

Higher-order statistical moments of predicted sonic boom waveforms through turbulence



Joel B. Lonzaga
NASA Langley Research Center
Hampton, Virginia
joel.b.lonzaga@nasa.gov

185th Meeting, Acoustical Society of America
Acoustics 23 Sydney
Dec 4-8, 2023

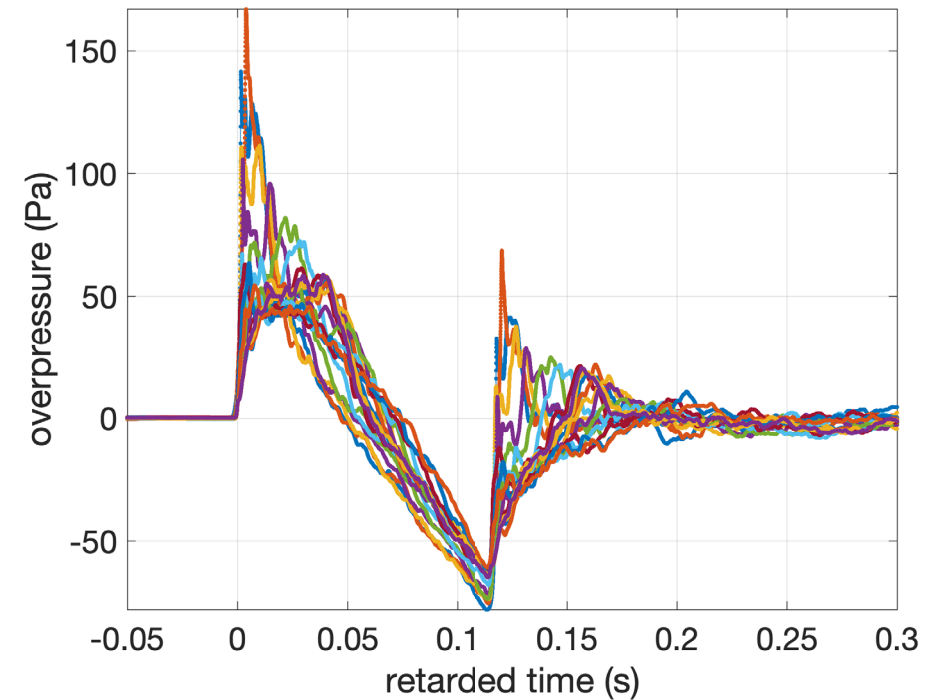
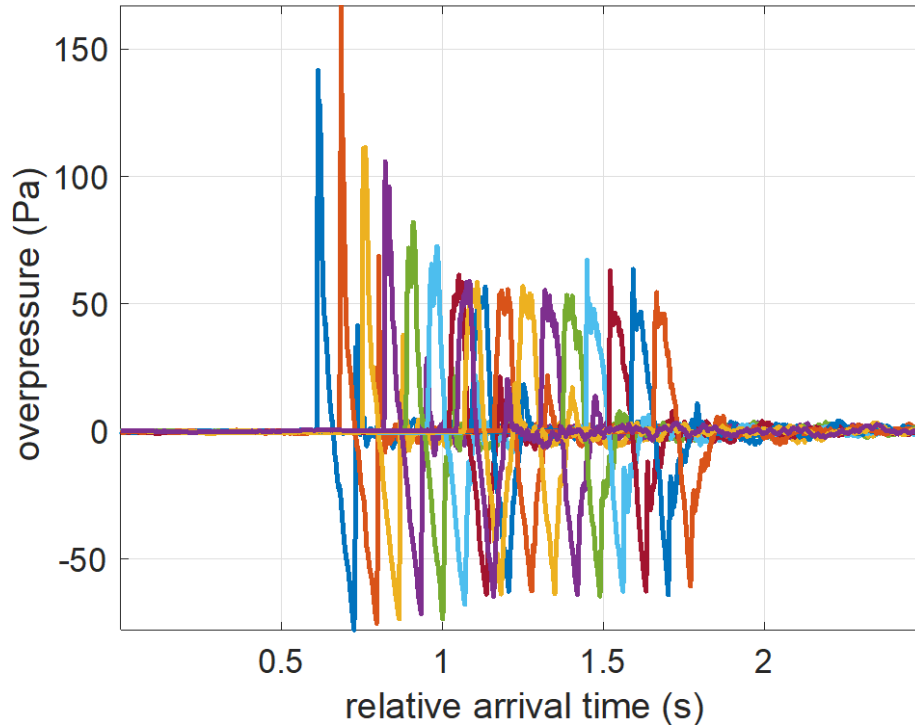


Outline



- Motivation of theoretical work
- Use of the Multiple Scattering Theory
 - Acousto-turbulence interaction
 - Review of the mean scattering effects
- Derivation of the root-mean-square acoustic pressure
- Incorporation of the scattering effects into a Burgers equation
- Modeling using the Burgers equation
- Qualitative comparison with measured data
- Conclusion

Motivation: Turbulence Effects on Measured Sonic Booms



Single Flight: Sonic Boom Arrivals

- SonicBAT Flight Test in Edwards AFB
- Measured booms from 18 microphones within a linear array, each ~30 m apart
 - Without turbulence, booms should have been nearly identical
- The variability among the booms is observable, especially when booms are aligned



Green's Function in Fourier Domain



➤ Linear Acoustic Wave Equations for Inhomogeneous Media

- $$\left(\nabla^2 - \frac{1}{\bar{c}^2} \frac{\partial^2}{\partial t^2}\right) g = -\delta(\mathbf{r} - \mathbf{r}')\delta(t - t') - \frac{2}{\bar{c}^2} \tilde{n}(\mathbf{r}) \frac{\partial^2 g}{\partial t^2}, \quad g = g(\mathbf{r}, \mathbf{r}', t, t')$$

➤ Acoustic Pressure

- $$p(\mathbf{x}, t) = \int dt' d^3x' g(\mathbf{x}, \mathbf{x}', t, t') s(t') \delta(x')$$

➤ Green's Function

- $$G(\mathbf{k}, \omega; \mathbf{r}') = G_o(\mathbf{k}, \omega) - G_o(\mathbf{k}, \omega) \int \frac{d^3k'}{(2\pi)^3} N_e(\mathbf{k} - \mathbf{k}') G(\mathbf{k}', \omega; \mathbf{r}') e^{i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}'}$$
- $$G_o(\mathbf{k}, \omega) = \frac{1}{k^2 - \kappa_o^2 - i\epsilon}, \quad \kappa_o = \frac{\omega}{\bar{c}}$$

➤ Fourier transform

- $$g(\mathbf{r}, \mathbf{r}', t, t') = \int \frac{d\omega d^3k}{(2\pi)^4} e^{-i\omega(t-t') + i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} G(\mathbf{k}, \omega; \mathbf{r}')$$



Turbulence Effects in the Burgers Equation(1)



Green's Function Solution

- Based on Multiple Scattering Theory [Ref1:Lonzaga (2023)]
- Ensemble averaging and Bourret approximation [Ref2:Bourret (1963)]

- $$\langle G(\mathbf{k}, \omega) \rangle = \frac{1}{k^2 - \kappa_0^2 - \tilde{\Sigma}(k) - i\epsilon}$$

- Acousto-turbulence interaction energy

- $$\tilde{\Sigma}(k) = 4\kappa_0^4 \int \frac{d^3 k'}{(2\pi)^3} G_o(\mathbf{k}', \omega) \Phi(\mathbf{k} - \mathbf{k}')$$

- Exponential correlation function $\varphi(\mathbf{r} - \mathbf{r}') = \sigma^2 e^{-|\mathbf{r} - \mathbf{r}'|/\ell}$

- $$\Phi(\mathbf{k}) = \frac{8\pi\sigma^2\ell^3}{(k^2\ell^2 + 1)^2}$$

Turbulence Effects in the Burgers Equation(2)



➤ Mean acoustic pressure and dispersion relation:

- $\langle P(x, \omega) \rangle = A(\omega) e^{i\kappa_o x} e^{-\alpha_s x}$
- α_s is the scattering coefficient and obtained from the dispersion relation
- Dispersion relation: $k_e^2 - \kappa_o^2 - \tilde{\Sigma}(k_e) = 0$
- $k_e = \kappa_o - i\alpha_s$

➤ Integration of scattering coefficient into the Burgers equation [Ref3:Lonzaga (2019)]

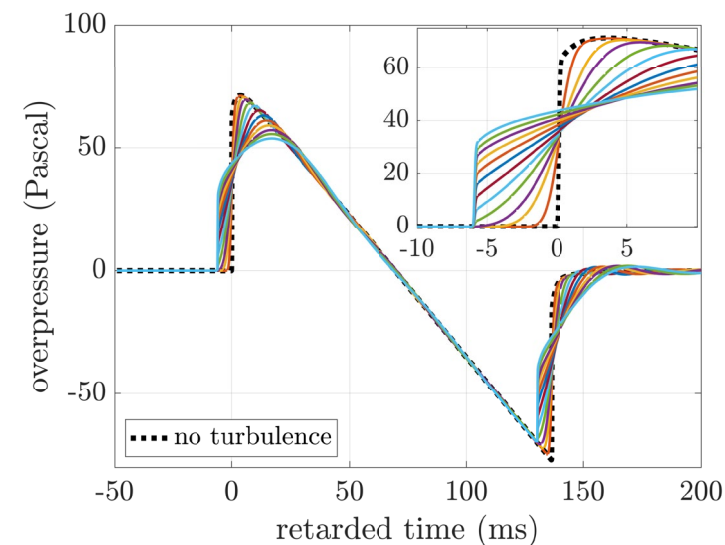
$$\frac{d\langle U \rangle}{dx} = \frac{i\omega\tilde{\beta}}{2} \int \frac{d\omega'}{2\pi} \underbrace{\langle U(x, \omega') \rangle \langle U(x, \omega - \omega') \rangle}_{\text{Nonlinearity}} - \underbrace{[\alpha_c + \alpha_v]}_{\text{Absorption}} \underbrace{+ \alpha_s}_{\text{Scattering from Turbulence}} \langle U(x, \omega) \rangle$$

Modeling Turbulence Effects Using Different Correlation Lengths

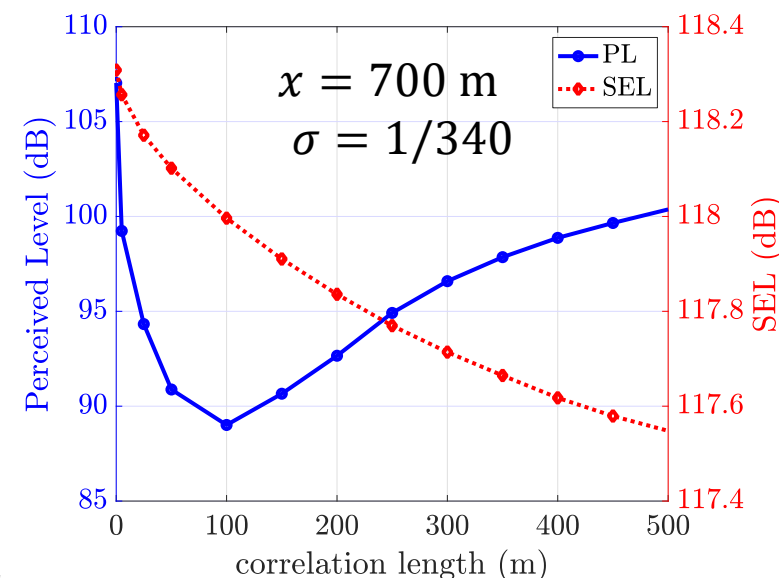


- Burgers equation was used to generate the model waveforms through turbulence fields
- Larger correlation lengths result in waveforms with smaller peak overpressures and larger rise times
- On average, turbulence weakens and thickens shocks
 - Energy reduction (SEL)
 - Loudness reduction (Perceived Level) until minimum is reached
 - Loudness starts to increase due to emergence of a weak shock to the left of the original shock
- Reduction consistent with acoustic measurements
- Results have been published with JASA [Ref1:Lonzaga 2023]

Sonic booms through turbulence with different correlation lengths



Sonic boom energy content (SEL) and loudness (PL)

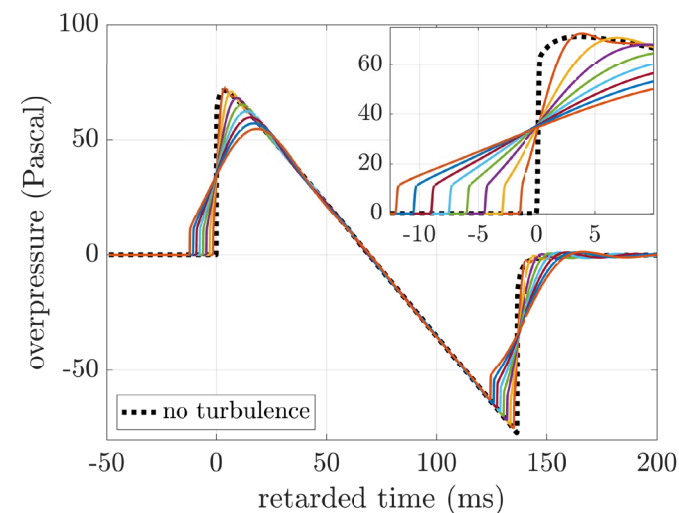


Modeling Turbulence Effects Using Different Variance

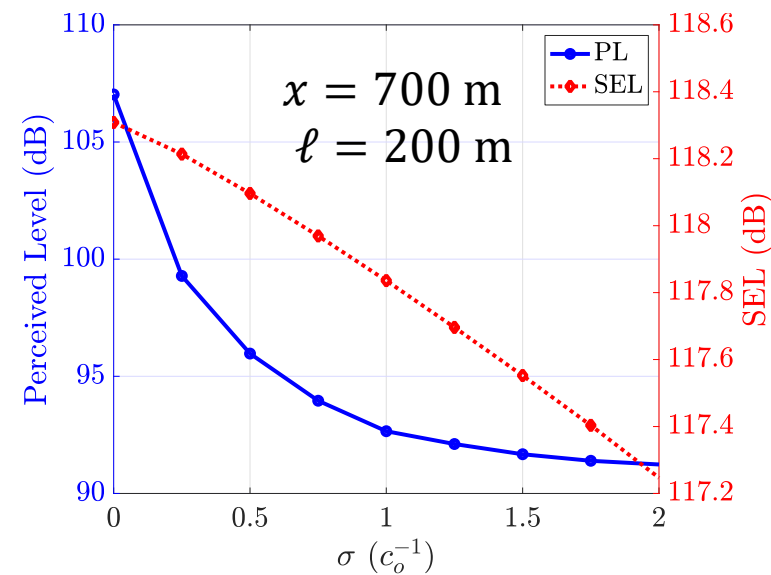


- Larger turbulence variances result in waveforms with smaller peak overpressures and larger rise times
- Turbulence weakens and thickens shocks
- Reduction of energy (SEL) and loudness (PL)
- Reduction consistent with sonic boom measurements

Sonic booms through turbulence with different variances



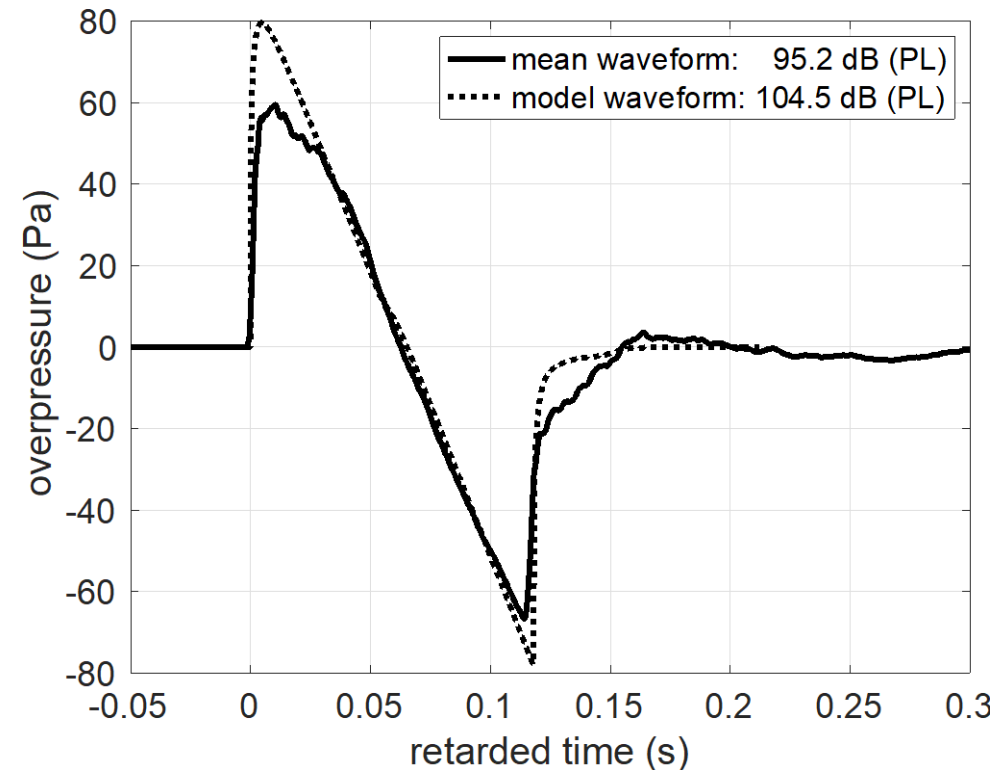
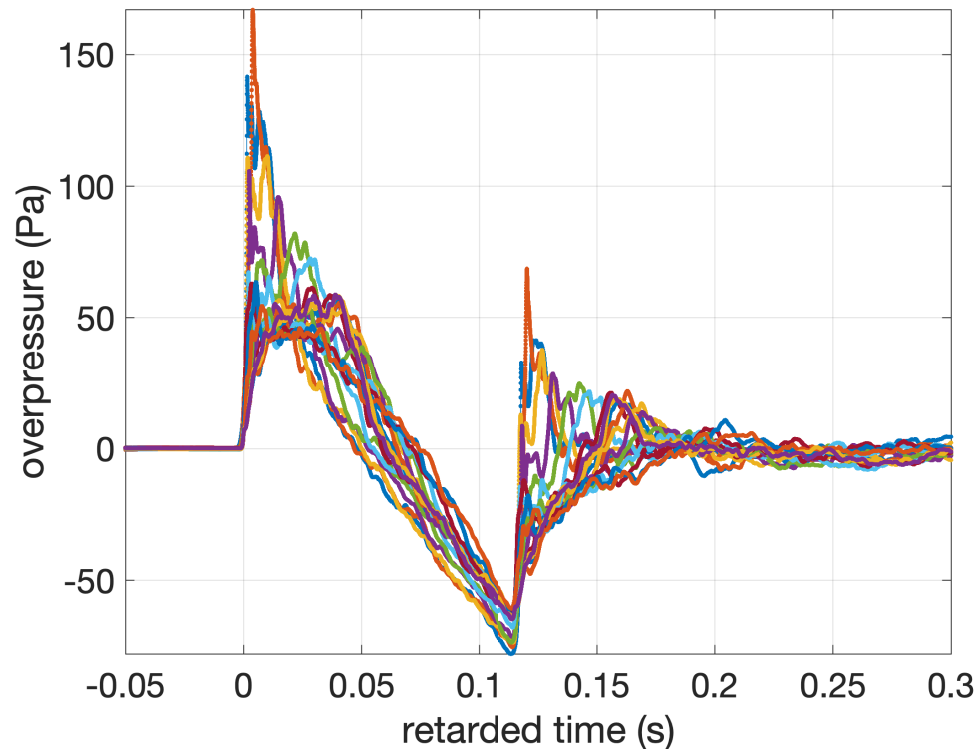
Sonic boom energy content (SEL) and loudness (PL)



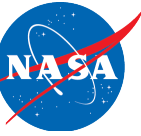
Mean-Valued Waveform From Recorded Sonic Booms



- Measured sonic boom waveforms from a steady supersonic flight pass,
 - Without turbulence, waveforms should be nearly identical
- Mean waveform is shown with model waveform without turbulence
- Loudness reduction of about 10 dB (PL)
 - Consistent with numerical simulations over a range of turbulence parameters during the flight test



Modeling the Variance of the Loudness of Sonic Booms



- Random acoustic pressure field as a sum of the mean $\langle P(x, \omega) \rangle$ and fluctuating $\tilde{P}(x, \omega)$ parts

$$P(x, \omega) = \langle P(x, \omega) \rangle + \tilde{P}(x, \omega)$$

- Variance of the acoustic pressure

$$\langle \tilde{P}^2(x, \omega) \rangle = \langle P^2(x, \omega) \rangle - \langle P(x, \omega) \rangle^2$$

- From Parseval's theorem, the energy content is given by

$$E(x) = \int \frac{d\omega}{2\pi} |P(x, \omega)|^2$$

- Loudness of the acoustic pressure is related to

$$L(x) = \int \frac{d\omega}{2\pi} |P(x, \omega)|^2 f(\omega)$$

- Weighting function $f(\omega)$

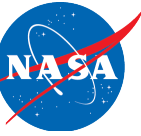
- For energy content and loudness of the mean-valued waveform, $P(x, \omega) \rightarrow \langle P(x, \omega) \rangle$

- For energy content and loudness of the root-mean-square waveform,

$$P(x, \omega) \rightarrow P_{\text{rms}}(x, \omega) = \sqrt{\langle P^2(x, \omega) \rangle}$$



Modeling the Variance Of The Acoustic Pressure



Calculation of the root-mean-square acoustic pressure $P_{\text{rms}}(x, \omega)$

- Use the Green's function

$$G(\mathbf{k}, \omega; \mathbf{r}') = G_o(\mathbf{k}, \omega) - G_o(\mathbf{k}, \omega) \int \frac{d^3 k'}{(2\pi)^3} N_e(\mathbf{k} - \mathbf{k}') G(\mathbf{k}', \omega; \mathbf{r}') e^{i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}'}$$

- Iterate the Green's function expression

- Calculate

1. $G(\mathbf{k}, \omega; \mathbf{r}') G(\mathbf{k}_1 - \mathbf{k}, \omega; \mathbf{r}')$

2. $\langle \hat{P}(\mathbf{k}, \omega) \hat{P}(\mathbf{k}_1 - \mathbf{k}, \omega) \rangle$

3. $\langle P^2(\mathbf{r}, \omega) \rangle = \int \frac{d^3 k_1}{(2\pi)^3} e^{i\mathbf{k}_1 \cdot \mathbf{r}} \int \frac{d^3 k}{(2\pi)^3} \langle \hat{P}(\mathbf{k}, \omega) \hat{P}(\mathbf{k}_1 - \mathbf{k}, \omega) \rangle$

- Derive the mean-square acoustic pressure

$$\langle P^2(x, \omega) \rangle = A^2(\omega) e^{2i\kappa_o x} e^{-\alpha_s x}$$

- Obtain root-mean-square acoustic pressure: $P_{\text{rms}}(x, \omega) = \sqrt{\langle P^2(x, \omega) \rangle}$

$$P_{\text{rms}}(x, \omega) = A(\omega) e^{i\kappa_o x} e^{-\alpha_s x/2}$$

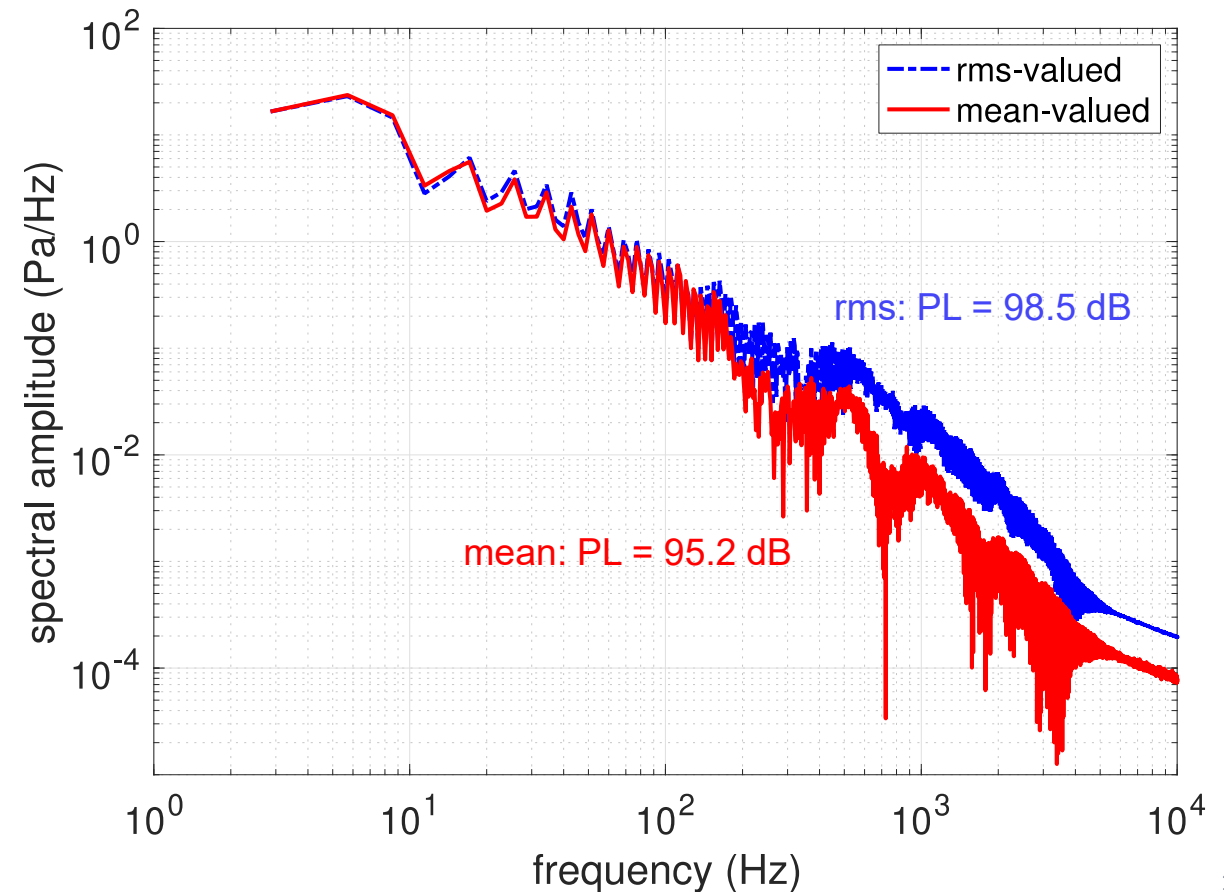
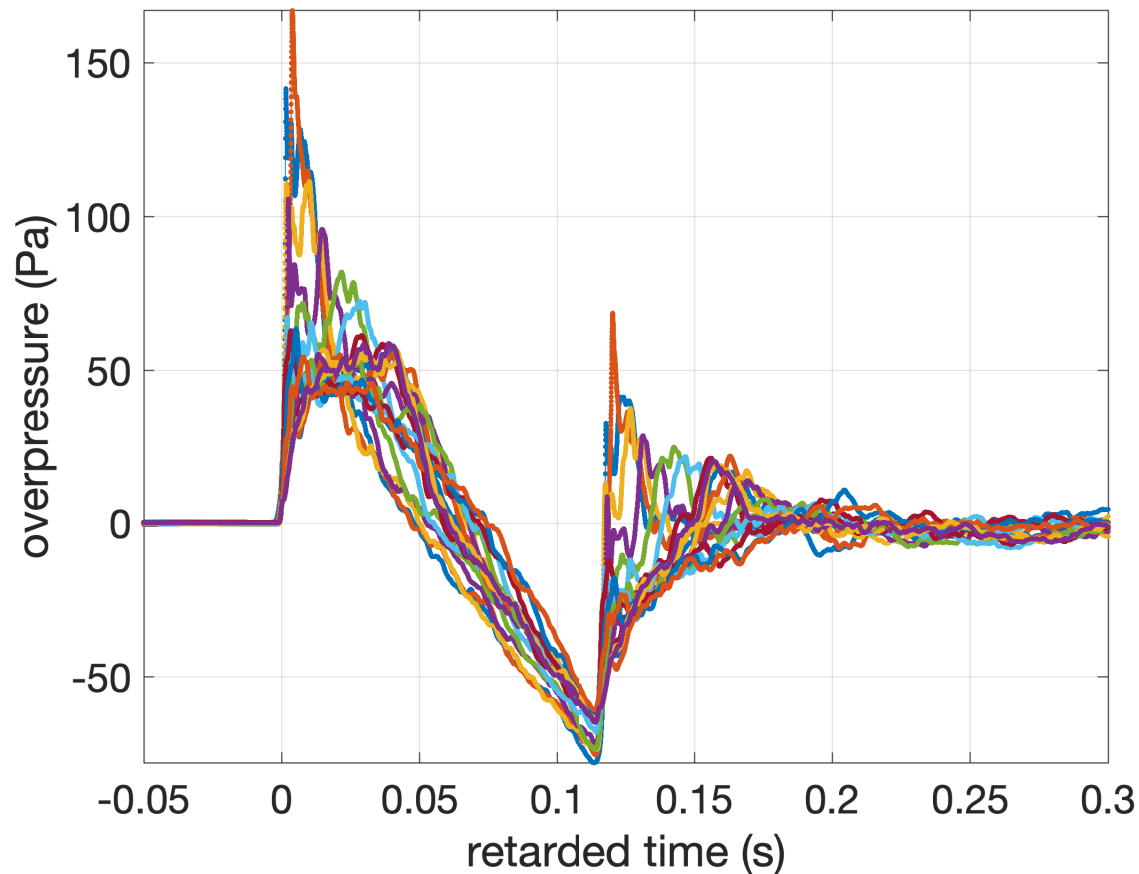
- Compare with the mean-valued acoustic pressure

$$\langle P(x, \omega) \rangle = A(\omega) e^{i\kappa_o x} e^{-\alpha_s x}$$

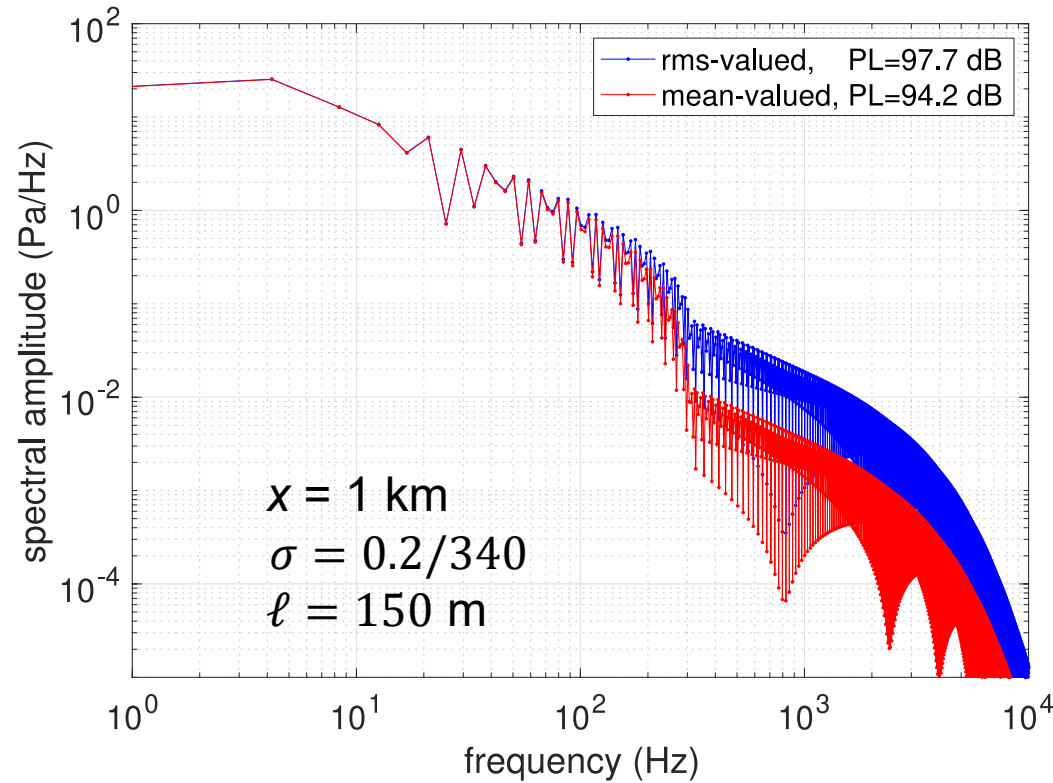
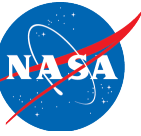
Measured Waveforms: RMS Acoustic Pressure



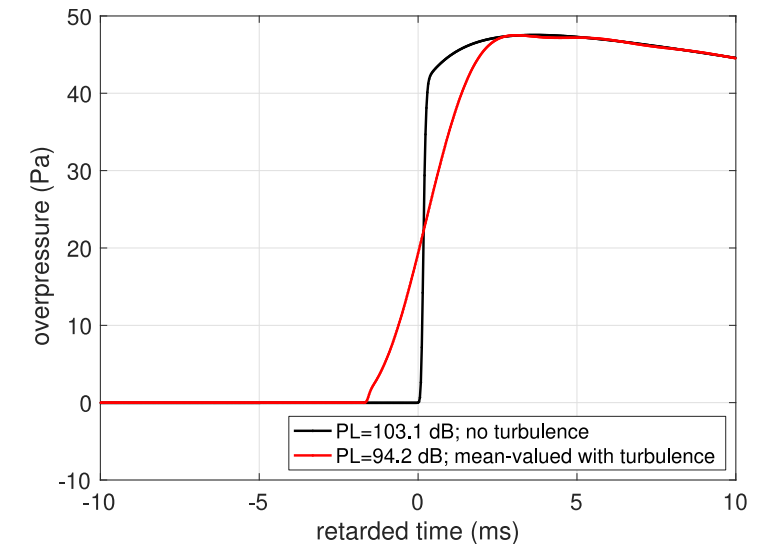
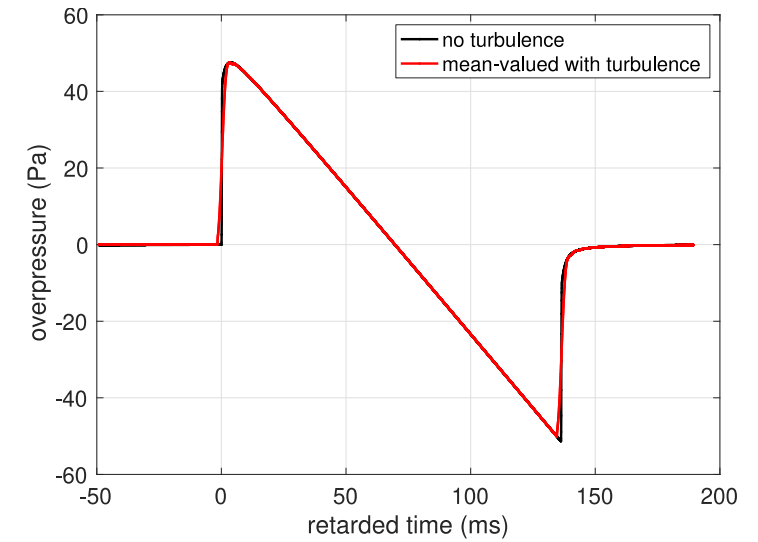
- Measured sonic boom waveforms from the SonicBAT Flight Test
- RMS spectrum has PL = 98.5 and is shown along with the mean-valued spectrum that has a PL = 95.2
- Demonstrating applicability of the methodology



Modeling RMS Acoustic Pressure



- RMS acoustic pressure has
- larger spectral amplitude at higher frequencies
 - larger PL value
- than the mean-valued acoustic pressure



- Ubiquity of turbulence effects in measured sonic booms
- Mean and variance of the loudness needed to bound loudness prediction
- Need for an efficient, Burgers-based sonic boom propagation solver
- ***Development of Multiple Scattering Theory to predict***
 - ***Mean-valued acoustic pressure and loudness***
 - ***RMS acoustic pressure and loudness***
- Future work to resolve if the loudness probability density function is Gaussian-distributed