

Optimization of Plug Nozzles Using CFD Informed Design Space Querying

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This paper presents an axisymmetric CFD approach based on Reynolds-averaged Navier-Stokes (RANS) calculations for the shape optimization of plug nozzle contours for both supersonic cruise and landing/takeoff (LTO) conditions. Three key external plug design parameters were defined as independent variables in the optimization scheme, while nozzle performance was evaluated using thrust coefficient (C_{fg}) and discharge coefficient (C_d) as the primary dependent variables. By varying these parameters, a wide range of axisymmetric conic section plug profiles can be achieved, simplifying the problem. The paper also discusses the supersonic plug nozzle optimization approach, design space seeding, meshing, and CFD simulations for the optimization process. The results demonstrate that the approach is effective in exploring the parameter space and reducing computational expenses, making it a viable option for optimizing nozzle plug contours. Using this optimization toolchain, the study yielded preliminary designs for LTO, cruise, and LTO-cruise balanced plug nozzles.

I. Nomenclature

A. Variables

A	= Area
C_p	= Heat capacity
C_d	= Discharge coefficient
C_{fg}	= Thrust coefficient
D	= Distance
F	= Thrust
L	= Length
$\dot{m}$	= Massflow
n	= Number of cells in a specified domain
P	= Conic focal parameter
p	= Pressure
r	= radial coordinate
T_o	= Total temperature
V	= Velocity
x	= Axial coordinate
y^+	= Dimensionless wall distance
γ	= Ratio of specific heats

B. Subscripts

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<i>cell</i>	=	CFD cell centered quantities
<i>cowl</i>	=	Nozzle cowl lip location
<i>cruise</i>	=	Evaluated at cruise simulation conditions
<i>e</i>	=	Nozzle exit conditions
<i>i</i>	=	Nozzle inflow state
<i>ideal</i>	=	Conditions present with isentropic processes
<i>k</i>	=	Arbitrary counting variable
<i>LTO</i>	=	Evaluated at landing and takeoff simulation conditions
<i>Pressure</i>	=	Pressure component
<i>t1</i>	=	Tangency control point one
<i>Visc</i>	=	Viscous component

II. Introduction

Plug nozzles have gained significant attention in recent years, primarily driven by the growing interest in supersonic transport. For a more comprehensive historical overview of plug nozzles, please refer to Ref [1]. They may present acoustic advantages over traditional convergent and convergent-divergent nozzles, particularly in the crucial landing and takeoff (LTO) scenarios, which are under close regulatory scrutiny. A thorough examination of plug nozzle flow, performance and acoustic characteristics can be found in Ref [2] and Ref [3]. Moreover, they may maintain the cruise performance of traditional nozzles. Acoustic analysis, however, remains computationally expensive, with large eddy simulation (LES) being the primary toolchain employed. To reduce the computational expense, lower fidelity tools are necessary to understand the LTO and cruise aerodynamic performance of candidate nozzles, avoiding wasted LES analysis on poorly performing nozzle configurations. An axisymmetric computational fluid dynamics (CFD) approach based on Reynolds-averaged Navier-Stokes (RANS) is a far cheaper alternative to LES. This paper focuses on the RANS-based shape optimization of nozzle plug contours for both supersonic cruise and LTO conditions, using three conic plug geometric parameters as independent variables, and thrust coefficient (C_{fg}) and discharge coefficient (C_d) as the primary dependent variables. Other theory-based optimization techniques for plug nozzle performance at supersonic cruise have been studied in Ref [4]. All cruise analysis was conducted at a nozzle pressure ratio (NPR) of approximately 5.9 and a freestream Mach number of 1.4, while LTO analysis was performed at an NPR of 2.0 and Mach 0.01, this unusually low Mach number was chosen to have applicability to future experimental results performed in a test cell with mostly static flow. Some cases were analyzed at an NPR of 7.57 and Mach 0.01, simulating the maximum attainable pressure ratio without experiencing under-expansion during takeoff conditions.

III. Geometry

To simplify the problem, the plug geometry was reduced to three critical design parameters, which can be varied to yield a wide range of plug profiles. To enforce crown tangency conditions, facilitate smoothness, and reduce the number of curve parameters, it was decided that all the plugs should be some form of conic. The chosen geometric parameters to be varied are tangency point location (L_{t1}), plug length (L) and conic parameter (P) as shown in Fig. 1. For each plug geometry and NPR, the appropriate L_{cowl} for the extendable cowl was calculated to facilitate theoretical perfect expansion and integrated into the meshed plug geometries, although a more thorough experimental study of the effect on parallel cowl extensions on nozzle performance can be seen in Ref [5].

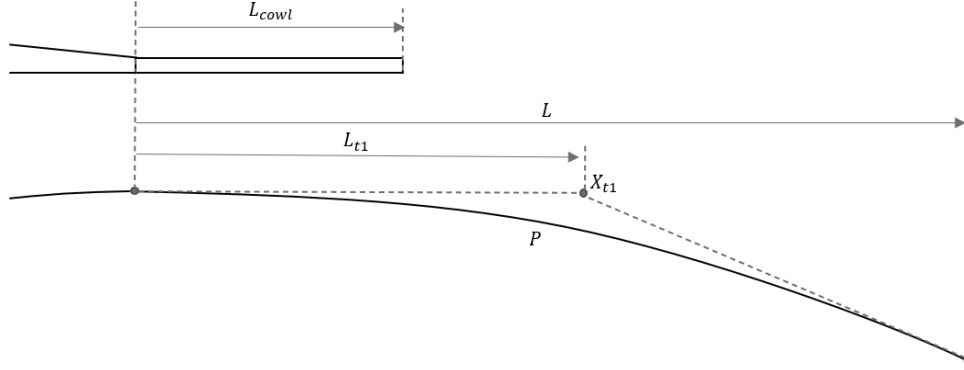


Fig. 1 Geometric definition of a generic axisymmetric plug nozzle

By varying these simple conic parameters, a wide range of plug geometries can be captured, including interesting profiles such as bulbous and pure conic plugs. However, inverted conic (concave) plugs were not considered in this optimization, as early CFD results indicate poor performance at the low-pressure ratios seen in LTO flight conditions. Initial crown-plug curvature was found to be too great for any flow to remain attached on a concave plug nozzle with a constant radius cowl; for a detailed numerical study of flow separation phenomena of plug nozzles see Ref [6]. Inverted conics and tangency enforcing plug-crown blending operations should be considered only with inward turning cowls, seen in Fig. 3 in Ref [1], as method of characteristics (MoC) and similarly shaped plugs require isentropic turning to properly expand nozzle flow. Furthermore, with flow leaving the nozzle axially, inverted conics perform very poorly. Fig. 2 below exhibits ordered families of plug nozzles, with a single variable undergoing variation within each family, exemplifying the diverse geometries attainable through the manipulation of three straightforward parameters within the confines of design space considerations. Additionally, Fig. 3 explores random variations the geometric control parameters, showcasing the wide array of possible geometric outcomes.

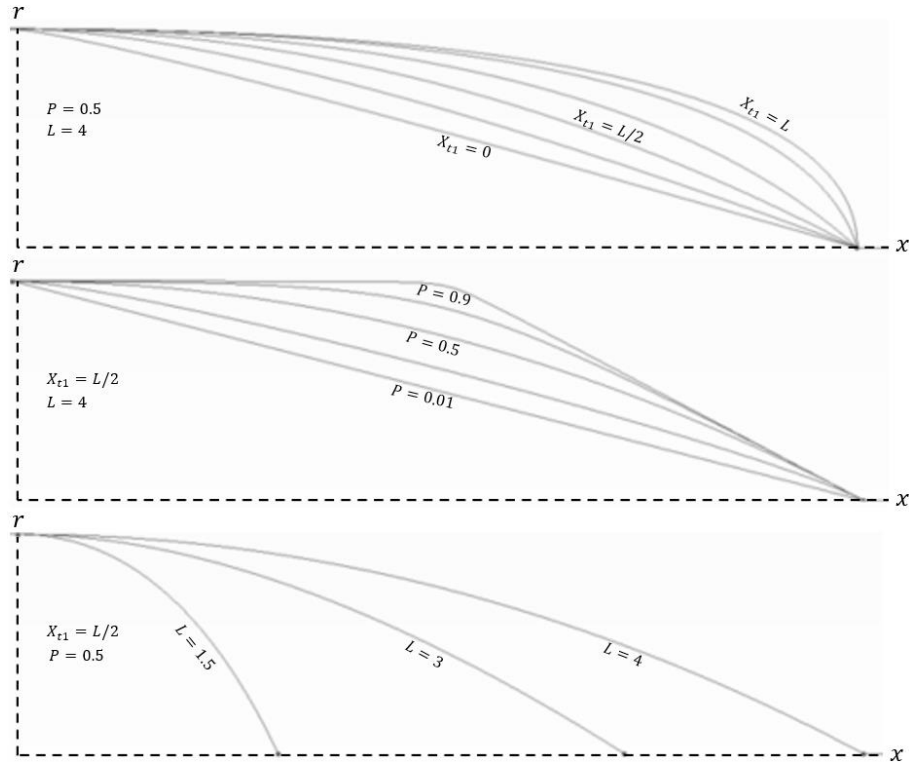


Fig. 2 X-r plane half views of sample plug nozzles with varying X_{t1} (Top), P (Center) and L (Bottom)

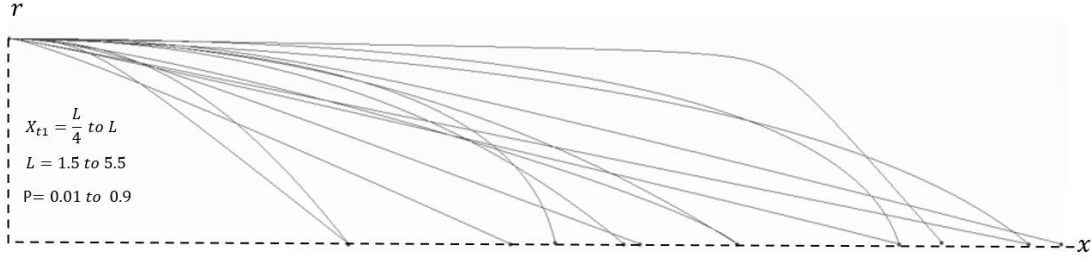


Fig 3: Random variation of geometric plug nozzle parameters within realistic ranges

IV. Solution Process

Three different optimization methods were evaluated for addressing the problem at hand: gradient descent, genetic algorithm, and Latin hypercube (LHC) design space querying, which is the method proposed in this paper. Gradient descent was dismissed due to its tendency to converge on local optima or "false valleys," its sensitivity to the initial design space conditions, and the complexity involved in performing performance analysis and optimization at the end of each CFD solution, requiring a "looped model" that constantly updates the mesh and model based on the present analysis. While the genetic algorithm is intriguing and novel, it also faces the same "looped model" complexity and requires artificial genes, mutation, disease, and familial interaction code, adding to the time scale and complexity of the optimization process. Moreover, this algorithm may be slow to converge and overly sensitive to evolutionary selection pressure parameters. LHC design space querying was chosen for its ability to avoid false valley solutions, the reduced computation time for surrogate models used for performance results, and its simplicity in comparison to a looped optimization model. The studies can be batched as "runs" to enable the creation of all meshes, CFD execution over those meshes, and performance analysis via Python. Although this process is slightly more involved, it significantly reduces the front-end coding time and allows for closer monitoring of the solution process, minimizing the "black box" effect that can occur in complex optimization problems. A process chart illustrating the CFD-informed LHC seeded design space querying process used for optimizing plug nozzles is provided in Fig. 4.

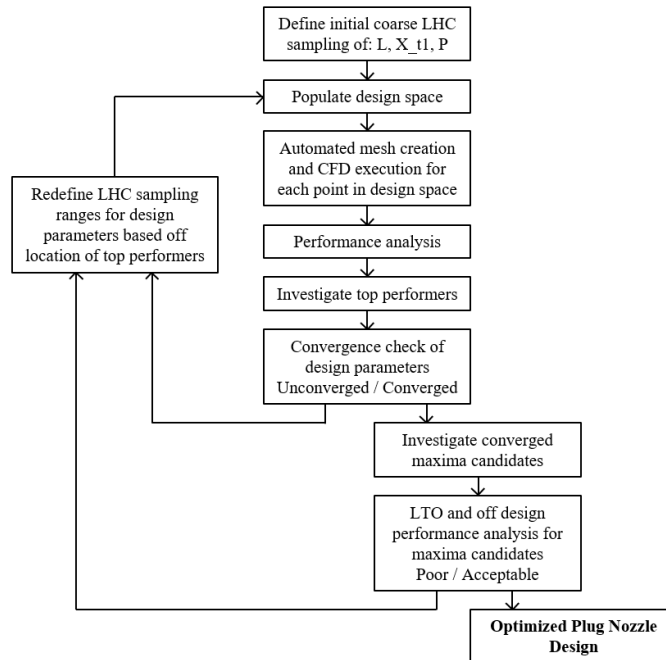


Fig. 4 Block diagram for selected solution process

V. The Software Toolchain

A. Design Space Seeding

To effectively explore the design space of geometric variables, Latin Hypercube seeding was utilized for the plug length L_{plug} and conic parameter P , while the axial location of the tangency control point (X_{t1}) was randomly assigned a fraction of the plug length. The tangency control point X_{t1} was not included in the LHC seeding process due to its unique bounds, which are determined by the plug length for each unique point. An example of such a seeded design space with 68 unique design conditions can be seen below in Figure 5, with each axis representing a design parameter. At the conclusion of each run, the results were used to color and size each individual plug nozzle design with its C_{fg} and C_d respectively. With these results, markers were assigned to an arbitrary number of top performers, and a new bounding box for the concurrent run is defined. This design space seeding is then re-executed on this new, smaller design space in the hope of locating a plug nozzle configuration that is nearer to the global C_{fg} and C_d optimum.

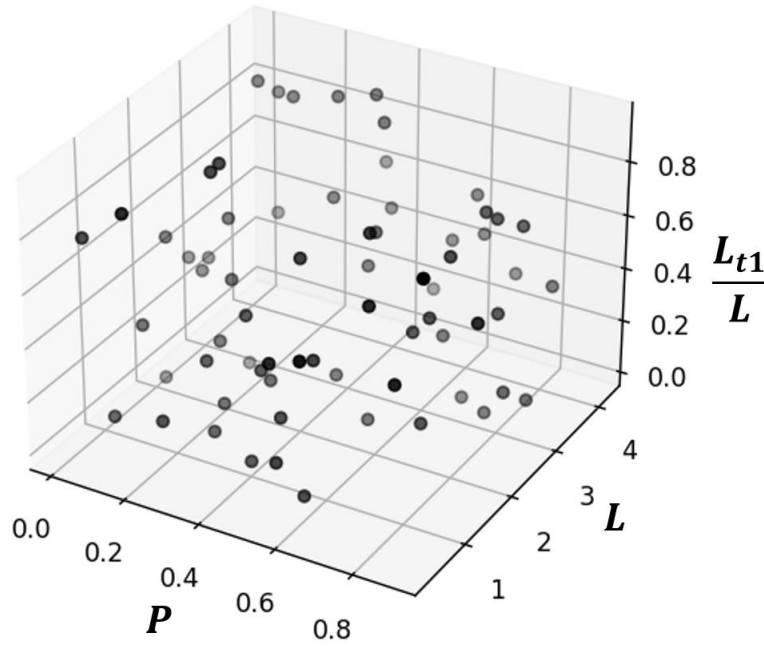


Figure 5: Sample LHC-seeded design space, lighter shades indicating greater in-page depth

B. Meshing

The initial unstructured meshes were generated in Pointwise, yielding between 18,000 and 22,000 cells, contingent upon plug geometry. All meshes feature multiple stretched “T-rex” cell layers on wall boundaries to ensure a y^+ value at or below unity. The meshes were generated through the implementation of a pilot mesh and Glyph (.glf) script that was executed within Pointwise, with matrix input from Python. This methodology allows the automated creation of an arbitrary number of meshes for a wide range of conic plug geometries. To further refine the simulations, ANSYS Fluent’s density-based mesh adaption schemes were employed after the initial 20,000 cell mesh had converged. This allowed for the reduction of y^+ values, refinement of shear layers, and resolution of complex shock structures. With the utilization of multiple refinement cycles, the mesh sizes were increased to approximately half a million cells, concentrated primarily on the interior walls, shear layer, and shock locations.

C. CFD

The computational fluid dynamics (CFD) simulations were conducted using the ANSYS Fluent 2022 R1 commercial software. All simulations were performed as axisymmetric, density-based implicit calorically

perfect gas models, utilizing the k-omega shear stress transport (k- Ω SST) turbulence model [7] and assuming fully turbulent steady state flow. Cruise analysis was conducted at a nozzle pressure ratio (NPR) of approximately 5.9 and a freestream Mach number of 1.4, while LTO analysis was performed at an NPR of 2.0 and freestream Mach number 0.01. All CFD simulations were run with both inflow total and far-field static temperatures of 273 K, representing “cold flow” conditions, and a far-field pressure of 101,325 Pa. Additional CFD runs were carried out at LTO flight conditions for various NPRs to assess the performance of the best-performing plug nozzle contour candidates across the entire throttle range. Simulation residuals were closely monitored, and all simulations were run well into convergence to enable effective cross-comparison. This was achieved using a linear step scaling of the Courant number (CFL) from 1 to 5 at first-order spatial accuracy for the first 3,000 iterations, followed by an additional 3,000 iterations at second-order accuracy with linear step scaling of the CFL from 5 to 10. Finally, 4,700 iterations were run with the density-based adaptive mesh refinement option engaged. The refinement frequency was set to 600 iterations, and the maximum refinement level was 10, corresponding to seven refinement cycles. Approximate post-refinement cell size is shown below in Fig 6. for an arbitrary plug geometry candidate at LTO flight conditions, NPR of 7.57 and a fully extended cowl.

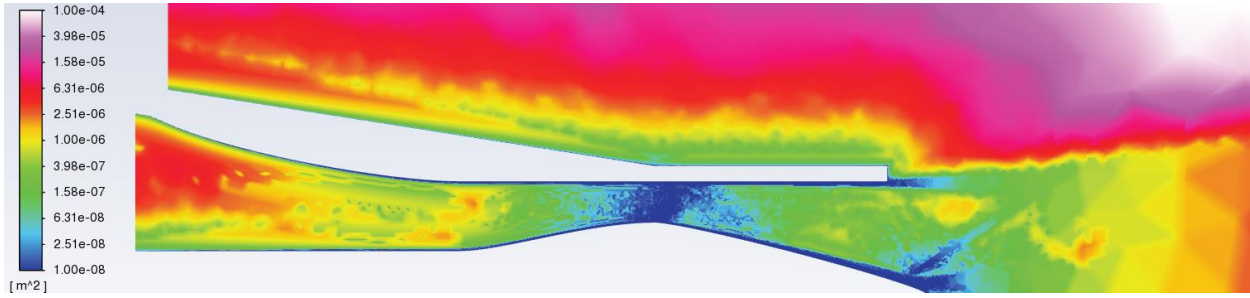


Figure 6: Cell area (log scale) for arbitrary nozzle candidate, NPR = 7.57, M = 0.01, Nominal length = 8 [in]

D. Performance Analysis

In this study the gross thrust coefficient (C_{fg}) and discharge coefficient (C_d) were the main dependent quantities of interest yielded from the CFD simulations, with greater importance given to C_{fg} . The gross thrust coefficient, which is a metric for nozzle efficiency, is defined as the ratio of the actual calculated thrust (F) to the ideal thrust (F_{ideal}) assuming isentropic flow through the nozzle. Discharge coefficient is defined as the ratio of the actual mass flow rate through the nozzle to the theoretical ideal mass flow. The expression for C_{fg} used in this study can be seen in Eq. (1), with substitutions shown in Eqs. (2) and (3).

$$C_{fg} = F/F_{ideal} \quad (1)$$

$$F = \dot{m}V_i + A_i(p_i - p_{Plenum}) - F_{Pressure} - F_{Visc} \quad (2)$$

$$F_{ideal} = \dot{m} \sqrt{2C_p T o_i \left(1 - \left(\frac{1}{NPR} \right)^{\frac{\gamma-1}{\gamma}} \right)} \quad (3)$$

Similarly, discharge coefficient is calculated using Eq. (4), substituting CFD calculated massflow ($\dot{m}$) and theoretical inviscid massflow using Eq. (5) below.

$$C_d = \dot{m}/\dot{m}_{ideal} \quad (4)$$

$$m_{ideal} = A_e P_{Plenum} NPR \sqrt{\frac{\gamma}{C_p T O_i} \left(1 + \left(\frac{\gamma - 1}{2}\right)\right)^{-\left(\frac{\gamma + 1}{2(\gamma - 1)}\right)}} \quad (5)$$

All calculations of velocity quantities yielded from CFD solutions are computed using massflow-weighted averages, while pressure quantities were computed using area-weighted averages as seen below in Eqs. (6) and (7).

$$V_i = \frac{\sum_{k=1}^n V_{cell_k} \dot{m}_{cell_k}}{\sum_{k=1}^n \dot{m}_{cell_k}} \quad (6)$$

$$P_i = \frac{\sum_{k=1}^n P_{cell_k} A_{cell_k}}{\sum_{k=1}^n A_{cell_k}} \quad (7)$$

VI. Results

Using the processes outlined above, optimizations were carried out for cruise and take off conditions. The design spaces for both contours are shown in Fig. 8 and Fig. 9. It is important to note that the test cases become more densely populated as they approach the global optima, indicated by a black "x". This toolchain operates in "runs", starting with a coarse design space, yielding results, and refining the design space accordingly. The low computational cost of the method made it unnecessary to use a mathematically rigorous optimization scheme such as gradient descent, and a less sophisticated Latin hypercube design space exploration was deemed sufficient. An optimized plug nozzle contour was achieved with geometric parameters converged to approximately 1 part in 1000 within a few days of wall time on a single Intel® Xenon® W-11855M 6-Core CPU. With the mesh density and convergence parameters used in this study, cases can be executed at a rate of roughly 30 per day. Using this outlined process, optimized plug nozzles named profiles K1b and K7 were yielded for LTO and Cruise, respectively. CFD flowfields for the optimized profiles, in addition to the LTO-cruise compromise profile, K9, can be seen below in Fig 7.

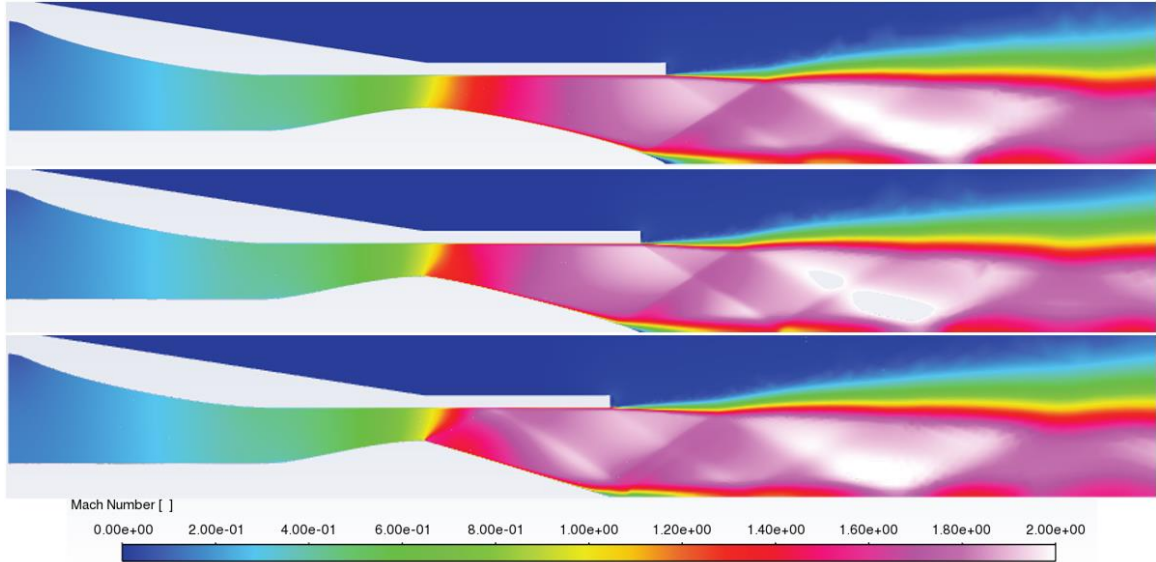


Fig. 7 Mach Number contours for optimized plug nozzle candidates at maximum NPR (7.57), LTO optimized “K1” (Top), performance tradeoff “K9” (Middle) and cruise optimized “K7” (Bottom)

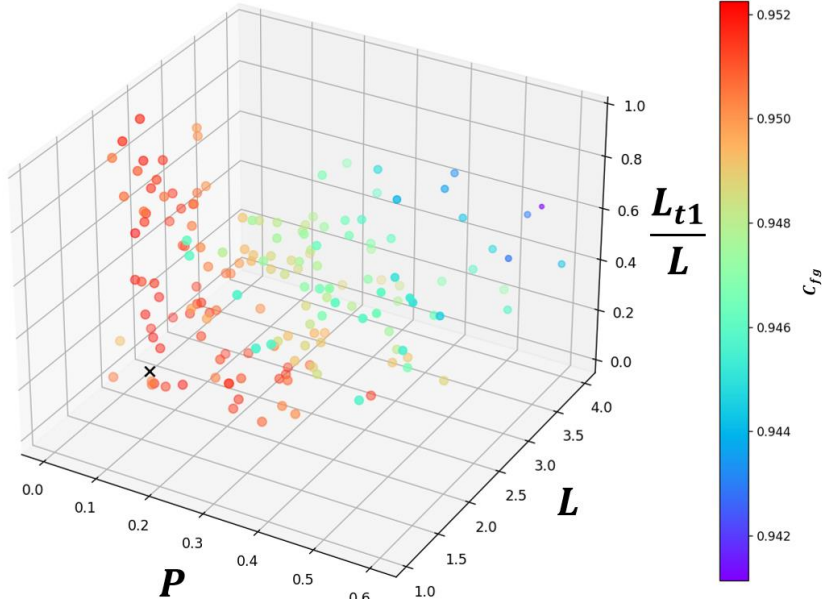


Fig. 8 Cruise optimized plug nozzle design space results, colored by C_{fg} , sized by C_d , with K7 optima marked by black "x"

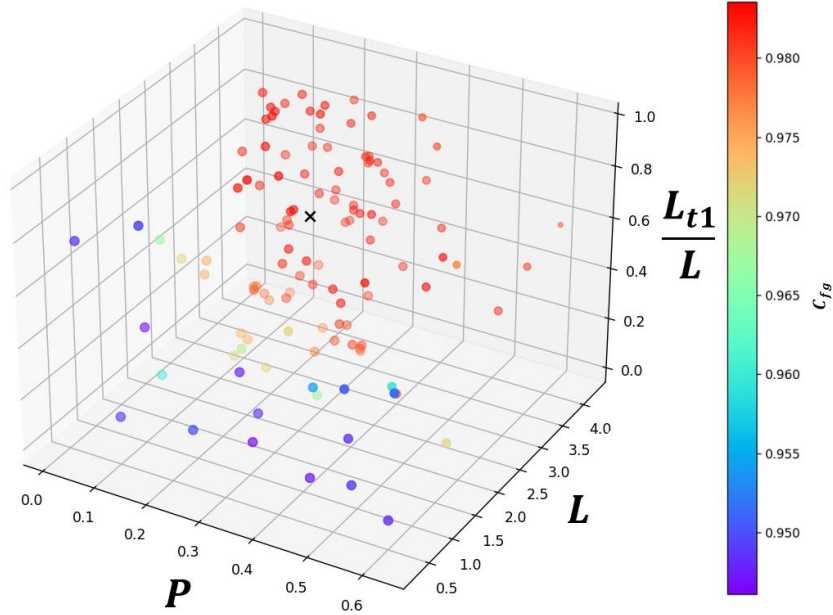


Fig. 9 LTO optimized plug nozzle design space results, colored by C_{fg} , sized by C_d , with K1b optima marked by black "x"

In practice, the chosen design will be a compromise between LTO and Cruise performance, with greater importance assigned to the cruise conditions to address range concerns. A Pareto front was constructed from the data seen in Figure 10 to choose a nozzle with good all-around performance. Figure 11 shows the LTO and Cruise performance for each design, with the theoretical optima marked by the intersection of the dashed grey LTO optima and Cruise optima lines. The minimal value of Eq. (8) yields the nozzle with the best overall performance, named K9, with the smallest Euclidian distance from the optima shown by the intersection of dashed grey lines in Figure 10.

$$D_{optimal} = \sqrt{(\Delta C_{fg_{cruise}})^2 + (\Delta C_{fg_{LTO}})^2} \quad (8)$$

Plug nozzle candidate K9 has a $\Delta C_{fg_{LTO}}$ of -0.28% from the LTO optimized candidate K1b, and a $\Delta C_{fg_{cruise}}$ of -0.20% from the cruise optimized candidate K7, indicating minimal loss in performance in either flight regime. Operating range performance was evaluated at sea-level static (SLS) for the three candidate nozzles K1b, K7 and K9, seen in Fig. 11. Additionally shown in Fig. 11, candidate K7 performs best at the higher pressure ratios encountered at cruise, while K1b performs best at the low pressure ratios of LTO operation and K9 performs well at both, but not best at either, offering good compromise. Uncertainties in CFD simulations arise from several factors including the turbulence model choice [7], grid discretization errors, boundary conditions, model assumptions (e.g., calorically perfect and chemically frozen gas states) and numerical approximations. While uncertainty quantification wasn't examined in this study, it's essential to recognize associated sensitivities of output quantities, especially concerning converged global optima.

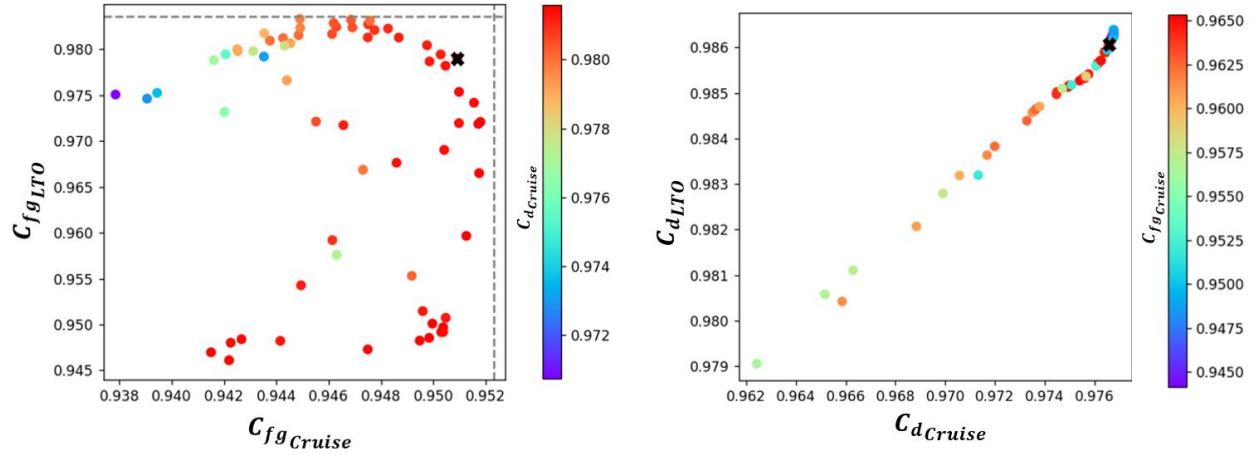


Fig. 10 LTO-Cruise C_{fg} (left) and C_d (right) performance tradeoffs Pareto front, with K9 marked by “x”

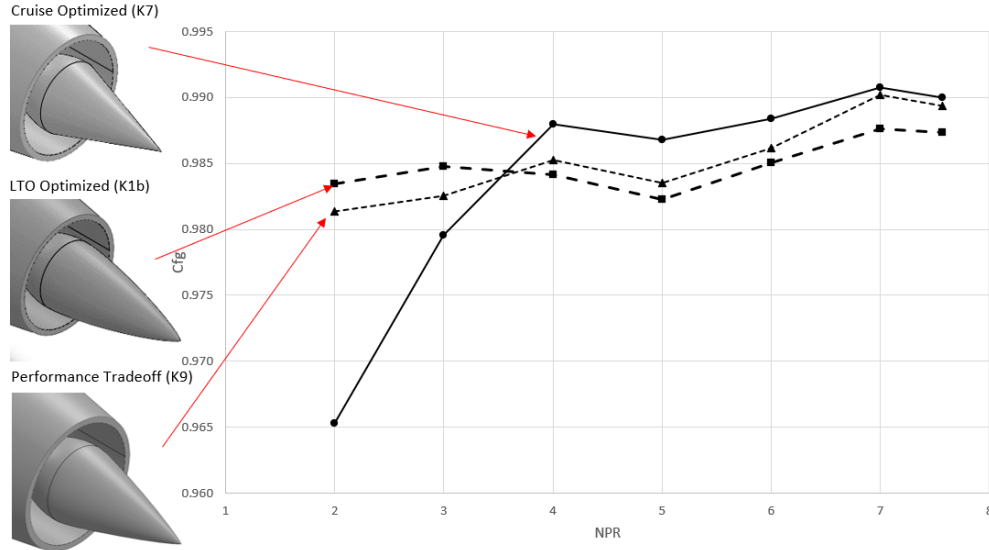


Fig. 11 SLS operating range performance for candidate nozzles, $M=0.01$

VII. Conclusion

This paper presents a method for the geometric optimization of nozzle plug contours for supersonic cruise and LTO conditions, using an axisymmetric CFD informed design space exploration. The strategy focuses on using three plug geometric parameters as independent variables, with thrust coefficient (C_{fg}) and discharge coefficient (C_d) as the primary dependent variables. The results show that the approach can effectively explore the design space and reduce computational expense, making it a viable option for the shape optimization of plug nozzle contours. Both LTO and cruise optimized plug nozzles were created using this optimization. A design that balanced performance at both cruise and LTO was also generated using this toolchain, which shows good performance over its operating range. In addition to demonstrating design optimization using the procedure described in this paper, one important finding of this work is a relatively small sensitivity of nozzle performance metrics to the design parameters under consideration, when kept within reasonable ranges. Another important consideration is the existence of numerical uncertainties that may outweigh any benefits realized here through the optimization scheme outlined utilizing relatively low fidelity tools. To validate these designs, there are preliminary plans in place at NASA Glenn Research Center to manufacture and test these optimized plugs in a small scale nozzle facility. In addition, the optimized nozzle profiles will be incorporated into future efforts aimed at evaluating acoustic performance using high-fidelity LES analysis. With the addition of experimental and LES results, profiles K7, K1b, and K9 will be thoroughly investigated to provide further validation datasets.

VIII. Acknowledgements

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IX. References

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