

Flutter and Stochastic Gust Constraint Studies of the Undeflected CRM using MPhys

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Aircraft design must ultimately address gust and flutter characteristics to achieve airworthiness. As modern aircraft continue to become more lightweight and flexible, these aeroelastic phenomena are more likely to drive the design process and should thus be considered early. Designing for ride quality due to typical gust spectra is also of interest to ensure a smooth passenger experience. The present work demonstrates the new implementation of stochastic gust constraints in MPhys/OpenMDAO, in addition to flutter and static stress constraints, among others. Design studies are shown for the NASA Common Research Model, including both fuel burn minimization as well as multiobjective optimization of fuel burn and landing gross weight. Two sets of optimization problems are covered, differing mainly by the gust quantity of interest, which include tip displacement and root bending moment. Fuel burn penalties imposed by the flutter and gust constraints are also discussed.

I. Introduction

Aeroelastic constraints have long been of vital importance to aircraft design, where high aspect ratio, lightweight wing structures tend to yield more desirable performance yet are more prone to structural failure. For instance, flutter and gust responses, in addition to static stresses, are often accounted for in modern aircraft design optimization workflows. Enforcement of flutter constraints is often done throughout a multitude of possible conditions in a flight envelope; such a comprehensive analysis is ultimately required for flight certification. Other certification requirements [1] may involve both discrete and continuous gust responses [2].

In general, discrete gust analyses may utilize time-domain loads due to a one-minus-cosine gust velocity perturbation, ensuring that structural components do not exceed material yield strengths. On the other hand, a continuous gust analysis [3] considers the stochasticity of atmospheric velocity. This is done through the use of empirical gust spectra like those of Dryden or von Kármán [2], which describe frequency content of atmospheric velocity perturbations. Such an analysis, when carried out in the frequency-domain, may provide a closed-form, probabilistic estimate of quantities such as wingtip displacement, root bending moment, or von Mises stress, which can then be used in nonlinear constraint formulations. Ride-quality metrics, in addition to certification-related metrics, may also be considered to help ensure a smoother ride for the pilots, passengers, or cargo. Such metrics may become more relevant as modern transport aircraft become more lightweight and flexible.

The present work demonstrates optimization of the NASA Common Research Model (CRM) jig shape subject to flutter and stochastic gust constraints, among other constraints such as static trim and stress. The analysis is implemented with the help of the MPhys Python library [4, 5]. MPhys is a collaborative, open-source effort, which seeks to streamline the development of multiphysics problems by offering standardized interfaces between disciplines, among other utilities. Being built on top of OpenMDAO [6], MPhys allows complex, multidisciplinary problems to be modularized.

A particular strength of MPhys is that various physics solvers or geometry parameterization methods may be easily swapped. To take advantage of this strength, the present work demonstrates aeroelastic constraints at multiple flight conditions using multiple fidelities of aerodynamics. In particular, steady responses are obtained using a Vortex Lattice Method (VLM) or the FUN3D [7–10] steady Stabilized Finite Element (FUN3D/SFE) [11, 12] solver. Either aerodynamic solver is coupled to an in-house structural solver, PyShell, using the Matching-based Extrapolation of Loads and Displacements (MELD) transfer scheme [13]. FUN3D analyses currently use Euler equations, with future plans to move to RANS, as the focus of this work is on the integration of gust constraints in MDO. However, an empirical profile drag (comprised primarily of viscous drag) is added to both the VLM and inviscid CFD aerodynamics (with VLM also including empirical wave drag) to obtain more realistic fuel burn estimates. Unsteady responses for gust

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and flutter constraints are modeled with a Double Lattice Method (DLM). For the baseline configuration, the DLM is also compared to a higher fidelity analysis, which uses the FUN3D/SFE Linearized Frequency Domain (LFD) [14, 15] solver. In this work, a particular focus will be on the gust analysis, which was recently implemented using MPhys.

This paper is organized as follows. First, the stochastic gust constraint formulation is described in Sect. II. Section III then describes the optimization problem setup for the CRM. Section IV presents baseline static aeroelastic, flutter, and gust responses. Two different gust constraints are considered: (1) the Root Mean Square (RMS) tip displacement and (2) the combined steady and unsteady root bending moment with the inclusion of rigid body modes. Section V presents three sets of optimization results: (1) single-objective optimization with tip RMS gust constraints, (2) multi-objective optimization with the same gust constraints, and (3) single-objective optimization with root bending moment gust constraints. The paper ends with conclusions and future work.

II. Gust RMS Calculation with Sensitivities

For a given frequency ω , the structural equations of motion for a gust response can be written as

$$\left(-\omega^2 [\bar{\mathbf{M}}] + i\omega [\bar{\mathbf{C}}] + [\bar{\mathbf{K}}] - q_\infty [\bar{\mathbf{Q}}_{ss}]\right) \{\boldsymbol{\eta}\} = q_\infty \{\bar{\mathbf{Q}}_{gs}\}, \quad (1)$$

where $\bar{\mathbf{M}}$, $\bar{\mathbf{C}}$, and $\bar{\mathbf{K}}$ are the $[N_{mode} \times N_{mode}]$ generalized mass, damping, and stiffness matrices, q_∞ is the freestream dynamic pressure, $\boldsymbol{\eta}$ is the generalized displacement vector, while $\bar{\mathbf{Q}}_{ss}$ and $\bar{\mathbf{Q}}_{gs}$ are the frequency-dependent structure-on-structure and gust-on-structure Generalized Aerodynamic Force (GAF) matrix and vector. Here, the GAFs are computed by DLM or LFD for $O(1-10)$ reduced frequencies and interpolated across $O(1000)$ frequencies. In addition, note that gust GAFs also depend on a given gust velocity amplitude that scales the GAFs linearly. The left and right sides can be written as

$$[\mathbf{LHS}]_{(N_{mode} \times N_{mode})} = -\omega^2 [\bar{\mathbf{M}}] + i\omega [\bar{\mathbf{C}}] + [\bar{\mathbf{K}}] - q_\infty [\bar{\mathbf{Q}}_{ss}], \quad (2)$$

and

$$\{\mathbf{RHS}\}_{(N_{mode} \times 1)} = q_\infty \{\bar{\mathbf{Q}}_{gs}\}. \quad (3)$$

The resulting generalized displacements for a given frequency, as well as its partial derivative with respect to an input x , can be written as

$$\{\boldsymbol{\eta}\} = [\mathbf{LHS}]^{-1} \{\mathbf{RHS}\}, \quad (4)$$

and

$$\partial_x \{\boldsymbol{\eta}\} = [\mathbf{LHS}]^{-1} (\partial_x \{\mathbf{RHS}\} - \partial_x [\mathbf{LHS}] \{\boldsymbol{\eta}\}). \quad (5)$$

Partial derivatives of modal displacements can thus be found using partial derivatives of the **LHS** and **RHS** with respect to the generalized structural matrices and GAFs.

The Power Spectral Density (PSD) of a gust response is then computed as

$$PSD(\omega) = |\{\mathbf{S}\}^T \{\boldsymbol{\eta}\}|^2 f_{atm}(\omega), \quad (6)$$

where $\mathbf{S}$ represents a quantity of interest induced by unit generalized displacements while $f_{atm}(\omega)$ represents a forming filter based on the frequency content of atmospheric velocity variations (in this work described by the Dryden model). In order to simplify PSD derivatives, an intermediate variable “ z ” can be defined as

$$z = |\{\mathbf{S}\}^T \{\boldsymbol{\eta}\}|, \text{ i.e. } PSD(\omega) = z^2 f_{atm}. \quad (7)$$

For inputs that only affect $\boldsymbol{\eta}$, which make up the majority of terms (except $\mathbf{S}$ and parameters in f_{atm}), the partial derivative of z can be written as

$$\partial_x z = \frac{1}{z} \left[\text{real} \left(\{\mathbf{S}\}^T \{\boldsymbol{\eta}\} \right) \text{real} \left(\{\mathbf{S}\}^T \partial_x \{\boldsymbol{\eta}\} \right) + \text{imag} \left(\{\mathbf{S}\}^T \{\boldsymbol{\eta}\} \right) \text{imag} \left(\{\mathbf{S}\}^T \partial_x \{\boldsymbol{\eta}\} \right) \right], \quad (8)$$

with the PSD partial derivative given as (where $\partial_x f_{atm}$ is zero for all inputs except the few that affect f_{atm})

$$\partial_x PSD(\omega) = 2z\partial_x z f_{atm} + z^2 \partial_x f_{atm}. \quad (9)$$

Finally, the trapezoidal method is used to integrate over the PSD to find the RMS of the gust quantity of interest (described by $\mathbf{S}$), i.e.,

$$RMS = \sqrt{\int PSD(\omega) d\omega} \quad (10)$$

$$= \sqrt{\frac{1}{2} \sum_{i=0}^{n-1} (PSD_i + PSD_{i+1}) (\omega_{i+1} - \omega_i)}. \quad (11)$$

The partial derivative of the RMS can thus be computed as

$$\partial_x RMS = \frac{1}{4RMS} \sum_{i=0}^{n-1} (\partial_x PSD_i + \partial_x PSD_{i+1}) (\omega_{i+1} - \omega_i). \quad (12)$$

A caveat is that the imaginary parts of RMS partial derivatives with respect to GAFs must then be flipped due to the lack of complex differentiability [15]. These calculations, from equations of motion to RMS of a quantity of interest, are done within an OpenMDAO component with inputs including the generalized structural matrices, GAFs, and the $\mathbf{S}$ vector (as well as flow parameters such as gust velocity).

III. Design Problem Definition

The undeflected CRM [16] is the focus of optimization demonstrations considered here, where effects of including gust and flutter constraints will be demonstrated. 1g cruise and 2.5g pull-up maneuver flight conditions are modeled; the gust and flutter conditions, when included, vary among optimization cases. While the coupled analysis model neglects the fuselage and tail, fuel and other weights take a typical mission into account. Table 1 lists some presumed parameters for weight and fuel calculations, as well as atmospheric conditions. Note that the reserve fuel mass is presumed to reside in the fuselage, while the rest of the fuel resides in the wing and is thus taken into account in the static structural analyses. The structural analyses also include a concentrated mass to model the engine and additional masses distributed along the leading and trailing edges to account for the presence of control surfaces. The modal structural analyses for flutter and gust constraints currently presume zero fuel mass in the wing; taking different fuel weights into account would likely require the structural mass matrix be handled within OpenMDAO—rather than at the solver-level, as it is currently.

Table 2 lists the objective, constraints, and design variables to be considered for the model. Overall, the parameterization includes 14 shape design variables, 720 structural sizing design variables, two aerodynamic design

Table 1 General parameters used for fuel and weight calculations; atmospheric conditions are also given.

Description	Value	Units
Design range	14,306.7	km
Specific fuel consumption	0.53	kg/(kg _f *s)
Fuel density	810	kg/m ³
Reserve fuel mass	15,000	kg
Fixed weight	1.4×10^6	N
Cruise Mach number	0.85	-
Cruise altitude	37,000	ft
Cruise fuel used	50	% of mission fuel burn
Maneuver Mach number	0.64	-
Maneuver altitude	Sea level	-
Maneuver fuel used	100	% of mission fuel burn

variables, and an additional design variable used to match fuel weights used in the structural analysis and resulting flight metrics (referred to as fuel mismatch). There are 3 equality constraints and 8-10 inequality constraints; combined with the fuel burn objective, analytic derivatives of 12-14 outputs are needed at each optimization iteration. Among the inequality constraints are flutter and gust constraints at one or two Mach numbers. Other constraints are associated with the geometry or static aeroelastic load cases. Several constraints rely on Kreisselmeier-Steinhauser (KS) aggregates [17], including those for stress, flutter, and structural geometry. Two of these affect geometry of the stiffeners, which are modeled through stiffness smearing [18]. In general, KS aggregation combines numerous quantities (such as stresses throughout a structural mesh) into a single quantity; as a result, a gradient-based optimizer only needs to compute the derivative of a single quantity of interest rather than many. There is a trade-off when setting the KS parameter, as increasing values tend to reduce conservativeness (and therefore more precisely enforce a constraint boundary), but at the cost of higher nonlinearity of the function space, which may be difficult for an optimizer to handle.

Table 3 summarizes the different optimization cases to be considered. Case 1a solves the optimization problem with VLM aerodynamics without enforcing flutter or gust constraints. Case 1b includes gust and flutter constraints at two different Mach numbers: 0.5 and 0.85. For the gust constraints, the tip z-displacement RMS values are not to exceed their baseline values. Case 2 considers the same constraints as 1b, but solves for the fuel burn (FB) versus landing gross weight (LGW) pareto front. Case 3a includes only one gust and one flutter constraint, both at Mach 0.85 cruise conditions. The gust constraint is based on the root bending moment RMS combined with the steady value, where rigid body modes are included in the modal analysis. The static load cases are computed using VLM. In case 3b, the VLM aerodynamics are replaced by those of the FUN3D Euler solver, where drag is augmented with empirical profile drag when computing fuel burn estimates.

Table 2 Objective, design variables, and constraints for the optimization problems.

	Description	Specification	Quantity
minimize:	Cruise fuel burn		
with respect to:	Root chord differential	$\in [-3, 3] \text{ m}$	1
	Tip chord differential	$\in [-1, 1] \text{ m}$	1
	Span differential	$\in [-10, 10] \text{ m}$	1
	Sweep differential	$\in [-10, 10] \text{ deg}$	1
	Wing thickness	$t_{wing} \geq 0.5 \times t_{wing,0}$	5
	Wing twist differential	$\in [-10, 10] \text{ deg}$	5
	Fuel scaler	$\in [0, 1]$	1
	Shell thickness	$\in [0.003, 0.03] \text{ m}$	240
	Stiffener thickness	$\in [0.003, 0.03] \text{ m}$	240
	Stiffener height	$\in [0.03, 0.15] \text{ m}$	240
	Static angle of attack	$\in [-10, 10] \text{ deg}$	2
	Total		746
subject to:	Fuel mismatch	$= 0$	1
	Static load factor	$\eta = [1, 2.5]^T$	2
	Wing area	$A \geq 205.8 \text{ m}^2$	1
	Available fuel mass	$\geq 63,464.28$	1
	Trailing edge spar depth	$\geq 0.19 \text{ m}$	1
	Stiffener aspect ratio	$KS_{AR} \leq 1$	1
	Stiffener vs. shell thickness	$KS_{t/t_s} \leq 1$	1
	Pull-up stress	$1.5KS_{\sigma} \leq 1$	1
	Flutter	$KS_{fI} \leq 0$	1-2
	Gust	Varies	1-2
	Total		11-13

Table 3 Summary of optimization problems, where FB and LGW refer to fuel burn and landing gross weight.

Label	Objective(s)	Steady Aerodynamics	Unsteady Aerodynamics	Flutter Constraints	Gust Constraints
1a	FB	VLM	-	0	0
1b	FB	VLM	DLM	2	2 (tip displacement)
2	FB, LGW	VLM	DLM	2	2 (tip displacement)
3a	FB	VLM	DLM	1	1 (root bending moment)
3b	FB	FUN3D Euler	DLM	1	1 (root bending moment)

IV. Baseline Aeroelastic Responses

This section investigates the aeroelastic responses of the baseline design. Section IV.A discusses the static aeroelastic solution using FUN3D Euler aerodynamics. Section IV.B then presents flutter results using DLM and FUN3D LFD aerodynamics. Section IV.C presents gust analysis results, where the tip z-displacement is the gust quantity of interest. Finally, Sect. IV.D presents gust analysis results, where the root bending moment is the gust quantity of interest. The analysis includes two rigid body modes in the pitch and plunge degrees of freedom.

A. Static aeroelastic solution

The two static aeroelastic flight conditions are evaluated by coupling the FUN3D Euler solver to the in-house PyShell structural solver. The MELD transfer scheme is used to couple the aerodynamic and structural loads and displacements. When evaluating KS stress aggregates for optimization, a KS parameter of 40 is used. Figure 1 shows pressure coefficient and normalized von Mises stresses of the baseline design at cruise conditions.

Note that later optimizations use a VLM in place of FUN3D as a lower fidelity alternative. For comparison, initial cruise lift to drag ratios using VLM and FUN3D are 18.3 and 23.2, respectively, where load factors are 0.87 and 1.01. When augmented with empirical profile drag, which is done to produce more realistic fuel burn values, the FUN3D lift to drag ratio is 17.6. With VLM deflection being slightly less than FUN3D (1.58 versus 1.87 m), the KS stress aggregates are 0.43 and 0.46 for VLM and FUN3D. Meanwhile, the respective root bending moment values are 8.51×10^6 and 1.07×10^7 N-m for VLM and FUN3D.

B. Flutter response

The present p - k flutter analysis procedure was previously detailed in Jacobson *et al.* [14] and has been used for gradient-based optimization with DLM and LFD aerodynamics [15] in OpenMDAO/MPhys. When used for optimization, a KS aggregate of modal damping is used to avoid any damping values within a “keep-out” zone in the dynamic pressure-damping space. The KS parameter value is chosen to be 200. Figure 2 shows the damping and frequency of

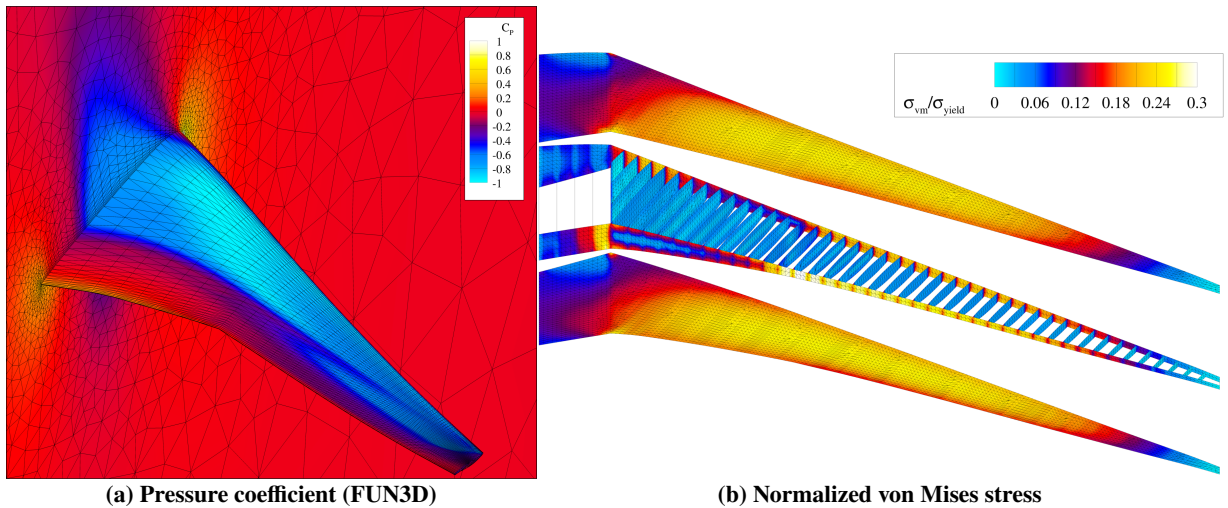


Fig. 1 Baseline static aeroelastic solution at cruise conditions using FUN3D Euler aerodynamics.

the baseline design at a Mach 0.85 cruise condition. Results are shown for both DLM and Euler LFD (1 degree angle of attack). The two fidelities agree closely for this case, likely because the transonic dip often seen with CFD-based aerodynamics is slightly higher than 0.85.

To demonstrate the availability of analytic derivatives, Fig. 3 shows derivatives [19] of the flutter KS aggregate with respect to structural thicknesses. Here, the flutter KS, which should not exceed zero, is reduced when the ribs and trailing edge spar faces near the Yehudi break are made thicker. Also seen are regions of positive derivatives, for instance at the outboard lower skins; increases of mass and stiffness in these regions in fact make the wing less dynamically stable.

C. Tip RMS gust response

To demonstrate the current gust analysis implementation in MPhys, the baseline design DLM-based gust solution is explored, where tip z-displacement serves as the first gust quantity of interest. Mach number, velocity, density, design gust velocity, and gust turbulence scale are set to 0.85, 254 m/s, 0.4 kg/m³, 25.9 m/s, and 762 m, respectively. Figure 4 shows the resulting generalized displacement solutions (i.e., η in Eq. 1) for a frequency range of 0 to 30 Hz. For reference, the $\mathbf{S}$ vector for tip z-displacement (i.e., z-displacement induced at a tip node due to unit generalized displacement) is $[0.0376, -0.0029, 0.0392, 0.0305, 0.0173, 0.0467, 0.0227, -0.0243, 0.0564]^T$.

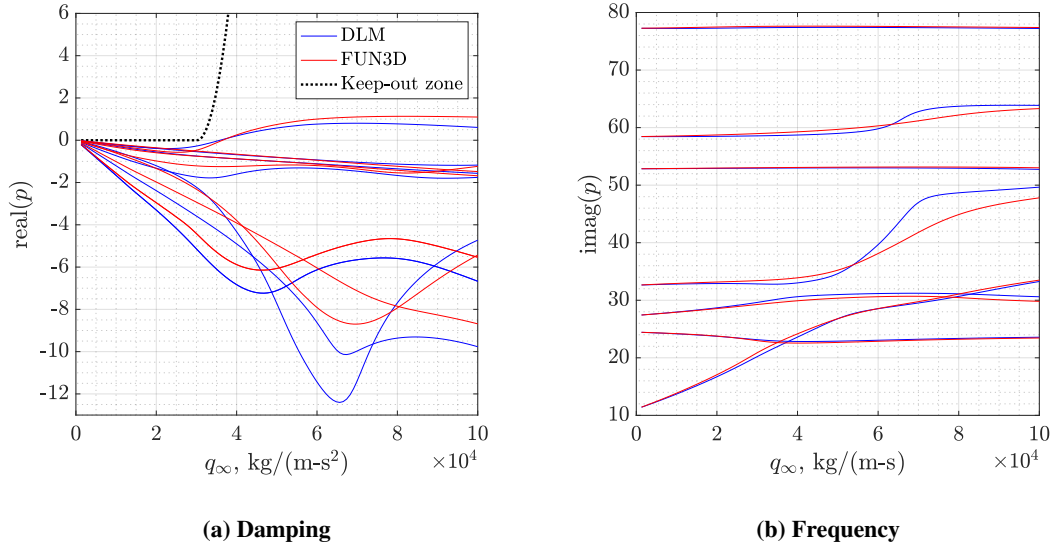


Fig. 2 p - k flutter results for the baseline design (Mach 0.85). While only seven modes are shown, nine are used in later optimizations.

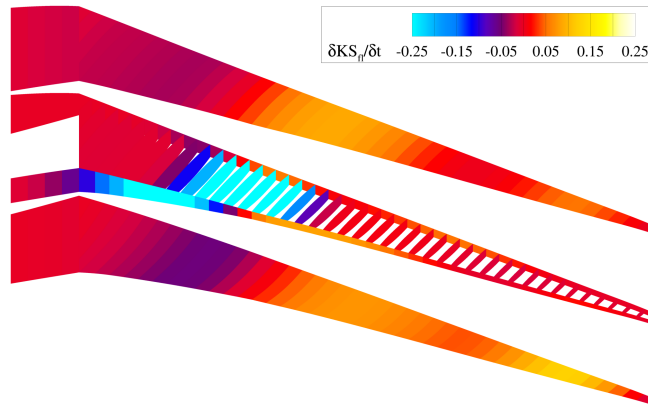
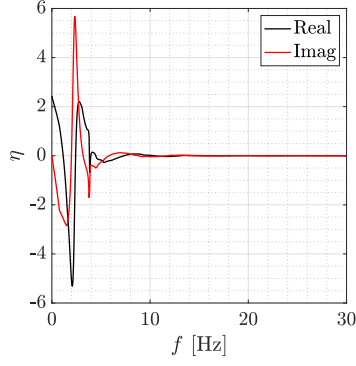
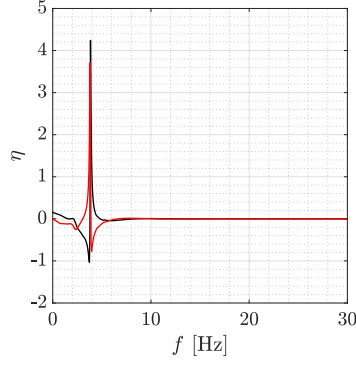


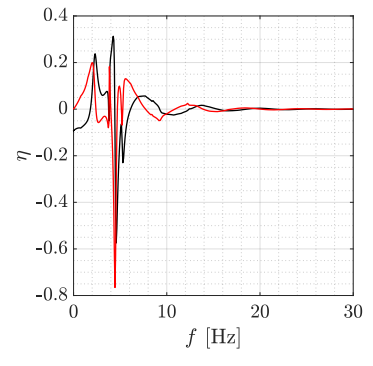
Fig. 3 Sample flutter KS derivative contours with respect to structural thickness design variables (baseline design, Mach 0.85).



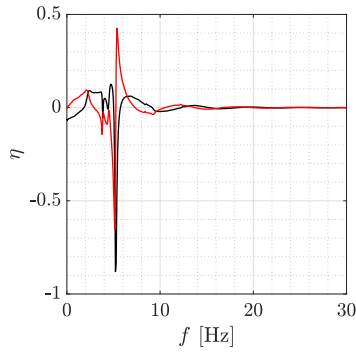
(a) Mode 1



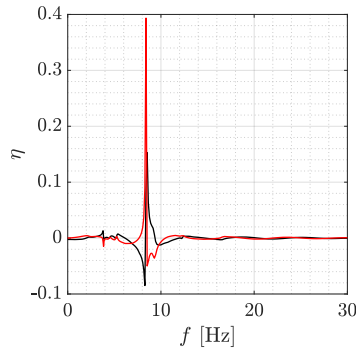
(b) Mode 2



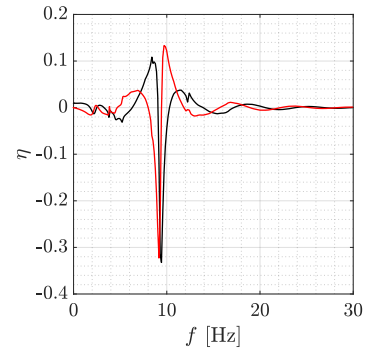
(c) Mode 3



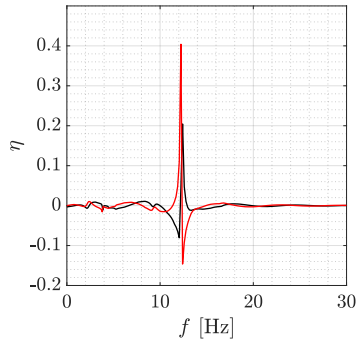
(d) Mode 4



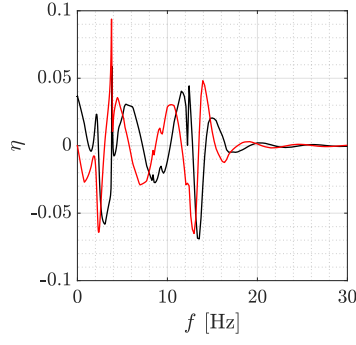
(e) Mode 5



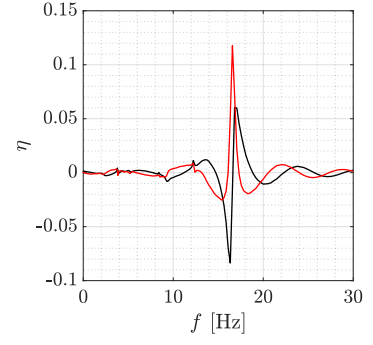
(f) Mode 6



(g) Mode 7



(h) Mode 8



(i) Mode 9

Fig. 4 Generalized displacement solutions across the frequency range.

Figure 5 shows the resulting squared product of $\mathbf{S}$ and $\boldsymbol{\eta}$ (black), the Dryden gust spectrum (red), and the tip displacement PSD (blue). The resulting tip displacement RMS to be used in a design constraint is roughly 2.17 m. Also shown in this figure are the gust RMS contours at each point on the structure; these contours illustrate that the RMS is largely driven by quasisteady deformations of the first mode.

Figure 6 shows gust PSD and wingtip RMS as Mach number is varied while sound speed and dynamic pressure are held fixed. Here, note that small magnitude responses at higher frequencies may be sensitive to the reduced frequencies selected (in combination with linearly interpolated GAFs across the frequency range). The gust RMS may in turn vary by a small amount.

An interesting feature of the gust analysis is that, when flow conditions are set to those of flutter onset, the PSD has a peak that approaches infinity at the flutter frequency. Figure 7 shows this phenomenon. It is unclear, however, why the response is at its maximum at flutter onset, with the peak in the PSD shrinking as the dynamic pressure is further increased. An example of this is shown by the red “post-flutter” curve, which uses the same density as the blue flutter onset result but with velocity increased to 300 m/s. Finally, to illustrate the availability of analytic gust RMS derivatives, Fig. 8 shows derivative contours with respect to shell thicknesses on the wingbox.

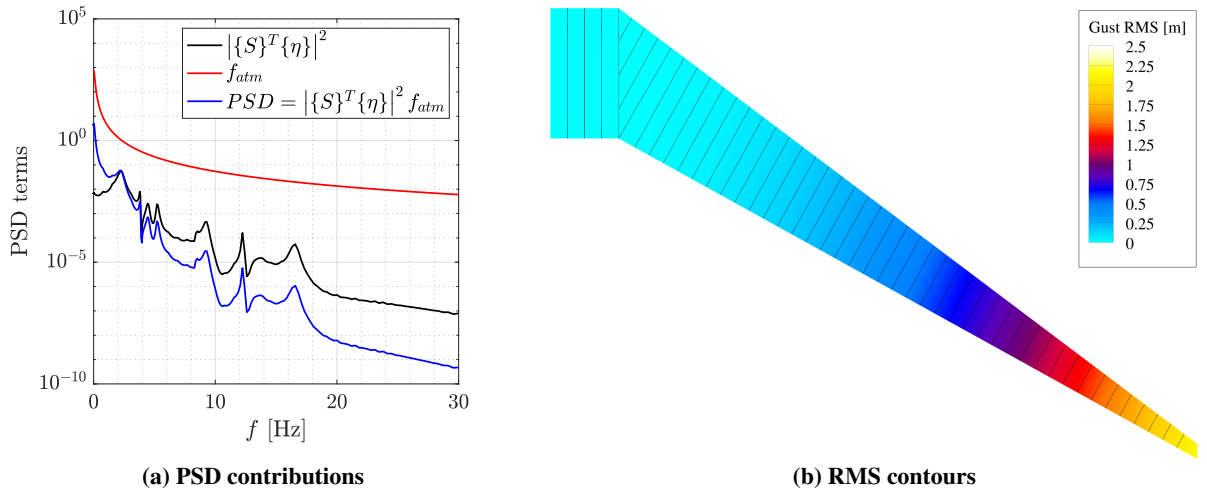


Fig. 5 PSD contributions by Dryden gust spectrum and equations of motion at Mach 0.85.

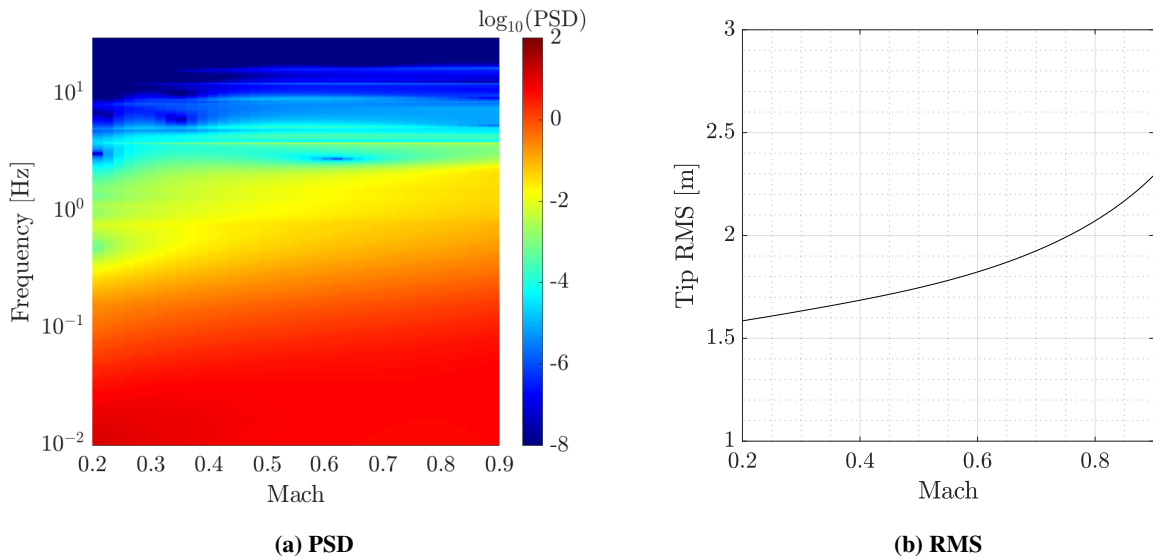


Fig. 6 Variation of gust PSD and RMS across Mach number.

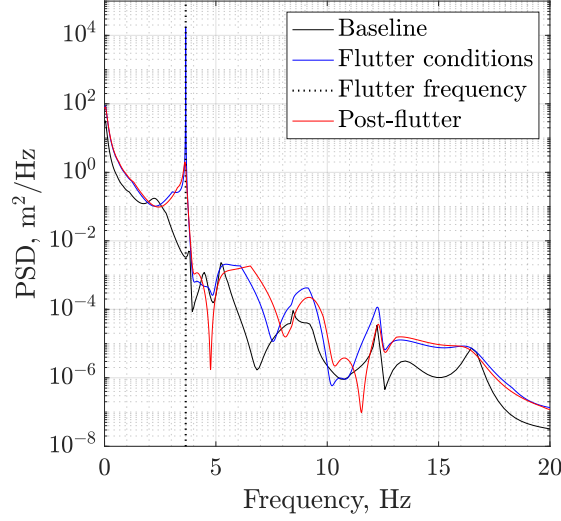


Fig. 7 Gust response at flutter conditions ($\rho = 0.9178 \text{ kg/m}^3$, $v = 279.6 \text{ m/s}$) compared to baseline conditions ($\rho = 0.4 \text{ kg/m}^3$, $v = 254 \text{ m/s}$). The post-flutter result uses the flutter onset density with a higher velocity of 300 m/s.

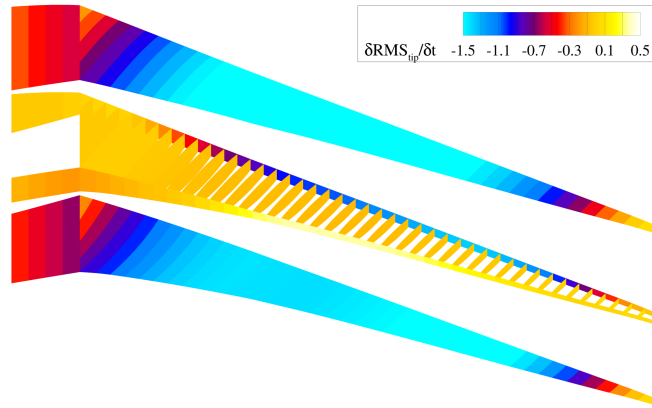


Fig. 8 Sample gust RMS derivative contours with respect to structural thickness design variables (baseline design, Mach 0.85).

D. Root bending moment gust response

The baseline gust response is next observed for a case where root bending moment is the gust quantity of interest. Pure pitch and plunge rigid body modes are included to more realistically model actual flight conditions. Shown in Fig. 9 is the resulting PSD of root bending moment using DLM aerodynamics. For comparison with higher fidelity aerodynamics, the LFD [20] PSD about the undeflected and statically deflected geometries are also included.

As observed in the PSD, the RMS of the root bending moment, when including rigid body modes rather than modeling a clamped wing, is driven by a peak in the PSD at around 0.5 Hz. This is in contrast to the previous tip displacement case, which is largely driven by the zero-frequency response. As shown in Fig. 10, this is because zero-frequency modal displacements for structural modes are now roughly zero. Those of the rigid body modes are nonzero, which contributes no root bending moment (i.e., the first two terms of $\mathbf{S}$ are now zero).

When comparing DLM to Euler LFD, both aerodynamic fidelities demonstrate similar trends in the PSD. The peak in the PSD, however, is significantly higher with LFD (with undeflected LFD having a higher peak than deflected LFD,

presumably due to having a stronger shock). Due to this difference, the resulting LFD RMS values are 1.07×10^7 and 1.21×10^7 N-m with and without deflection, compared to 7.43×10^6 N-m with DLM. Note, however, that the FUN3D angle of attack here is 1 degree, and a value of 0 could yield smaller RMS values. The DLM gust response, on the other hand, is of course independent of angle of attack.

Figure 11 demonstrates the availability of analytic derivatives of the gust response. When using this gust response in optimization, the steady root bending moment (without rigid body modes) is added to the unsteady RMS value. This figure therefore shows steady and combined derivatives as well. Note that while the use of root bending moment in optimization is believed to be more meaningful than tip displacement, the authors ultimately plan to constrain stresses instead, which would more directly prevent material yield stresses from being encountered. This will require significantly more effort to implement in MPhys/OpenMDAO, as Rational Function Approximation (RFA) must be used to obtain accurate elemental stresses from the modal structural representation.

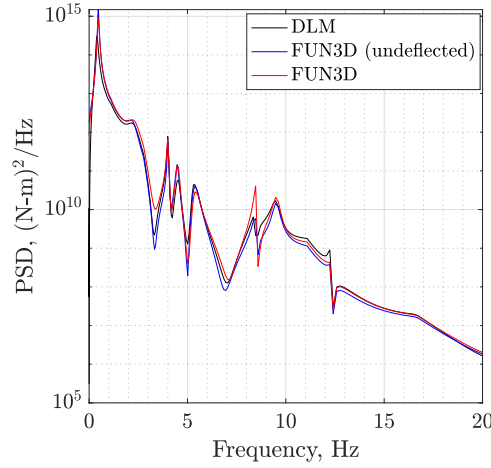


Fig. 9 PSD of the root bending moment at cruise conditions.

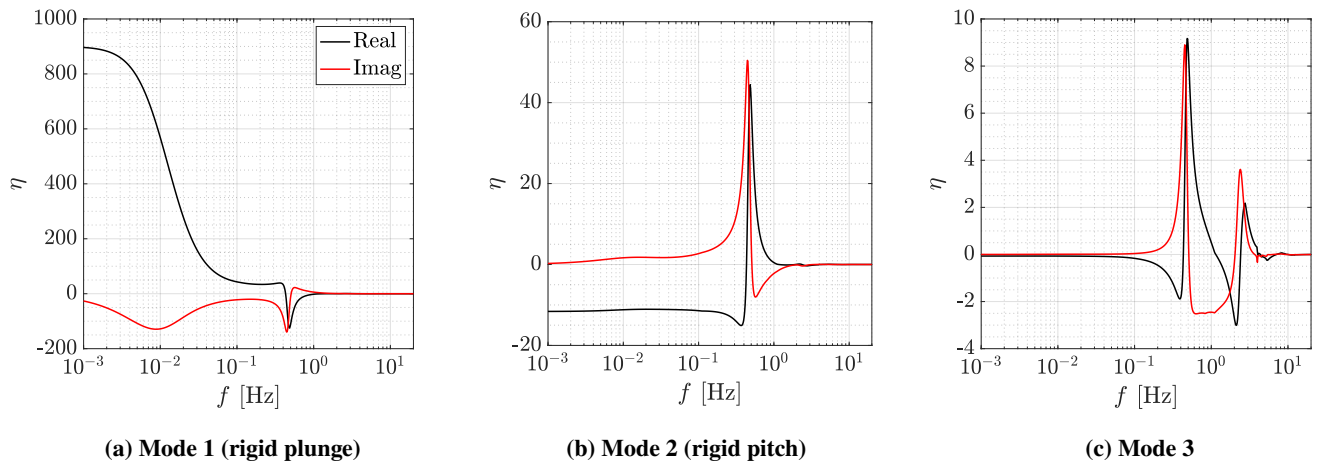


Fig. 10 Modal displacements corresponding to root bending moment with rigid body modes (first three modes out of nine).

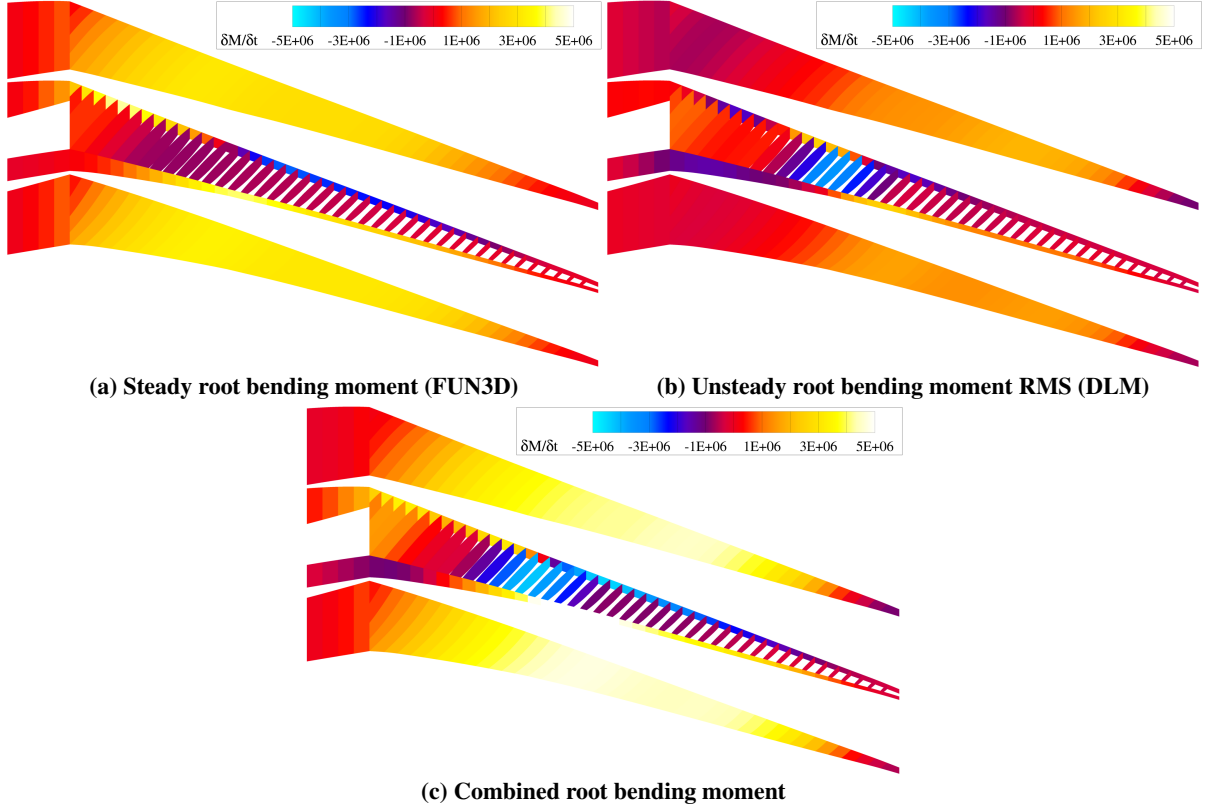


Fig. 11 Sample root bending moment derivative contours with respect to structural thickness design variables (baseline design, Mach 0.85).

V. Optimization Results

This section presents several sets of optimization results as summarized by Table 3. In Sect. V.A, single-objective optimization with and without tip RMS gust constraints (cases 1a and 1b in Table 3) is presented. In Sect. V.B, multi-objective optimization (case 2) is performed to investigate the trade-offs between fuel burn and landing gross weight objectives. Finally, Sect. V.C presents results using root bending moment gust constraints with the inclusion of rigid body modes (cases 3a-3d). The first two sets of optimization are limited to panel-based aerodynamics, while the third includes FUN3D aerodynamics as well.

A. Cases 1a-1b: Design problem with tip RMS gust constraints

Figure 12 shows Sequential Least Squares Quadratic Programming (SLSQP) optimization results using the objective, design variables, and constraints listed in Table 2. The gust and flutter constraints enforced at Mach 0.85 and 0.5 use the same reduced frequency array (11 values from 0 to 2) and number of modes (9). For the gust constraints, the maximum imposed tip RMS was taken from the Mach sweep shown in Fig. 6. For the flutter KS constraints, the dive speed, flutter margin, and KS parameter are taken to be 185 m/s, 1.2, and 200, respectively.

Overall, convergence is achieved in roughly 250 iterations (249 and 247 function and gradient evaluations, respectively), with the maximum constraint value reaching roughly 1×10^{-5} . The fuel burn converges to roughly 108,000 kg, with the flutter and gust constraints imposing roughly a 900 kg penalty. An optimality metric, which makes use of the OpenMDAO recorder with derivatives saved, is reduced by close to four orders of magnitude. Because the SLSQP optimization algorithm in the OpenMDAO SciPy driver does not provide an optimality metric, this metric uses a nonnegative least squares algorithm [21] to compute Lagrange multipliers, with side and nonlinear constraints greater than -0.01 (negative being feasible) treated as active. Notably, the flutter and gust constraints are inactive at Mach 0.85 but active at Mach 0.5. Note, however, that the use of higher fidelity LFD in the future is expected to yield a more active transonic flutter (as well as gust) constraint. The constraints on stiffener aspect ratio and available fuel mass are also inactive, while all others are active.

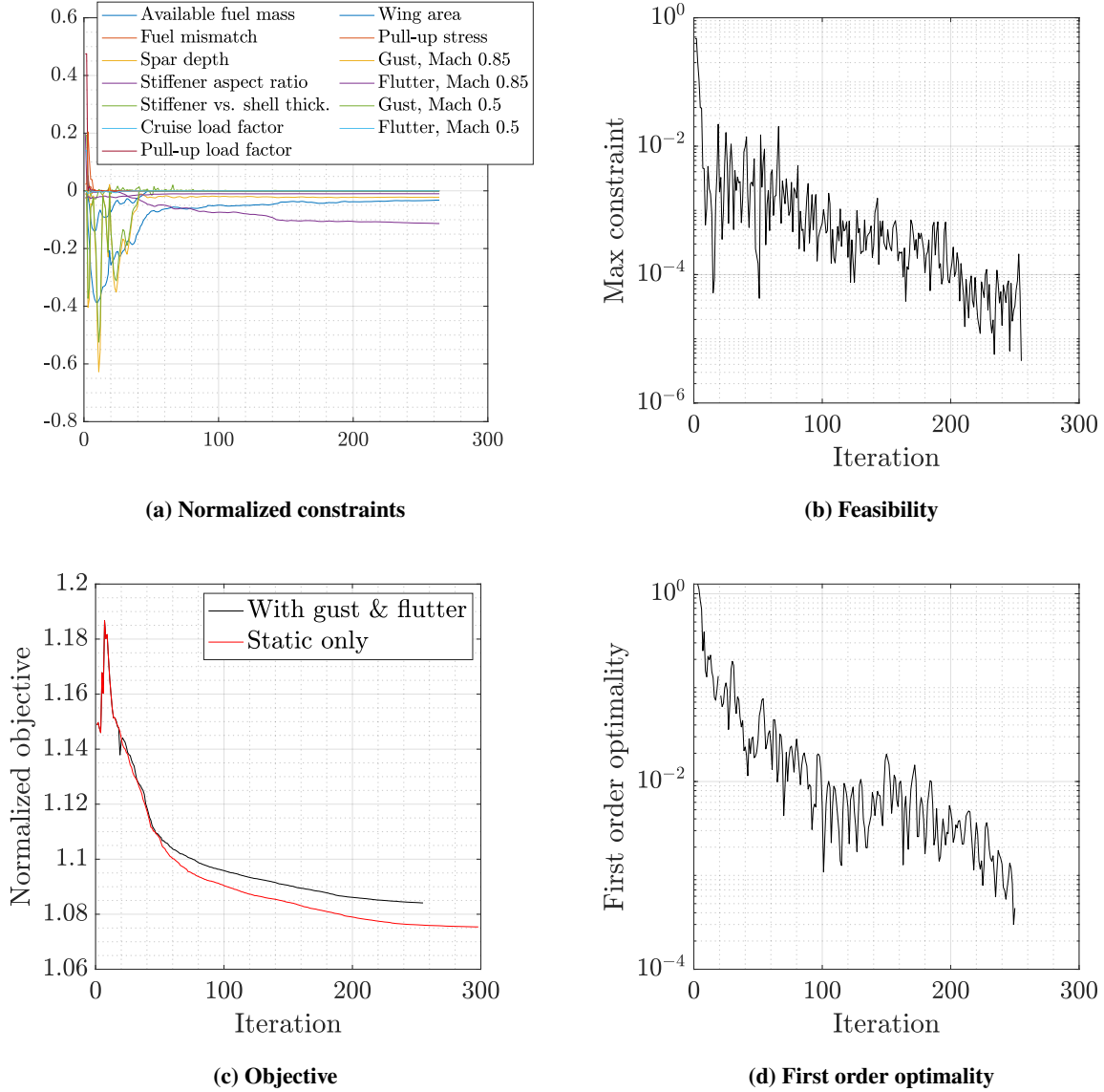


Fig. 12 Optimization convergence for the CRM.

B. Case 2: Multi-objective optimization with tip RMS gust constraints

Figure 13 shows multiobjective optimization results with fuel burn and landing gross weight acting as the competing objectives. The fuel-burn-only objective shown previously yields a 13.7% LGW penalty when compared to the LGW-only objective. Perhaps surprisingly, the latter yields a 63% fuel burn penalty when compared to the fuel-burn-only case. For this LGW-only case, a lightweight, short, aerodynamically-inefficient wing was obtained. All flutter and gust constraints (as well as those on available fuel mass and stiffener aspect ratio) are inactive at this low aspect ratio design. This is not surprising for the gust constraint, with the tip RMS likely being closely tied with wing span. It is also intuitive that the flutter response would also be more stable for shorter, low aspect ratio wings.

C. Cases 3a-3d: Design problem with root bending moment gust constraints

Optimizations with a root bending moment constraint are next demonstrated. For these new constraints, the steady cruise value (obtained using either VLM or FUN3D) is combined with the unsteady RMS value from DLM (with

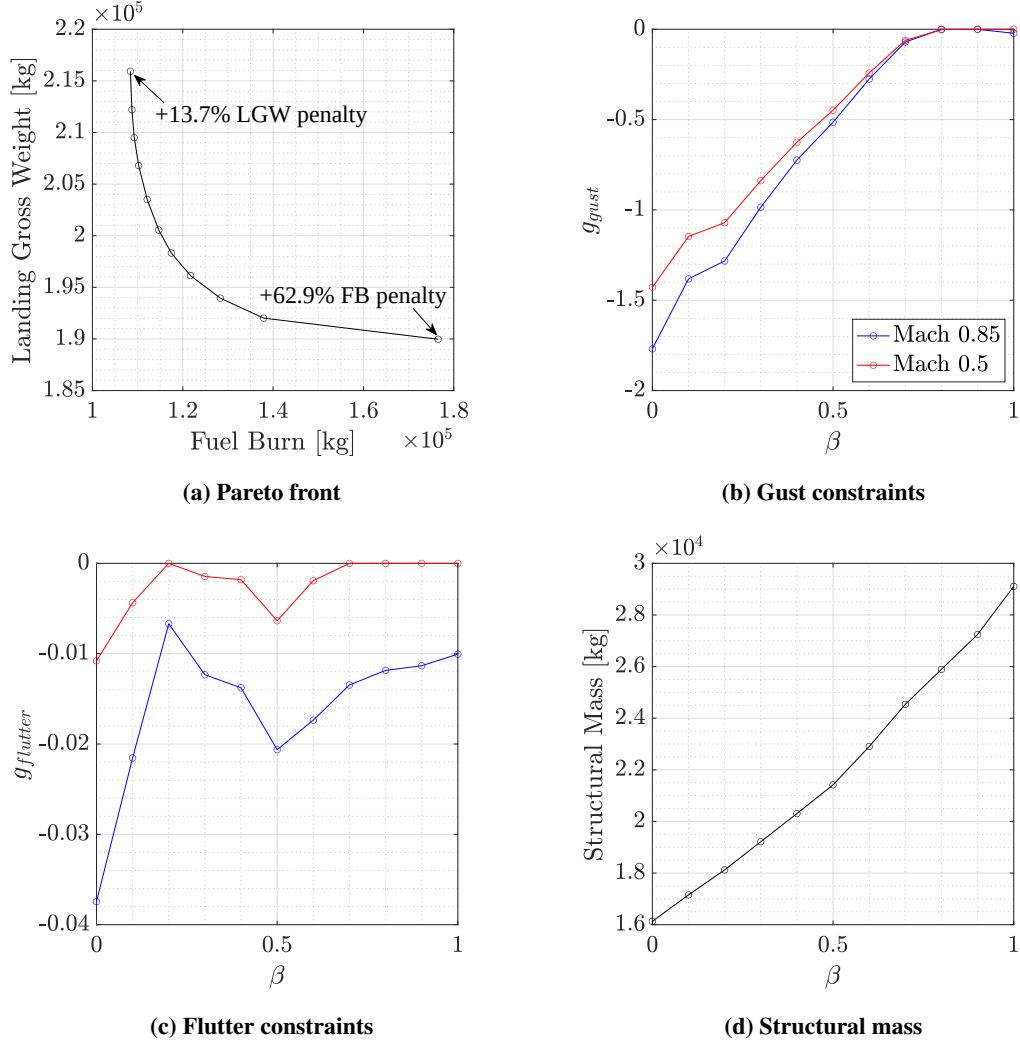


Fig. 13 Multi-objective optimization between fuel burn ($\beta = 1$) and landing gross weight ($\beta = 0$).

future plans to use LFD). For flutter, only the Mach 0.85 constraint is retained while the Mach 0.5 one is discarded. With these modified gust and flutter constraints, all other design variables and constraints are the same as the previous optimizations.

For the gust constraint, a somewhat arbitrary upper bound of 1.6×10^7 N-m is chosen. This was done based on steady root bending moment and KS stress aggregate values at cruise and pull-up of the baseline design (with VLM providing steady aerodynamic loads). At cruise conditions (untrimmed at a 1 degree angle of attack), root bending moment and stress KS values are 8.51×10^6 N-m and 0.43, respectively. Corresponding values at pull-up (untrimmed at a 4 degree angle of attack) are 1.9×10^7 N-m and 0.65, respectively. By assuming linear proportionality of stress KS and root bending moment, estimated root bending moment values corresponding to a KS value of 1 are 2×10^7 and 3×10^7 N-m. The average between the two flight conditions yields an estimated upper limit of 2.5×10^7 N-m, and a 1.5 factor of safety brings this value down to roughly 1.6×10^7 N-m.

For reference, the baseline design when using VLM and DLM yields a combined root bending moment of roughly the chosen limit of 1.6×10^7 N-m. The same operation with steady FUN3D aerodynamics would yield 2.47×10^7 N-m and 2.63×10^7 N-m limits for cruise and pull-up, also yielding roughly a 1.6×10^7 N-m constraint upper bound. However, with the LFD RMS being 1.07×10^7 N-m (roughly the same as the steady value) compared to 7.43×10^6 N-m with DLM, the DLM-based constraint is nonconservative. Nonetheless, while rather arbitrary when compared to actual stresses, implementation of this constraint is seen as an intermediate step towards enforcing stresses directly.

These optimizations were run using the SLSQP algorithm in pyOptSparse [22], which allowed optimizations to be restarted as needed using hotstart files. In addition, remote components, recently added to the MPhys library, were also used to help lessen the time needed for optimizations. These are OpenMDAO explicit components that allow a smaller optimization on a login node to launch HPC jobs that evaluate large MPhys/OpenMDAO groups as needed. This is done using pbs4py* for HPC job management and ZeroMQ [23] for network communication with the remote group on the HPC job. By keeping track of function and gradient wall times, the component can determine whether to evaluate a design on the current job or to launch a new HPC job. The cruise, pull-up, and DLM MPhys analyses were each evaluated on separate HPC jobs using separate remote components. OpenMDAO parallel groups and parallel derivative coloring was used so that the three remote components evaluate function and derivative information in parallel. Overall, this allowed each optimization to be run on three processors on the login node, continuously in the background for longer than the maximum job time and using more than the maximum allowable nodes per job.

Figure 14 shows convergence metrics for the present optimization problem, including the objective function, maximum constraint violation, and first-order optimality. The steady, unsteady RMS, and combined root bending

* Available at <https://github.com/nasa/pbs4py>

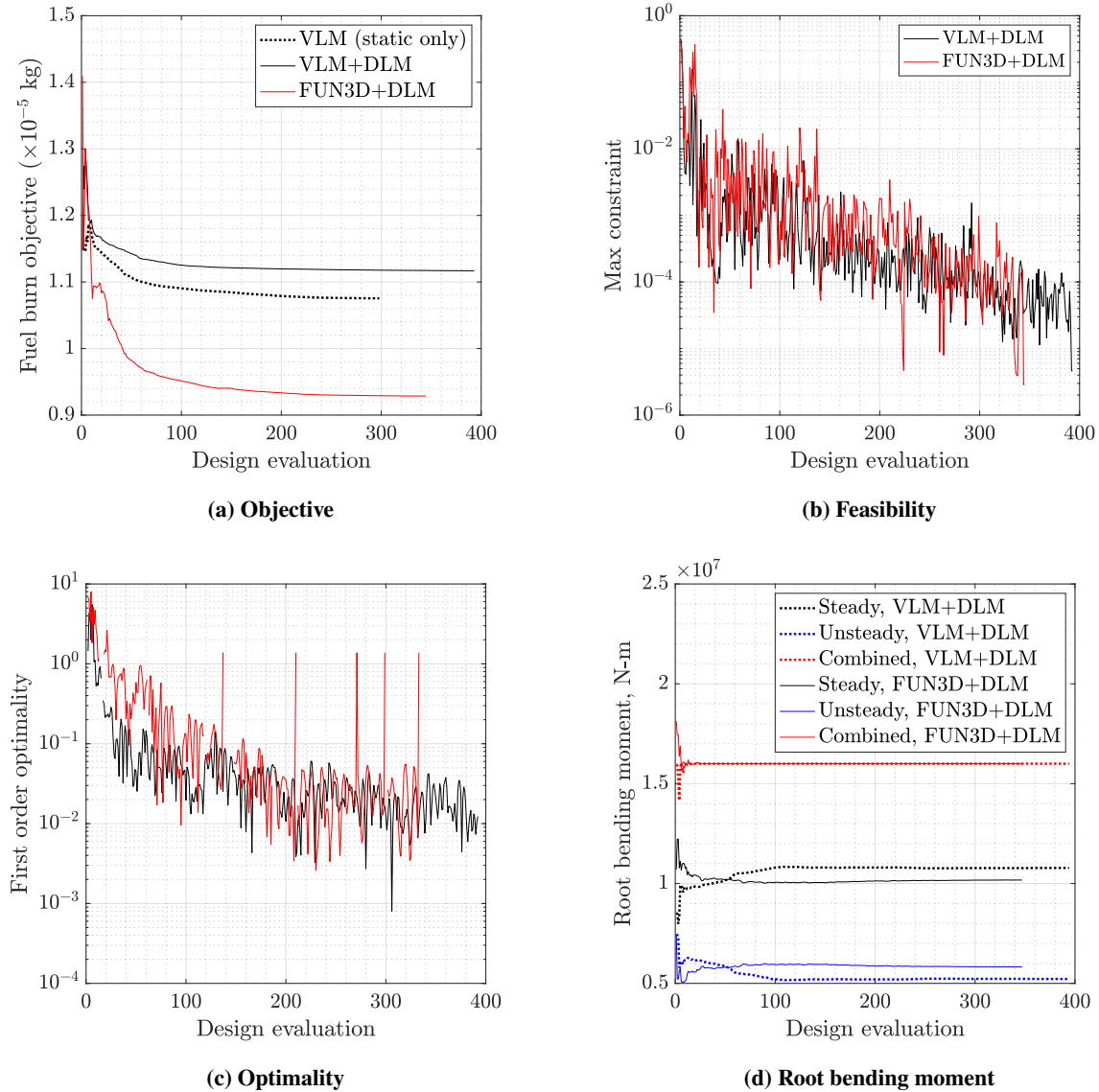


Fig. 14 Optimization convergence for the CRM.

moments are also provided. In the first three plots, solid black and red lines use VLM and FUN3D Euler CFD (augmented with empirical profile drag), respectively, for the static aerodynamic loads. The VLM-based case that excluded flutter and gust constraints (dotted black), shown previously in Sect. V.A, is shown for reference.

The present VLM-based case (solid black) yielded a final fuel burn of roughly 111,600 kg, with the gust and flutter constraints imposing a fuel burn penalty of roughly 4,100 kg. The case took the most iterations, requiring 392 and 390 function and gradient calls, respectively. All constraints except stiffener aspect ratio and available fuel mass are active. The FUN3D case with drag augmented to account for viscous drag (solid red) yielded a fuel burn of 92,900 kg; a comparable case without flutter and gust constraints has not yet been carried out.

As suggested by the optimality metric, this particular case had a few problematic iterations (for example near designs 140 and 200), where the optimizer evaluated several line search iterations. This did not occur in a separate FUN3D case without profile drag (not shown), and is presumably due to inadequate convergence of the flow adjoint solve, which could likely be resolved by altering FUN3D settings. Apart from these iterations, function to gradient evaluation ratio was otherwise close to one, like in the other cases. Overall, all cases satisfied constraints to within roughly 1×10^{-5} . In the cases with the root bending moment constraint, steady values remained similar or increased while the unsteady RMS component decreased. Finally, Fig. 15 shows the optimized planforms for each case. In general, inclusion of gust and flutter constraints yields a shorter and more swept wing. The FUN3D-based optimization yielded a more highly swept wing than VLM, with the sweep, root chord, and tip chord design variables all reaching their upper bounds.

VI. Conclusion

Optimization of the CRM has been demonstrated with over 700 design variables and over 10 constraints. The particular focus was dynamic aeroelastic constraints for flutter and stochastic gust responses. With the help of the OpenMDAO-based MPhys library, aerodynamic fidelity was easily swapped between panel- and CFD-based aerodynamic solvers while using the same structural solver. Single- and multi-objective optimization was demonstrated with and without the flutter and gust constraints using different aerodynamic fidelity levels. LFD-based flutter and gust responses of the baseline design were also shown, although this higher fidelity of unsteady aerodynamics has yet to be used for full CRM optimizations.

The authors plan to improve upon several aspects of the optimization problems. On the aerodynamic side, the use of Reynolds-Averaged Navier-Stokes (RANS) equations is planned to more accurately account for viscous drag. Obtaining the unsteady aerodynamic responses (for both gust and flutter) from the LFD solver at transonic Mach numbers, in addition to DLM at subsonic Mach numbers, is also planned. Modeling the fuselage and tail aerodynamically is another improvement planned, and would likely introduce challenges in geometry parameterization as well as displacement and load transfers at the wing-fuselage junction. On the structural side, enforcement of stresses (rather than root bending moment or tip displacement) due to atmospheric gusts is also planned. Other areas with room for growth are inclusion of control surfaces for Gust Load Alleviation (GLA) as well as the addition of more flutter constraints throughout the flight envelope.

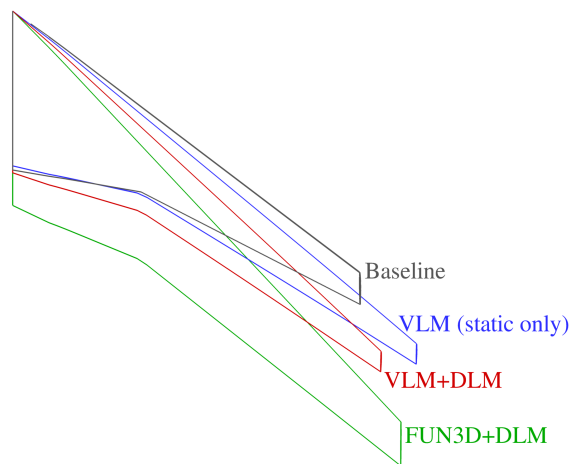


Fig. 15 Baseline and optimized planforms.

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