

Aeroacoustic Analysis using Dynamic Mode Decomposition of Unsteady Pressure-Sensitive Paint Measurements

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In this paper, we present a new method for the diagnosis and analysis of aeroacoustic phenomena. Taking advantage of the well-known property that the measurements of the Unsteady Pressure-Sensitive Paint (uPSP) have much higher spatial resolution compared to those of the conventional pressure transducers, the method is based on the visualization and analysis of the outputs of the Dynamic Mode Decomposition (DMD) of the uPSP measurements. The uPSP measurements were collected with four Phantom high-speed cameras in the Ascent Transient Aerodynamics Test (ATAT) of the Space Launch System (SLS) Block 1 cargo vehicle in the 11-by-11-foot transonic test section of the Unitary Plan Wind Tunnel (UPWT) at NASA Ames Research Center in September 2019. The method presented in this paper is demonstrated by investigating an interesting phenomenon observed in the SLS ATAT – the vortex shedding tone generated by the forward attachment of the Solid Rocket Booster (SRB). As examples, the DMD outputs of the uPSP measurements in a subsonic test in the SLS ATAT are presented. It is shown the information retrieved from the DMD outputs of the uPSP measurements can be effectively used in the identification and diagnosis of the aeroacoustic phenomena. The work described in this paper is a part of NASA’s development of a new state-of-the-art uPSP capability in production wind tunnels. Funding was provided by the NASA Aerosciences Evaluation and Test Capabilities Portfolio Office.

Nomenclature

AETC	= NASA Aerosciences Evaluation and Test Capabilities Portfolio Office
ATAT	= Ascent Transient Aerodynamics Test
c	= speed of sound (ft/s)
D_A	= distance sound travels in a period of sinusoidal acoustic signal (ft)
DMD	= Dynamic Mode Decomposition
DFT	= Discrete Fourier Transform
d_{LV}	= distance launch vehicle travels in a period of sinusoidal acoustic signal (ft)
$d_{upstream}$	= offset of linear model (deg)
E_a, e_a	= vector/scalar of error term(s) of linear model (deg)
FFT	= Fast Fourier Transform
f_A	= frequency of sinusoidal acoustic signal (Hz)
$f_{A,m}$	= measured frequency of acoustic wave by observer (Hz)
f_S	= sampling frequency of the uPSP measurements (Hz)
$f_{VortexShedding}$	= vortex shedding frequency (Hz)

H_a	= data matrix of linear model
$k_{upstream}, k_{downstream}$	= slopes of linear model (deg/inch)
M	= Mach number
N_a	= number of selected grid nodes
P_{Ea}	= variance matrix of vector of error terms of linear model
PSD	= Power Spectral Density
$P_{\hat{\theta},upstream}$	= variance matrix of vector of estimated parameters of linear model
R	= specific gas constant of air ($R = 1716.46 \text{ ft} \cdot \text{lb} / \text{slug} \cdot ^\circ\text{R}$)
SLS	= Space Launch System
SPOD	= Spectral Proper Orthogonal Decomposition
SRB	= Solid Rocket Booster
SVD	= Singular Value Decomposition
T_A	= period of sinusoidal acoustic signal (s)
$T_{static,^\circ R}$	= static temperature of free stream flow ($^\circ\text{R}$)
$T_{total,^\circ F}, T_{total,^\circ R}$	= total temperature of free stream flow ($^\circ\text{F} / ^\circ\text{R}$)
t	= instant when the sinusoidal acoustic signal is at the peak (s)
$U_{A,m}$	= measured speed of acoustic wave by observer
UPWT	= NASA Unitary Plan Wind Tunnel
$U_{upstream}, U_{downstream}$	= speeds of waves traveling in upstream/downstream directions (ft/s)
U_∞	= speed of free stream flow (ft/s)
uPSP	= Unsteady Pressure-Sensitive Paint
v_{LV}	= speed of launch vehicle (ft/s)
x_a, x_b	= x coordinates of grid nodes (inch)
γ	= ratio of specific heats of air ($\gamma = 1.4$)
$\Delta f_{vortexShedding}$	= uncertainty of estimated vortex shedding frequency (Hz)
$\Delta k_{upstream}, \Delta k_{downstream}$	= uncertainties of estimated slopes of linear model (deg/inch)
$\Delta U_{upstream}, \Delta U_{downstream}$	= uncertainties of estimated speeds of waves (ft)
$\Delta \lambda_{upstream}, \Delta \lambda_{downstream}$	= uncertainties of estimated wavelengths of waves (ft)
$\theta_{upstream}$	= vector of parameters of linear model
$\lambda_{A,m}$	= measured wavelength of acoustic wave by observer (ft)
$\lambda_{upstream}, \lambda_{downstream}$	= wavelengths of waves traveling in upstream/downstream directions(ft)
σ	= standard deviation of error term of linear model (deg)
Φ_a, ϕ_a	= vector/scalar of phase(s) of DMD mode at vortex shedding frequency (deg)

Notice to Readers

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I. Introduction

In this paper, we present a new method for the diagnosis and analysis of aeroacoustic phenomena. Taking advantage of the well-known property that the measurements of the Unsteady Pressure-Sensitive Paint (uPSP) have much higher spatial resolution compared to those of the conventional pressure transducers, the method is based on the visualization and analysis of the outputs of the Dynamic Mode Decomposition (DMD) of the uPSP measurements. NASA is developing a new state-of-the-art uPSP capability in production wind tunnels (Refs. [1-11]). The work described in this paper is a part of the campaign.

The uPSP measurements were collected with four Phantom v2512 high-speed cameras at 10,000 frames per second in the Ascent Transient Aerodynamics Test (ATAT) of the Space Launch System (SLS) Block 1 cargo vehicle in the 11-by-11-foot transonic test section of the Unitary Plan Wind Tunnel (UPWT) at NASA Ames Research Center in September 2019. Fig. 1 shows the scale model installed in the wind tunnel with uPSP applied to the model surface.

The new method presented in this paper is demonstrated by investigating an interesting phenomenon observed in the SLS ATAT – the vortex shedding tone generated by the forward attachment of the Solid Rocket Booster (SRB). As examples, the DMD outputs of the uPSP measurements in a subsonic test in the SLS ATAT are presented. It is shown the information retrieved from the DMD outputs of the uPSP measurements can be effectively used in the identification and diagnosis of the aeroacoustic phenomena.

This paper is organized as follows. In Section 2, the DMD of uPSP measurements is described and the DMD outputs of the uPSP measurements in the subsonic test are presented. In Section 3, the new method is demonstrated by the estimation of the vortex shedding frequency, the wavelength and propagation speed of the traveling waves with the DMD outputs of uPSP measurements in the subsonic test. Based on the information retrieved from the DMD outputs of the uPSP measurements, it is shown that the wave observed in the DMD mode at the vortex shedding frequency, which is originated at the forward attachment of the SRB and travels in the upstream direction, is an acoustic wave. In Section 4, the measured propagation speeds of the acoustic wave in the wind tunnel tests are discussed. Finally, the conclusions are presented in Section 5.

II. DMD of uPSP Measurements

The DMD is based on the eigendecomposition of a best-fit linear operator that approximates the dynamics present in the data. Given the timeseries of data, DMD computes a set of modes, each of which is associated with a fixed oscillation frequency and growth/decay rate (Refs. [12, 13]).

The conventional DMD algorithm is based on the Singular Value Decomposition (SVD). For the data with zero mean, the DMD is equivalent to the Discrete Fourier Transform (DFT) (Refs. [14, 15]). Since the uPSP is mainly used to determine the unsteady property of the flow, the DMD of the uPSP measurements is implemented in two steps (Ref. [16]): first, subtract the mean value from the uPSP measurement; then apply the Fast Fourier Transform (FFT) on the resulting data with zero mean. The DMD of the uPSP measurements with FFT has two advantages: (1) The FFT algorithm is well known for its computational efficiency. Compared to the SVD-based DMD algorithm, the DMD with FFT reduces the computation time. (2) Since the data size is big, it is advantageous to process the uPSP data in parallel. Compared to the SVD-based DMD algorithm, the DMD with FFT can be easily implemented in parallel processing.

The uPSP measurements collected with the Phantom v2512 high-speed cameras were processed with the uPSP data processing software (Refs. [17, 18]). A 5×5 box filter was first implemented to the raw measurements on the pixels of the cameras. The filtered measurements on the pixels of the cameras were projected to the grid nodes on the surface of the scale model and converted to the engineering unit. The first 1,024 frames of the uPSP measurements on the grid nodes were used as the inputs to DMD and the outputs were generated with the execution in parallel of a code in C, with libraries of FFTW for FFT (Ref. [19]) and MPI/OpenMP for parallel processing (Ref. [20]), on the NASA Pleiades supercomputer.

As examples, the DMD outputs of the uPSP measurements in a subsonic test, which was taken on September 24, 2019 as one of the Mach sweep runs in the SLS ATAT, are presented in this paper. In the test, the nominal Mach number was 0.73, and the angle of attack and the angle of the sideslip were both set to zeros.

Figs. 2(a) and 2(b) show the magnitude and phase of the DMD mode of uPSP measurements in the subsonic test at the vortex shedding frequency respectively on the grid nodes of the SLS core, plotted in a plane of x coordinate and azimuth angle of the nodes. Note that the uPSP measurements are unavailable in some areas on the core due to the blockage by the Solid Rocket Boosters (SRBs). The vortex shedding tone was generated when the flow passed the forward attachments of the SRBs to the SLS core. As expected, Fig. 2(a) shows that the DMD mode on the grid nodes close to the forward attachments of SRBs has larger magnitudes. The phase of the DMD mode ranges from -180° to $+180^\circ$. As shown in Fig. 2(b), the grid nodes with the phase close to -180° are plotted in blue and the grid nodes with

the phase close to $+180^\circ$ are plotted in yellow. The cyclical patterns are observed in Fig. 2(b), corresponding to the traveling waves at the vortex shedding frequency.

In Figs. 3(a) and 3(b), the DMD modes of uPSP measurements in the subsonic test, corresponding to the real and imaginary parts of the DFT coefficients at the vortex shedding frequency, are visualized on the surface of the scale model. From the visualization of the DMD mode, it is observed that two waves at the vortex shedding frequency, originated at the forward attachments of the SRBs, travel in the upstream and downstream directions respectively. In the next section, this phenomenon will be analyzed with a new method based on the visualization and analysis of the DMD outputs of the uPSP measurements.

III. Analysis of DMD Outputs of uPSP Measurements in the SLS ATAT

In this section, the parameters of the subsonic test are determined first, and then the new method is demonstrated by the estimation of the vortex shedding frequency, the wavelengths and propagation speeds of the traveling waves at the vortex shedding frequency with the DMD outputs of uPSP measurements in the subsonic test. The wavelength of the traveling wave is estimated with two methods: (1) from the plot of phase of the DMD mode; and (2) from the slope of the phase of the DMD mode versus the x coordinate of the grid nodes along a streamline. The estimated propagation speeds of the waves at the vortex shedding frequency traveling in upstream and downstream directions, together with the corresponding uncertainties, are presented.

A. Determination of Parameters of the Subsonic Test

The measured Mach number (denoted as M) and the total temperature of free stream flow (in $^\circ\text{F}$, denoted as $T_{total,^\circ\text{F}}$) of the subsonic test can be retrieved from the wind tunnel data file. Based on the inputs M and $T_{total,^\circ\text{F}}$, we can determine the static temperature of free stream flow (in $^\circ\text{R}$, denoted as $T_{static,^\circ\text{R}}$), the speed of sound (in ft/s, denoted as c) and the speed of free stream flow (in ft/s, denoted as U_∞) as following:

$$T_{static,^\circ\text{R}} = T_{total,^\circ\text{R}} / (1 + 0.5 \cdot (\gamma - 1) \cdot M^2) \quad (1)$$

$$c = \sqrt{\gamma \cdot R \cdot T_{static,^\circ\text{R}}} \quad (2)$$

$$U_\infty = M \cdot c \quad (3)$$

where γ is the ratio of specific heats of air and $\gamma = 1.4$; R is the specific gas constant of air and $R = 1716.46 \text{ ft} \cdot \text{lb} / \text{slug} \cdot ^\circ\text{R}$; $T_{total,^\circ\text{R}}$ is the total temperature of free stream flow in $^\circ\text{R}$ and

$$T_{total,^\circ\text{R}} = T_{total,^\circ\text{F}} + 459.67 \quad (4)$$

The parameters of the subsonic test, including the input parameters from the wind tunnel data file and the parameters derived with Eqs. (1)-(4), are listed in Table 1.

Table 1 Parameters of the Subsonic Test

Test Date		09/24/2019
Nominal Mach Number		0.73
Inputs from wind tunnel data file	Measured Mach Number, M	0.7296
	Total Temperature of Free Stream Flow ($^\circ\text{F}$), $T_{total,^\circ\text{F}}$	74.15
Derived Parameters	Total Temperature of Free Stream Flow ($^\circ\text{R}$), $T_{total,^\circ\text{R}}$	533.82
	Static Temperature of Free Stream Flow ($^\circ\text{R}$), $T_{static,^\circ\text{R}}$	482.4
	Speed of Sound (ft/s), c	1076.7
	Speed of Free Stream Flow (ft/s), U_∞	785.6

B. Estimation of the Vortex Shedding Frequency

Fig. 4 shows the average Power Spectral Density (PSD) of the timeseries of uPSP measurements in the subsonic test on the grid nodes of the SLS core. As shown in the figure, in the frequency range we are interested in, the peak of the fundamental component of the vortex shedding can be observed. Note that the 1st harmonic of the vortex shedding can also be identified in the plot of the average PSD. The estimated vortex shedding frequency, denoted as $f_{\text{vortexShedding}}$, is determined as the frequency corresponding to the peak of the average PSD. Considering the DMD outputs were generated with the first 1,024 frames of the uPSP measurements, the uncertainty of the estimated vortex shedding frequency, denoted as $\Delta f_{\text{vortexShedding}}$, is determined as

$$\Delta f_{\text{vortexShedding}} = f_s / 1024 = 9.7656 \text{ Hz} \quad (5)$$

where f_s is the sampling frequency of the uPSP measurements and $f_s = 10,000 \text{ Hz}$.

The estimated vortex shedding frequency and the uncertainty will be used in Section 4.E to estimate the propagation speeds of the traveling waves at the vortex shedding frequency and determine the corresponding uncertainties.

C. Estimation of the Wavelength from the Plot of Phase of the DMD Mode

Fig. 5 shows the phase of the DMD mode in the subsonic test at the vortex shedding frequency of the grid nodes in an area close to the SRB at the negative azimuth side. As shown in the figure, two red lines link the grid nodes on the edges of the blue and yellow regions along the streamline. For the grid node on the red line located downstream to the forward attachment of the SRB, the color changes from yellow to green and to blue along the streamline, indicating the phase of the DMD mode decreases from $+180^\circ$ to 0° and to -180° ; for the grid node on the red line located upstream to the forward attachment of the SRB, the color changes from blue to green and to yellow along the streamline, indicating the phase of the DMD mode increase from -180° to 0° and to $+180^\circ$. These observations are consistent with the description in Section 2 that the waves at the vortex shedding frequency, originated at the forward attachments of the SRB, travel in the upstream and downstream directions respectively.

Let $x_{a,\text{end1}}$ and $x_{a,\text{end2}}$ denote the x coordinates (in inch) of the grid nodes at the ends of the red line located upstream to the forward attachment of the SRB, and let $x_{b,\text{end1}}$ and $x_{b,\text{end2}}$ denote the x coordinates (in inch) of the grid nodes at the ends of the red line located downstream to the forward attachment of the SRB. The wavelengths (in ft) of the waves traveling in the upstream and downstream directions, denoted as $\lambda_{\text{upstream}}$ and $\lambda_{\text{downstream}}$ respectively, can be determined as

$$\lambda_{\text{upstream}} = (x_{a,\text{end2}} - x_{a,\text{end1}}) / 12 \quad (6)$$

$$\lambda_{\text{downstream}} = (x_{b,\text{end2}} - x_{b,\text{end1}}) / 12 \quad (7)$$

The uncertainties of the estimated wavelengths from Eqs. (6) and (7), denoted as $\Delta \lambda_{\text{upstream}}$ and $\Delta \lambda_{\text{downstream}}$, are determined as the difference of x coordinates (in inch, converted to ft) of two consecutive grid nodes on the two red lines respectively.

D. Estimation of the Wavelength from the Slope of the Phase of the DMD Mode versus the X Coordinate

Fig. 6(a) shows a group of selected grid nodes on the SLS core, located upstream to the forward attachment of the SRB. The selected grid nodes in the group are along a streamline whose azimuth is -121.3° . Fig. 6(b) shows the phases of the DMD mode at the vortex shedding frequency, illustrated as blue dots, versus the x coordinates of the selected grid nodes in the group. It is observed the phase of the DMD mode at the vortex shedding frequency basically varies as a linear function of the x coordinate of the grid node.

Let $x_{a,1}, x_{a,2}, \dots, x_{a,N_a}$ be the x coordinates (in inch) of the selected grid nodes and $\phi_{a,1}, \phi_{a,2}, \dots, \phi_{a,N_a}$ be the corresponding phases (in deg) of the DMD mode at the vortex shedding frequency of the selected grid nodes, where

N_a is the number of selected grid nodes. A linear model between the phase of the DMD mode and the x coordinate is assumed, as shown below,

$$\Phi_a = H_a \theta_{upstream} + E_a \quad (8)$$

where Φ_a is an $N_a \times 1$ vector of the phase of the DMD mode

$$\Phi_a = [\phi_{a,1} \quad \phi_{a,2} \quad \cdots \quad \phi_{a,N_a}]^T \quad (9)$$

H_a is an $N_a \times 2$ data matrix of the linear model, which includes the x coordinates of selected grid nodes shown in Fig. 6(a),

$$H_a = \begin{bmatrix} x_{a,1} & 1 \\ x_{a,2} & 1 \\ \vdots & \vdots \\ x_{a,N_a} & 1 \end{bmatrix} \quad (10)$$

and $\theta_{upstream}$ is a 2×1 vector of parameters of the linear model, including the slope and offset, denoted as $k_{upstream}$ and $d_{upstream}$ respectively,

$$\theta_{upstream} = [k_{upstream} \quad d_{upstream}]^T \quad (11)$$

E_a is an $N_a \times 1$ vector of the error terms of the linear model, which includes the modeling error and the measurement noise of the phase of the DMD mode at the vortex shedding frequency, as shown below,

$$E_a = [e_{a,1} \quad e_{a,2} \quad \cdots \quad e_{a,N_a}]^T \quad (12)$$

It is assumed the expected value of E_a is an $N_a \times 1$ vector of zeros and the variance matrix of E_a is given in the following equation:

$$P_{E_a} = \sigma^2 \cdot I_{N_a \times N_a} \quad (13)$$

where σ is the standard deviation of the error term. $I_{N_a \times N_a}$ is an $N_a \times N_a$ identity matrix.

The vector of estimated parameters of the linear model, denoted as $\hat{\theta}_{upstream}$, can be determined with the least-squares fit:

$$\hat{\theta}_{upstream} = (H_a^T H_a)^{-1} H_a^T \Phi_a \quad (14)$$

The variance matrix of the vector of estimated parameters are given by the equation below:

$$P_{\hat{\theta},upstream} = \sigma^2 \cdot (H_a^T H_a)^{-1} \quad (15)$$

where the standard deviation of the error term, σ , is determined as the maximum of the absolute values of the elements of the vector of the estimation error, denoted as $\hat{E}_a$, as shown below:

$$\hat{E}_a = \Phi_a - H_a \hat{\theta}_{upstream} \quad (16)$$

With the estimated parameters given in Eq. (14), the linear model between the phase of the DMD mode at the vortex shedding frequency and the x coordinate is illustrated as the red line in Fig. 6(b).

Let $\hat{k}_{upstream}$ and $\Delta \hat{k}_{upstream}$ denote the estimated slope (in deg/inch) of the linear model and its corresponding uncertainty. $\hat{k}_{upstream}$ is determined as the 1st element of the 2×1 vector $\hat{\theta}_{upstream}$ given in Eq. (14). Note that

$\hat{k}_{upstream}$ is positive, indicating a wave traveling in the upstream direction. $\Delta\hat{k}_{upstream}$ is determined as the square root of the element (1, 1) of the 2×2 matrix $P_{\hat{\theta},upstream}$ given in Eq. (15).

The wavelength (in ft) of the wave traveling in the upstream direction can be determined as

$$\lambda_{upstream} = 360/\hat{k}_{upstream}/12 \quad (17)$$

The uncertainty $\Delta\lambda_{upstream}$ (in ft) is determined from $\Delta\hat{k}_{upstream}$ as

$$\Delta\lambda_{upstream} = 360/12 \cdot \Delta\hat{k}_{upstream}/\hat{k}_{upstream}^2 \quad (18)$$

Fig. 7(a) shows another group of selected grid nodes located downstream to the forward attachment of the SRB. The azimuth of the selected grid nodes is -60.3° . Fig. 7(b) shows the phase of the DMD mode at the vortex shedding frequency, illustrated as blue dots, versus the x coordinate of the selected grid nodes shown in Fig. 7(a). With the parameters estimated from the least-squares fit, the linear model between the phase of the DMD mode at the vortex shedding frequency and the x coordinate is illustrated as the red line in Fig. 7(b).

Let $\hat{k}_{downstream}$ and $\Delta\hat{k}_{downstream}$ denote the estimated slope (in deg/inch) of the linear model, as shown in Fig. 7(b), and its corresponding uncertainty. Note that $\hat{k}_{downstream}$ is negative, indicating a wave traveling in the downstream direction. With the same method discussed above for the wave traveling in the upstream direction, $\hat{k}_{downstream}$ and $\Delta\hat{k}_{downstream}$ are determined with the least-squares fit outputs.

The wavelength (in ft) of the wave traveling in the downstream direction can be determined as

$$\lambda_{downstream} = 360/|\hat{k}_{downstream}|/12 \quad (19)$$

where $|\cdot|$ indicates the absolute value. The uncertainty $\Delta\lambda_{downstream}$ (in ft) is determined from $\Delta\hat{k}_{downstream}$ as

$$\Delta\lambda_{downstream} = 360/12 \cdot \Delta\hat{k}_{downstream}/\hat{k}_{downstream}^2 \quad (20)$$

E. Estimation of the Propagation Speed of the Traveling Wave

The estimated propagation speeds (in ft/s) of the waves at the vortex shedding frequency traveling in upstream and downstream directions, denoted as $U_{upstream}$ and $U_{downstream}$ respectively, are computed with the following equations:

$$U_{upstream} = \lambda_{upstream} \cdot f_{vortexShedding} \quad (21)$$

and

$$U_{downstream} = \lambda_{downstream} \cdot f_{vortexShedding} \quad (22)$$

where $\lambda_{upstream}$ and $\lambda_{downstream}$ are estimated wavelengths (in ft) of the waves at the vortex shedding frequency traveling in upstream and downstream directions, which are determined in two ways, as shown in Eqs. (6), (7), (17) and (19). $f_{vortexShedding}$ is the estimated vortex shedding frequency (in Hz), as described in Section 3.B.

The uncertainties of the estimated propagation speeds of the waves (in ft/s) at the vortex shedding frequency traveling in upstream and downstream directions, denoted as $\Delta U_{upstream}$ and $\Delta U_{downstream}$ respectively, are determined as

$$\Delta U_{upstream} = \Delta\lambda_{upstream} \cdot f_{vortexShedding} + \lambda_{upstream} \cdot \Delta f_{vortexShedding} \quad (23)$$

$$\Delta U_{downstream} = \Delta\lambda_{downstream} \cdot f_{vortexShedding} + \lambda_{downstream} \cdot \Delta f_{vortexShedding} \quad (24)$$

The uncertainties $\Delta\lambda_{upstream}$ and $\Delta\lambda_{downstream}$ for the estimated wavelengths from the plot of the phase of the DMD mode at the vortex shedding frequency are discussed in Section 3.C, and the uncertainties for the estimated wavelengths from the slope of the phase of the DMD mode versus the x coordinates are given in Eqs. (18) and (20). The uncertainty $\Delta f_{VortexShedding}$ is determined with Eq. (5).

The estimated propagation speeds of the waves at the vortex shedding frequency traveling in upstream and downstream directions ($U_{upstream}$ and $U_{downstream}$), together with the corresponding uncertainties ($\Delta U_{upstream}$ and $\Delta U_{downstream}$), based on the estimated wavelengths from the plot of phase of the DMD mode at the vortex shedding frequency and from the slope of the phase of the DMD mode at the vortex shedding frequency versus the x coordinate of selected grid nodes along a streamline respectively, are listed in Table 2. The ratios of the propagation speeds of the waves to the speed of sound ($U_{upstream}/c$ and $U_{downstream}/c$) and the ratios of the propagation speeds of the waves to the speed of free stream flow ($U_{upstream}/U_{\infty}$ and $U_{downstream}/U_{\infty}$), together with the corresponding uncertainties ($\Delta U_{upstream}/c$, $\Delta U_{downstream}/c$ and $\Delta U_{upstream}/U_{\infty}$, $\Delta U_{downstream}/U_{\infty}$), are also computed and listed in Table 2. Note that the speed of sound c (in ft/s) and the speed of free stream flow U_{∞} (in ft/s), determined with Eqs. (2) and (3), are listed in Table 1.

Further investigations show that the estimated propagation speed of the wave traveling in the upstream direction, $U_{upstream}$, is approximately equal to the difference of the speed of the sound, c , and the speed of the free stream flow of the subsonic test, U_{∞} . It turns out that the wave observed in the DMD mode at the vortex shedding frequency, which is originated at the forward attachment of the SRB and travels in the upstream direction, is an acoustic wave. In Section 4, the measured propagation speeds of the acoustic wave in the wind tunnel tests are discussed. It will be shown that, for a subsonic wind tunnel test, the measured propagation speed of the acoustic wave, by an acoustic signal detector which is fixed to the launch vehicle and located upstream to the acoustic signal generator, which is also fixed to the launch vehicle, is exactly equal to the difference of the speed of sound, c , and the speed of the free stream flow, U_{∞} .

As shown in Table 2, the estimated propagation speed of the wave traveling in the downstream direction ($U_{downstream}$) is approximately 76% of the speed of the free stream flow (U_{∞}) of the subsonic test. The value is consistent with the expected aerodynamic convection speed, confirming the wave observed in the DMD modes at the vortex shedding frequency traveling in the downstream direction is due to the aerodynamic convection.

Table 2 Estimated Propagation Speeds of the Waves at the Vortex Shedding Frequency Traveling in Upstream and Downstream Directions, together with the Corresponding Uncertainties

Waves	Variables	Estimated Values from the Plot of the Phase of the DMD Mode	Estimated Values from the Slope of the Phase of the DMD Mode vs the X Coordinate
Wave traveling in upstream direction	Propagation Speed of the Wave (ft/s), $U_{upstream}$	299.7 ± 26.8	301.9 ± 8.3
	Ratio of Speed of the Wave to Speed of Sound, $U_{upstream}/c$	0.28 ± 0.02	0.28 ± 0.01
	Ratio of Speed of the Wave to Speed of Free Stream Flow, $U_{upstream}/U_{\infty}$	0.38 ± 0.03	0.38 ± 0.01
Wave traveling in downstream direction	Propagation Speed of the Wave (ft/s), $U_{downstream}$	587.6 ± 31.5	598.9 ± 9.3
	Ratio of Speed of the Wave to Speed of Sound, $U_{downstream}/c$	0.55 ± 0.03	0.56 ± 0.01
	Ratio of Speed of the Wave to Speed of Free Stream Flow, $U_{downstream}/U_{\infty}$	0.75 ± 0.04	0.76 ± 0.01

IV. Measured Propagation Speed of the Acoustic Wave in Wind Tunnel Tests

In this section, we discuss the measured propagation speed of the acoustic wave in the wind tunnel tests. First, we assume an observer in a reference frame fixed to the ground and consider the measured propagation speed of the acoustic wave by the observer in two cases: (1) the launch vehicle stands on the launch pad; (2) the launch vehicle flies horizontally at a subsonic speed. Next, we assume the observer is moved from the reference frame fixed to the ground to a reference frame fixed to the wind tunnel and the launch vehicle, then the two cases considered above correspond to the “wind off” and “wind on” wind tunnel tests respectively.

Assume there is a sinusoidal acoustic signal generator fixed on the launch vehicle, whose frequency is denoted as f_A . The distance that the sound travels in a period of the sinusoidal acoustic signal, denoted as D_A , can be written as

$$D_A = c \cdot T_A = c/f_A \quad (25)$$

where T_A is the period of the sinusoidal acoustic signal. It is also assumed there is an acoustic signal detector fixed on the launch vehicle and located upstream to the sinusoidal acoustic signal generator.

Now we consider an observer, in a reference frame fixed to the ground, attempting to measure the propagation speed of the acoustic wave with the acoustic signal detector in two cases: (1) the launch vehicle stands on the launch pad; (2) the launch vehicle flies horizontally at a subsonic speed.

A. Measured Propagation Speed of the Acoustic Wave when the Launch Vehicle Stands on the Launch Pad

In the 1st case, we assume the launch vehicle stands on the launching pad. Fig. 8(a) shows a snapshot, at an instant t , of the launch vehicle and the wavefronts of the sinusoidal acoustic signal at the peak. It is assumed the sinusoidal acoustic signal is at the peak at the instant t , so the signal is also at the peak at instants $t - T_A$ and $t - 2T_A$. For the observer in the reference frame fixed to the ground, the launch vehicle is located at the same position at the instants $t - T_A$ and $t - 2T_A$. At the instant t , the wavefront of the sinusoidal acoustic signal at the peak transmitted at the instant $t - T_A$ is shown as the smaller circle in Fig. 8(a), whose radius is equal to D_A and whose center is located at the position of the launch vehicle at the instant $t - T_A$; the wavefront of the sinusoidal acoustic signal at the peak transmitted at the instant $t - 2T_A$ is shown as the larger circle in Fig. 8(a), whose radius is equal to $2D_A$ and whose center is located at the position of the launch vehicle at the instant $t - 2T_A$.

For the signal detector fixed on the launch vehicle, the measured period of the sinusoidal acoustic signal (the duration between the instants of two consecutive peaks) is equal to T_A , or equivalently, the measured frequency of the sinusoidal acoustic signal $f_{A,m1} = 1/T_A$. The measured wavelength of the acoustic wave (the distance between the wavefronts of two consecutive peaks) is equal to $\lambda_{A,m1} = D_A$. The measured propagation speed of the acoustic wave is determined as

$$U_{A,m1} = \lambda_{A,m1} \cdot f_{A,m1} = D_A \cdot 1/T_A \quad (26)$$

Combining Eqs. (25) and (26) leads to

$$U_{A,m1} = c \cdot T_A \cdot 1/T_A = c \quad (27)$$

Therefore, in the 1st case that the launch vehicle stands on the launching pad, as shown in Fig. 8(a), the measured propagation speed of the acoustic wave, $U_{A,m1}$, is equal to the speed of sound, c , for the observer in a reference frame fixed to the ground.

B. Measured Propagation Speed of the Acoustic Wave when the Launch Vehicle Flies at a Subsonic Speed

Now we consider the 2nd case that the launch vehicle flies horizontally to the left at a subsonic speed v_{LV} . Note that v_{LV} is constant and $v_{LV} < c$. Fig. 8(b) shows a snapshot, at the instant t , of the launch vehicle and the wavefronts of the sinusoidal acoustic signal at the peak. Like in the 1st case discussed in Section 4.A, we also assume in the 2nd

case the sinusoidal acoustic signal is at the peak at the instant t , so the signal is also at the peak at instants $t - T_A$ and $t - 2T_A$. The positions of the launch vehicle at the instants $t - T_A$ and $t - 2T_A$ are also illustrated in Fig. 8(b) for convenience. The distance that the launch vehicle travels in a period of the sinusoidal acoustic signal, denoted as d_{LV} , can be written as

$$d_{LV} = v_{LV} \cdot T_A \quad (28)$$

The wavefront of the sinusoidal acoustic signal at the peak transmitted at the instant $t - T_A$ is shown as the smaller circle in Fig. 8(b), whose radius is equal to D_A and whose center is located at the position of the launch vehicle at the instant $t - T_A$; the wavefront of the sinusoidal acoustic signal at the peak transmitted at the instant $t - 2T_A$ is shown as the larger circle in Fig. 8(b), whose radius is equal to $2D_A$ and whose center is located at the position of the launch vehicle at the instant $t - 2T_A$.

For the observer in a reference frame fixed to the ground, both the speed of sound, c , and the speed of the launch vehicle, v_{LV} , are constant. Since both the acoustic signal generator and the acoustic signal detector are fixed on the launch vehicle, the measured period of the sinusoidal acoustic signal (the duration between the instants of two consecutive peaks) is still equal to T_A , or equivalently, the measured frequency of the sinusoidal acoustic signal $f_{A,m2} = 1/T_A$. From the geometry shown in Fig. 8(b), the measured wavelength (the distance between the wavefronts of two consecutive peaks) by the acoustic signal detector, which is located upstream to the sinusoidal acoustic signal generator, can be written as

$$\lambda_{A,m2} = D_A - d_{LV} \quad (29)$$

The measured propagation speed of the acoustic wave is determined as

$$U_{A,m2} = \lambda_{A,m2} \cdot f_{A,m2} = (D_A - d_{LV}) \cdot 1/T_A \quad (30)$$

Combining Eqs. (25), (28) and (30) leads to

$$U_{A,m2} = (c \cdot T_A - v_{LV} \cdot T_A) \cdot 1/T_A = c - v_{LV} \quad (31)$$

Therefore, in the 2nd case that the launch vehicle flies horizontally at the subsonic speed v_{LV} , as shown in Fig. 8(b), the measured propagation speed of the acoustic wave, $U_{A,m2}$, by the acoustic signal detector located upstream to the sinusoidal acoustic signal generator, is equal to the difference of the speed of sound, c , and the speed of the launch vehicle, v_{LV} , for the observer in a reference frame fixed to the ground.

When the observer is moved from the reference frame fixed to the ground to a reference frame fixed to the wind tunnel and the launch vehicle, the two cases discussed above correspond to the “wind off” and “wind on” wind tunnel tests respectively.

For the “wind off” wind tunnel test, there is no difference between the two reference frames. For the “wind on” wind tunnel test, in the reference frame fixed to the wind tunnel and the launch vehicle, the speed of the launch vehicle is 0 and the free stream flow flies to the right at the constant speed U_∞ . Considering the fact

$$U_\infty = v_{LV} \quad (32)$$

Eq. (31) can be rewritten as

$$U_{A,m2} = c - U_\infty \quad (33)$$

As shown in Eq. (33), for a subsonic wind tunnel test, the measured propagation speed of the acoustic wave, $U_{A,m2}$, by an acoustic signal detector which is fixed to the launch vehicle and located upstream to an acoustic signal generator, which is also fixed to the launch vehicle, is exactly equal to the difference of the speed of sound, c , and the speed of the free stream flow, U_∞ .

V. Conclusions

A new method for the diagnosis and analysis of the aeroacoustic phenomena is presented in this paper. Taking advantage of the well-known property that the measurements of the uPSP have much higher spatial resolution compared to those of the conventional pressure transducers, the method is based on the visualization and analysis of the DMD outputs of the uPSP measurements. The effectiveness of the method is demonstrated by investigating an interesting phenomenon observed in the SLS ATAT – the vortex shedding tone generated by the forward attachment of the SRB. As examples, the DMD outputs of the uPSP measurements in a subsonic test in the SLS ATAT are presented and used in the estimation of the vortex shedding frequency, the wavelengths and propagation speeds of the traveling waves at the vortex shedding frequency. Based on the information retrieved from the DMD outputs of the uPSP measurements, it is shown that the wave observed in the DMD mode at the vortex shedding frequency, which is originated at the forward attachment of the SRB and travels in the upstream direction, is an acoustic wave. It is also shown that, for a subsonic wind tunnel test, the measured propagation speed of the acoustic wave, by an acoustic signal detector which is fixed to the launch vehicle and located upstream to the acoustic signal generator, which is also fixed to the launch vehicle, is exactly equal to the difference of the speed of sound and the speed of the free stream flow.

The estimated propagation speed of the traveling wave at the vortex shedding frequency is based on the estimated wavelength of the traveling wave, which is determined with two methods: (1) from the plot of phase of the DMD mode; and (2) from the slope of the phase of the DMD mode versus the x coordinate of the grid nodes along a streamline. The two methods have properties compensating each other: the first method is straight-forward and good for a quick check; the second method is more complicated and provides more details of the wave speed in a local area, especially when the wave speed varies along the streamline. It is observed the estimation precision of the wavelength and the wave speed is limited by the noise of the phase of the DMD mode, which is due to the noise of the uPSP measurements. Preliminary results of the investigation show that the Spectral Proper Orthogonal Decomposition (SPOD) of the uPSP measurements can attenuate the noise of the uPSP measurements effectively (Refs. [21, 22]). The noise of the phase of the SPOD mode is reduced significantly compared to that of the phase of the DMD mode. More research work on the SPOD of the uPSP measurements will be done in the future.

Acknowledgment

The work described in this paper is a part of NASA's development of a new state-of-the-art uPSP capability in production wind tunnels. Funding was provided by the NASA Aerosciences Evaluation and Test Capabilities (AETC) Portfolio Office. The authors would like to thank James Bell, Paul Bremner, Nicholas Califano, Bob Ciotti, David Ellsworth, Ross Flach, Patrick Heaney, Jaffar Iqbal, Alan Landman, Bruce Laverde, Kenneth Lyons, Blair McLachlan, Patrick Moran, Bron Nelson, Jack Ortega, Jayanta Panda, David Piatak, Victoria Pollard, James Ramey, James Ross, Timothy Sandstrom, Martin Sekula, Francesco Soranna, Thomas Steva, Craig Streett, Paul Stremel, Lucy Tang and Thomas Volden for their expertise and advice for the research work of this paper.

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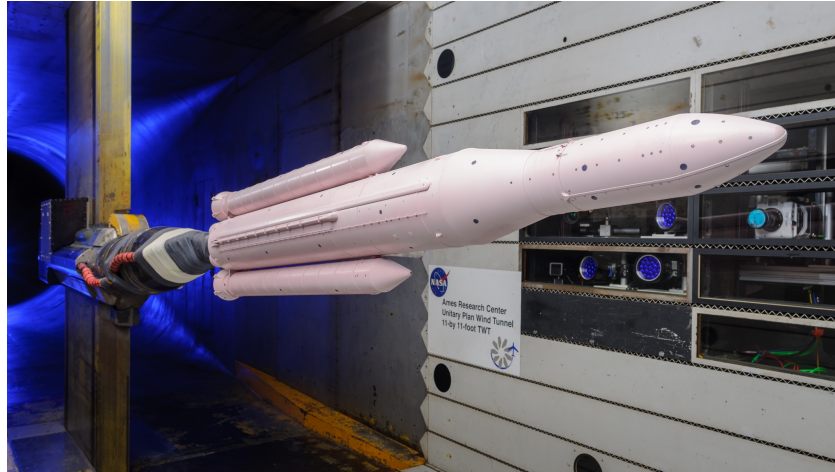
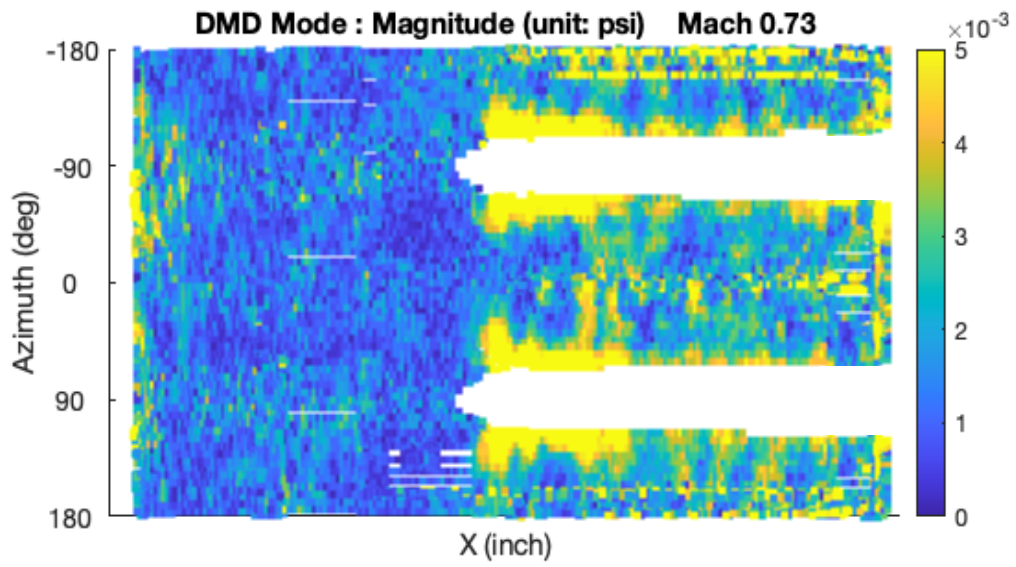
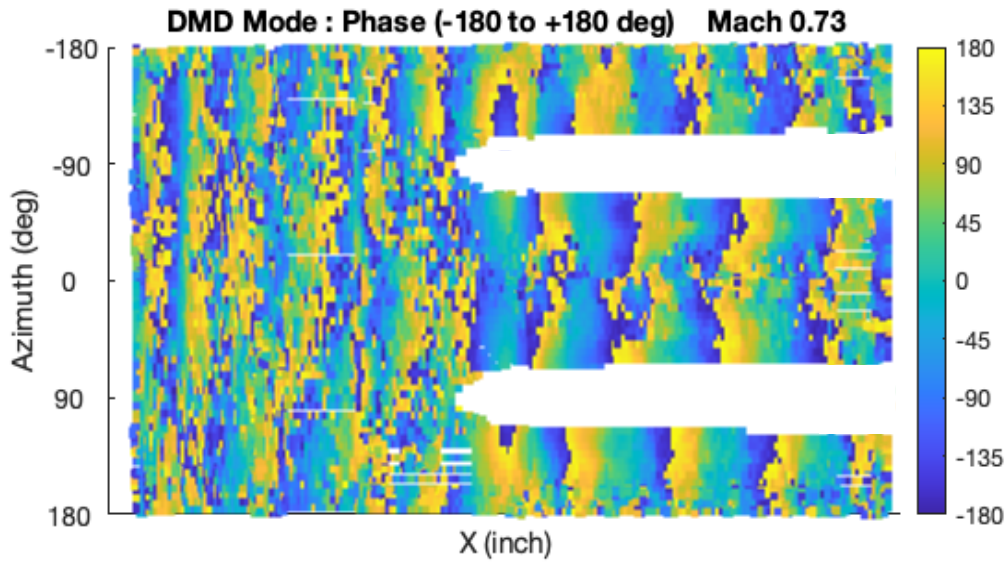


Fig. 1 Scale model of the SLS Block 1 cargo vehicle with uPSP in the 11-by-11-foot transonic test section of the UPWT at NASA Ames Research Center during the ATAT.

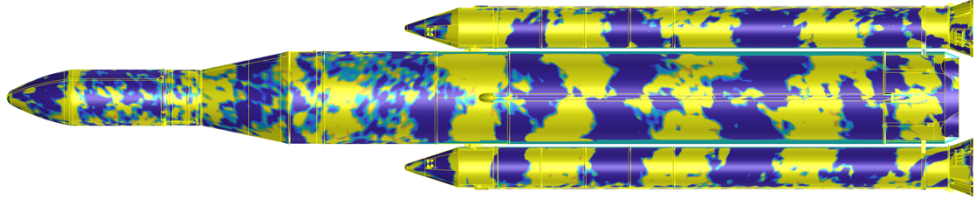


(a)

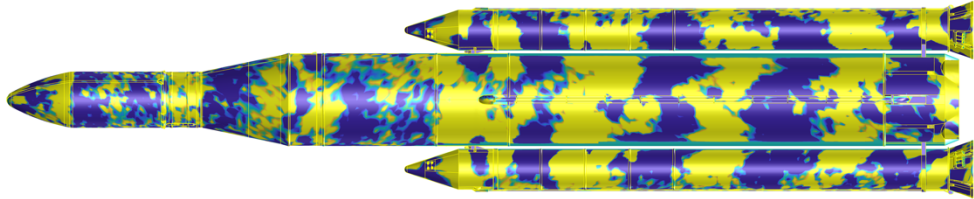


(b)

Fig. 2 Magnitude (a) and phase (b) of the DMD mode of uPSP measurements in the subsonic test at the vortex shedding frequency on the grid nodes of the SLS core.



(a)



(b)

Fig. 3 DMD modes of uPSP measurements in the subsonic test, corresponding to the real (a) and imaginary (b) parts of the DFT coefficients at the vortex shedding frequency, on the surface of the scale model.

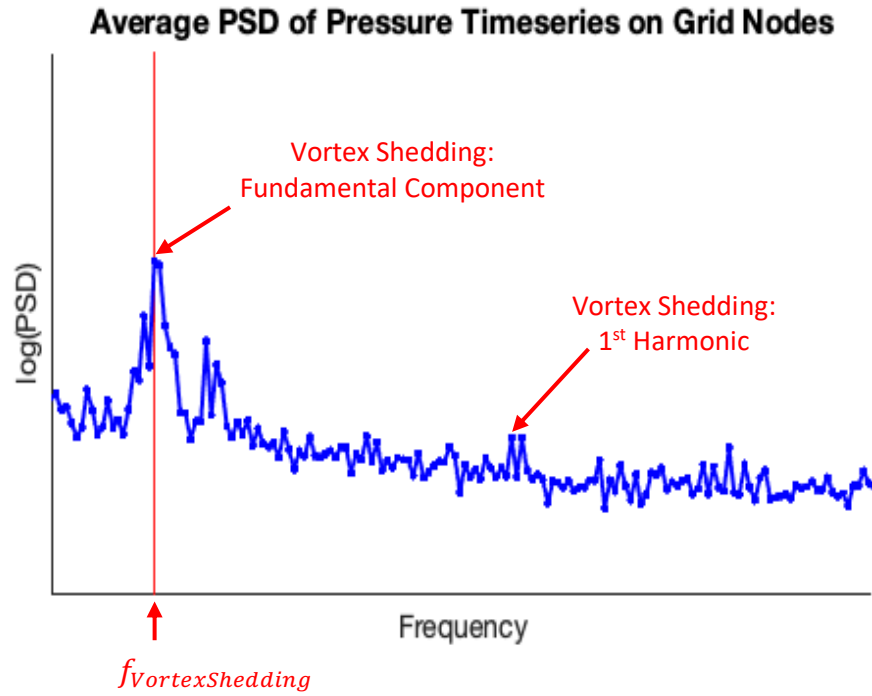


Fig. 4 Average PSD of the timeseries of uPSP measurements in the subsonic test on the grid nodes of the SLS core.

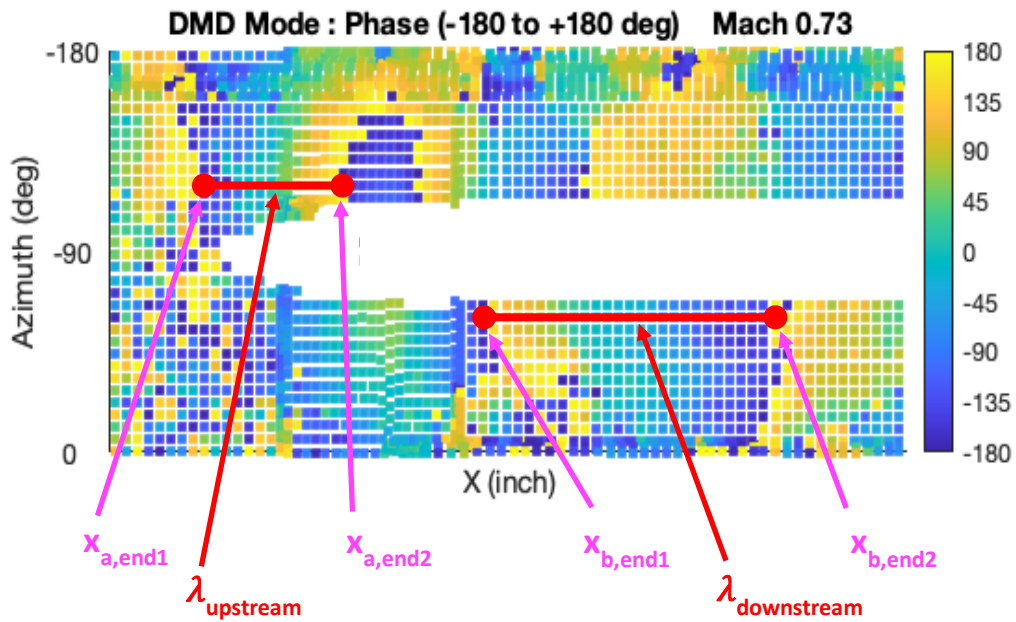
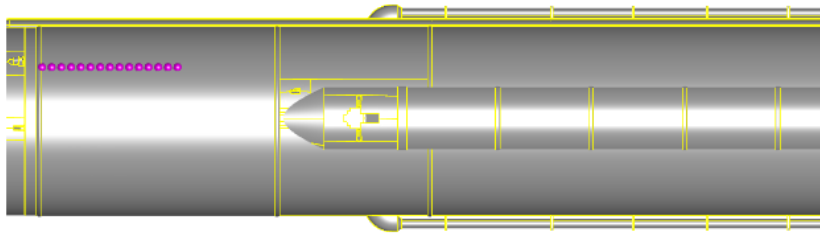
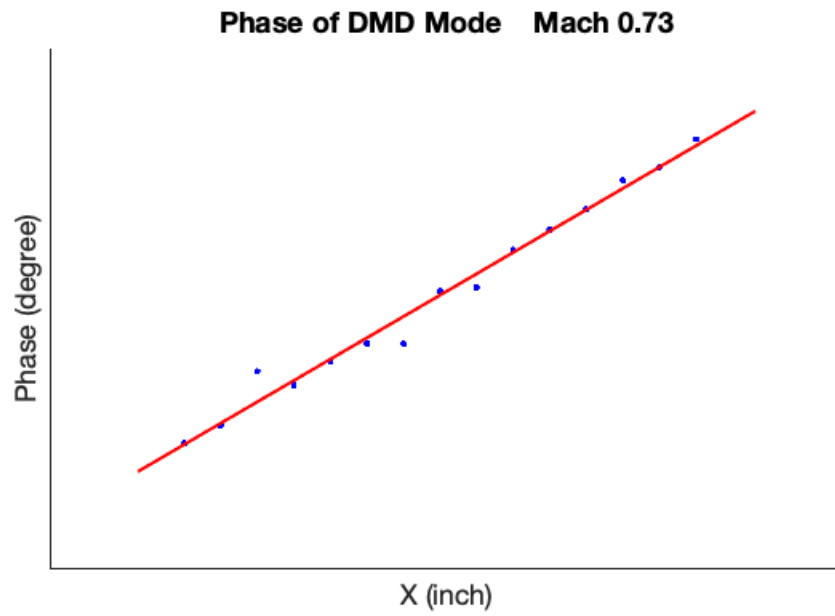


Fig. 5 Phase of the DMD mode of uPSP measurements in the subsonic test at the vortex shedding frequency of the grid nodes in an area on the SLS core close to the SRB at the negative azimuth side. The red lines link the grid nodes on the edges of the blue and yellow regions along the streamline.

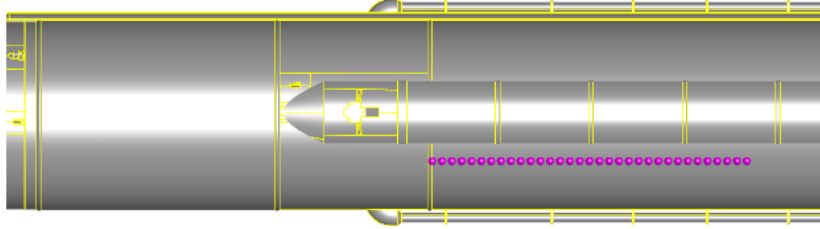


(a)

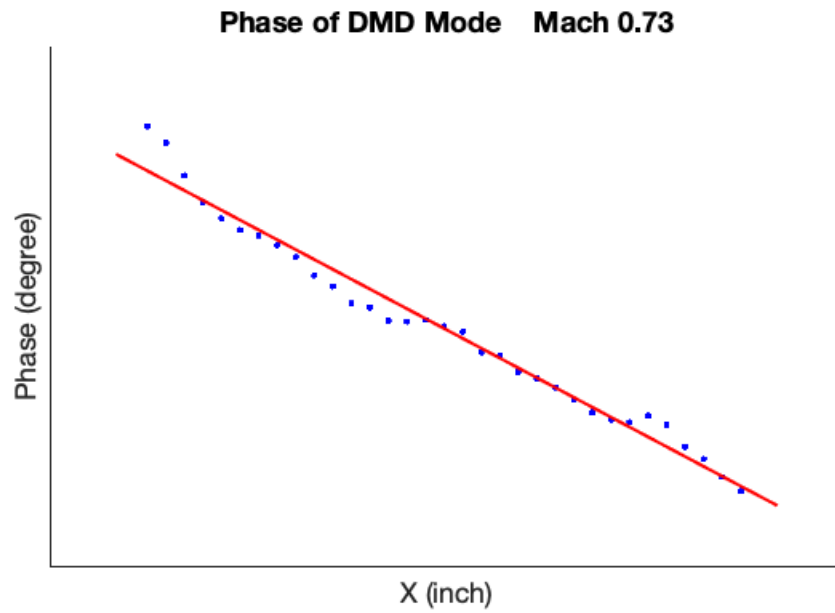


(b)

Fig. 6 (a) Selected grid nodes on the SLS core, located upstream to the forward attachment of the SRB; and (b) the phases of the DMD mode at the vortex shedding frequency of the selected grid nodes (blue dots) and the linear model, with estimated parameters from the least-squares fit, of the phase of the DMD mode versus the x coordinate (red line).

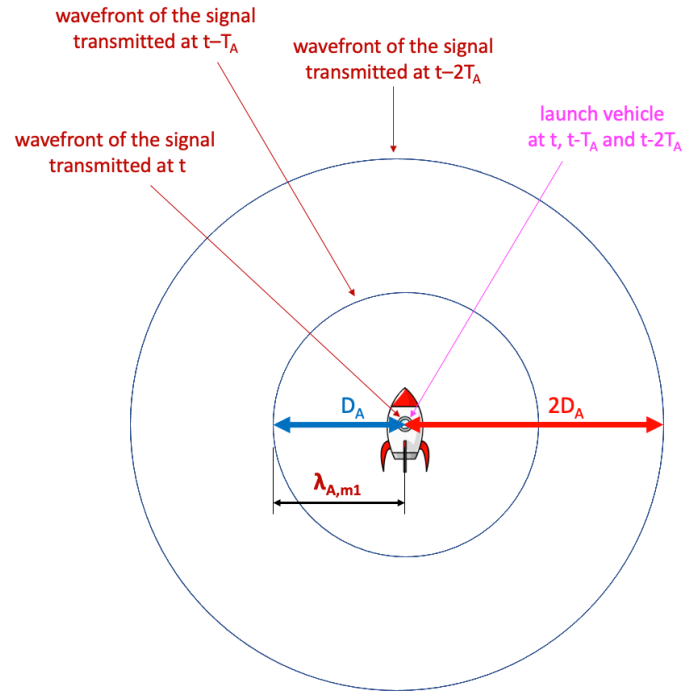


(a)

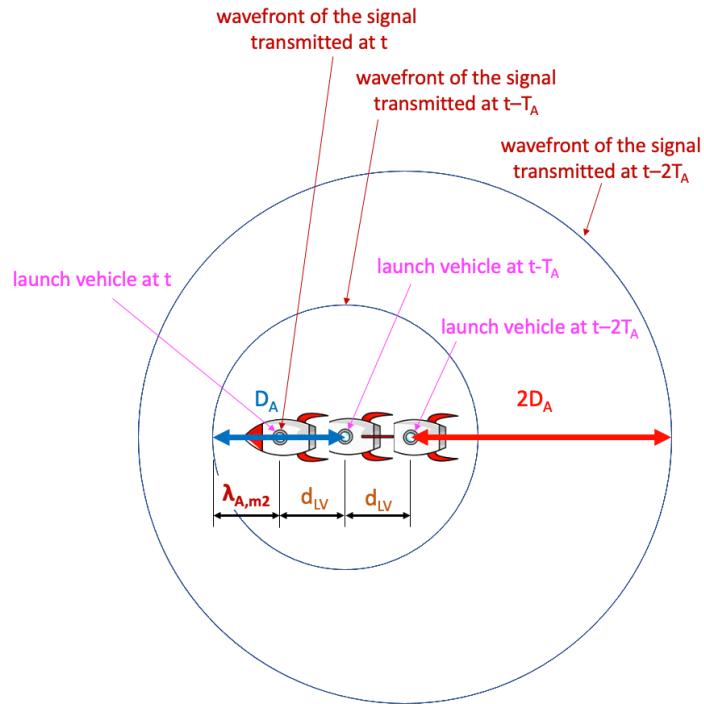


(b)

Fig. 7 (a) Selected grid nodes on the SLS core, located downstream to the forward attachment of the SRB; and (b) the phases of the DMD mode at the vortex shedding frequency of the selected grid nodes (blue dots) and the linear model, with estimated parameters from the least-squares fit, of the phase of the DMD mode versus the x coordinate (red line).



(a)



(b)

Fig.8 Illustration of the measured propagation speed of the acoustic wave in two cases: (1) the launch vehicle stands on the launch pad; (2) the launch vehicle flies horizontally at a subsonic speed.