_{22}=SU_{12}, X_{33}=SU_{13}, X_{23}=SU_{14}, X_{12}=SU_{15}$
 $X_{13}=SU_{16}$

Matrix properties:

- ☐ $Su = E_m * Epsm_1$ (failure is based on strain and is assumed linear until failure)

Intellectual Debt** : *answers first, explanations later*

- Traditional debt shifts control: from borrower to lender, and from future to past, as later decisions are constrained by earlier bargains.
- **Answers without theory — intellectual debt — also will shift control in subtle ways.**

- Implicit Assumption: Von Mises ($\text{sqrt}(3)$); elastic up to failure — **enable reduction from 26 to 19 input features**
- Meta data required for ML model to identify Intellectual Debt (Why need robust Information Management System)
- ML data schema for data & models established --- see SciTech 23, National Harbor, MD, Jan 23-27, 2023

Specific Laminates Used For Training

- Total of 5000 laminates which are divided 80/10/10 into training, validation, testing i.e., training dataset has 4000 laminates with 10 points each for fatigue curve
- In this set, each fatigue life curve spans the life from 1E3 cycles to 1E9 cycles
- Mix of laminates with **random angles** (40%) and **specific custom angles** (60%)
- Custom angle set consisted of
 - $[0]_4$, $[90]_4$, $[0/90]_2$, $[\pm 45]_2$, $[0/\pm 45/90]$
 - $[\pm 15]_2$, $[\pm 30]_2$, $[\pm 60]_2$, $[\pm 75]_2$
 - $[\pm 15/\pm 30]$, $[\pm 15/\pm 45]$, $[\pm 15/\pm 60]$, $[\pm 15 / \pm 75]$
 - $[\pm 30/\pm 45]$, $[\pm 30/\pm 60]$, $[\pm 30/\pm 75]$
 - $[\pm 45 / \pm 90]$

ANN Architecture



MLP

- For stiffness $[A, B, D]$ matrices, a multi-layer Keras-based ANN was trained 3 layers; 40 neurons/layer]. Mean square error (MSE) for both training and validation was ~ 0.002 .
- For fatigue $[S-N]$ predictions, a neural net ensemble with 30 sub-models were trained and combined using a linear-fit. Each individual neural net (sub-model) consisted of 6 hidden layers with 48 neurons in each layer.
- The output from these 30 sub-models was then combined using a simple linear stacked model.

RNN

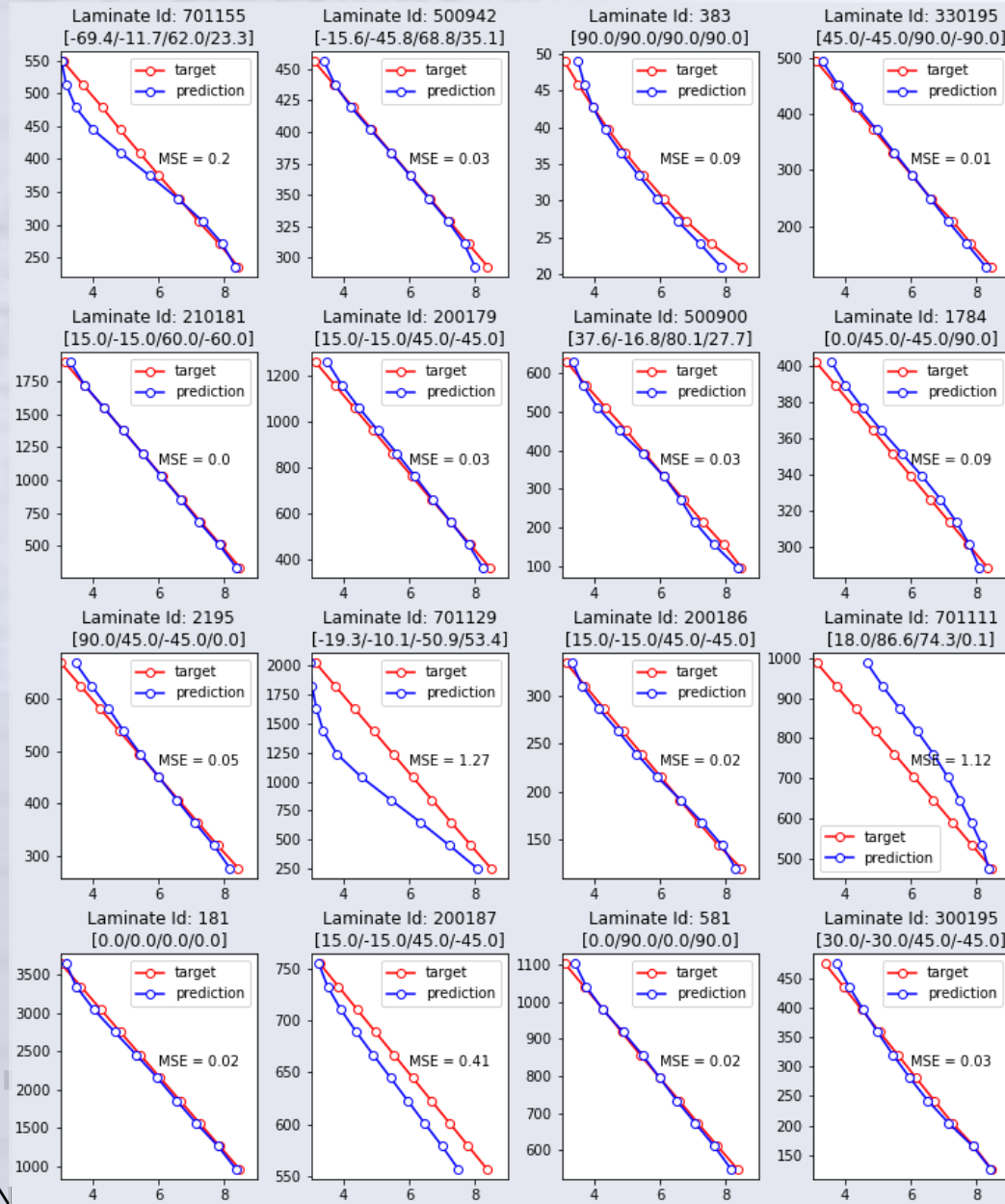
- For fatigue $[S-N]$ predictions, the neural net consisted of one dense and one LSTM (Long Short-Term Memory) layer with 20 neurons each.
- It is a relatively simple architecture.
- The model and the data do not fit the traditional definition of RNN. However, given a specific laminate (volume fraction, ply angles, constituent material properties) MAC/GMC predicts the fatigue life (number of cycles to failure) for a given fatigue load, thus providing a series of implicitly related points (S vs. N) for a given laminate.

Results: Initial Stiffness Prediction of Random Laminate

Item	Target (MAC/GMC)	Prediction	% Error
A11	112106	111403	0.6
A12	30077	30491	1.4
A13	-29619	-28294	4.5
A22	82447	80919	1.9
A23	-4348	-4838	11.3
A33	30532	29936	1.9
D11	5978	6073	1.6
D12	2473	2509	1.4
D13	-1514	-1468	3.0
D22	10299	10029	2.6
D23	1832	1810	1.2
D33	2511	2465	1.8

- Laminate [71.1/-21.9/-53.2/-9.3/]s;
- Fiber vol. fraction = 0.69
- Initial [A] and [D]
 - **Trace** of [A] very accurate
 - **Off diagonals** have larger errors but values themselves are smaller (e.g., <0.01-0.1 of A11; less significant) compared to diagonal terms
 - All terms in [D] < 3% error
- No learning curve relative to running MAC/GMC
 - Computational speed-up relative to physics-based model is minimal
 - But overall user effort greatly reduced!!

MLP Validation Results: S-N Curves [Mixed Angles]



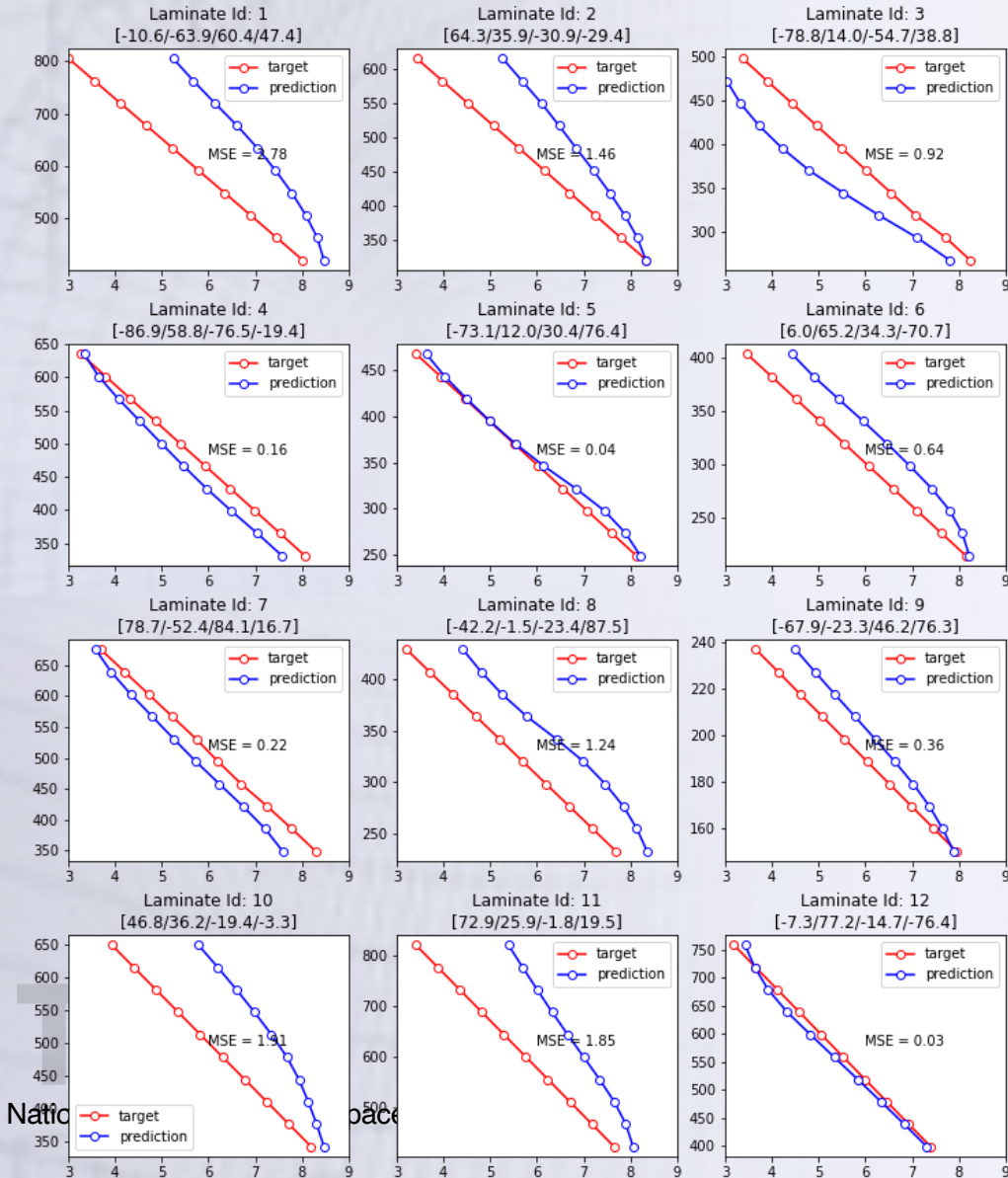
- Fatigue predictions are a **good estimate** of the actual lives (computed from the physical model)
 - ❑ Probability that a prediction is within $\pm 10\%$ $\log(N) = 0.85$.
--- this is equivalent to approx. $\pm 300\%$
 - ❑ Probability that a prediction is within $\pm 20\%$ $\log(N) = 0.94$
 - ❑ Probability that a prediction is within $\pm 30\%$ $\log(N) = 0.97$
- Predictions are made at a fraction of cost (CPU time) compared to the actual physics-based model
 - ANN = **0.01 to 1.27 MSE** for the sixteen shown
- Accuracy is consistent with what is obtained by measurements, but ANN predicts the results almost instantaneously

MPL Validation on Consistent Set of Laminates



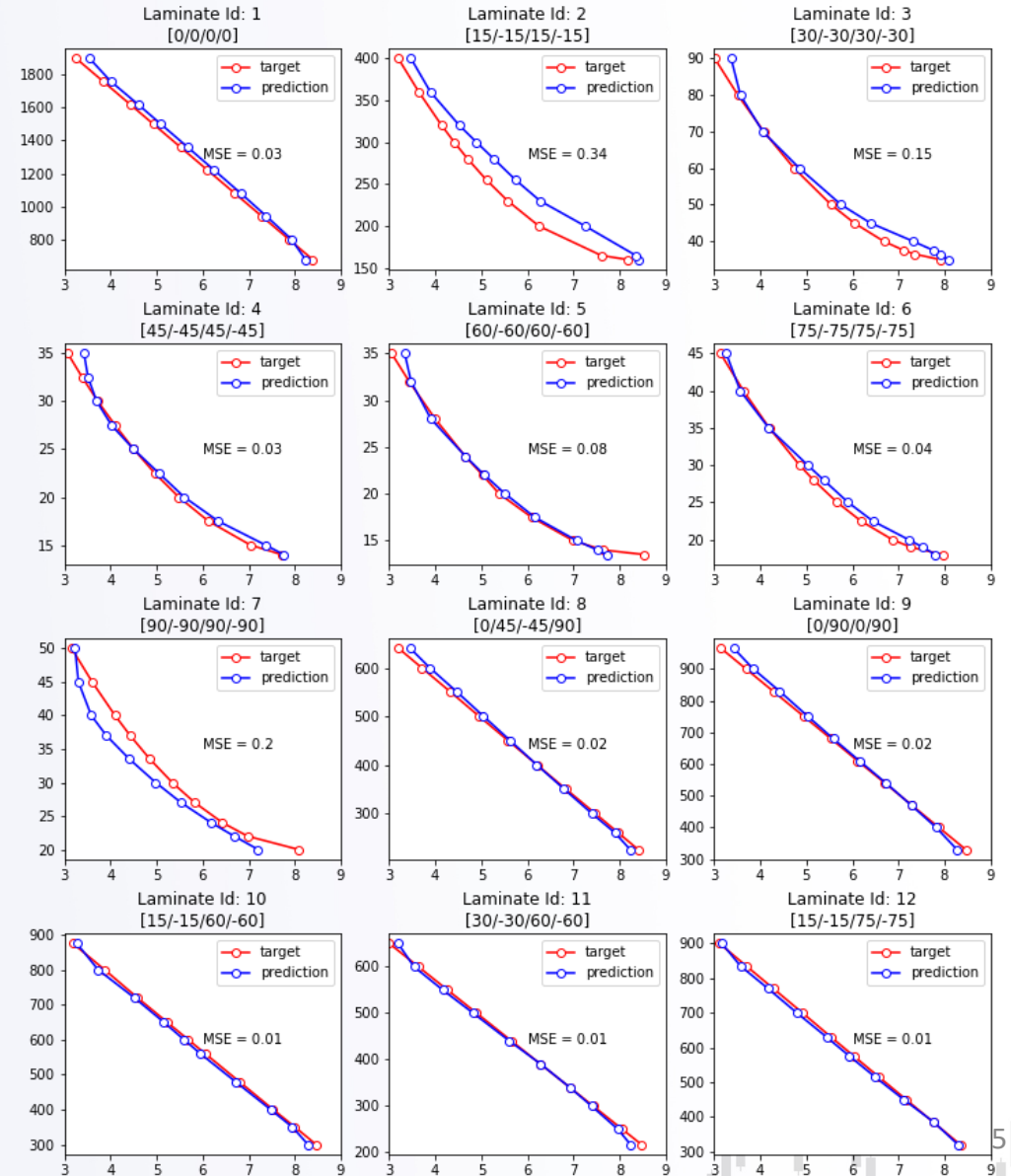
ANN = 0.03 to 2.78

Random Angles (Poor)

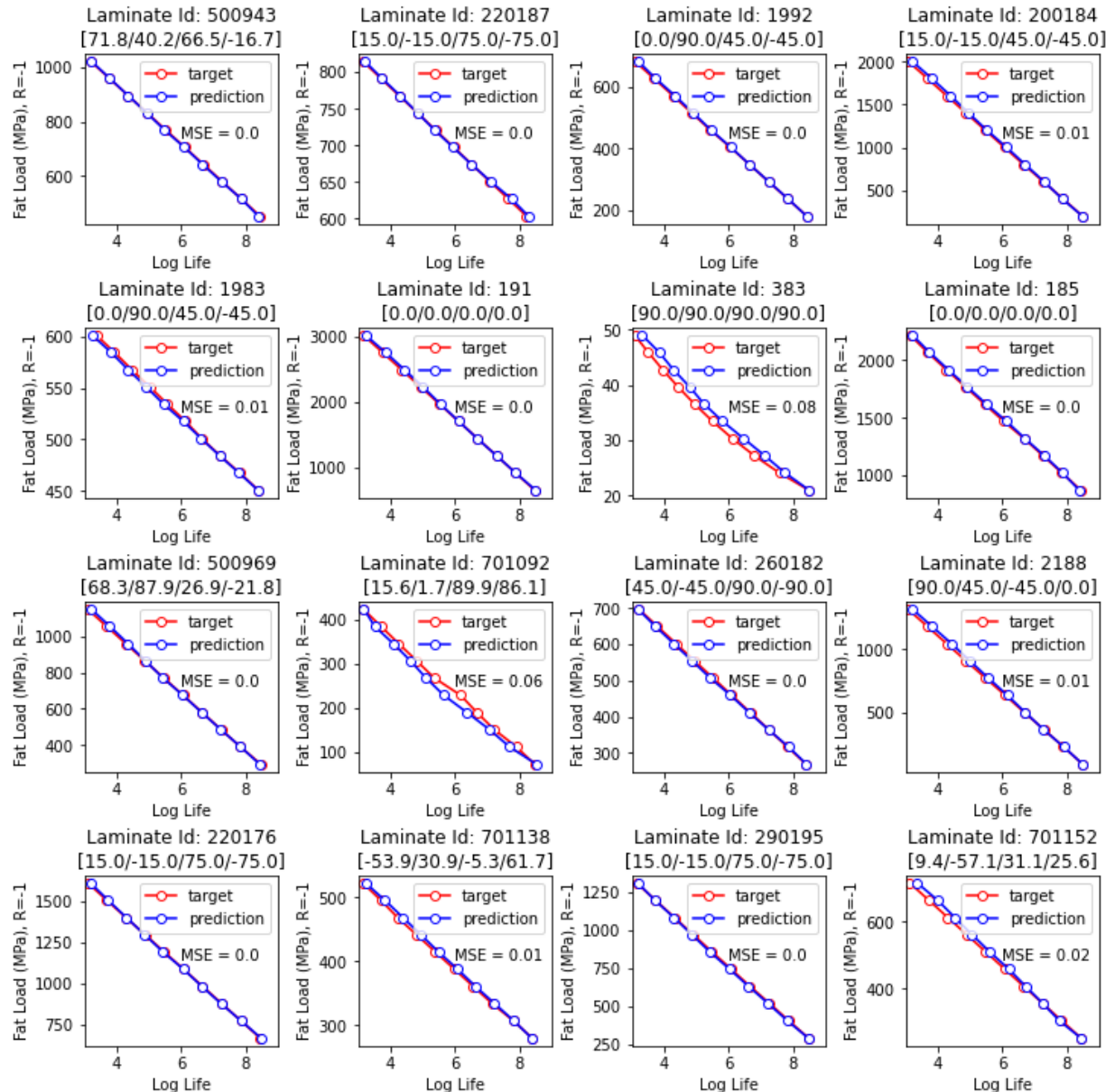


ANN = 0.01 to 0.34

Custom Angles (Excellent)



RNN Validation Results: S-N Curves[Mixed Angles]



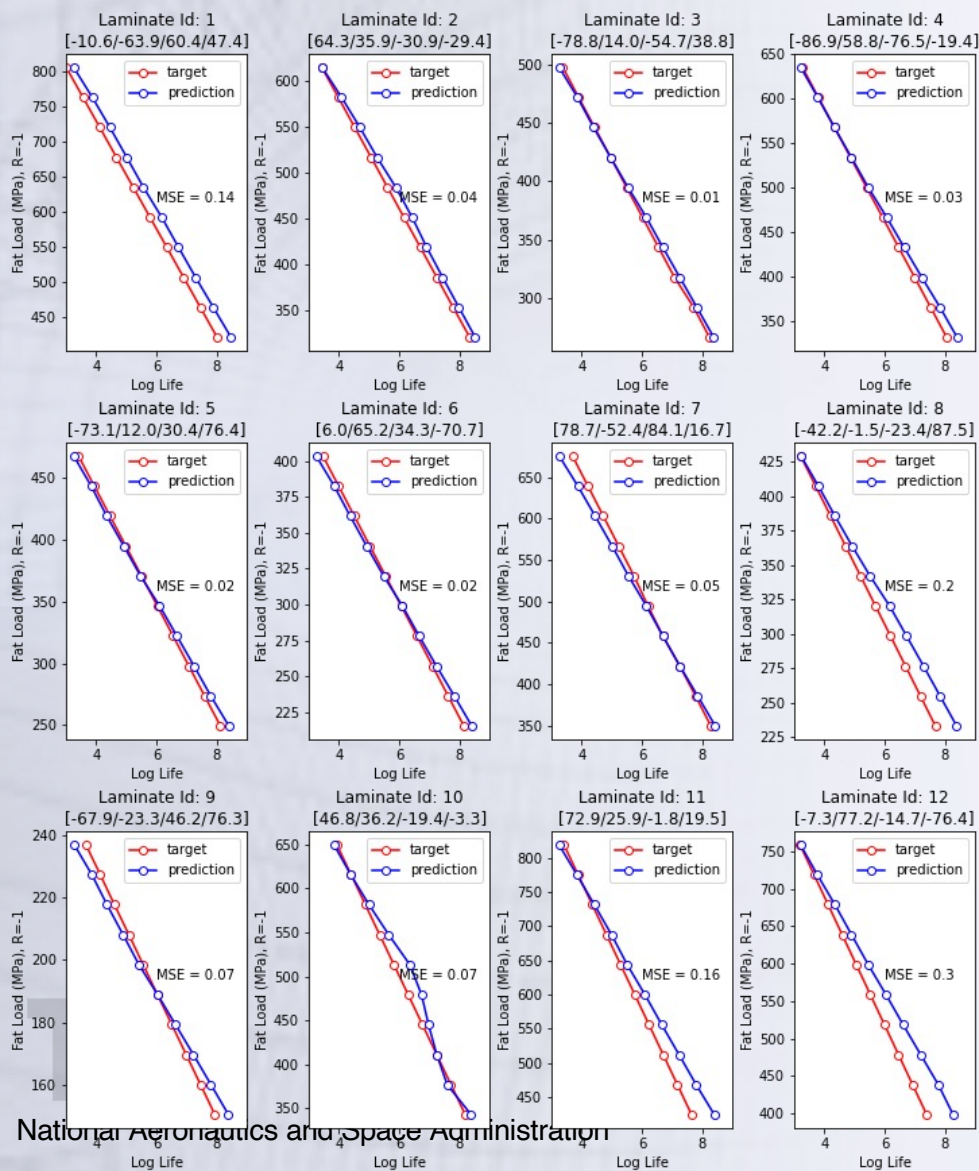
- Fatigue predictions are a **very good** estimate of the actual lives (computed from the physical model)
 - ❑ Probability that a prediction is within $\pm 5\%$ = 0.92 --
- this is equivalent to approx. $\pm 200\%$
 - ❑ Probability that a prediction is within $\pm 10\%$ = 0.98
 - ❑ Probability that a prediction is within $\pm 20\%$ = 1.0
 - ❑ Probability that a prediction is within $\pm 30\%$ = 1.0
- Predictions are made at a fraction of cost (CPU time) compared to the actual physics-based model
 - RNN = **0.0 to 0.08 MSE** for the sixteen shown
- Accuracy is consistent with what is obtained by measurements, but ANN predict the results almost instantaneously

RNN Validation on Consistent Set of Laminates



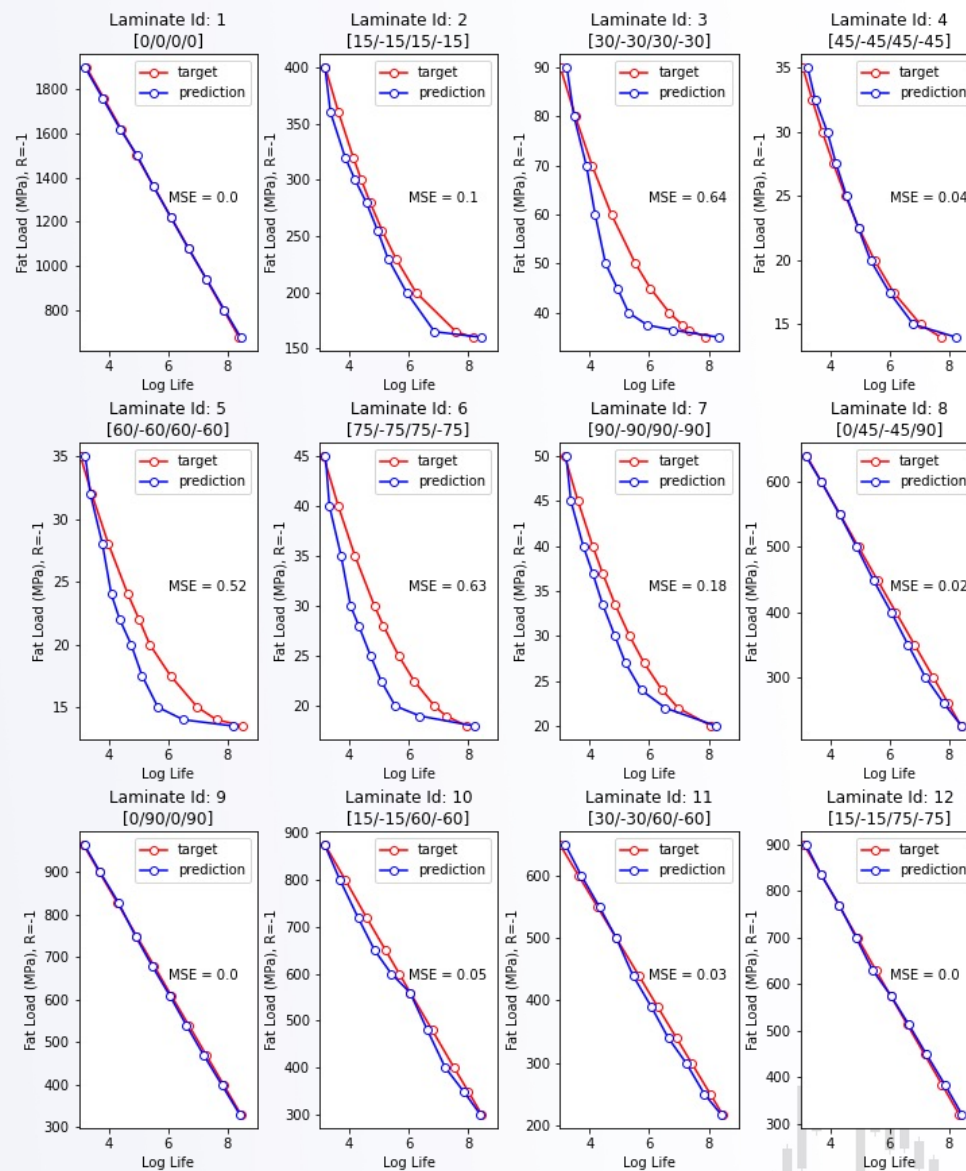
ANN = 0.01 to 0.3

Random Angles (Excellent)



ANN = 0.0 to 0.64

Custom Angles (Very Good)



MSE Results Given Different Mixture Of Datasets

μ is Mean and σ is Standard Deviation

	Random (40%) & Custom (60%)		Random (60%) & Custom (40%)	
	MLP	RNN	MLP	RNN
Total	$\mu = 0.26$ $\sigma = 0.93$	$\mu = 0.04$ $\sigma = 0.22$	$\mu = 0.07$ $\sigma = 0.18$	$\mu = 0.003$ $\sigma = 0.12$
Random	$\mu = 0.59$ $\sigma = 1.48$	$\mu = 0.05$ $\sigma = 0.3$	$\mu = 0.03$ $\sigma = 0.17$	$\mu = 0.001$ $\sigma = 0.04$
Custom	$\mu = 0.08$ $\sigma = 0.19$	$\mu = 0.03$ $\sigma = 0.13$	$\mu = 0.12$ $\sigma = 0.19$	$\mu = 0.04$ $\sigma = 0.18$

- Clearly RNN is overall better performing than MLP
- Impact on RNN and MLP are expected when you change the percentage of laminate types in training dataset
 - Including more random cases improves prediction for random and decrease performance of Custom

* Random cases: Angles can be non integer and nontypical
 Custom cases: Have commonly used angles (15, 30, 45, 60, 90, etc.)

Graphical User Interface (GUI)



Define Input Parameters

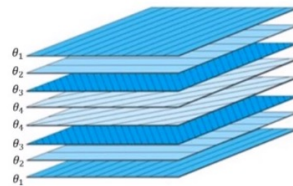
Inputs Analysis

Fatigue Life Estimator v1.0

Run #2

Laminate Definition

	Value	Lower Bound	Upper Bound
θ_1 (°)	62	-90.0	90.0
θ_2 (°)	-80	-90.0	90.0
θ_3 (°)	75	-90.0	90.0
θ_4 (°)	-5	-90.0	90.0
V_f	0.64	0.4	0.7



Fiber Properties

	Value	Lower Bound	Upper Bound
E_L (GPa)	643	70.0	700.0
E_T (GPa)	116	70.0	200.0
ν_L	0.33	0.2	0.4
ν_T	0.22	0.2	0.4
G_L (GPa)	133	25.0	200.0
S_{U11} (MPa)	9000	2500.0	9500.0
S_{U22} (MPa)	1500	300.0	2500.0

Matrix Properties

	Value	Lower Bound	Upper Bound
E (GPa)	3.4	2.5	4.5
ν	0.21	0.2	0.45
ϵ_1	0.026	0.015	0.03
ϵ_2	0.025	0.015	0.03
β	5.6	4.0	12.0
SFL (MPa)	30	15.0	30.0
XML (MPa)	110	80.0	160.0



Exit

ANN Results
Est of A, B, D and S-N Curve based on Input

Inputs Analysis

Fatigue Life Estimator v1.0



ABD Matrix

Run #1 Run #2

A_{11} (N/mm)	72614.66	119006.28
A_{12} (N/mm)	22799.16	32120.79
A_{13} (N/mm)	787.1	3864.44
A_{22} (N/mm)	72491.18	254740.18
A_{23} (N/mm)	692.59	36356.0
A_{33} (N/mm)	23904.56	35012.41
B_{11} (N/mm ²)	0.0	0.0
B_{12} (N/mm ²)	0.0	0.0
B_{13} (N/mm ²)	0.0	0.0
B_{22} (N/mm ²)	0.0	0.0
B_{23} (N/mm ²)	0.0	0.0
B_{33} (N/mm ²)	0.0	0.0
D_{11} (N/mm ³)	9968.03	2794.87
D_{12} (N/mm ³)	1606.27	4102.64
D_{13} (N/mm ³)	617.1	1656.23
D_{22} (N/mm ³)	2712.82	25486.94
D_{23} (N/mm ³)	560.95	5411.64
D_{33} (N/mm ³)	1700.24	4345.13

Nx (MPa)

574.7

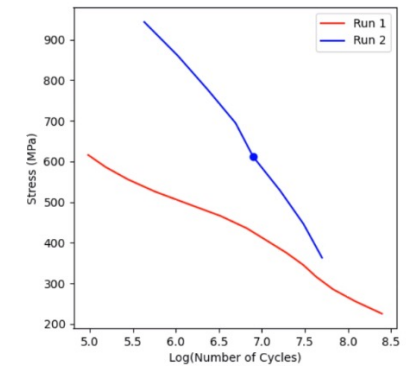
Number of Cycles
7909161

Execute

Clear

Save to Excel

Save to Image



Plot S vs log(n)

Plot S vs n

Exit

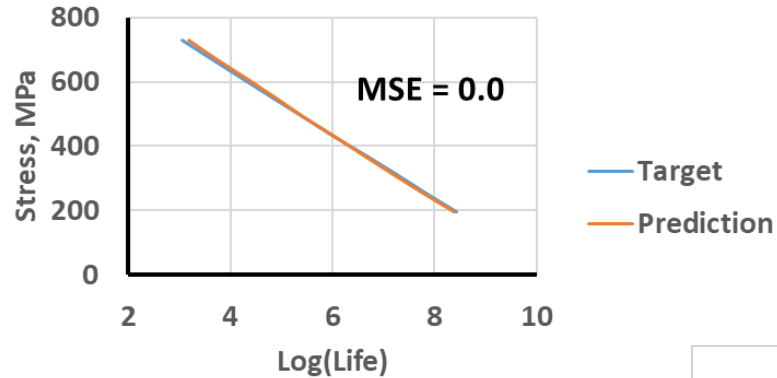
- Enables multiple cases to be run
- Input/output per case can be saved to Excel

****Ultimate democratization toolset** – micromechanics-based fatigue life analysis with no training needed

Graphical User Interface (GUI)

Demo #1

Laminate [0/±45/90]_s; FVR = 0.55



Fiber Properties

E_L , GPa	300
E_T , GPa	70
ν_L	0.2
ν_T	0.22
G_L , GPa	40
SU11, MPa	4000
SU21, MPa	1000

Matrix Properties

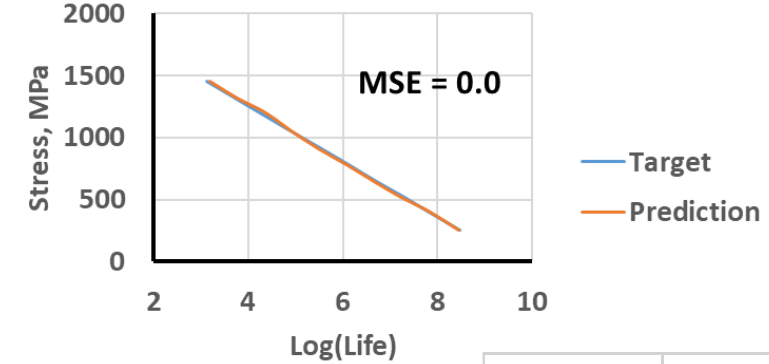
E_r , GPa	4
ν	0.35
ϵ_1	0.025
ϵ_2	0.027
β	7
SFL, MPa	25
XML, MPa	110

< 2.5% error

Initial Stiffness	Target	Pred
A11	7.10E+04	7.30E+04
A12	2.30E+04	2.30E+04
A13	8.70E-10	7.90E+02
A22	7.10E+04	7.20E+04
A23	-8.00E-08	6.90E+02
A33	2.40E+04	2.40E+04
D11	9.80E+03	9.97E+03
D12	1.60E+03	1.61E+03
D13	6.00E+02	6.17E+02
D22	2.60E+03	2.71E+03
D23	6.00E+02	5.61E+02
D33	1.66E+03	1.70E+03

Demo #2

Laminate [62/-80/75/-5]_s; FVR = 0.64



Fiber Properties

E_L , GPa	643
E_T , GPa	116
ν_L	0.33
ν_T	0.22
G_L , GPa	133
SU11, MPa	9000
SU21, MPa	1500

Matrix Properties

E_r , GPa	3.4
ν	0.21
ϵ_1	0.026
ϵ_2	0.025
β	5.6
SFL, MPa	30
XML, MPa	110

< 0% error

Initial Stiffness	Target	Pred
A11	1.20E+05	1.20E+05
A12	3.10E+04	3.20E+04
A13	1.90E+03	3.80E+03
A22	2.60E+05	2.60E+05
A23	3.90E+04	3.60E+04
A33	3.30E+04	3.50E+04
D11	2.80E+03	2.80E+03
D12	4.20E+03	4.10E+03
D13	1.70E+03	1.70E+03
D22	2.50E+04	2.50E+04
D23	5.40E+03	5.40E+03
D33	4.30E+03	4.30E+03

Summary

- Developed **deep learning ANN multiscale surrogate** models to predict micromechanics-based laminate stiffnesses (ABD) and fatigue life of eight-ply, symmetric, regular, PMC laminates.
- ANN surrogate models requires a large amount of data for training to achieve a certain level of accuracy.
 - Generally, the amount of experimentally measured data available is limited because of the costs involved.
 - Consequently, synthetic/virtual data needs to be generated using some physics-based model (can be computationally intensive)
 - In general, RNN models were better able to capture the form of the fatigue curves with less computational expense.
- **Advantages of ANN Life Estimator**
 - Tool can provide reasonable **estimate** of desired properties and life of PMC composite **8-ply symmetric laminates**.
 - With significantly less user effort / learning
 - At a fraction ($\sim 10^{-4}$) of the cost (CPU time, User Input) compared to the actual physics-based model
 - Fatigue GUI - Ultimate democratization toolset (no training needed)
 - Viable industrial tool for estimating composite life
- **Disadvantages of ANN Life Estimator**
 - Like all macro composite models – need to be recharacterized/trained for various types of laminates (if any dependent feature is not included as input, e.g., number and thickness of each ply)
 - Underlying surrogate models require a large amount of training data (albeit real or virtual)
 - Needs careful tracking of meta data associated with surrogate models (assumptions, intellectual debt)

Thanks for Your Attention



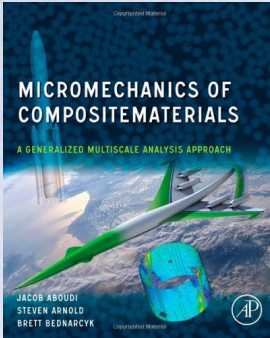
Contact: Steven.M.Arnold@nasa.gov

- Thanks to Josh Stuckner, Multiscale and Multiphysics Branch at NASA Glenn Research Center for many helpful discussions.

Key References



Aboudi, J., Arnold, S.M., and Bednarczyk, B.A. (2013)
Micromechanics of Composite Materials: A Generalized Multiscale Analysis Approach, Elsevier, Oxford, UK., pp 1-984.



Outline

- 1) Introduction
- 2) Constituent Material Modeling
- 3) Fundamentals of the Mechanics of Multiphase Materials
- 4) The Method of Cells Micromechanics
- 5) The Generalized Method of Cells Micromechanics
- 6) The High Fidelity Generalized Method of Cells Micromechanics
- 7) Multiscale Modeling of Composites
- 8) Fully Coupled Thermomechanical Analysis of Multiphase Composites
- 9) Finite Strain Micromechanical Modeling of Multiphase Composites
- 10) Micromechanical Analysis of Smart Composite Materials
- 11) Higher-Order Theory for Functionally Graded Materials
- 12) Wave Propagation in Multiphase and Porous Materials
- 13) Micromechanics Software

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“Written with both **students and practitioners** in mind and is coupled with a fully functional MATLAB code to enable solution of technologically relevant micromechanics problems. The many illustrative example problems and exercises highlight key concepts and rely heavily on the MATLAB code”



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1. Introduction
2. Lamination Theory Using Macromechanics
3. Closed Form Micromechanics
4. Failure Criteria and Margins of Safety
5. The Generalized Method of Cells (GMC) Micromechanics Theory
6. The High-Fidelity Generalized Method of Cells (HFGMC) Micromechanics Theory
7. Progressive Damage and Failure

Features:

- Thermoelastic Material Behavior
- Emphasis on Local fields via Strain and Stress Concentration Tensors;
- General **MATLAB open-source code provided**
 - four micromechanics theories MT, MOC, GMC, HFGMC
 - four failure criteria, along with consistent treatment of Margins of Safety (MoS)
 - Emphasis on PMC & CMC, order and disorder microstructures
 - <https://github.com/nasa/Practical-Micromechanics>
- Extensive Practical Examples; ~ 15 Exercises /Chapter (Solution Manual available to professors)