The abcd Reliability Growth Model

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INTRODUCTION

This paper presents a modification of the well-known Duane-Crow reliability growth model. In the abcd reliability growth model, the initial period of exponential decline of the failure rate in the Duane-Crow model may be followed by a period of constant failure rate. Data often show that an exponential decline in failures is followed by a constant failure rate. If a growth model including only the initial period of exponential decline is applied to increasingly longer failure rate data sets, the data will include longer periods of constant failure rate, and the estimated reliability growth rate will decline from an initially high value down toward zero. Using the Duane-Crow model without extending it to include a possible period of constant failure rate may create the mistaken impression that the initial reliability growth continues forever, but at an ever-decreasing rate.

1 THE DUANE RELIABILITY GROWTH MODEL

Duane observed in 1964 that if n(t) is the number of failures occurring until time t, a plot of the cumulative failure rate, n(t)/t, versus the cumulative test time, t, usually follows a straight line when plotted on log-log graph paper. This occurs when failures were fixed by redesigns that improved reliability. Duane's log-log plot is described by the equation

$$\log [n(t)/t] = \log k - a \log t \quad (1)$$

The usual form of the Duane reliability growth model is exponential.

$$n(t)/t = k\, t^{-\alpha} \quad (2)$$

The failure rate is n(t)/t. k is a constant. The reliability growth rate is α, the downward slope of n(t)/t versus t. Measured values of α usually vary from 0.2 to 0.6,[1] which is between 0 and 1. Since each failure contributes 1/t to the failure rate, a failure rate decline of greater than $\alpha = 1$ is not possible. $\alpha = 0$ corresponds to a constant failure rate.

Crow used a 56-failure data set to illustrate reliability growth.[1] A Duane log-log graphical model fit to this data gives

$$n(t)/t = 0.640\, t^{-0.283} \quad (3)$$

Figure 1 plots the Crow 56-failure data set with the Duane mathematical model of equations (1) and (2).

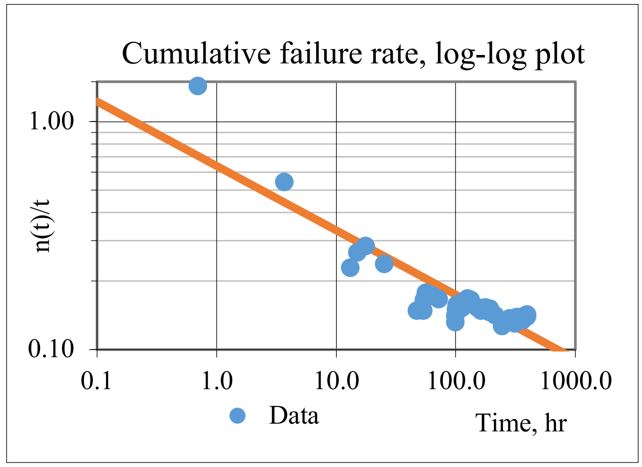

Figure 1. Duane log-log cumulative failure rate plot.

The downward slope is due to early reliability growth. The final data points for n(t)/t are above the fitted line, suggesting that reliability growth has slowed or stopped. If the later failures are ignored, the downward slope is much steeper.

The downward slope of n(t)/t versus t varies between 0 and 1. Suppose k failures occur before t = 1 and testing continues without further failures. Then the failure rate is $n(t)/t = k/t = k\, t^{-1}$. The downward slope of n(t)/t versus t is $\alpha = 1$, the most rapid possible failure rate decline. As time increases, $n(t)/t = k/t = k\, t^{-\alpha}$ approaches 0 for all α.

Given the two parameters k and α, and if they remain constant, the expected cumulative failure rate can be calculated for any time t. It is also possible to solve this equation to find the test time needed to achieve any specific lower failure rate, even approaching zero. These calculations are misleading if the failure rate becomes constant after an initial period of reliability growth. This an obvious and known problem that is usually set aside in reliability growth analysis.[1]

Figure 2 shows the cumulative failure rate, n(t)/t, of the Crow data set plotted versus time linearly, not in the usual Duane log-log graph.

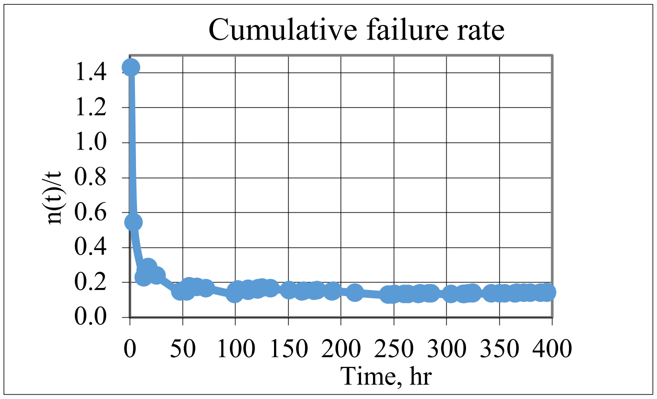

Figure 2. Cumulative n(t)/t versus time, not log-log.

The Duane reliability growth model assumes that reliability growth continues, and the failure rate decreases as long as testing is done. This is clearly not the case for the Crow data set. Reliability growth stops early when a constant low failure rate is reached. However, a major redesign could both increase ultimate reliability but also add new significant failure modes to be removed in another period of reliability growth.

2 THE CROW RELIABILITY GROWTH MODEL

Crow in the 1970's provided a theoretical basis for the Duane model. He assumed that the failures of a system during development testing occur according to a non-Homogeneous (time-varying) Poison Process (NHPP) with a power law mean value function, m(t).[1] The mean number of failures over time is assumed to be

$$m(t) = k\, t^{\beta} \qquad (4)$$

where β is between zero and one.

The instantaneous failure rate is the time derivative of the number of failures.

$$\lambda(t) = d[m(t)]/dt = k\, \beta\, t^{\beta-1} \qquad (5)$$

This is known as the Weibull distribution failure rate, although the full Weibull distribution is more complex. The mathematically expected cumulative failure rate is given by

$$\text{Expected } [n(t)/t] = m(t)/t = k\, t^{\beta-1} \qquad (6)$$

The Crow and Duane reliability growth models are equivalent, with the Duane α equal to Crow's 1 - β. The parameter k is the same in both. The parameter β is the ratio of the current instantaneous failure rate, λ(t), to the average cumulative failure rate, m(t)/t.

$$\beta = \lambda(t)/[m(t)/t] = k\, \beta\, t^{\beta-1}/k\, t^{\beta-1} \qquad (7)$$

The typical β of 0.4 to 0.8 corresponds to a decreasing failure rate and positive reliability growth.[1] The reliability growth parameters can be estimated from failure time data. Suppose that N failures are observed during the test time (0, T), and that they occur sequentially at times s1, s2, … , sN. The maximum likelihood estimate of β is

$$\beta^* = N / \sum \ln (T/s_i) \qquad (8)$$

where ln is the natural logarithm and the summation ∑ is over i = 1 to N. The maximum likelihood estimate of k is

$$k^* = N / T^{\beta^*} \qquad (9)$$

The Crow model analysis of the Crow 56-failure data set using the equations (8) and (9) found k = 0.217 and β = 0.927.

$$[n(t)/t] = k\, t^{\beta-1} = 0.217\, t^{-0.073} \qquad (10)$$

The β corresponds to an α = 1 - β = 0.073, which is much less than the α = 0.283 found using the Duane graphical method. Figure 3 shows the cumulative failure rate data n(t)/t plotted versus time, t, in a log-log graph, along with the Duane line fit and Crow model fit.

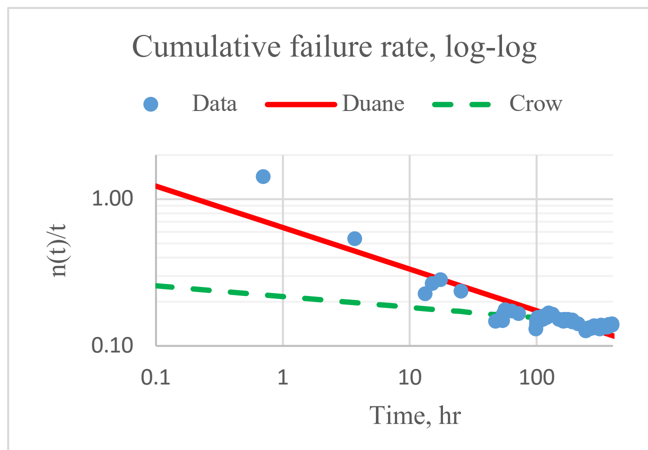

Figure 3. The failure rate n(t)/t data and the Duane line and Crow model fits.

Crow mathematical model fits the early data less well than the Duane graphical log-log fit. The Crow model is much less influenced by the early infant mortality data and gives a much more pessimistic projection of future reliability growth. Nevertheless, the reliability growth claim is that "While growth is small, hypothesis testing indicates it is significantly different from 0. Thus, growth is occurring, and the failure intensity (failure rate) is decreasing."[1] This seems a serious misinterpretation of the data. Both models have the same fundamental problem, the assumption that reliability growth continues without end.

3 THE abcd RELIABILITY GROWTH MODEL

The Duane-Crow exponential reliability growth model does not include the later constant failure rate period that often appears after long testing. It is incorrect to assume that reliability growth will continue forever. The abcd reliability growth modelling approach combines the initial reliability growth period with a later constant failure rate period to form the abcd model. The constant failure rate period includes two types of failures, those due to acceptable low-rate limited life failures that will not be corrected, and others due to unacceptable failures that will be corrected later.[1]

The abcd mathematical model for reliability growth followed by constant failure rate is

$$n(t)/t = a\,t^{-b} + c \text{ from } t = 0 \text{ to } t_d. \quad (11a)$$

$$= c + d \text{ after } t_d, \text{ where } d = a\,t_d^{-b} \quad (11b)$$

The failure rate is n(t)/t, a is a constant and b is the reliability growth rate, the downward slope of n(t)/t versus t. The parameter c is the constant failure rate due to failure modes that will not be corrected. The time t_d is when reliability improvement no longer occurs, and the total failure rate becomes constant. The longer t_d, the more failures will be found and fixed. The parameter d is the constant failure rate due to failure modes that could be corrected later, equal to the remaining reliability growth potential.

The abcd model is computed for the Crow reliability growth data set.

$$\text{Failure rate} = 1.37\,t^{-0.99} + 0.14 \text{ from } t = 0 \text{ to } t_d = 100 \quad (12a)$$

$$= 0.01 + 0.14 = 0.15 \text{ after } t_d = 100 \quad (12b)$$

The remaining correctable failures, d, are few since the reliability growth time, $t_d = 100$ is long compared to the initial failure rate MTBF = 1/n(t) ~ 10 hr. Figure 4 shows the exponential fit of the Crow data out to $t_d = 100$.

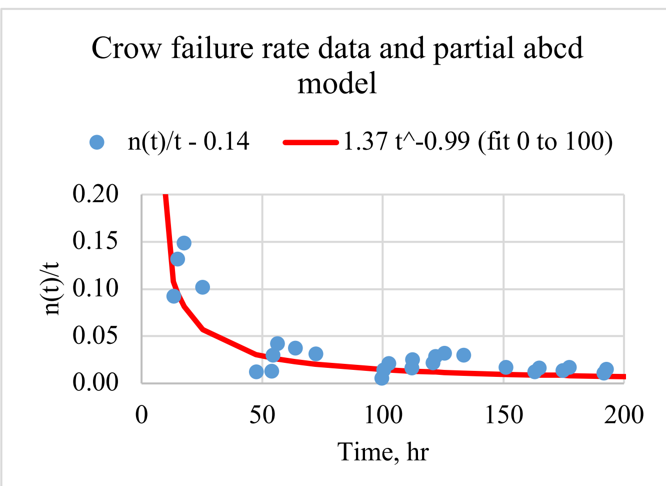

Figure 4. Crow data and partial abcd model.

The long-term constant failure rate c = 0.14 was removed from the data before the exponential curve fit was calculated.

The complete abcd model for the 56 failure /crow data set is shown in Figure 5.

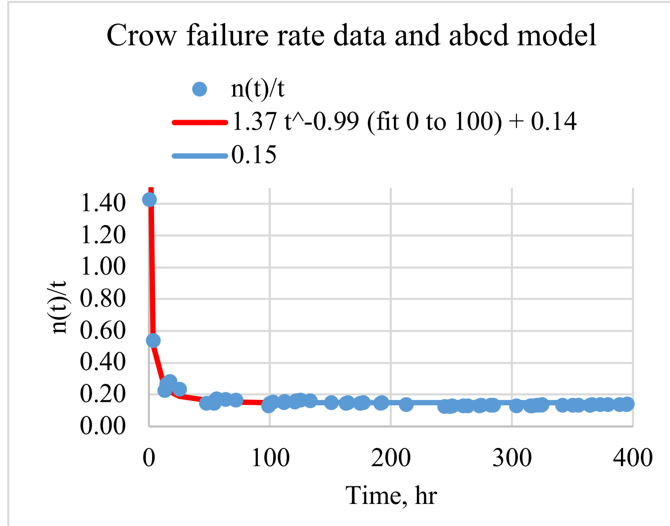

Figure 5. Crow data and complete abcd model.

The model fit to the Crow data closely reproduces the data. The initial failure rate at t = 1 is 1.37 + 0.14 =1.51, approximately equal to the first data point at time 0.7 hours, 1.43. At $t_d = 100$, failure rate = c + d = 0.14 + 0.01 = 0.15, which is close to the failure rates of 0.14 and 0.15 at time 99.6 and 100.3 hours.

4 *ANALYSIS OF ADDITIONAL DATA*

The Duane Crow reliability growth model assumes that reliability growth continues without limit, ultimately removing all the failure modes and reaching a zero-failure rate. However, there is always a non-zero final failure rate which is equal to the total number of failures divided by the test time. If this final failure rate is subtracted before fitting the exponential reliability growth curve, the fitted growth curve with the failure rate is added back will be closer to the data. Extending the Duane Crow model to include the period of constant failure rate in the abcd model is useful when the final failure rate is significant.

To check this, several failure time data sets were obtained from a military handbook on reliability test methods.[2] The data sets having the most failures were chosen to illustrate the abcd model. The data sets have 41, 19, and 16 failures.

Figure 6 shows the data points n(t)/t, for the 41-failure data and the abcd model fit.

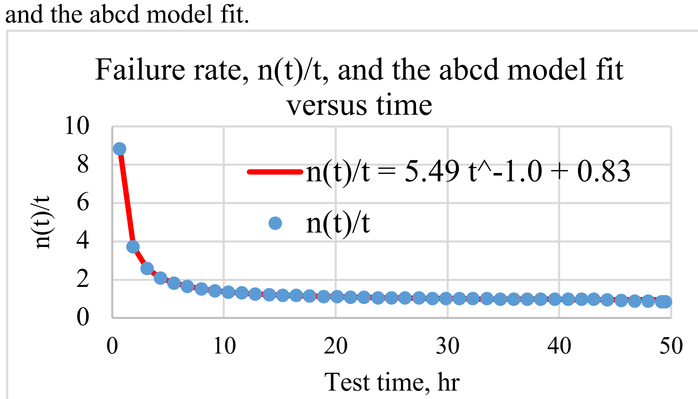

Figure 6. The 41-failure n(t)/t and the abcd model fit.

Unlike the Crow data, the 41-failure data set has a continually declining failure rate. The last failure is at t = 43 and n(t)/t continues to decline until t = 50, due to the division of the total number of failures by increasing test time. The end of both the test and the reliability growth period is at td = 50 and the final failure rate c + d = 0.83. The computed abcd model provides a close fit to the data.

Figure 7 shows the data points n(t)/t, for the19-failure data and the abcd model fit.

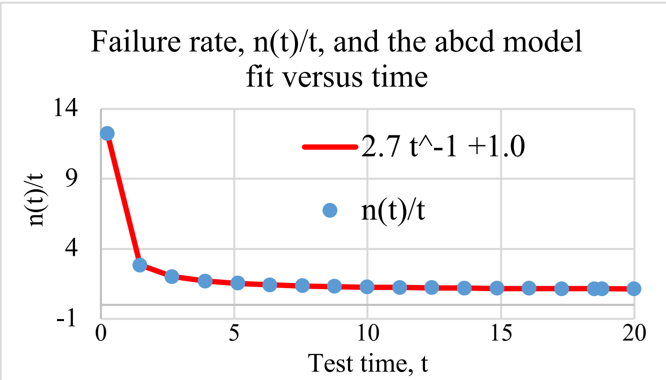

Figure 7. The 19-failure data, n(t)/t and the abcd model fit.

The 19-failure data set is like the Crow data set in that there is a final constant failure rate, c = 1.0, after td = 15. Also similarly, d ~ 0.0 indicating both that reliability growth due to redesign has ceased and that the decline in n(t)/t due to the 1/t factor has become small.

The 16-failure data set is like the 19-failure data set and the Crow data set in having a final constant failure rate, c = 0.88, after td = 16 and d ~ 0.00. Figure 8 shows the data points n(t)/t, for the16-failure data and the abcd model fit.

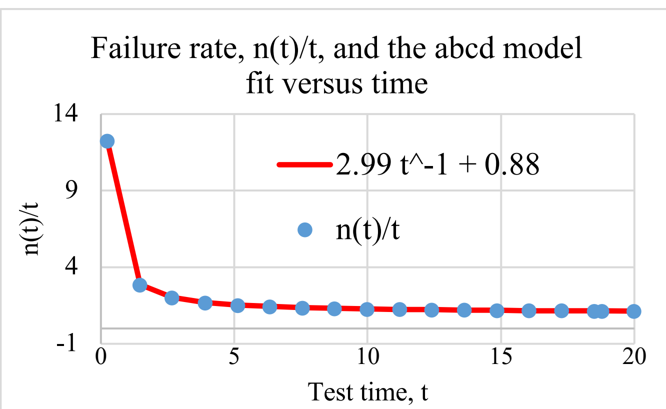

Figure 8. The 16-failure data and the abcd model fit.

5 THE FINAL FAILURE RATE IS USUALLY LARGE

Extending the Duane Crow model to include the period of constant failure rate in the abcd model is useful when the final failure rate is significant. To check this, most of the failure time data sets in two military handbooks on reliability test methods were checked.[1, 2] Table 1 shows the initial and final failure rates and the percentage of the initial rate in the final rate.

Table 1. Initial and final failure rates in reliability growth data.

Number of failures	Data set	Initial failure rate	Final failure rate	Final percentage
56	Crow, ref 1, p. 114	1.43	0.14	9.8%
41	Ref 2, p. 121	8.82	0.83	9.4%
40	Ref 2, p. 128	6.60	0.81	12.2%
27	Ref 1, p. 83	0.38	0.09	23.4%
19	Ref 2, p. 122	12.50	0.87	7.0%
18	Ref 2, p. 113	5.40	0.82	15.2%
16	Ref 2, p. 128	4.29	0.78	18.2%
15	Ref 2, p. 111	1.50	0.02	1.1%
15	Ref 2, p. 114	5.79	0.73	12.6%
8	Ref 2, p. 124	2.86	0.82	28.7%
7	Ref 2, p. 114	4.18	0.72	17.2%
7	Ref 2, p. 124	3.51	0.68	19.3%
6	Ref 2, p. 115	5.40	0.58	10.7%
5	Ref 2, p. 115	4.37	0.74	16.8%

In all but one case the final failure rate is significant. It is an average of 14.4% of the initial failure rate. In all but one case it is 7% or higher. Assuming that acceptable or uncorrected failures can be neglected in modeling reliability growth data does not seem justified.

6 DISCUSSION

The familiar bathtub curve used in reliability shows the failure rate initial declining with time, remaining constant for a long period, and finally increasing. Operating equipment is usually assumed to have a constant failure rate. This makes it surprising that in the widely used Duane-Crow reliability growth model, reliability growth is not assumed to end in a constant failure rate. Even where the failure rate is still declining at the end of testing, the remaining failure rate should be removed from the data to compute a good fit to the data.

The most surprising result shown in the abcd models of the 56, 41, 19, and 16 failure data sets is that b, the exponential decline rate is exactly 1. This is much different that the very variable growth rates found in the Duane Crow model. There, the reliability growth rate declines as longer periods of constant failure rate data are included. $n(t)/t = a\ t^{-1}$ is easily explained. Suppose failure 1 occurs at t = 1. The initial n(t)/t = a 1/1 but as t increases, the cumulative failure rate due to the first failure declines as a/t. The first failure mode probably has a high failure rate and a short MTBF, but for reliability growth that failure mode is removed. Later failures are usually not numerous enough or rapid enough to overcome this decline. A reliability growth exponent of 1 is significant because it indicates the reliability growth process of find and fix is being well implemented. If failure modes are not removed or if fixes introduce new serious failure modes, the failure mode would not decline at the maximum possible rate.

7 APPLICATIONS

Implementing reliability growth helps cure a significant problem. New system designs often have unexpected and unacceptable high initial failure rates. This may be due to improper specifications, design errors, workmanship problems, unreliable components, or flawed materials. When excessive failure rates are encountered, a "find and fix" program should identify and remove the high probability failure modes that appear in early testing. The reliability growth testing may be terminated when the system has an acceptable low failure rate. Testing may be extended to improve the estimation of the final constant failure rate. The duration of the reliability growth phase and the subsequent reliability testing is a design decision that can be optimized to reduce cost.[4]

The abcd reliability growth model can help in analyzing completed tests, tracking ongoing tests, and planning future tests. If a completed reliability growth test is analyzed, the time td when a constant failure rate begins, and the constant failure components c and d can be identified and the abcd model used to analyze the reliability growth patten. A similar analysis can be applied during reliability growth. The current failure rate can be removed from the data and the exponential failure decline model used to check if the reliability growth exponent is approximately b = 1. A slower failure rate decline would suggest that the find and fix process may be inefficient.[3]

8 SUMMARY & CONCLUSIONS

The abcd parameters of the model are determined by the failure test data, which is the time of each successive failure. If the failure rate reaches a constant value, that value is c. If Reliability growth testing is terminated at td, the remaining uncorrected failures have a failure rate of d. The values of a and b are determined by fitting a t^{-b} to n(t)/t -c. Typically, b = 1.0, reflecting the fastest possible reliability growth, where each failure mode contributes only once to the failure count and its effect on the average failure rate declines linearly with time.

The abcd model was developed by combining an initial phase of exponential Duane Crow reliability growth model with a constant long term failure rate with two potential components. The failure rate $n(t)/t = a\ t^{-b} + c + d$, where $a\ t^{-b}$ describes exponential reliability growth, c is the constant accepted failure rate, and d represents the potential failure rate reduction obtainable by further reliability growth testing.

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BIOGRAPHY

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Harry Jones is a NASA systems engineer working in life support. He previously worked on missiles, satellites, Apollo, digital video communications, the Search for Extra Terrestrial Intelligence (SETI), and the International Space Station (ISS).