

# Acoustic Signature Uncertainty Quantification for the X-59

Marian Nemec

Computational Aerosciences Branch

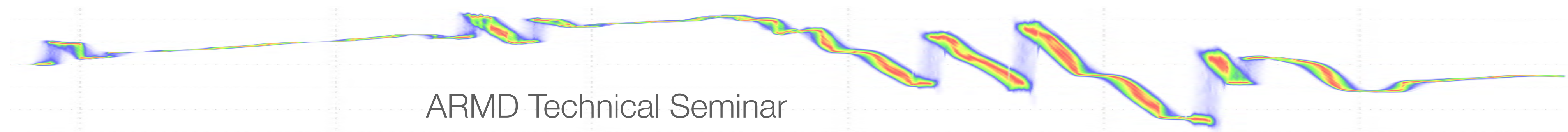
NASA Advanced Supercomputing Division, Ames Research Center

Joint work with

Michael Aftosmis, Tim Barth & Sriram Rallabhandi, NASA Ames

Garo Bedonian, RPI & Liam Smith, Georgia Tech

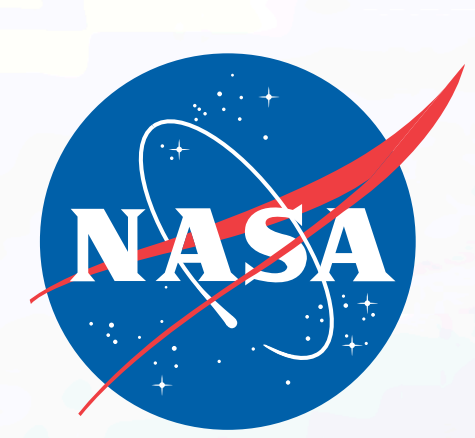
Wade Spurlock, STC



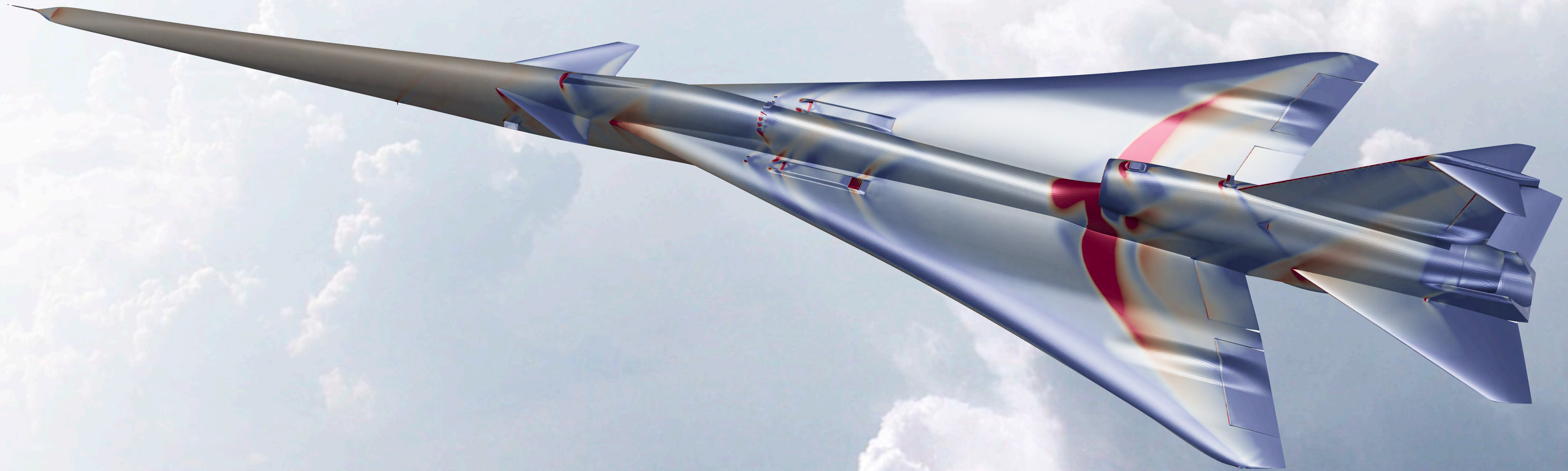
ARMD Technical Seminar

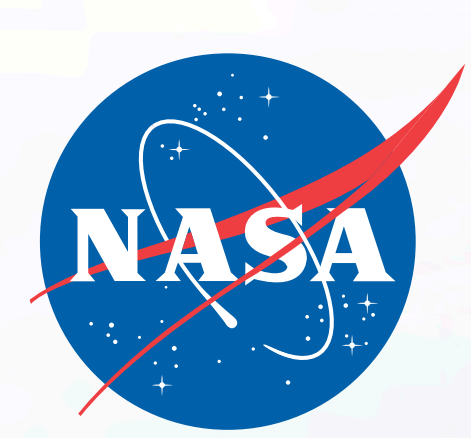
September 19, 2023





Goal is to quantify uncertainty in the level of noise reaching ground due to uncertainty in flight conditions and the atmosphere during supersonic cruise





# Outline

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1. Background & Motivation

2. Objectives

3. Methodology

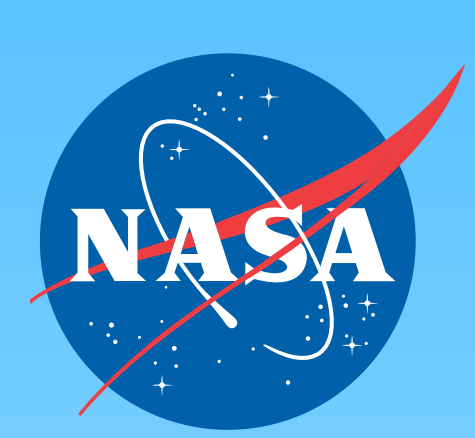
- Uncertainty quantification
- Low-boom simulations
- Atmospheric uncertainty in propagation

4. Results

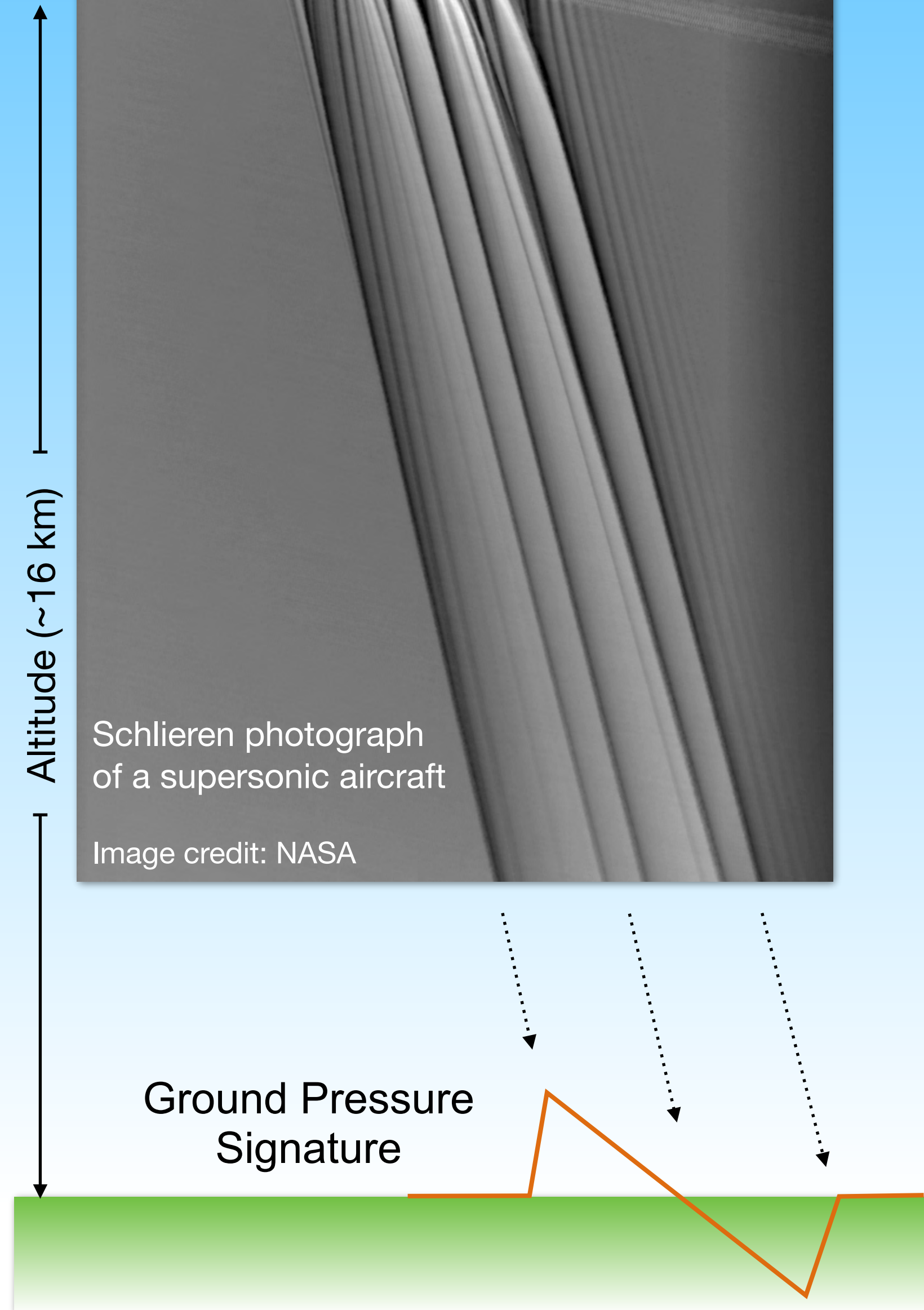
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- Uncertainty in atmospheric conditions
- Uncertainty in X-59 operating conditions and atmosphere

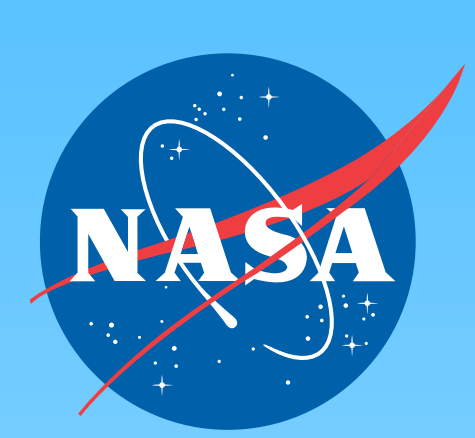
5. Summary & Outlook





# Sonic Booms

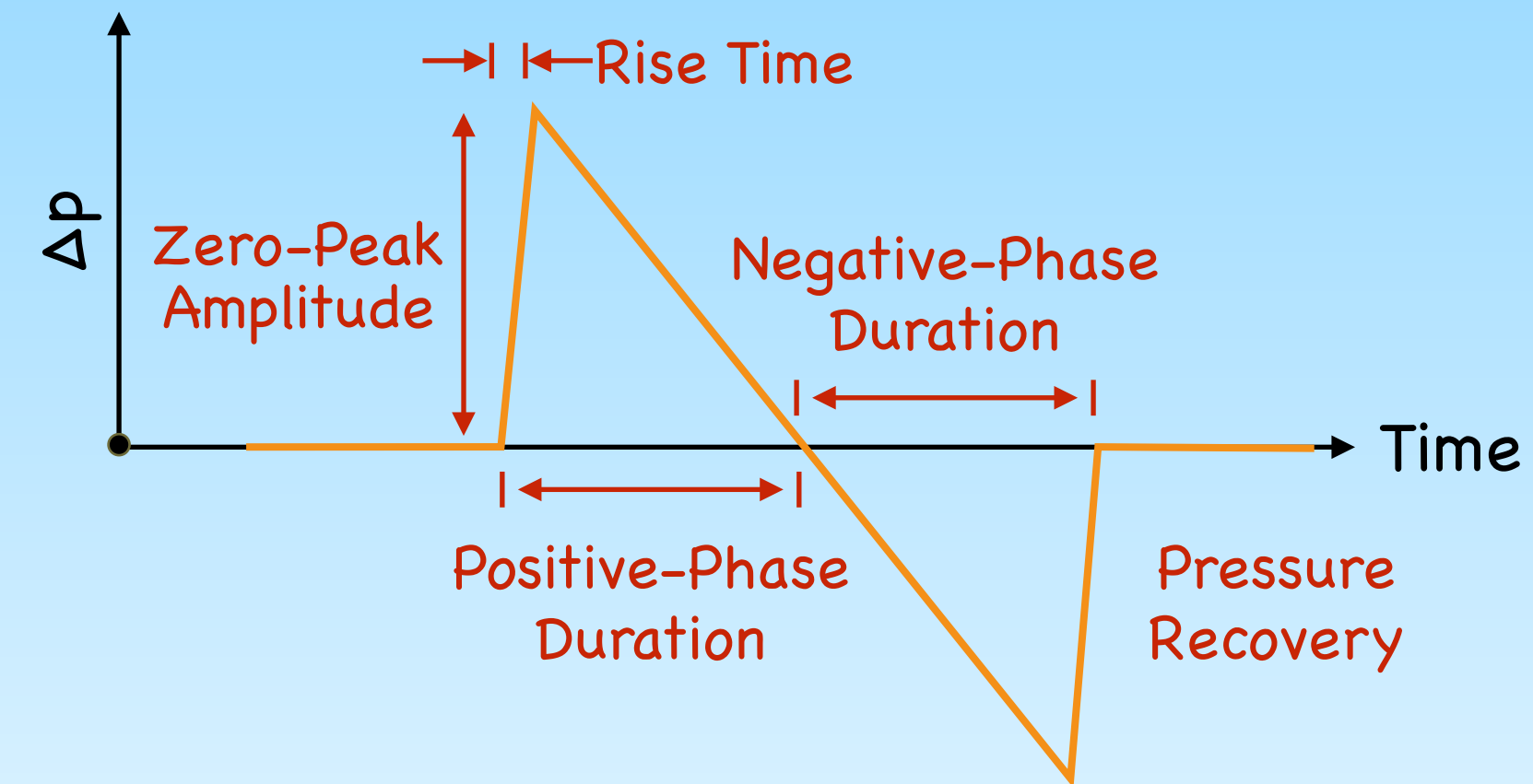
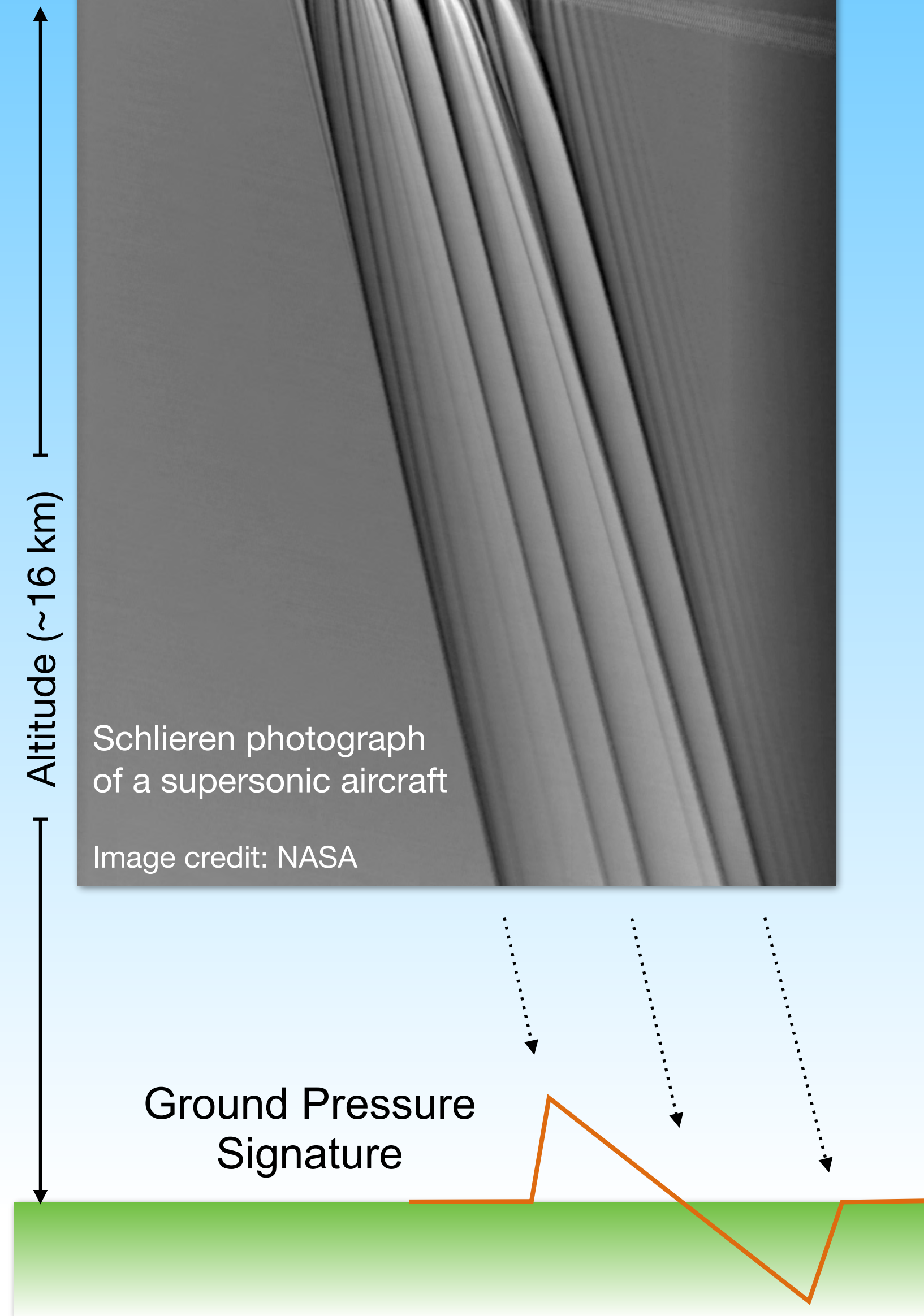


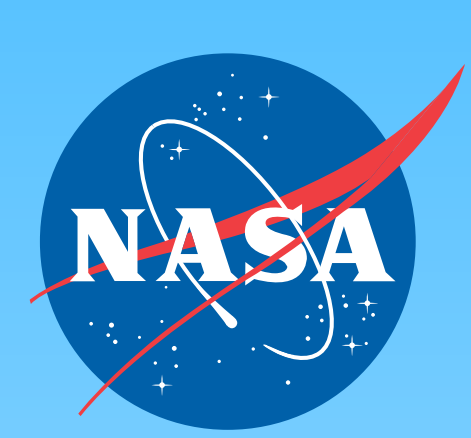


# Sonic Booms

Sound characteristics are a function of the ground pressure signature

- Classical signatures are N-waves





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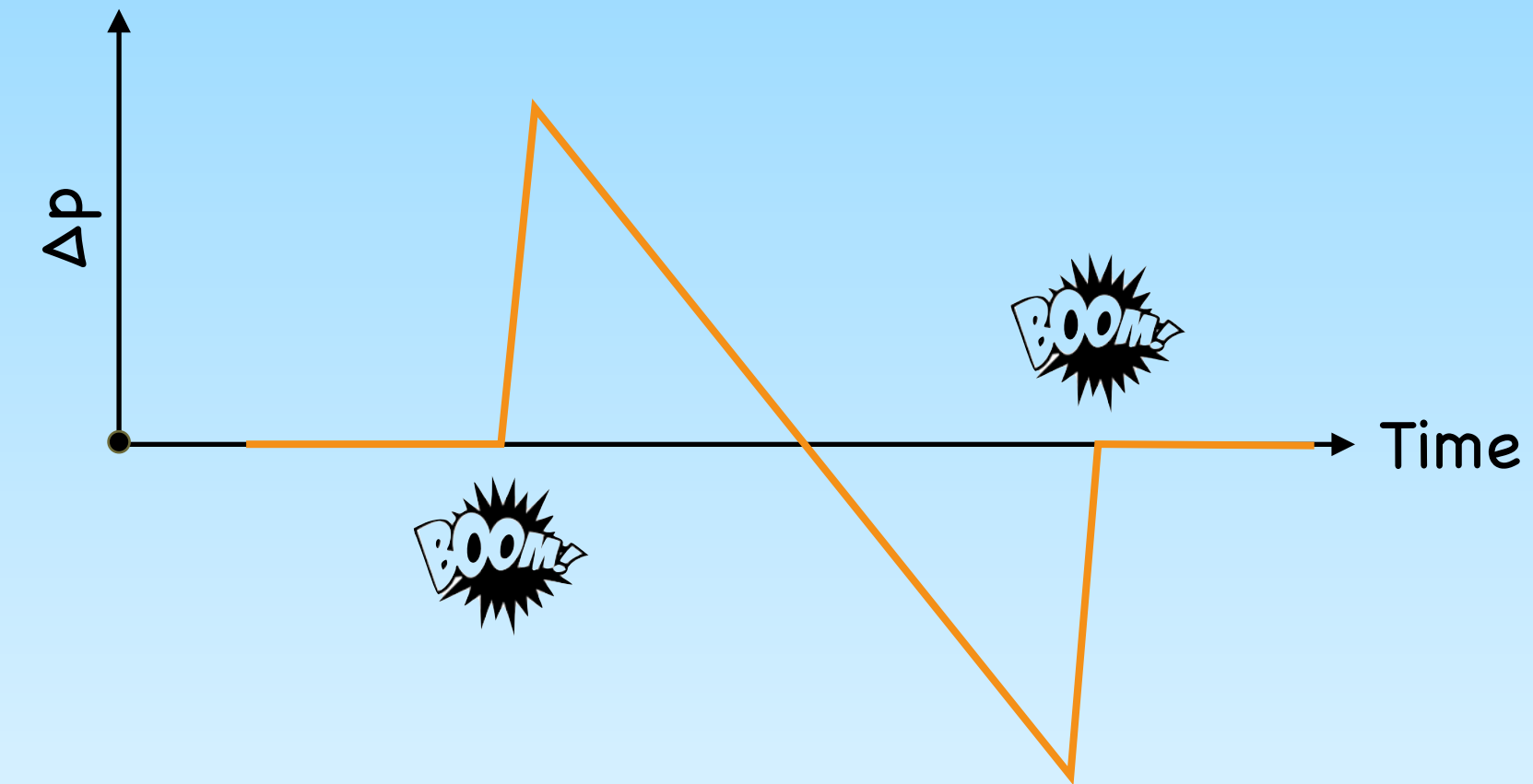
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Altitude (~16 km)

Schlieren photograph of a supersonic aircraft

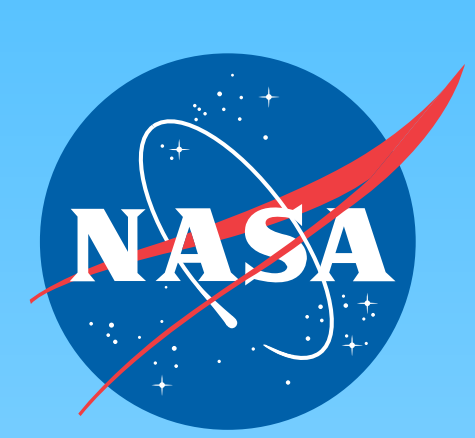
Image credit: NASA

Ground Pressure Signature



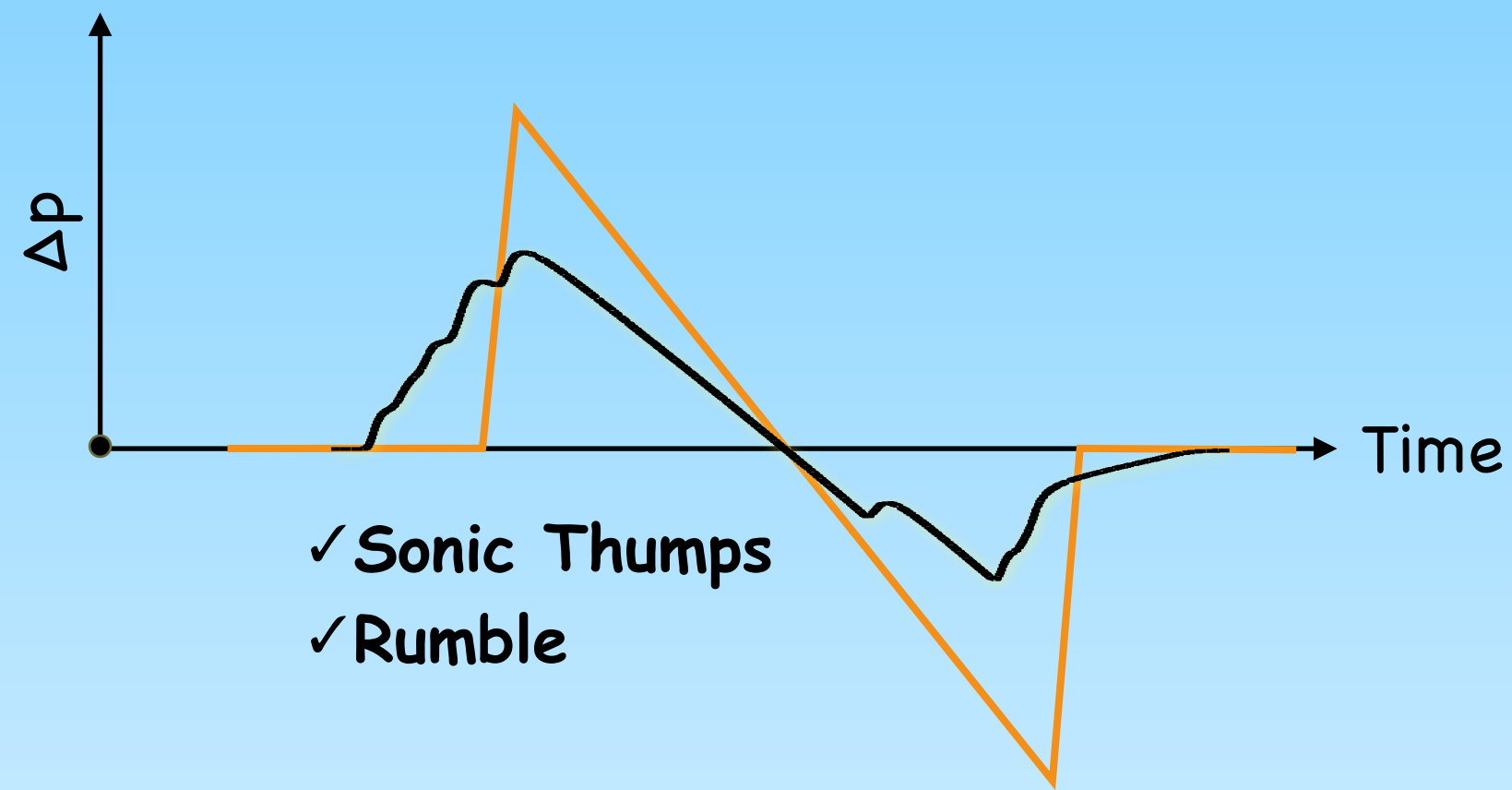
## Loudness metrics

- Perceived Loudness PL (dB)
- Weighted Sound Exposure Level: A/B/C/D/ESEL (dB)
- Indoor Sonic Boom Annoyance Predictor
  - $ISBAP = PL - 0.4201(CSEL - ASEL)$



# X-59: Low-Boom Design

- Aerodynamic shaping to eliminate sonic booms



- Strategy is to increase rise time, decrease amplitude, increase duration and smooth recovery
- Requires designing aircraft with signatures that do not coalesce into N-waves
- Extensive reliance on high-fidelity simulations based on Computational Fluid Dynamics for analysis and design
  - Shock dominated, strongly nonlinear flowfield

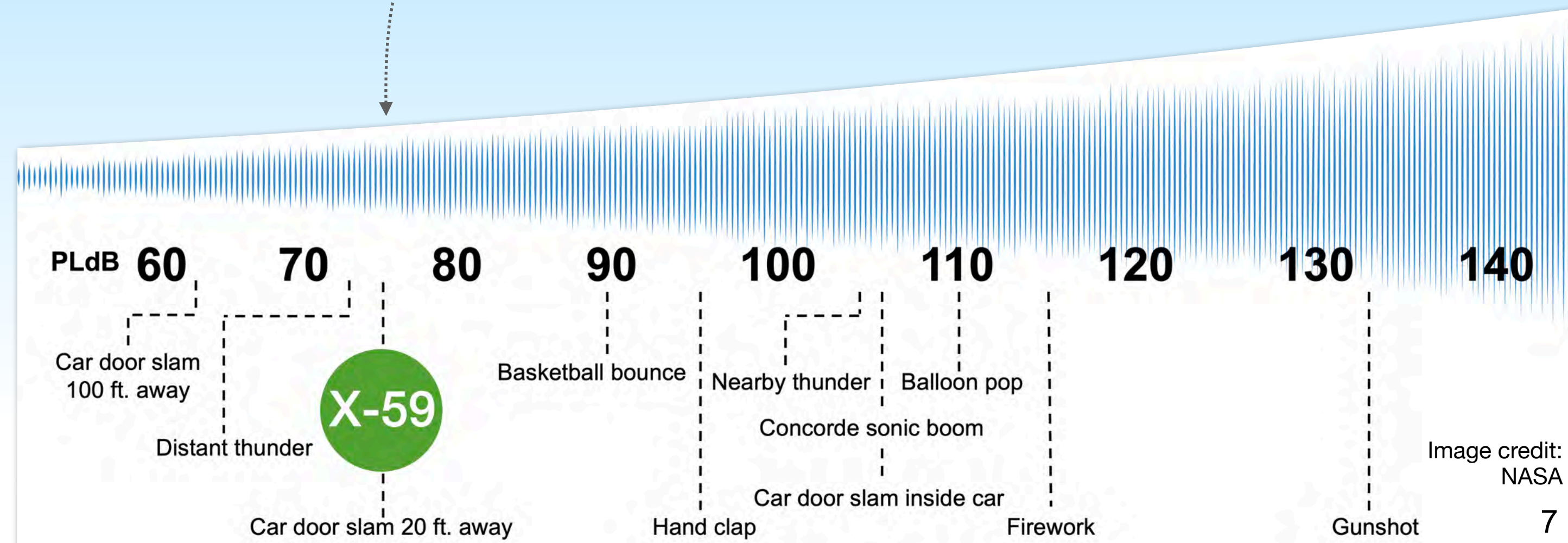
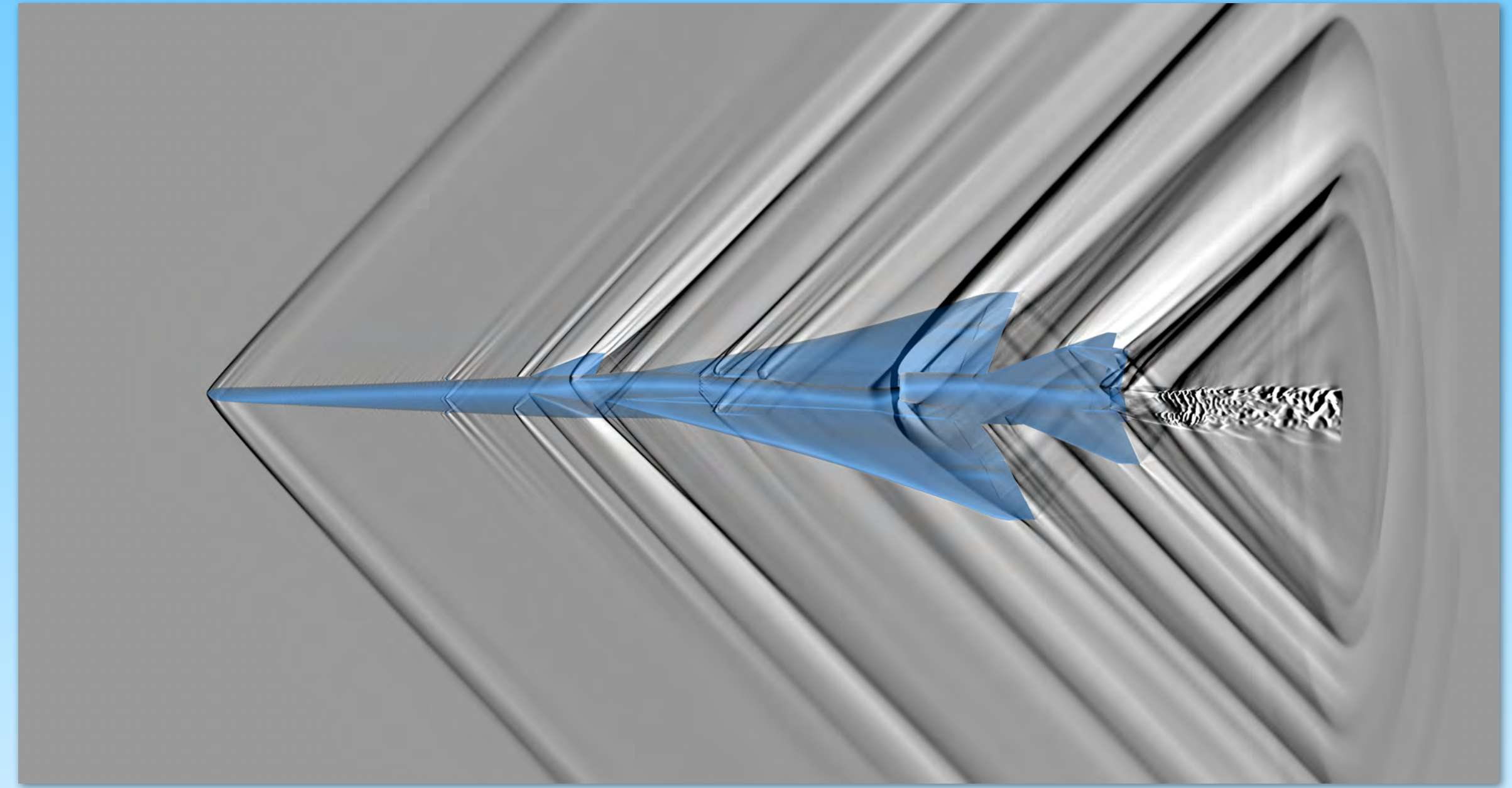
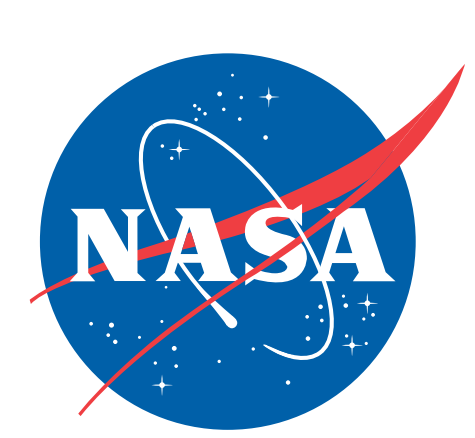
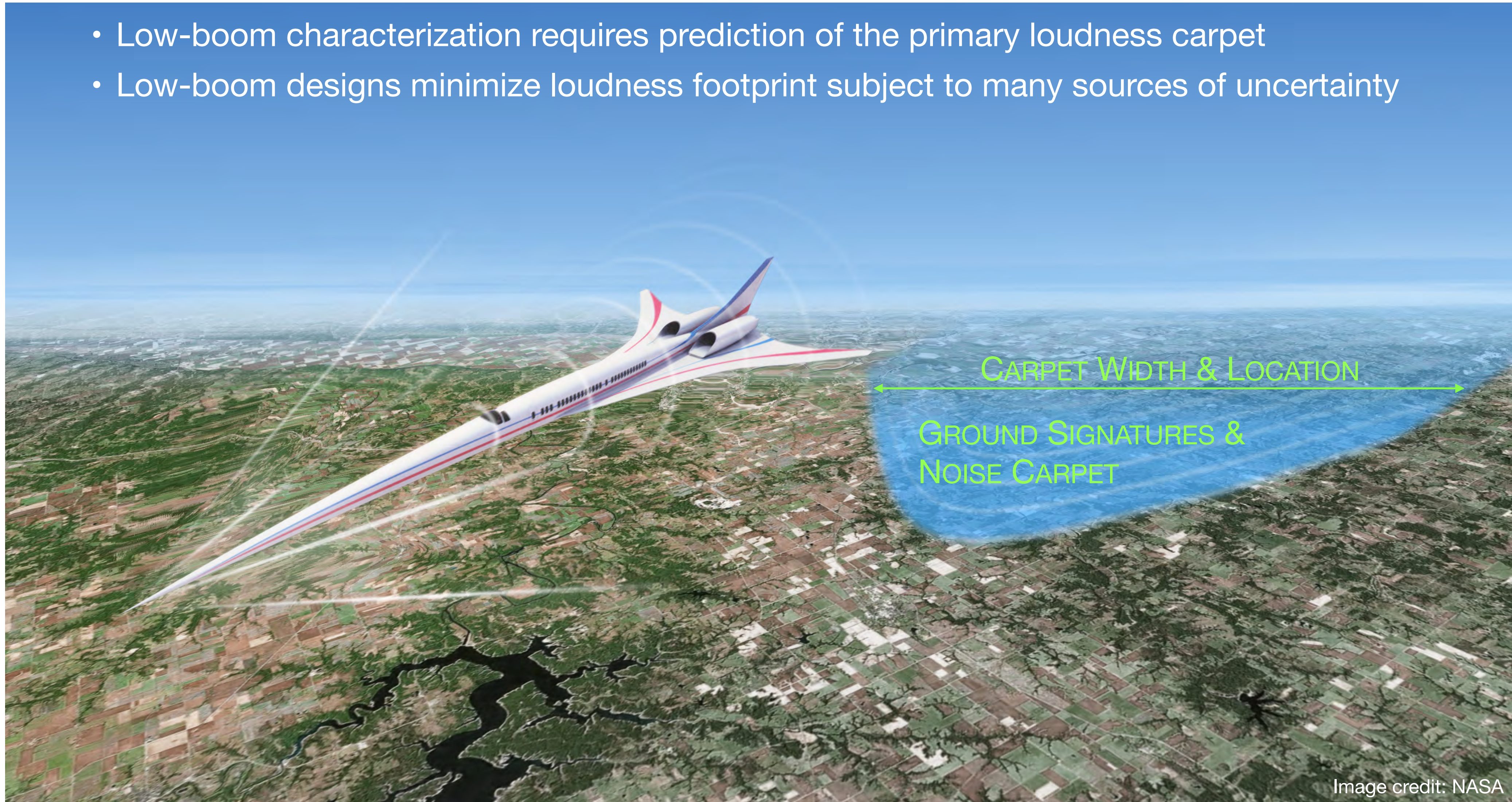


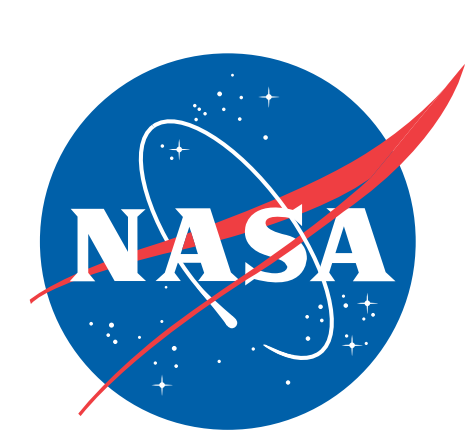
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# Predicting En Route Noise

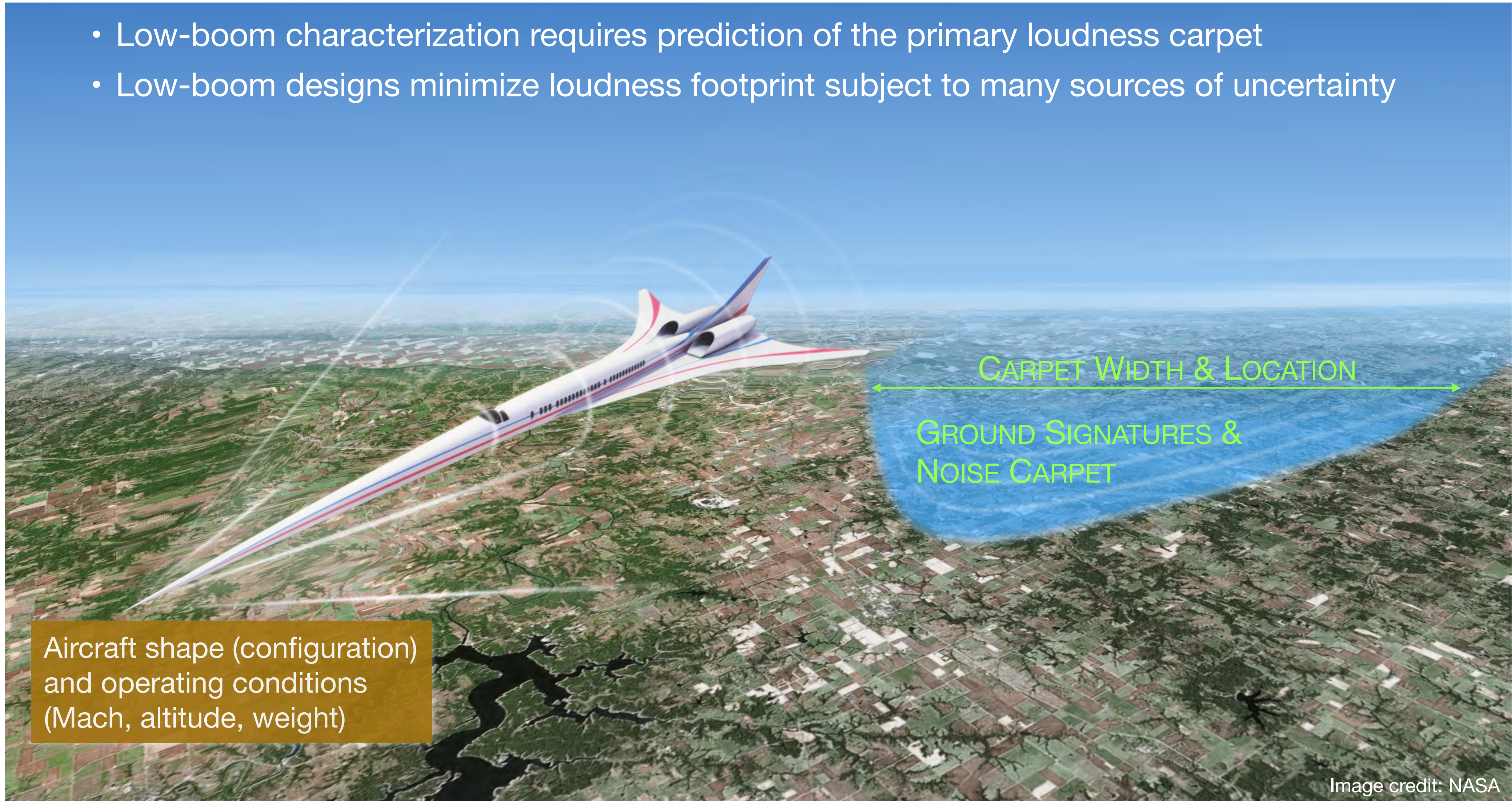
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- Low-boom designs minimize loudness footprint subject to many sources of uncertainty

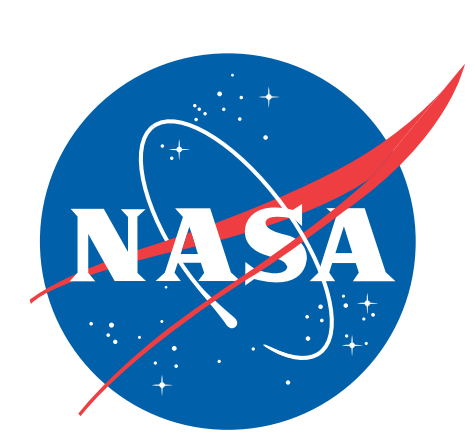




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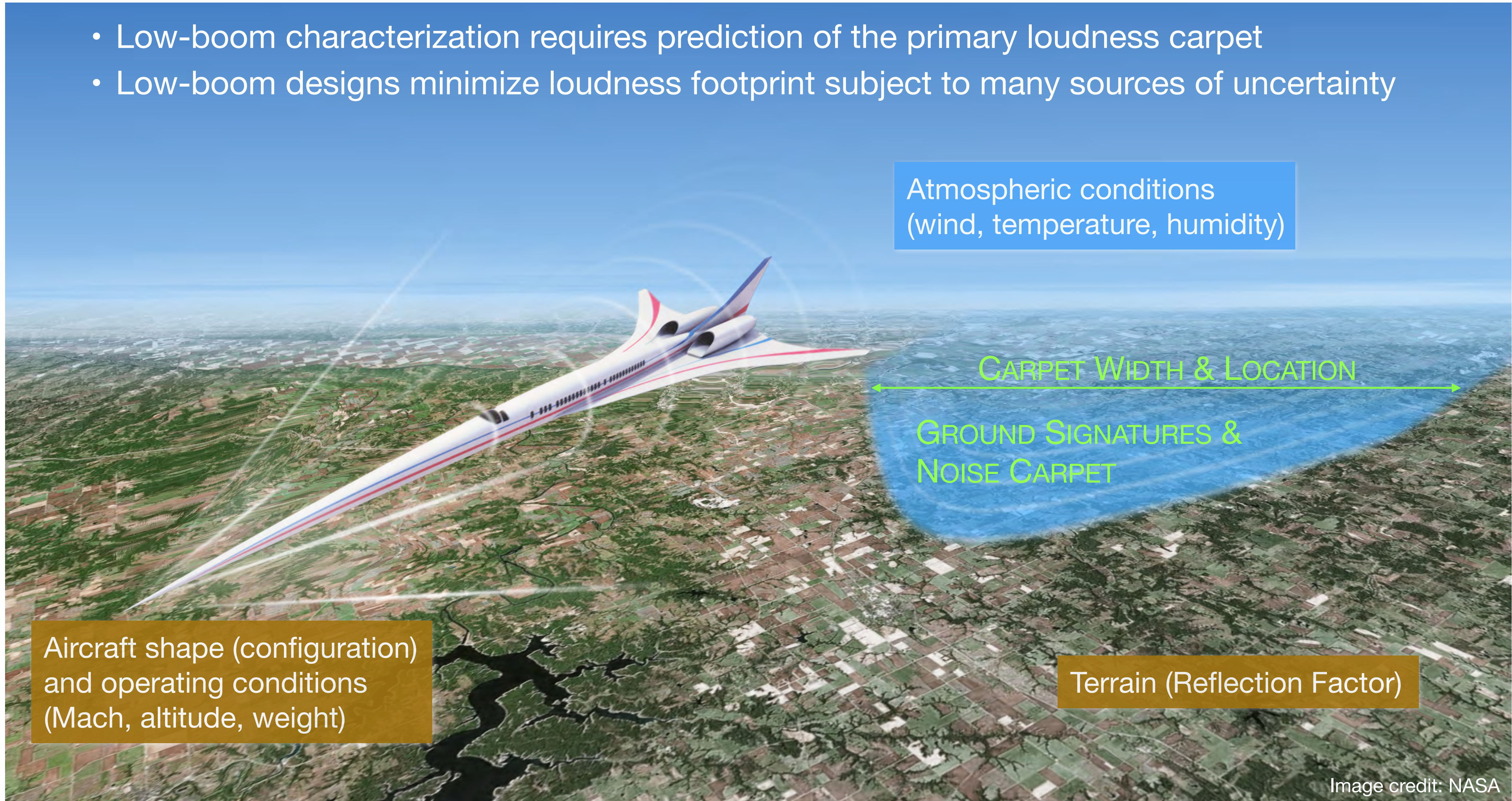
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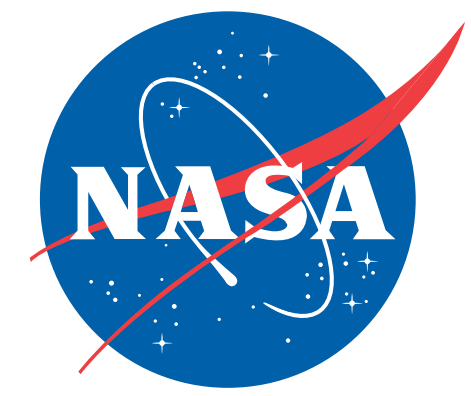




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# Objectives

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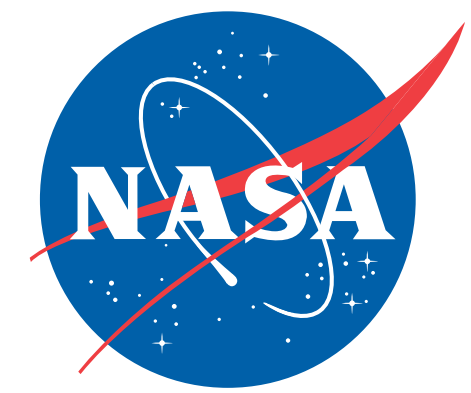
## 1. Characterizing effect of input uncertainties on low-boom outputs

- Mean, standard deviation and probability density function
  - Nearfield and ground signatures
  - Loudness metrics: Perceived Loudness, A/B/C/D/E SEL, ISBAP
  - Noise carpet and carpet width
- Input uncertainties include cruise conditions and the atmosphere

## 2. Assessing and minimizing sources of error

- Directly coupling UQ with high-fidelity simulations
- Tracking discretization error in simulations and numerical error in evaluation of statistics
- Estimating model-form error

**Toward En Route Noise Certification by Simulation-Based Analysis**



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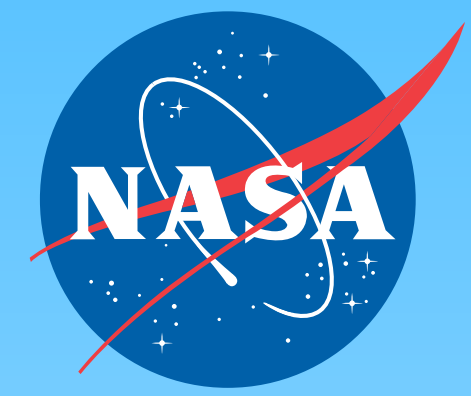
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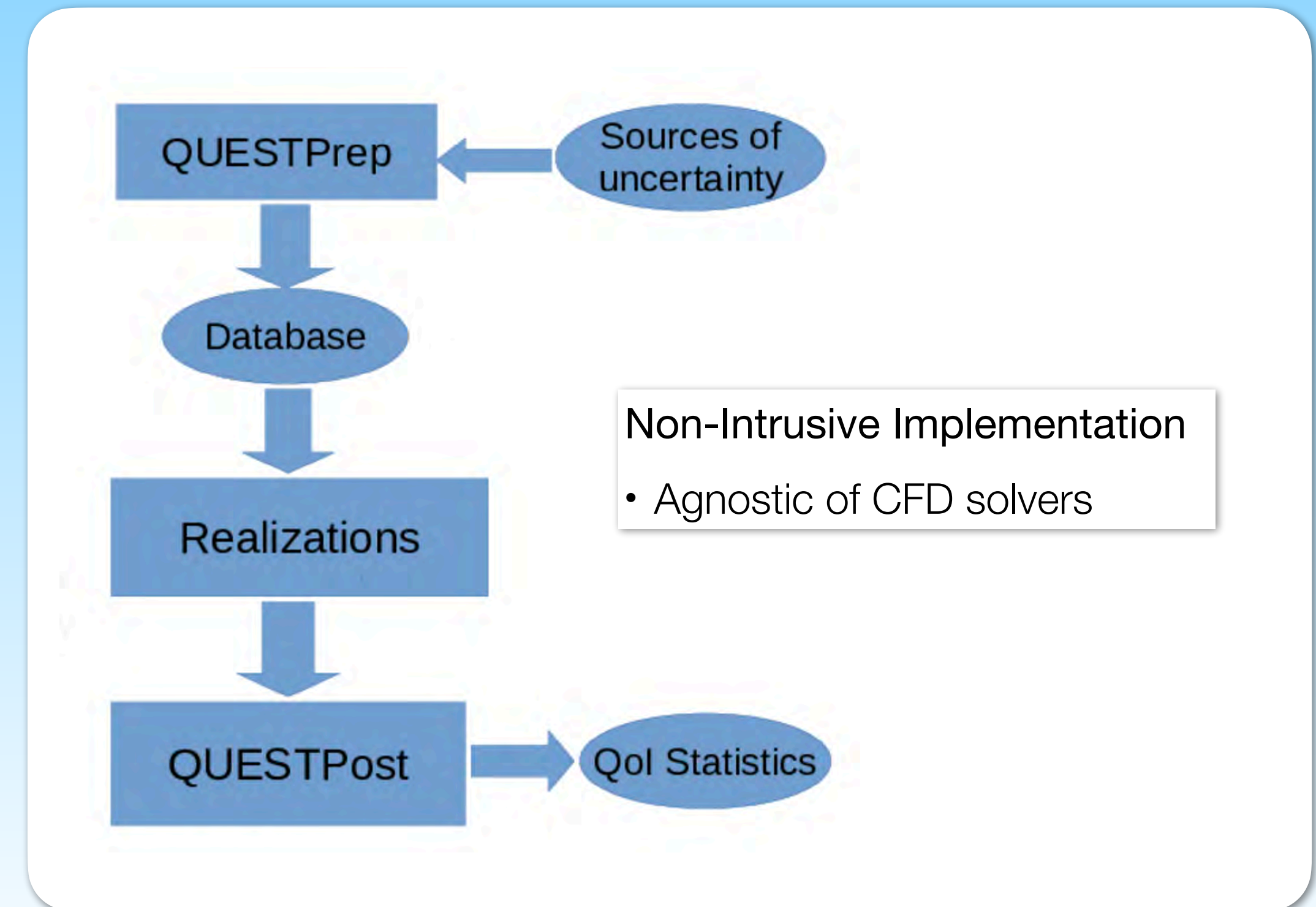
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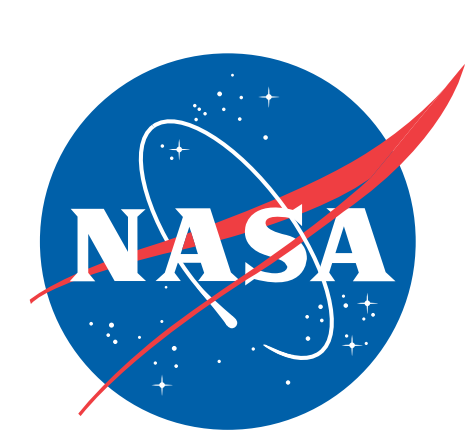


# Uncertainty Quantification with QUEST

## Quantified Uncertainty with Error bias estimation Software Tools

- Developed by Tim Barth (ARC-TNP) in ARMD-TACP Transformational Tools and Technologies (TTT) project
- Implements comprehensive set of UQ methods
  - Dense and sparse quadratures
  - Multi-level Monte-Carlo
  - Kernel density estimation for p.d.f. evaluation
  - Specialized numerics to handle stochastic discontinuities, e.g. those due to shocks
- Intrinsic error estimates for multi-level quadratures and MLMC sampling
- Intrinsic post-processing of discretization errors



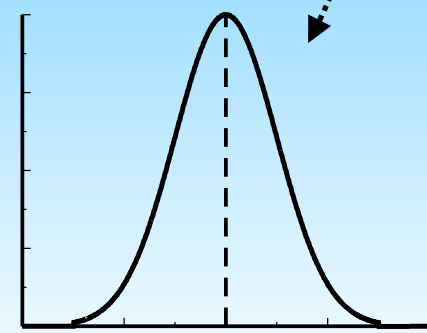


# Evaluation of Statistics for Outputs of Interest

## Expectation (Mean)

- $\xi$  → Input Random Variable, e.g. Mach Number
- $f(\xi)$  → Input Probability Density Function
- $J(\xi)$  → Exact Output, e.g. Perceived Loudness

$$E[J(\xi)] = \int J(\xi) f(\xi) d\xi$$



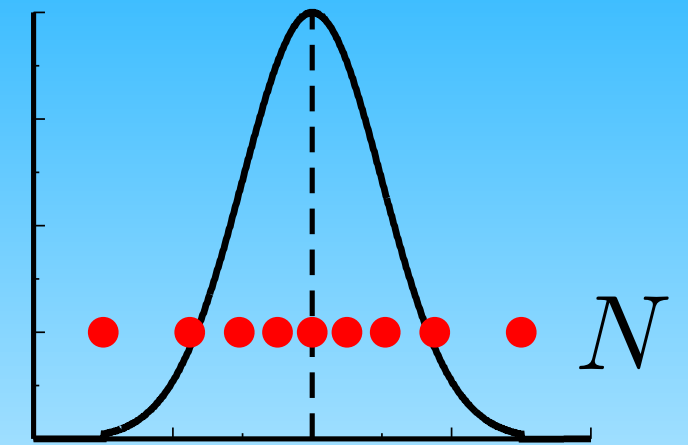
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$N$  - point Numerical Quadrature

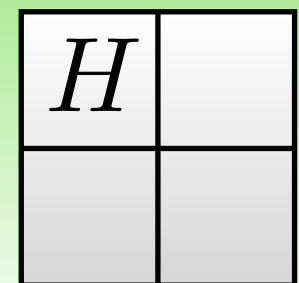
Approximate Outputs

$$J_H(\xi_1), \dots, J_H(\xi_N)$$

$$E_N[J_H] = \sum_{i=1}^N (w J_H)_i$$



## Error in Expectation Nearfield and Farfield Discretization



Cell-Size & Time-step

$$\varepsilon_H = J - J_H$$

## Numerical Error in Expectation

$$\varepsilon = E[J] - E_N[J_H]$$

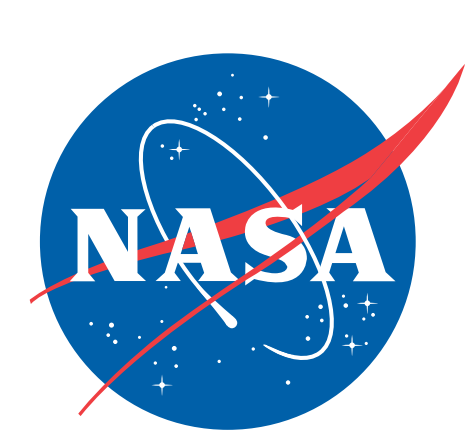
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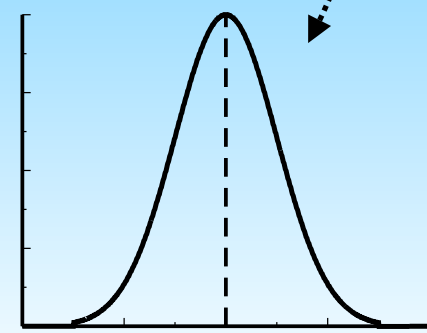


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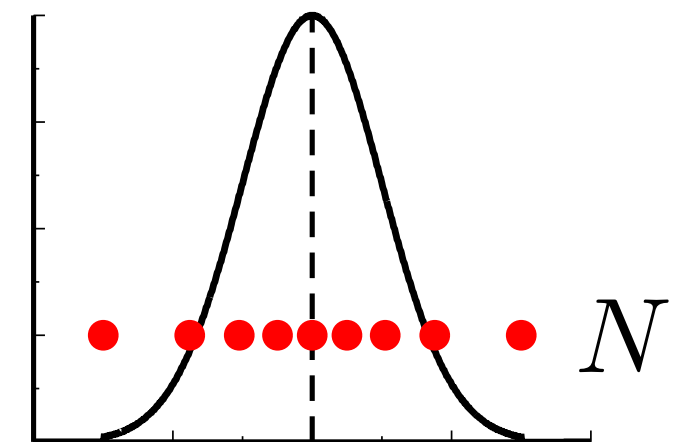
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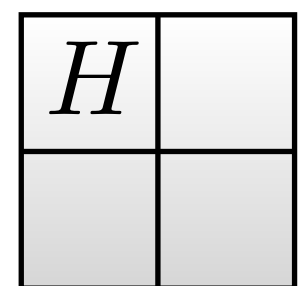
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### Nearfield and Farfield Discretization



Cell-Size & Time-step

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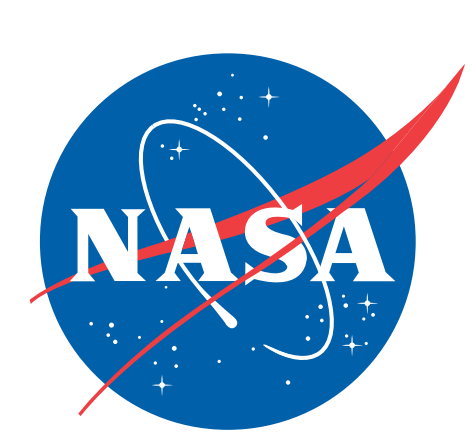
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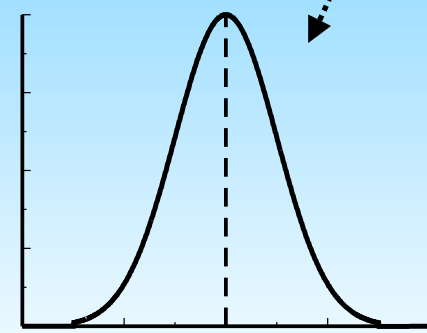


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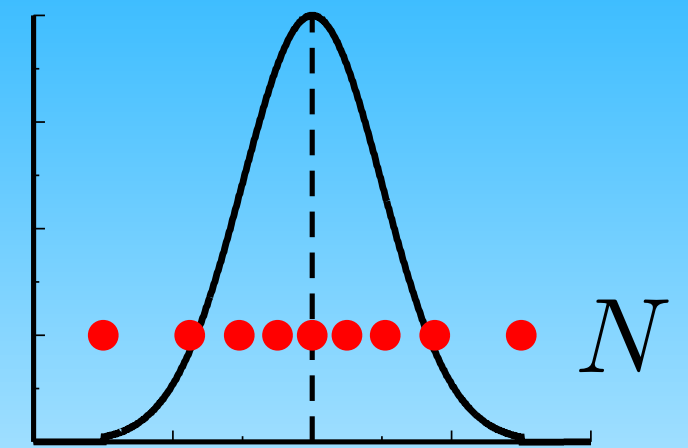
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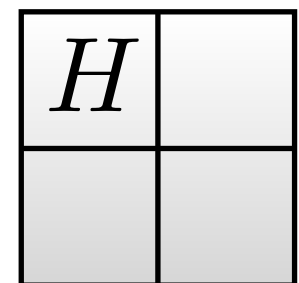


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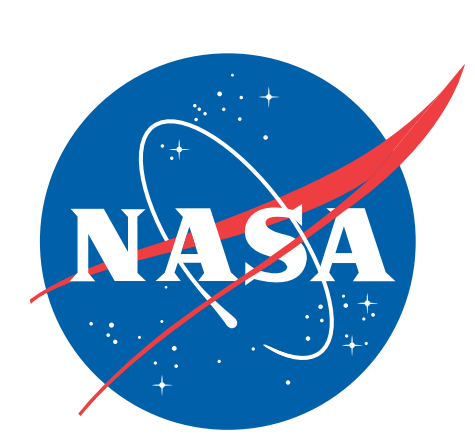
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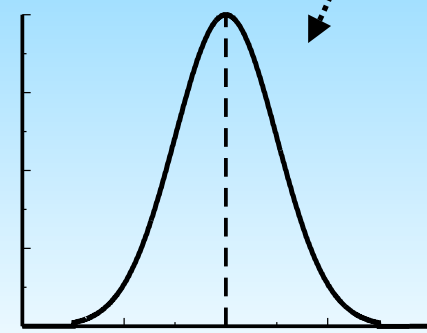


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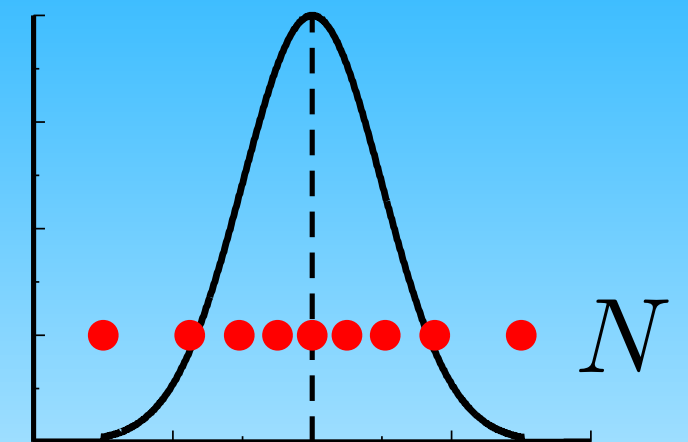


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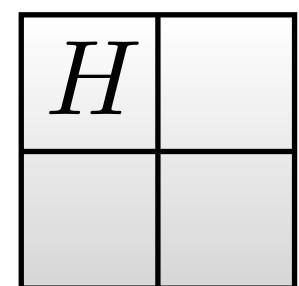


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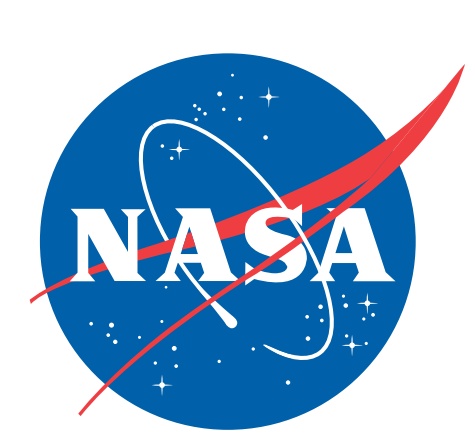
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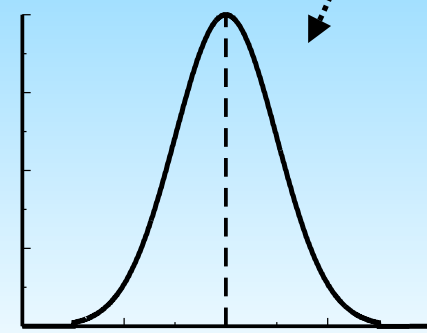


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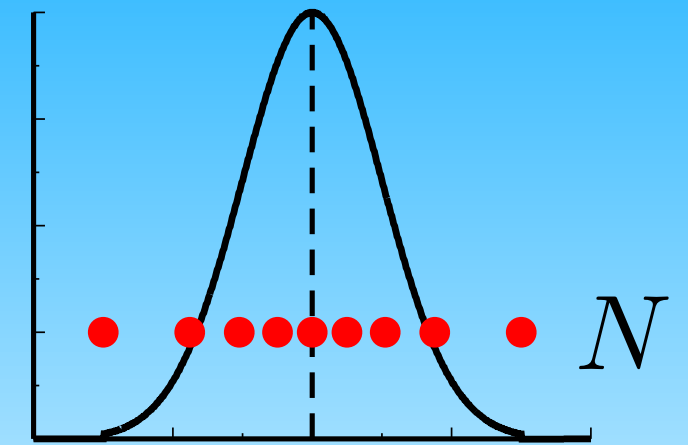
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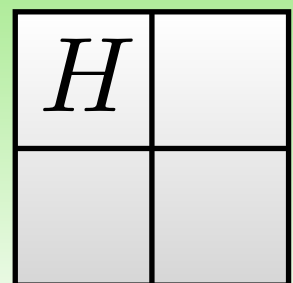
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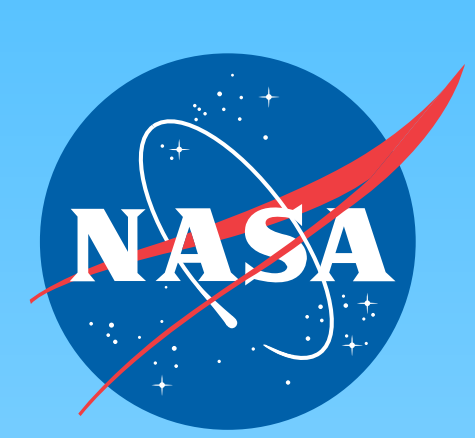
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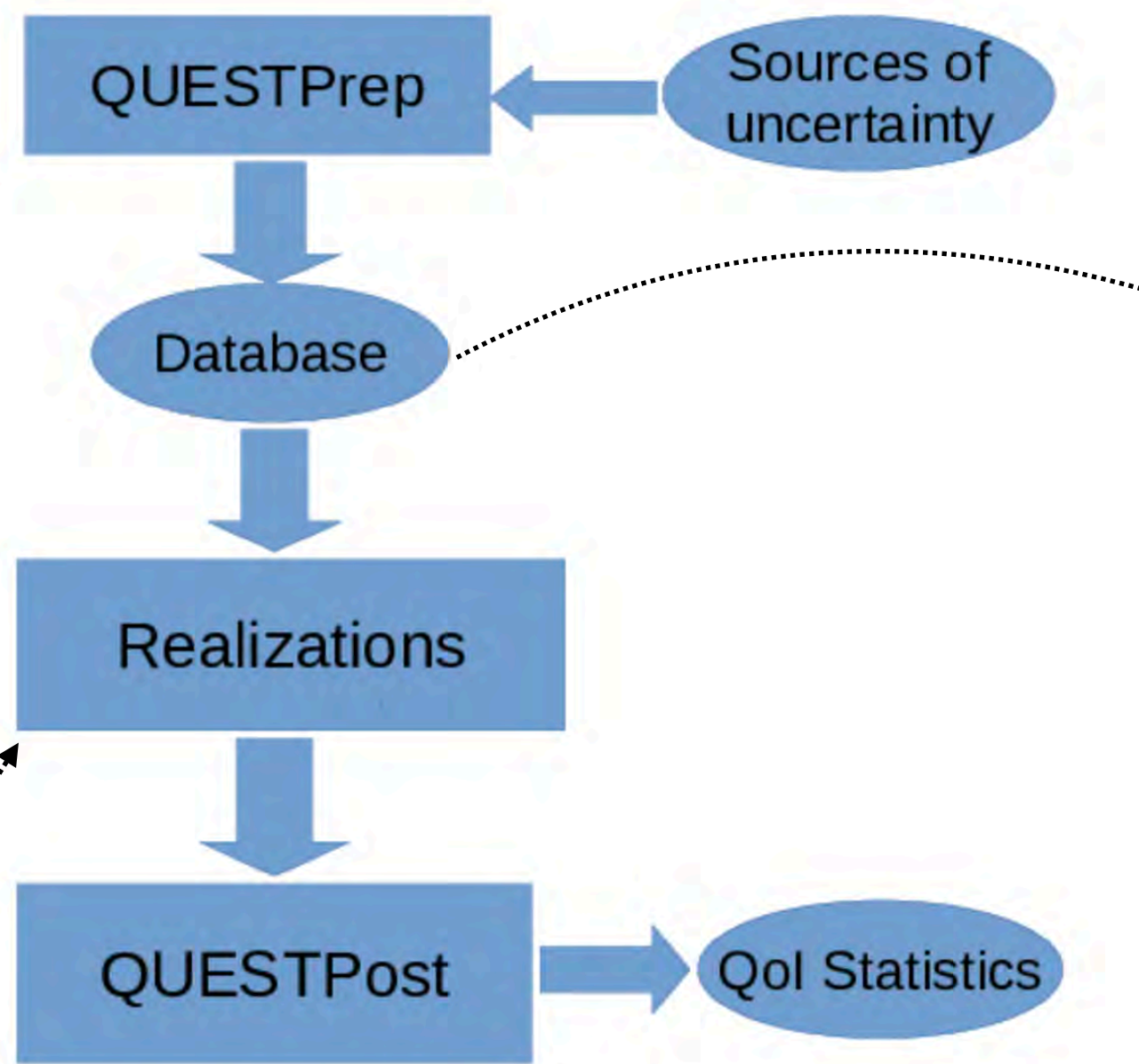
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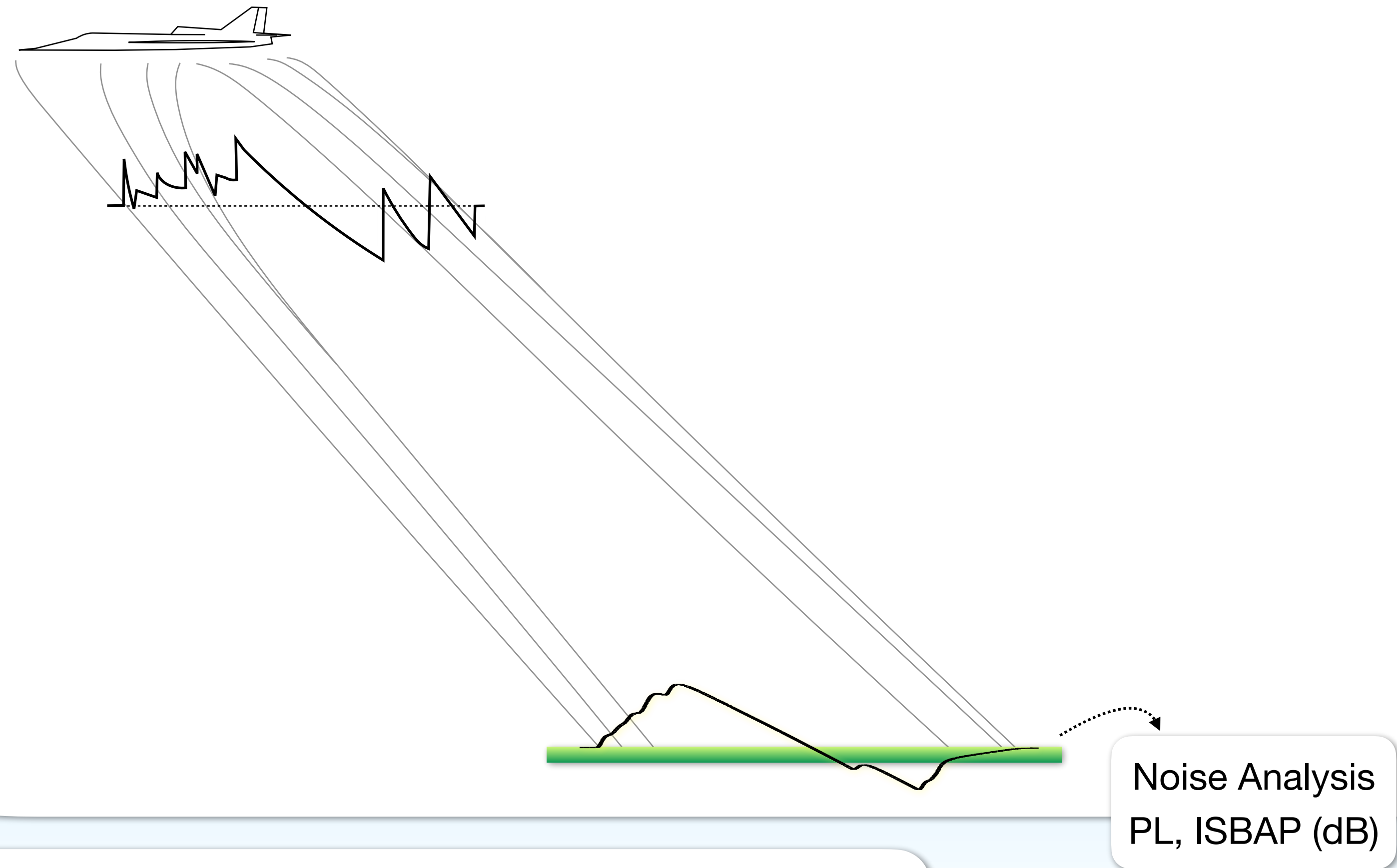


# Approach: Nearfield-Farfield Decomposition

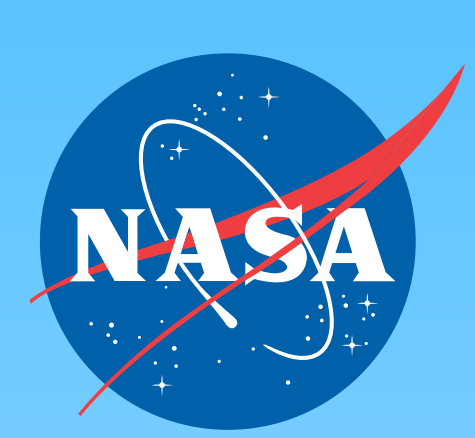
## QUEST



## Acoustic Signature Simulation

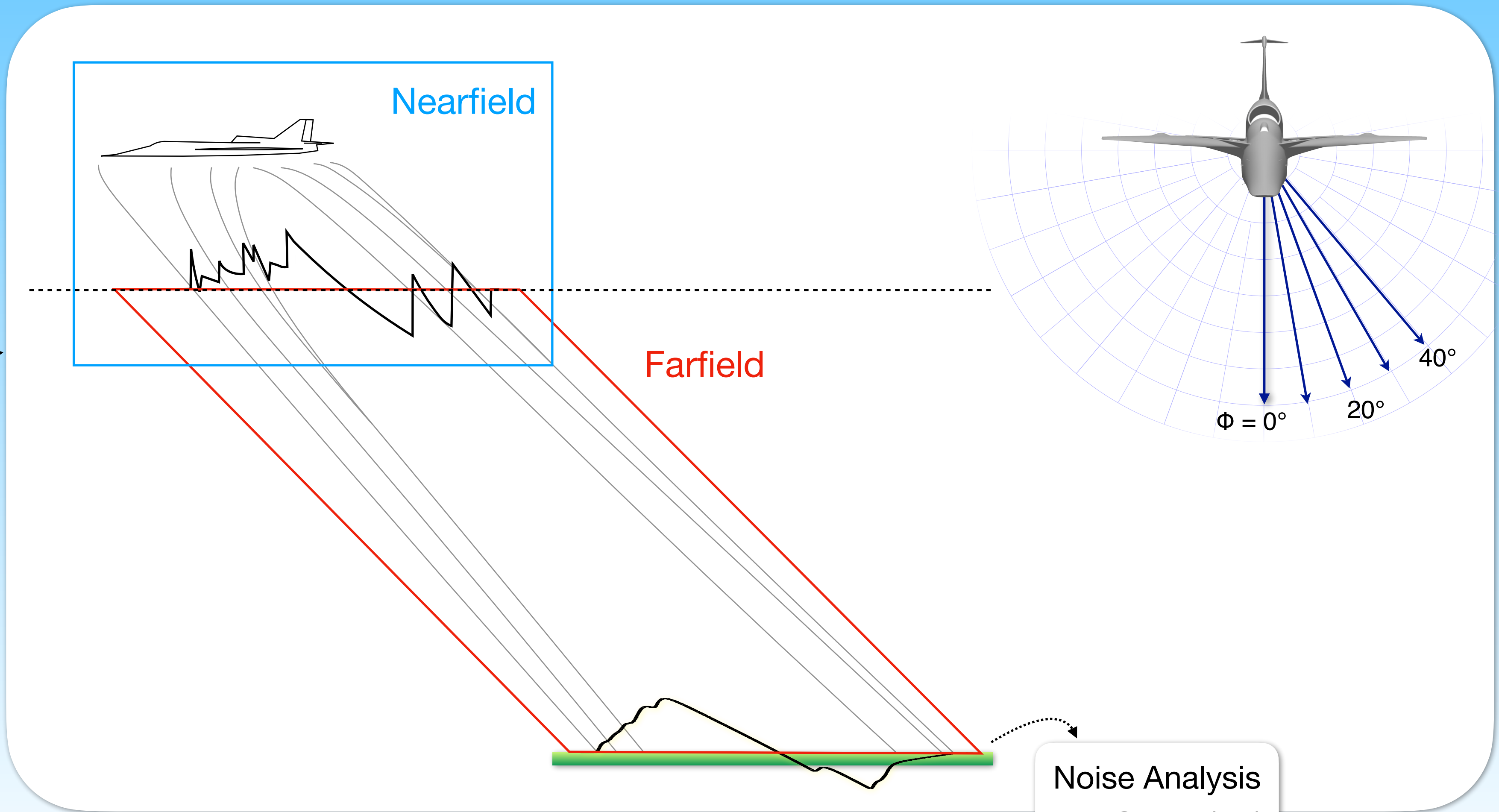
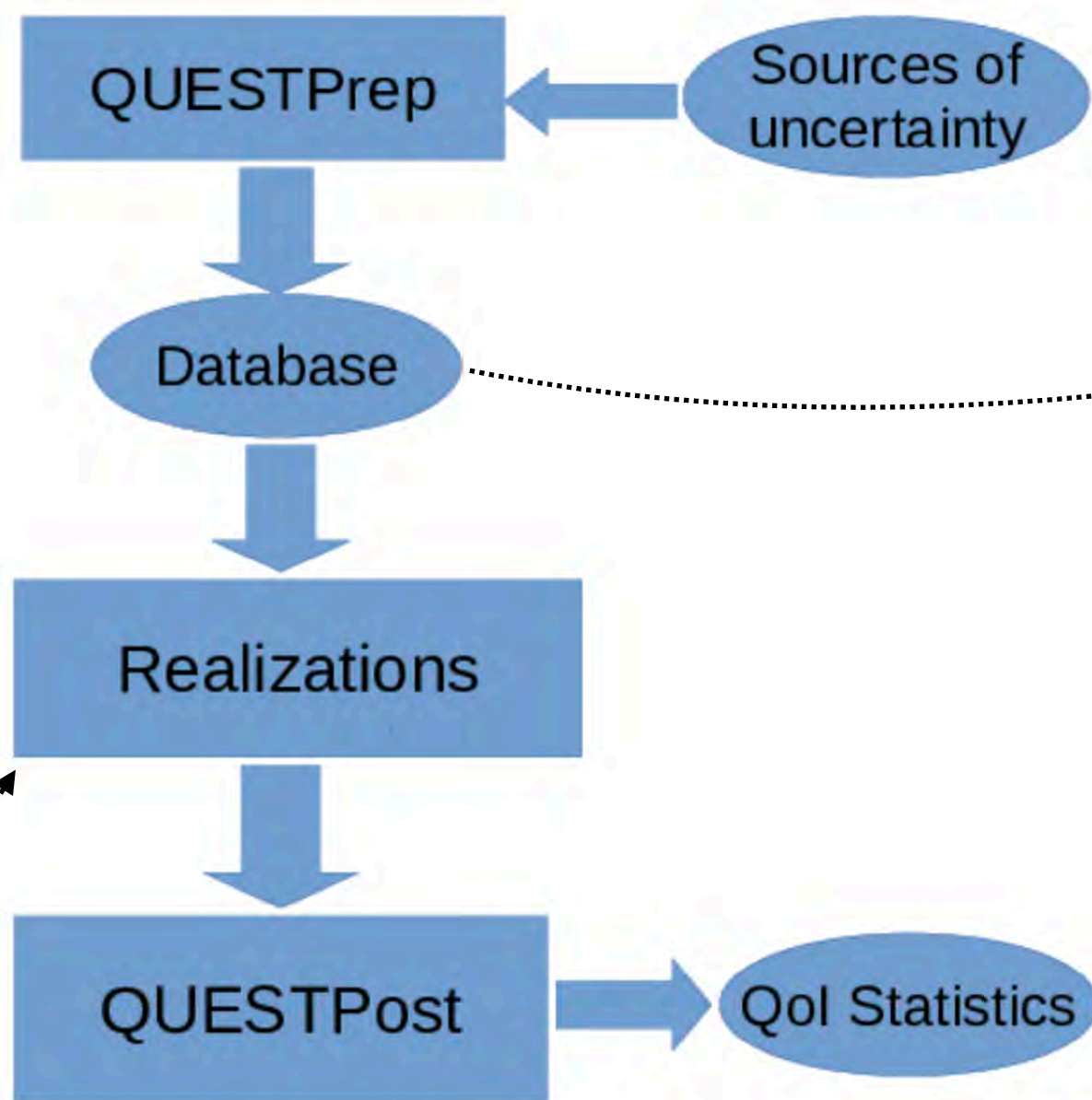


**Realizations, e.g. noise metric**  $J_H(\xi_1), J_H(\xi_2), \dots, J_H(\xi_N)$   
**Discretization Error**  $\varepsilon_H(\xi_1), \varepsilon_H(\xi_2), \dots, \varepsilon_H(\xi_N)$



# Approach: Nearfield-Farfield Decomposition

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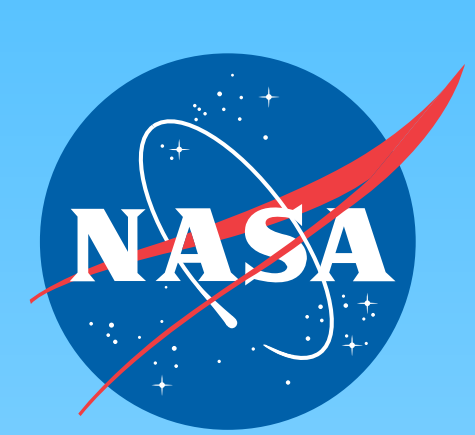


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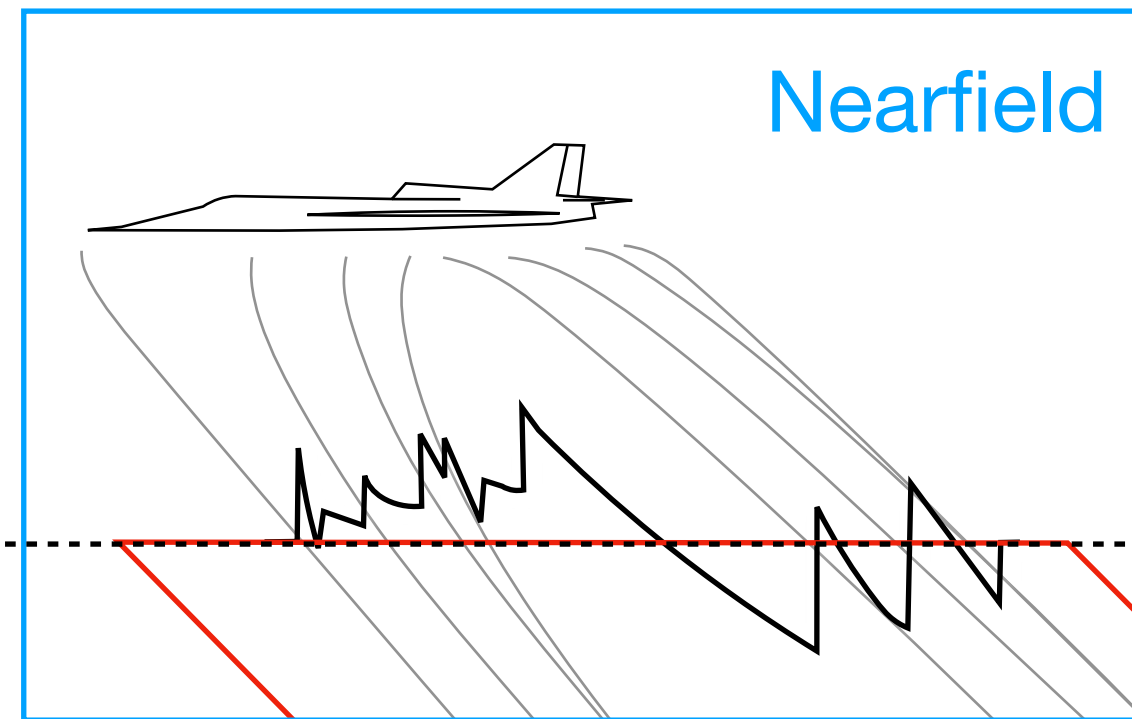
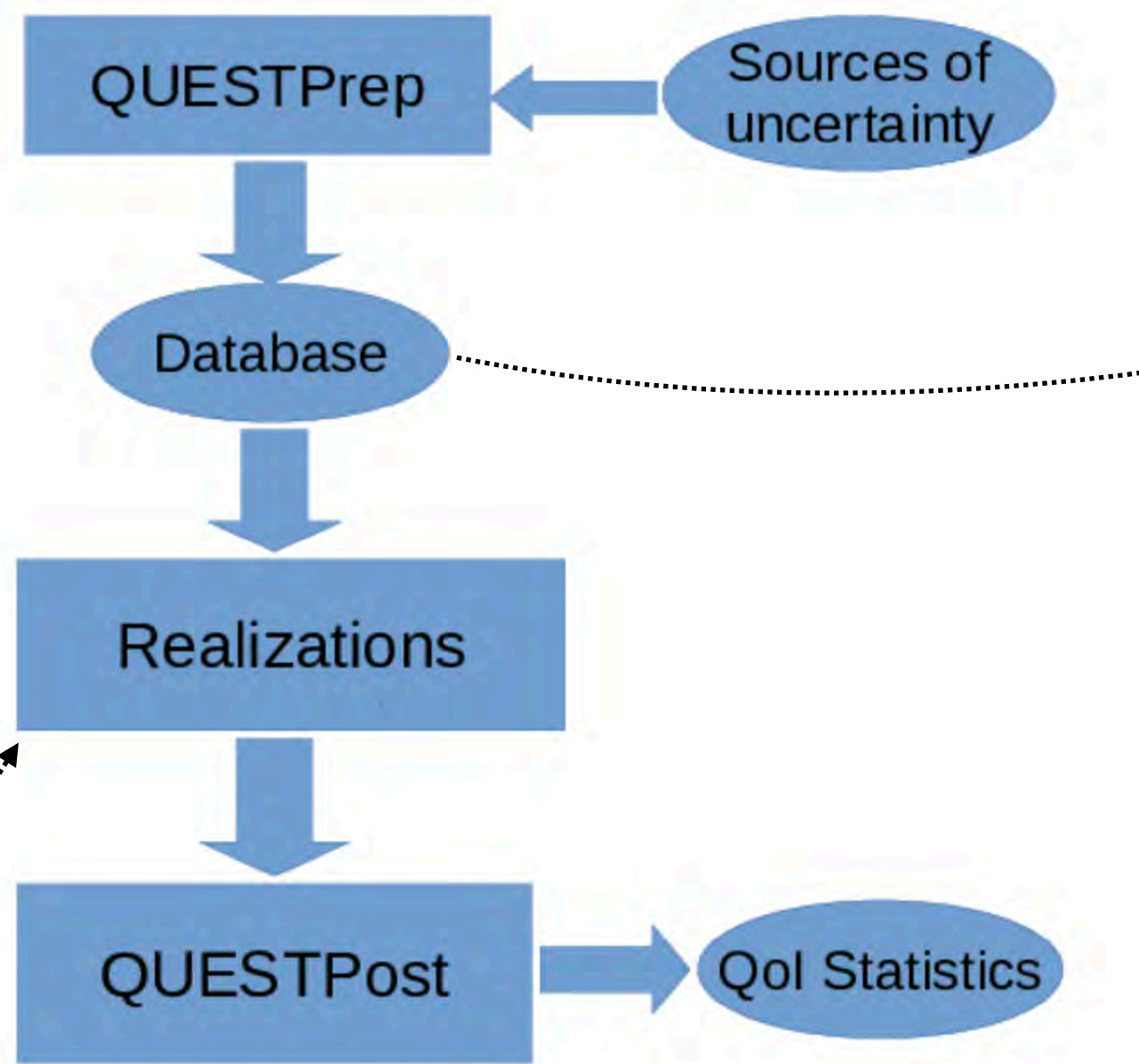
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Noise Analysis  
 PL, ISBAP (dB)



# Approach: Nearfield-Farfield Decomposition

## QUEST



3D effects important

- Aircraft shape
- Boundary layers, plumes, secondary flow paths

Need to accurately simulate complex near-body shocks to predict nearfield signatures

**Farfield**

- Stratification
- Absorption
- Winds

Pressure signatures are independent

- Geometric acoustics
- Propagation distances > 20 km
- State of atmosphere critical

Need to accurately propagate discontinuous pressure signatures to ground and predict loudness metrics

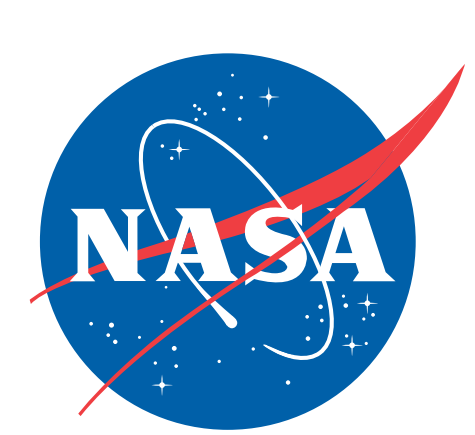
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Discretization Error

$$\varepsilon_H(\xi_1), \varepsilon_H(\xi_2), \dots, \varepsilon_H(\xi_N)$$



# Nearfield Solver: Cart3D

## Flow Solver

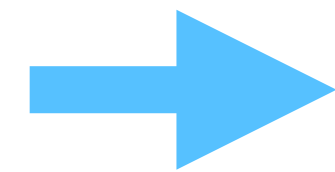
- Inviscid flow
- 2nd-order upwind method
  - van Leer flux, Barth-Jespersen Limiter
- RK-5, Multigrid acceleration
- Domain decomposition: OpenMP & MPI

## Automatic Meshing

- Multilevel Cartesian mesh with embedded boundaries
- Handles arbitrarily complex vehicle shapes

## Output-Driven Mesh Adaptation

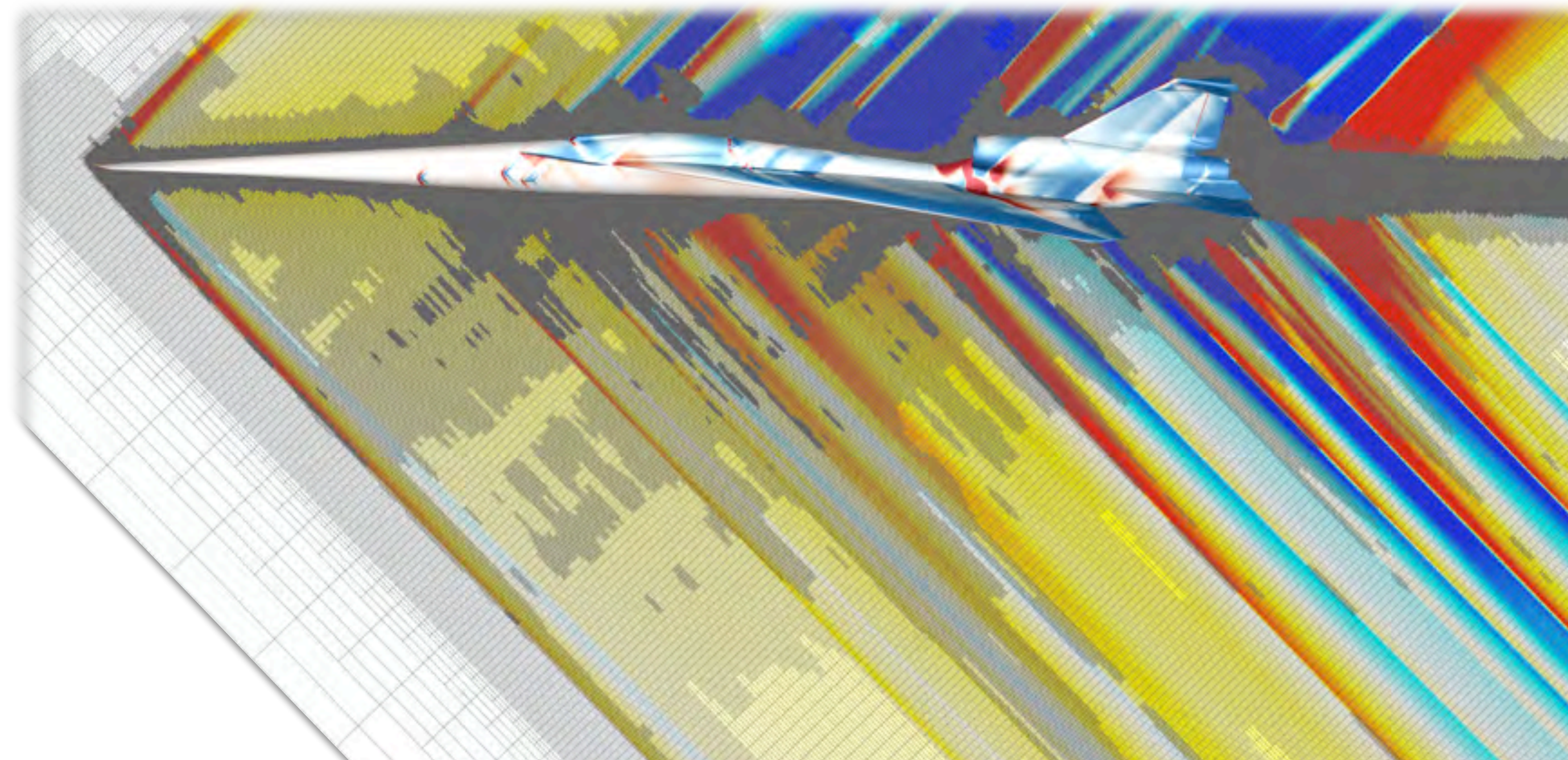
- Method of adjoint-weighted residuals
- Mesh is refined in locations with highest impact on pressure signatures



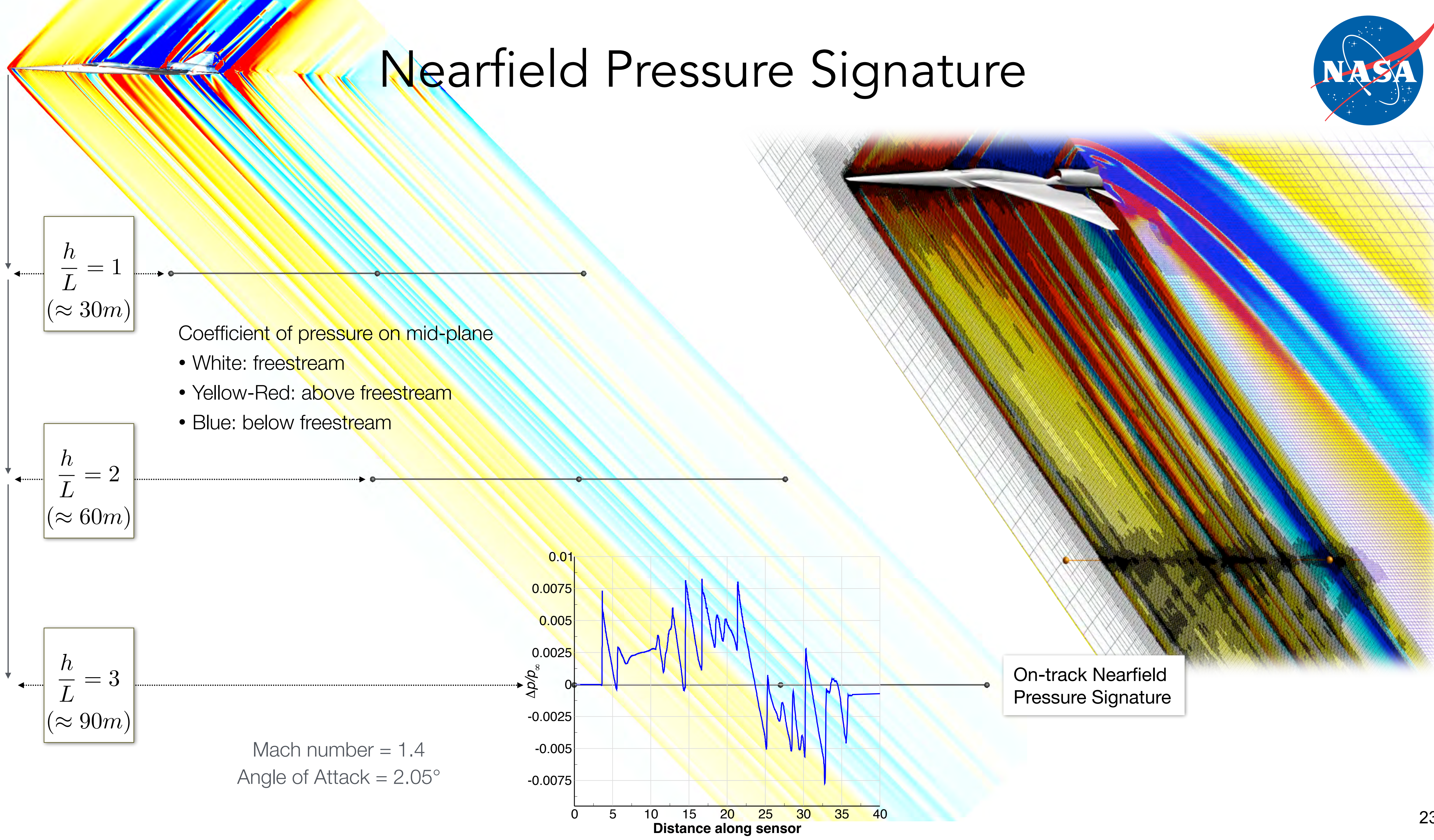
## Every nearfield simulation includes

- Mesh refinement study to demonstrate mesh convergence
- Reliable bound on remaining discretization error

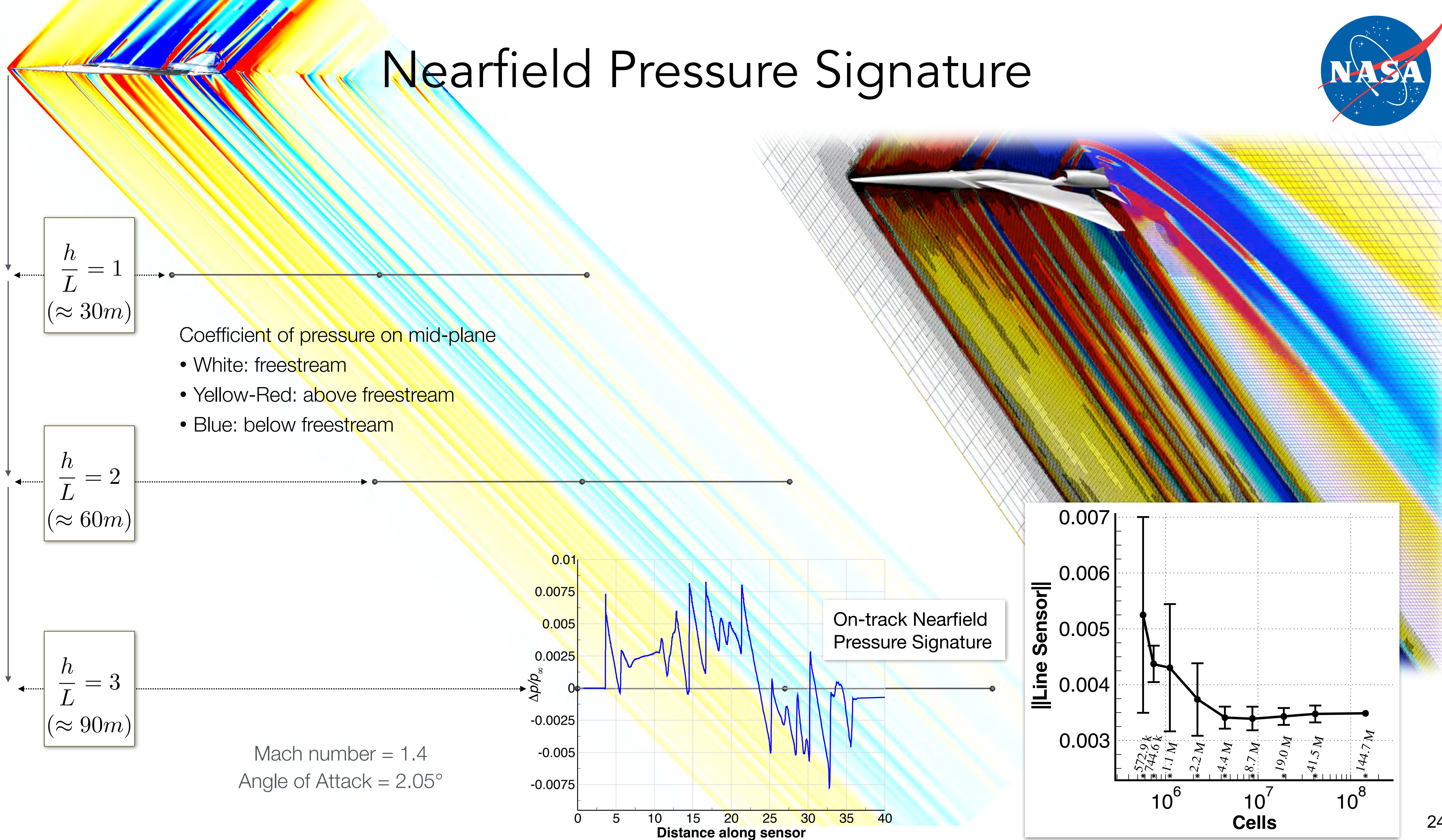
Credibility, Efficiency & User-Independent Results

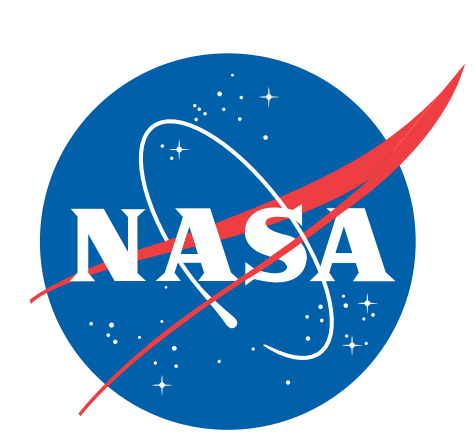


# Nearfield Pressure Signature



# Nearfield Pressure Signature





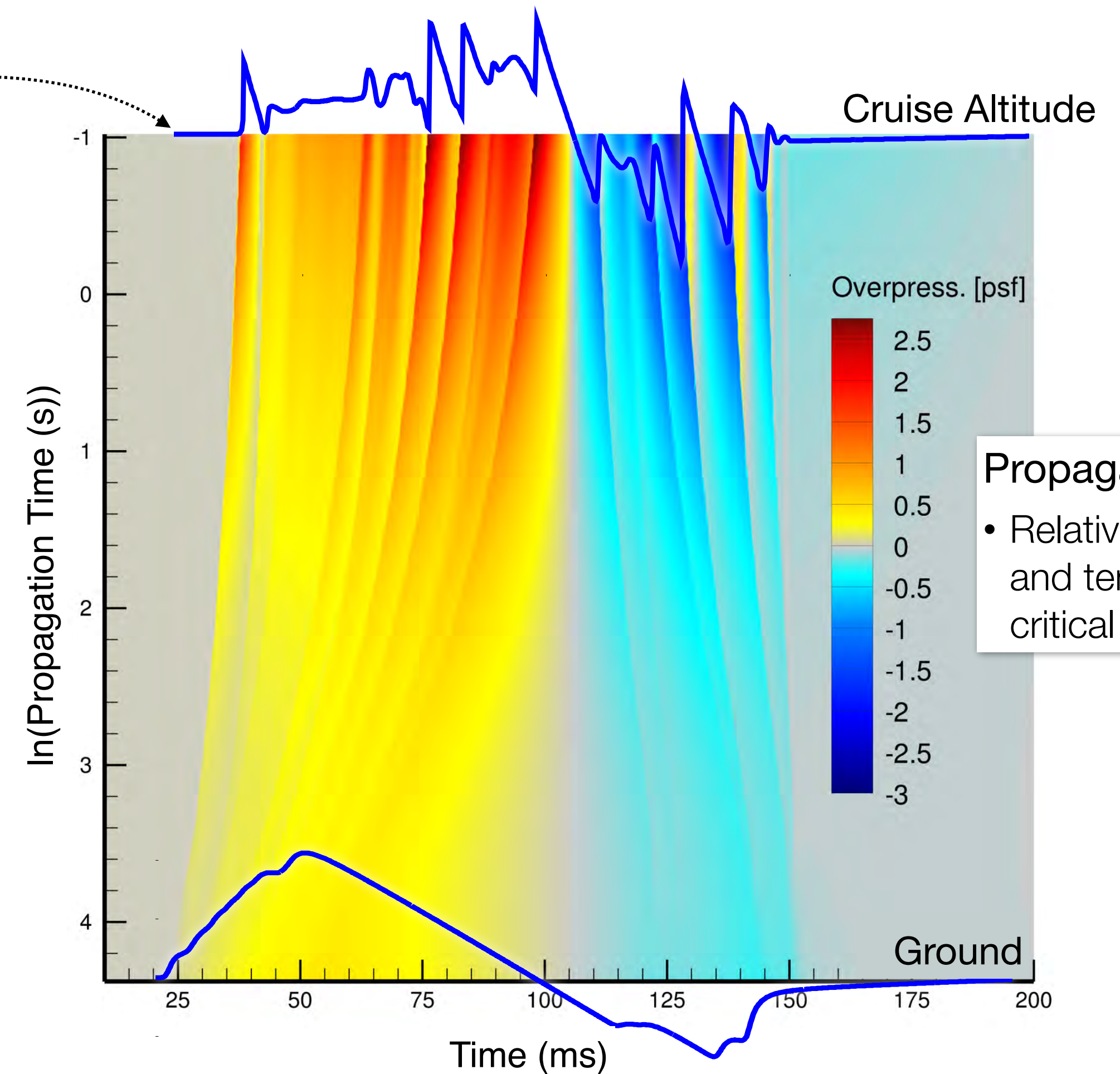
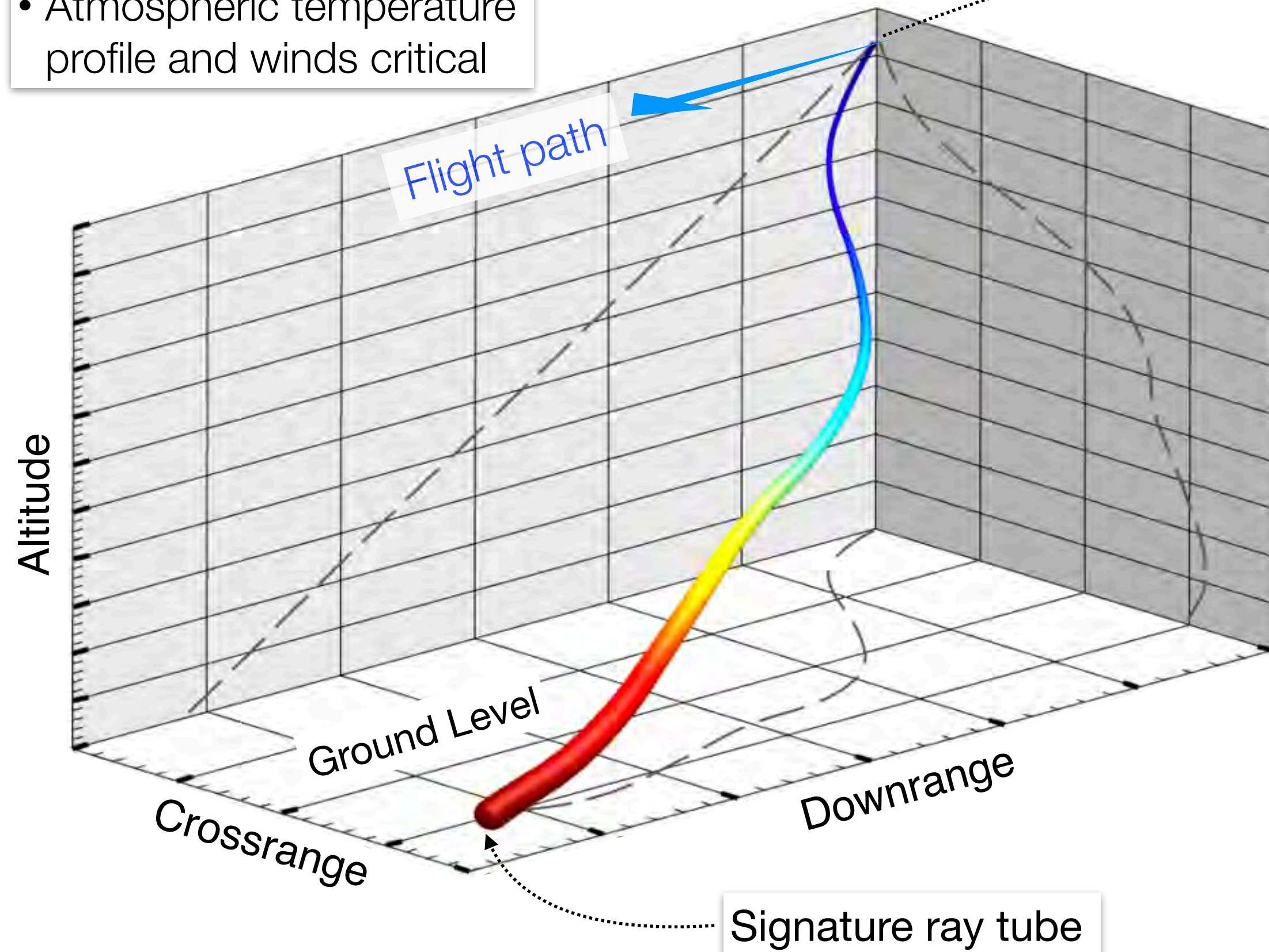
# Farfield Solver: sBOOM

Ray tracing coupled with augmented Burgers' equation

- Includes nonlinearity, thermoviscous absorption and relaxation
- 2nd-order finite volumes: Godunov's flux, van Leer Limiter, RK2, uniform mesh

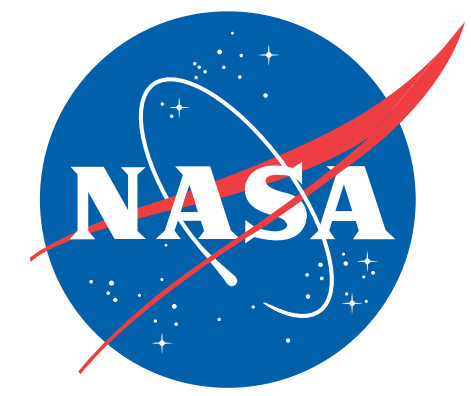
## Ray tracing

- Atmospheric temperature profile and winds critical



## Propagation

- Relative humidity and temperature critical



# Atmospheric Uncertainty in Propagation

Signature strongly influenced by local atmospheric conditions

- Ray tracing depends on temperature profile
- Relative humidity influences waveform attenuation via molecular relaxation
- Winds primarily affect arrival time and ground intercept

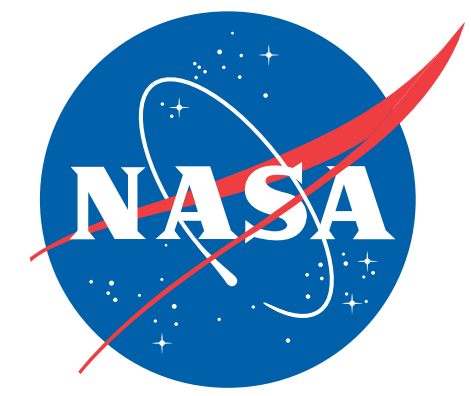
Detailed databases of atmospheric profiles for temperature, relative humidity, pressure and wind available

- Climate Forecast System Reanalysis v2 (CFSv2)
- European Centre for Medium-Range Weather Forecasts (ECMWF) Reanalysis v5 (ERA5)

Challenge is to accurately and efficiently model stochastic fluctuations in the atmosphere

- Atmospheric profiles are correlated fields, but we need independent random variables (as few as possible)
- Random process should approximate mean and standard deviation of observations





# Atmospheric Observations

Current focus is on variability in atmospheric profiles at Edwards AFB

- Location for initial X-59 acoustic validation flights
- Historical observations from 2003 to 2022 from ERA5 dataset
- Profiles available every 6 hours every day
- Selected August at 18:00 UTC (10 am PT) as an example
  - 620 atmospheric profiles
- This study considers temperature (T) and relative humidity (RH)

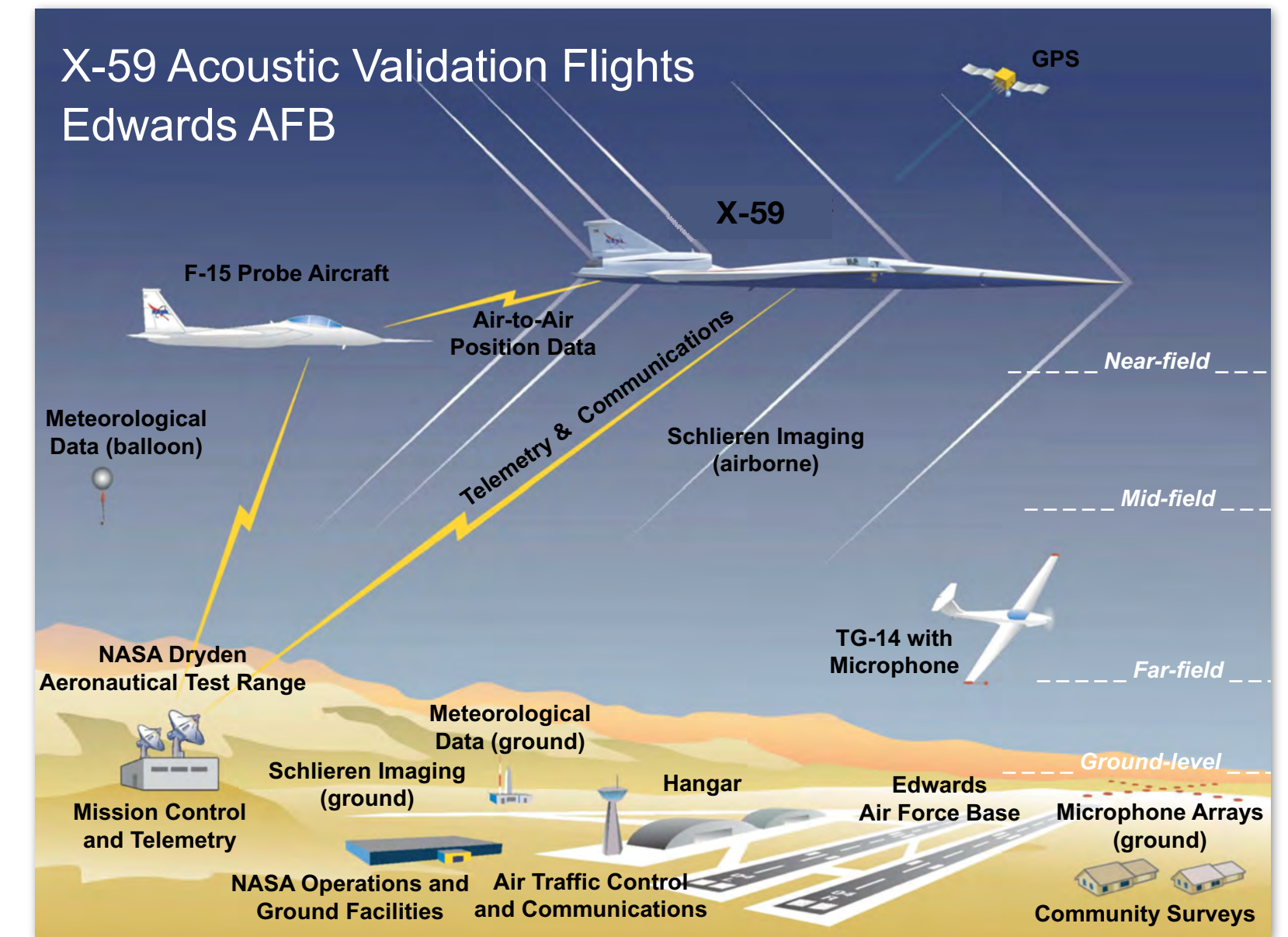
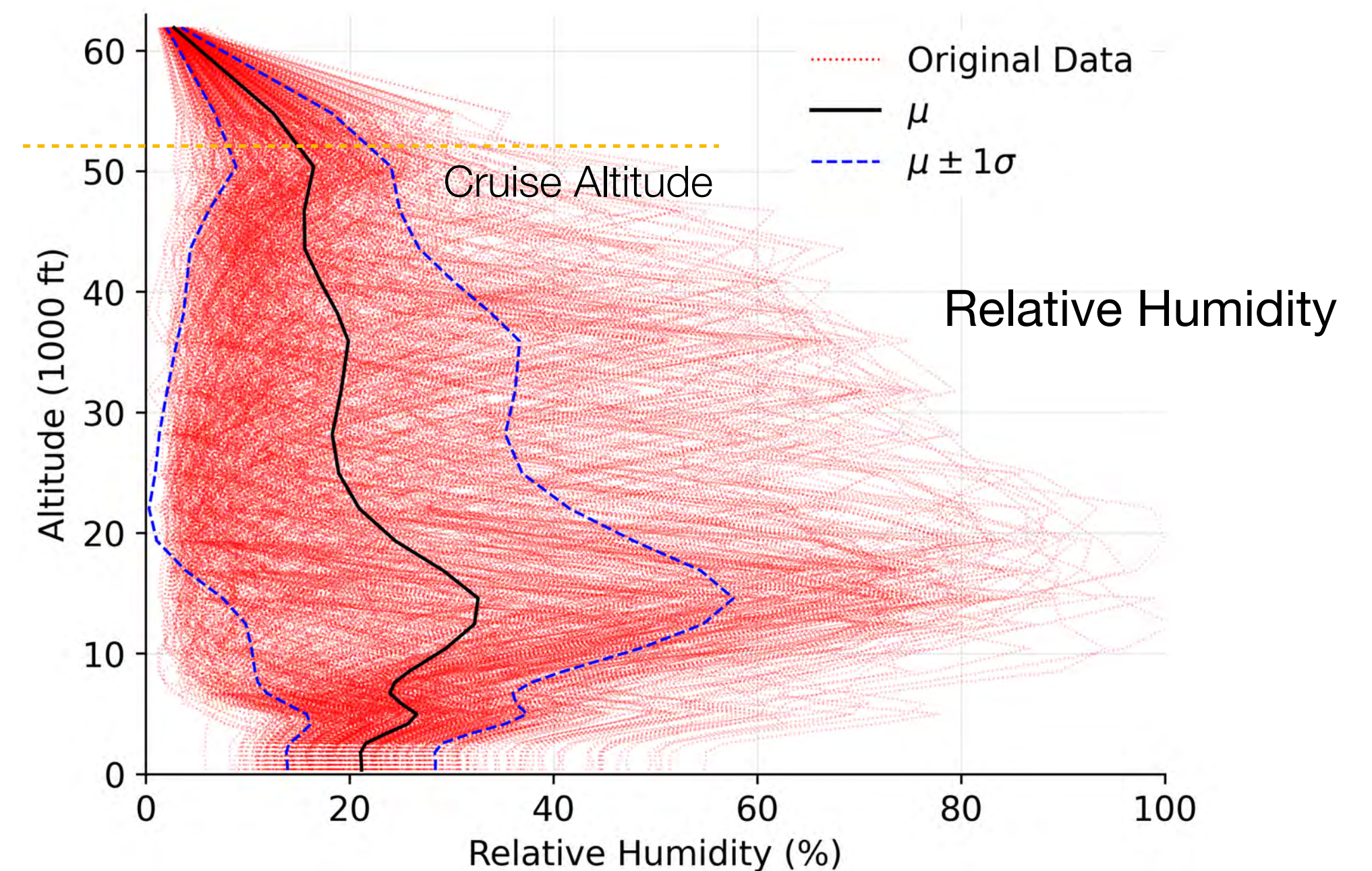
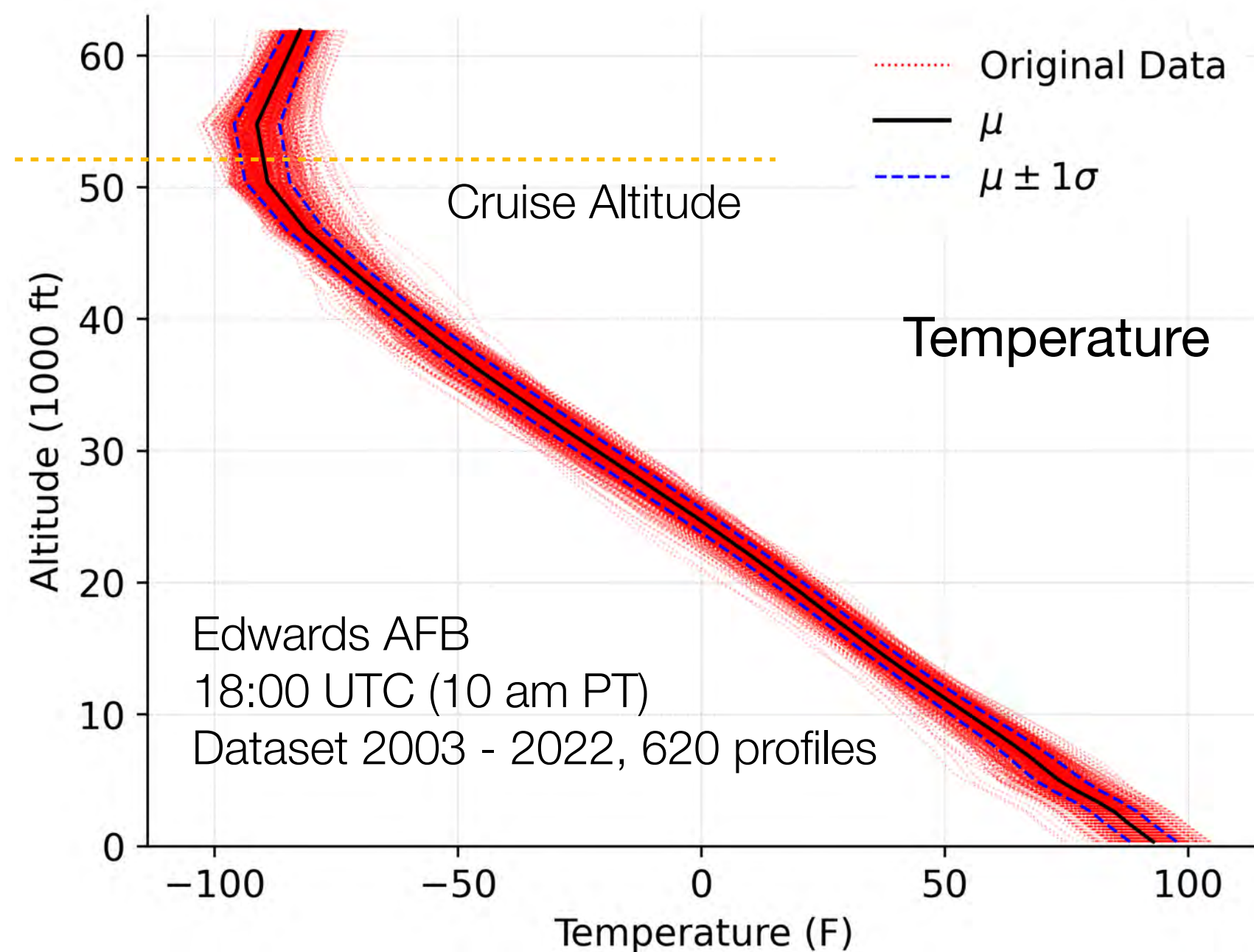
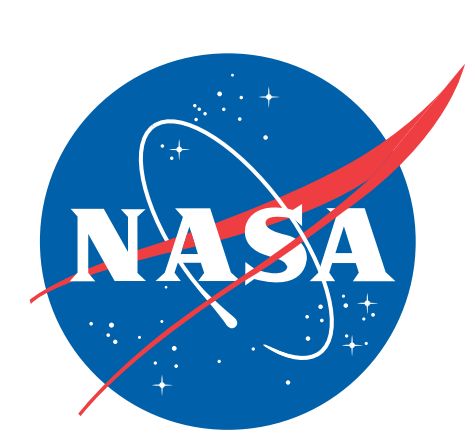
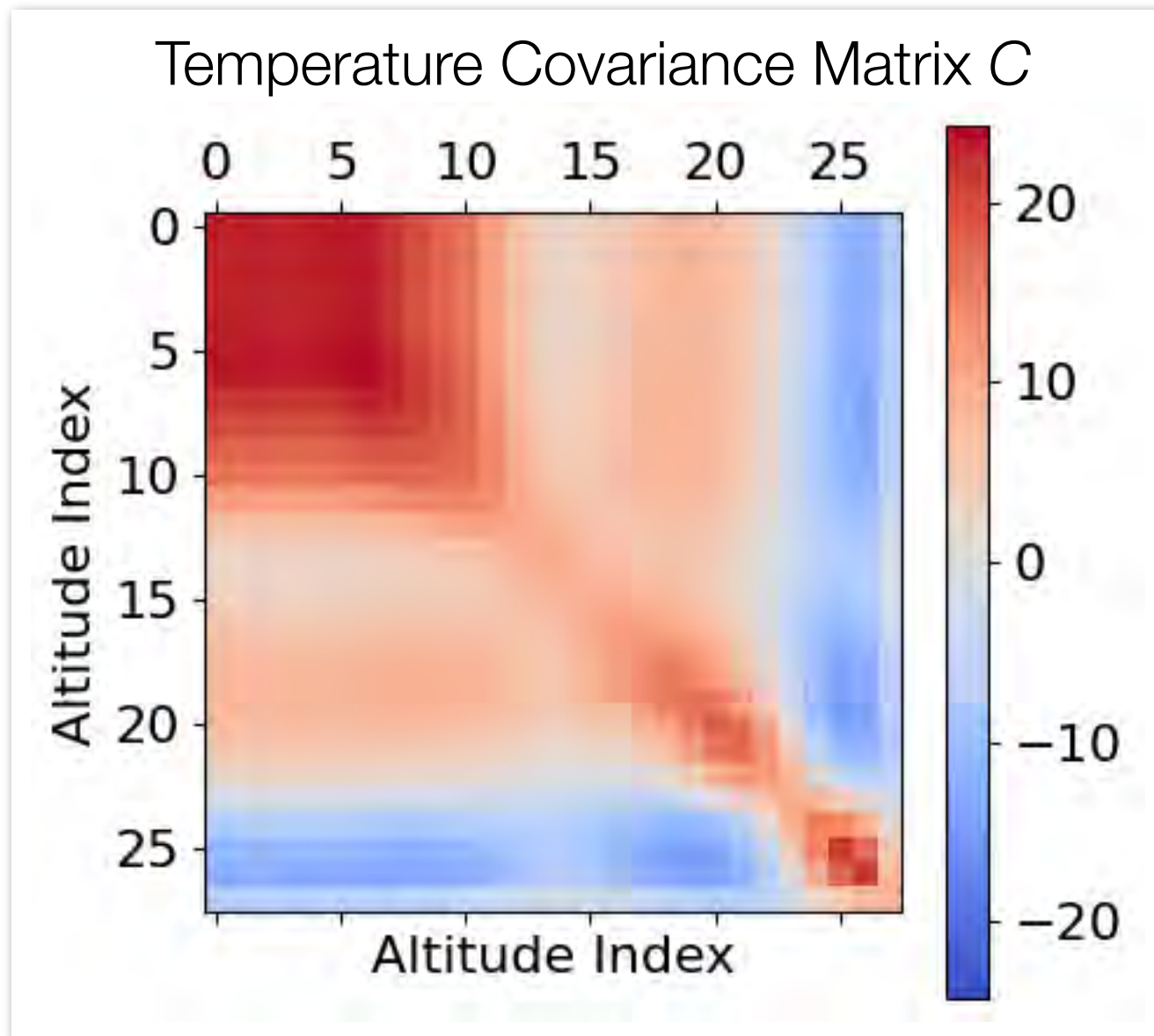


Image Credit: NASA



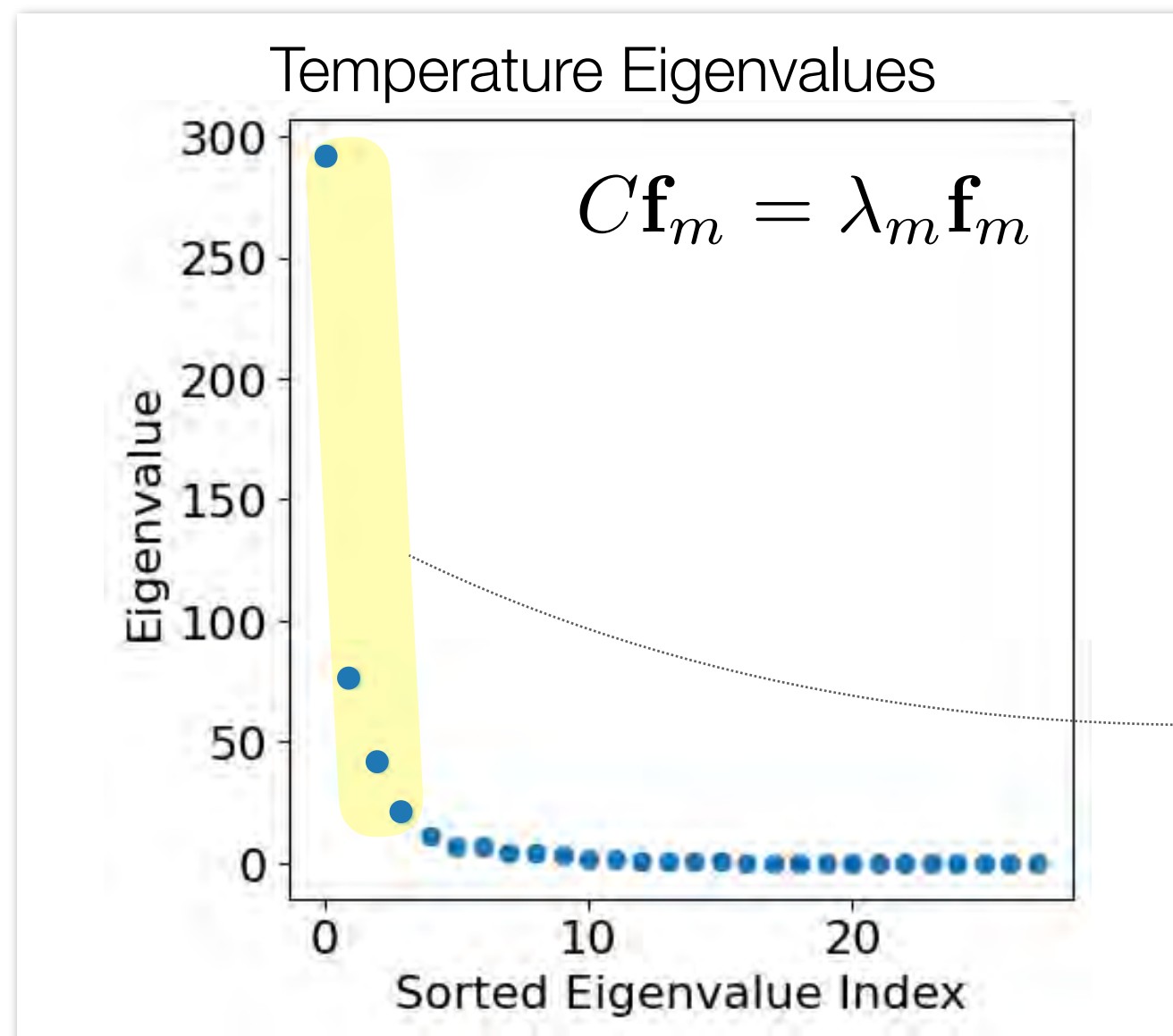


# Parameterization of Atmospheric Profiles



## Karhunen-Loeve (KL) Expansion

- Karhunen-Loeve expansion is used to model uncertainty in atmospheric data
  - Ideal for centered, Gaussian random fields
- Main idea is to use the covariance of the observed data to anchor the parameterization
  - Diagonalize discrete covariance matrix
  - Select largest eigenvalues to model fluctuations in atmospheric properties



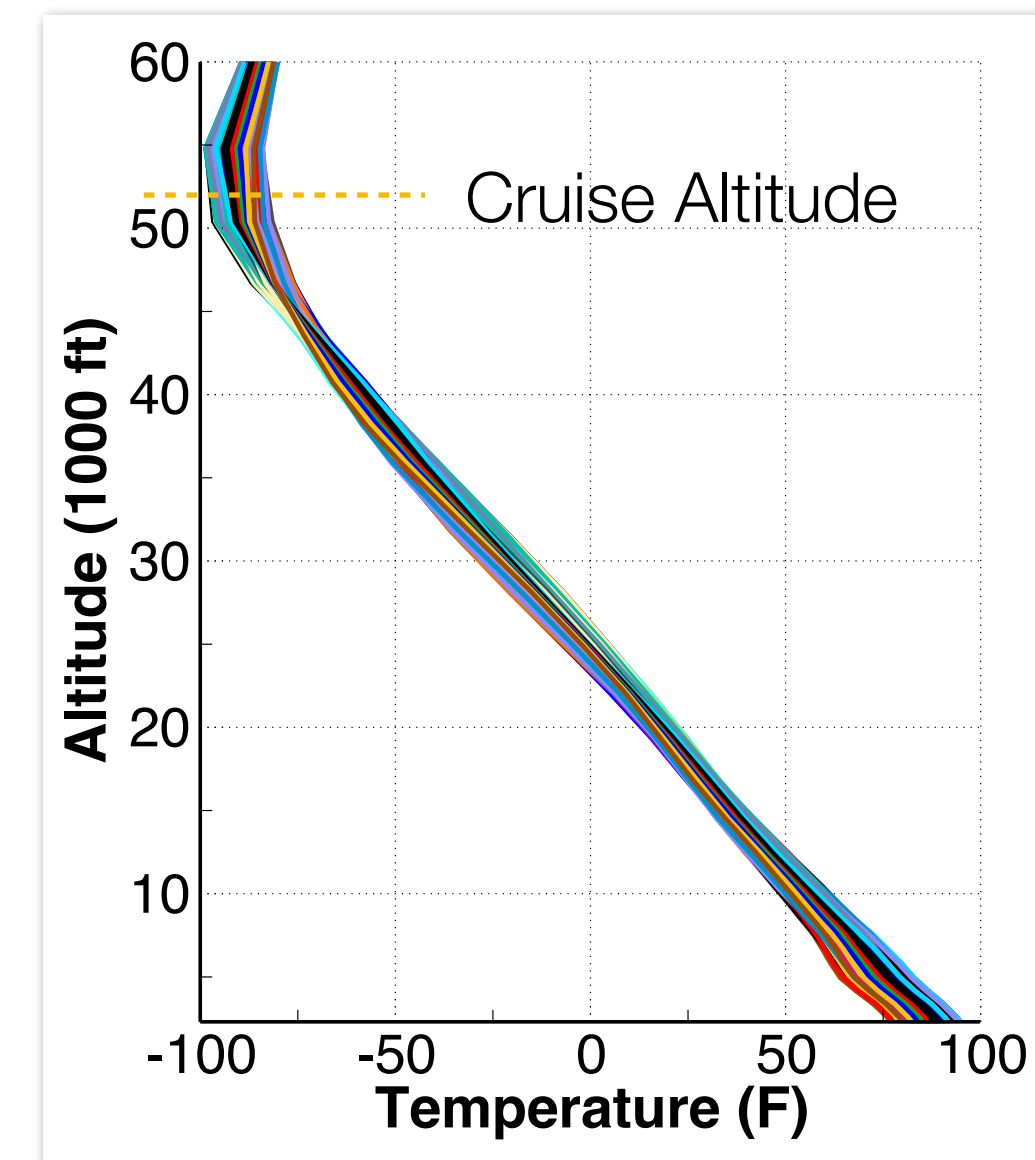
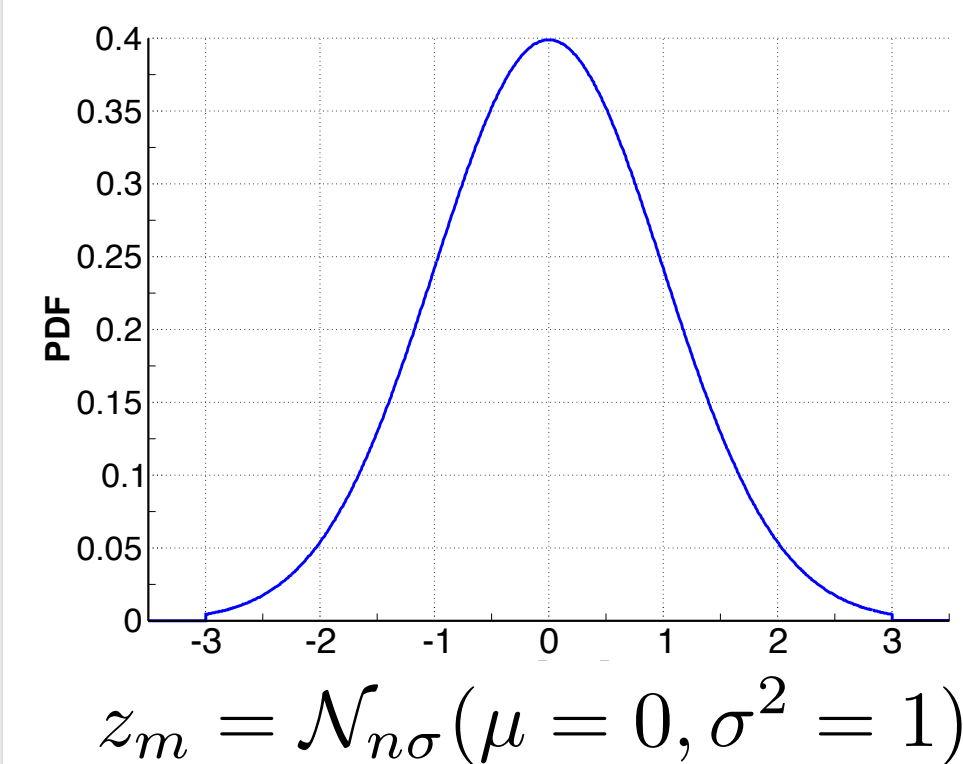
## Discrete random-process temperature profile

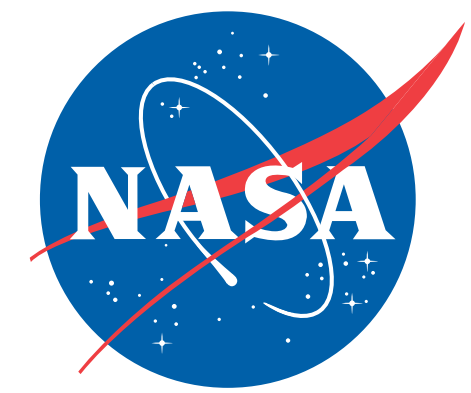
$$\mathbf{T}(\xi) = \bar{\mathbf{T}} + \sum_{m=1}^M z_m(\xi) \sqrt{\lambda_m} \mathbf{f}_m$$

Mean from  
observed data

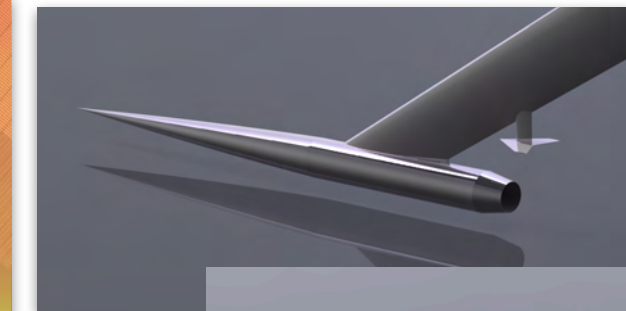
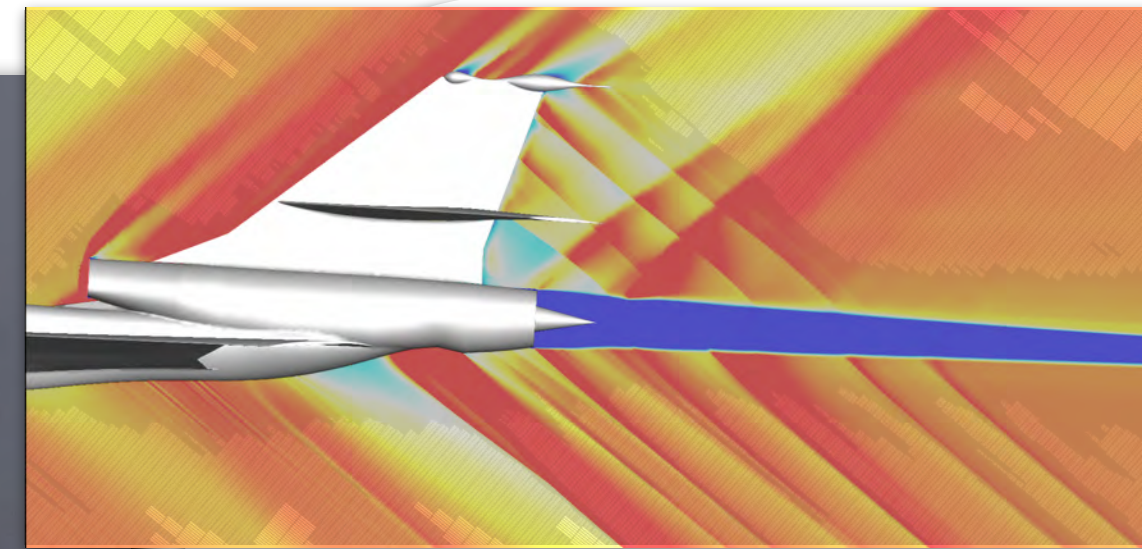
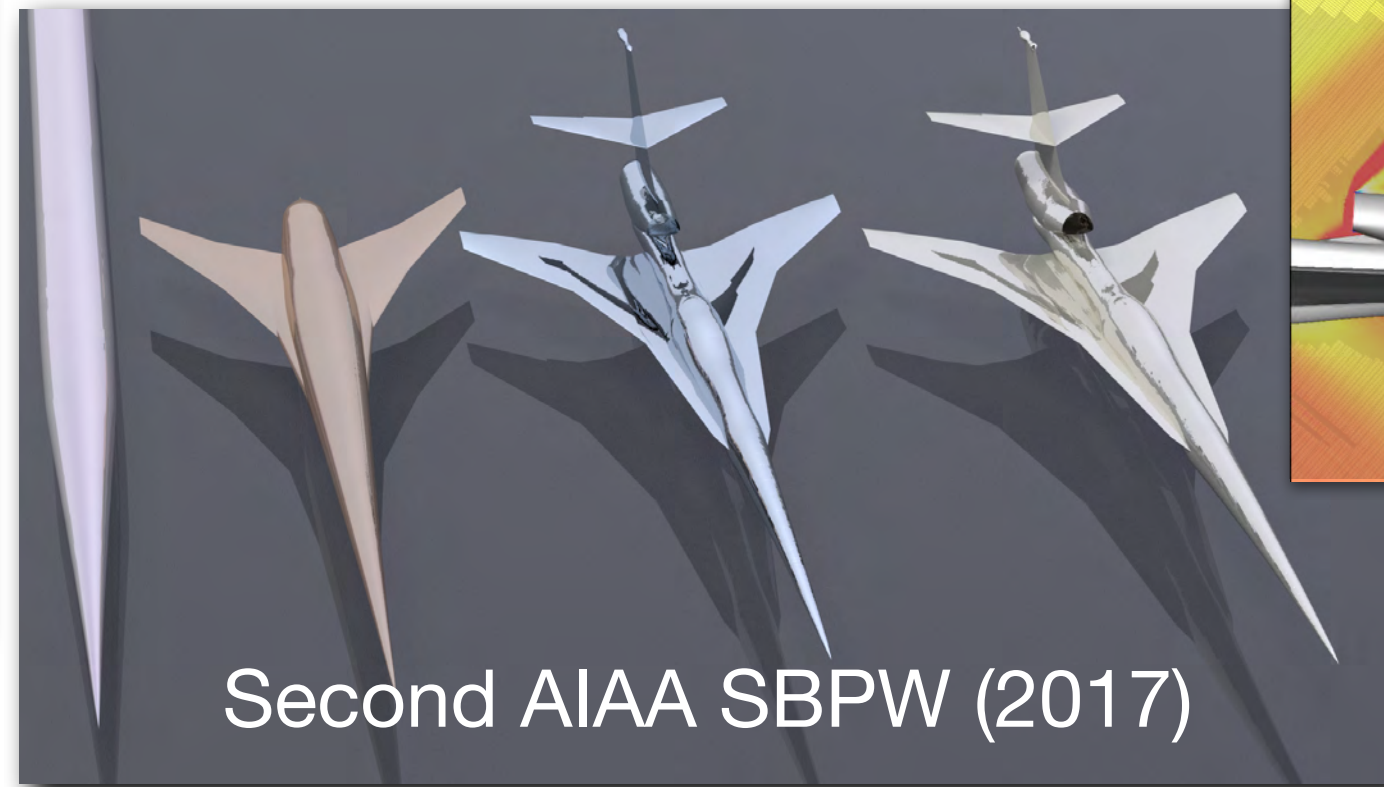
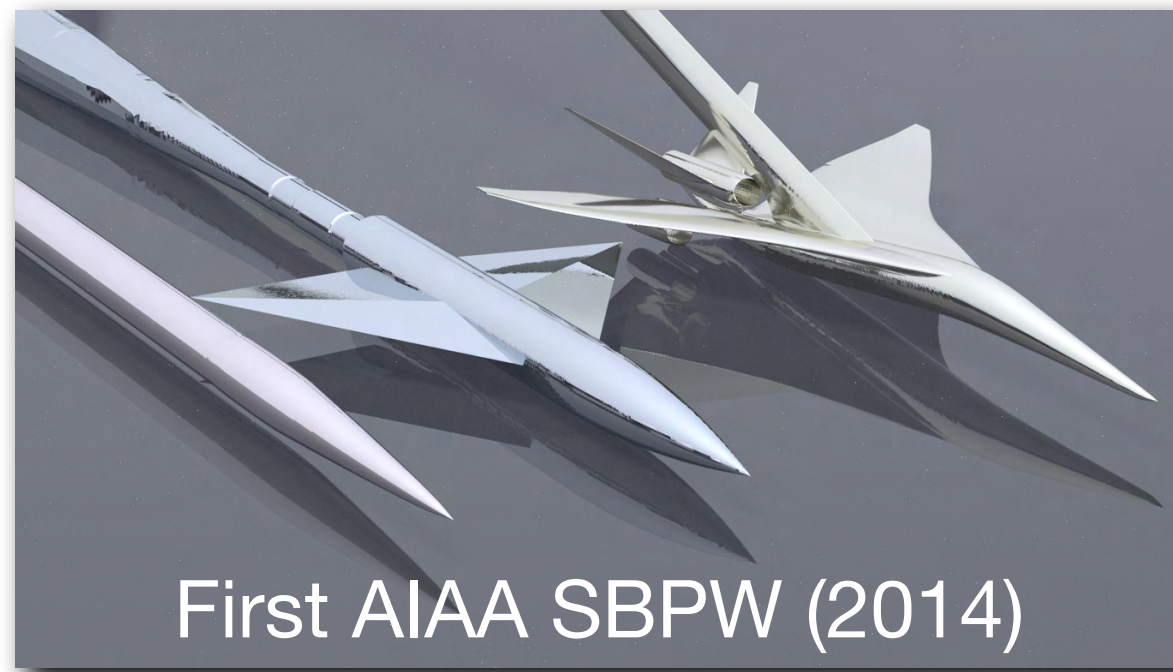
$M$  largest eigenvalues  
of covariance matrix  $C$

## Independent Random Variables

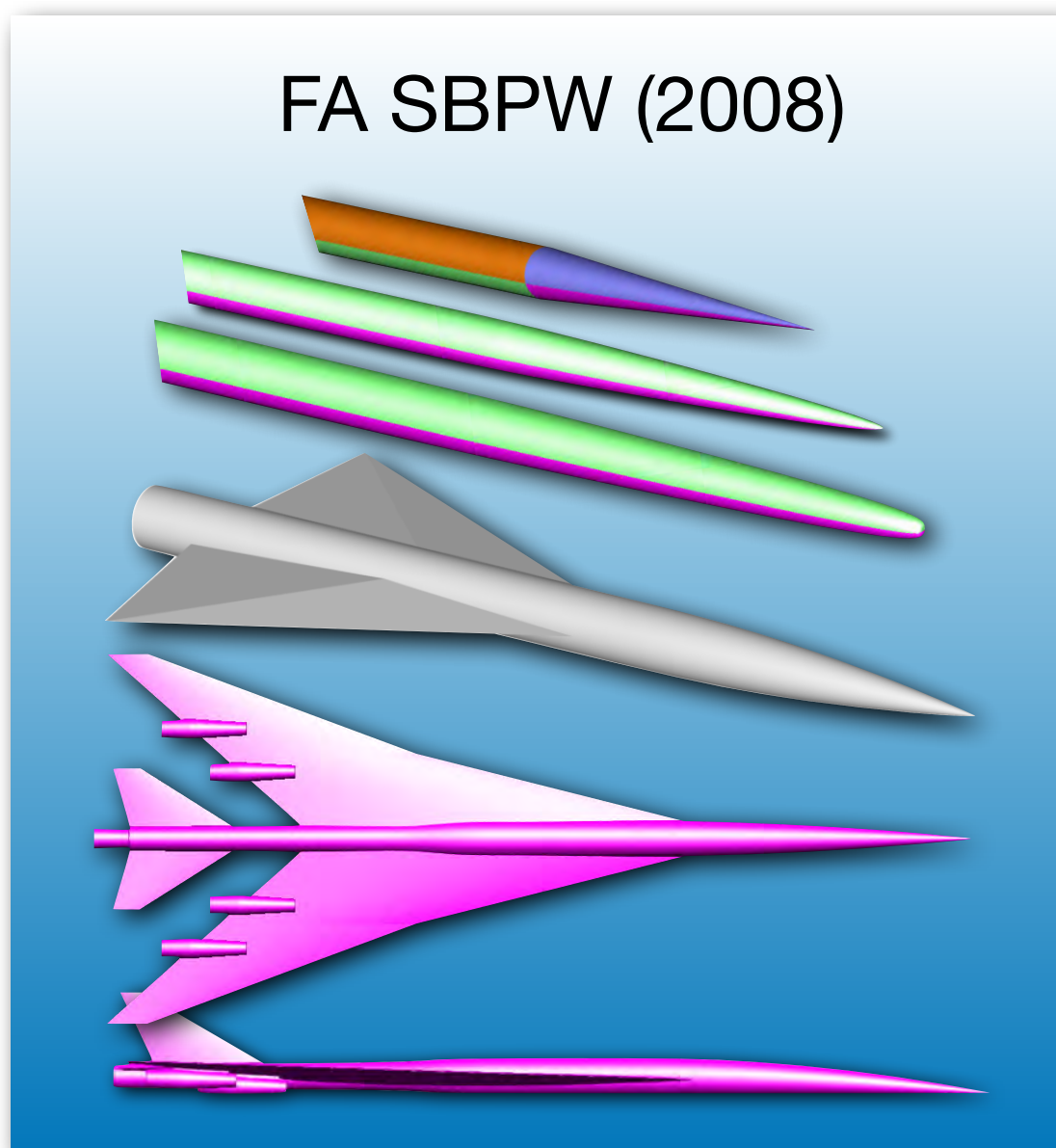




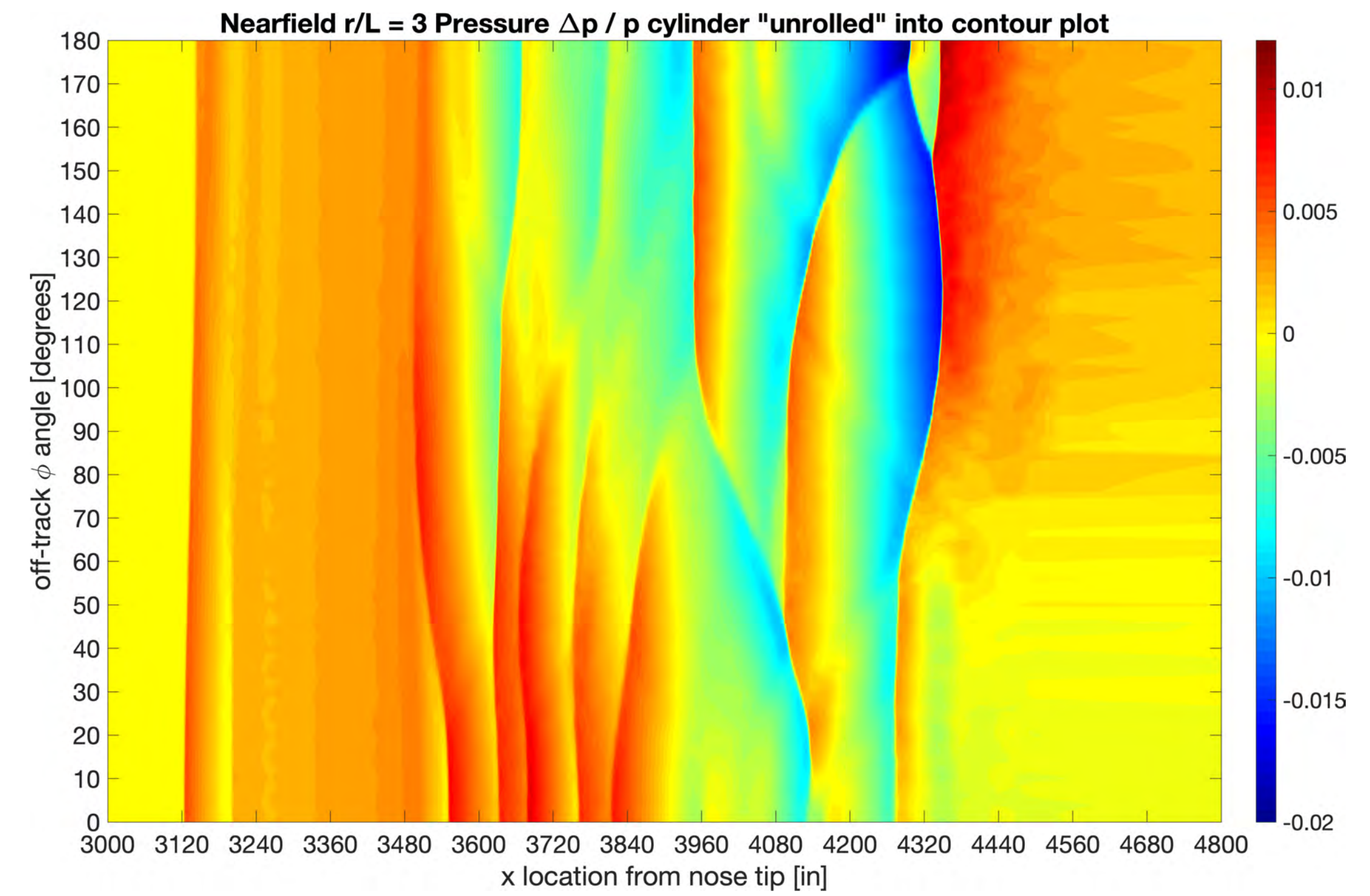
# Many Verification and Validation Studies

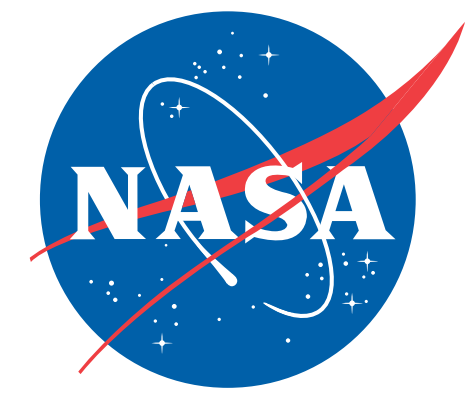


Third AIAA SBPW (2019)



**Progress**  
from **nearfield-only**  
**on-track simulations** on  
simplified geometries with no  
propulsion to **full ground noise**  
**carpet X-59 simulations** with  
propulsive and secondary  
flow paths





# Outline

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## 1. Background & Motivation

## 2. Objectives

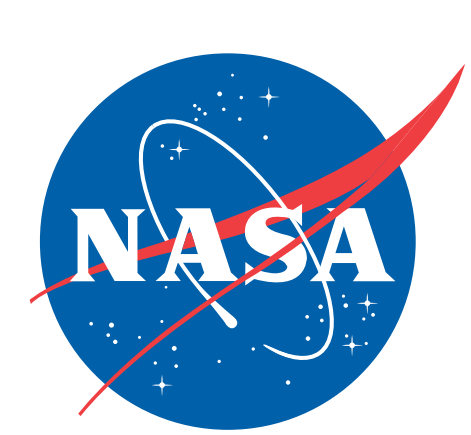
## 3. Methodology

- Uncertainty quantification
- Low-boom simulations
- Atmospheric uncertainty in propagation

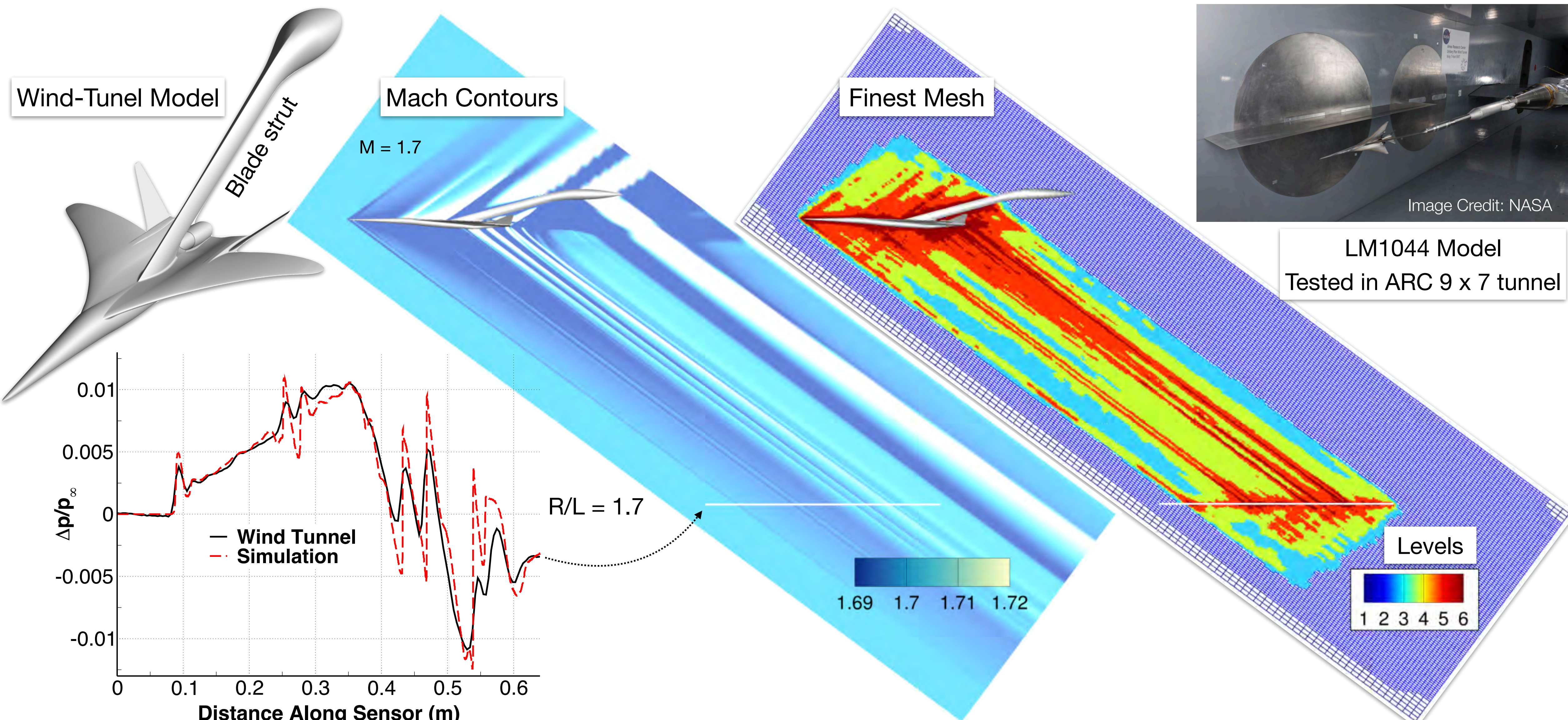
## **4. Results**

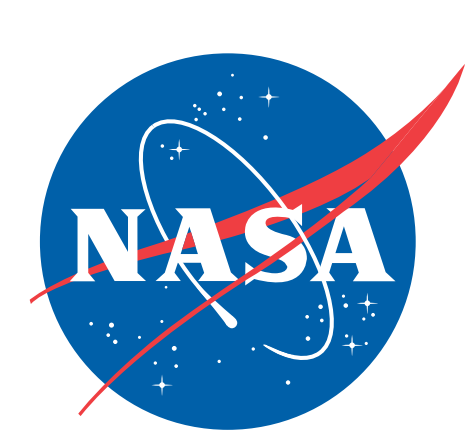
- Nearfield signature uncertainty estimates
- Uncertainty in atmospheric conditions
- Uncertainty in X-59 operating conditions and atmosphere

## 5. Summary & Outlook

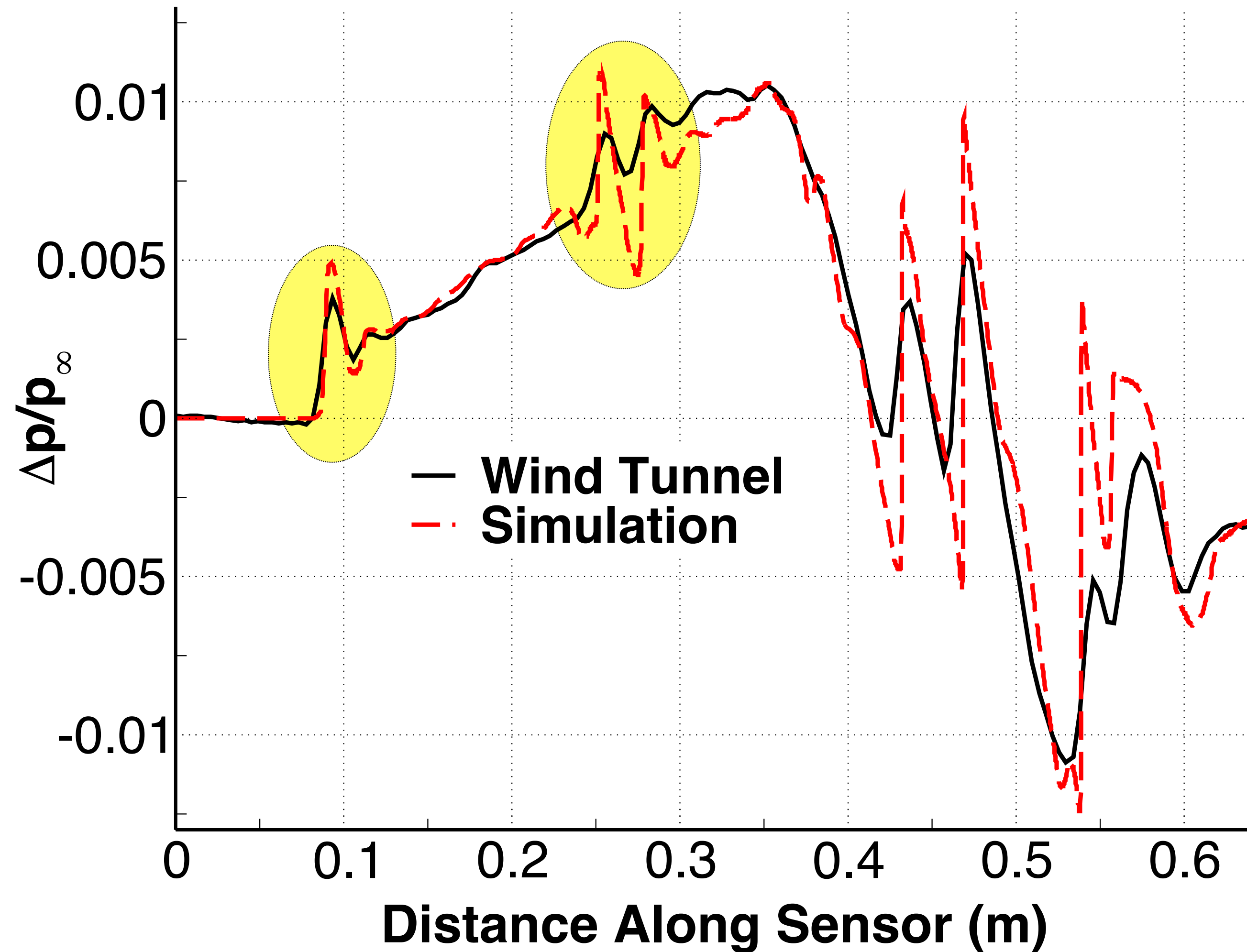


# Nearfield Signature Uncertainty Quantification

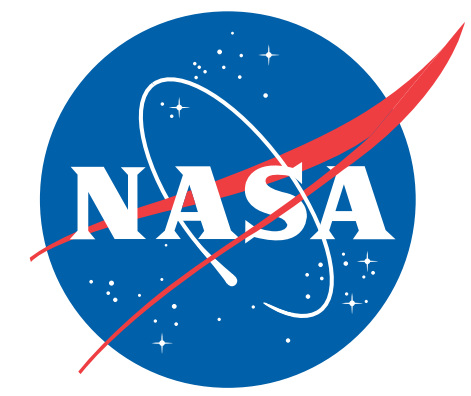




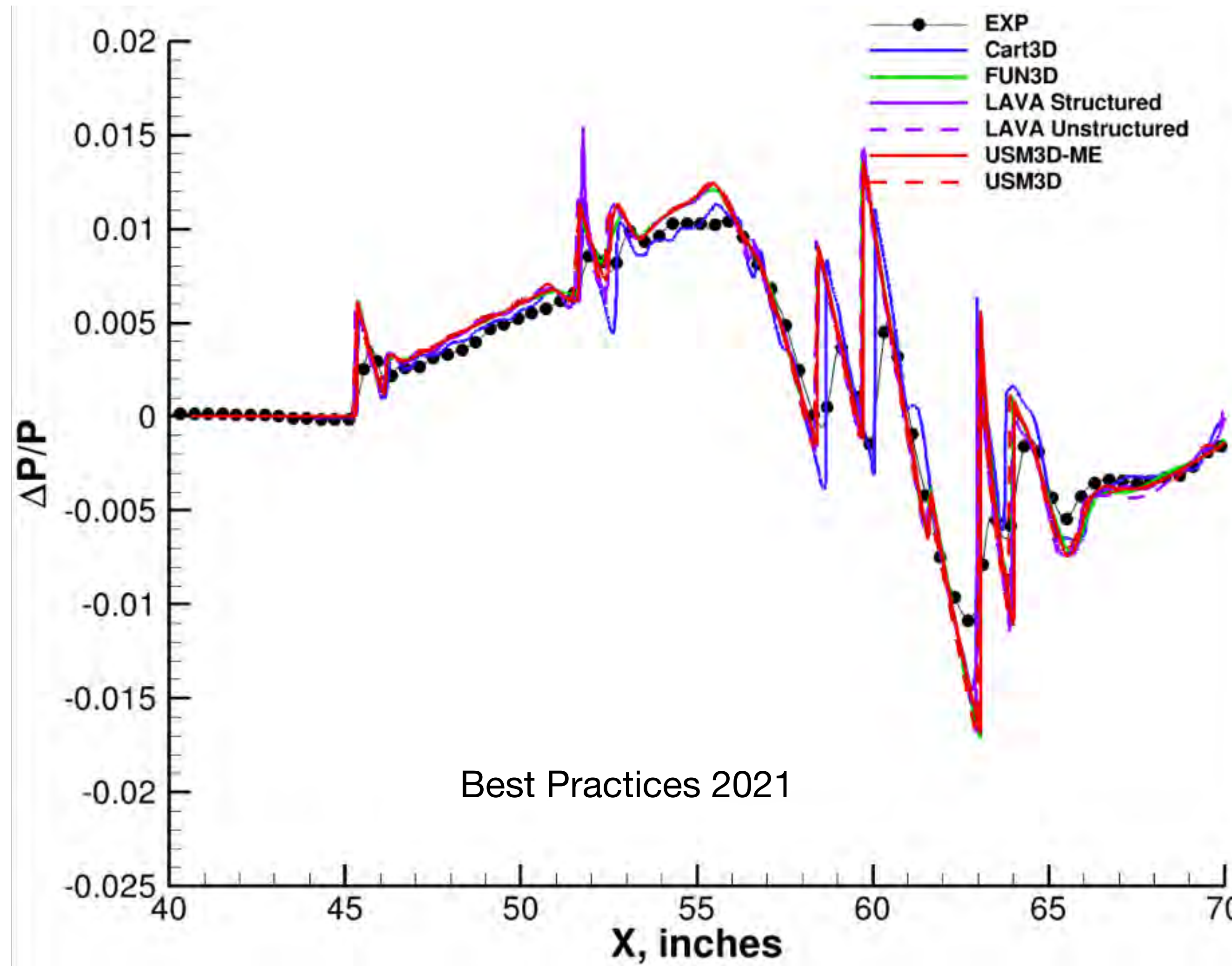
# Nearfield Signature UQ



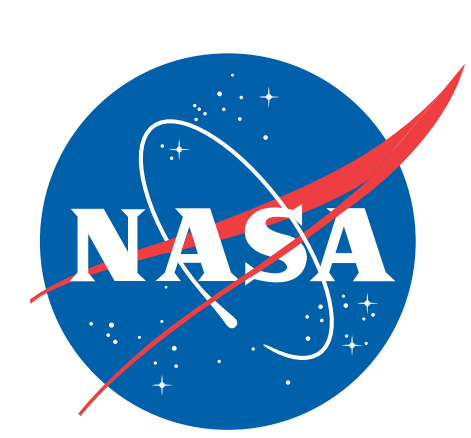
- Comparison of wind tunnel measurements to deterministic simulation reveals significant discrepancies starting at the leading shock, through the rooftop and the recovery region
- Predicted signature misses shock amplitude and rise time



# Nearfield Signature UQ

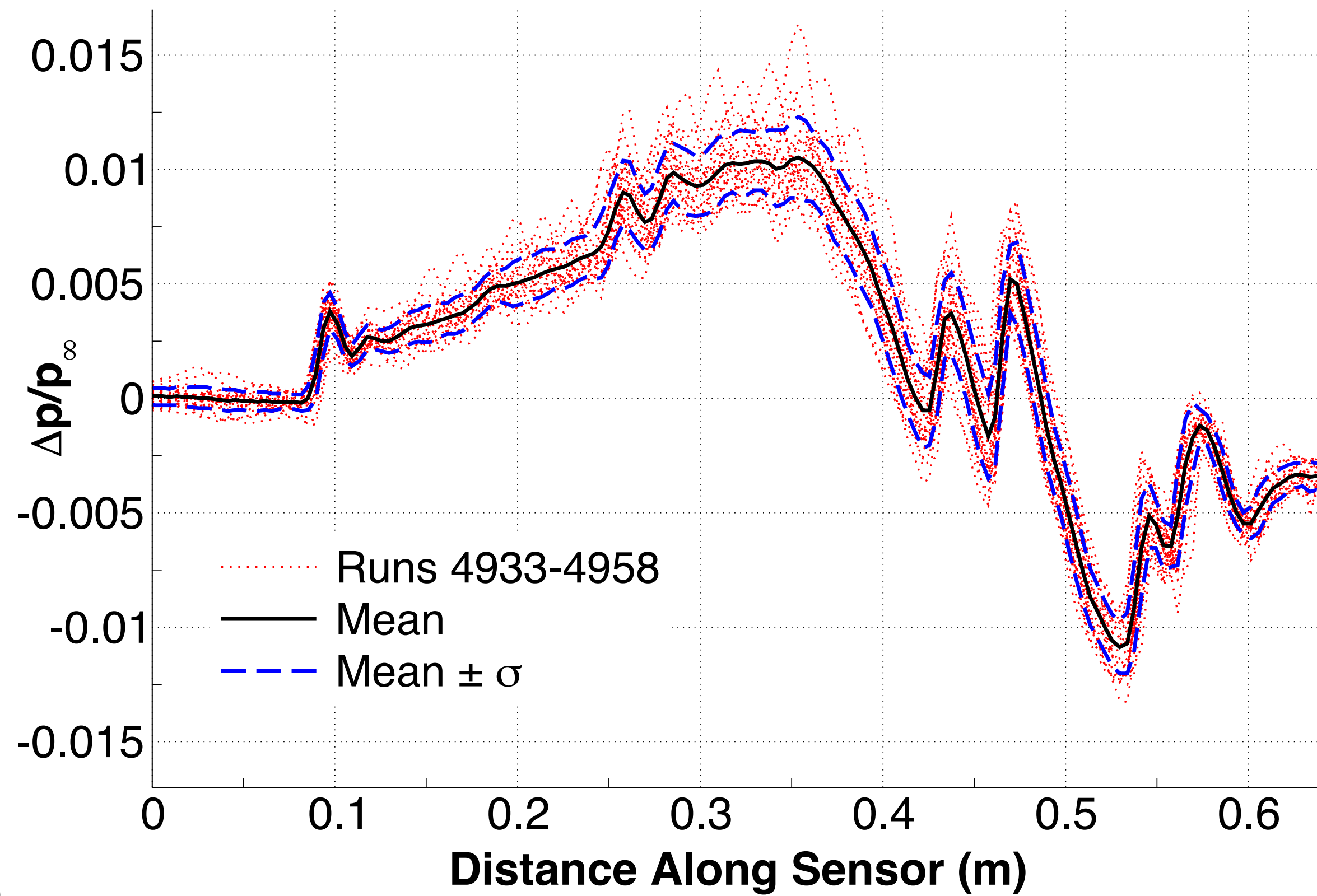


- Consistent predictions between codes
- Higher modeling fidelity does not significantly improve agreement with measurements
- All predicted signatures miss shock amplitude and rise time

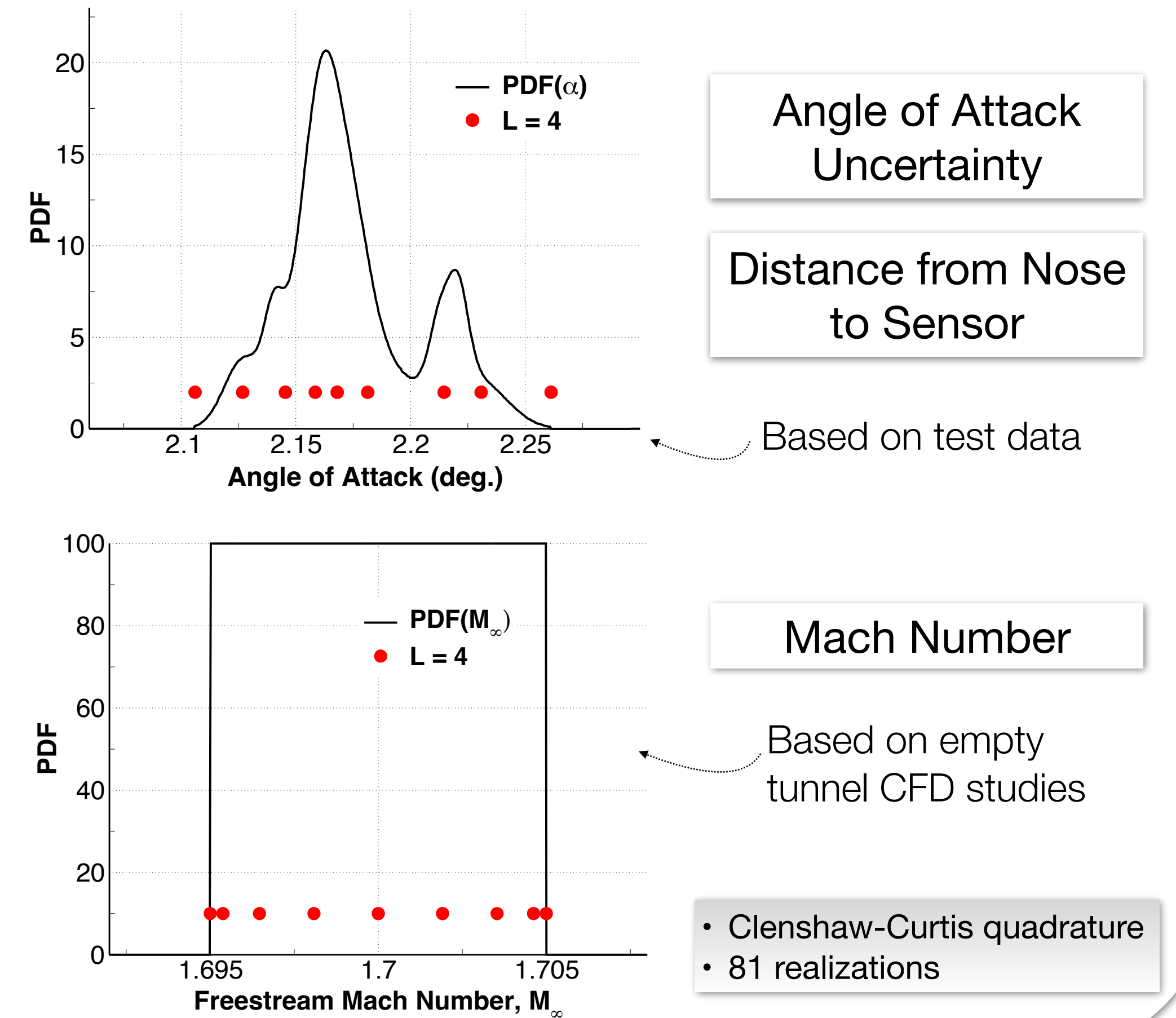


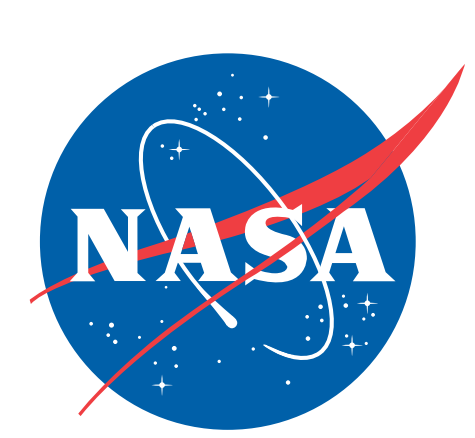
# Experimental Measurements & Random Variables

### Measured Signatures



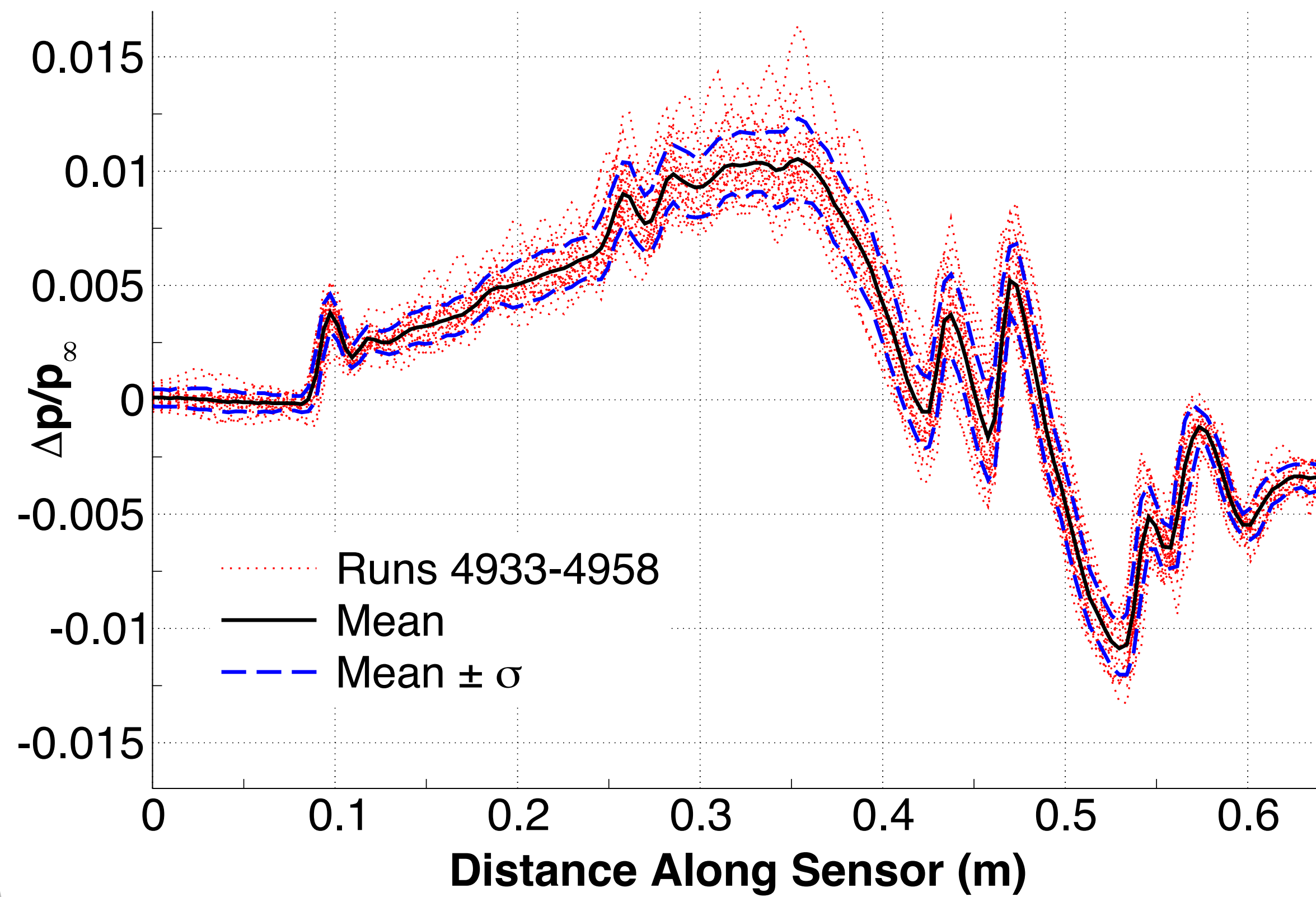
### Input Random Variables



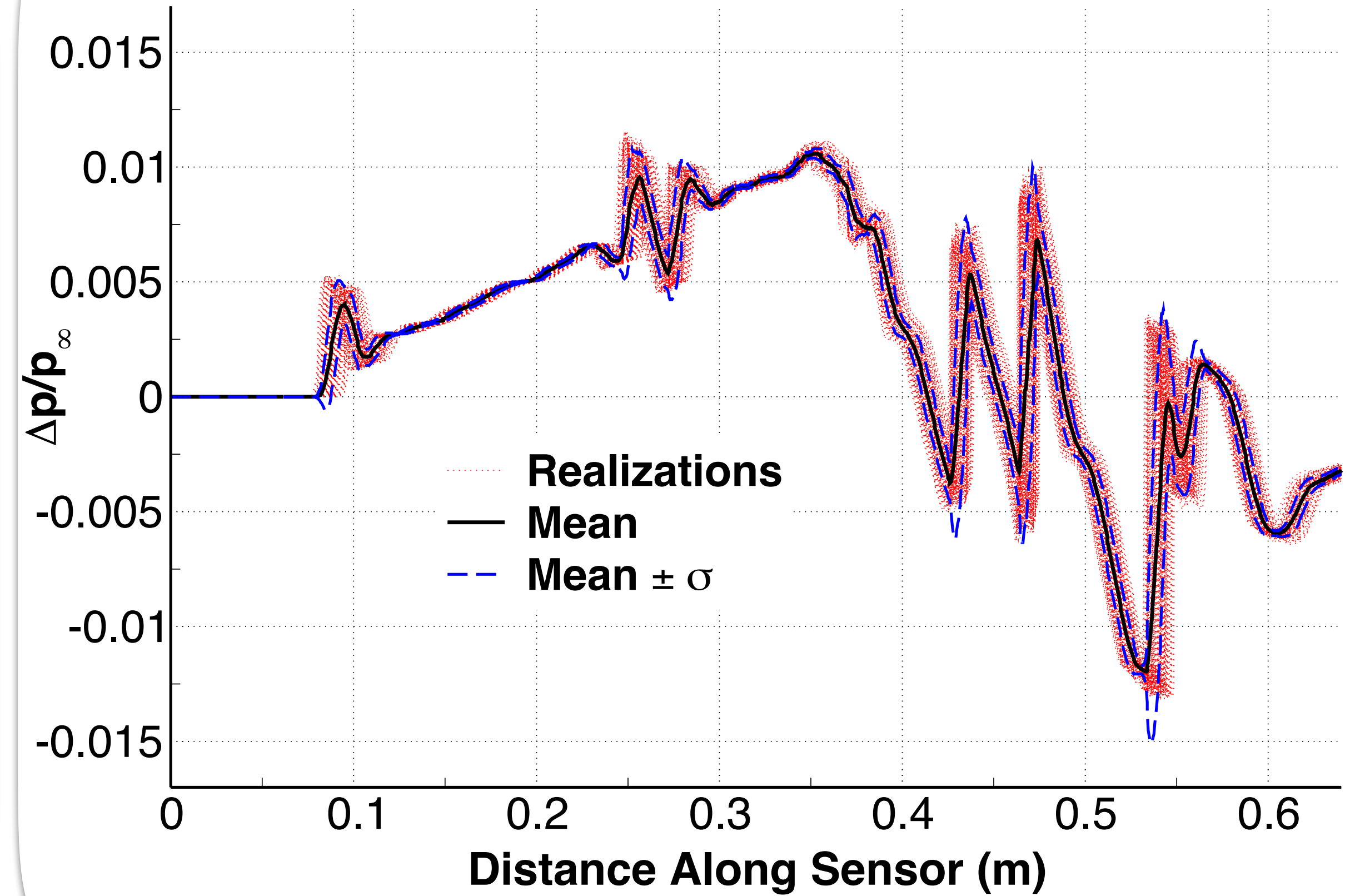


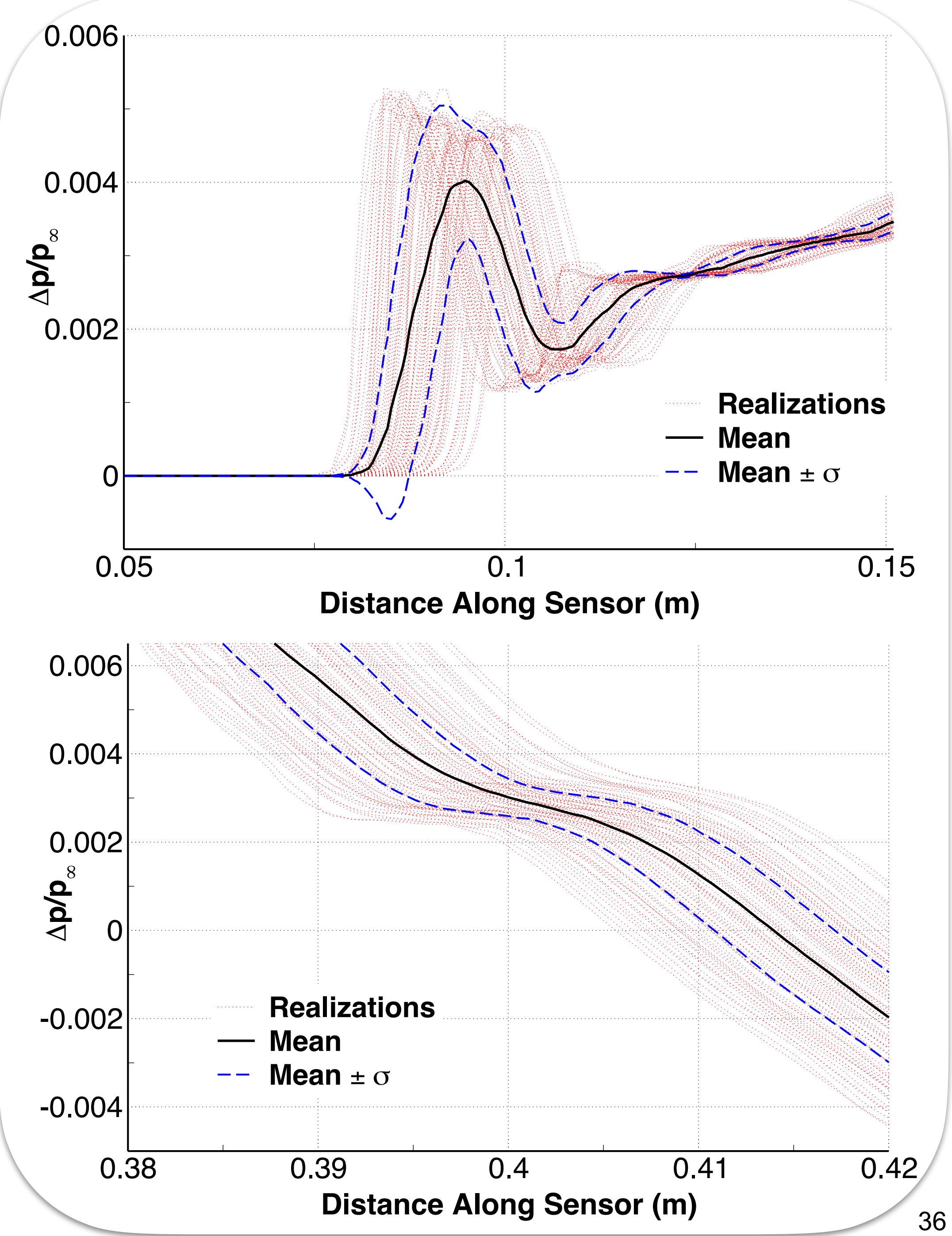
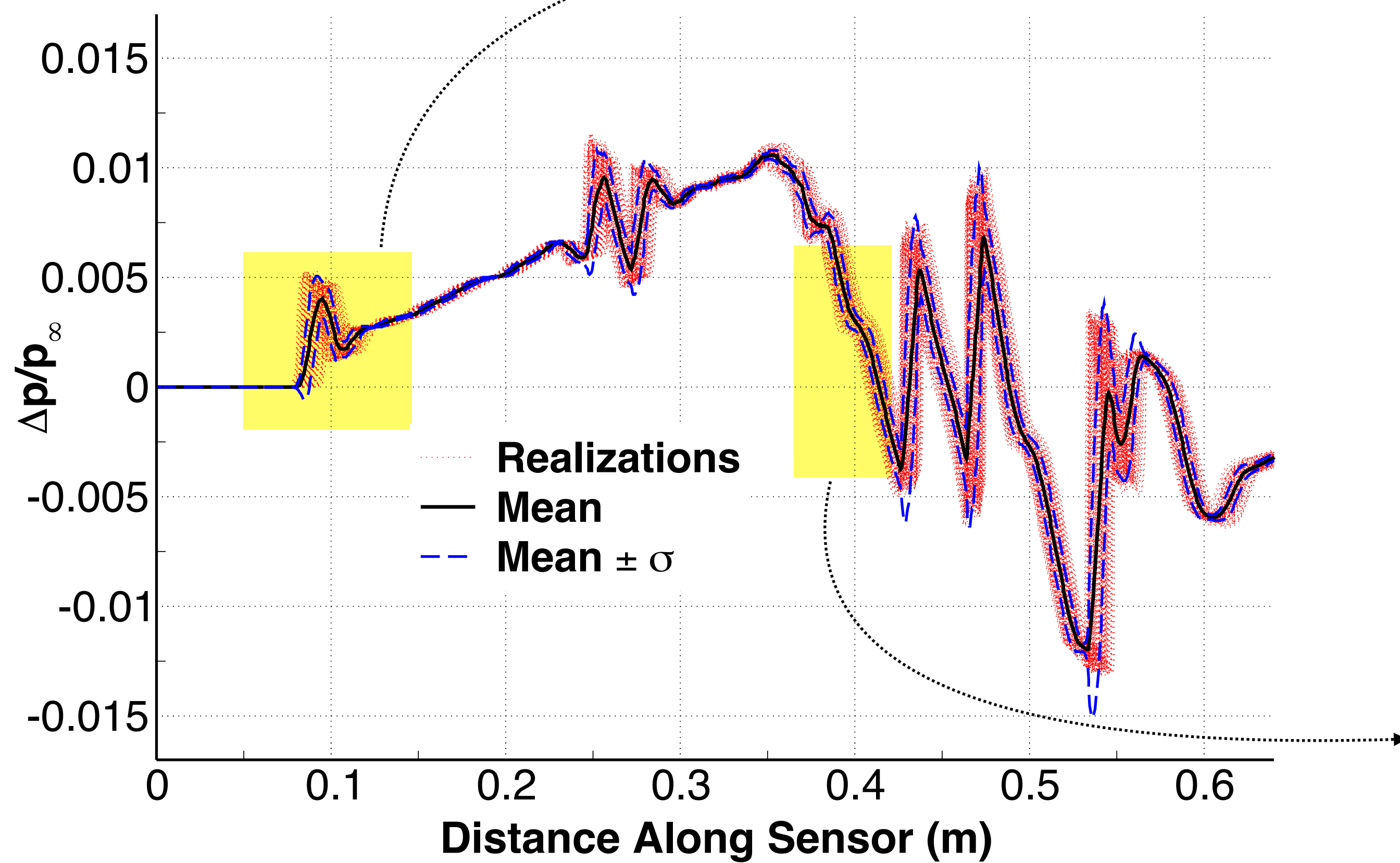
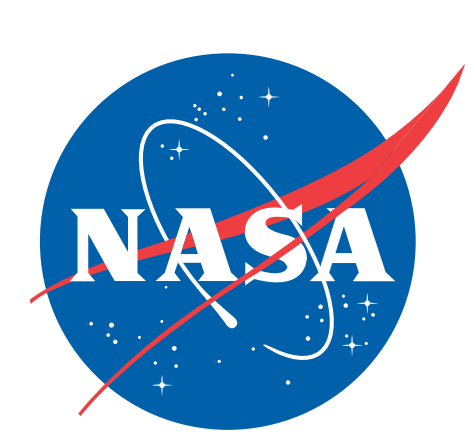
# Uncertain Measurements & Simulations

### Measured Signatures

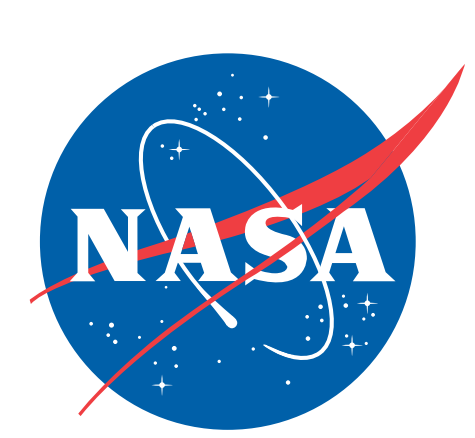


### Simulated Signatures

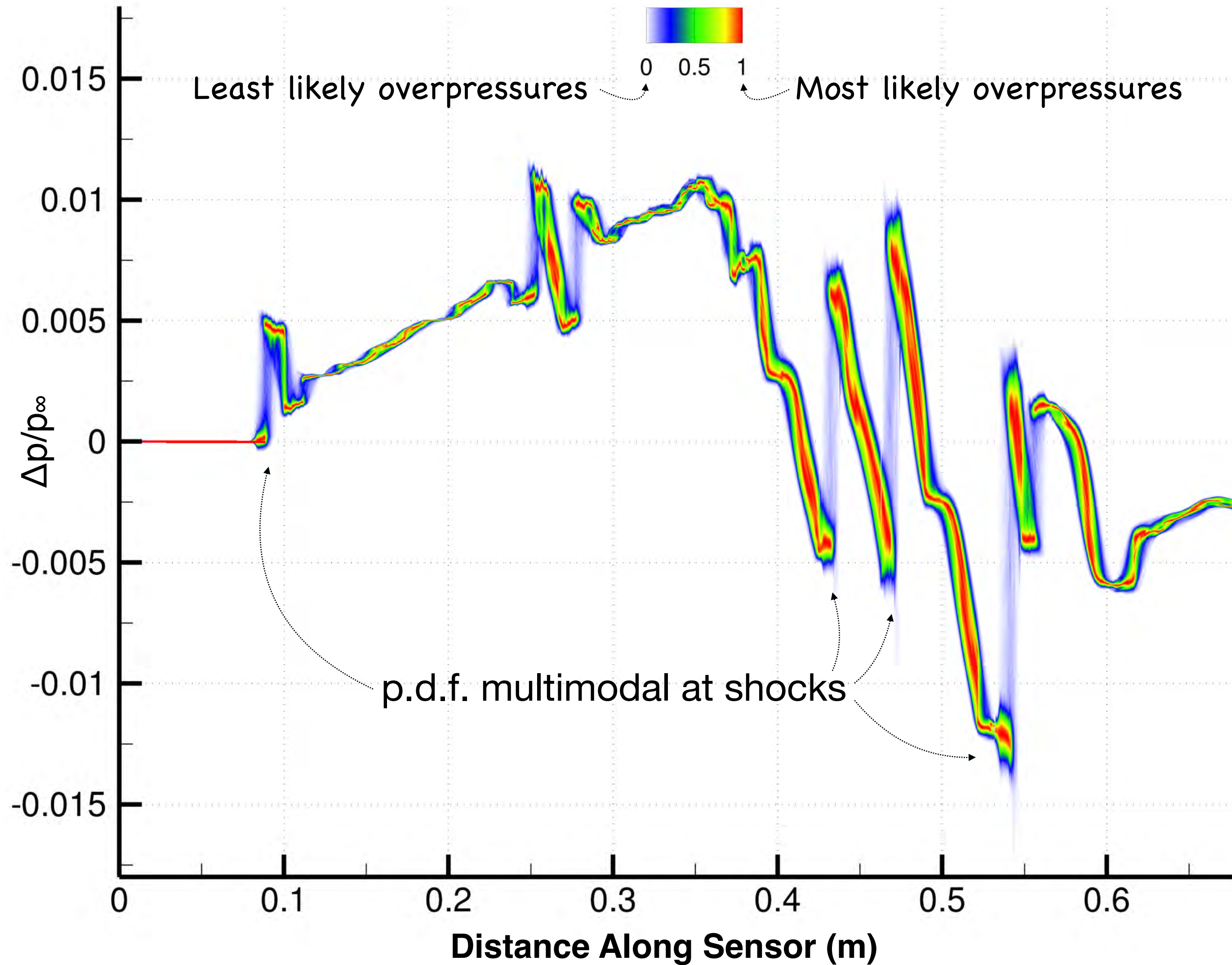




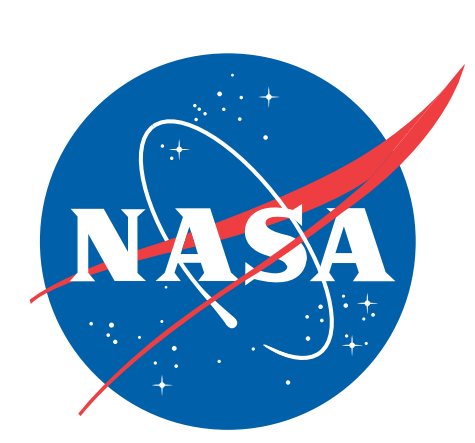
- Similar to measured signatures, the mean reduces shock amplitude and relaxes shock rise-time
- Standard deviation mischaracterizes data spread at discontinuities
- Moment statistics characterize uncertainty well in smooth regions



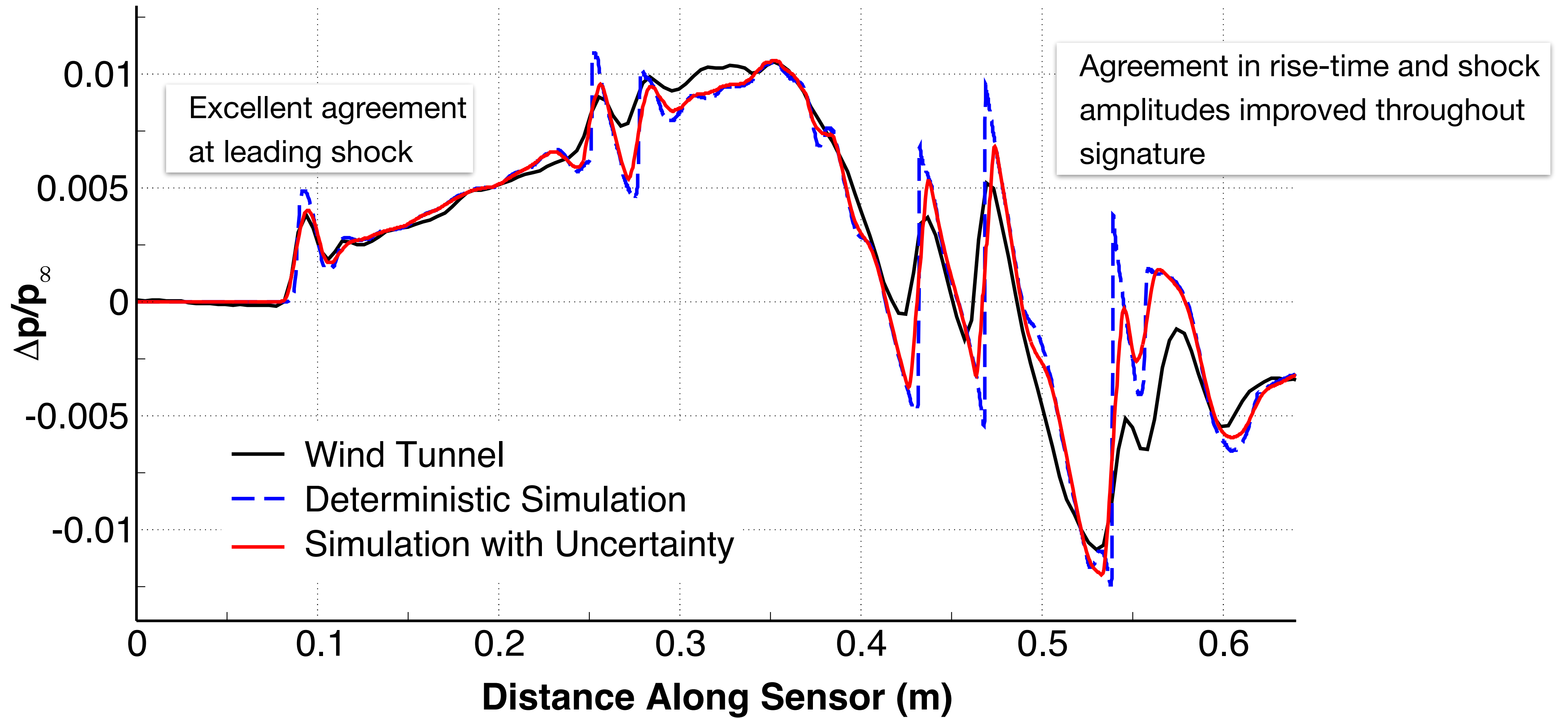
# Signature Probability Density Function

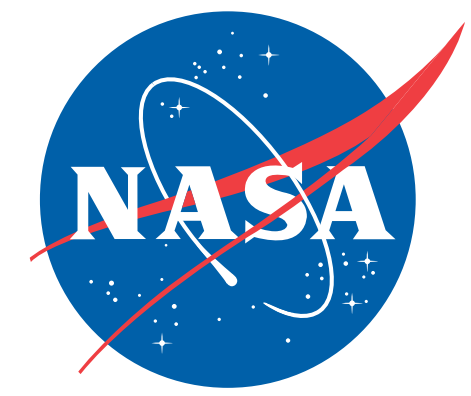


- p.d.f. provides complete characterization of uncertainty
- computed from the same database (81 realizations) as moment statistics



# Comparison of Mean Signatures

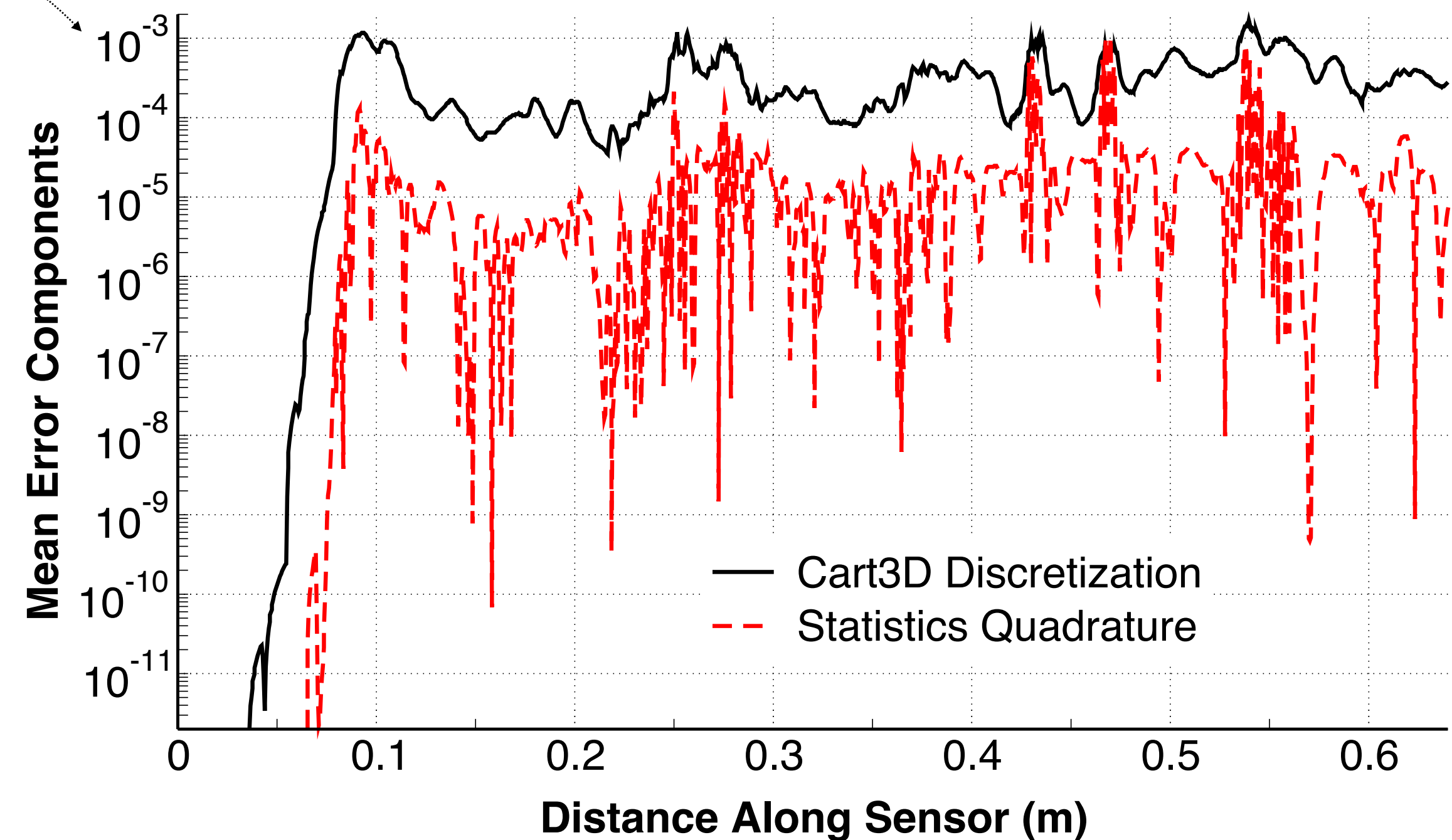
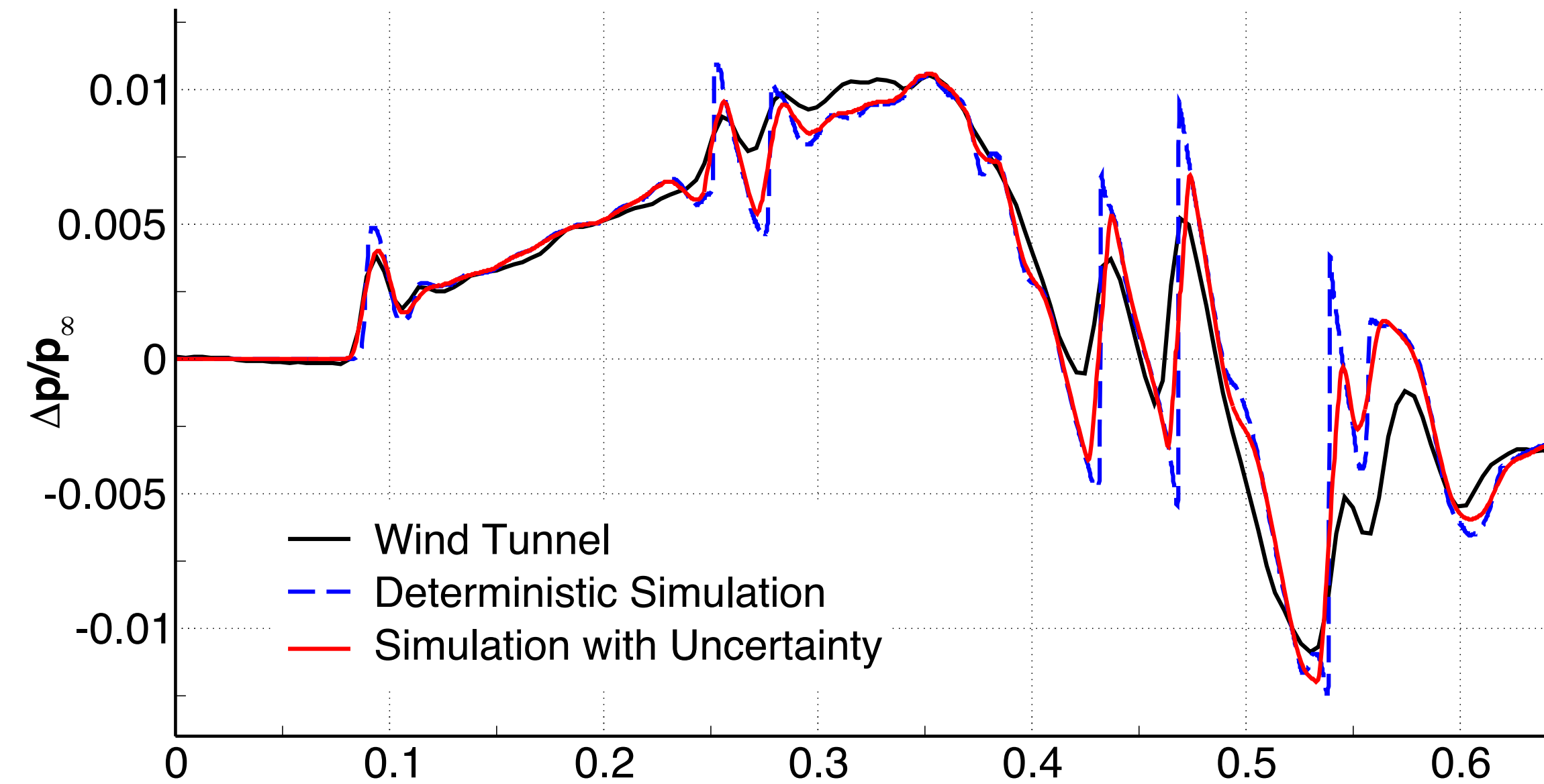


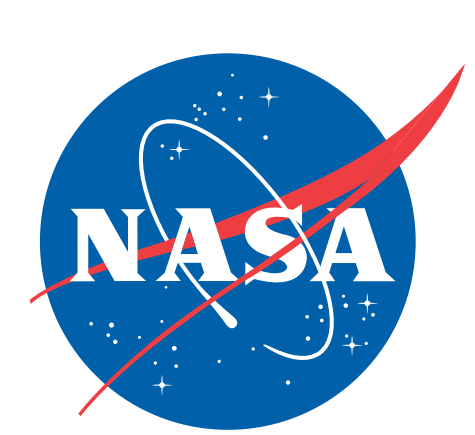


# Error Assessment

## Error Decomposition for Mean Signature

- Error distributions peak at shocks
- Discretization error from Cart3D simulations is approximately order-of-magnitude larger than error from statistics quadrature
  - ▶ Reducing CFD discretization error (increasing mesh refinement) is more important than increasing the number of quadrature points
  - ▶ Control of numerical errors critical for bounding model-form error





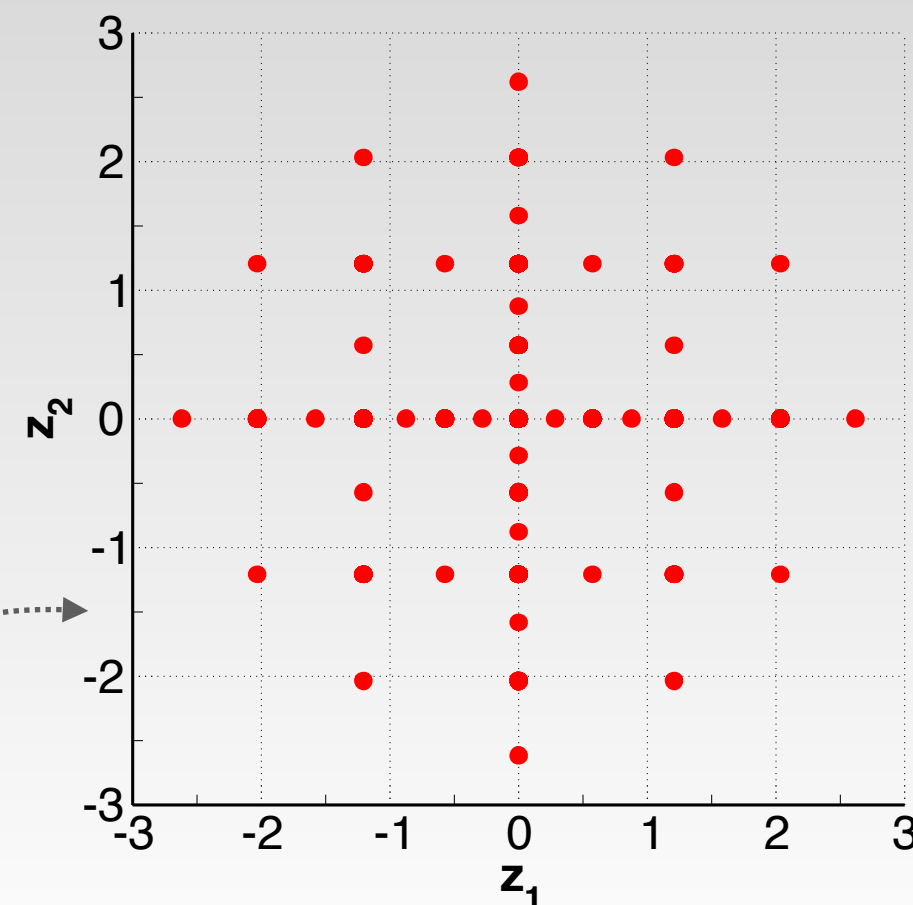
# Atmospheric Uncertainty in Propagation

## Characterize uncertainty in ground signature and loudness due to variability in state of the atmosphere

- Temperature (T) and relative humidity (RH) profiles only
- X-59 (C612A) nearfield signature, Mach number 1.4
- Location Edwards AFB
- Assume flights in August, around 10 am PT

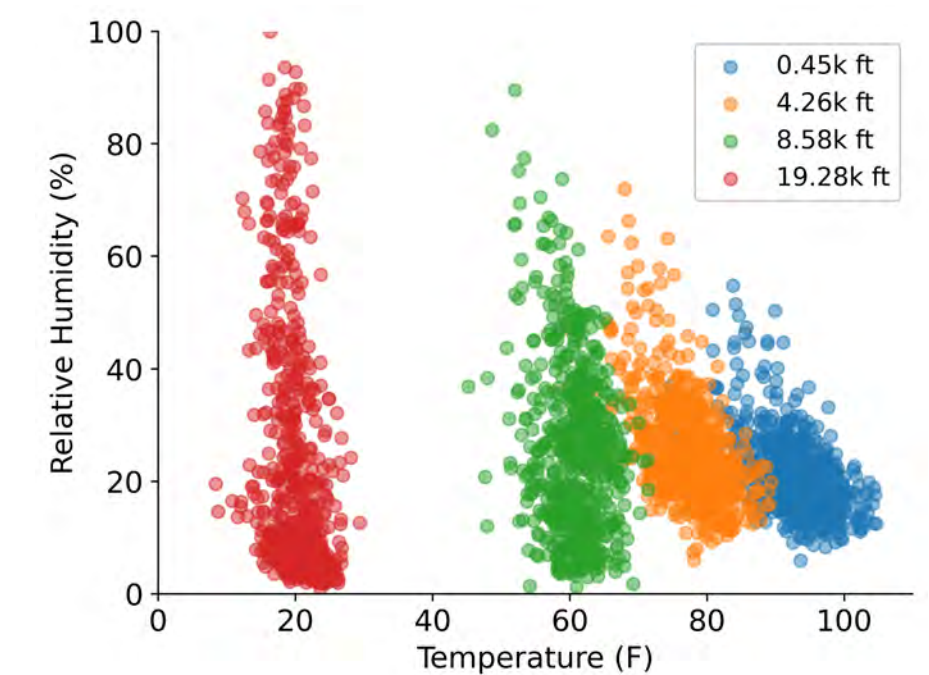
### UQ Inputs

- KL expansion with 6 eigenvalues (M=6) using temperature and dew-point temperature to model T and RH
  - ▶ 6 independent normal variables ( $z_1-z_6$ )
- L=4 sparse Gauss-Patterson quadrature
  - ▶ 545 atmospheric profiles for T and RH



### KL Expansion for Combined T and RH

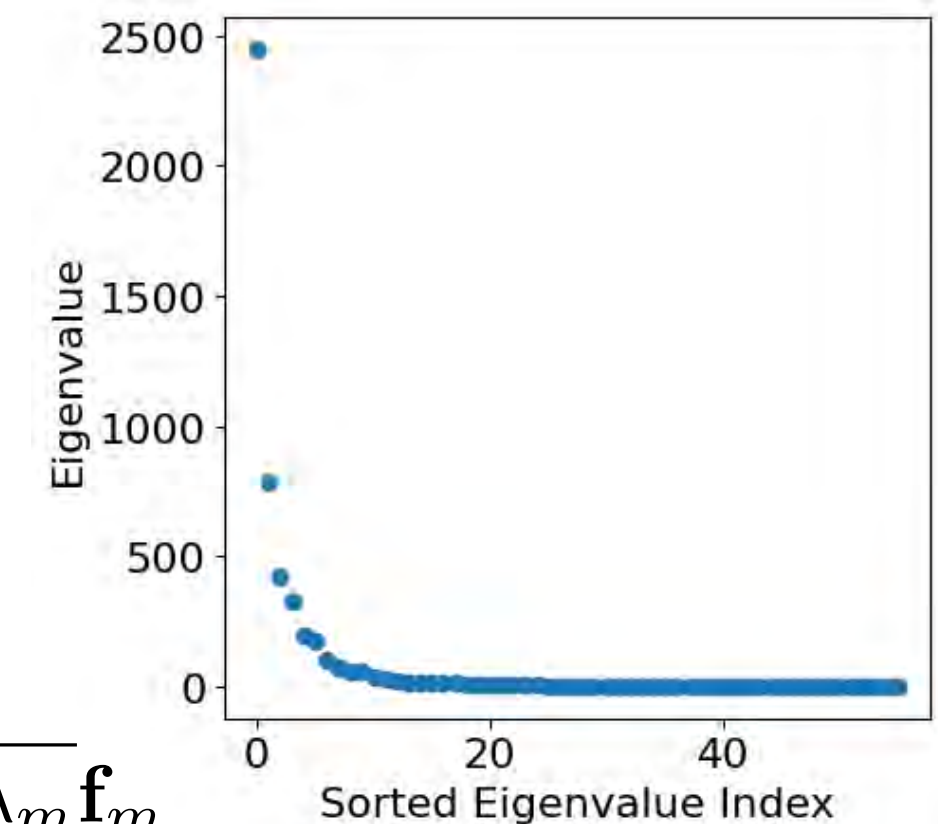
- RH distributions are skewed and bounded
  - ▶ Poor fit for KL expansion
- Use conversion from RH to Dew Point Temperature
- Fit both Dry Bulb and Dew Point Temperatures with one KL expansion

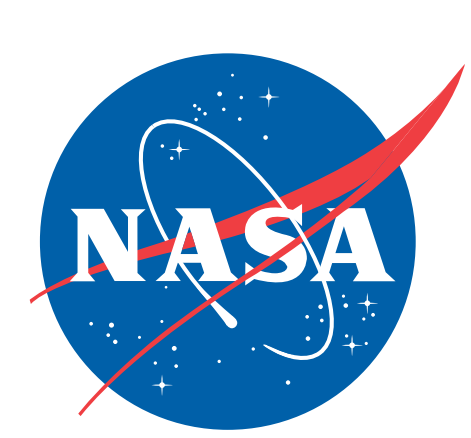


$$\mathbf{RH} = f(\mathbf{T}_{db}, \mathbf{T}_{dp})$$

$$\mathbf{T}(\xi) = \begin{bmatrix} \mathbf{T}_{db} \\ \mathbf{T}_{dp} \end{bmatrix}$$

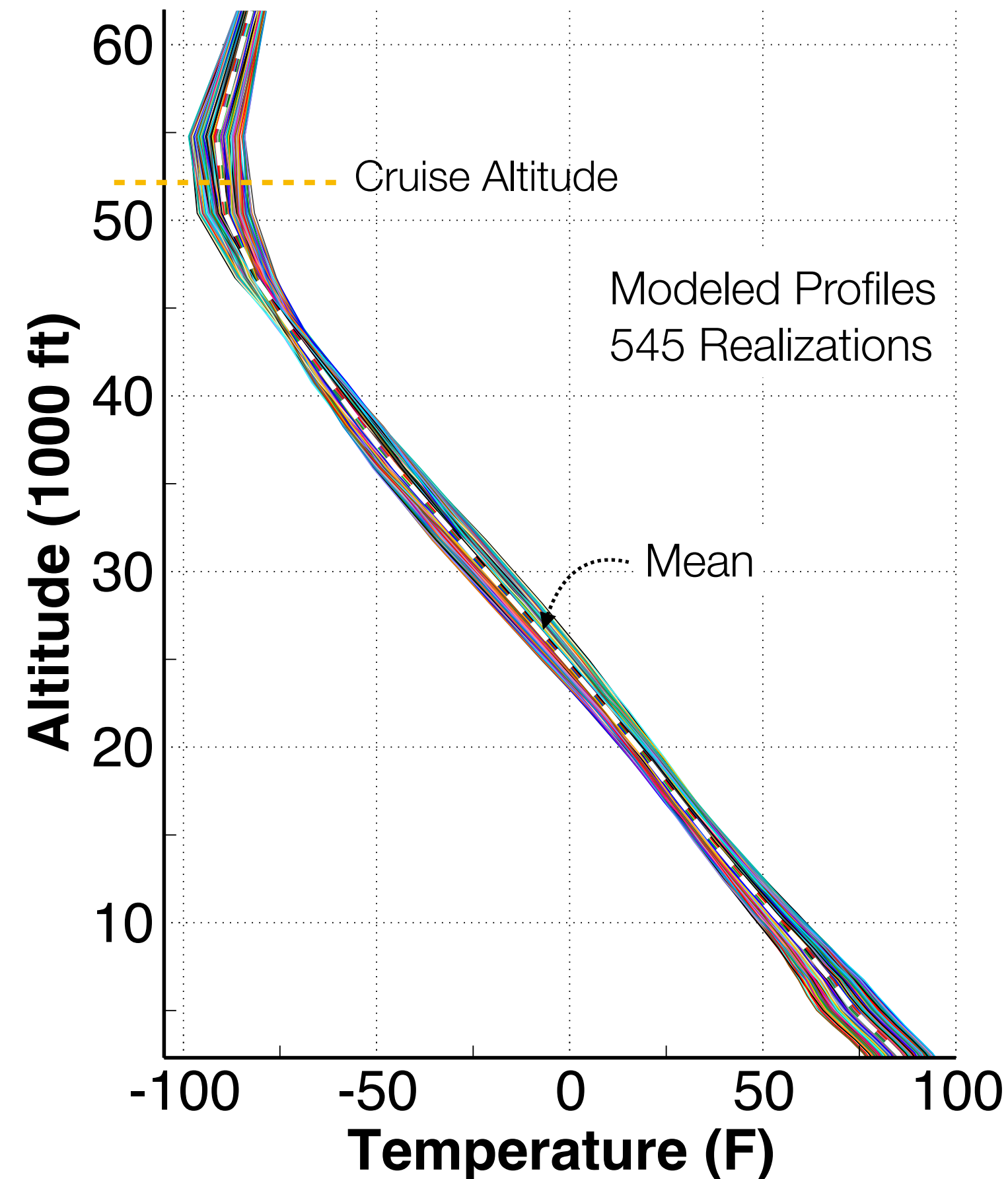
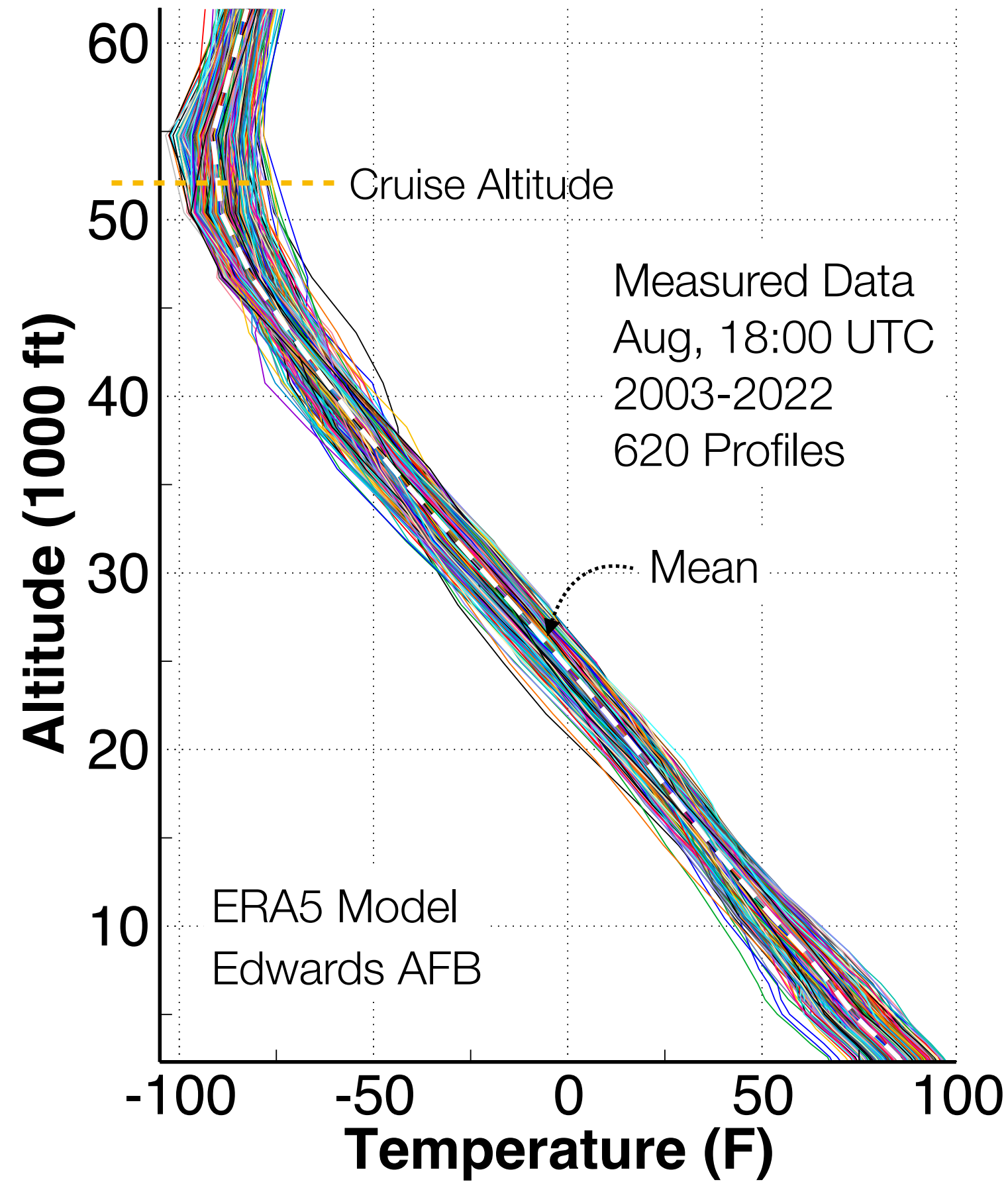
$$\mathbf{T}(\xi) = \bar{\mathbf{T}} + \sum_{m=1}^M z_m(\xi) \sqrt{\lambda_m} \mathbf{f}_m$$



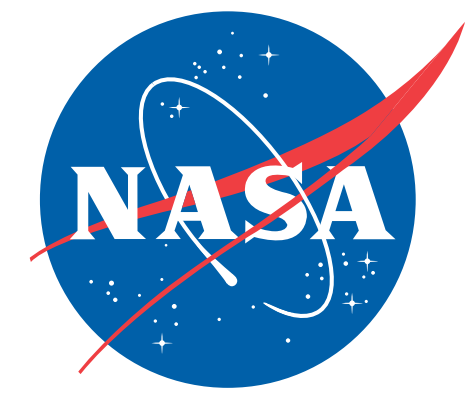


# Comparison of Modeled and Measured Profiles

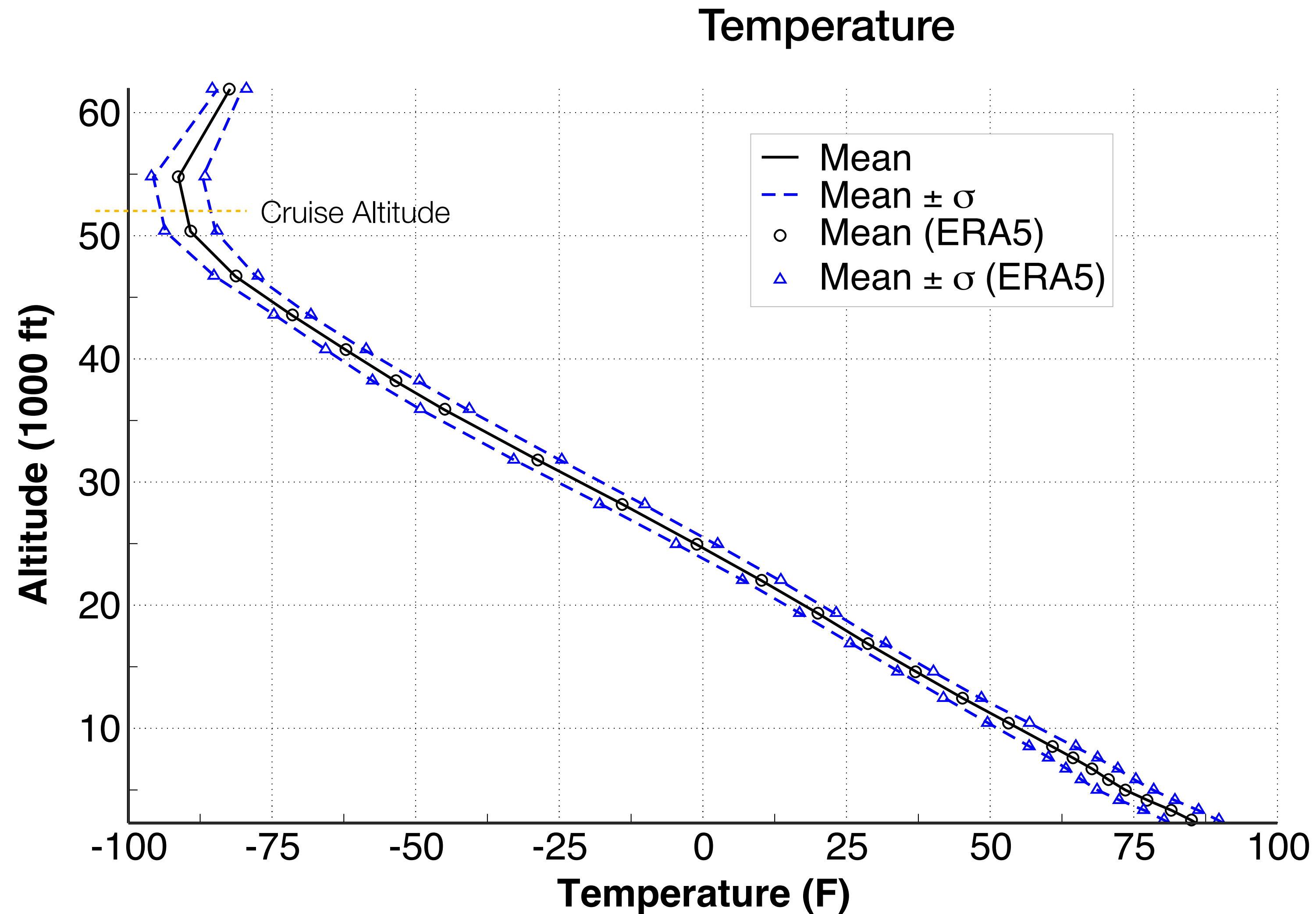
## Temperature



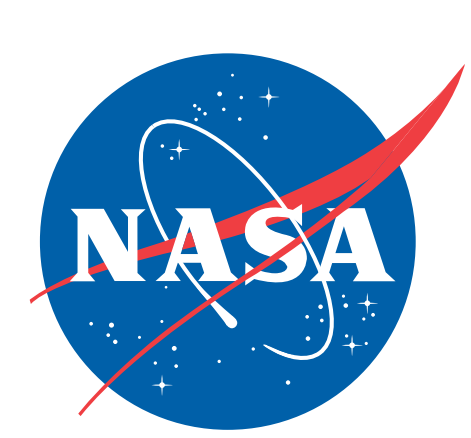
- KL-6 parameterization
- Level 4 Gauss-Patterson sparse quadrature
- Good agreement between modeled and measured profiles
  - Infrequent outliers not captured well
  - Excellent agreement near the mean



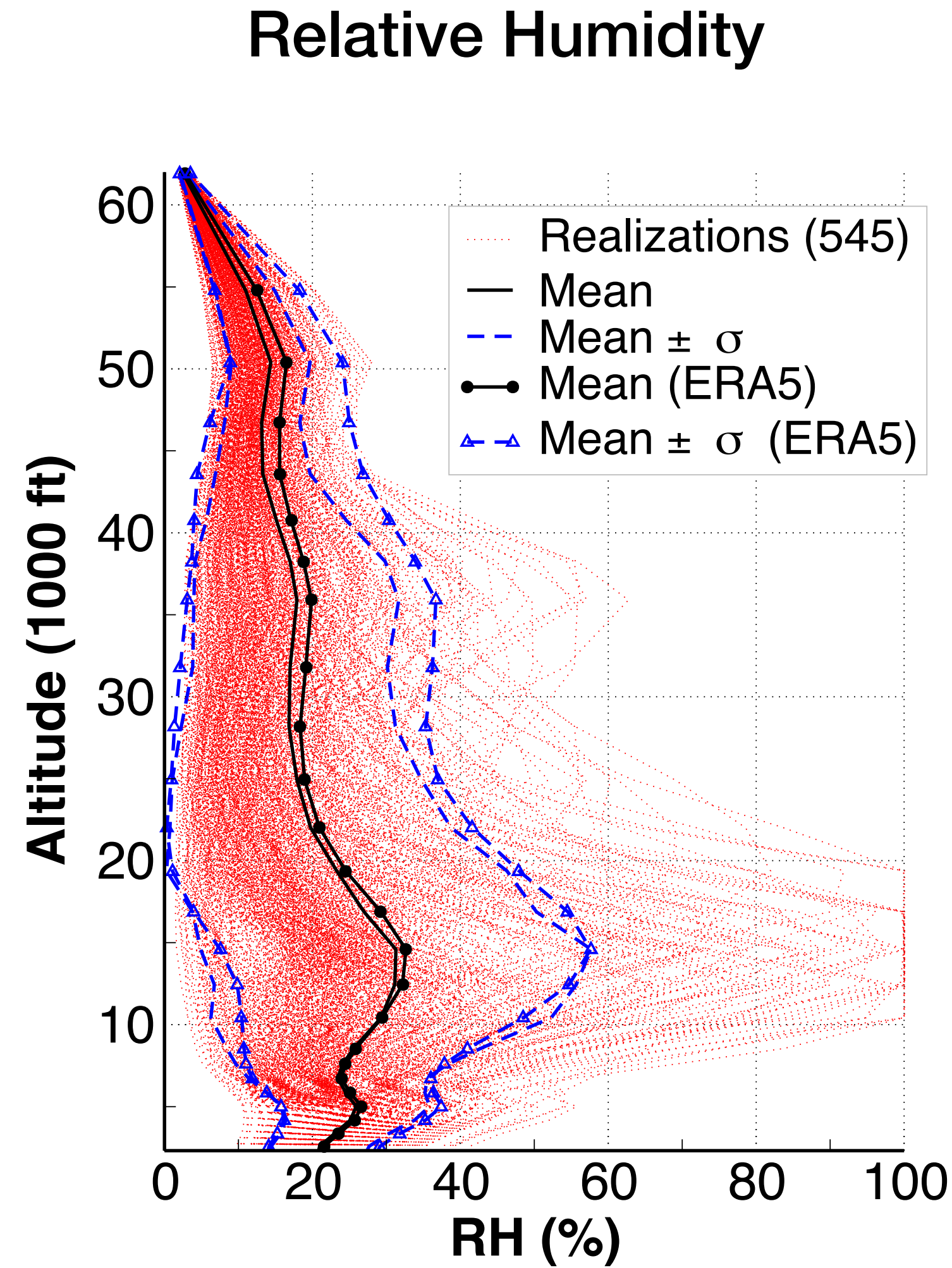
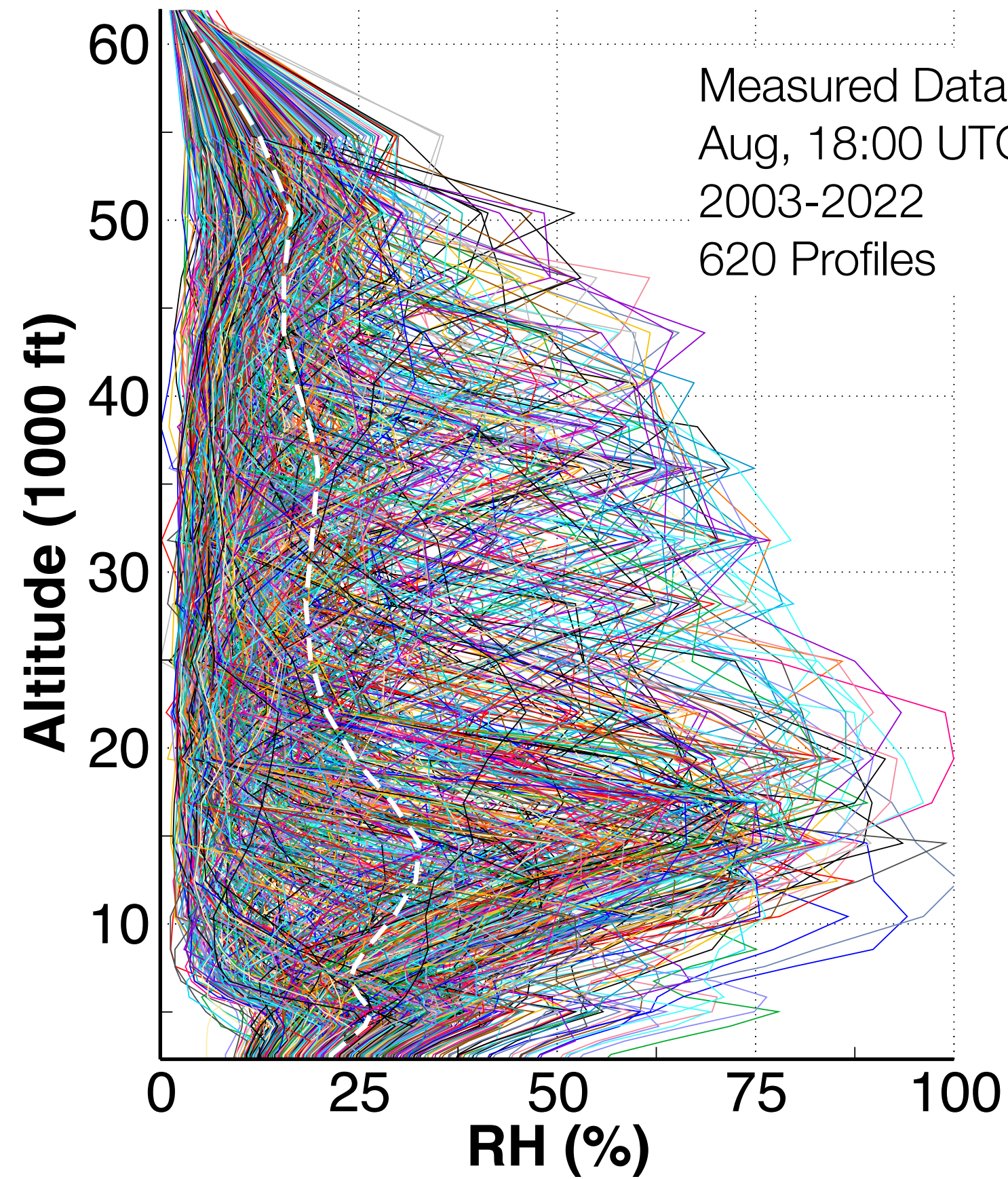
# Comparison of Modeled and Measured Profiles



- KL-6 parameterization
- Level 4 Gauss-Patterson sparse quadrature
- Mean profiles agree exactly (by construction)
- Agreement in standard deviation is excellent

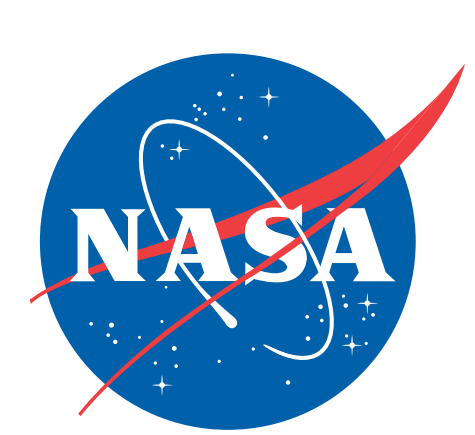


# Comparison of Modeled and Measured Profiles



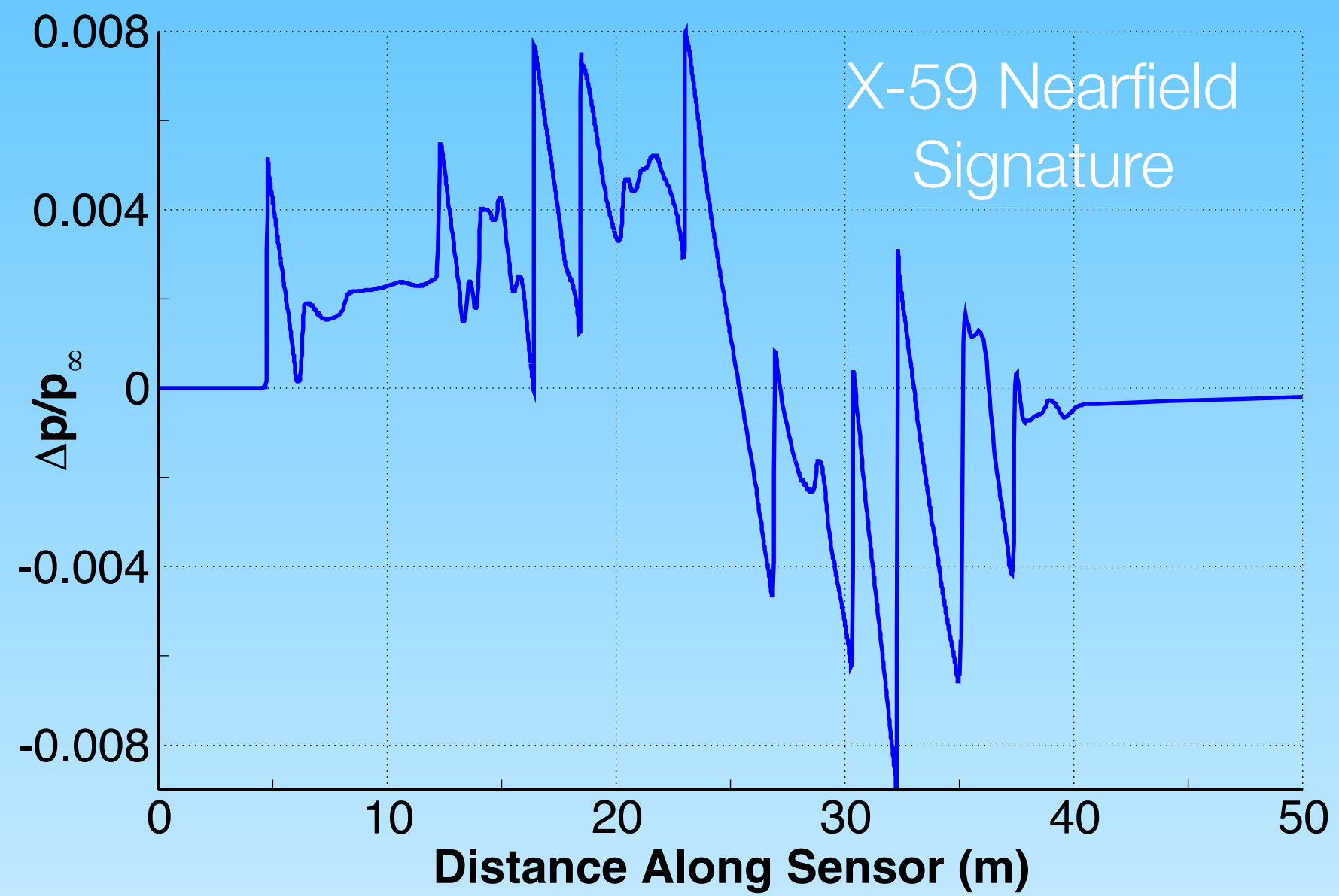
- KL-6 parameterization
- Level 4 Gauss-Patterson sparse quadrature
- Good agreement between modeled and measured profiles, especially at lower altitudes (below 25k ft)
  - Outliers not captured well between 40k-50k ft
  - 8 profiles exceed 100% and need clipping

Overall, KL-6 parametrization is an excellent compromise for accurately capturing main features of T & RH data at relatively low computational cost



# Propagation through Uncertain Atmosphere

## sBOOM Propagation Inputs



*Altitude 52,026 ft*

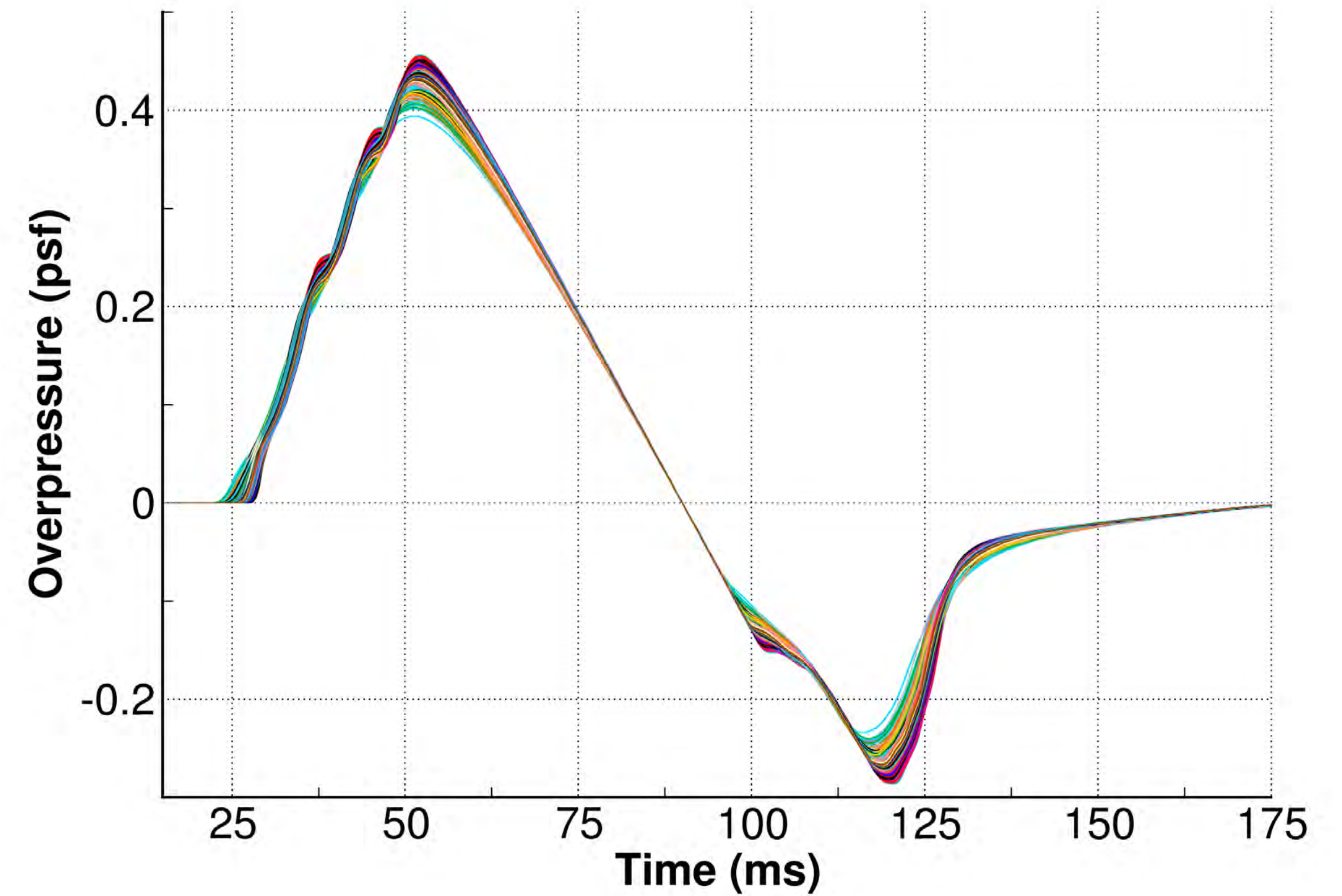
*Ground Elevation 2,311 ft*

*Terrain Reflection Factor 1.9*

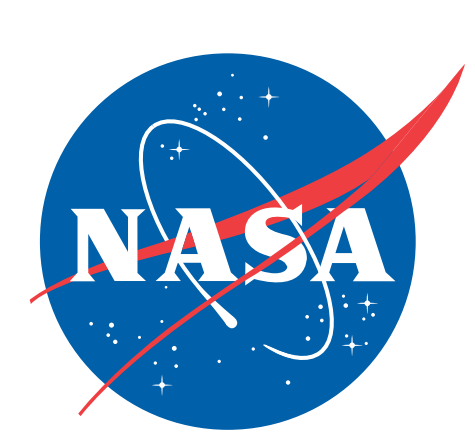
*Sampling Frequency 100kHz*



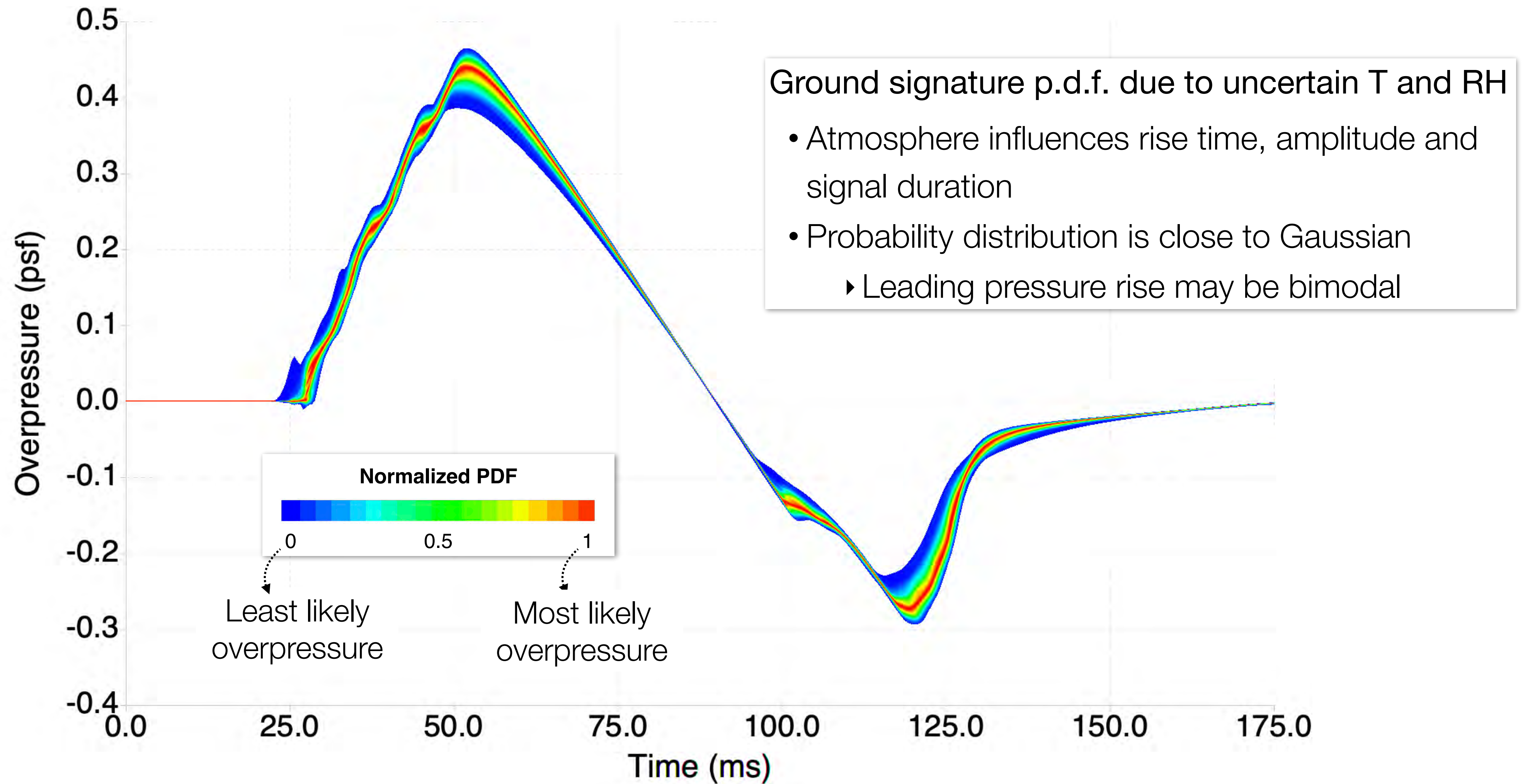
## Ground signatures resulting from propagation through 545 T and RH atmospheric profiles

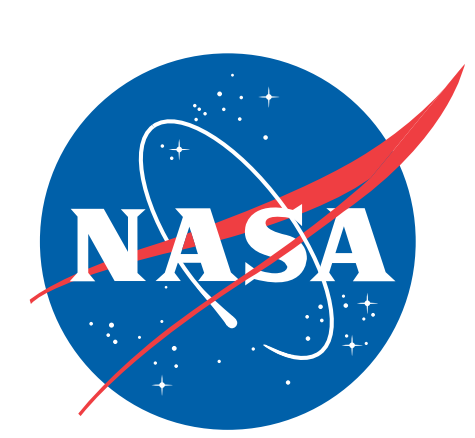


- Evaluate ground signature and loudness statistics from these realizations

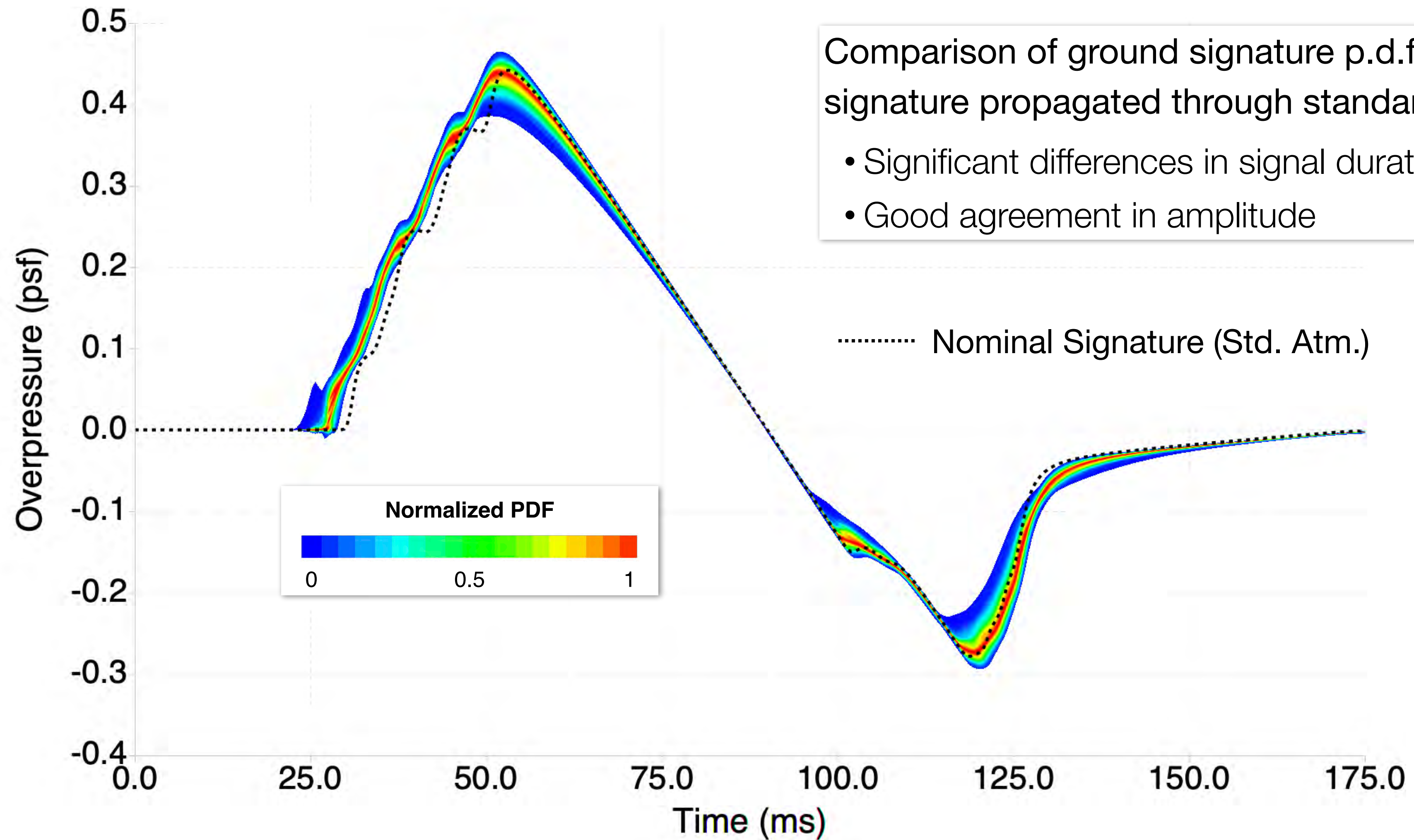


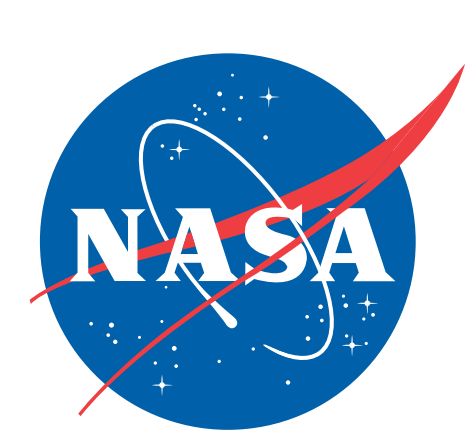
# Propagation through Uncertain Atmosphere



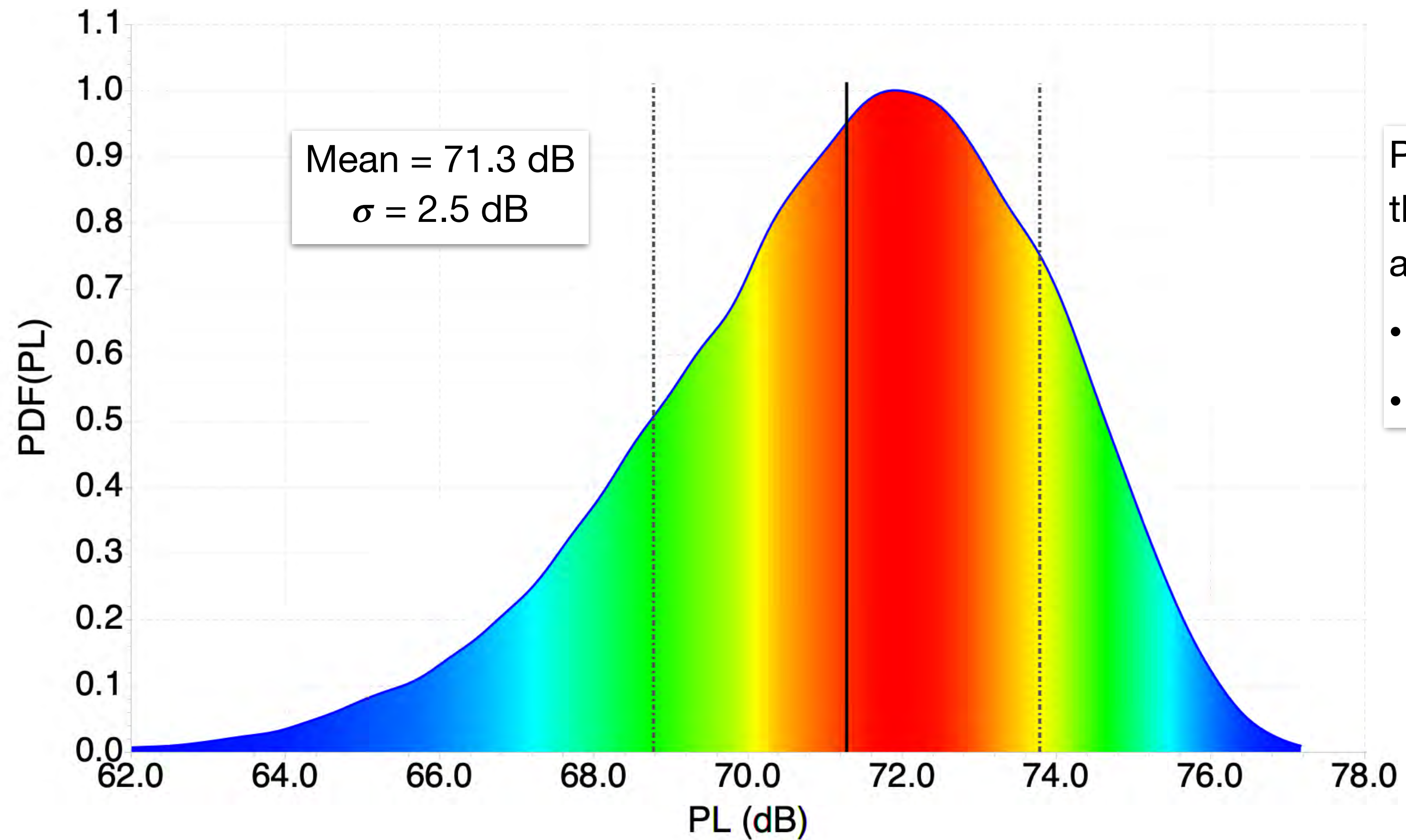


# Propagation through Uncertain Atmosphere



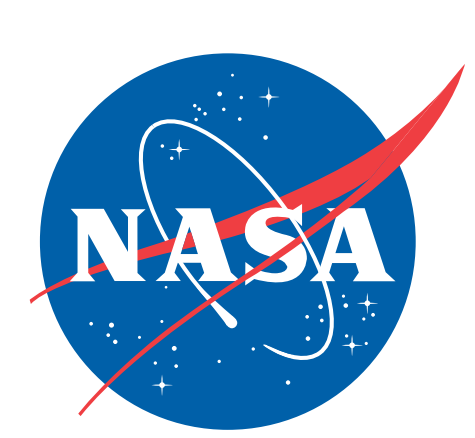


# Perceived Loudness (PL) P.D.F.

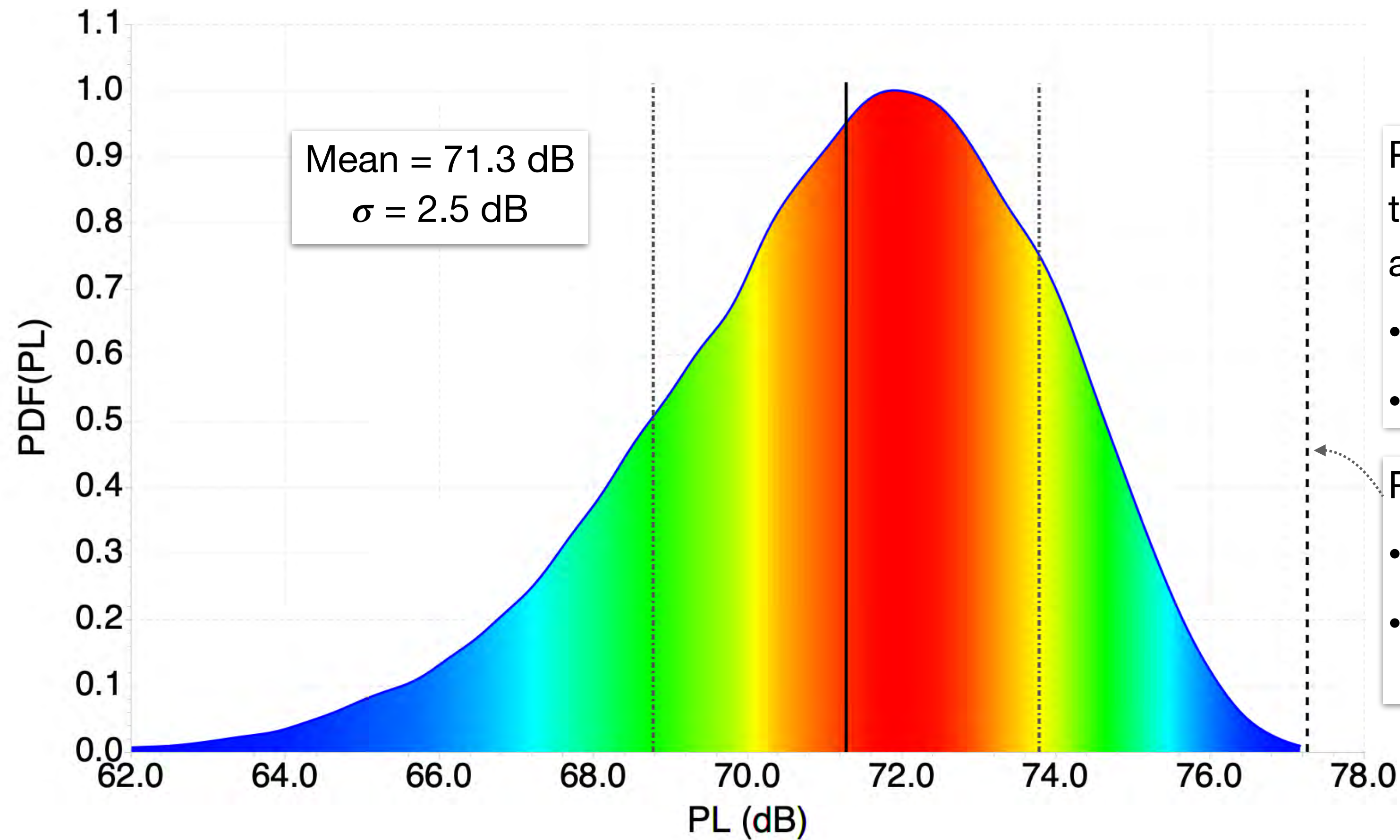


Probability density function fully characterizes the uncertainty in PL due to variability in T and RH atmospheric profiles

- Distribution is skewed to the left
- One standard deviation is well below 75 dB



# Perceived Loudness (PL) P.D.F.

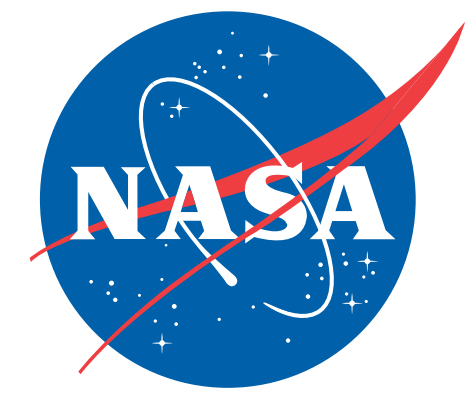


Probability density function fully characterizes the uncertainty in PL due to variability in T and RH atmospheric profiles

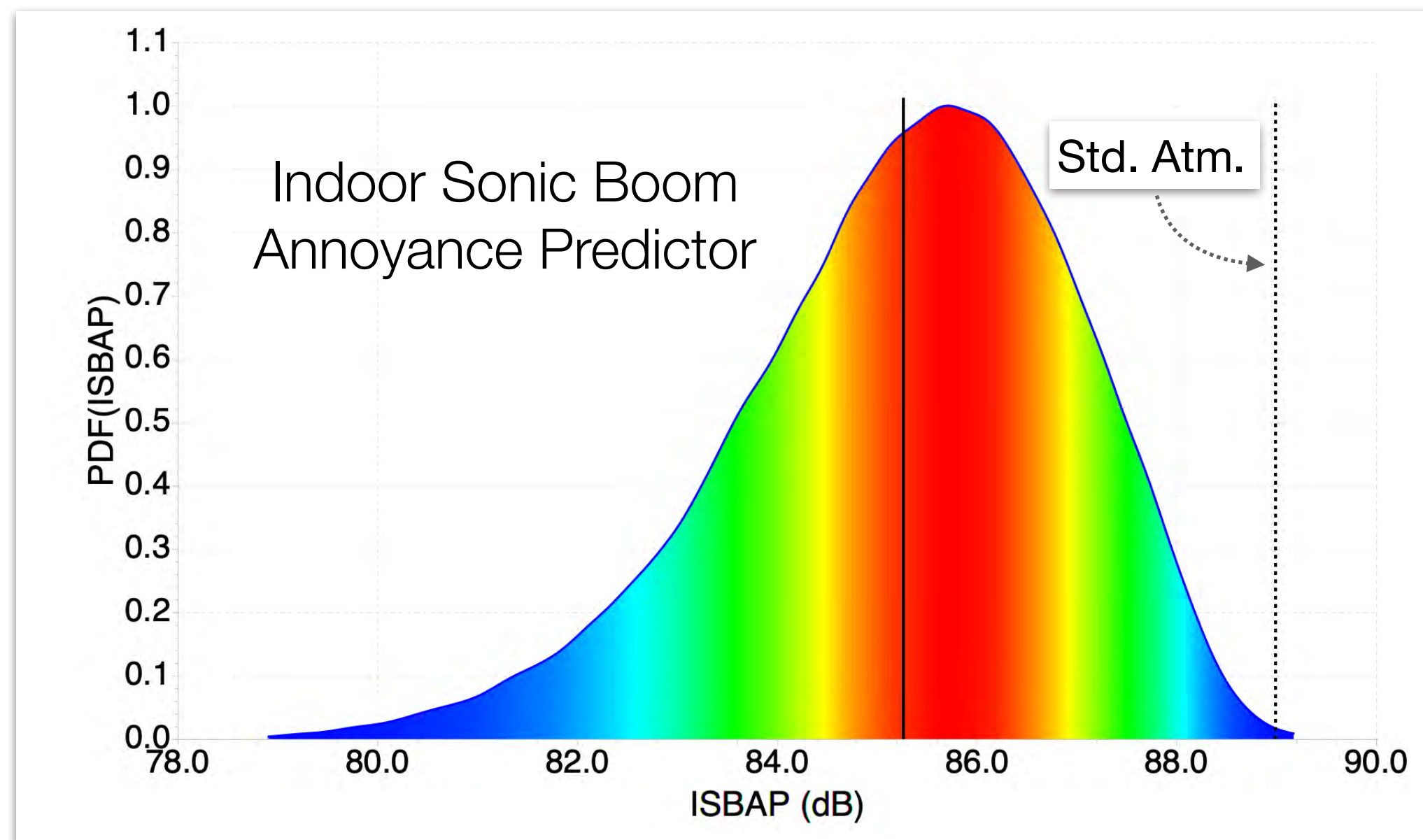
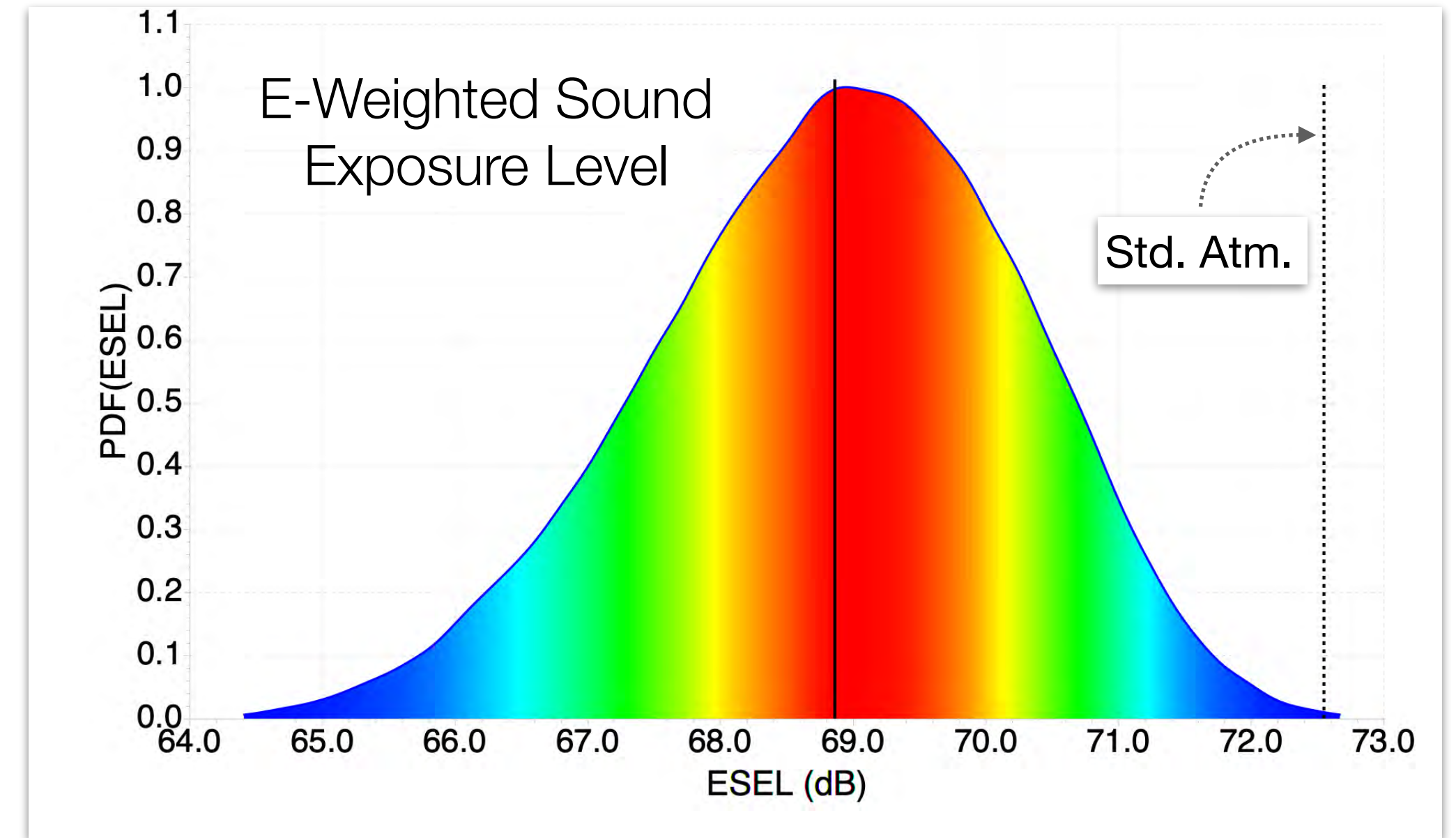
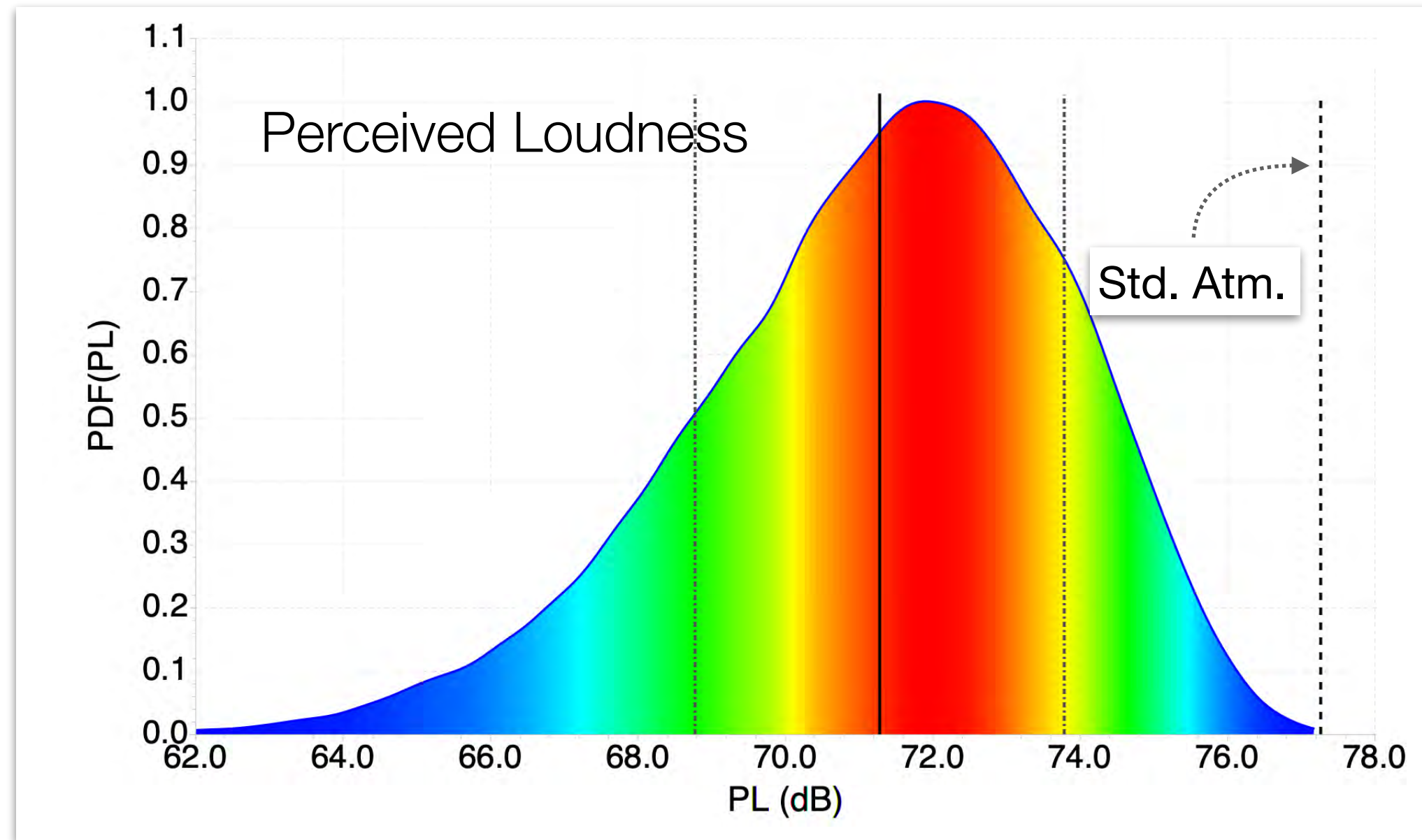
- Distribution is skewed to the left
- One standard deviation is well below 75 dB

PL from propagation through Std. Atm.

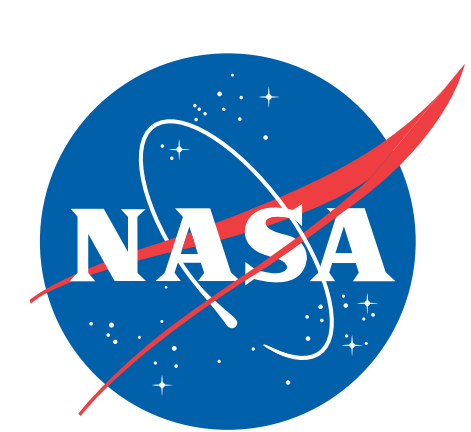
- Atmospheric effects reduce mean PL by 6 dB
- Probability less than 0.05% of matching Std. Atm. PL at Edwards



# PL, ESEL and ISBAP Distributions



- Unimodal distributions, skewed to the left
  - Similar shapes
- Loudness values obtained from propagation through standard atmosphere are highly unlikely at Edwards

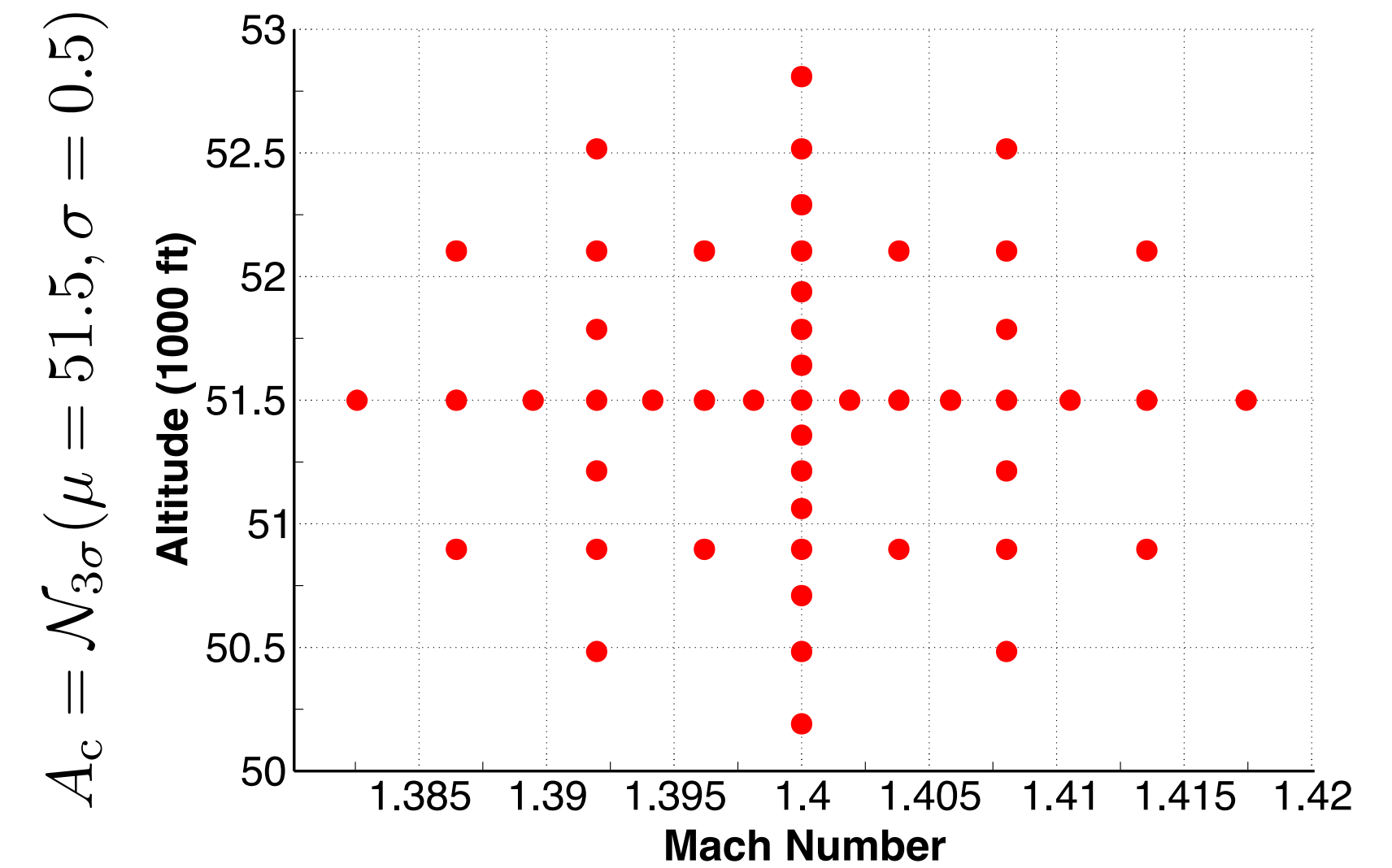


# Uncertain Operating Conditions and Atmosphere

Characterize uncertainty in X-59 ground signature and loudness due to uncertainty in cruise Mach number and altitude, and the atmosphere

- 8 independent random variables
  - Assume normal distributions for Mach number ( $M_\infty$ ) and altitude ( $A_c$ )
  - Reuse KL-6 parameterization of T and RH profiles
- Assume flights at Edwards AFB in August, around 10 am PT

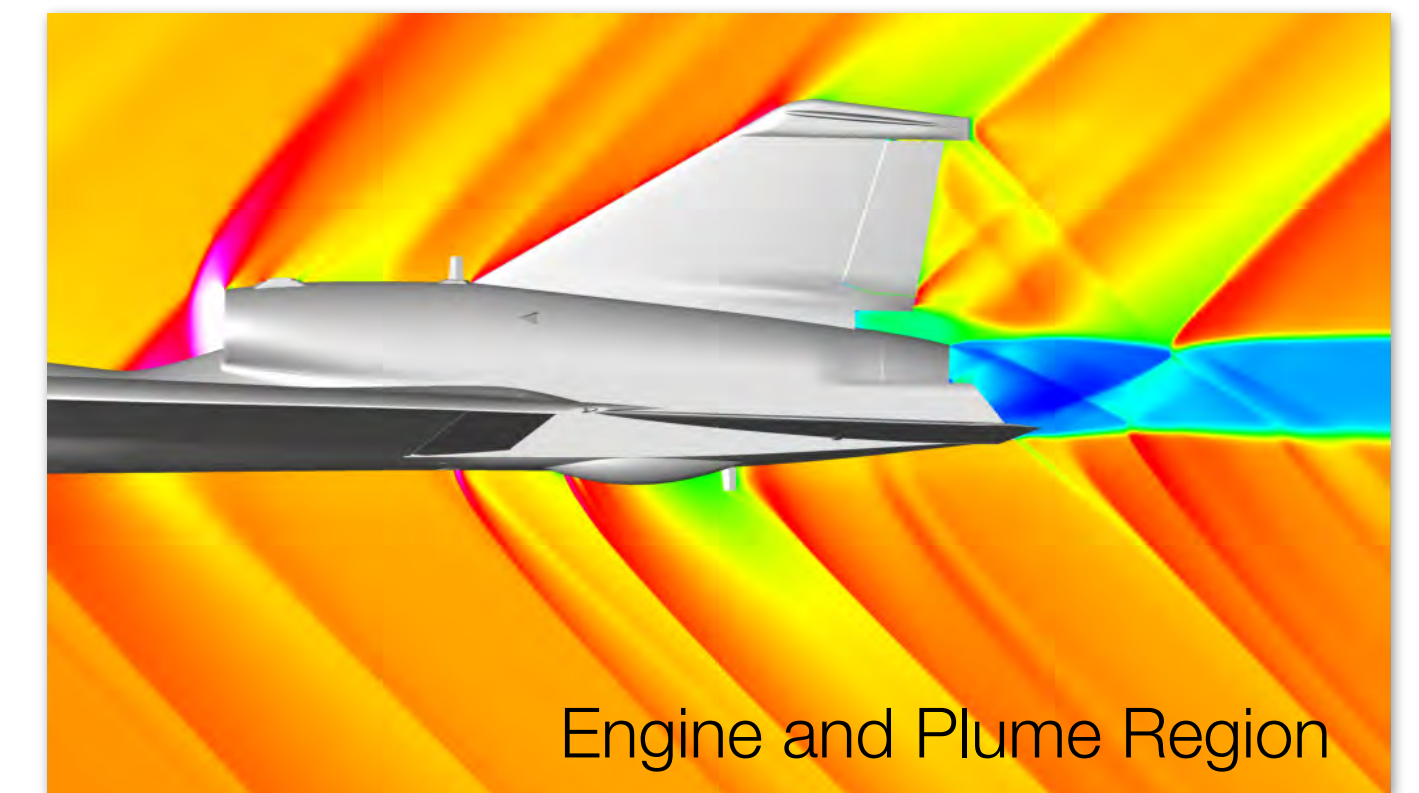
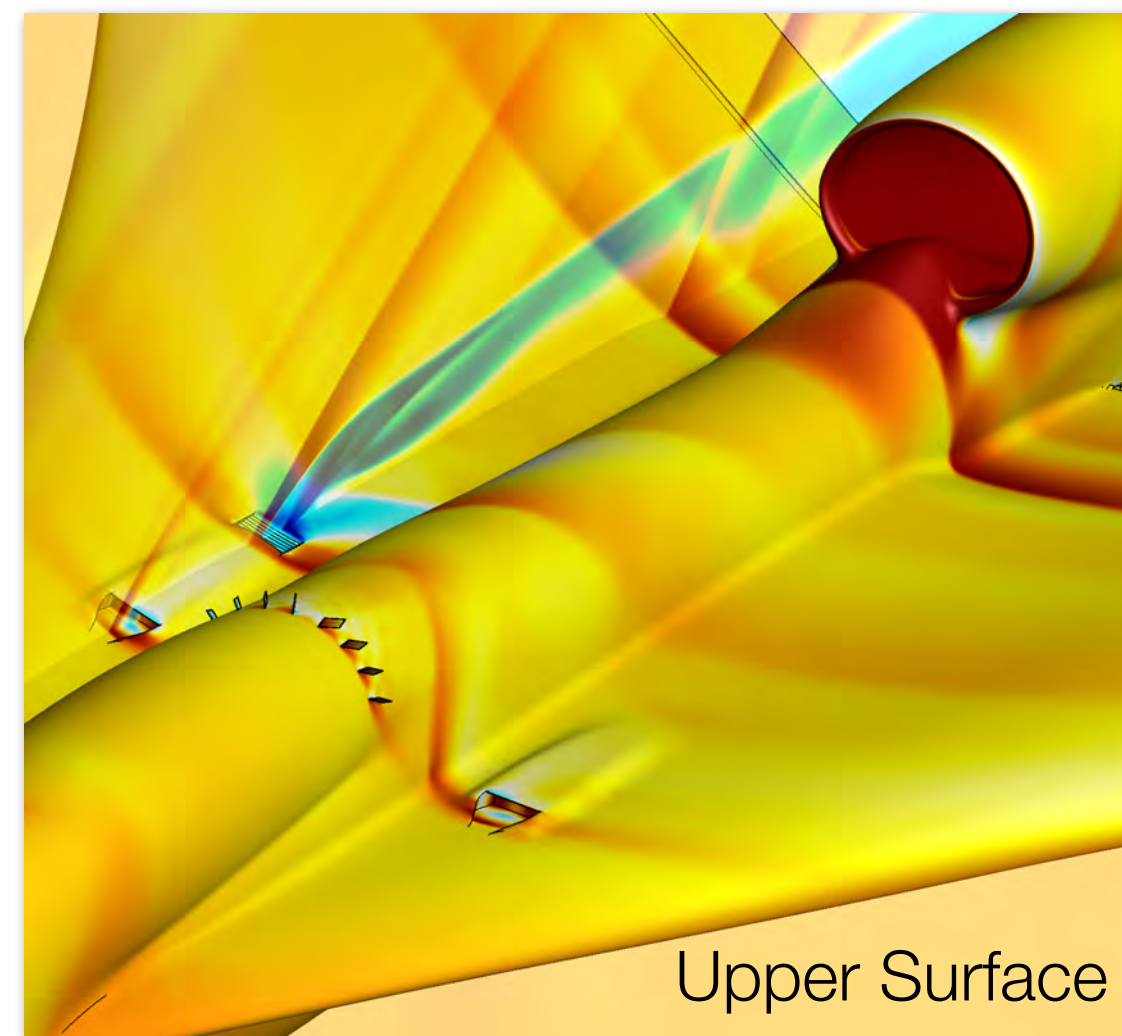
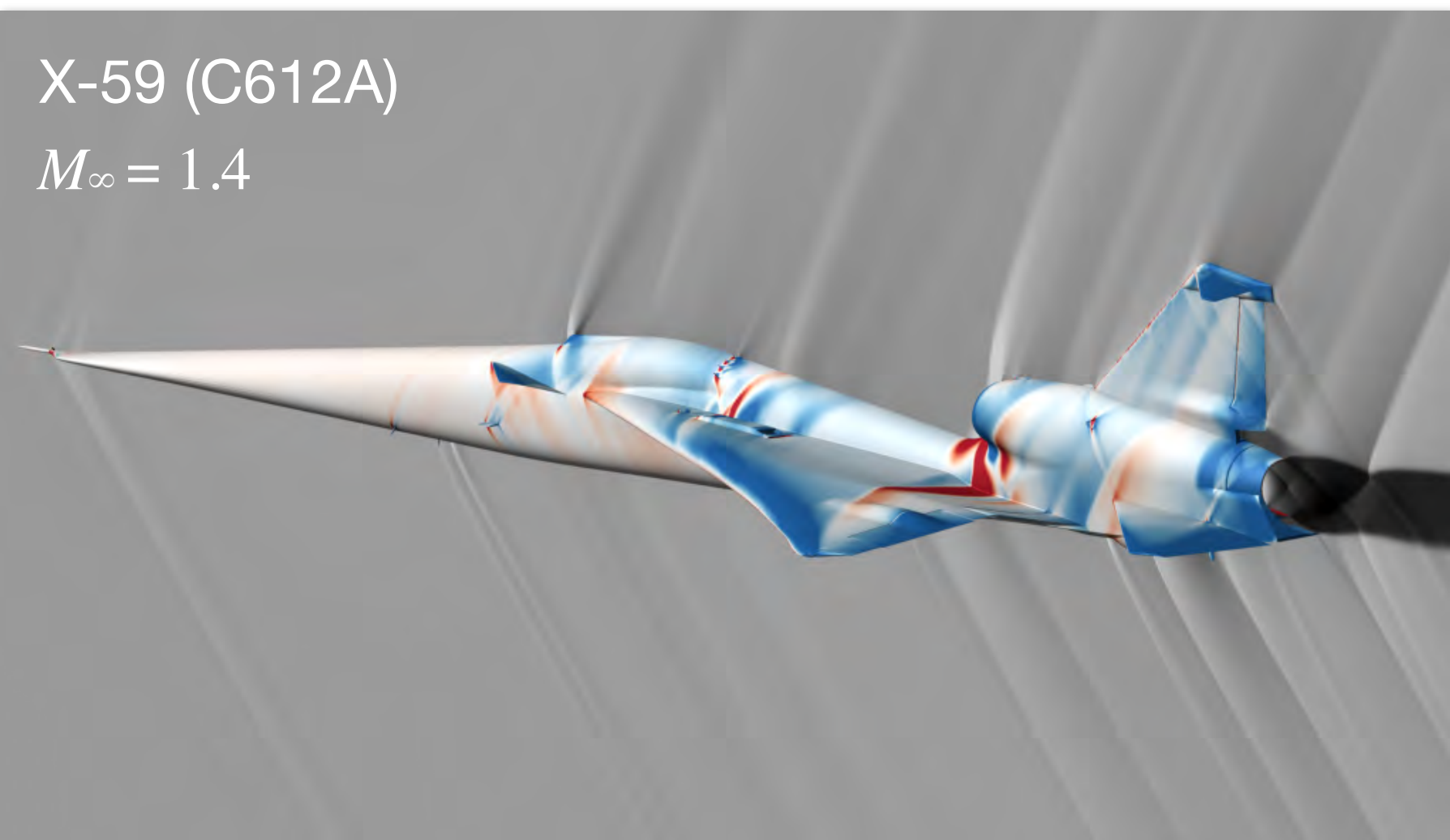
Level 4 Sparse Gauss-Patterson Quadrature

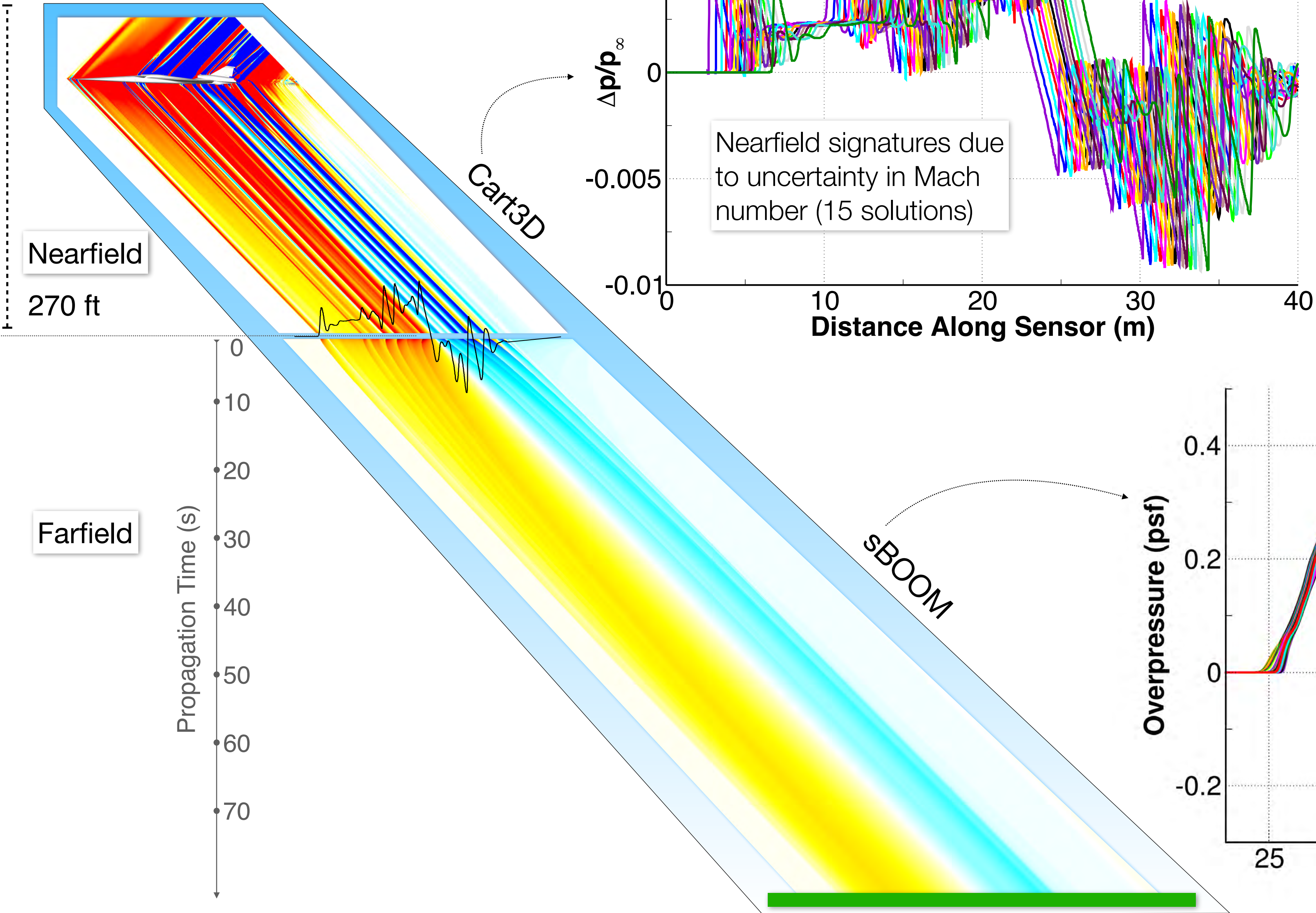
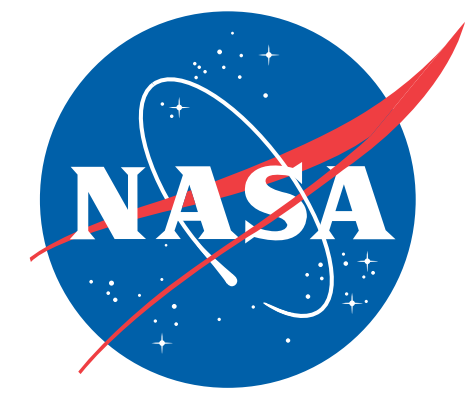


$$M_\infty = \mathcal{N}_{3\sigma}(\mu = 1.4, \sigma = 0.0066)$$

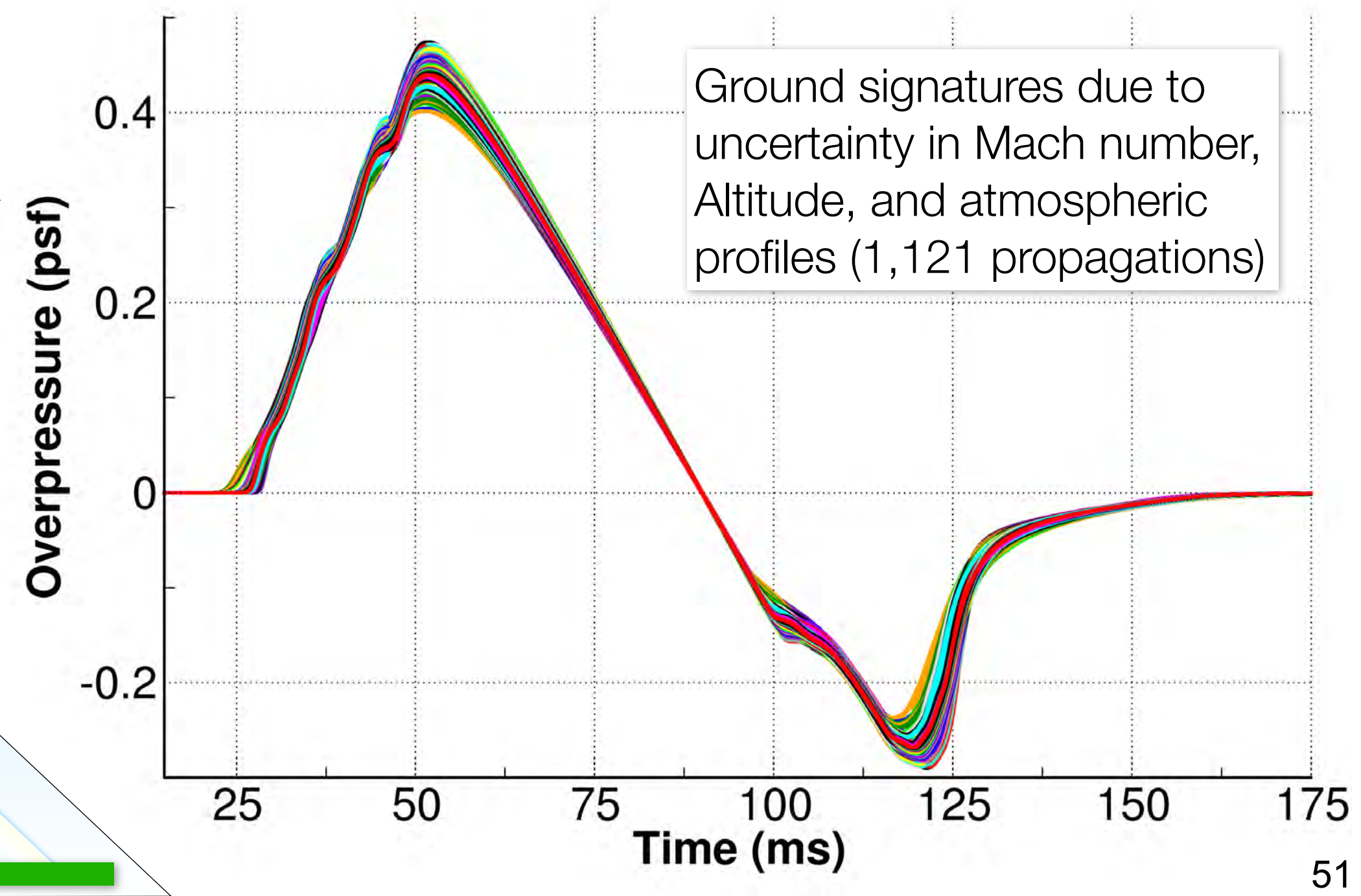
X-59 (C612A)

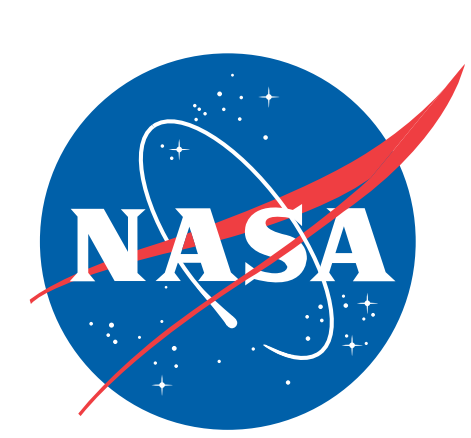
$M_\infty = 1.4$



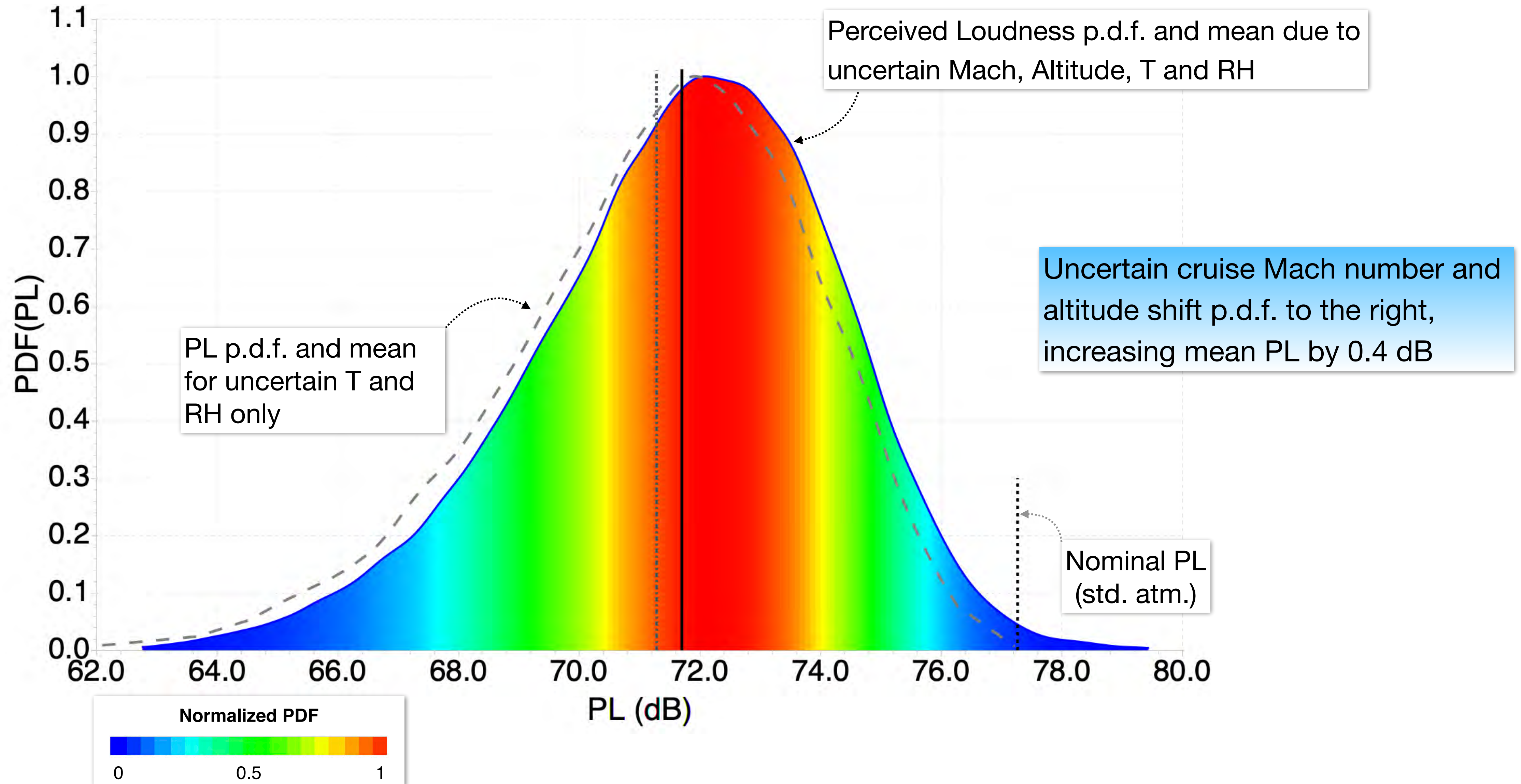


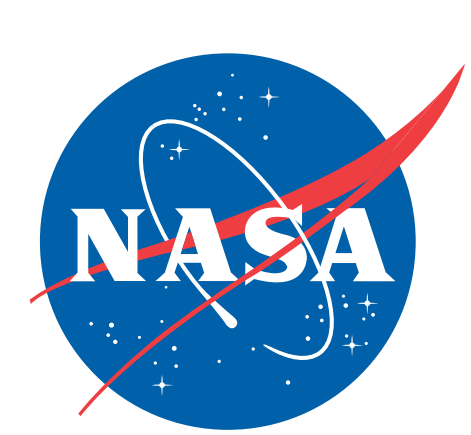
- ### Simulation Details
- 8 input random variables
  - L=4 sparse Gauss-Patterson quadrature
    - 1,121 simulations required
  - Separate variables into two groups: those that affect the nearfield and those that do not
    - 15 nearfield CFD solutions
    - 1,121 farfield propagations



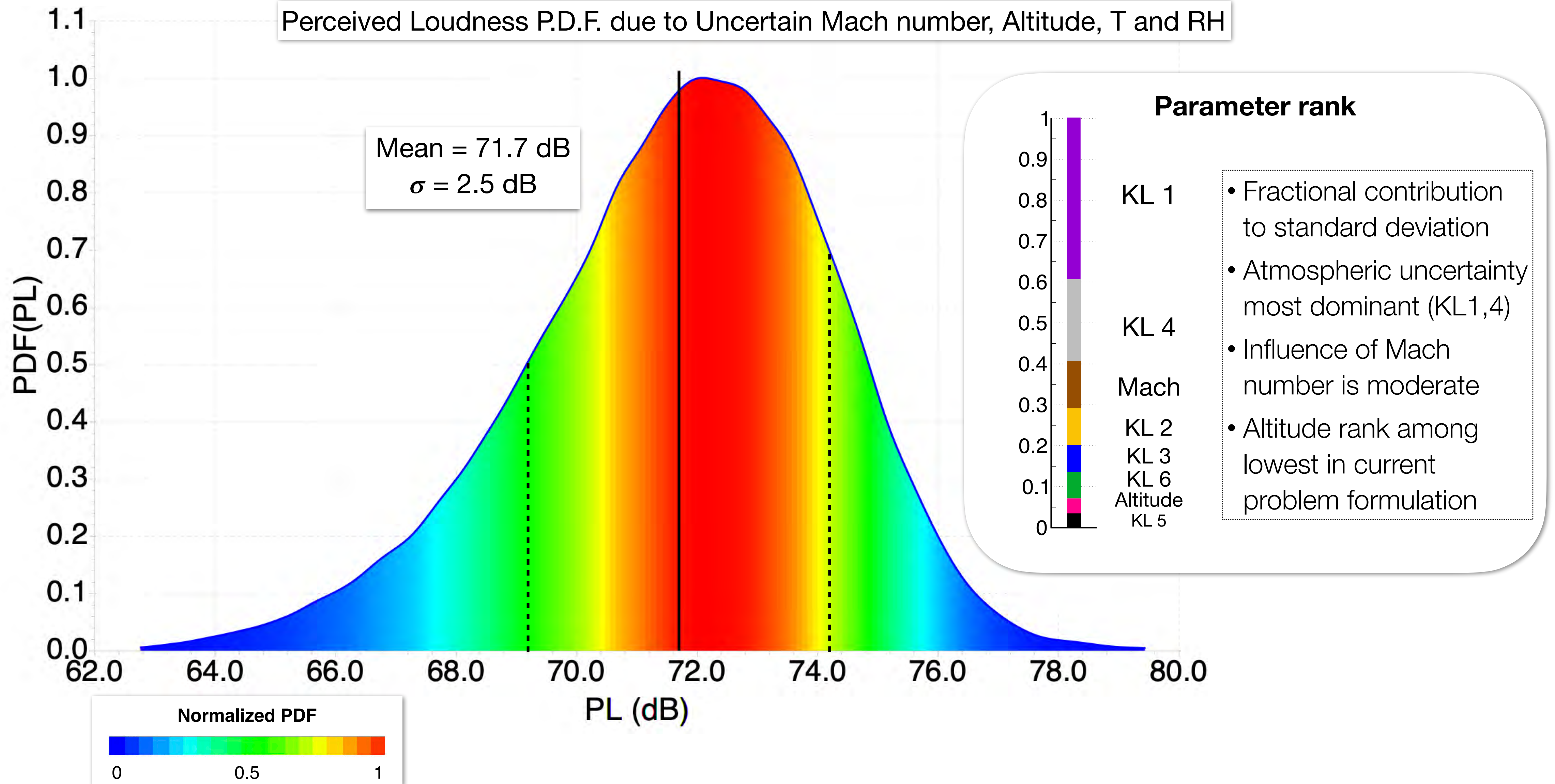


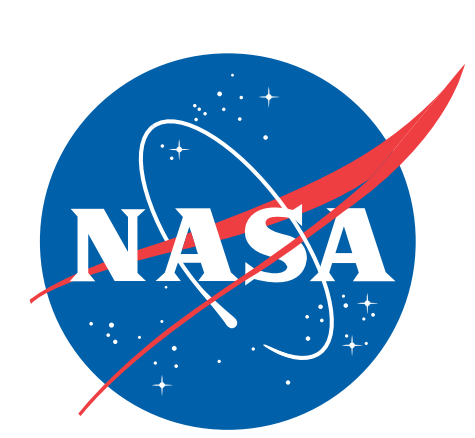
# Influence of Operating Conditions on PL





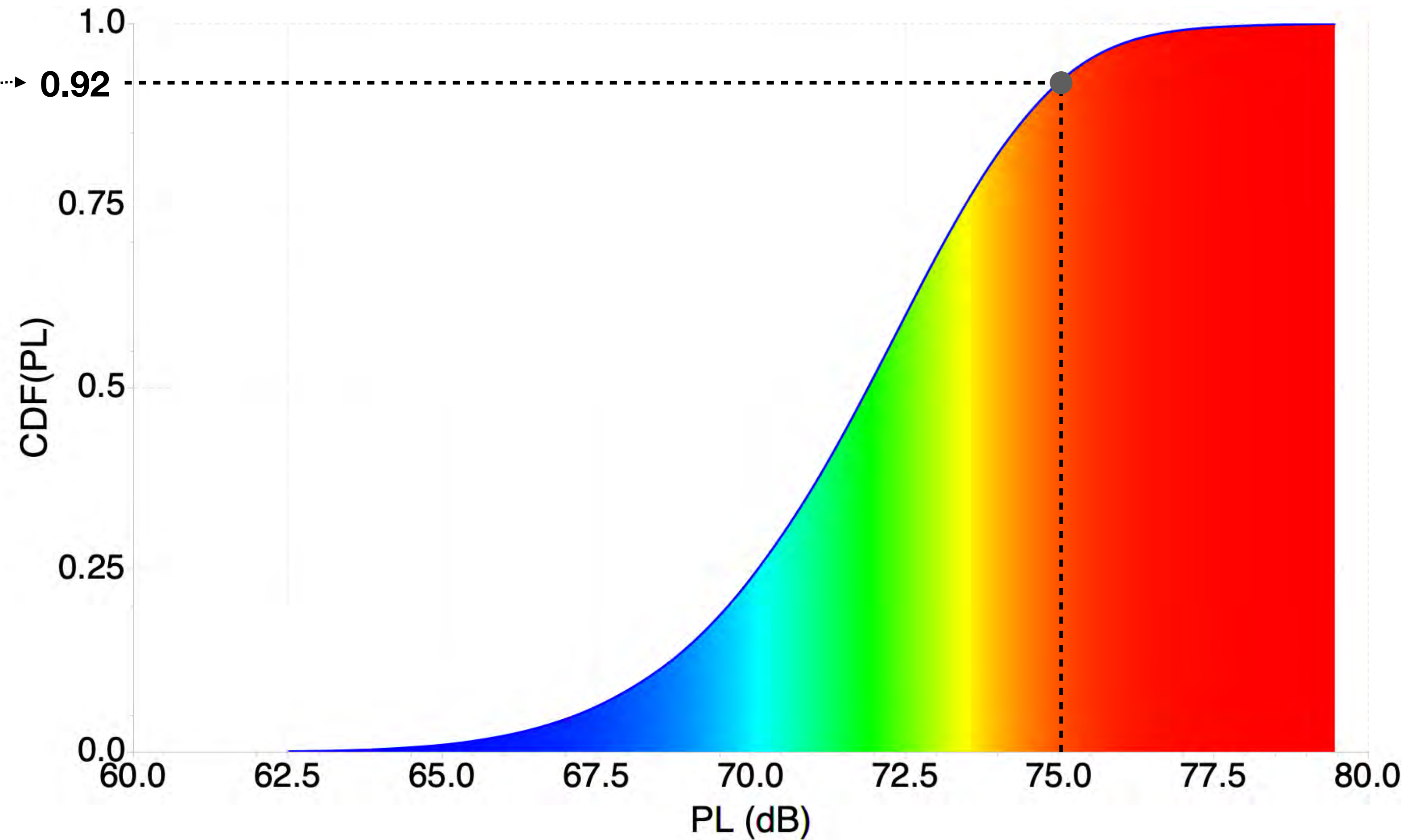
# Ranking of Uncertain Inputs

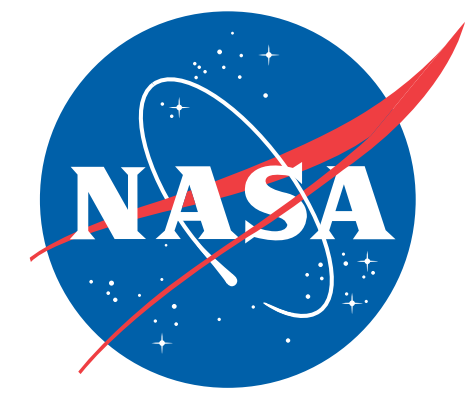




# Cumulative Distribution Function for PL

- 92% chance of success
  - Probability of exceeding 75 PL dB at Edwards AFB for flights in August at 18:00 UTC due to uncertainty in cruise Mach number, altitude and atmospheric conditions is 8%
- Estimate neglects variability in aircraft mass and trim, and atmospheric turbulence

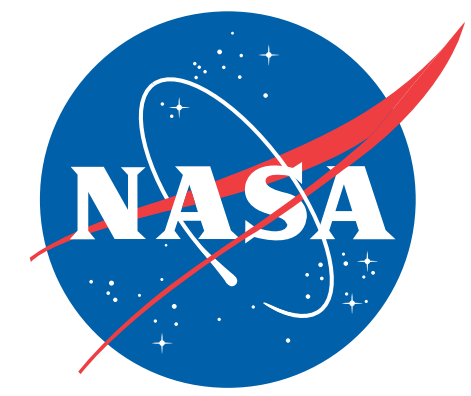




# Summary

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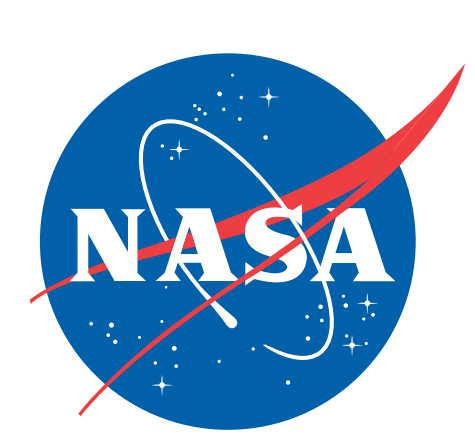
1. Demonstrated importance of evaluating the p.d.f. for nearfield signatures
    - p.d.f. is bimodal at shocks, rendering moment statistics inappropriate
  2. Showed significant improvement in predictions when test uncertainties are included in simulations
    - Important step toward assessing model-form errors
  3. Developed an efficient parameterization of atmospheric profiles using Karhunen-Loeve expansion
    - Demonstrated importance of including atmospheric uncertainty in loudness predictions
- 
- Demonstrated efficient uncertainty quantification for loudness metrics under conditions expected during X-59 test flights
    - Affordable cost on real-world problems, despite direct coupling with high-fidelity simulations and complex aircraft geometry
  - Enabling technology to improve confidence in predicted acoustic signatures of future supersonic transports



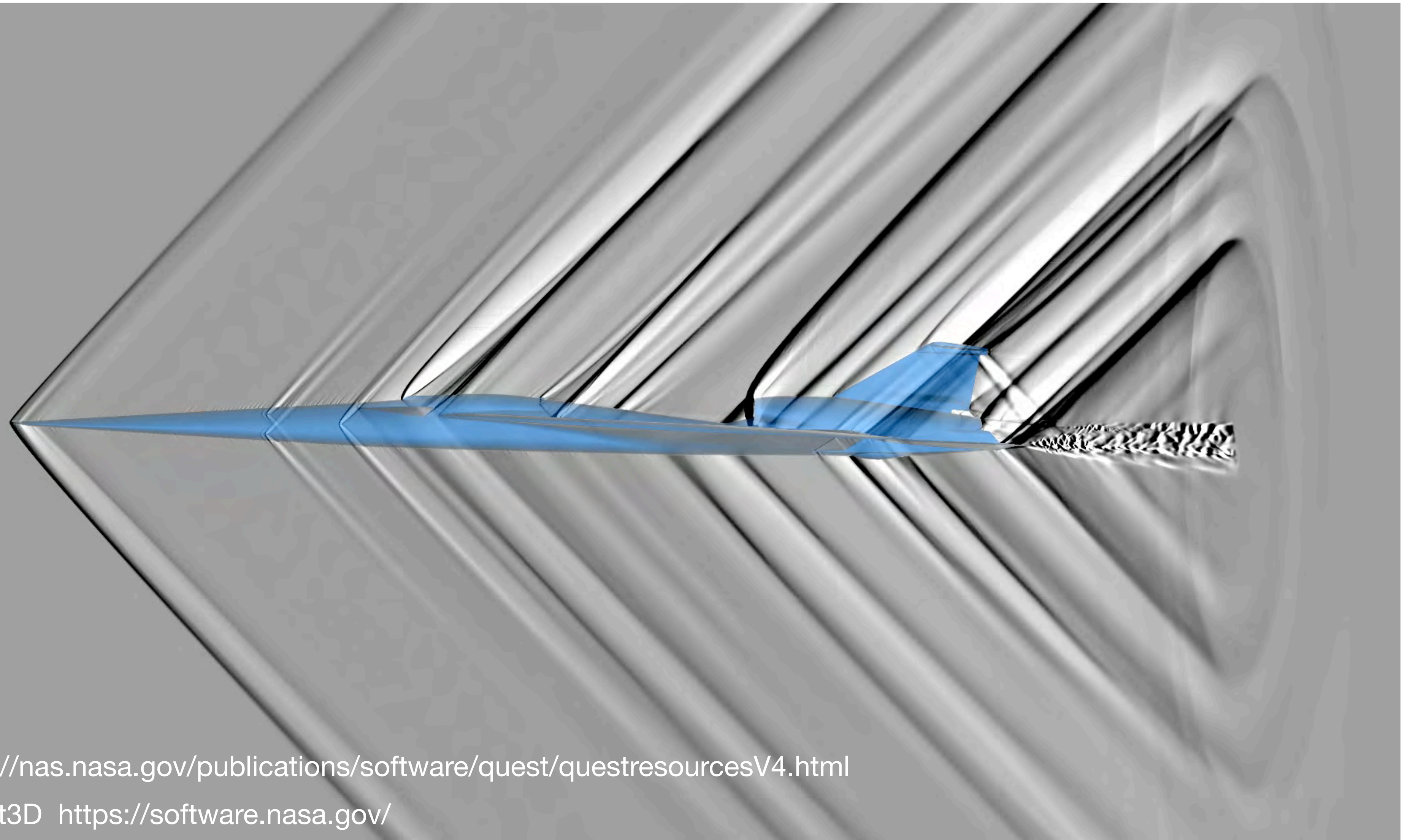
# Acknowledgements

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- NASA's ARMD Commercial Supersonic Technology Project
- NASA's ARMD Transformational Tools and Technologies Project
- NASA High-End Computing Program and NASA Advanced Supercomputing (NAS) Division at Ames Research Center for computing resources
- NASA OSTEM internship program





# Questions



QUEST <https://nas.nasa.gov/publications/software/quest/questresourcesV4.html>

sBOOM & Cart3D <https://software.nasa.gov/>