High Reliability at Minimum Cost

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SUMMARY & CONCLUSIONS

This paper investigates the minimum cost of improving the reliability of complex technical systems. The two major methods to improve reliability are redesigning the system for higher reliability or providing redundant components to replace failed elements. The costs of redesign for reliability or adding redundancy are estimated. The most cost-effective combination for high reliability can be identified.

The cost of increasing the intrinsic reliability of a system can be modeled as cost proportional to 1/(system failure rate)^a, where the exponent "a" measures the difficulty of increasing reliability. The "a" exponent can vary from 0.25 to about 2.5. Operational reliability can also be increased by using redundant systems. The failure rate for N parallel redundant units is (system failure rate)^N. The cost of redundancy is N times the system cost. The total redundant system cost is proportional to N/(system failure rate)^a.

The cost of redundancy increases as N gets larger, but larger N allows a higher system failure rate, which reduces the system design cost. There is a certain N, a certain level of redundancy, that has the minimum cost to achieve the required overall redundant system failure rate. The minimum cost for the redundant system is achieved at the optimum level of redundancy. The N for minimum cost is equal to -a ln (redundant system failure rate). The minimum cost of the N redundant systems is proportional to N * (original system failure rate)^a. The optimum redesigned individual system failure rate is proportional to exp (-1/a), so the greater the difficulty, the higher the optimum individual system failure rate. Increasing the intrinsic reliability of a system encounters diminishing returns and at some point it becomes more cost-effective to add redundancy. The difficulty of increasing intrinsic system reliability determines the optimum design for high reliability at minimum cost.

1 THE COST OF IMPROVING RELIABILITY

Designing a system for higher reliability is usually considered very difficult but the first steps are sometimes surprisingly easy. New systems are often designed by narrowly focusing on their peak operating performance so that long-term operational reliability can be neglected. If reliability was not emphasized in the initial design, simply selecting more reliable components, derating components to reduce stresses, and carefully limiting the operational environment can greatly improve reliability. On the other hand, commonly used commercial systems, such as automobiles and computers, have had their reliability greatly improved by continued intense effort and further improvements are very difficult. Improving reliability has increasing costs and diminishing returns.

It is useful to have a mathematical formula for the cost of increasing reliability. The formula should reflect several basic constraints. The cost of any gain in reliability should always be greater than zero. The cost for higher reliability increases with higher reliability. The cost of reliability equal to one should be infinite [1]. The engineering facts are no free lunch, diminishing returns, and sooner or later, everything fails. Several different mathematical rules have been proposed for the cost of improving reliability.

1.1 *Rechtin's logarithmic rule of thumb*

Rechtin's rule of thumb is, "Reducing the failure rate by a factor of 2 takes as much effort as the original development." [2] If the original cost is C0 for the original failure rate, F0, of say 4 percent, it costs a second equal amount C0 to achieve F1 = F0/2 = 2 percent and a third amount C0 to go from F1 = F0/2 = 2 percent to F2 =F0/4 = 1 percent. The mathematical equation for the total Rechtin cost C of a failure probability F is:

$$\text{Rechtin cost } C(F) = C0\ [1 + \log_2 (F0/F)] \qquad (1)$$

The Rechtin rule cost increases as the logarithm to the base two of the ratio of failure probability improvement.[3] A failure rate improvement ratio of 10 requires 4.32 times the original cost. Similar suggested cost functions have the cost of failure probability improvement increasing as the natural logarithm of the ratio of failure improvement [1, p. 277].

1.2 *Misra et al. exponential cost of reliability*

An exponential function for the cost of reliability was suggested by Misra et al. [1, pp. 276, 278]

$$\text{Exponential cost } C(F) = a \exp (b/F) \qquad (2)$$

The factor "a" is a constant, the reliability cost increase exponent. Following Aggrawall [1 p. 278], suppose we know the cost C0 for failure probability F0,

$$C(F0) = C0 = a \exp (b/F0) \qquad (3)$$

and suppose cost is C1 for very low reliability, F1 ~ 1,

$$C(F1 \sim 1) = C1 \sim a \exp b \qquad (4)$$

Then,

$$a = C1 \exp -b, \text{ and} \quad (5)$$

$$b = [F0/(1-F0)] \ln (C0/C1) \quad (6)$$

Substituting for a and b in (2) and rearranging,

$$\text{Exponential cost } C(F) = C1\ (C0/C1)^{F0\,(1-F)/F(1-F0)} \quad (7)$$

This checks, since for F = F0, C(F) = C0, and for F = F1 ~ 1, C(F) = C1. Suppose F0 is small, << 1, and we need a lower failure probability, F < F0,

$$C\ (F) \sim C1\ (C0/C1)^{F0/F} \quad (8)$$

The cost increases as the power of F0/F, the failure rate reduction ratio. For this exponential power cost rule, the cost increases much more rapidly with the ratio of failure improvement than for the Rechtin or other logarithmic rules.

1.3 *Proportional cost of reliability*

In Rechtin's logarithmic rule, cost increases roughly as $\log_2 (1/F)$, where F is the failure rate improvement ratio. In Misra's exponential power law rule, cost increases as $C^{1/F}$. An intermediate proportional cost rule would be,

$$\text{Proportional cost } C(F) = C/F^{a} \quad (9)$$

Several similar reliability cost functions have been suggested with the exponent of F, a = 1, or a > 0, or 0 <a < 1 [3] [1, p. 277]. If the cost is C0 for failure probability F0, C(F0) = C0 =C/F0^a, so C = C0 F0^a and,

$$\text{Proportional cost } C(F) = C0\ (F0/F)^{a} \quad (10)$$

For a =1, the cost increase ratio, C(F)/C0, is equal to the failure rate improvement ratio, F0/F.

1.4 *Mettas exponential cost of reliability*

Mettas has also proposed an exponential cost of reliability [4] [5],

$$C = \exp [(1 - f)\ (R-R\ \min)/(R\ \max-R)] \quad (11)$$

Here f is the feasibility of increasing a component's reliability, between 0 and 1, R min is the initial reliability value of the component, and R max is its maximum achievable reliability. Converting from reliability, R, to failure probability, F = 1 – R, Metttas' formula becomes,

$$C(F) = C(0) \exp [(1 - f)\ (F0 - F)/(F-F\ \min)] \quad (12)$$

Checking, C(F) = C(F0) for F = F0 and C(F) is infinite for F = F min. Setting f = 0 for minimum feasibility and setting the minimum failure rate bound F min = 0, the formula becomes,

$$C(F) = C(0) \exp [(F0 - F)/F] \quad (13)$$

The simplified equation (13) is like Misa's equation (2) above. Mettas' equation (12) adds the feasibility factor, f, and a minimum failure rate bound, F min, approaching which drives cost to infinity.

1.5 *Military equipment cost of reliability*

Alexander considered the cost of increasing reliability in military equipment. [6]. He found "that reliability improvements are possible, that the greater the improvement the more costly the necessary investment, and that the improvement probably rises proportionally faster than the investment." The military equipment considered was surprisingly easy to improve in reliability. Specifically, "These data indicate *increasing returns* to reliability investments: a 10 percent increase in reliability would cost 5 percent more in total RDT&E expenditures, whereas a doubling of reliability would cost 20 percent more; and a five-fold reliability gain would require at least a 50 percent increase in development costs." (Emphasis added.).

Here the word "reliability" corresponds to the MTBF (Mean Time Before Failure). The MTBF ratio and cost ratio data points are, (1,1), (1.1, 1.05), (2, 1.2), and (5, 1.5). Since the failure rate is the inverse of the MTBF, the failure rate ratio and cost ratio data points are (1,1), (0.91, 1.05), (0.5, 1.2), and (0.2, 1.5). These points are plotted, and a power curve fitted in Figure 1.

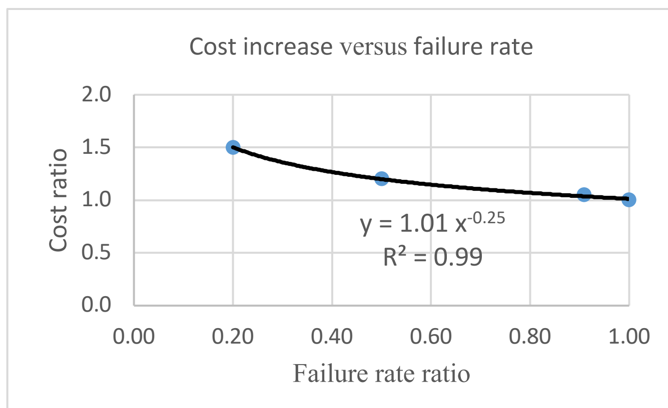

Figure 1. Military equipment cost increase versus failure rate decrease ratio.

This is a proportional cost as in formula (10) with a = 0.25.

$$\text{Military equipment cost } C(F) = C0\ (F0/F)^{0.25} \quad (14)$$

1.6 *Internet web site cost of reliability*

Shirazi discussed the cost for high reliability for internet sites, "A common Site Reliability Engineering (SRE) estimate is that the more reliability you want, the more it costs, with a rule of thumb that each additional 9 of reliability (eg., moving from 99% to 99.9% reliability) costs 10 *times* (10x) more to achieve."

[7] (Emphasis in original.) As the reliability increases 0.9, 0.99, and 0.999, the failure probability decreases 0.1, 0.01, and 0.001, and the cost increases 1, 10, 100. This is a proportional cost of formula (10) with a = 1.

$$\text{Internet site cost } C(F) = 1\ (0.1/F)^{1} \qquad (15)$$

2 *GENERAL FORMULA FOR THE COST OF HIGHER RELIABILITY*

Four different mathematical functions have been proposed for the increasing cost to reduce the failure rate, F; $\log_2 (1/F)$, exp (1/F), and $(1/F)^a$ for a = 0.25 and 1. The $\log_2 (1/F)$ and exp (1/F) can be approximated by $(1/F)^a$ for different specific a exponents, so that the proportional function cost of reliability seems able to model all the proposed formulas.

Figure 2 shows the cost increase curves for $\log_2 (1/F)$, exp (1/F), and $(1/F)^a$ for a = 0.25 and 1.

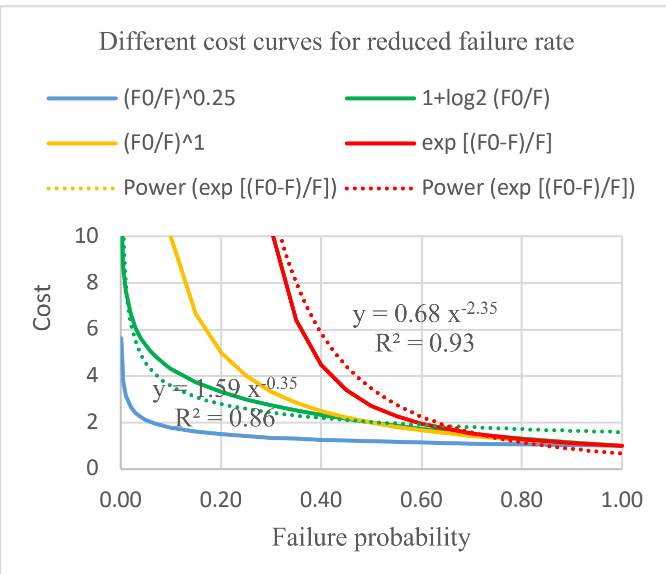

Figure 2. Cost increase curves for log, exp, and proportional cost growth.

The slowest cost increase is for $(1/F)^{0.25}$, and the next slowest for $1 + \log_2 (1/F)$, which can be approximated by $1.59\ (1/F)^{0.35}$. The third slowest increase is for $(1/F)^1$, and the fastest increase is for exp [(F0-F)/F], which can be approximated by $0.68\ (1/F)^{2.35}$. It seems that a proportional cost increase formula with a variable exponent can describe the proposed range of cost increases required to reduce the failure rate.

$$\text{Proportional cost } C(F) = C0\ (F0/F)^{a} \qquad (16)$$

The cost increase exponent "a" can vary from 0.25 to about 2.5.

3 *DATA ON THE COST OF HIGHER RELIABILITY*

The proportional cost formula for the increasing cost of reduced failure probability could be used to model actual data and to estimate the value of the cost increase exponent, a. Unfortunately, data on the incremental cost of gradually increasing the reliability of a single system seems unavailable. However, there are studies that have measured the added cost of a single step increase in reliability for a group of similar systems. This data on defense systems directly shows the cost of improved reliability.

3.1 *Killingsworth/McQueary Defense Systems*

Killingsworth et al. reported McQueary's data showing the relationship between the additional investment in reliability and achieved reliability for about a dozen defense systems. [8] The additional investment varied from about equal to the original investment to 1,000 times as much, with the resulting reliability improvement ratios from 0.5 to 10. The systems varied in complexity from pumps and gyros to entire aircraft. The smaller, less costly systems could accept much higher investment ratios and achieve much greater reliability improvement ratios.

The data fell close to a straight line on a log-log graph and the linear regression equation was

$$\text{LN(Reliability Improvement Ratio)} = 0.4719\ \text{LN (Investment/Original Cost)} - 1 \qquad (17)$$

The cost-reliability relation was also given as:

$$\text{Investment} = \text{Original Cost} * (\text{Reliability Improvement Ratio}/0.3659)^{2.119} \qquad (18)$$

In the previously used notation

$$CF - C0 = C0\ [(F0/F - 1)/0.3659)]^{2.119} \qquad (19)$$

$$CF/C0 = [(F0/F - 1)/0.3659)]^{2.119} + 1 \qquad (20)$$

This equation fit to the data is similar to the proportional cost model proposed here, but an approximation using the form $C0\ (F0/F)^a$ would have the cost increase exponent a = 3.5.

3.2 *Lubas Defense Systems*

Lubas reported another study that confirmed the empirical relationship between reliability investment and reliability improvement for five defense systems. [9] The reliability improvement ratio ranged from 0.24 for a doubling of cost to 6.75 for a cost 2,980 times higher. Cutting the failure probability in half requires roughly an order of magnitude increase in cost, a factor of 11.6 times.

Again the data fell close to a straight line on a log-log graph

$$\text{LN(Improvement in Reliability)} = 0.343\ \text{LN (Investment/Original Cost)} - 0.81 \qquad (21)$$

This can be transformed to the following

$$C(F)/C0 = 10.6\ (F0/F - 1)^{2.92} \qquad (22)$$

Again, the equation fit to the data is similar to the proportional cost model. An approximation using the form C0 $(F0/F)^a$ would have the cost increase exponent a = 4.2.

4 THE COST OF REDUNDANCY

It would be straightforward if any system could be redesigned to have the required reliability, but the steeply increasing cost of reliability can make this impractical. If greater reliability is needed than a single system can provide, the usual, solution is to provide redundant units. If the probability that a system fails is F, the probability that two redundant units both fail is F^2, assuming that the failures are independent. If there are N redundant units, each with failure probability F, the overall failure probability is F^N.

This suggests that very low failure probabilities can be obtained using multiple redundancy, but often common cause failures due to design errors or mistaken requirements can occur in all the redundant units. Thorough testing and long operational experience can remove most common cause failures, but new designs may have up to one-tenth of their total failure probability due to common cause failures. If there is a failure probability of 0.1 F due to common cause failures, no amount of redundancy can reduce the failure probability below 0.1 F.

The cost to provide redundancy is simply the cost of the N redundant units. Suppose that each system has a failure probability of Fs and that N redundant units are required to provide the required final failure rate of Ff. We have,

$$Ff = Fs^{N}, \text{ and } Fs = Ff^{1/N} \quad (23)$$

The cost of a system with failure probability Fs is,

$$C(Fs) = C0\ (F0/Fs)^a \quad (24)$$

The cost of N redundant systems with the required final failure rate of Ff is,

$$CN = N\ C(Fs) = N\ C0\ (F0/Fs)^a \quad (25)$$

The cost increases as N gets larger, but larger N allows larger Fs, which reduces cost. There is a certain N, a certain level of redundancy, that has the minimum cost to achieve Ff. Equation 25 can be rewritten in terms of Ff and N by using (23)

$$CN = N\ C(Fs) = N\ C0\ F0^a/Ff^{a/N} \quad (26)$$

Taking the derivative of (26) and setting it to zero, the optimum number of redundant systems, N, for minimum cost is

$$\text{Opt } N = -\ a \ln (Ff) \quad (27)$$

N increases with higher a, the difficulty of improving system reliability, and with lower Ff, the final redundant failure probability. The minimum total cost for N systems is,

$$\text{Min } CN = N\ C0\ F0^a\ e \quad (28)$$

Where N is the Opt N for minimum cost in (27) and e is 2.718, the base of the natural logarithms. The cost increases with N, with higher initial system cost, C0, with higher initial system failure probability, F0, and with the proportional difficulty of improving reliability, a. Figure 3 plots the normalized minimum cost increase for different levels of difficulty, a, and decreasing final failure probability, Ff.

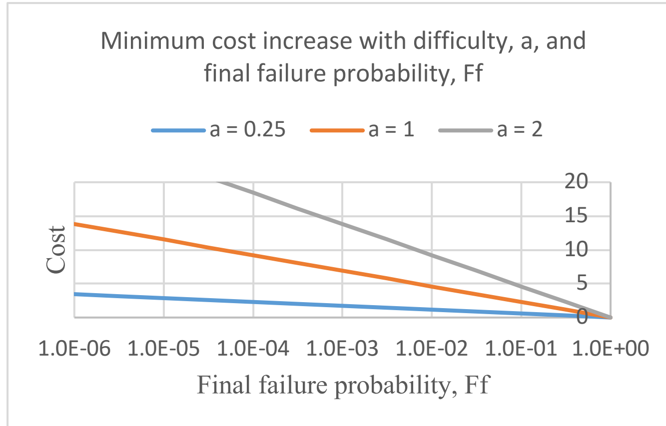

Figure 3. Minimum cost increase versus difficulty and final failure probability.

The minimum cost plotted in Figure 3 is normalized with C0 =1, F0 =1, and divided by e, so the Min CN plotted = N = - a ln (Ff). The minimum cost is directly proportional to the optimum N for minimum cost. The minimum cost and the optimum N are directly proportional to a, the difficulty of increasing reliability in the proportional cost model (9), and to the logarithm of Ff, the final failure probability.

5 OPTIMUM SYSTEM RELIABILITY

The optimum N and minimum cost CN correspond to an optimum system failure probability Fs,

$$Fs = Ff^{1/N} \quad (29)$$

For Ff and optimum N,

$$Fs = e^{-1/a} \quad (30)$$

The difficulty of achieving Fs is proportional to a in the exponential proportional cost of reliability model,

$$\text{Proportional cost } C(F) = C0\ (F0/F)^a \quad (16)$$

Considering the possible high cost of increasing reliability, the minimum cost of reliability may be achieved using lower reliability systems with higher redundancy.

6 CONCLUSION

Many of the suggested formulas for the increased cost of higher reliability can be approximated by the proposed proportional cost formula, where cost is proportional to the (original failure rate/reduced failure rate) raised to an exponential power. The proportional cost formula is supported by data and appears useful in estimating the cost of reliability. The magnitude of the cost increase exponent is important. A larger cost increase exponent indicates the cost increases more rapidly as the failure rate is reduced.

In designing to achieve high reliability with minimum cost, the magnitude of the cost increase exponent directly determines a specific optimum combination of system reliability improvement and redundancy. A small reliability cost increase exponent indicates that significant effort should be made in system reliability improvement, while a large exponent indicates that redundancy is probably more cost effective. The value of the cost increase exponent directly determines the optimum combination of system reliability and redundancy for minimum cost.

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