

Enhancements to Linear Stability-Based, CFD-integrated Transition Prediction for High-Speed Flows

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Combining linear stability calculations with computational fluid dynamics (CFD) simulations has great potential for the automated modeling of high-speed flows, especially when adequate information about the configuration and the disturbance environment is available. However, a significant impediment to the applicability of this technique is the lack of an efficient method to calculate the crucial amplification ratio corresponding to the onset of transition in hypersonic flows. This ratio, also known as the "transition N-factor," is dependent upon the freestream disturbance environment as well as the surface properties of the test article. In response to the need for an engineering solution to predict the transition N-factor within conventional hypersonic wind tunnels, this paper presents a data-driven correlation that expands the existing correlations from straight circular cones with a narrow range of half angles to a broader array of axisymmetric configurations. Furthermore, when tested against a chosen dataset that was not used in its calibration, the suggested correlation shows good predictive accuracy with an RMS error of only 6.9%. Although similar accuracy may also be achieved via existing correlations based on similar datasets, predictions based on the proposed correlation have the advantage of not requiring an extensive amount of configuration-specific data. Practical applications often have access to the input parameters needed for this correlation, such as the freestream disturbance intensity, Mach number, and body-based slenderness Reynolds number. Additionally, this correlation outperforms the traditional assumption of a constant N-factor, particularly for configurations with blunted nose geometries. The development of this correlation is grounded in an extensive dataset encompassing conical models with body half-angles varying between 5 degrees and 16 degrees, Mach numbers ranging from 5 to 14, and nosetip-based Reynolds numbers approaching the transition reversal limit for blunt-nosed cones.

Nomenclature

A	=	Disturbance amplitude [%]
A_{cs}	=	Cross-sectional area [m ²]
c	=	Test section circumference [m]
$\bar{c}$	=	Normalizing parameter [nondimensional] (Eq. 1)
C_F	=	Tunnel wall skin-friction coefficient
C_R	=	Receptivity coefficient
f	=	Disturbance frequency [kHz]
K	=	Facility-specific constant (Eq. 6)
k_p	=	Temperature-related constant (Eq. 8)
k_q	=	Temperature-related constant (Eq. 8)
L_n	=	Nozzle length [m]
P	=	Pressure [kPa]
$\mathbf{q}'$	=	Perturbation flow variables (density, velocity, and temperature) [metric standard units]
N	=	Logarithmic amplification factor [nondimensional]
Re_∞	=	Freestream unit Reynolds number [m ⁻¹]
Re_D	=	Tunnel diameter-based Reynolds number [nondimensional]

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Re_N	=	Nose radius-based Reynolds number [nondimensional]
Re_{sl}	=	Slenderness-based Reynolds number [nondimensional]
Re_x	=	Local length-based Reynolds number [nondimensional]
r	=	Radial coordinate [m]
t	=	Time [s]
T	=	Temperature [K]
Tu_∞	=	Equivalent turbulence intensity [%]
x	=	Axial coordinate [m]
y	=	Azimuthal coordinate [m]
α	=	Streamwise wave number [1/m]
β	=	Azimuthal wave number [1/m]
γ	=	Intermittency
γ_g	=	Ratio of specific heats [nondimensional]
γ_0	=	Spectral slope of tunnel disturbances w.r.t disturbance frequency [(P'/P)Hz ^{1/2}] (Eq. 3)
(ξ, η, ϕ)	=	Body-fitted coordinates [nondimensionalized by characteristic length]
ρ	=	Density [kg m ⁻³]
σ_1	=	Receptivity correlation parameter [nondimensional] (Eq. 25)
σ_2	=	Receptivity correlation parameter with bluntness correction [nondimensional] (Eq. 28)
ϕ	=	Azimuthal angle [°]
ω	=	Disturbance angular frequency [rad s ⁻¹]
<i>subscripts</i>		
D	=	Relating to test section diameter
e	=	Relating to boundary layer edge
max	=	Relating to the saturation amplitude of disturbances
sl	=	Relating to body slenderness
tr	=	Value required for transition onset
w	=	wall conditions
x	=	Corresponding to given x location
0	=	Stagnation conditions
∞	=	Freestream value
<i>superscripts</i>		
+	=	Defined in wall units
'	=	Perturbation quantities
$\overline{Q}$	=	mean of quantity Q

I. Introduction

The prediction of laminar-to-turbulent has been identified as a key capability across speed regimes [1], and is particularly important for high-speed flows where transition can be a dominant contributor to the overall prediction uncertainty in the aerothermodynamic environment of the flight vehicle [2]. Despite its importance, however, transition prediction in high-speed flows is still a significant challenge. While substantial investments have been devoted to this research area [3], there is a need for continued research to develop a dependable, cost-effective, and resilient predictive capability for transition in high-speed flows.

One source of the challenges associated with high-speed transitional flows is the phenomenological differences between the instigation of transition in subsonic flows and hypersonic flows. Flows over smooth surfaces and with sufficiently low levels of free-stream disturbances experience transition to turbulence as a result of the amplification of modal instabilities. In low-Mach-number flows without significant crossflow, planar Tollmien-Schlichting waves are the most amplified. These waves are fluctuations in velocity, and so are purely kinematic with no thermodynamic component. At high Mach numbers, Mack-mode instabilities become the most-amplified instability mode [4]. At supersonic Mach numbers, Mack's first-mode instabilities are usually the most amplified, and in the hypersonic regime, Mack's second-mode instabilities are dominant. The latter instabilities are partially driven by the thermodynamic state of the flow [5].

The most widely used transition models in the CFD community are the Langtry-Menter $\gamma - Re_\theta$ model [6] and the Menter γ model [7], which are coupled to the Menter's Shear Stress Transport (SST) turbulence model [8], and the Coder

Amplification Factor Transport (AFT) model [9], which is coupled to the Spalart-Allmaras (SA) turbulence model [10]. These models were developed primarily to address the dominant transition mechanisms in low-Mach-number flows. Because the instability modes of interest for hypersonic flows are of a different kind than those for which these models were developed, it is difficult to extrapolate them to hypersonic conditions. Models specifically for the high-speed flow environment have been proposed, relying either on correlations to account for the thermodynamic properties of the flow as in Ref. [11] or on alternate frameworks that model the transport of fluctuations in the laminar flow as in Refs. [12–14]. These tools are still limited in their applications to canonical test cases. High-speed flows can be reliably examined via scale-resolving simulations, as in Refs. [15] and [16], but these methods are too costly to be practical in a number of applications.

An intermediate-cost alternative to transport models and scale-resolving simulations is CFD-integrated stability analysis. Stability computations are capable of identifying the growth of instabilities in flows across the speed regime. Coupled stability predictions have been presented in the literature by researchers at ONERA and DLR [17–19], as well as by Shi et al. [20] and Halila et al. [21]. Recently, efforts undertaken by NASA Langley have demonstrated CFD-integrated stability analysis for low-speed flows using NASA’s OVERFLOW [22] and FUN3D [23] codes and have demonstrated CFD-integrated analysis for supersonic and hypersonic flows using OVERFLOW [24]. In the analysis of high-speed flows, it was noted that improvements to the prescription of intermittency for high-speed cases was needed and that the lack of a means of estimating the N-factor at which transition onset occurs was a major obstacle to the maturation of this method. This work is concerned with the latter challenge.

There are several existing correlations that can be used to predict transition in high-speed flows [25–28], but these correlations are not formulated in such a way that allows them to be compatible with integrated transition prediction. The present work seeks to derive and evaluate a correlation for the N-factor at transition onset for simple axisymmetric geometries in conventional hypersonic ground test facilities for use in coupled CFD-stability computations. The present correlation has been formulated with the following goals and requirements in mind: First, the inputs required for the correlation should be consistent with the detail and scope of the transition correlations used in typical transport-based transition models for high-speed flows. Because the coupling of stability analysis with CFD is expected to be used in similar applications as the transport-based models, it is necessary that the required inputs do not overburden the users by requiring excessive, detailed information they may not possess. In practice, this means that an individual ground-test facility should be characterized by small number of transition-related parameters (analogous to the freestream turbulent kinetic energy and turbulent eddy viscosity, which two-equation RANS models such as the SST model require and are commonly used in the context of transition modeling in low-speed flows). Second, the correlation should be tuned with data from multiple ground-test facilities and test campaigns. This approach is necessary to mitigate potential ambiguity about the applicability of the trends observed in this study, accounting for variations in geometric and facility parameters, such as conical bodies with a broad range of half angles and multiple types of conventional facilities. Next, while some geometric parameters are known to impact the transition N-factor, there may be situations in which these parameters are not available to the user or in which they are not applicable. Thus, as a third requirement of the correlation, we also consider it desirable to avoid relying on geometry-specific information, or to provide options to enable CFD-coupled predictions even when the user lacks specific details of the test setup, such as the nose radius, body half-angle, test section shape, etc. This goal is identified with the aim of expanding the applicability of the correlation in future work, and with the knowledge that a reliance of specific geometric information may limit the applicability of this relationship for complex configurations. Finally, the correlation should allow for predictions of the expected value of N_{tr} before any stability analyses or CFD simulations are carried out for a given configuration.

The correlation developed in the remainder of this work is closed form and the only required input parameters correspond to the freestream Reynolds number, Mach number, wall temperature, total temperature, a single disturbance parameter that is described in subsequent sections, and optionally, the nose radius. The tunnel noise environment as interpreted in this correlation is characterized by a single scalar value which is easily calculated from routine flow quality analysis of an experimental facility. This correlation is applicable to straight-cone configurations in conventional hypersonic ground test facilities. The remainder of this work is organized as follows. Section II provides an overview of the correlations available in the contemporary literature, what aspects of them are successful, and what aspects need improvement in order to be applicable in the context of CFD-coupled stability analysis. In section III, the facilities and transitional flow experiments included in this study are described. In section IV.A, a high-level description of e^N stability analysis is provided, and the general approach to the correlation being developed is presented. Section IV.B outlines the components that make up the overall correlation, beginning with the characterization of the freestream disturbance environment in conventional high-speed wind tunnels (sections IV.C and IV.D, continuing with the estimation of the boundary-layer disturbance amplitude at transition onset (section IV.E), and finally, the receptivity coefficient from

the freestream disturbance field to incipient instabilities in the boundary layer (section IV.F). In section V, the CFD predictions based on this correlation are evaluated against data for selected configurations that were not included in the training database for the correlation. The final section VI presents concluding remarks, including our plans to expand the applicability of the present correlation.

II. Existing Correlations for Hypersonic Transition

Pate's correlation [25] introduced several concepts that are included in subsequent correlations and identified the importance of acoustic radiation from tunnel walls in driving the transition process in high-speed flows. Pate's correlation is applicable to sharp, slender cones, and is given by

$$Re_T = \frac{48.5(C_F)^{-1.4}\bar{c}}{\sqrt{\delta^*/c}}, \quad (1)$$

where Re_T is the transition Reynolds number, C_F is the skin friction coefficient on the tunnel walls, δ^* is the boundary layer displacement thickness on the tunnel walls, $\bar{c}$ is a nondimensional normalizing parameter related to the facility and model size, and c is the test section circumference. In this context, the transition Reynolds number refers to the local value of the length-based Reynolds number at which transition onset occurs. Pate's correlation is empirical and is constructed to predict the transition onset Reynolds number rather than N-factor or disturbance amplitude. Additionally, Pate's correlation does not account for bluntness-induced changes to the transition location and requires the computation of the tunnel wall skin friction coefficient and the displacement thickness. In addition to an ambiguity about the spatial location where these metrics should be evaluated, the effort required to generate the necessary input parameters makes the use of Pate's correlation significantly more labor-intensive for the user than a typical transport-based transition prediction. Pate reports standard deviation of the error between the correlation and training data of 11.6% in terms of Re_T . Marineau et al. [26] developed a correlation based on Mack's amplitude method for 7°-half angle conical geometries at Mach 10. Marineau's correlation entails an iterative calculation of the transition N-factor, N_{tr} , and it may be expressed in the form

$$N_{tr} = \ln \left(\frac{1.25 A_{max}}{A_0(f_{tr})} \right). \quad (2)$$

In this equation, A_{max} and A_0 are, respectively, the maximum and initial magnitude of disturbances within the boundary layer measured via the root mean square (RMS) of pressure fluctuations normalized by the local static pressure, and f_{tr} is the frequency of the individual mode that leads to transition. To estimate N_{tr} using this method, the user supplies an initial guess for the transition location. From this estimate, the local edge Mach number and most amplified frequency at the estimated location are calculated. With this information, the value of A_{max} is estimated using experimental data gathered in the AEDC tunnel 9 facility and the value of A_0 is calculated according to

$$A_0 = C_R \gamma_0 \sqrt{(\Delta f)}, \quad (3)$$

where C_R is the receptivity coefficient, γ_0 is the spectral amplitude of the tunnel disturbance environment, and Δf is the frequency bandwidth of the second-mode instabilities. Similar to A_{max} , these quantities are determined via facility-specific correlations that, in the case of Marineau et al., were derived using data collected in the Tunnel 9. These correlations are presented graphically in Ref. [26]. This correlation was developed for a relatively narrow set of facilities and conditions and only used experimental data for a cone with a 7° half-angle and a nominal freestream Mach number of 10. Because of this, the applicability of this correlation outside of the above range of parameters remains unknown. While the iterative nature of N_{tr} prediction does not make it incompatible with coupled stability analysis, it would effectively represent a subiteration in this context that could increase the overall time and effort required to use the method. Marineau reports a standard error between the correlation and training data of 14% in terms of transition location, or 3.7% when the error is normalized by 1.5 m model length.

In a review of freestream disturbance levels and 7° half-angle cone experiments in conventional ground test facilities, Marineau et al. [27] presented a number of findings regarding second-mode disturbances across 14 facilities. First, they found that

$$A_0 \sim (Re_\infty/3.5L_n)^{-1}, \quad (4)$$

where L_n is the length of the nozzle employed in the facility. Second, they found that the maximum second mode amplitude scales linearly with the boundary layer edge Mach number, i.e.,

$$A_{max} \sim M_e. \quad (5)$$

In this review, the authors stop short of developing a correlation for N_{tr} , but the findings provide useful insight for others seeking to do so.

Lee et al. [28] presented a correlation for the N-factor in ground test facilities that includes many of the observations of Marineau et al. [27], and is constructed using experimental data from five different experimental facilities. Their approach utilizes individual correlations to identify the necessary information rather than using the linear stability theory (LST). However, the resulting relationship is not constructed in a way that would make it generally applicable. Instead, it is presented as a series of relationships that allow extrapolation of existing experimental data given certain parameters. These relationships are summarized as follows: if the nose bluntness Reynolds number, Re_N , is less than 1×10^4 then the following relationship must be satisfied

$$N_{tr} = \frac{1}{K_2} \ln(Re_N / K_1), \quad (6)$$

$$C_R = 274.6 \left(\frac{T_w}{T_0} \right)^{7.987} (Re_N)^{c_2}, \quad (7)$$

and

$$Re_{tr} = k_p Re_N + k_q, \quad (8)$$

where Re_N is the Reynolds number based on nose radius as mentioned above. The principal challenge associated with Lee's correlation is the definition of the constants K_1 , K_2 , and to a lesser extent, the remaining constants, c_2 , k_p , and k_q . The values of the first two constants are facility-specific, meaning that at least two run conditions for a given configuration in a given facility must be observed experimentally and also simulated via LST. From these, the values of K_1 and K_2 can be determined by fitting Eq. 6 to the experimental data. The remaining constants must be similarly estimated from the experimental data, and while Lee et al. [28] provide a partial accounting of the dependencies of these constants (for instance, they report that k_p is a function of wall temperature when the Mach number is held constant), the description is insufficient to form a widely-applicable, closed-form correlation for N_{tr} . These relationships are developed based on the experimental data for 7° and 8° half-angle cones in wind tunnels with nominal Mach numbers of 6 and 10 only. In addition to the ambiguity in its formulation, this correlation was established using a narrower range of conditions than optimal. Similar to the findings of Marineau et al. [27], the trends identified by Lee et al. [28] provide valuable insights into the relationships between key flow parameters and the transition N-factor. However, on their own, they may not be sufficient for predicting the N-factor for an axisymmetric configuration.

III. Experimental Facilities and Test Cases

This section lists and describes the ground test facilities and experimental measurements used to develop the correlation for N_{tr} . Most of these facilities provide both freestream characterization data and experimental transition data which is used to formulate the N-factor correlation as described in the Introductions. However, a few remaining facilities either lack adequate characterization of their freestream flow conditions or have not been utilized in the transition experiments used to develop the correlations targeted in the present paper. Nonetheless, they still contribute valuable insights to this study, and hence, are included in this section. For instance, no experiments conducted in the Texas A&M Actively Controlled Expansion Tunnel are used in the correlation, but that facility provides characterization data for lower unit Reynolds numbers than most of the other facilities. Many of the facilities examined here were also included in the tunnel disturbance analysis presented in Ref. [27], and the interested reader is referred to that resource for additional information about the disturbance environment in modern ground-test facilities. In this work, we target the characterization of the freestream disturbance field in the test section in terms of the root mean square (RMS) amplitude of the pressure disturbance, which is usually measured via pitot probes, but can also be predicted via direct numerical simulations (DNS) of the wind tunnel nozzle and the test section under certain assumptions. A summary of the facilities in which experimental data contributing to this study were gathered is provided in Table 1. This study incorporates a total of 48 individual experimental data points. Among these, 41 are employed for developing the correlation, whereas the remaining 7 are utilized to assess its performance on an independent data set.

TAMU Actively Controlled Expansion Tunnel

The Texas A&M Actively Controlled Expansion (ACE) tunnel is a variable Mach number facility which is housed alongside the ex-LaRC Mach 6 Quiet tunnel at TAMU. The facility is a conventional wind tunnel that is capable

Table 1 Summary of experimental data sets included in the present correlation.

Facility	Mach Number(s)	Facility Type	Cone Half-Angle
LaRC 20-inch Mach 6	6	Blow Down Tunnel	5°
LaRC Variable Density Tunnel	8	Blow Down Tunnel	10° and 16°
AEDC Tunnel 9	10 and 14	Blow Down Tunnel	7°
TU Braunschweig HLB	5.9	Ludwig Tube	7°
German Aerospace Center HEG	7.5	Shock Tunnel	7°
VKI Longshot Tunnel	6.0	Gun Tunnel	7°
Sandia HWT	5 and 7.8	Blow Down Tunnel	7°
CUBRC LENS 1	7 to 10	Shock Tunnel	5° and 7°
AFRL Mach 6 LT	6	Ludwig Tube	ogive-cylinder

of testing at Mach numbers between 5 and 8. Kostak et al. [29] provide the total temperature, mass, and velocity fluctuations as well as the total pressure fluctuations in the facility. This facility was used in the ground testing of the Boundary Layer Transition (BOLT) flight test geometry, and therefore, may be included in our future work that aims to increase the complexity of flow configurations considered by the present correlation.

Langley Research Center 20-inch Mach 6 Tunnel

The latest characterization of the NASA 20-inch Mach 6 wind tunnel was described by Chou [30], although previous measurements were also carried out by Rufer [31] and even earlier by Stainback [32]. A DNS characterization of the acoustic disturbances dominating the freestream disturbance environment in a 2D version of the wind tunnel nozzle and the comparison of the DNS predictions with the measurements in [30] has been reported by Hildebrand et al. [23, 33]. The facility is a blow down tunnel with run times up to 20 minutes and a maximum temperature of 555.6 K. The Reynolds number can be set by modifying the total pressure. The range of unit Reynolds numbers for which facility has been characterized ranges from 3.28×10^6 to 26.5×10^6 /m. This facility has been used for numerous transitional flow experiments at Mach 6, including those by Horvath et al. [34], which provide spatially well-resolved transitional heat transfer data for straight-cone and flared-cone geometries including the effects of nose bluntness. Table 2 provides details of the experimental cases from this facility that were examined in Ref. [34].

Table 2 Flow parameters, disturbance metrics, and N_{tr} for cases examined in the LaRC 20-inch Mach 6 Tunnel.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
6.0	9.19e6	60.56	298.76	0.00254	0.329	4.1
6.0	10.5e6	60.56	298.76	0.00254	0.329	4.2
6.0	14.1e6	60.56	298.76	0.00254	0.329	4.6
6.0	17.7e6	60.56	298.76	0.00254	0.329	4.7
6.0	25.59e6	60.56	298.76	0.00254	0.329	5.5
6.0	25.59e6	60.56	298.76	0.79375	0.329	6.0
6.0	25.59e6	60.56	298.76	1.5875	0.329	6.9

Langley Research Center Mach 8 Variable-Density Tunnel

The NASA Langley Mach 8 Variable-Density Tunnel (VDT) was capable of test section Mach numbers between 7.2 and 8.1 with a test section diameter of 0.46 m. This facility was capable of significantly higher freestream Reynolds numbers than the other facilities considered here, and the characterization and experimental data associated with this facility are older in comparison with the other facilities included herein. Therefore, despite the large number of experimental data points provided by Stainback et al. [35], eight experimental data points are chosen for consideration in this work, four of which correspond to experiments on a 10° half-angle cone, and the other four correspond to a 16°

half-angle cone. Table 3 provides details of the experimental cases examined from this facility.

Table 3 Flow parameters, disturbance metrics, and N_{tr} for cases examined in the LaRC VDT.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
8.0	33.9e6	128.0	298.76	2.5	0.273	10.1
8.0	20.4e6	136.67	298.76	2.5	0.273	8.9
8.0	45.3e6	135.0	298.76	2.5	0.273	10.7
8.0	40.9e6	128.0	298.76	2.5	0.273	11.4
8.0	42.6e6	139.4	298.76	2.5	0.273	13.4
8.0	27.8e6	139.11	298.76	2.5	0.273	12.7
8.0	37.5e6	142.0	298.76	2.5	0.273	13.6

AEDC Tunnel B

The Arnold Engineering and Development Complex (AEDC) Tunnel B hypersonic test facility is a conventional, closed-circuit hypersonic flow facility capable of operating at Mach 6 and Mach 8, with unit Reynolds numbers between $1.6 \times 10^6 \text{ m}^{-1}$ and $12.1 \times 10^6 \text{ m}^{-1}$ [36]. While no experiment conducted in this facility is included in this work, hot-wire and pressure RMS data for this tunnel are available, and so it is included here as an additional data point for identifying trends in the freestream disturbance amplitude as a function of Mach number and Reynolds number.

AEDC Tunnel 9

The AEDC Tunnel 9 hypersonic test facility is a conventional blowdown facility capable of operating at Mach 7, 8, 10 and 14, with unit Reynolds numbers between 13.1×10^6 and $24.9 \times 10^6 / \text{m}$ [36]. This facility has been characterized extensively for the prediction of 2^{nd} -mode induced transition, and has had multiple transition experiments conducted in it for which accompanying stability analysis has been conducted [26, 37]. Table 4 provides details of the experimental cases examined from this facility.

Table 4 Flow parameters, disturbance metrics, and N_{tr} for cases examined in AEDC's Tunnel 9.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
10.0	1.81e6	54.0	298.76	0.01524	0.41	3.9
10.0	6.40e6	54.0	298.76	0.01524	0.41	5.1
10.0	15.1e6	54.0	298.76	0.01524	0.41	5.6
14.0	1.7e6	46.5	298.76	0.01524	0.41	3.6
14.0	2.6e6	46.5	298.76	0.01524	0.41	3.8
14.0	3.6e6	46.5	298.76	0.01524	0.41	3.9
14.0	7.2e6	46.5	298.76	0.01524	0.41	4.5
14.0	12.1e6	46.5	298.76	0.01524	0.41	5.3

TU Braunschweig HLB

The Hypersonic Ludwig Tube Braunschweig is a Mach 5.9 Ludwig tube facility located at the Technical University of Braunschweig. The facility is capable of running at Reynolds numbers from 2.2×10^6 to $22 \times 10^6 / \text{m}$, with run times of approximately 80 ms. The facility is characterized by Munoz et al. [38] and via DNS by Duan et al. [39]. Experimental data for a hypersonic straight cone conducted in this facility are presented by Estorf et al. [40]. Table 5 provides details of the experimental cases examined from this facility.

Table 5 Flow parameters, disturbance metrics, and N_{tr} for cases examined in the DLR HLB.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
5.9	17.8e6	57.7	298.76	0.024	0.395	4.5
5.9	14.2e6	57.7	298.76	0.024	0.395	4.4
5.9	10.7e6	57.7	298.76	0.024	0.395	4.2
5.9	8.9e6	57.7	298.76	0.024	0.395	4.1
5.9	7.1e6	57.7	298.76	0.024	0.395	4.0

German Aerospace Center High Enthalpy Shock-Tunnel Göttingen

The German Aerospace Center (DLR) High Enthalpy Shock-Tunnel Göttingen (HEG) is a piston-driven shock tunnel capable of achieving Mach 10 flow in the test section. This facility is one of the few shock tunnel facilities included in the present study and has been characterized by Wagner et al. [41]. This facility housed a 7° half-angle cone experiment, described in Ref. [42], which is included in the training database for the current study. Table 6 provides details of the experimental cases examined from this facility.

Table 6 Flow parameters, disturbance metrics, and N_{tr} for cases examined in the DLR HEG.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
7.5	4.0e6	270.0	293.0	2.5	0.293	6.2
6.0	6.4e6	245.0	293.0	2.5	0.293	6.8

Von Karman Institute Longshot Gun Tunnel

The Von Karman Institute (VKI) Longshot facility is a gun tunnel capable of achieving Mach 6 flow in the test section. This facility represents the only gun tunnel included in the present study. This type of facility is less common than blowdown facilities, but a number of gun tunnel facilities are currently operational around the world, and so it is appropriate that at least one of those be included here. Disturbance characterization and experimental test data for this facility are reported by Grossir et al. [43]. Table 7 provides details of the experimental cases examined from this facility.

Table 7 Flow parameters, disturbance metrics, and N_{tr} for cases examined in the VKI Longshot Tunnel.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
6.0	1.80e6	60.1	259.0	0.0126	0.26	5.6
6.0	2.28e6	60.1	259.0	0.0126	0.26	5.8
6.0	2.71e6	60.1	259.0	0.0126	0.26	6.1

Sandia National Lab Hypersonic Wind Tunnel

The Sandia National Laboratories (SNL) Hypersonic Wind Tunnel (HWT) is a hypersonic blowdown facility with three interchangeable nozzles allowing it to produce test-section Mach numbers of 5, 8, and 14 [44]. At the conditions considered in this work, Mach 5 and 8, the facility's maximum stagnation temperature is 900 K. In Mach 5 operation, the Reynolds number range is $3 - 30 \times 10^6 \text{ m}^{-1}$, and in Mach 8 operation, the Reynolds number range is $3 - 20 \times 10^6 \text{ m}^{-1}$. Experimental transition results for a hypersonic straight cone studied in this facility are presented by Casper et al. [45, 46]. In addition to the experimental transition data, PSE-based stability analyses on a 7-degree half-angle sharp cone are conducted by Casper et al. [46]. Table 8 provides details of the experimental cases examined from this facility.

Table 8 Flow parameters, disturbance metrics, and N_{tr} for cases examined in the Sandia HWT.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
5.0	8.50e6	60.56	298.76	0.005	0.313	3.9
5.0	10.0e6	60.56	298.76	0.005	0.313	4.1
5.0	11.4e6	60.56	298.76	0.005	0.313	4.2
5.0	15.4e6	60.56	298.76	0.005	0.313	4.2
7.8	6.2e6	60.56	298.76	0.005	0.313	4.8
7.8	7.0e6	60.56	298.76	0.005	0.313	4.9
7.9	9.0e6	60.56	298.76	0.005	0.313	5.0
7.9	13.0e6	60.56	298.76	0.005	0.313	5.3

CUBRC LENS I Facility

The final facility housing a straight-cone experimental database included in this work is the CUBRC LENS 1 facility. This facility is not included in the data used to develop the correlation targeted in the present study but is used as an independent data set to evaluate the performance of the correlation. Disturbance characterization data for the LENS I is not available, so for the cases examined here its disturbance environment is estimated to be similar to that of the DLR HEG. Both facilities are shock tunnels and the Mach numbers analyzed here are in the mid-hypersonic range (approximately Mach 8 to Mach 10) for both. Experimental data and stability-calculated N_{tr} values are reported by MacLean et al. [47, 48] for a 7° half-angle cone with several different nose bluntness radii and Reynolds numbers. Table 9 provides details of the experimental cases examined from this facility for independent analysis of the correlation.

Table 9 Flow parameters, disturbance metrics, and N_{tr} for cases examined in the CUBRC LENS I.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{tr}
9.90	2.36e6	217.1	298.76	6.35	0.293	6.6
9.95	2.34e6	215.1	298.76	2.5	0.293	5.8
9.92	2.42e6	216.3	298.76	0.05	0.293	5.2
9.93	3.13e6	215.9	298.76	6.35	0.293	7.0
10.0	1.79e6	213.0	298.76	0.05	0.293	5.2
7.16	10.2e6	231.7	298.76	2.5	0.293	7.5

AFRL Mach 6 Ludwig Tube

To assess whether the correlation can be extended to geometries with nonzero pressure gradients, we apply it to a tangent ogive-cylinder configuration, as investigated by Hill et al. [49]. This configuration is a 1-meter-long cylinder with a 31.1 mm radius in the cylindrical section. The tip angle for this configuration is 14° and the ogive radius of curvature is 1.047 m. This configuration is described by Hill et al. as a sharp-tipped ogive, but for the present computations it was necessary to use a small but finite nose radius for the configuration. In the present study, this radius was chosen to be 0.025 mm. The experimental study was undertaken in the AFRL Mach 6 Ludwig Tube facility. Characterization data for this facility is not yet available, and so the freestream disturbance environment was estimated to be approximately similar to the Sandia HWT which operates in a similar Mach and Reynolds number range. The inflow parameters for this test case are given in Table. 10.

Determination of Experimental N-Factors

Some of the experimental transition data used to develop the targeted correlation are presented alongside LST or PSE data that was already available, and hence, can be used to determine the N-factor at which the onset of transition occurs. The experimental transition locations for the remaining data points had to be supplemented with N_{tr} computations based on stability analysis. To that end, laminar CFD simulations were conducted using NASA's

Table 10 Flow parameters, disturbance metrics, and N_{lr} for the case examined in the AFRL Mach 6 Ludwig Tube.

M_∞	$Re_\infty(1/m)$	$T_\infty(K)$	$T_w(K)$	R_N (mm)	$Tu_{ref}(\%)$	N_{lr}
6.1	6.55e6	158.4	300	0.25	0.313	3.1

OVERFLOW flow solver [50] and the computed laminar baseflow definitions were used to perform LST computations using the LASTRAC code that is also developed and maintained by NASA LaRC [51]. An illustrative stability computation is depicted in Fig. 1, which shows the N-factor envelope for one of the cases from the DLR HEG facility. This figure also includes PSE results for this example case, demonstrating that PSE and LST agree closely for the straight-cone configurations that constitute the majority of the cases studied in the present work. The underlying laminar CFD simulations for each of these cases utilized an axisymmetric mesh of 1153×535 points in the streamwise and wall-normal directions, respectively. The wall-normal mesh was constructed to include approximately 200 points inside the boundary layer throughout the majority of the streamwise extent of the body. These meshes are shock-aligned and also adapted to the boundary-layer growth, and these features are identified using precursor laminar simulations on grids that are not feature-adapted. For experimental data in which N_{lr} is not reported, but an experimental transition location is reported, that transition location is taken as authoritative. When axial profiles of surface quantities are reported but no determination of x_{lr} is given, the transition location is estimated to occur at the intersection between a straight line drawn tangent to local slope in the center of the transition region and the extrapolated laminar portion of the flow. This process is sketched in Fig. 1 (a), but the experimental transition locations provided by the experimental authors (that agree with the present estimation) are used for this data set.

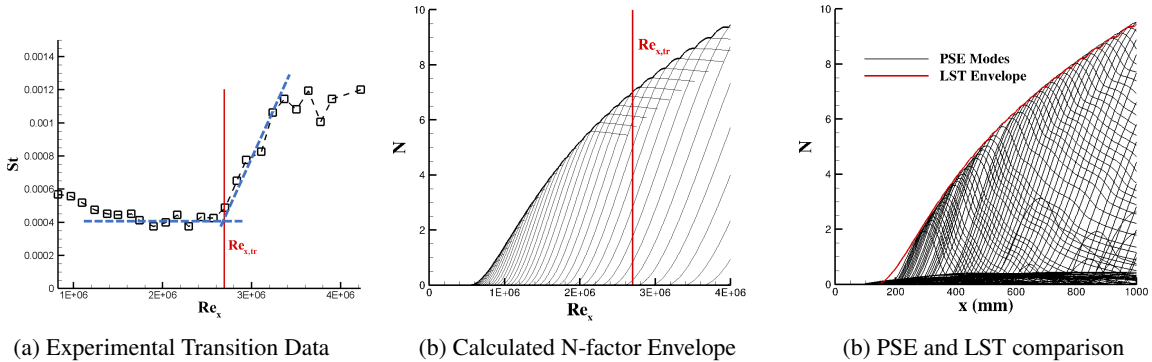


Fig. 1 Determination of transition N-factor for a 7° half-angle cone case from Mach 8 experiments in the DLR HEG by Wagner et al [42]. The transition location reported by the experimentalists (a) is compared to N-factor envelope based on LST calculations (b). LST results are shown for frequencies between 100-900 kHz in increments of 10 kHz. In (c), individual modes calculated with PSE calculated for frequencies between 100-900 kHz in increments of 5 kHz are compared to the N-factor envelope calculated with LST, demonstrating agreement between the two methods.

IV. N-Factor Prediction for High-Speed Flows

This section discusses the prerequisites for predicting N_{lr} in high-speed flows, utilizing the criteria mentioned in section I, and provides an explanation of the current correlation. The correlation is constructed using the experimental test cases described in section III. First a summary of e^N method to predict the onset of transition based on the amplification of boundary layer instabilities is provided, and a description of the quantities which must be estimated is given. Next, freestream disturbances experienced by the flow in each test case are estimated in subsection IV.C. The following subsections detail the estimation of key parameters based on the experimental test case data, supplemented with data in the literature [27]. Section IV.G combines outcomes of the previous section in a summary of the overall correlation and provides accuracy metrics for the resulting relationship. In this work, error is calculated by evaluating the

percent error between the experimental and predicted N_{tr} values, and the standard deviation of this across all of the test cases is reported. This convention is in line with that used in the prior literature [25, 26]. Compared to the contemporary literature, the present correlation is constructed based on a larger and more diverse database and is closed-form. This choice is driven by the target application of coupled stability computations, in which the user is ideally ‘hands-off’ after supplying the necessary input parameters to the solver. Throughout this section, choices and assumptions that are made specifically for compatibility with the target application of coupled CFD-stability computations are identified in the text.

A. e^N Linear Stability Analysis

The e^N method attempts to correlate the measured transition location in a flow with the local value of the N-factor envelope. Linear stability computations are used to predict the growth or decay of instabilities across a broad range of frequencies (or frequency-azimuthal wavenumber combinations in the case of 3D disturbances). The amplification rates of these instabilities depend on the laminar base flow, which is denoted here as $\mathbf{q}(\xi, \eta, \zeta, t) = (\rho, u, v, w, T)$ in terms of the primitive variables of the Navier-Stokes equations. The overall flow variables correspond to the sum of a mean and time-varying state, i.e., $\mathbf{q}(\xi, \eta, \zeta, t) = \bar{\mathbf{q}}(\eta) + \mathbf{q}'(\xi, \eta, \zeta, t)$. The perturbations, $\mathbf{q}'$ are assumed to be periodic in time (t) and azimuthal direction (ζ) and locally periodic in the streamwise (ξ) direction, and therefore, we can write

$$\mathbf{q}' = \hat{\mathbf{q}} \exp \left(i \left[\int \alpha d\xi + \beta \zeta - \omega t \right] \right). \quad (9)$$

Equation 9 can be substituted into the linearized Navier Stokes equations and solved as an eigenvalue problem after invoking suitable approximations related to slow spatial variations of the base flow. The growth rate of the disturbance at the spatial location of interest can be found as the imaginary part of the resulting eigenvalues, α , as in

$$\sigma(\xi, \omega, \beta) = -\Im(\alpha). \quad (10)$$

A complete description of the linear stability computation can be found in Ref. [51]. The amplification ratio, often referred to as the N-factor, for a given ω - β combination is found as the integral of the growth rate over the streamwise distance from some upstream point where the mode becomes unstable. This takes the form

$$N_i(\xi, \beta, \omega) = \int_{\xi_1}^{\xi} \sigma(\xi', \omega, \beta) d\xi', \quad (11)$$

and the N-factor envelope corresponds to the maximum amplitude growth at a given point on the surface of the body,

$$N(\xi) = \max[N_i(\xi, \beta, \omega)]. \quad (12)$$

In practical stability computations, discrete increments of ω and β are studied with the goal of adequately representing the continuum spectrum. The value of N_{tr} for a given transition scenario corresponds to the value of envelope N-factor that optimally correlates with the onset of transition across the various cases of interest; it is a function of both the transition scenario and the free-stream disturbance environment.

B. Form of the N-Factor Correlation

To express the transition N-factor (N_{tr}) for high-speed flows in terms of quantities that are readily accessible to CFD practitioners, we begin with the definition of N as the logarithmic amplification ratio

$$N = \ln \left(\frac{A}{A_0} \right), \quad (13)$$

where A is the amplitude of the local disturbance and A_0 is the initial amplitude of the boundary-layer instability wave where the amplification process begins. For convectively unstable flows, this initial amplitude depends on the freestream disturbance environment and the receptivity process. Following this definition, at the point of transition where $N = N_{tr}$,

$$N_{tr} = \ln \left(\frac{A_{tr}}{A_0} \right). \quad (14)$$

The prediction of N_{tr} therefore requires a reasonable estimation of the initial and critical (transitional) disturbance amplitudes. Marineau et al [26] have shown that the maximum amplitude of second-mode instabilities in a single facility

is insensitive to the unit Reynolds number, but scales nonlinearly with the Mach number. Therefore, the dependencies for this parameter may be described as

$$A_{tr} = A_{tr}(M_\infty) \quad (15)$$

The estimation of the initial disturbance amplitude is considered next. The boundary layer on the surface of a hypersonic vehicle does not directly experience the freestream disturbance environment of the flow. Instead, the incoming freestream disturbances first pass through the leading-edge shock before interacting with the boundary layer to excite the instability waves that eventually amplify and interact nonlinearly to induce the onset of transition. The process by which freestream disturbances seed the boundary-layer instabilities is called boundary layer receptivity, and can be represented via a receptivity coefficient, as in

$$A_0 = C_R A_\infty. \quad (16)$$

As described by Eq. 16, the receptivity coefficient and the freestream disturbance amplitude must both be known in order to estimate the transition N-factor. Characterizing the freestream disturbance environment in compressible flows is a complex task, which we will discuss in the following subsection. Marineau established a connection between the receptivity coefficient and the nosetip Reynolds number, which can be represented as the product of the freestream unit Reynolds number and the nosetip radius, among other factors. However, to offer engineering estimates for N_{tr} for complex configurations where scalar geometric parameters may be difficult to identify, it is also desirable for the correlation to be independent of the geometry in question. The approach taken in this work is to correlate the disturbance amplitude at transition onset using available data from the literature, and then to incorporate this information alongside the N-factor envelopes for specific test cases to back out the initial amplitude A_0 , and finally, to identify a suitable correlation between the freestream and facility parameters and the calculated values of A_0 . Because A_∞ can be calculated from the facility characterization data, the receptivity coefficient C_R can then be deduced via Eq. 16. At this point, the dependencies of the resulting correlation for C_R can be expressed as,

$$C_R = C_R(Re_\infty, M_\infty, T_w/T_T, A_\infty, Re_N). \quad (17)$$

Combining Eqs. 13, 14 and 15 leads to the following general form for the transition N-factor,

$$N_{tr} = \ln[A_{tr} M_\infty] - \ln[C_R A_\infty]. \quad (18)$$

The next subsection will examine the characterization of A_∞ , which measures the amplitude of the freestream disturbances in an appropriate part of spectrum. For simplicity, we focus on the overall spectrum first and, in this work, the amplitude of the freestream disturbances is taken to be the RMS of the pressure fluctuations, normalized by the freestream static pressure. However, it should be noted that this is not the only metric to define the amplitude of the freestream disturbances, and future work will investigate alternate options to describe A_∞ .

C. Freestream Disturbance Characterization

In low-speed flows, the transition N-factor is often predicted using Mack's correlation [4], which relates the free-stream turbulence intensity (FSTI) to N_{tr} . This is appropriate for transition in a number of low-speed wind tunnels where acoustic disturbances are sufficiently low, and therefore, the FSTI alone is sufficient to characterize the disturbance environment. This ceases to be the case in wind tunnels operated at sufficiently high supersonic and hypersonic Mach numbers, and the acoustic environment within the tunnel becomes increasingly important.

Beginning with the work of Morkovin [52] and Kovasznay [53], the freestream disturbance environment in compressible flows is known to consist of four modes. In addition to fast and slow acoustic modes, there is a vorticity mode, and an entropy mode. As discussed by George and Sujith [54], these modes are linearly decoupled from each other in a uniform base flow. The presence of multiple modes poses a challenge in the characterization of the free-stream disturbance environment. The difficulties due to the multimodal character of the disturbance environment are compounded by the fact that many quantities cannot be measured directly because the presence of instrumentation produces shock waves in the flow. Generally, the available data for free-stream disturbances consists of one or more measures, such as the root mean square (RMS) and power spectral density (PSD) of mass flux perturbations, $m' = (\rho u)'$, total temperature, T'_0 , and total pressure, P'_0 . Using the ansatz of Stainback and Wagner [55], one can relate the total pressure fluctuations measured by a pitot probe to static pressure fluctuations upstream of the pitot shock. Interpreting these data is nontrivial, and numerous means of decomposing the experimental pitot probe and hot wire anemometer data into the canonical acoustic, vorticity, and entropy modes have been suggested, as in Refs. [32, 39, 56].

Despite the multimodal nature of the freestream disturbance environment, it is often much more straightforward to measure only the RMS of pressure fluctuations. This approach is common in modern characterizations of freestream disturbances in wind tunnels, as it allows for high-frequency sensors to be used which can calculate frequency-specific data such as the power spectral density (PSD) of pressure disturbances. Because the acoustic mode generally dominates the others in conventional hypersonic facilities, this approach is often considered sufficient characterization for practical purposes. The correlation presented here is based on RMS pressure fluctuation, because it is readily available for many facilities. In future work, similar approaches will be explored that utilize the energy of the disturbances and may be more inclusive of the multimodal character of the disturbance environment in compressible flows.

Tunnel Noise Correlations

It is standard practice to describe a tunnel's disturbance environment is by calculating an equivalent turbulence intensity based on the tunnel's free-stream noise level. Pressure RMS is readily available for most facilities and is easily measured via pitot probes or surface microphones, and so this method entails both low investment and reduced complexity. The two correlations seen most often in the literature are those of Wang [57] and of He and Morgan [58], which are respectively expressed as

$$Tu_{\infty} = \frac{NL}{\gamma_g M_{\infty}}, \quad (19)$$

and

$$Tu_{\infty} = \frac{NL}{M_{\infty} + 2}, \quad (20)$$

where NL is the RMS tunnel noise level, i.e., $P'/\bar{P}$. Because acoustic disturbances are believed to dominate the overall free-stream disturbance environment at Mach numbers above 2.5 [59], the tunnel noise level alone may be sufficient to obtain a reasonable estimate for N_{tr} . Both relationships are presented here to acknowledge the existing literature, but in practice they are interchangeable for stability applications. For air flows, these relationships agree in their predictions of Tu_{∞} within 16% for Mach numbers between 3 and 10 and agree exactly at Mach 5. In the context of developing transition correlations for high-speed flows, the precise choice among these correlations seems relatively unimportant so long as a consistent choice is adopted, at least within the above range of Mach numbers. For this work, Eq. 20 is used to report freestream turbulence intensity, but Tu_{∞} has greater relevance to transport equations-based transition models and, in the current work on stability-coupled models, we use the pressure-disturbance amplitude to characterize the freestream disturbances. Nonetheless, even in the present context, the equivalent turbulence intensity remains useful as a means of providing a consistent terminology between low-speed and high-speed transition analyses.

D. Disturbance Amplitude In Test Facilities

For the purposes of the correlation being developed here, it is desirable for a facility to be characterized by a single parameter, similar to the practice for transport-based transition models. However, the freestream disturbance amplitude is not constant with respect to Reynolds number. In Fig. 2, we plot the characteristic disturbance amplitudes calculated using Eq. 20 for the facilities described previously as a function of the unit Reynolds number. Fig. 2 shows that the freestream disturbance amplitude at diameter-based Reynolds numbers above approximately 3.6×10^5 decreases linearly with the square root of Reynolds number, so the local value can be estimated using a linear fit. Because the slope of this curve for each facility is similar, disturbance magnitude can be estimated using only a facility-specific reference parameter. This value is chosen as the y-intercept for the linear fit based on the square root of Reynolds number and can be calculated for any facility so long as RMS pressure fluctuations have been measured at one condition. This is of particular utility for facilities characterized via DNS, where only a small number of freestream flow conditions are usually analyzed. Turbulence intensity can be estimated as

$$Tu_{\infty} \approx -5 \times 10^{-5} \sqrt{Re_D} + Tu_{\infty,ref}. \quad (21)$$

For facilities where there is no information from which this parameter can be estimated, the authors recommend a default value of 0.3 for $Tu_{\infty,ref}$ or estimating the value of this parameter based on a similar facility. The range of Reynolds numbers in which the linear approximation is valid varies on a facility-by-facility basis, but each of the individual test cases examined in this work falls within the range where this relationship is valid for its corresponding test facility. It is found that the final correlation is moderately sensitive to the value of this reference parameter for a given test case. For a typical test case, the difference between the predicted value of N_{tr} decreases by around 10% when $Tu_{\infty,ref}$ is increased from 0.3 to 0.4.

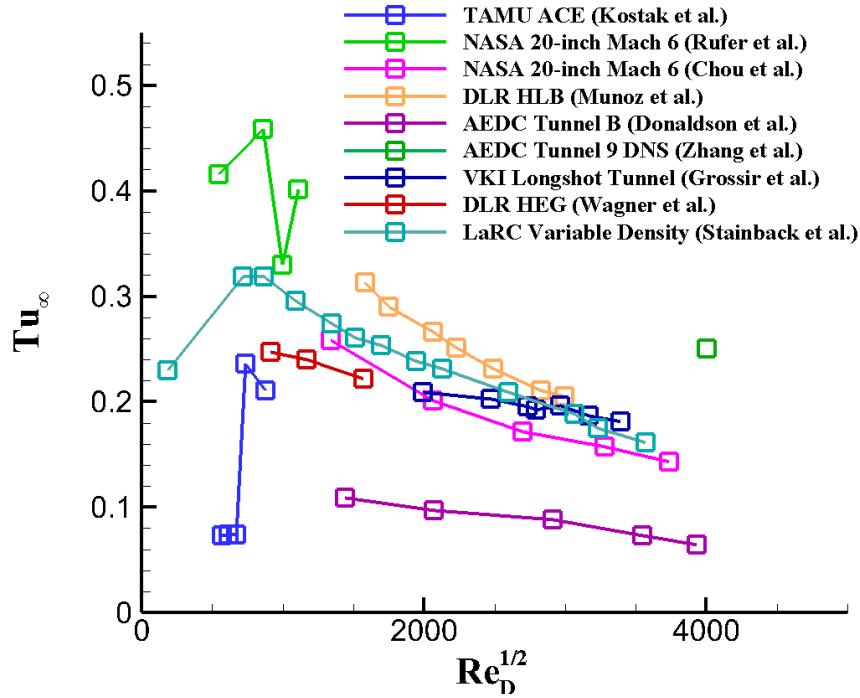


Fig. 2 Freestream disturbance magnitude metrics plotted against freestream unit Reynolds number for several ground test facilities.

E. Determination of A_{tr}

The next parameter required for the N-factor correlation is A_{tr} , i.e., the disturbance amplitude at transition onset. Marineau et al. [27] provided data concerning the saturation amplitude, A_{max} , for RMS pressure fluctuations on 7° cones. They also utilized measurements from the AEDC Tunnel 9 facility to demonstrate the link between the saturation amplitude and the amplitude of boundary-layer disturbances at the onset of transition,

$$A_{tr} = 1.25A_{max}. \quad (22)$$

Lee et al. [28] took a similar approach in their work and used a piecewise linear fit to correlate the saturation amplitude as a function of the edge Mach number, M_e . In this work, a generalized sigmoid function is used to provide a smooth approximation of the data. Because the data are measured in the cone boundary layer, they are collected in terms of the edge Mach number, M_e , but the correlation is evaluated here in terms of the freestream Mach number, M_∞ . This introduces small errors for slender geometries but prevents an undesirable dependency on geometric information which will be unavailable outside of specific conical geometries. At present, this simplification has been found not to introduce significant errors into the correlation, and it allows for the prediction of N_{tr} in a closed form manner. For some configurations such as cones with large ($>16^\circ$) half angles, it may be necessary that the boundary layer edge Mach number be known rather than assumed to be similar to the freestream Mach number. In the event that this is required, it could be obtained via a preliminary laminar CFD simulation or estimated using either Taylor-Macoll relationships (based on a representative body half-angle) or the local estimations employed in transport model-based transition models such as Refs. [11, 60]. The correlation for A_{max} is defined as

$$A_{max} \approx 2 + \frac{26.1}{1 + 50e^{-6(M_\infty - 4)} + 20e^{-1.8(M_\infty - 4)}} \quad (23)$$

and is shown graphically in Fig. 3. While some scatter in the experimental data for the saturation amplitude is observed, Eq. 23 provides a means of calculating A_{max} independent of the other key parameters so that C_R is the only remaining parameter which must be solved for. Moreover, this existing experimental data set includes data points similar in number and diversity of test conditions to the experimental N-factor database examined in this work. If a modified workflow was

developed in which M_e is known, Eq. 23 could simply be solved in terms of M_e rather than M_∞ , and this substitution would not necessitate changes to the rest of the correlation. The experimental data on which this correlation is based correspond to data for the 7° cones analyzed in Ref. [27], not the test cases described in section III although some cases from that work are further analyzed here as well.

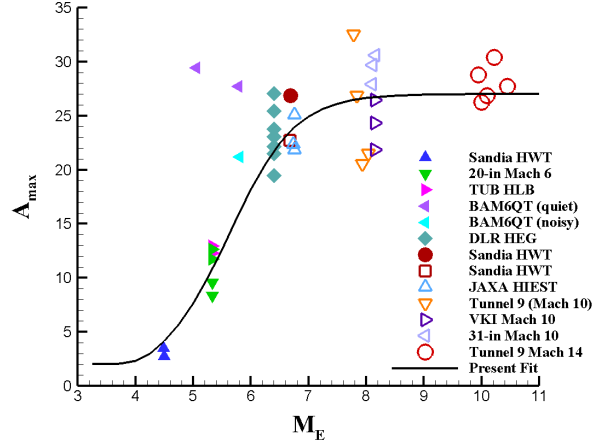


Fig. 3 Correlation for A_{max} based on the boundary layer edge Mach number. Experimental data are originally presented in Marineau et al. [27]. In the present correlation, this is evaluated in terms of M_∞ rather than M_e to allow for closed-form N_{tr} prediction.

From this figure, the differences in the predicted value of A_{max} estimated using M_∞ rather than M_e can be calculated for a given cone angle. It can be observed that the difference in predicted A_{max} between a prediction based on M_e as opposed to M_∞ for a typical slender ($\approx 10^\circ$) cone is similar the scatter observed among the experimental data at a given Mach number, and that both the scatter in the experimental data and the difference between M_e and M_∞ decrease with decreasing M_∞ .

F. Receptivity Coefficient

The only remaining parameter which must be solved for in the derivation of the N_{tr} correlation is the initial amplitude, A_0 , of the boundary-layer disturbance near the onset of instability amplification. Because all of the the other key parameters have been defined, any outstanding independent variables that are known to influence N_{tr} but are not used to calculate A_{max} or A_∞ must be accounted for in the calculation of C_R . In addition to the freestream or edge Mach number and the freestream unit Reynolds number, which have been partially considered in Eq. 23 and in the estimation of the freestream RMS pressure respectively, N_{tr} is generally understood to depend on the temperature of the flow, of the body, and on nose bluntness. For this correlation, the curve fit for C_R includes total temperature to wall temperature ratio in addition to Mach number and Reynolds number. Nose radius is also known to impact transition in high-speed flows, but not all configurations have a clearly identifiable nose radius. To alleviate this potential ambiguity, the correlation is bifurcated at this point. One version of the correlation that includes an explicit nose bluntness correction will be generated, and a second that does not explicitly depend on nose radius will be developed as well.

To calculate C_R , both A_0 and A_∞ must be known. A_∞ can be estimated based on Eq. 21, and A_0 can be calculated by substituting Eqs. 22 and 23 into Eq. 14. The transition N-factors for the experimental database are known, thus the equation can be solved for A_0 , yielding

$$A_0 = N_{tr} - \ln(A_{max} * 1.25). \quad (24)$$

With A_0 known for the experimental test cases, the value of, C_R for each case can be calculated from Eq. 16, which is the definition of C_R . For both versions of the correlation, a parameter that collapses the data with minimal scatter is identified by hand by generating several combinations of the independent variables (Mach number, Reynolds number, temperature and optionally nosetip radius) and estimating C_R based on those parameters, and choosing the one with the lowest associated error.

Turbulence Intensity without Geometry Information

Without invoking any geometric information regarding either the test article or facility, the experimental receptivity coefficient data were found to collapse according to

$$C_R = 1.64887 \exp(-0.123205 \sigma_1) \quad (25)$$

where

$$\sigma_1 = \left(\left(Re_{sl} M_\infty^{-1.1} \right)^{0.18} \left(\frac{T_0}{T_w M_\infty} \right)^{0.17} \right)^2. \quad (26)$$

and Re_{sl} is the slenderness Reynolds number, defined as

$$Re_{sl} = \frac{\rho U_\infty A_{cs}}{\mu L}. \quad (27)$$

In this equation, A_{cs} is the cross-sectional area of the body at its thickest point, and L is the axial body length. This length scale is chosen because it provides some information regarding the characteristic streamwise slope of the body (similar to what would be provided by cone half-angle), but is also applicably to any geometry.

Based on the correlation for C_R described in Eq. 25, the RMS error in N_{tr} between the correlation and the experimental data considered in the correlation is 7.5%, corresponding to a standard error in N_{tr} of 0.51. This value refers to the overall error associated with test cases considered in the development of the correlation, which are all those described in section III except the CUBRC and AFRL Mach 6 Ludwig tube cases. This version of the correlation is the most generally applicable. All of the inputs required for this version are always available during external aerodynamics CFD simulations, and the turbulence intensity information is either available for a number of facilities or can be easily estimated for the other facilities. This expression for the correlation is summarized in Fig. 4. As expected, this correlation incurs the most significant errors in cases with large nose radius-based Reynolds number such as the blunted cones from the LaRC 20-inch Mach 6 tunnel and the CUBRC LENS facility. The problem of accounting for the impact of nose bluntness without requiring a geometry-specific parameter as a modeling input is a significant challenge for laminar-turbulent transition prediction for hypersonic flows using stability-based and transport-equations-based methods.

In the course of this work, we also developed a correlation using the unit Reynolds number. However, this choice hinges on an arbitrary reference length of 1 meter for all cases. While this reference length has been employed in prior studies dealing with unit Reynolds number effects (Potter and Whitfield [61], Lee et al. [28]), we believe that to be an inappropriate choice and, instead, advocate for the use of strictly nondimensional parameters that are rooted in the essential characteristics relevant to boundary-layer transition. However, because the physical factors underlying the unit Reynolds number effects are intricate and not entirely understood as yet, we have defined preliminary nondimensional features by using geometric parameters that can be accessed relatively easily and that are also potentially relevant to the transition process. We encourage future research to investigate alternative nondimensional parameters, especially as additional data becomes available. However, the currently chosen parameters seem to offer a reasonable initial fit in the context of our current study and the dataset at hand. A similar correlation based around the unit (per meter) Reynolds number with no case-based length scale has error metrics similar to those for the present version. The RMS of the error in N_{tr} was 8.1% and the standard error in N_{tr} was 0.55.

Turbulence Intensity with Geometry Information

A simple correction based on adding the nose radius Reynolds number to σ_1 (Eq. 25) slightly improves the correlation for cases with bluntness. Thus, when Re_N is known, the data can be fitted using the following expression,

$$C_R = 1.16575 \exp(-0.07371 \sigma_2) \quad (28)$$

where the corrected value σ_2 is given by

$$\sigma_2 = \sigma_1 + Re_N^{0.3}. \quad (29)$$

Based on the correlation 28 for C_R , the RMS error in N_{tr} between the correlation and the underlying experimental data (which might be viewed as the model training error) is 6.9%, corresponding to a standard error in N_{tr} of 0.45. While some geometries can't be described by a single nose radius, a representative nose radius could be estimated for many configurations to improve the mean accuracy of the correlation for a more complex configuration as compared to the baseline version that ignores the nose bluntness. Bluntness modifies N_{tr} both by modifying the stability characteristics

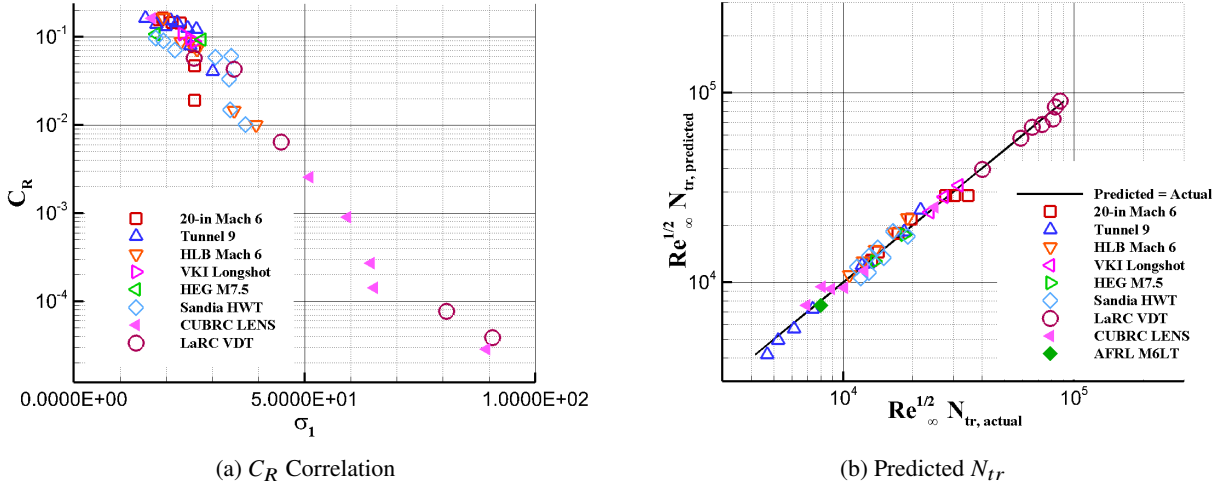


Fig. 4 Receptivity coefficient correlation (a) and comparison between the computed and actual value of N_{tr} (b) for the correlation based on turbulence intensity and not considering the nose bluntness correction. N-factors are multiplied by the square root of the Reynolds number for each test case.

of the boundary layer compared to the equivalent sharp test case and by modifying the receptivity of the boundary layer. LST can only capture the former, and so an explicit correction to the C_R correlation is used to capture the latter. It is observed that the gains in accuracy between the correlation that considers nose bluntness and the version presented in Fig. 4 are limited to the cases with the largest nose radius-based Reynolds numbers, and the predictions for the sharp cones that constitute a large portion of the database are not noticeably improved by this inclusion. While facility test section and cone half-angle were considered in the process of developing this correlation, they were not found to improve the accuracy of the correlation. The apparent contrast between this finding and the general wisdom outlined by Pate [25] may be explained in that the impact of facility size is already captured in the tunnel characterization data. In most cases, fluctuating quantities are measured at the centerline, and there is an observed trend wherein larger facilities tend to have smaller freestream noise levels for similar Reynolds numbers than otherwise comparable facilities. The mechanism, as described by Pate [25], that results in that correlation's dependence on facility size is therefore already accounted in the characterization data themselves, and an explicit correction is unnecessary. This correlation is summarized in Fig. 5.

G. Overall Correlation

A closed-form correlation to predict N_{tr} for second-mode-driven transition over simple axisymmetric configurations has been developed. This final closed-form correlation is developed by substituting the approximations for A_∞ , A_{max} , and C_R derived in sections IV.D through IV.F into the definition of N_{tr} (Eq. 18). As inputs the correlation requires the freestream Reynolds number, Mach number, total temperature, body surface temperature, reference tunnel RMS pressure disturbance amplitude, and optionally the geometry nose radius. The tunnel RMS pressure parameter can be optionally expressed in terms of equivalent turbulence intensity. This correlation can be integrated directly into the existing coupled CFD-stability prediction capability being matured by LaRC [24], allowing for application of coupled stability analysis to any problem for which the present correlation may be applied. Summarizing Eqs. 13 through 29, the correlation without considering nose radius is expressed as

$$N_{tr}(M_\infty, Tu_{\infty,ref}, T_w, T_0, Re_D, Re_{sl}) = \ln \left[1.25 \left(2 + \frac{26.1}{1 + 50e^{-6(M_\infty-4)} + 20e^{-1.8(M_\infty-4)}} \right) \right] - \ln[(1.64887 \exp(-0.123205\sigma_1))(-5 \times 10^{-5} \sqrt{Re_D} + Tu_{\infty,ref})] \quad (30)$$

where,

$$\sigma_1 = \left((Re_{sl} M_\infty^{-1.1})^{0.18} \left(\frac{T_0}{T_w M_\infty} \right)^{0.17} \right)^2, \quad (31)$$

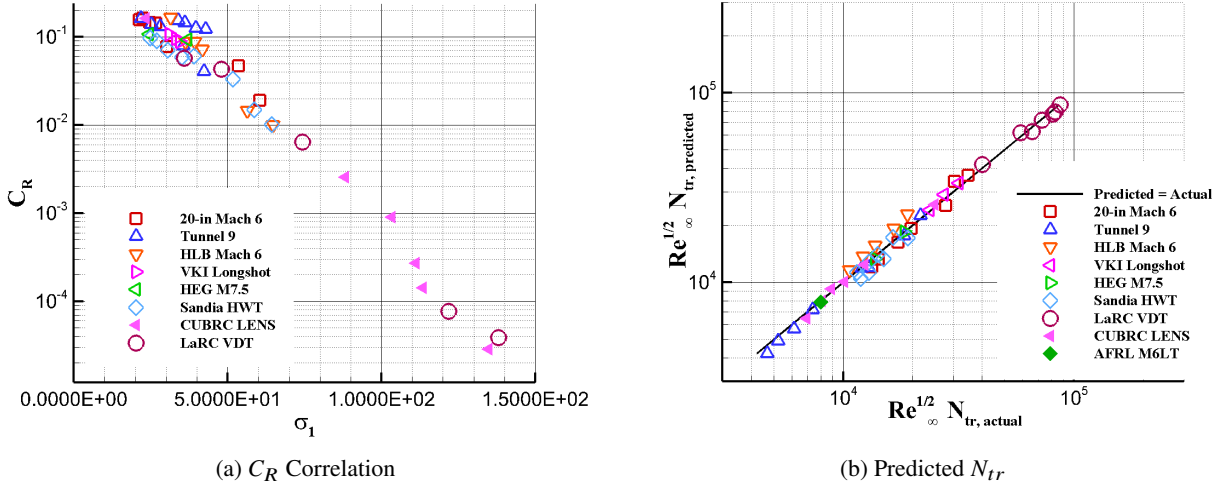


Fig. 5 Receptivity coefficient correlation (a) and comparison between the computed and actual value of N_{tr} (b) for the correlation based on turbulence intensity and considering the nose bluntness correction. N-factors are multiplied by the square root of the Reynolds number for each test case.

and the correlation is expressed as

$$N_{tr}(M_\infty, Tu_{\infty, ref}, T_w, T_0 Re_D, Re_{sl}, Re_N) = \ln \left[1.25 \left(2 + \frac{26.1}{1 + 50e^{-6(M_\infty - 4)} + 20e^{-1.8(M_\infty - 4)}} \right) \right] - \ln[(1.16575 \exp(-0.07371\sigma_2))(-5 \times 10^{-5} \sqrt{Re_D + Tu_{\infty, ref}})] \quad (32)$$

when including nose radius as an input parameter, where

$$\sigma_2 = \left((Re_{sl} M_\infty^{-1.1})^{0.18} \left(\frac{T_0}{T_w M_\infty} \right)^{0.17} \right)^2 + Re_N^{0.3}. \quad (33)$$

This correlation includes some simplifications for ease of use and breadth of applicability. The freestream and edge Mach numbers are assumed to be similar. This assumption allows for prediction of N_{tr} without specific knowledge of the local flow quantities, but may introduce errors for configurations with larger characteristic half-angles than those included here that may exhibit large differences between the freestream Mach number and the Mach number at the boundary layer edge. The freestream RMS pressure has been approximated based on two facility-specific parameters, and maximum disturbance amplitude information for a 7° cone has been applied to axisymmetric shapes of arbitrary half-angle. This correlation does not include geometries whose nose-radius-based Reynolds number place them beyond the transition reversal for blunt-nosed bodies, and an extension of the proposed correlations to account for the transition reversal would require modifications to how nose bluntness is considered (i.e. replacement of Eqs. 28 and 29). For the geometries considered in this work, these simplifications are acceptable, but there is some potential that a future test case may highlight applications for which these assumptions must be replaced with more information-intensive relationships.

V. Results for Test Data

To demonstrate the added value of the proposed correlation in the context of coupled CFD-stability transition prediction, it is applied to a test case from the LaRC 20-inch Mach 6 tunnel that has been previously analyzed using coupled OVERFLOW-LASTRAC analysis [24]. In previous analysis, the computation required that a precursor LST analysis be performed to determine N_{tr} . In the current computation, this precursor analysis is not necessary. Instead the correlation developed here is used to specify N_{tr} . The axial heat transfer coefficient distribution predicted using coupled stability analysis and the present correlation is presented in Fig. 6 along with the N-factor profiles for this case.

To evaluate the generalizability of the correlation process, we use a selected set of data points that were not used to develop these correlations. If the error associated with the predictions for this independent data set is similar to the

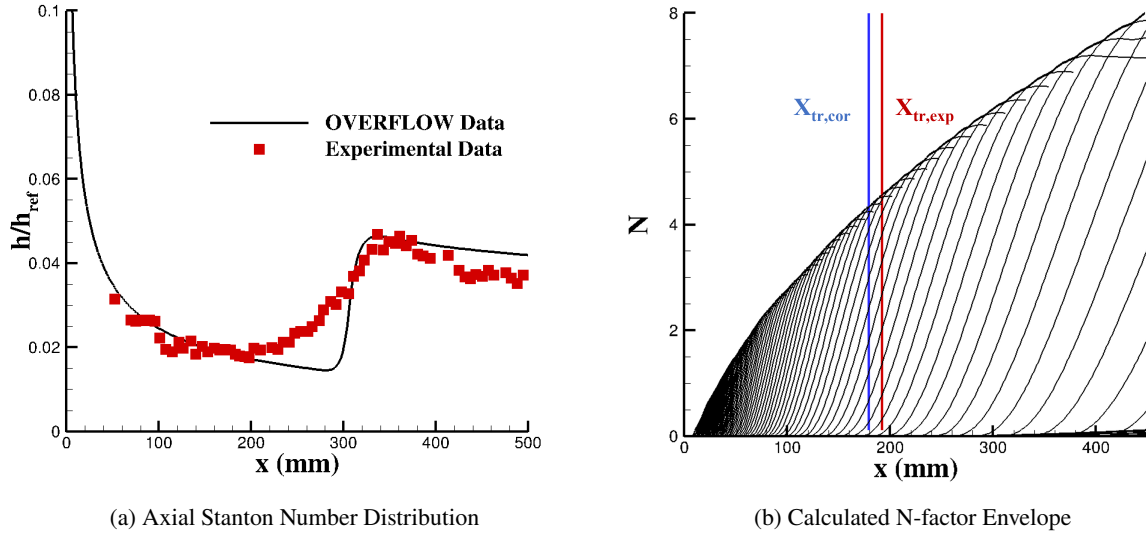


Fig. 6 Results showing the axial Stanton number distribution (a) and the N-factor envelopes (b) for a 5° half-angle cone studied in the LaRC 20-inch Mach 6 tunnel. Results are shown for frequencies between 100-900 kHz in increments of 10 kHz. The experimental values of heat transfer reported in Ref. [34] are shown with red symbols for comparison.

scatter internal to the model, that would help provide confidence that overfitting has not occurred, and the correlation is applicable to the wider set of data than the training database is representative of. On the other hand, if the independent verification produces significantly greater error than the scatter within the training database, then the correlation for the transition N-factors cannot be reliably extrapolated beyond the facilities included in the development of the correlation. The independent test cases examined here include the straight cone experiments considered by MacLean et al. [47, 48] corresponding to a 7° straight cone with various nose radii in the CUBRC LENS I facility, with the freestream Mach numbers varying from 6.58 to 10. These data cover a reasonable range of Mach numbers as well as a wide range of unit Reynolds numbers and nose bluntness Reynolds numbers.

When considering only the independent data that was not used to develop the correlation, the characteristic errors associated with the present correlation are as follows: Using the correlation based on the turbulence intensity and without the bluntness correction, the RMS error between the correlation and the independent test data considered in the correlation is 7.9%. Using the correlation based on turbulence intensity and with the bluntness correction the RMS error between the correlation and the experimental data considered in the correlation is 6.2%. For both of the correlations, the errors associated with the data used to develop the correlation are extremely similar to those between the predicted and experimental values used to tune the correlation, suggesting that other comparable experiments could be predicted with similar levels of accuracy. Additionally, the CUBRC tunnel has characteristic freestream enthalpies higher than the facilities used to tune this correlation, and so this result represents a successful extrapolation of the correlation outside of the training database for one of its independent variables.

The heat transfer profile for the transitional flow using automated and integrated stability analysis is presented in Fig. 7. It is observed that the experimentally determined transition location agrees well with the transition location calculated by the automated analysis using the present correlation for N_{tr} . Good agreement is observed between the CFD predictions and the experimental data, and only one input parameter was required for this calculation beyond the information used to calculate a purely laminar CFD solution. This parameter, namely, the freestream RMS pressure fluctuation in the wind tunnel (expressed here as the equivalent parameter Tu_∞), was successfully estimated using the knowledge of similar facilities. The successful prediction of transition onset demonstrates that the current correlation can generate accurate results using a workflow and level of configuration knowledge similar to that of existing RANS-based transition prediction tools, such as transport-based transition models.

Comparisons between the experimental and correlated transition data for the ogive-cylinder cases studied in the AFRL Mach 6 Ludwig tube are presented in Fig. 8. Close agreement is observed between the predicted value of

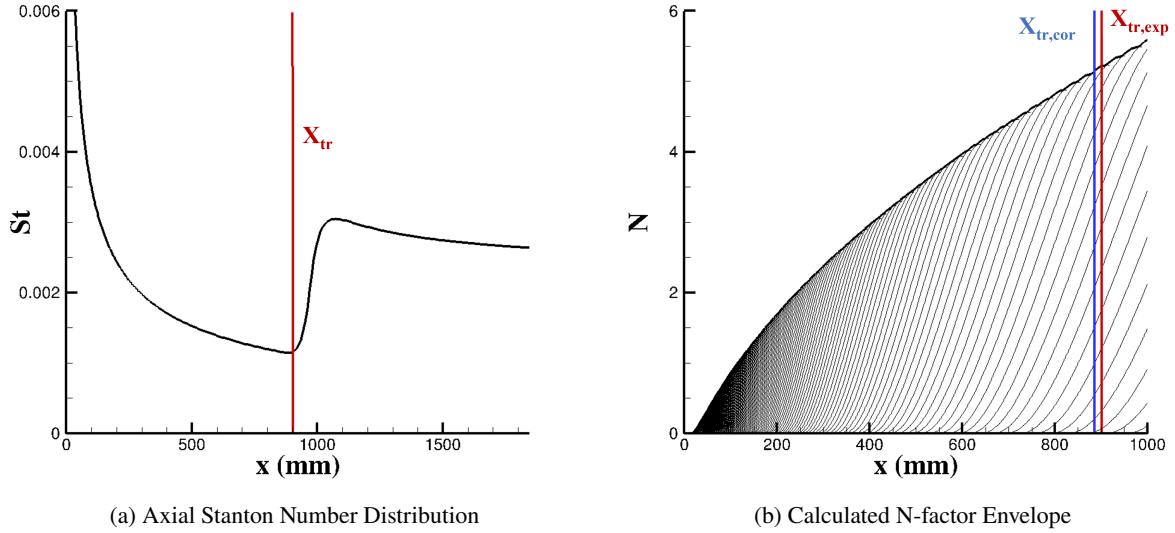


Fig. 7 Results showing the axial Stanton number distribution (a) and the N-factor envelopes (b) for a 7° half-angle cone studied in the CUBRC LENS 1 facility at Mach 9.92. Results are shown for frequencies between 100-900 kHz in increments of 10 kHz. The x-position corresponding to the transition N-factor based on the experiment is shown in red, and the value calculated with the present correlation with nose bluntness is shown in blue.

$N_{tr} = 3.02$ and the experimental value of $N_{tr} = 3.1$. Examination of more ogive-type cases would be necessary to establish confidence in the correlation's ability to predict transition for these geometries in general, but this preliminary result appears promising. Moreover, the present correlation is sufficiently modular that multiple versions of varying scope could be developed. For instance, the present version produces good predictions for straight-cone cases with and without nose bluntness prior to the onset of transition reversal at sufficiently high values of the nose bluntness parameter. A second set of training data that includes roughly equal numbers of straight-cone, flared-cone, and ogive-cylinder cases could be used to develop the analogs of Eqs. 25 and 26 that are better suited to predicting N_{tr} for a wider range of configurations.

VI. Conclusions and Future Work

A correlation for predicting the critical N-factor necessary for transition to occur in high-speed flows over axisymmetric configurations has been developed in this work and defined by Eqs. 30 through 33. It was tuned using experimental data that considers Mach numbers between 5 and 14, Reynolds numbers between $1.5 \times 10^6 \text{ m}^{-1}$ and $4.5 \times 10^7 \text{ m}^{-1}$, and utilized a database of transition experiments in conventional ground test facilities involving axisymmetric configurations with half-angles ranging from 5° to 16° . Two versions of the correlation are developed based on whether or not a characteristic nose bluntness is known and/or relevant. The amount and detail of input information required for this correlation is significantly lower than those of Pate [25], Lee et al. [28], or Marineau et al. [26], and the overall accuracy of the present correlation is found comparable to or better than those existing methods. The RMS error between the 38 experimental cases used to develop the correlation and the predicted transition N-factor values is 7.5% when the nose radius is not considered, and 6.9% when it is. The present correlation also considers a more inclusive set of conditions than either of the existing N-factor correlations and so is applicable across a wider variety of flow conditions and geometric parameters, and initial results for an ogive-cylinder geometry suggest the correlation can be applied to that class of geometry with similar errors. Moreover, the correlation is found to produce accurate predictions of transition N-factor for an independent data set based on engineering estimations of the characteristic freestream noise parameter. This represents a capability beyond the existing state-of-the-art, as the methods of Marineau and Lee require facility-specific tuning or precursor LST, preventing a blind evaluation such as the one presented for this correlation. This finding suggests the correlation may be robustly applied to axisymmetric flow

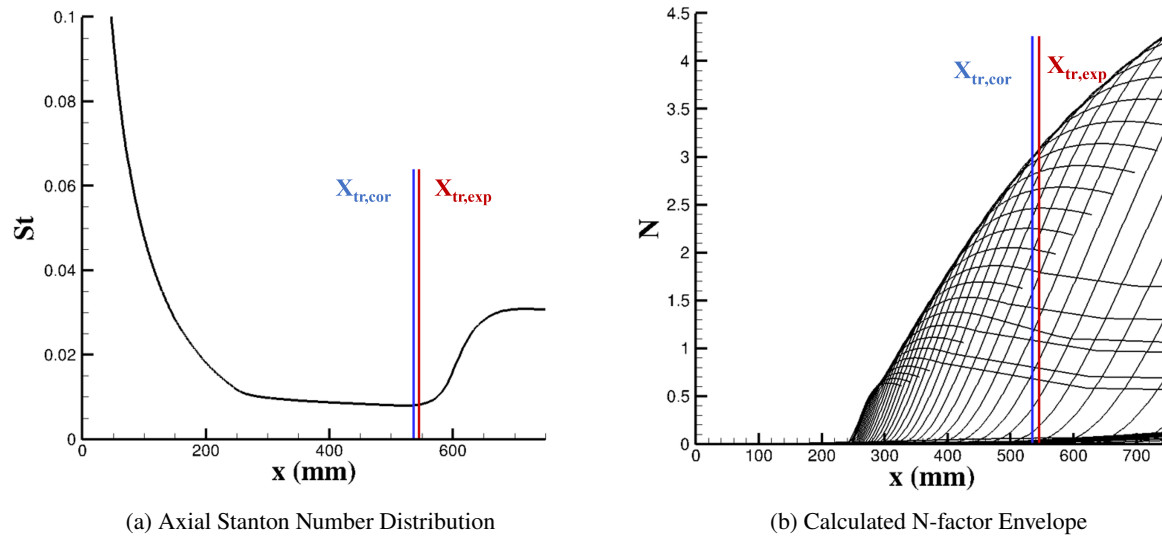


Fig. 8 Results showing the axial Stanton number distribution (a) and the N-factor envelopes (b) for the ogive-cylinder case studied in the AFRL Mach 6 Ludwig tube. Results are shown for frequencies between 100 kHz and 300 kHz in increments of 10 kHz. The x-position corresponding to the transition N-factor based on the experiment is shown in red, and the value calculated with the present correlation with nose bluntness is shown in blue.

scenarios in conventional tunnels within the range specified.

Future work will focus on expanding the applicability of the correlation by adding transition data from other types of facilities, specifically quiet tunnels, ballistic ranges, and flight tests to the training database. These efforts, especially the prediction of in-flight transition, are expected to require additional data to become available, but the HIFiRE-1 flight test database analyzed by Li et al. [62] is expected to provide a starting point for this effort. Additionally, the efficacy of the correlation will be evaluated for additional cases with nonzero pressure gradients and additional modifications will be made as needed to allow for the robust prediction of transition for more complex configurations. The present correlation is valid for configurations with a nose radius Reynolds number less than that which induces the bluntness reversal, wherein the transition N-factor stops increasing with increasing Re_N and begins decreasing with increasing Re_N at values of Re_N greater than 1×10^4 . Additional adjustments to the geometry correction based on experimental data representative of this reversal will be included in the future to expand the applicability of the present correlation. Additionally, efforts to advance coupled stability in other ways, such as the development of improved intermittency models for coupled stability analysis of high-speed flows, and the integration of PSE-based prediction into the coupled framework, are also planned.

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