

Transition Analysis for the Pazy Wing

Balaji Shankar Venkatachari*

Analytical Mechanical Associates Inc., Hampton, VA 23666

Nathaniel Hildebrand,[†] Meelan M. Choudhari,[‡] and Pawel Chwalowski[§]

NASA Langley Research Center, Hampton, VA 23681

The Pazy wing, designed as a benchmark for highly flexible wings that experience large deformations, presents intriguing nonlinear phenomena due to its unique geometry with spanwise varying static deformation and low Reynolds number flow conditions. These phenomena include limit cycle oscillation related to laminar separation bubbles, boundary-layer transition, and dynamic stall. The primary focus of this study is to investigate transition behavior on the sagged Pazy wing under static loading by using the shear-stress-transport (SST)-based Langtry-Menter $\gamma - Re_{\theta t}$ transition model in the NASA OVERFLOW and FUN3D solvers. Based on the steady-state results obtained via different variants of the Langtry-Menter model, we investigate the likely transition behavior over a range of incidence angles. The predicted flowfield on the sagged Pazy wing is strongly three dimensional and the application of turbulence index as a viable indicator of the transition front in such flows is also explored. The study demonstrates the limitations of the Langtry-Menter model when applied to low Reynolds number flows. We introduce a spanwise periodic configuration to simulate a narrow-span section of the Pazy wing, specifically centered on one of the 14 nearly equidistant ribs spanning the wing. The computational results demonstrate that the spanwise periodic configuration provides a straightforward environment for studying transition characteristics as well as demonstrating systematic grid convergence and successful code-to-code comparisons. The research offers useful insights into the transition characteristics of flexible wings under various loading conditions, contributing to the field of aeroelastic simulations.

Nomenclature

c	= chord length [m]
C_D	= drag coefficient
C_f	= skin-friction coefficient
C_L	= lift coefficient
C_m	= pitching moment coefficient
C_p	= surface pressure coefficient
M	= Mach number
Re	= unit Reynolds number [m^{-1}]
Re_c	= Chord-based Reynolds number
$Re_{\theta t}$	= transition onset momentum thickness Reynolds number based on local boundary-layer edge conditions
T	= temperature [K]
Tu	= freestream turbulence intensity
(u, v, w)	= streamwise, wall-normal, and spanwise velocity components [$m\ s^{-1}$]
(x, y, z)	= Cartesian coordinates [m]
y^+	= near wall grid spacing in wall units
α	= angle of attack [deg]
γ	= intermittency
μ	= dynamic viscosity [kg/(ms)]

*Sr. Research Scientist, Email: balaji.s.venkatachari@nasa.gov, Senior member, AIAA

[†]Aerospace Technologist, Computational AeroSciences Branch, Email: nathaniel.j.hildebrand@nasa.gov, AIAA Member

[‡]Aerospace Technologist, Computational AeroSciences Branch, Email: m.m.choudhari@nasa.gov, Fellow, AIAA

[§]Research Aerospace Engineer, Aeroelasticity Branch, Email: pawel.chwalowski@nasa.gov, AIAA Senior Member

μ_t = turbulent eddy viscosity [kg/(ms)]

Subscripts

amb = ambient value
 ∞ = farfield reference state
 L = local value

Abbreviations

CFD = computational fluid dynamics
LM = Langtry-Menter
RANS = Reynolds-averaged Navier-Stokes
SST = shear-stress transport

I. Introduction

The Pazy wing model was designed to be a benchmark for a very flexible wing that experiences large deformations and provides experimental data for validation of numerical tools used in the investigation of aeroelastic effects under nonlinear regimes [1–4]. The data have been used by the Large Deflection Working Group from the third Aeroelastic Prediction Workshop.** The simple yet unique geometry and the targeted flow conditions involving low Reynolds number produces interesting nonlinear phenomena such as limit cycle oscillation related to laminar separation bubbles (LSB), boundary-layer transition, and dynamic stall. Even under static conditions alone, aerodynamic behavior of the Pazy wing can provide valuable information for the CFD modeling of 3D LSBs and the process of laminar-to-turbulent transition across a broad range of incidence angles. With a continuing shift toward more light-weight wing structures, particularly for unmanned aerial vehicles (UAV), it is crucial to fully understand the viscous flow characteristics over flexible wings and to predict them accurately. This is true especially in the case of large deformations.

UAVs, helicopters, turbine blades, helicopter propellers, and airplane wings all exhibit laminar separation at low Reynolds numbers, followed by turbulent reattachment of the boundary layer. As a result, the CFD tools utilized for those applications must possess the ability to accurately predict the transition region inside a separation bubble as well as the proper size of the bubble. While reduced-order methods such as XFOIL [4] can be useful for designing airfoils that operate at low Reynolds number, such tools become less reliable for complex 3D configurations involving unsteady behavior. Direct numerical simulations and wall-resolved large-eddy simulations have the potential to provide the most complete information regarding separation bubbles and the process of transition within it, but due to their excessive computational cost, these methods will remain impractical for the regular CFD user at least for the next few years. Methods based on stability theory, such as the parabolized stability equations (PSE) [6], integrated with Reynolds-averaged Navier-Stokes (RANS) solvers or RANS-based transition models [7–11] may be the best alternatives for understanding and predicting the behavior of the separation bubbles and underlying transition mechanisms for complex geometries.

Modern transport-equation-based transition models incorporate information about the transition process using local information without requiring detailed boundary-layer profiles or integral boundary-layer parameters. They achieve this by solving additional transport equations and relying on appropriate correlations to determine the onset of transition. The similarity between the equation sets for the transition model and the turbulence model, respectively, along with the local dependence of each term in those models makes it easy to implement them in unstructured-grid-CFD codes. However, the majority of RANS-based models only work well in a limited range of flow regimes because the correlations used in these models are derived from a small number of experiments, and the method as a whole does not mimic the dominant physics of the boundary-layer transition process. As a consequence, even though models such as the SST-based Langtry-Menter (LM) model [8] have the potential to predict transition due to various scenarios such as natural transition in a low disturbance environment, bypass transition due to high intensity freestream turbulence, and separation-induced transition, one must be aware of its range of applicability in order to obtain meaningful predictions in a given practical application. Prior studies of the LM model (see Refs. [12–

** <https://nescacademy.nasa.gov/workshops/AePW3/public/wg/largedeflection>

14)) have indicated that it can yield inaccurate predictions of transition onset locations, as well as the skin-friction levels achieved in the turbulent regions immediately downstream of the transition location at both low (see Refs. [12,13]) ($Re_c < \mathcal{O}(10^5)$) and sufficiently high Reynolds numbers [14] ($Re_c > 10^7$). At low Reynolds numbers, transition is often caused by laminar separation bubbles and the onset happens much earlier than due to natural transition. Although the LM model includes a simplified treatment for separation bubbles, it may not be sufficient to provide accurate predictions of the transition onset location and bubble length at lower Reynolds number conditions. Thus, the Pazy wing configuration can serve as a suitable testing ground for understanding the behavior of the RANS-based transition models for a spanwise quasiperiodic configuration at the relatively low Reynolds numbers of interest ($Re_c = 1.2 \times 10^5 - 3.4 \times 10^5$) for the third Aeroelastic Prediction Workshop.

In this paper, we describe the findings of an assessment of the SST-based $\gamma - Re_{\theta t}$ LM model using the NASA OVERFLOW [15] and FUN3D [16] solvers in predicting the transition behavior of the Pazy wing under different flow conditions. We also present a comparison of the computational predictions with data available from the experiment [3] as well as prior simulations based on the γ model [4]. FUN3D is expected to be the flow solver of choice for future work on the Pazy wing by NASA, as it has the capability to perform aeroelastic computations. However, we mainly use OVERFLOW in the present study and presented comparisons with FUN3D computations for selected conditions. One of the goals is to assess the performance of the RANS-based transition models in the presence of short-scale spanwise oscillations in the wing geometry. In addition, we explore a potential simplification of the Pazy wing configuration to study its aerodynamic behavior in a controlled as well as cost-efficient manner, so as to facilitate easier code-to-code comparisons. The paper is organized as follows. Section II provides a description of the Pazy wing model and the flow conditions of the experimental campaign. Details about the two flow solvers are presented in Section III. Section IV outlines the results obtained on the complete geometry, while Section V is focused on the results obtained for the simplified representation of the Pazy wing model. Finally, concluding observations are presented in Section VI.

II. Pazy Wing Model and Flow Conditions

The Pazy wing model has a rectangular planform, with no taper or sweep [1]. The cross section of the wing is nominally based on the symmetric NACA 0018 airfoil. The wind tunnel model has a rib structure, with a Polyester foil wrapping that structure. Because of the unique way in which it has been manufactured, the cross section of the wing deviates significantly from the original NACA 0018 airfoil shape, and the foil sags in between the ribs, resulting in an alternating pattern of reduced thickness sections and kinks along the span of the wing. The as-built geometry also includes a wingtip rod for attaching weights to modify the dynamical structure of the wing. An overall schematic of the wind-tunnel geometry is depicted in Fig. 1. In Fig. 1, the sectional cuts at the quarter chord location and two different spanwise locations are shown. The spanwise sectional curve at $y = 0.328$ m corresponds to the sagged portion, while the curve at $y = 0.346$ m represents the rib. The wing has an overall span of 0.56 m, with a chord length of 0.1 m. The clamping section has a diameter of 0.12 m and a height of 0.021 m. Given that we are only interested in analyzing the aerodynamic characteristics under static conditions, the wingtip rod is excluded from the geometry used for computations. More details on the model geometry and the wind-tunnel measurements can be found in Refs. [1–3].

The experimental campaign was carried out at the Open Jet Facility at Delft University of Technology, which has a freestream turbulence intensity of around 0.5%. Infrared thermography measurements of the wing surface were made during the experiment to infer the boundary layer state [3]. For the computations reported in this paper, the freestream turbulence intensity was assumed to be 0.3% instead of 0.5%, to be consistent with the earlier CFD study by Ritter et al. [4]. We assume sea level conditions for the test section environment, with $\rho_\infty = 1.225$ kg/m³, $P_\infty = 101325$ Pa, and $T_\infty = 288.15$ K. The computations are carried out for two separate freestream conditions: (i) the experimental conditions of inflow velocity $U_\infty = 18$ m/s ($M_\infty = 0.053$), resulting in a chord Reynolds number of 120,000, and (ii) for a higher inflow velocity of 50 m/s ($M_\infty = 0.1469$) that corresponds to a chord Reynolds number of 342,165. The latter freestream condition corresponds to the highest inflow velocity possible in the wind tunnel and it has been utilized for the majority of CFD results presented in this paper, with the lower Reynolds number condition being used merely to illustrate the changes in the flowfield. There were two reasons for this decision: (i) the accuracy of transition models such as the Langtry-Menter model can degrade at low Reynolds numbers (see

Refs. [12, 13]); and, more importantly, (ii) a comprehensive set of CFD predictions at this higher Reynolds number are available for comparison (see the work by Ritter et al. [4]). Angles of attack, α , ranging from 0° to 10° in increments of 2° have been investigated to date, while data for α up to 16° are available from the experiments.

III. Flow Solvers

Version 2.3e of the NASA OVERFLOW code [15] is used in this study for the RANS-based transition computations as well as for obtaining the baseflow needed for stability analysis. OVERFLOW is an implicit, structured, overset grid Navier-Stokes solver that is capable of computing both time-accurate and steady-state solutions via a variety of options for the spatial as well as temporal discretizations. RANS-based transition models available in OVERFLOW 2.3e include: (i) the two-equation Langtry-Menter transition model (LM2009) [8] based on the 2003 version of Menter’s SST RANS model [17], along with two variants that account for crossflow induced transition, namely the roughness-based model (LM2015) by Langtry et al. [18] and the helicity-based extension (LM-CFHE) by Grabe et al. [19]; (ii) Coder’s 2019b version of the amplification factor transport (AFT2019b) equation-based model [10] that uses the SA model [20]; (iii) the SA-based Medida-Baeder transition model [9]; and (iv) Menter’s SST-based γ model [11]. The Langtry-Menter transition model and its variants, LM2015 and LM-CFHE, will be extensively used in this paper. In contrast, the results reported by Ritter et al. [4] were computed using a recently developed, one equation γ model, that is similar to the one proposed by Menter [11], and did not account for crossflow effects or have any special treatment for laminar separation such as the LM model. The steady-state computations reported in this paper were carried out by using third-order discretization [22] for the convective terms in all transport equations, a second-order accurate representation for gradient evaluation, the unfactored successive symmetric overrelaxation (SSOR) implicit solution algorithm [23, 24], and low-Mach preconditioning. It may be noted that the LM-CFHE and γ models have been implemented in a separate branch of the solver that is not presently available in the public release version of OVERFLOW 2.3e.

FUN3D is a finite-volume, unstructured, node-centered, upwind-biased RANS flow solver with implicit time advancement in both local and global time-stepping modes [16, 21]. Currently, the SST-based, two-equation LM2009 model, along with the LM2015 and LM-CFHE modifications for crossflow induced transition, is the only RANS-based transition model available in the release version of FUN3D. The inviscid fluxes are calculated with a Roe-Riemann solver [22]. For second-order accuracy, the interface values are obtained by extrapolation of the centroidal values of the control volumes (based on gradients computed at the mesh vertices with an unweighted least-squares technique). The viscous fluxes are discretized in a manner that velocity gradients on dual faces are calculated with the Green-Gauss theorem. The convective terms of the turbulence equations are solved using a first-order spatial discretization scheme. Pseudotime integration is performed with a backward Euler scheme. The linear system of equations at each time step is numerically solved by using either a point implicit procedure or an implicit line relaxation scheme as described in Nielsen et al. [25]. FUN3D will eventually be the CFD solver of choice for future NASA studies on the Pazy wing geometry, as it has the capability to do aeroelastic computations. For performing the aeroelastic computations, FUN3D can either be coupled with an external structural solver or use the internally available linear structural dynamics modal solver.

In both OVERFLOW and FUN3D, no other limiters were employed besides those defined by the original authors of the turbulence or transition models.

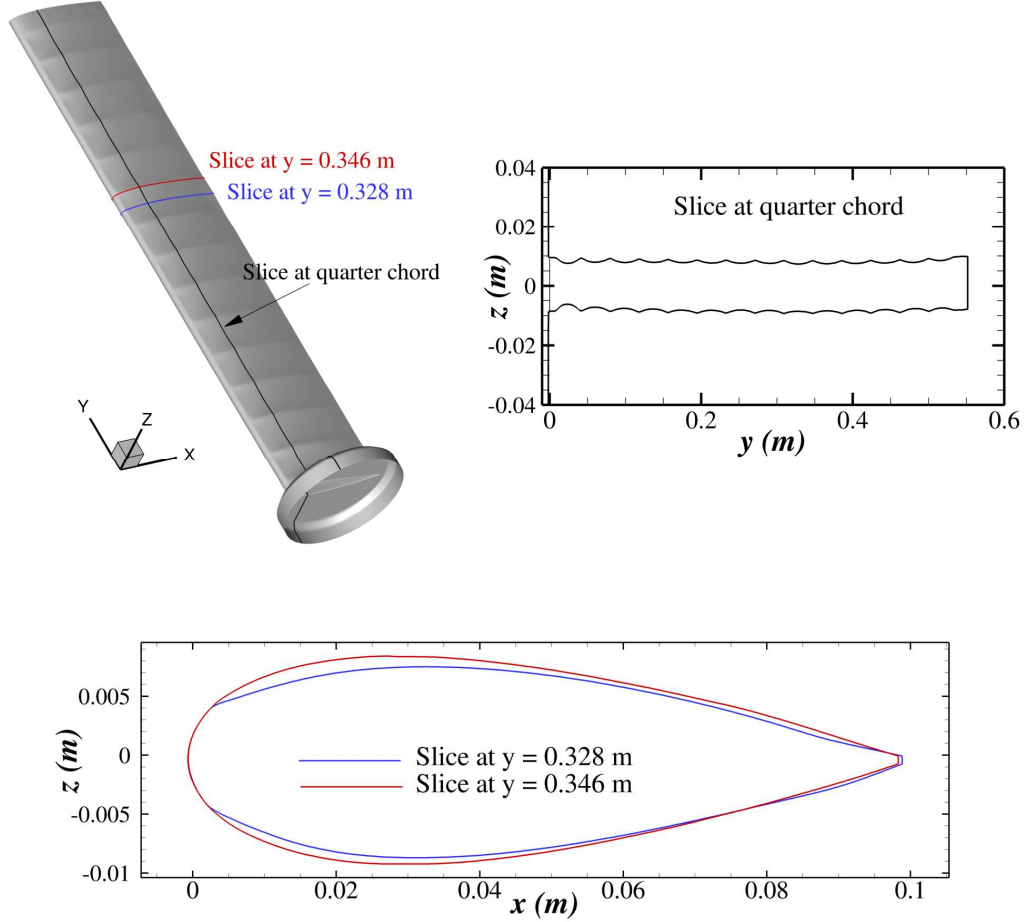


Fig. 1. Outer surface of the Pazy wing, shown along with sectional cuts along two different spanwise locations and the quarter chord location.

IV. Full-Span Results

Results for the complete, sagged Pazy wing geometry are reported in this section. The OVERFLOW computations have been carried out using a structured overset grid. The wingtip rod from the experimental test article has been excluded from the computational geometry, as it is not expected to have a significant influence on the transition characteristics, except possibly in the immediate vicinity of the wing tip. The configuration used for the computation includes the clamping section and the wall (treated as a slip-wall) upon which the clamping section is mounted. The mesh used for the OVERFLOW computations includes three overset near-body blocks (one for the clamping section, one for the wingtip, and one for the wing). The generation of off-body grids and hole cutting were carried out by using the OVERFLOW domain connectivity function (DCF) approach. The near-body grids over the wing had 265 points in the wall-normal direction with a near-wall spacing of 1.3×10^{-6} m ($y^+ \sim 0.25$ based on the quarter chord), an average growth rate of 1.04, and six of the initial grid points near the wall being uniformly spaced. The mesh contained 457 points around the wing in the chordwise direction, with grid spacings varying from 0.036 mm (closer to the leading and trailing edges) to 1 mm. In the spanwise direction, it had 361 points with the spacing ranging from 1.1 mm to 1.8 mm. The near-body grid had an overall grid count of approximately 65 million points and extended outward from the body by a distance of approximately 1.5 chords, with the bulk of the grid points focused within a distance of two boundary-layer thicknesses (estimated at the midchord location). The inflow, outflow, and far-field boundaries of the outer-body grids were placed at approximately 300 chords away from the center of the wing. This mesh had an overall grid count of 70 million points and corresponded to the baseline mesh used in our computations.

The FUN3D computations were carried out using a hybrid unstructured grid, that was generated and also used by Ritter et al. [4]. Unlike the structured mesh used for OVERFLOW computations, this grid included the wingtip rod, however its inclusion is not expected to impact the comparisons as discussed previously. The surface of the wing (excluding the wingtip rod and the clamping section) contained structured (quadrilateral) meshes that had a chordwise spacing varying from 0.3 to 0.7 mm and an average spanwise spacing of 1.0 mm. The quadrilateral meshes were extruded into the boundary layer to a distance of about 10.3 mm, resulting in 44 layers that had a near wall spacing of 2.4×10^{-6} m ($y^+ \sim 0.46$ near midchord station) along with a significantly higher growth rate of 1.16 in comparison with the structured mesh described earlier. The hexahedral meshes eventually morphed into tetrahedral elements elsewhere. The far-field boundary was placed at approximately 80 chords away from the center of the wing. The hybrid mesh had an overall grid count of 7.2 million nodes (12 million elements). The smaller grid count for the unstructured mesh is more in line with the grid sizes targeted for eventual aeroelasticity applications.

The following boundary conditions were used in the computations reported here: adiabatic wall boundary conditions on the wing model; inviscid wall boundary conditions on the wind-tunnel bottom wall upon which the model is mounted; and freestream boundary conditions were used for the remaining boundaries.

Both OVERFLOW and FUN3D results reported here are based on steady-state computations. For the Langtry-Menter transition models, the option to sustain freestream turbulence [26] was used, to avoid the turbulence decay upstream of the body. The inflow turbulence intensity was set to 0.3%, matching the computations made by Ritter et al. [4], and $\mu_t / \mu = 1.0$ based on best practices for using the turbulence sustaining terms. The LM2015 model accounts for crossflow effects based on the root-mean-square height of surface roughness. Due to the lack of characterization of the foil surface, the roughness height on the Pazy wing model was assumed to equal $3.3 \mu\text{m}$, i.e., similar to that of a painted surface in the benchmark crossflow experiment on the NLF(2)-0415 swept airfoil [27].

A. Influence of the Transition Model

As a first step to evaluate if crossflow effects played a role for this geometry, computations for $\alpha = 8^\circ$ were carried out with OVERFLOW using the baseline LM2009 model that accounts for natural transition without any crossflow effects, along with the two variants that include the crossflow effects, namely LM2015 and LM-CFHE. The skin-friction contours on the upper (suction) surface of the wing for $Re_c = 342,165$ obtained with the different variants of the LM model are shown in Fig. 2. In this plot, the lighter, orange colored region with lower values of skin friction can be identified as the laminar region, while the darker, red colored region corresponds to the fully turbulent flow; and the white colored region with negative skin friction (bounded by black lines corresponding to zero skin friction) corresponds to regions of separated flow. From Fig. 2, it is easily observed that all three plots show the same features and that the overall skin-friction contours are very similar across all model variants, indicating that crossflow effects are not significant in this case. Across the majority of the span, transition occurs very close to the leading edge because of the flow separation (white colored regions). However, a delayed onset of transition is observed in narrow spanwise regions in the immediate vicinity of the ribs, again due to a separate set of separation bubbles that is further downstream of the spanwise elongated bubbles within the sagged portions of the wing. There is a third set of separated regions in the flow near the trailing edge. Even though crossflow effects are found to be unimportant, they are retained in the results presented henceforth in order to allow an assessment of the overall model. Without any loss of generality, these results have been obtained by using the LM-CFHE model alone.

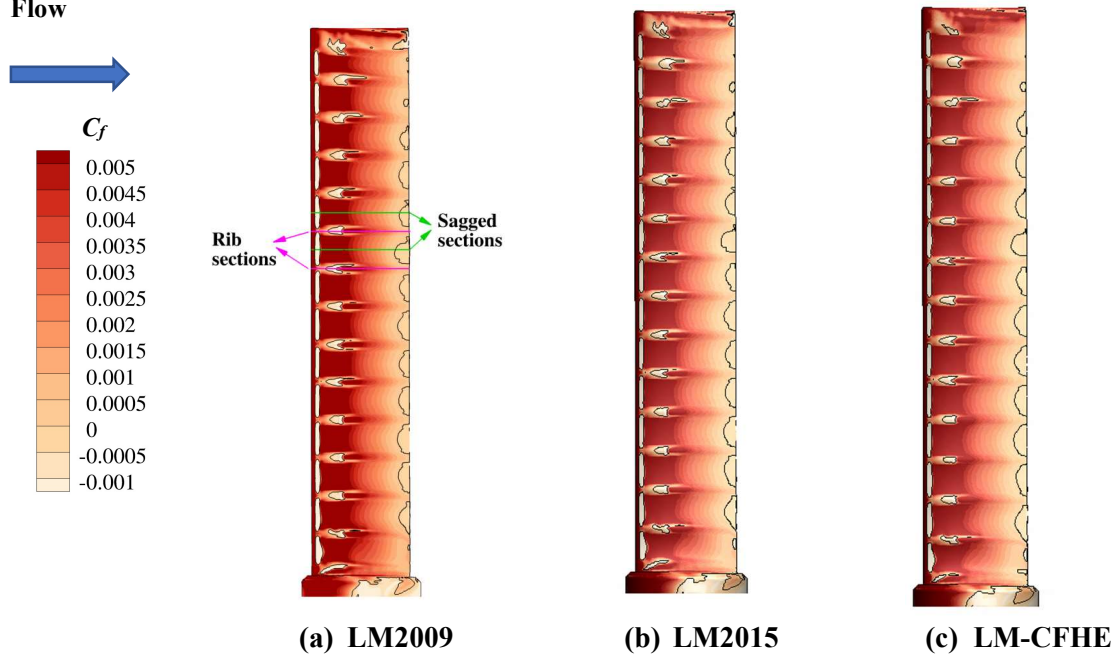


Fig. 2. Contours of the skin friction coefficient for the sagged Pazy wing obtained with variants of the LM transition model for $Re_c = 3.4 \times 10^5$ and $\alpha = 8^\circ$. The black line corresponds to the zero-crossing location of the skin friction value.

To provide a better understanding of the differences in flow behavior along the ribs and the sagged portion of the wing, the chordwise distributions of the skin friction and pressure coefficients at two different spanwise locations (see Fig. 1) are shown in Fig. 3 for $Re_c = 3.4 \times 10^5$ and $\alpha = 8^\circ$. The suction peak on the upper surface of the wing is stronger along the center of the sagged portion ($y = 0.328$ m), which is likely to be related to the kink in the geometry near the leading edge as seen from the blue curve at the bottom of Fig. 1. This strong suction peak is followed by a relatively rapid increase in pressure that causes the boundary layer to separate immediately and to undergo transition prior to reattachment near approximately 10 percent of the chord length, as seen from the rapid increase in the skin friction coefficient in Fig. 3(a). On the other hand, the suction peak at the rib location ($y = 0.346$ m; solid red color curve in Figs. 1 and 3) is not as strong and the pressure increase past the suction peak is slower in comparison with the sagged region. Thus, even though the skin friction distribution has a local minimum just past the 5% chord location, the onset of separation is delayed until 16% chord, which is followed by separation-induced transition at around 30% chord. Consequently, the separation bubble length along the rib (equivalent to approximately 14 percent of the chord length) is significantly longer than the shorter (and farther upstream) bubble length of 5 percent chord along the sagged region. The lower surface remains mostly laminar across the entire span of the wing as a result of the relatively low Reynolds number combined with a largely favorable pressure gradient at all spanwise locations. The present CFD results for the skin-friction levels, separated flow pattern, and the approximate location of the separation bubbles are similar to those reported by Ritter et al. [4] (see Fig. 13 from Ref. [4]), who had used the one equation γ transition model (without any crossflow correction) in place of the two-equation LM-CFHE model employed here.

B. Angle of Attack Variation Studies

In this section, we describe results based on the angle-of-attack variation, along with a comparison of the results obtained using two different flow solvers on different grids. All computational results are again presented as contour plots of the skin friction coefficient on the upper surface (suction side) of the wing. Thermography images from the experiment [3] are available for a qualitative comparison but are not included here. As the flow remained mostly laminar for the entire chord along the lower surface (pressure side) of the wing, the results presented here focus on the upper surface alone.

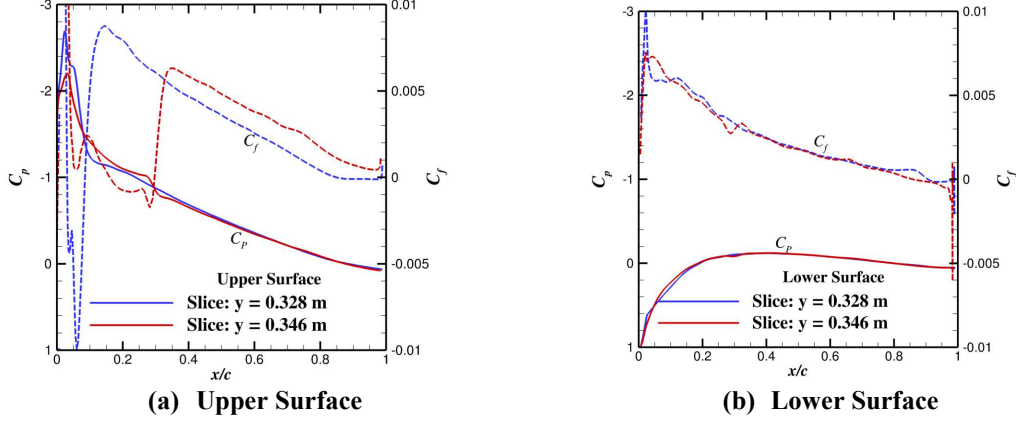


Fig. 3. Chordwise distribution of the pressure and skin-friction coefficients at two different spanwise locations (one at the rib ($y = 0.346$ m) and other in the sagged portion) obtained with the LM-CFHE transition model for $Re_c = 3.4 \times 10^5$ and $\alpha = 8^\circ$.

Figures 4–6 show the skin-friction contours, along with chordwise distributions of C_p and C_f at two different spanwise stations, for $\alpha = 0^\circ$, 6° , and 10° , respectively. All these results were obtained for $Re_c = 3.4 \times 10^5$, and by using the LM-CFHE transition model as implemented in the OVERFLOW and FUN3D solvers. The general observation from Figs. 4–6 is that, for all angle-of-attack conditions investigated here, there is a sequence of spanwise elongated separation bubbles near the leading edge within the sagged portion of the wing, and another set of bubbles that is narrow in span but elongated in the chordwise direction. At an angle of attack above six degrees, one can also notice the occurrence of a separated region near the trailing edge that moves upstream and makes the flow very unsteady, an aspect that has not been investigated in this work. The overall behavior is very similar to the results for $\alpha = 8^\circ$ from the previous section (Figs. 2 and 3).

At zero degrees angle of attack (Fig. 4), one observes separation bubbles near the leading edge in the sagged portions of the wing, and their reattachment is laminar. The flow eventually becomes turbulent when the separated flow that appears across the entire span of the wing near the 60% chord location (see Figs. 4(b) and 4(c)) reattaches. At higher angles of attack (see Figs. 5 and 6), the spanwise elongated separation bubbles that appear near the leading edge of the sagged portions of the wing reattach as a turbulent boundary layer. Also, at these higher angles of attack, the flow remains laminar along the ribs, further downstream of that in the sagged portions, until the flow separates and reattaches as a turbulent boundary layer. Furthermore, the separated flow that forms along the ribs appear as separate pockets instead of a single, spanwise-contiguous separated regions seen at zero degrees angle of attack.

With an increase in the angle of attack, the upstream separation bubbles in the sagged portions of the wing become more elongated in the spanwise direction while the separation bubbles concentrated around the ribs appear to shrink in size. In general, the onset of transition near the ribs is further downstream than that within the sagged portion and tends to move further upstream as the angle of attack is increased. This trend can be seen by comparing subplots (b) and (c) of Figs. 4–6.

Although two different flow solvers are used with two entirely different grid systems, the results shown in Figs. 4–6 indicate a good match between the results obtained with FUN3D and OVERFLOW for the flow conditions investigated. This agreement includes the levels of relevant skin-friction contours, the overall pattern of flow separation, and the locations of the separated flow regions. There are some minor differences near the trailing edge region closer to the wing tip between the two solutions, possibly due to the inclusion of the wingtip rod in the FUN3D computations. Similarly, at $\alpha = 10^\circ$, the OVERFLOW solution shows a larger separation near the wing root section (closer to the clamping section). However, these differences are considered to be reasonable for the purposes of the present analysis. Even though the grid used for FUN3D is coarser than that used for OVERFLOW in the wall-normal direction, it does not appear to affect the overall quality of the results as seen from the good match between C_f and C_p curves shown in Figs. 4–6, thereby indicating that the grids are sufficient for the model chosen and the models are working as intended. Furthermore, the skin friction contours shown here, as well as the shapes and locations of the separation bubbles, are qualitatively consistent with those from the computations by Ritter et al. [4] (see Fig. 13 from Ref. [4]).

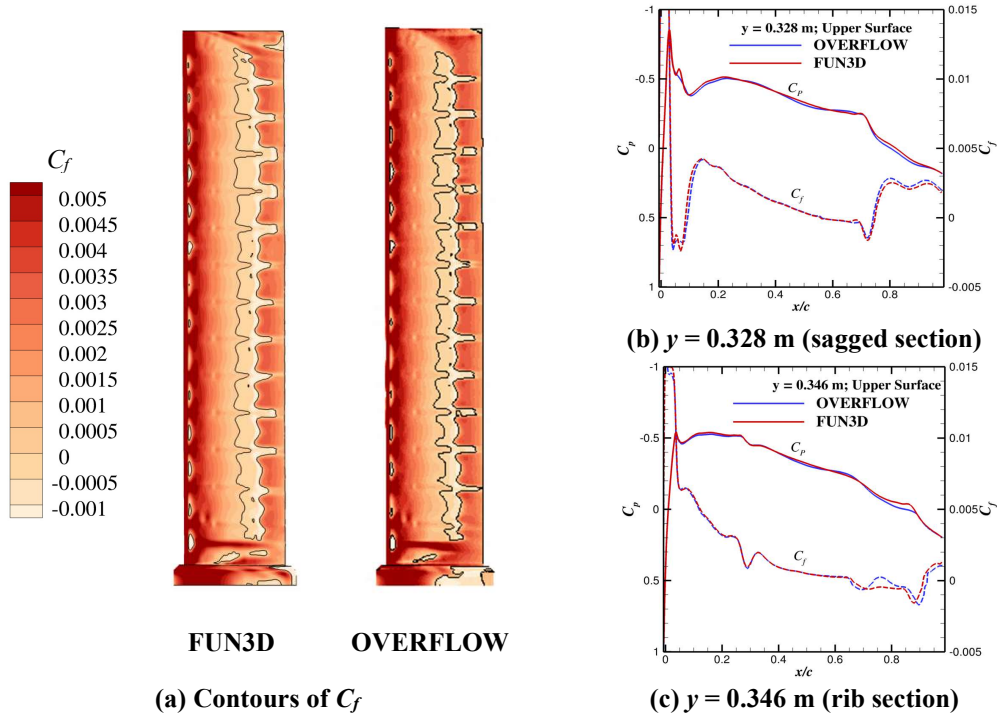


Fig. 4. Contours of the skin friction coefficient and chordwise distribution of the pressure and skin friction at two different spanwise locations for the upper surface of the sagged Pazy wing obtained with FUN3D and OVERFLOW for $\alpha = 0^\circ$. The black lines in the contour plot corresponds to the zero-crossing location of the skin friction value.

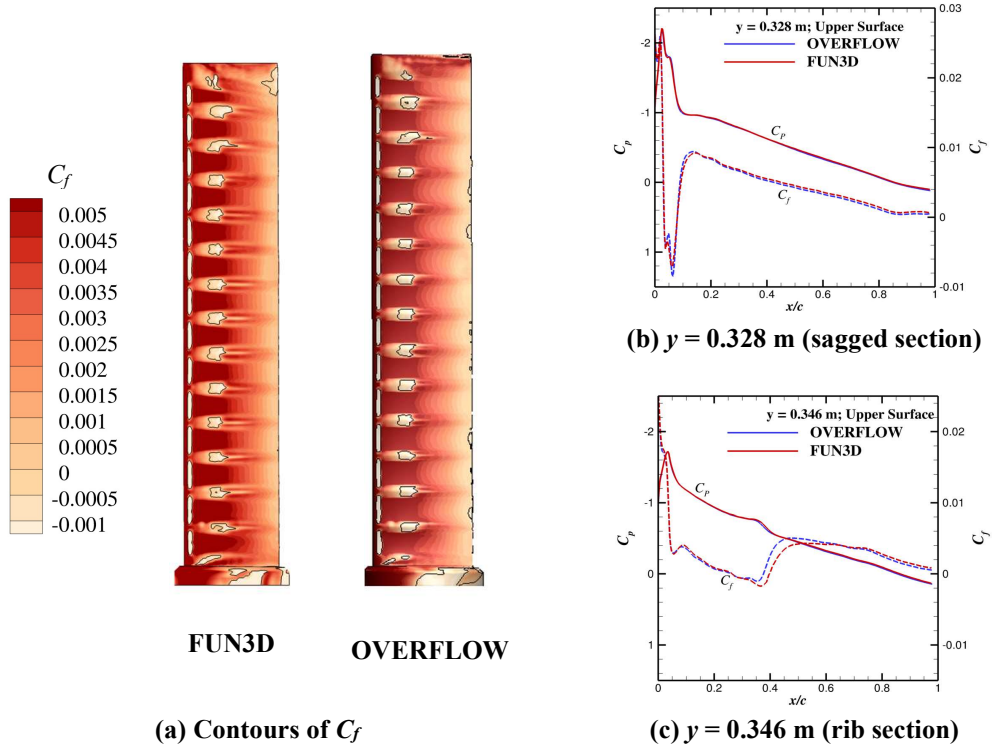


Fig. 5. Same as Fig. 4, but for $\alpha = 6^\circ$.

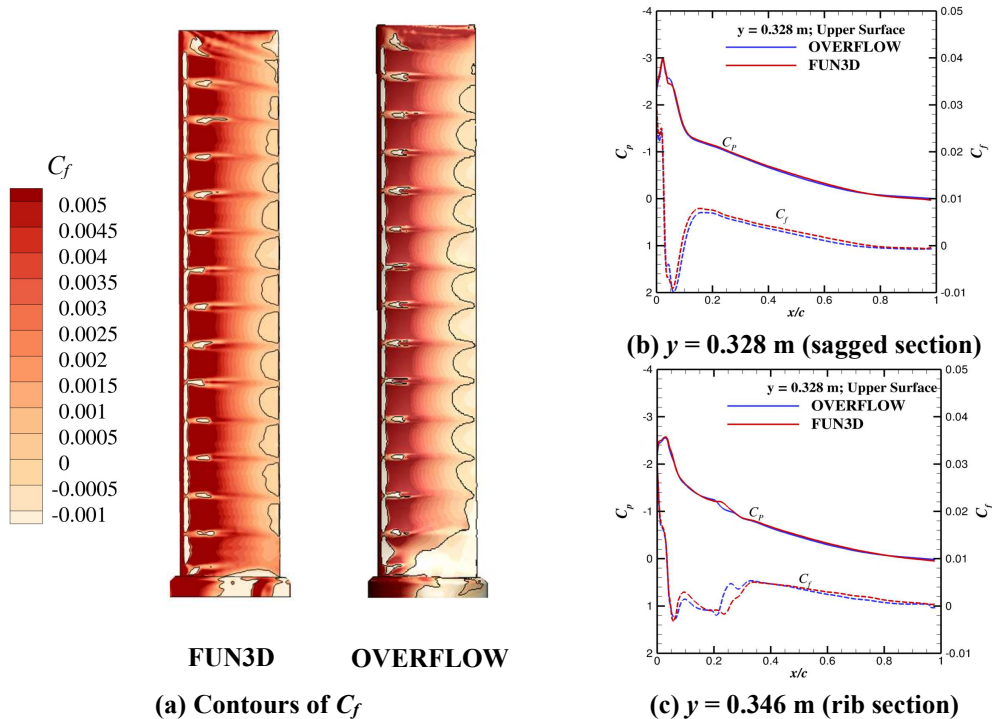


Fig. 6. Same as Fig. 4, but for $\alpha = 10^\circ$.

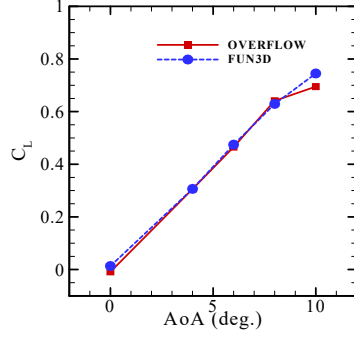
While the difference between the Reynolds number used in the computations and the actual one from the experiment prevents a direct comparison between the results from the CFD and experiment, one can make a qualitative comparison of the observed flow features. For $\alpha = 0^\circ$, the CFD computations predict small separation bubbles in the sagged portions of the wing near the leading edge that are not seen in the IR images from the experiments (see Fig. 16 from Ref. [4]). However, the IR images do indicate the spanwise contiguous separated flow region around the midchord region, albeit starting slightly upstream of that seen from the CFD. For $\alpha = 6^\circ$ and $\alpha = 10^\circ$, the flow patterns from CFD closely resemble those seen from the experiment, with the exception that, at $\alpha = 6^\circ$, the pockets of separation along the rib locations appeared to split into two individual regions in the experiment, as opposed to the single, consolidated bubble seen in the CFD results.

The aerodynamic coefficients computed with both OVERFLOW and FUN3D for $Re_c = 3.4 \times 10^5$ and various selected angles of attack are given in Fig. 7. As the angle of attack is increased from 0° to 4° , the drag coefficient appears to remain nearly flat, before increasing more rapidly with a further increase in the angle of attack. On the other hand, both the lift coefficient and the magnitude of the negative-valued pitching moment coefficient increase approximately linearly with α , at least up to $\alpha = 8^\circ$. While the results from Ritter et al. [4] also indicated a nearly constant drag coefficient for $\alpha < 4^\circ$, likely as a result of the large pockets of flow separation throughout the wing span for these angles of attack, additional investigation is necessary to fully understand the reasons behind this behavior of the drag coefficient. More important, the aerodynamic coefficients predicted by both CFD solvers closely track each other across the entire range of α investigated in this work. The consistency of transition model implementation in these solvers provides further confidence toward their utilization during future analyses of aeroelastic effects on this configuration.

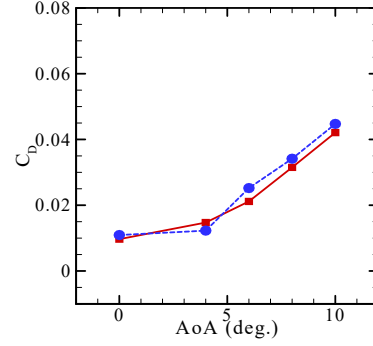
C. Reynolds Number Variation Studies

In this section, we investigate the influence of Reynolds number on the behavior of the LM model. Figures 8 and 10 show the skin-friction contour plots, along with the turbulence index contour plots for $\alpha = 0^\circ$ and 6° , obtained with the LM-CFHE transition model for $Re_c = 1.2 \times 10^5$ and 3.4×10^5 , respectively, for the upper surface of the Pazy wing. For those same conditions, Figs. 9 and 11 show the chordwise

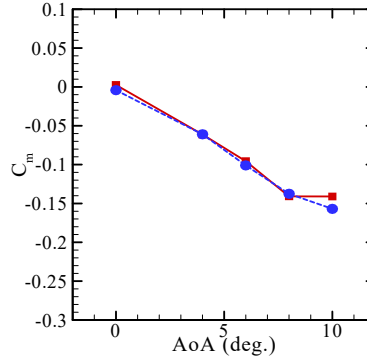
evolution of the coefficient of pressure and skin friction along with that of the turbulence index for the upper surface at two different spanwise stations (one in the sagged portion of the wing and another along the rib). The turbulence index was originally proposed by Spalart and Allmaras [20] and is frequently used as an indicator for the progress of transition in the CFD computations. The turbulence index value mostly lies between 0 and 1, with its value exceeding unity in a narrow region post transition. The definition of the turbulence index differs between the SA [20] and the SST [28] turbulence models. Recently, an improved implementation of the turbulence index for the SST turbulence model was proposed [28] and has been implemented within OVERFLOW. This newly proposed index was expected to guarantee that the index remained at a value of 1.0 in the fully turbulent region, while the original definition [29] often resulted in turbulent-index values significantly larger than 1 in the fully turbulent region. In this paper, we explore the application of this newly proposed turbulence index to see where transition onset is indicated. In the contour plots of the turbulence index within this section, the blue colored regions represent laminar flow (turbulence index closer to 0) and red colored regions indicate regions with turbulence index ≥ 1 indicating transitional or turbulent flow.



(a) Lift Coefficient



(b) Drag Coefficient



(c) Pitching Moment Coefficient

Fig. 7. Force and moment coefficients for transitional computations of the sagged Pazy wing made with FUN3D and OVERFLOW.

For $\alpha = 0^\circ$ (Figs. 8 and 9), the solution for the higher Reynolds number ($Re_c = 3.4 \times 10^5$) indicates a series of short-length, spanwise elongated separation bubbles close to the leading edge within the sagged portions of the wing. The pressure recovery in the aft portion of the wing gives rise to an additional region of separation that extends from $(x/c) \approx 0.6$ to $(x/c) \approx 0.75$ within the sagged portion and approximately over $(x/c) \in [0.60, 0.90]$ near the rib. In most cases, the separated regions over the sagged and rib regions are connected to each other, although there exist a few ribs where they are disjoint. In contrast to the above-mentioned pattern of dual regions of separation within the sagged portion, the solution for the lower Reynolds number case ($Re_c = 1.2 \times 10^5$) does not include any leading edge bubbles, and there is a single region of separation that begins at around $(x/c) \approx 0.4$, i.e., significantly upstream in comparison with the higher Re_c

computations. The reattachment occurs near $(x/c) \approx 0.70$. Furthermore, the chordwise locations of both separation and reattachment are nearly uniform across the wing span, with rather small variations between the sagged and rib regions. As a result of these differences in separation patterns, the C_p curves shown in Fig. 9(a) also indicate significant variation between the two Reynolds numbers.

For $\alpha = 0^\circ$, the IR images from the experiment (Fig. 7 from Ref. [3] or Fig. 16 from Ref. [4]) indicates both the starting and ending locations of the spanwise separated region to be similar to those seen from the low Re_c computations, but the observed ridges in the skin friction contours in the immediate vicinity of the rib locations are only present in the higher Re_c computations. Additionally, the short-length, spanwise elongated separation bubbles close to the leading-edge within the sagged portions of the wing, seen in the higher Re_c computations, are not observed in the IR images from the experiment. Similarly, the isolated pockets of reduced skin friction found along the span of the wing close to the leading-edge within the sagged portions of the wing, seen in lower Re_c computations (see Fig. 8), are also not observed in the IR images from the experiment.

In the experiment, the transition onset was inferred from the IR images to be around $(x/c) \approx 0.8$ across the entire span of the wing (see Fig. 8 from Ref. [3]). The flow is already turbulent based on the skin-friction curves from the CFD computations shown in Fig. 9(b). Figure. 9(b) also shows that the values of the improved turbulence index from Ref. [28] remain significantly above unity in the fully-turbulent region downstream of reattachment. Because this improvement was aimed at ensuring that the value of the turbulent index would be confined to the range of $[0,1]$, these results suggest the need for further refinements of the turbulence index definition for computations utilizing SST-based transition models. Also, given that the flow scenarios shown here involve long separation bubbles [30] that extend over several percent of the overall chord length, the onset of transition is expected to occur significantly downstream of the separation point. However, the rapid rise in the turbulence index was found to occur relatively close to the separation point. Further work is necessary to determine if this discrepancy is mainly due to the errors associated with the transition model or if it points to the need to further refine the definition of the turbulence index. Hence, further improvement to the definition of the turbulence index is warranted before it can be used to identify the transition onset location, especially for flows involving separation bubbles.

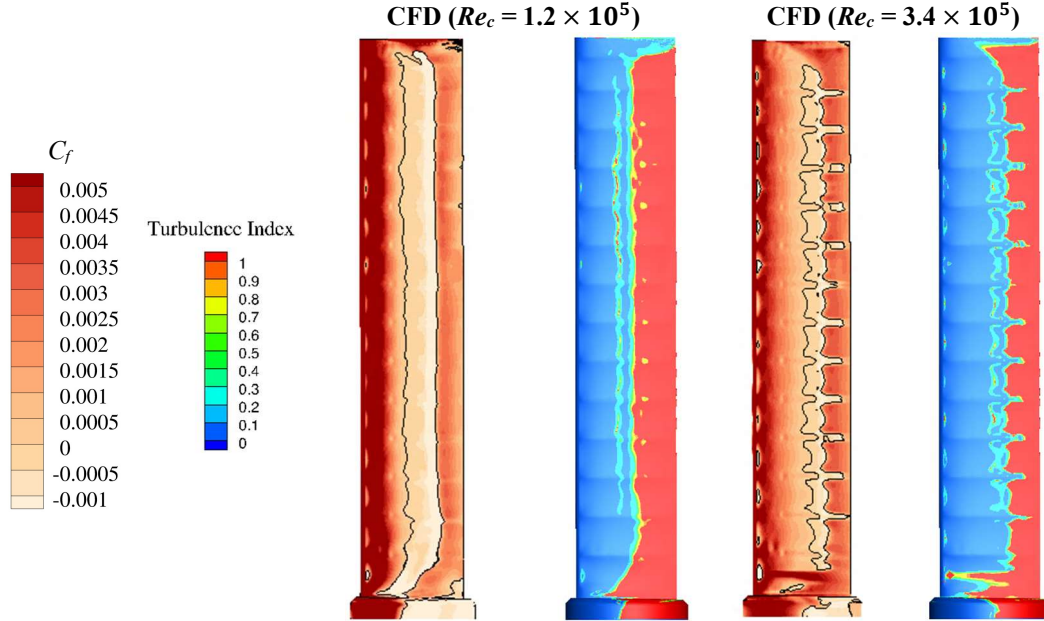


Fig. 8. Contours of the skin friction coefficient and turbulence index over the upper surface of the sagged Pazy wing using OVERFLOW. Results include CFD predictions for $\alpha = 0^\circ$ at two different values of Re_c . The black lines correspond to the zero-crossing location of the skin friction value.

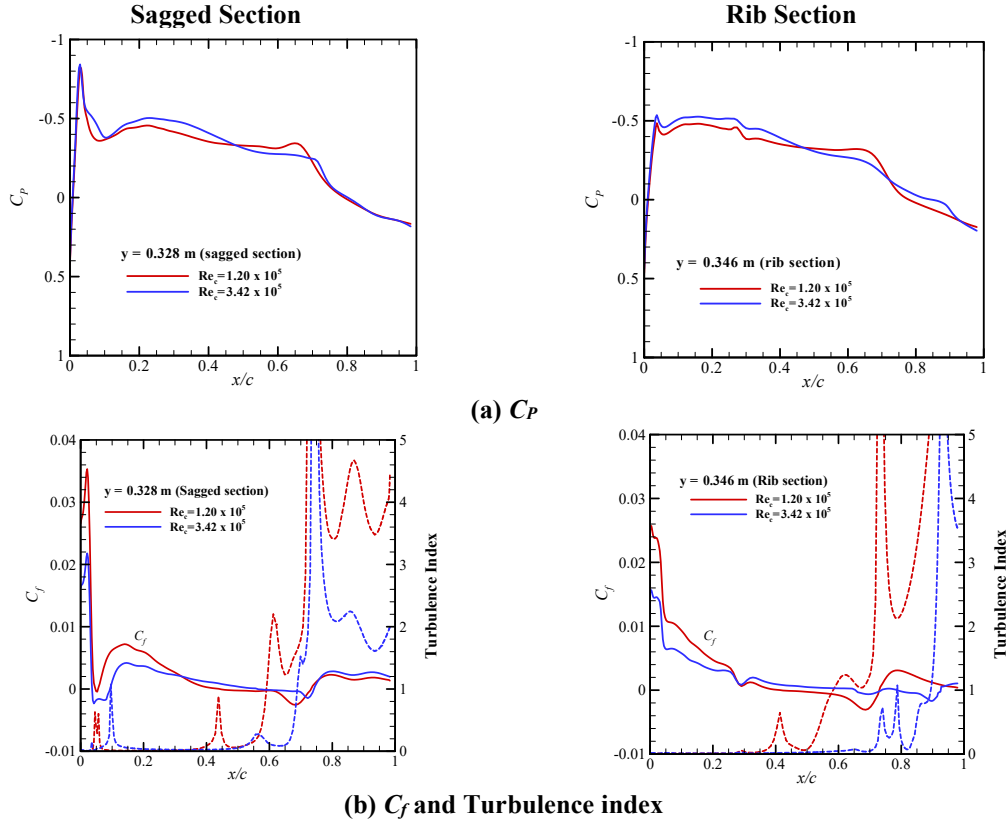


Fig. 9. Chordwise evolution of the pressure and skin friction coefficients from OVERFLOW, shown along with turbulence index (dashed line) for the upper surface of the Pazy wing at two different spanwise locations ($y = 0.328$ m: sagged region; and $y = 0.346$ m: rib location) for $\alpha = 0^\circ$ and two different values of Re_c .

Similar differences pertaining to the effect of the flow Reynolds number also occur at $\alpha = 6^\circ$ as seen from Figs. 10 and 11. At $\alpha = 6^\circ$, the separation bubbles near the leading edge within the sagged portions of the wing are larger in size in the computations at the lower Re_c . Similarly, the separation bubble along the ribs has a larger chordwise extent in the lower Re_c computations. The IR images from the experiment appear to be closer to the skin-friction contour plot from the higher Re_c computations, with the exception that, as described in Section IV(B), the pockets of separation along the rib locations appeared to be split into two individual regions in the experiment, as opposed to the single consolidated bubble seen in the CFD results at the higher Re_c .

From the IR image, it was inferred in the experiment that (see Fig. 8 from Ref. [3]) the transition onset in the sagged portion was around $(x/c) \approx 0.05$ and around $(x/c) \approx 0.45$ along the rib locations and its immediate spanwise vicinity. These locations are where the separated flow reattaches as a turbulent boundary layer in the computations at the higher Re_c , as can be seen from the skin friction curves shown in Fig. 11(b). However, from Fig. 11(b), it can be seen that for $\alpha = 6^\circ$ also, the turbulence index remains significantly above unity in the fully-turbulent region downstream of reattachment, thus preventing its use for identifying the transition onset location.

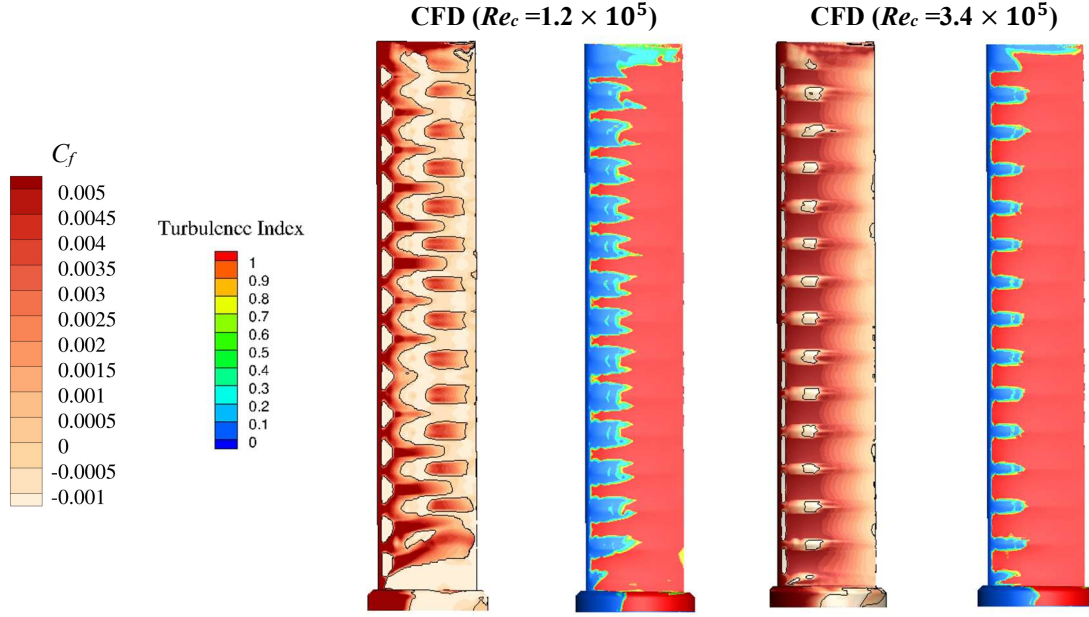


Fig. 10. Same as Fig. 8, but for $\alpha = 6^\circ$.

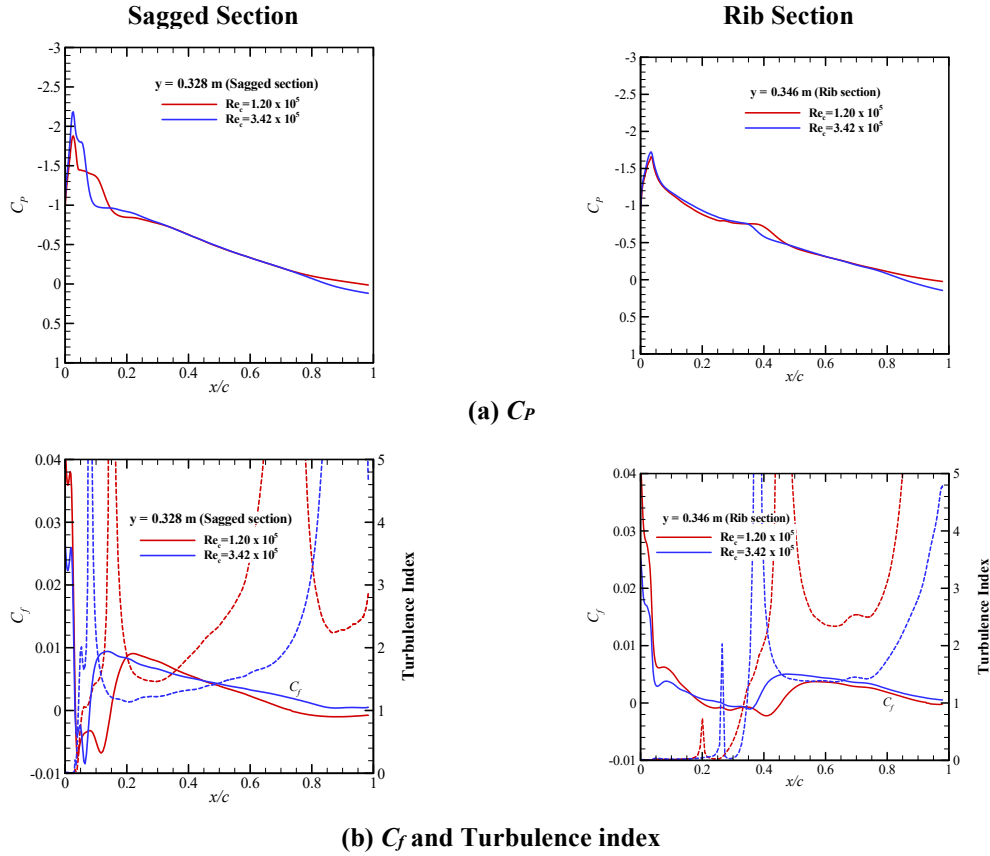


Fig. 11. Same as Fig. 9, but for $\alpha = 6^\circ$.

Based on the above discussion regarding the comparisons between the predicted skin friction contours for each of the two Reynolds number conditions and the flow patterns observed from the infrared image

obtained during the experiment (Fig. 16 from Ref. [4]), as well as the inferred transition onset locations from the experiment (Fig. 8 from Ref. [3]), the predictions for the higher Re_c provide a better qualitative match with the experimental data for $\alpha = 0^\circ$ and $\alpha = 6^\circ$. The CFD results for the higher Reynolds number also compare well between the present study based on the LM family of two-equation transition models and the previous study by DLR with the one-equation γ model [4]. Although a definitive assessment of the transition models is lacking at this stage, the present findings appear to be consistent with the earlier works [13,14] that had highlighted the shortcomings of the Langtry-Menter model in predicting laminar separation bubbles at low Re_c . Whereas those earlier studies involved two-dimensional configurations, the present work suggests that similar inaccuracies may also occur for strongly 3D transition fronts.

V. Results for Periodic Configuration

Given that both the experimental flow visualizations and the computational predictions presented in the previous section display quasiperiodic patterns across the span, a simplified periodic representation of the Pazy wing geometry was created to enable more systematic studies. Besides providing a convenient framework for quantitative code-to-code comparisons, the periodic representation also allows a cost-effective grid convergence study of the candidate transition models. Furthermore, such a simplification also allows for more comprehensive analysis of the flow, such as the application of stability analysis tools, which may help calibrate the performance of RANS-based transition models at low Reynolds number conditions.

The simplified periodic representation of the Pazy wing was created by isolating a unit spanwise cell centered on one of the ribs in the midspan region and the thickness distribution was filtered by retaining only four spanwise Fourier harmonics, yielding a close representation of the original geometry within that cell. This geometry was then used to generate a family of volume grids described later in this section. At each chordwise section, the airfoil contour was discretized with 601 points and a single spanwise wavelength was discretized with 64 points. In this representation, the rib was located in the middle of the domain, and the two spanwise boundaries represent the middle of the sagged portion on either side of the rib.

A. Grid Convergence Studies

Five levels of grids belonging to the same family were generated to study the periodic representation of the sagged Pazy wing geometry. Each of these grids was a single-block structured grid. The number of grid points in every direction was doubled for every other grid level. Details of the number of points in various directions for the five levels of grids generated are given in Table 1. For reference, mesh level 3 had a near-wall spacing of 1.3×10^{-6} m ($y^+ \sim 0.25$ based on quarter chord location and $Re_c = 3.4 \times 10^5$), with chordwise spacing varying from 0.01 mm to 0.77 mm and a spanwise spacing variation from 0.2 mm near the rib to 0.3 mm at the center of the sagged region. The following boundary conditions were used in the computations reported here: adiabatic wall boundary conditions on the wing model; periodic boundary conditions on the spanwise boundaries; and freestream boundary conditions for the remaining boundaries.

Table 1. Mesh dimensions.

Mesh Level	No. of points along airfoil contour	No. of points in spanwise direction	No. of points in wall-normal direction	Total No. of grid points (N)	Grid sizing parameter ($h = 1/\sqrt[3]{N}$)
Mesh 1	409	73	193	5,764,201	5.58E-3
Mesh 2	545	97	257	13,586,305	4.42E-3
Mesh 3	817	145	385	45,609,025	2.80E-3
Mesh 4	1089	193	513	107,820,801	2.10E-3
Mesh 5	1633	289	769	362,919,553	1.40E-3

Computations with the LM-CFHE model in OVERFLOW, conducted for $Re_c = 3.4 \times 10^5$ and $\alpha = 4^\circ$ using the five levels of meshes, are reported in terms of the contours of skin-friction coefficient in Fig. 12. In these contour plots, the two spanwise boundaries correspond to the middle of the sagged portion of the wing and the rib location is at the center of the domain. Based on the skin-friction contour plots, the overall

location and the shape of the separation bubbles appears to be visually converged for mesh levels 3 and above. Although not shown here, the force coefficients appear to indicate that asymptotic convergence has not been achieved yet. Additional runs using more refined meshes are needed to get a better understanding of the convergence behavior for these complex 3D flow configurations. But, from the standpoint of an engineering analysis, we note that the drag variation across the three finer meshes is already less than one drag count, which lends confidence that grid levels 3 and higher should suffice for the purposes of the present analysis. All of the OVERFLOW computations reported in the remaining portions of this section were carried out on mesh level 3.

B. Code-to-Code Comparison

In this section, we focus on code-to-code comparison of the results obtained using the LM model in OVERFLOW and FUN3D for the periodic representation of the sagged Pazy wing. In these computations, the Reynolds number is fixed at $Re_c = 3.4 \times 10^5$, while the angle of attack is varied from $\alpha = 0^\circ$ to $\alpha = 10^\circ$.

Because periodic boundary conditions could not be utilized in the FUN3D computations, symmetry boundary conditions were used in the spanwise direction. To ensure that this change does not affect the code-to-code comparison, an additional computation was carried out using OVERFLOW by employing symmetry boundary conditions on the spanwise boundaries for $Re_c = 3.4 \times 10^5$ and $\alpha = 6^\circ$. Figure 13 shows the prediction based on the periodic boundary condition via contours of skin friction, and contour lines of the same metric (black colored contour lines). Overlaid on top, in the same figure, are the skin friction contour lines obtained from computations with symmetry boundary conditions (green dashed lines). Figure 13 indicates that there is virtually no discernible difference between the results obtained using the two different boundary conditions, demonstrating that the OVERFLOW and FUN3D solutions obtained using different spanwise boundary conditions can be used toward meaningful comparisons between the results from these two solvers.

In Fig. 14, results from computations for angles of attack 0° and 6° obtained with the two different flow solvers are shown as contours of skin friction along the upper surface of the wing. Overall, the contour levels, as well as the location of the separation bubble compare well between the results from the two CFD solvers. However, because (1) FUN3D results were computed on a coarser level mesh (Mesh level 2) as opposed to that used for OVERFLOW (Mesh level 3) and (2) the specified spatial accuracy of FUN3D was lower than that of the OVERFLOW, we do see some differences in the shape of the separation bubble, both near the leading edge for the sagged portions of the grid as well further downstream along the rib location. Based on the full span results shown in the earlier section, these differences are expected to vanish when the results are computed on finer meshes. Additional FUN3D computations on finer resolution meshes are currently ongoing and they will be reported in a follow-on paper. We note that the skin-friction contours based on the OVERFLOW solution for this periodic configuration compare favorably well with the infrared thermography images from the experiments (Fig. 7 from Ref. [3] or Fig. 16 from Ref. [4]). While the predicted spanwise extent of the spanwise elongated separation bubble in the leading edge regions from the sagged portions of the wing appear to be larger than that in the experiments, the shape as well as the spatial extent of the other separation bubble that forms further downstream along the rib are similar between the experiment and the present CFD results. Overall, the agreement is satisfactory for the purpose of this initial assessment.

VI. Summary and Concluding Remarks

This paper was focused on selected results pertaining to the boundary-layer transition behavior for the sagged Pazy wing obtained using the SST-based Langtry-Menter $\gamma - Re_{\theta t}$ transition model in the NASA's OVERFLOW and FUN3D CFD codes. Through application of different variants of the (two-equation) Langtry-Menter transition model, it was shown that crossflow is not important for this 3D configuration and that the laminar-separation bubble induced transition is the main mechanism through which the boundary layer transitions across the span of the wing. Computations were performed at two different chord Reynolds

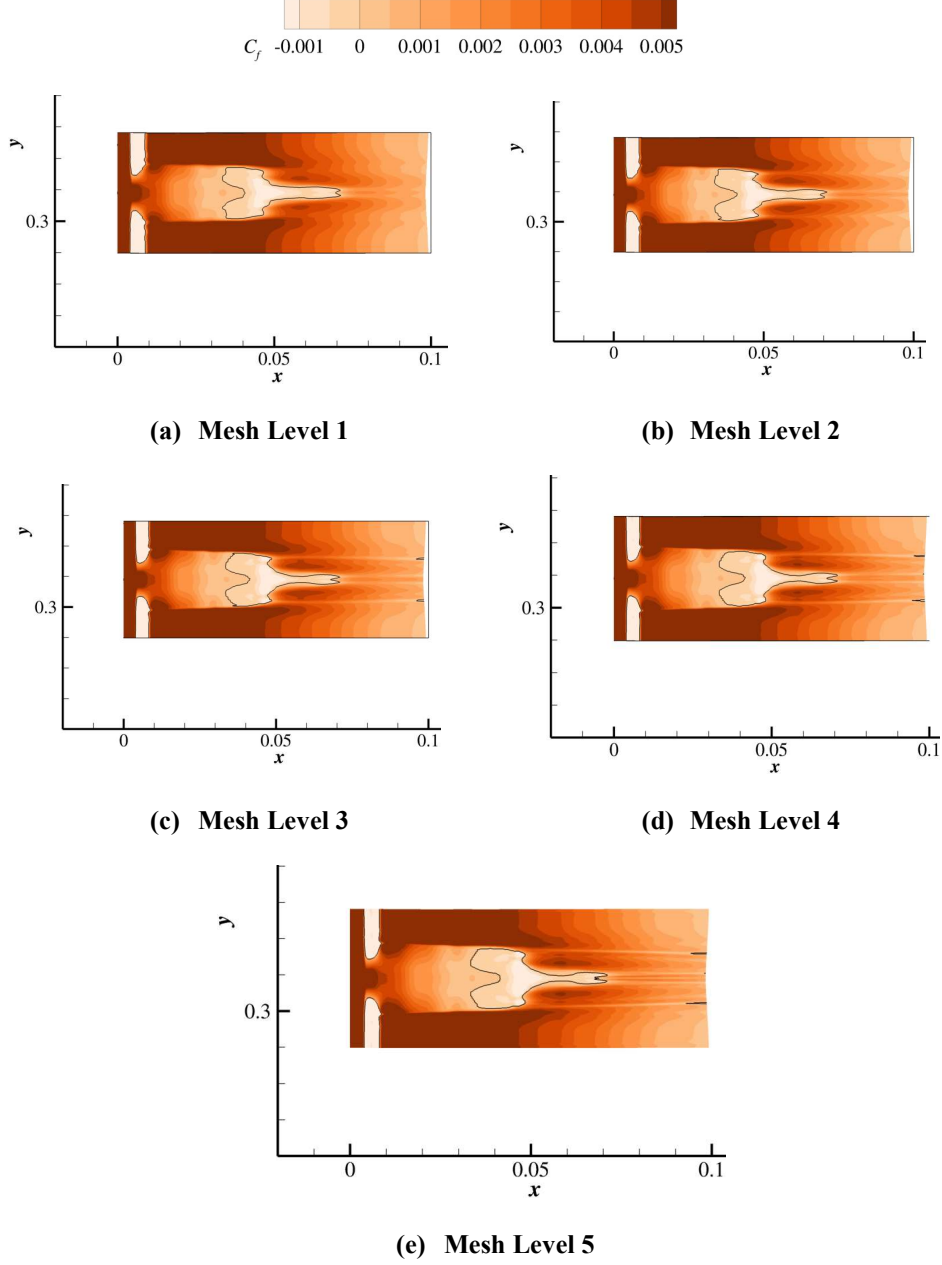


Fig. 12. Contours of the skin friction coefficient for the sagged Pazy wing obtained with the LM-CFHE transition model for $Re_c = 3.4 \times 10^5$ and $\alpha = 4^\circ$ on a family of grids. The black line corresponds to the zero-crossing location of the skin friction value.

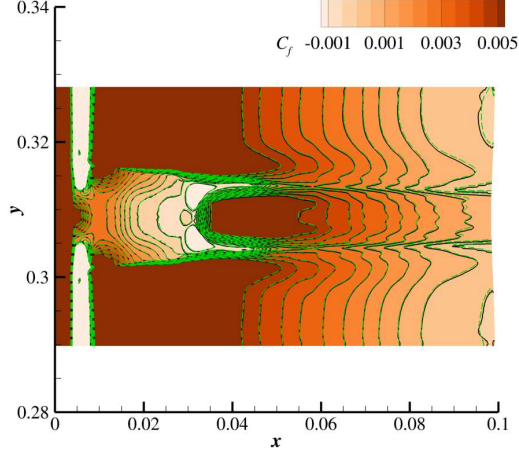


Fig. 13. Contours of the skin friction on the upper surface of the Pazy wing obtained using OVERFLOW and periodic boundary conditions, overlaid with contour lines obtained using symmetric boundary conditions for $Re_c = 3.4 \times 10^5$ and $\alpha = 6^\circ$. Black lines: Periodic boundary conditions; green dashed lines: symmetric boundary conditions.

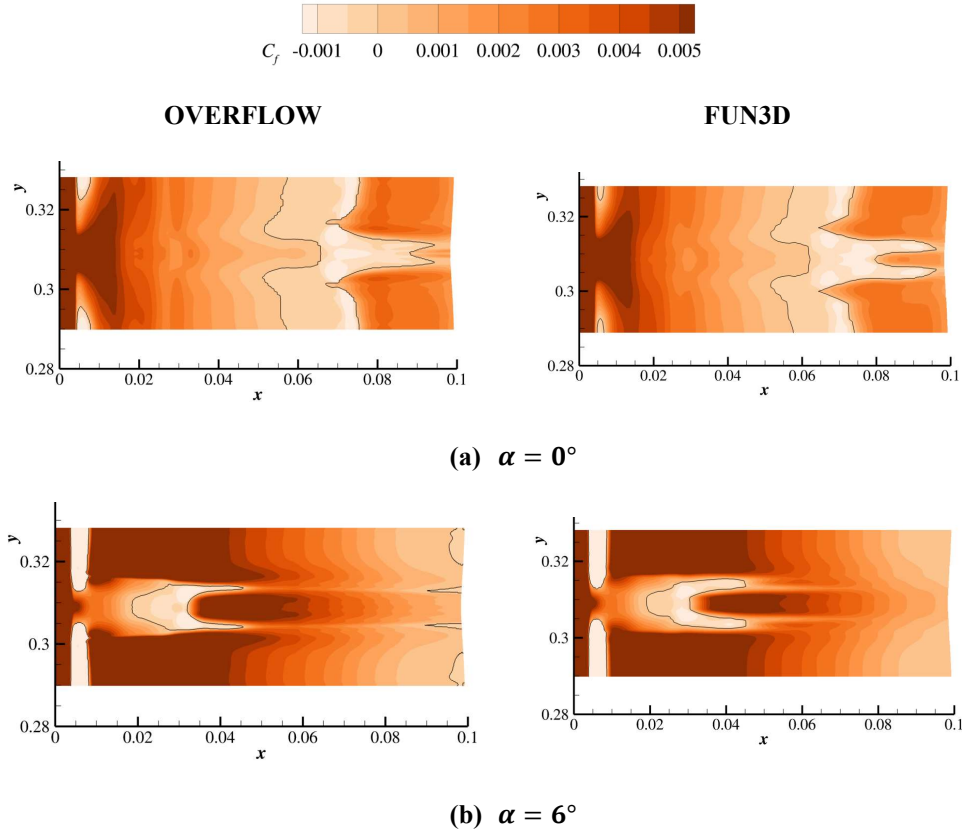


Fig. 14. Comparison between the contours of the skin friction coefficient obtained with two different solvers for the periodic representation of the sagged Pazy wing at $Re_c = 3.4 \times 10^5$. The black line corresponds to the zero-crossing location of the skin friction value.

Numbers. Although not shown, qualitative comparison with the IR thermography images obtained from the experiments supports the earlier findings [4] based on a single-equation γ transition model that a qualitative match with the experiment can only be obtained at Reynolds numbers that are significantly higher ($Re_c = 3.4 \times 10^5$) than the Reynolds number in the experiment ($Re_c = 1.2 \times 10^5$). These findings provide additional evidence for the previously documented inaccuracy of the Langtry-Menter model at low Reynolds number conditions [12–13]); however, additional experiments with detailed measurements of multiple quantities that allow for a more comprehensive assessment is needed to allow more definitive conclusions. As part of this work, we also briefly explored the potential for the recently proposed improvement to the definition of a turbulence index for SST-based CFD predictions [28] to serve as a reliable indicator for identification of the transition location based on a CFD solution. However, the limited observations presented herein have shown that the value of this index can become significantly larger than unity beyond transition and remain well above unity within the fully-turbulent region downstream of the flow reattachment. Furthermore, contrary to the expected behavior in long separation bubbles, the rapid rise in this index above the laminar values of close to zero occurred too close to the start of separation. These findings suggest that further investigations are necessary to determine the validity of this index for flows that include separation bubbles.

Based on how the CFD computations for the full-span Pazy wing as well as the flow visualizations from the experiment display quasiperiodic patterns across the span, a simplified, spanwise periodic representation of this geometry was created, to enable a more systematic study to understand the flow physics, identify optimum mesh spacings, and evaluate the transition model performance in a cost-effective manner. A systematic grid convergence study and a favorable preliminary code-to-code comparison of the CFD predictions based on the LM-CFHE transition model was also demonstrated using the simplified geometry.

Planned future research includes more detailed investigation into the unsteady behavior of the flow at higher angles of attack, additional CFD computations performed at the actual turbulence intensity level found in the experiment, and exploration of other RANS-based transition models. More importantly, aeroelastic calculations will be performed to further assess the effect of transition on the wing response. We also plan to perform a more detailed characterization of the behavior of the turbulence index as a metric to identify transition onset for conditions involving long separation bubbles. A study of the stability characteristics of the boundary layer over the simplified periodic representation would also be useful to provide further insights into the predictions of the Langtry-Menter model at low Reynolds number conditions and how they could be improved.

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