

Ground Tests on the BOLT Geometry at Mach 7: Cross-Facility Comparison and Stability Analysis

Cameron S. Butler^{*}, Bradley M. Wheaton[†],
Johns Hopkins University Applied Physics Laboratory, Laurel, MD, 20723, USA

Loïc Sombaert[‡], Mathieu Lugin[§], Reynald Bur[¶], François Nicolas^{||}, Sébastien Esquieu^{**},
DAAA, ONERA, Paris Saclay University, F-92190 Meudon, France
CEA-CESTA, 15 Avenue des Sablières, 33116 Le Barp, France

Fei Li^{††}, Meelan Choudhari^{‡‡},
NASA Langley Research Center, Hampton, VA, 23666, USA

Pedro Paredes^{§§},
National Institute of Aerospace, Hampton, VA, 23666, USA

Sebastian Willems^{¶¶},
Institute of Aerodynamics and Flow Technology, German Aerospace Center (DLR), 51147 Köln, Germany

Vincenzo Russo^{***},
Department of Aerospace Engineering, University of Maryland, College Park, MD, 20742, USA

Christoph Brehm^{†††},
Department of Aerospace Engineering, University of Maryland, College Park, MD, 20742, USA

Transition measurements have been obtained through two experimental campaigns conducted independently by the German Aerospace Center and the French Aerospace Lab with the French Alternative Energies and Atomic Energy Commission on subscale models of the BOLT-1 flight experiment geometry. This paper details a cross-facility comparison of measurements obtained at Mach 7, as well as subsequent computational analysis. Infrared (IR) thermography measurements obtained by both campaigns have facilitated a global comparison of the transition front across facilities at analogous conditions, which are found to be in good agreement. High-frequency surface pressure fluctuation data demonstrate significant amplification of instabilities with Mack-mode characteristics in the outboard regions of the acreage. These measurements are compared to stability analyses of varying fidelity. The computational methods employed to characterize the boundary-layer transition phenomena include the traditional line-marching implementation of the Parabolized Stability Equations (PSE), 2D eigenvalue analysis coupled with PSE, and Adaptive Mesh Refinement Wavepacket Tracking. While the 2D eigenvalue analysis is found to predict instabilities, which correlate in terms of frequency and acreage location to the experimental measurements, the predicted amplification for these instabilities is lower than would typically be expected for transition. Line-marching results

^{*}Senior Professional Staff, 11100 Johns Hopkins Rd., Laurel, MD, 20723, AIAA Member

[†]Principal Professional Staff, 11100 Johns Hopkins Rd., Laurel, MD, 20723, AIAA Associate Fellow

[‡]ONERA/CEA-CESTA, Ph.D. Student

[§]ONERA, Research Scientist

[¶]ONERA, Research Director

^{||}ONERA, Research Scientist

^{**}CEA-CESTA, Aerospace Engineer

^{††}Aerospace Technologist, Computational AeroSciences Branch

^{‡‡}Aerospace Technologist, Computational AeroSciences Branch, AIAA Fellow

^{§§}Associate Research Fellow, Resident at Computational AeroSciences Branch, NASA LaRC. AIAA Associate Fellow

^{¶¶}Research Engineer, Supersonic and Hypersonic Technologies Department

^{***}Graduate Research Assistant, AIAA Member

^{†††}Associate Professor, 3179 Glenn L. Martin Hall, Member AIAA

for traveling crossflow produce the best match to the experimental transition front, with a consistent transition N factor of approximately 3 – 3.5.

I. Introduction

This paper will highlight a subset of activities within the North Atlantic Treaty Organization (NATO) Science and Technology Organization (STO) Applied Vehicle Technology (AVT) 346 working group “Predicting Hypersonic Boundary-Layer Transition on Complex Geometries.” A subtask of this working group is focused on transition due to the interaction of multiple instability modes, and for this subtask, the Air Force Office of Scientific Research (AFOSR) Boundary Layer Transition (BOLT-1) geometry has been selected as a focus geometry. The BOLT-1 geometry has previously been shown to produce a complex flow field in which multiple boundary-layer instability modes are expected to occur simultaneously on the low-curvature concave surface of the geometry [1].

Within the working group, new ground-testing campaigns were recently conducted by the German Aerospace Center (DLR) and the French Aerospace Lab (ONERA) with the French Alternative Energies and Atomic Energy Commission (CEA) on subscale models of the BOLT-1 flight experiment geometry. This paper presents a comparison of transition measurements obtained in the two facilities at Mach 7. These measurements include global characterization of the transition front, as well as direct measurement of boundary-layer instabilities.

The ground test results will be compared to stability analyses performed using different methods. Two experimental conditions at Mach 7 were identified for further computational study to be conducted collaboratively by the Johns Hopkins University Applied Physics Laboratory (APL), the NASA Langley Research Center (LaRC), the University of Kentucky, and the University of Maryland. The present analysis shall employ transition-prediction tools that are suited to the study of flowfields with strong variation in multiple dimensions, as encountered on BOLT. The first analysis technique to be considered is a combination of 2D eigenvalue analysis and planar-marching PSE (PMPSE) methodologies, which allow for disturbances to vary strongly in a plane, as opposed to just the wall-normal direction [2-4]. The PMPSE will be performed using two different tools: the Sandia LSTPACK code and the NASA LaRC Matrix Forming Instability Analysis (MAFIA) code. The second technique used to study the BOLT flowfield stability is referred to as Adaptive Mesh Refinement Wavepacket tracking (AMR-WPT). Comparison of the PMPSE and AMR-WPT results will be made to legacy Linear Stability Theory (LST) methods using the NASA LaRC Stability and Transition Analysis Code (LASTRAC) and line-marching PSE (LMPSE) results from MAFIA, which assume spanwise homogenous flow.

II. Background and Motivation

The BOLT-1 (referred to in the remainder of this paper as simply “BOLT”) research effort began in 2017 when the United States Air Force Office of Scientific Research (AFOSR) proposed a new complex geometry as a challenge for the boundary-layer transition research community. This new geometry was significantly more complex than previous conical geometries studies within the research community, and was intended to assess its capabilities for prediction of the physics mechanisms of transition on a new geometry [5]. The geometry features low-curvature concave surfaces with highly-swept leading edges. The geometry has been the subject of three flight experiments as well as a significant community-wide ground test and computational research effort.

The first flight experiment research effort was led by the Johns Hopkins Applied Physics Laboratory (APL) beginning in 2017 and the fundamental research into transition mechanisms as part of the flight experiment design consisted of a large team of university partners including Purdue University [6], Texas A&M University [7, 8], and the University of Minnesota [9]. NASA LaRC also conducted numerous wind-tunnel experiments [10] and an experiment on roughness-induced transition for the flight [11]. The flight experiment occurred in 2021 but suffered an instability that precluded the vehicle from reaching the intended hypersonic conditions [12-15]. A repeat of the flight experiment called BOLT-1B is scheduled for mid-2024 using a new sounding rocket vehicle [16]. At the onset of the program in 2017, very little was known about the dominant transition mechanisms that would occur on this new and highly complex geometry.

Several subscale wind-tunnel experiments were conducted by the BOLT team very early in the project, with most of these entries focused on Mach 6 conditions. Experiments in the NASA LaRC 20-Inch Mach-6 Tunnel showed onset of transition on the outboard regions of the geometry [10]. At the Purdue Mach-6 Quiet Tunnel under noisy flow, the transition onset occurred in the same outboard region [6]. It was first noted that, under quiet flow conditions in the Purdue tunnel, the flowfield appeared vastly different than under noisy flow and was characterized by streamwise streak structures. Off-body measurements by Texas A&M [7] along with computations by the University of Minnesota [9]

confirmed the ability of certain streamwise vortex structures near the BOLT geometry centerline to harbor convective instabilities. It is important to note that the maximum quiet flow Reynolds numbers of both the Purdue and Texas A&M quiet facilities were, at the time, insufficient to observe the progression of these instabilities to transition under quiet flow. Additional experiments including full-scale tests at CUBRC are documented in Berridge et al. [17, 18].

Initial boundary-layer stability calculations of the BOLT geometry used standard linear theory stability codes LASTRAC (by NASA LaRC) and EPIC (by Texas A&M). It was shown for both codes in Moyes et al. [8] and Wheaton et al. [19] that both crossflow and second-mode growth were expected at various ground test conditions, and that these modes both had predicted growth within the same region of the geometry (the outboard region). However, in general, the individual second-mode and crossflow N-factors in this outboard region, when matched to available noisy tunnel data, have shown transition to occur at low N-factors [17].

It has remained challenging to develop a best practice approach for “stability methods” capable of use to match observed transition on the BOLT geometry. Knutson et al. [9] showed that numerical dissipation is important for developing a resolved mean flow on this geometry. Typical smaller grids with high dissipation numerical methods tend to dissipate the streamwise structures that are likely important to transition prediction on BOLT. And despite having measured near-centerline instabilities within the center vortex structure, it has remained difficult to measure instabilities within the outboard region of the BOLT geometry. The preflight BOLT test campaign was unable to find measurements of any instabilities with frequencies matching the predicted second-mode frequencies from the LASTRAC/EPIC despite attempts in multiple facilities. Because of the likely importance of spanwise variation in the flow, it is likely that 2D eigenvalue-type stability approaches are necessary to better match the experiments [20].

The motivation for the NATO AVT-346 working group was to use the BOLT geometry as a focus of study for mixed-mode transition, where multiple instability mechanisms are expected within the same region of the geometry. New wind-tunnel data were obtained by this group at DLR and ONERA on the BOLT geometry indicating a high-frequency instability mode at Mach 7 within the outboard region under noisy flow. The working group sought to generate new computations at these conditions and to explore further the best practices for mean-flow generation and stability analysis that might explain the frequencies observed in the new tests which were consistent with Mack-mode instabilities. The computational analysis presented herein reproduce the experimentally observed instability frequencies, but suggest that they are not the leading cause of transition for these conditions.

III. Experimental Setup

A. DLR H2K Campaign

1. Facility

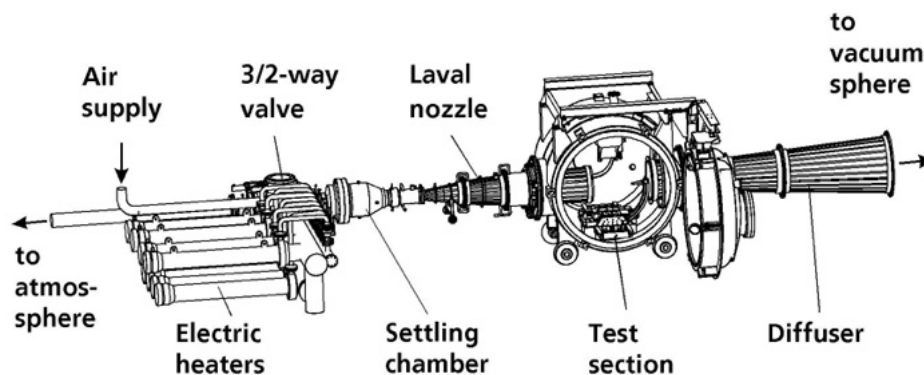


Fig. 1 Schematic of H2K.

This section describes one of the two experimental campaigns examined in this work, conducted in the Hypersonic Test Section (H2K) at the German Aerospace Center (DLR) in Cologne [21]. It is a blow-down wind tunnel with a free jet test section and a test time of 30 s (Fig. 1). Several water cooled contoured nozzles with an exit diameter of 600 mm are available for Mach numbers between 5.3 and 11.2. The test gas is dry air and heated with electrical resistance heaters beyond 1000 K. Figure 2 shows the test section with the model installed.

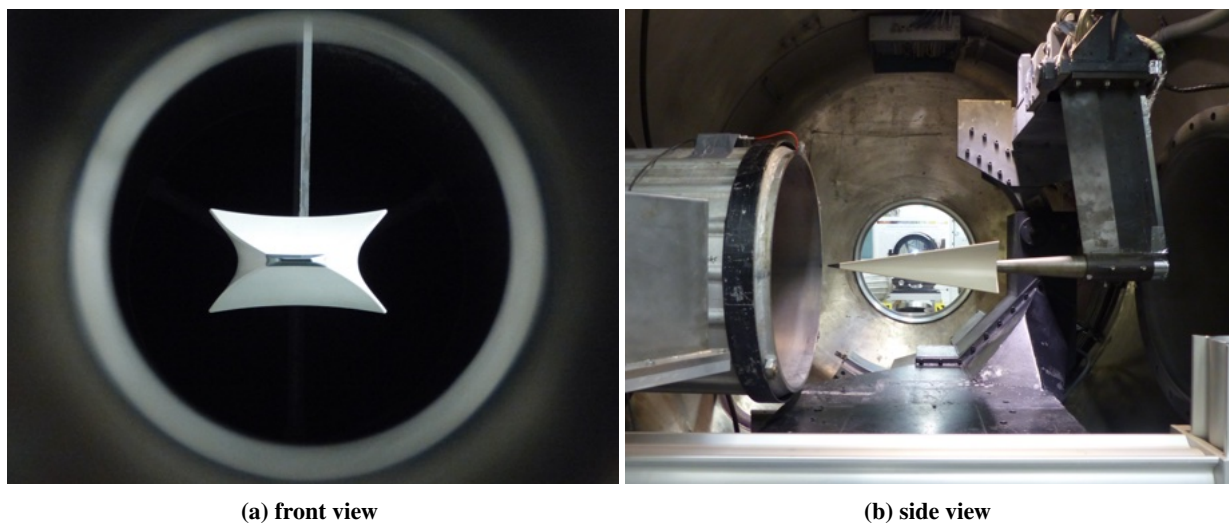


Fig. 2 BOLT model in H2K. [Source: DLR]

2. Model

The model used for the experiments (Fig. 3) is a 1:2 scale of the BOLT-1 geometry. The first design goal was to allow an instrumentation similar to the BOLT flight model. The second goal was to use as few parts as possible to reduce the number of intersections (and hence possible sources of disturbances) and to put the intersections where interferences with the measurements were unlikely. The third goal was to use polyether ether ketone (PEEK) – a low-conductivity, high-temperature plastic – wherever possible to allow infrared (IR) thermography of the surface. Therefore, the model consists of just three main parts: a steel nose, a (natural) PEEK main body, and a PEEK lid. Most of the sensors are integrated in the lid, which forms the top side of the model. The intersection to the main body is just below the upper shoulder on the sides. A steel flange forms the back of the model, with which it is mounted onto a stiff sting to avoid model vibrations during the tests. Because the model is attached to the standard H2K support arm, different angles of attack and yaw angles can be set easily.

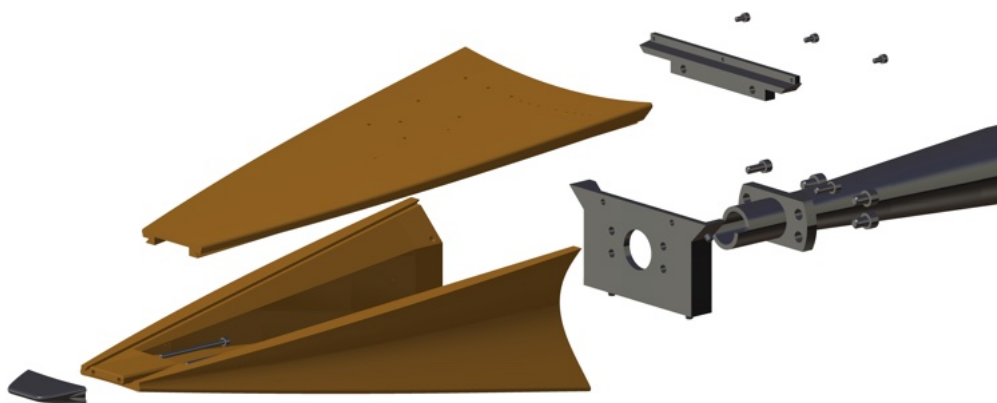


Fig. 3 Exploded view of the H2K wind tunnel model.

3. Instrumentation

The sensor instrumentation is based on the instrumentation of the flight model. There are 8 Piezotronics 132A31 sensors (PCB) for high-speed pressure measurements at the same locations as in the flight experiment. They are connected to PCB 482C05 signal conditioners. Their output signals are processed with ADLINK PXI-9816D/512 digitizers at 5 MHz sampling frequency. Ten Kulite® absolute pressure transducers XCE-080-0.35BarA (KPT) duplicate the last row and positions in the symmetry plane of midspeed pressure sensors of the flight model. They are connected

to a National Instruments (NI) PXIe-4331 bridge module and sampled at 100 kHz. The static pressure measurements are performed with 5 psi Initium pressure scanners from Pressure Systems, Inc. (PSI) at 50 Hz sampling rate. Figure 4 sketches the sensor positions on the model surface and the sensor numbering.

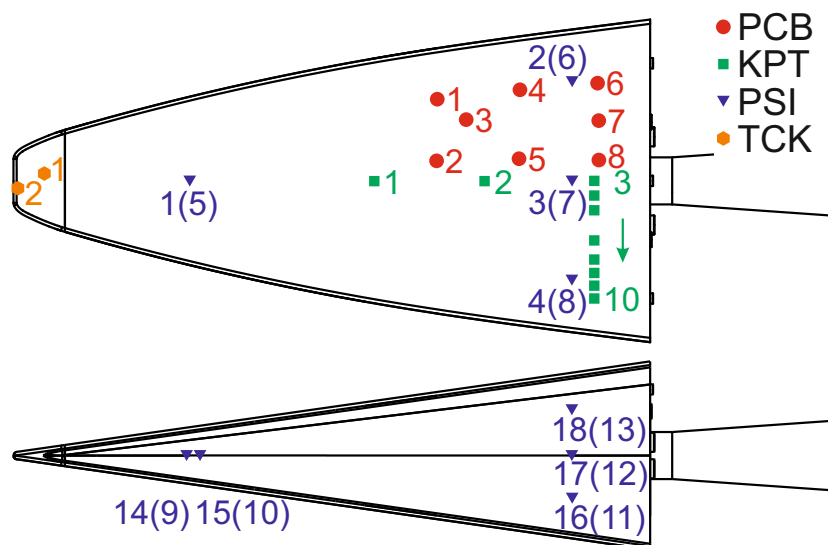


Fig. 4 Positions and numbering of the sensors of the H2K model. Numbers in brackets refer to the counterparts on the opposing surfaces.

The temperature distribution of the top side of the model was recorded with an IRCAM VELOX 1310k SM infrared camera with a resolution of 1280×1024 pixels and one side was monitored with an Infratec ImageIR 8300 infrared camera with 640×512 pixels resolution. Both cameras were sampled at 100 Hz. Schlieren images were taken with a Nikon Z6 with 6048×4024 pixels resolution and $\frac{1}{3200}$ s exposure time. Because the surface temperature of the metallic nose cannot be determined based on the IR measurements, there are two type K class 1 (TCK) thermocouples from MTB Sensor-Technik GmbH placed 3 mm and 21 mm from the leading edge inside the nose. They are sampled at 50 Hz with an NI PXIe-4353 temperature module.

The flow temperature in the settling chamber is also measured with a type K class 1 thermocouple from MTB sampled at 50 Hz with an NI PXIe-4353 temperature module. In the settling chamber, the pitot pressure is measured with a General Electric (GE) UNIK 5000 pressure sensor. Zero corrections of this sensor as well as the PSI and KPT sensors are done before each run using a Baratron consisting of a 398HD pressure transducer and a 270B-4 signal conditioner as reference. The signals of the GE and Baratron sensors are digitized with an NI PXI-6289 multifunction module at 50 Hz sampling rate.

4. Test conditions

Experiments in H2K were performed at Mach 5.3, 6.0 and 7.0 with unit Reynolds numbers equal to $3 \times 10^6 \frac{1}{m}$, $6 \times 10^6 \frac{1}{m}$ and $12 \times 10^6 \frac{1}{m}$. The angle of attack has been varied between -8° and 8° and the yaw angle between -4° and 4° , though this study considers only conditions for which angle of attack and yaw were 0° . The conditions of the runs used for the comparisons within this paper are given in Table II.

5. Processing

The PCB spectra from the H2K experiments are computed from 1 s of the normalized pressure data starting 5 s after wind tunnel start to guarantee stable flow conditions and damping of all model vibrations caused by wind tunnel start using Welch's method. The sequences are divided into 199 blocks with 50,000 samples each. Adjacent blocks overlap by 50 %. Each block is multiplied with the Hann function and afterward the power spectral density is computed. Finally, the root-mean-square average of all blocks is computed.

Table 1 Reservoir and freestream conditions for DLR H2K runs. (Here the viscosity of air is computed with the standard Sutherland's law and not the modification usually used at H2K, to be compatible with the R2Ch data and the simulations.)

Run	Metrology	M_∞	P_0 , bar	T_0 , K	P_∞ , Pa	T_∞ , K	U_∞ , m.s ⁻¹	Re_∞ , m ⁻¹	Re_L
23	IR + PCB	7.013	7.32	589.5	174.8	54.4	1037.0	3.27×10^6	1.41×10^6
24	IR + PCB	7.047	14.52	596.7	336.2	54.6	1043.7	6.28×10^6	2.72×10^6
25	IR + PCB	7.104	29.35	583.4	645.5	52.6	1032.8	12.94×10^6	5.60×10^6

The stereo IR camera setup is calibrated using a calibration target from Carl Zeiss GOM Metrology GmbH. The IR images of the videos are then mapped onto a 3D grid with the in-house tool ROBOT tracking the PCB and Kulite[®] sensors as markers. Heat-fluxes are computed solving the 1D heat equation for all nodes of the grid using the DLR in-house tool VisualHeatflow and the temperature-dependent material properties.

B. ONERA R2Ch Campaign

1. Facility

This section describes the joint ONERA/CEA-CESTA experimental campaign that was conducted within the cold hypersonic blow-down facility R2Ch, depicted in Fig. 5, located at the ONERA Meudon Center. It is a conventional wind tunnel, capable of sustaining hypersonic conditions for durations ranging from 10 to more than 60 seconds. A set of contoured axisymmetric nozzles with constant exit diameter (0.327 m) enable freestream Mach numbers of 5, 6, and 7. Reservoir pressure typically ranges from 1 to 80 bars. The gas (dry air) is heated by a Joule-effect heater to prevent air liquefaction in the test section. Reservoir temperatures can thereby exceed 700 K. Following the passage through the converging-diverging nozzle, the gas enters the test section and terminates in a 500 m³ vacuum tank. Determining freestream properties assumes isentropic relations and Sutherland's law of viscosity.

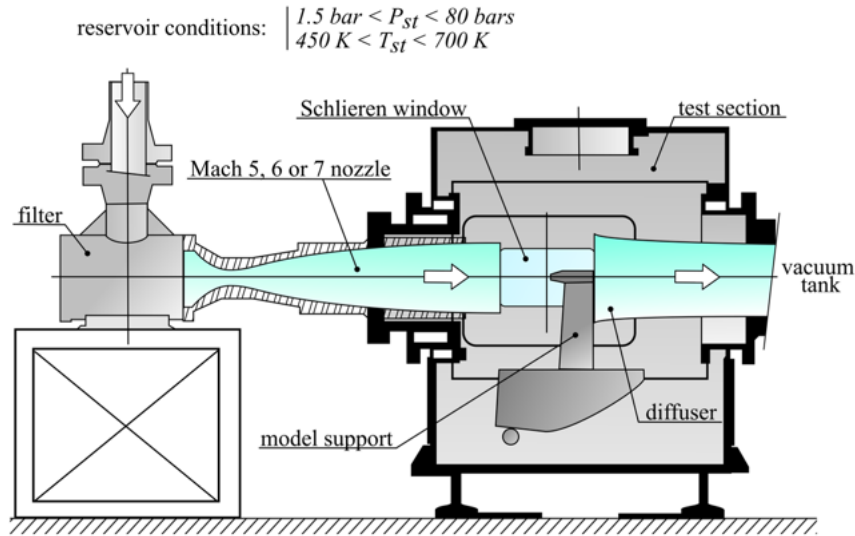


Fig. 5 Schematic of the R2Ch facility.

2. Model

A 1/3 scale BOLT model was designed for the campaign. It has an overall length of 288.67 mm. It is composed of three parts: one of the main surfaces of interest is made from black PEEK material, while the nose and the remaining body were fabricated using steel (Fig. 6). Efforts were made during machining to minimize steps at the junctions

Table 2 Reservoir and freestream conditions for ONERA R2Ch runs.

Run	Metrology	M_∞	P_0 , bar	T_0 , K	P_∞ , Pa	T_∞ , K	U_∞ , m/s	Re_∞ , m ⁻¹	Re_L
5314	IR	7	7.9	511.8	190.3	47.4	966.1	4.48×10^6	1.29×10^6
5348	PCB	7	8.5	516.4	204.4	47.8	970.3	4.74×10^6	1.37×10^6
5299	IR	7	17.5	539.5	423.8	49.9	991.8	9.13×10^6	2.64×10^6
5350	PCB	7	19.2	560.3	463.1	51.9	1010.7	9.36×10^6	2.70×10^6
5296	IR	7	41.2	582.8	994.6	54.0	1030.9	18.82×10^6	5.43×10^6
5349	PCB	7	40.4	588.1	977.0	54.4	1035.5	18.2×10^6	5.26×10^6

between the nose and the main surfaces, given that such steps could trigger flow transition. Pretesting measurements revealed relatively small step junction heights between the nose and PEEK surface (approximately 10 μm), which did not significantly impact flow behavior. No step junction was identified between the nose and the steel surface. Furthermore, careful attention was given to reducing surface roughness (steel main surface: $Ra < 0.2 \mu\text{m}$ and PEEK main surface: $Ra < 0.5 \mu\text{m}$, where Ra denotes the average of roughness height deviations from the mean).

The instrumentation involves two distinct high-frequency pressure measurement technologies: 11 PCB132-B38 sensors and 11 Kulite[®] transducers, all mounted on the steel surface. The cross-facility comparison detailed in this paper focuses mostly on the PCB transducers due to their suitability for hypersonic instability measurement. Static pressure data for the PCB transducers were acquired from 11 pressure taps symmetrically positioned on the PEEK surface. Although the PCB layout is based on the original APL BOLT model [22], differing from the DLR layout, it still enables meaningful comparisons (see Fig. 6). Note that the sensor locations in this figure have been scaled by the model lengths. To ensure accurate measurements on the curved surfaces of the BOLT model, an epoxy resin cap was applied to the top of the sensors and then machined to match the surface in order to avoid any steps. This resin facilitates a flush-mounted integration of the sensors and provides thermal protection without compromising their frequency response [23].

Three 4-channel PCB 482-C05 signal conditioners were employed to maintain a constant current source for the sensors. High-frequency signals were recorded over a 2 s duration at a sampling rate of 3.3 MHz, resulting in 6.6 million data points. This data acquisition was achieved using a National Instrument PXIe6376 card with 16-bit resolution. Power Spectral Densities (PSDs) were computed using Welch's method, where the signal was divided into 400 blocks, each consisting of 33333 samples, with a 50% overlapping factor ($\Delta f = 100 \text{ Hz}$). These blocks were then multiplied by a Hanning window. To differentiate aerodynamic content from the noise, prerun measurements were consistently conducted prior to each test.

A FLIR SC7600 midinfrared (2.5 – 5 μm) camera with 640 $\times$ 512 pixels resolution was used to measure the temperature distribution across the model during the experiments. To get precise temperature measurements, each pixel is related to its 3D coordinates by projecting the point of view of the camera on a 3D mesh. Using the PEEK properties that have been characterized by the French National Laboratory of Tests and Metrology (LNE) for different angles of observation, the local emissivity can be computed over the entire surface. The wall heat flux is then determined using FLUX1D, which solves the one-dimensional heat equation, assuming semi-infinite material thickness and temperature-dependent thermal properties. Further details on the method can be found in [24, 25].

3. Test conditions

Three Reynolds numbers are examined in the context of this cross-facility comparison: $Re_L \approx 1.3 \times 10^6$, 2.6×10^6 , and 5.4×10^6 . The test conditions for the R2Ch experiment are detailed in Table 2. To ensure the highest precision of the two diagnostics, it has been decided to use the data from two separate experimental runs for each Reynolds number of interest. One is dedicated to the acquisition of the IR camera and another focuses on PCB measurements.

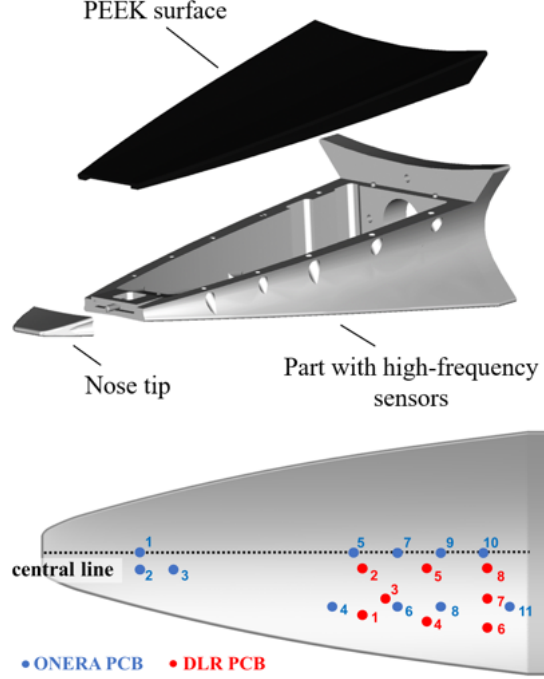


Fig. 6 Details on the ONERA 1/3 scale BOLT model with (top) an exploded view of the ONERA model and (bottom) a comparison between ONERA and DLR PCB layout.

Table 3 Freestream conditions for computational study.

Condition	Mach	$Re_\infty [\times 10^6 \text{ m}^{-1}]$	$Re_L [\times 10^6]$	$\alpha/\beta [^\circ]$	$P_\infty [\text{Pa}]$	$T_\infty [\text{K}]$	$U_\infty [\text{m/s}]$	$\rho_\infty [\text{kg/m}^3]$	$T_w [\text{K}]$
Run 23	7.013	3.076	1.33	0	174.8	54.41	1037.0	0.01119	298
Run 24	7.000	5.664	2.45	0	314.1	55.55	1045.8	0.01970	298

IV. Computational Methodology

The computational component of this study includes laminar base flow computations and stability analyses conducted independently by the APL and the NASA LaRC. The BOLT flowfield includes streaks that are known to be highly sensitive to the combination of numerical scheme and spatial grid resolution. The decision to perform parallel base-flow computations was made to gain added confidence in the laminar basic state and subsequent stability analyses. These computations were performed for the 1/2 scale model tested in H2K. Precise freestream conditions for the two cases of interest in this study are given in Table 3 meant to replicate the low and medium Reynolds number conditions studied experimentally. Note that the inflow density used for the Run 24 computations is approximately 8 percent lower than the experimental value.

A. Baseflow Computation

The fully-structured grid employed for the APL base-flow computations was generated using the GoHypersonic® LINK-3D mesh generation tool. This quarter-symmetry grid consisted of 328 million cells, with 1474 nodes along the centerline (streamwise direction) and 751 nodes in the spanwise direction between the centerline and leading edge. These gridding parameters are summarized in Table 4.

The APL laminar base-flow computations were carried out with US3D, an unstructured, finite-volume, compressible, Navier-Stokes solver developed by Candler et al. [26] at the University of Minnesota. First-order implicit time integration with the data-parallel line relaxation method was used to march the solutions to steady state. Inviscid fluxes were evaluated using a fourth-order, low-dissipation, kinetic-energy-consistent scheme with a dissipation switch [27]. The gas was modeled as calorically perfect with viscosity evaluated using Sutherland's law.

Table 4 Grid parameters used in this study.

	Cells [$\times 10^6$]	Streamwise Nodes	Spanwise Nodes (Main Surface)	Leading-edge Nodes (Spanwise)	Wall-normal Nodes
APL Grid	328	1549	751	85	251

In order to assess sensitivity of the laminar basic state to grid resolution and numerics, additional computations were performed using a NASA DNS-WENO code. The APL solution was used as a precursor simulation to interpolate onto the inflow boundaries for this calculation. Wu and Martin [28] provide a full discussion of the governing equations and numerical algorithms employed in the DNS-WENO code. The inviscid fluxes from the governing equations are computed using Jiang and Shu's [29] seventh-order weighted essentially nonoscillatory (WENO) finite-difference method, and the DNS-WENO code additionally contains an improved strategy that decreases the numerical dissipation via suitable limiters [30]. During the simulation, an absolute limiter on the WENO smoothness measurement and a relative limiter on the overall variation are used simultaneously. A fourth-order central difference scheme is used to discretize the viscous fluxes, and a third-order low-storage Runge-Kutta scheme is used for temporal integration [31]. The DNS-WENO grid was 571x833x256 (streamwise x spanwise x wall normal), starting just downstream of the nose.

B. Stability Analysis

1. Line-marching PSE

The majority of the stability results used for transition estimation in this paper was obtained using the line-marching form of linear, parabolized stability equations (LMPSE). The computational coordinates are defined as an orthogonal, body-fitted coordinate system, with (ξ, η, ζ) denoting the streamwise, wall-normal, and spanwise coordinates, respectively, and (u, v, w) representing the corresponding velocity components. The Cartesian coordinates are represented by (x, y, z) . Density and temperature are denoted by ρ and T , respectively. The PSE approximation to the linearized Navier-Stokes equations is based on assuming linear perturbations that are harmonic in time and spanwise direction and isolating the rapid phase variations in the streamwise direction via the following disturbance ansatz

$$\phi(\xi, \eta, \zeta, t) = \psi(\xi, \eta) \exp \left[i \left(\int_{\xi_0}^{\xi} \alpha(\xi') d\xi' + \beta\zeta - \omega t \right) \right], \quad (1)$$

where β denotes the spanwise wavenumber at the reference streamwise position, ξ_0 , ω is the angular frequency of the perturbation, and the unknown, streamwise varying wavenumber $\alpha(\xi)$ is determined in the course of the solution by imposing the additional constraint,

$$\int_{\eta} \hat{\mathbf{q}}^* \frac{\partial \hat{\mathbf{q}}}{\partial \xi} h_{\xi} h_{\zeta} d\eta d\zeta = 0, \quad (2)$$

which ensures that the amplitude functions $\hat{\mathbf{q}}(\xi, \eta) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$ vary slowly in the streamwise direction in comparison with the phase term $\exp \left[i \int_{\xi_0}^{\xi} \alpha(\xi') d\xi' \right]$. The curvature terms are enclosed in the streamwise and spanwise metric terms denoted by h_{ξ} and h_{ζ} , respectively. Substituting Eq. (1) into the linearized Navier-Stokes equations and invoking scale separation between the streamwise coordinate and the other two directions to neglect the viscous terms with streamwise derivatives, the PSE are obtained. Further details are given by Ref. [32].

Once a suitable set of integration trajectories are defined, the LMPSE analysis of the BOLT flowfield proceeds along similar lines as the LMPSE analysis in a planar/axisymmetric or infinite-span configuration. Specifically, the leading-order nonparallel effects and the effects of curvature are retained. Different integration trajectories are used for LMPSE analyses of the Mack's second-mode (MM) instabilities and the crossflow (CF) disturbances, respectively, as described later in the results section. In general, the CF disturbances have frequencies of less than 40 kHz and the MMs have frequencies above 150 kHz and less than 250 kHz. Computations are performed for a discrete set of frequencies (with an increment of 2 kHz for the CF disturbances and 5 kHz for MMs).

2. 2D Eigenvalue Analysis and Plane-Marching PSE

For flows with strong variation in the spanwise direction (as is the case here), instability mode shapes must be allowed to vary in both the wall-normal and spanwise directions. 2D eigenvalue analysis is analogous to classical LST as

a form of local eigenmode analysis, but allows the disturbance shape function to vary in a plane. In the plane-marching PSE (PMPSE) approach, the disturbance amplitude function of Eq. (1) depends on the three spatial directions,

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \hat{\mathbf{q}}(\xi, \eta, \zeta) \exp \left[i \left(\int_{\xi_0}^{\xi} \alpha(\xi') d\xi' - \omega t \right) \right], \quad (3)$$

and the parabolic integration of the disturbance equations involves the solution of a two-dimensional partial differential equation system. Further details regarding the numerical formulation may be found in the literature [4, 32-35]. The boundary-layer streaks formed near the leeward symmetry plane over the HIFiRE-5 elliptic cone [36] were the first instance of such analysis being applied to slender, hypersonic configurations and its relevance to multiple aspects of the BOLT and the BOLT-2 configurations has also been established [20, 37]. 2D eigenvalue analysis and PMPSE were conducted separately by both APL and NASA LaRC. APL utilized the Sandia LSTPACK code while NASA LaRC employed the MAFFIA code.

The APL stability domain was resolved with 401 nodes in the spanwise direction and 151 nodes in the wall-normal direction with clustering near the wall. Dirichlet boundary conditions were applied to both left and right boundaries with damping layers to linearly decay disturbances within 10 cells of the boundary.

3. Adaptive-Mesh-Refinement Wavepacket-Tracking Immersed-Boundary Method

The AMR-WPT method has been developed as an efficient alternative to conventional DNS to perform transition studies, and in combination with an Immersed-Boundary Method (IBM) can offer a high degree of flexibility, especially when treating complex geometries, essentially eliminating the time-consuming process of generating a computational mesh. The governing equations are the nonlinear disturbance flow equations that are obtained by writing the compressible Navier-Stokes equations in disturbance flow form. This allows one to decompose the flow field into a mean flow and disturbance flow solution. The AMR algorithm is designed to automatically track relevant features in the disturbance flow field, e.g., wavepackets. These wavepackets can be introduced through a pulse disturbance and then propagate along the boundary layer. With this approach, the grid resolution is concentrated around the wavepacket, thus significantly reducing the computational expense. The refinement strategy relies on a physics-based tracking criteria involving a combination of primitive variables, e.g., pressure, temperature and velocity, and their gradients (see Browne et al. [38, 39] for more details). In order to simulate the disturbance flow field around complex geometries on a Cartesian mesh, a higher-order IBM previously developed by Brehm et al. [40, 41] was modified to be applicable for the nonlinear disturbance equations. More details about the IBM approach in the context of solving the NLDE are provided in Ref. [42]. The AMR-WPT method has been successfully employed in prior works to simulate linear/nonlinear transitional wavepackets and particle interactions with high-speed boundary layers [38, 39, 43, 44], and demonstrated to accurately simulate amplification of the disturbances due to different types of instability mechanisms on more complex geometries, e.g., first and second modes [42], and vortical instabilities on a fin cone [34] and BOLT-2 [45].

For the current simulations, we employed a fifth-order accurate WENO scheme for the convective terms and a second-order accurate discretization of the viscous terms with a third-order accurate strong stability preserving Runge-Kutta scheme. Figure 7 gives an overview of the computational setup for these simulations including isocontours of the disturbance flow pressure field and the finest grid level concentrated around the wavepacket.

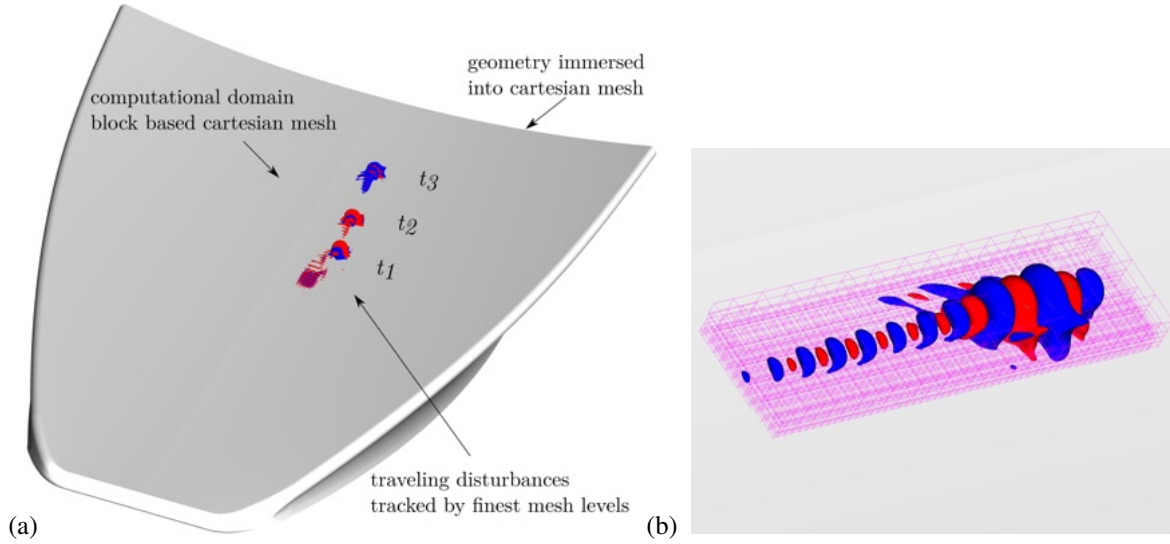


Fig. 7 (a) Schematic of Immersed-Boundary setup in a Cartesian mesh using WPT and (b) instantaneous pressure isosurfaces on the surface of BOLT, showing a wavepacket traveling downstream where the finest levels are seen to track only the main instability.

V. DLR H2K/ ONERA R2Ch Campaigns comparison

This section presents a comparative study between the DLR H2K and ONERA R2Ch experimental campaigns. Infrared thermography and high-frequency pressure fluctuation sensors reveal consistent results across facilities, evidenced by matching heat flux distributions and comparable surface pressure fluctuation spectra. Investigation across three Reynolds numbers provides insights into transition patterns and instability development, contributing to a comprehensive understanding of transition dynamics and providing valuable data for the stability analyses.

1. Heat-transfer measurements

To ensure a comparison that is independent of reservoir temperature variations between runs, the Stanton number distribution is computed based on IR heat transfer measurements using the following definition:

$$St = \frac{Q_w}{\rho_\infty u_\infty c_p (T_r - T_w)} \quad (4)$$

where

$$T_r = T_\infty \left(1 + r \frac{\gamma - 1}{2} M_\infty^2\right) \quad (5)$$

with a value of 1004 J/kg-K assumed for c_p , the specific heat of air. The wall recovery temperature (T_r) has been computed assuming a recovery factor, r , of 0.89. Figure 8 shows global St distributions for the three length-Reynolds numbers investigated in the two facilities (1.4×10^6 , 2.7×10^6 , and 5.5×10^6). These distributions from the two campaigns are qualitatively consistent and the heating patterns highlight complex flow features in both the central and outboard regions. Black dots seen in the DLR maps correspond to high-frequency sensor locations. Four markers are visible on the ONERA model, meant to prevent potential effects of model vibrations on the IR measurement. (Note: St values presented in this section from the ONERA tests are not definitive and may be subject to change after new processing.)

At low Re_L (Figs. 8a and 8b), the heat transfer around the centerline of the geometry is very low. This is characteristic for the BOLT geometry and is a result of the thick boundary layer developing in this region. When the 2D nose meets the highly-swept leading edges, a strong entropy gradient is generated downstream in the flow. This effect, combined with the large spanwise pressure gradient imposed by the curved bow shock in front of the body leads to the development of a pair of large counter-rotating vortices [46]. These vortices are affirmed for the present conditions by the laminar

CFD presented in Section VI. The centerline boundary layer is therefore lifted, while a relatively thin boundary layer is present outboard of these vortices. Higher heat transfer rates are observed in the swept leading-edge regions.

Beyond $x/L = 0.8$, the elevated St outboard of $z/L = 0.1$ observed across facilities potentially indicates the onset of transition. Numerous streaks with a higher heat flux are visible on the ONERA model at this low Re_L condition between $z/L = \pm 0.1$ and $z/L = \pm 0.175$, which do not appear in the DLR experiment. Their physical origin is yet to be identified. They could result from stationary CF instabilities or be part of the laminar flow topology. Two plausible explanations have emerged for this discrepancy between facilities. First, it could be caused by the difference in metrologies. Given that heat flux at these test conditions is very low, increased sensitivity of the IR sensor should lead to a more efficient capture of small surface temperature variations. Second, there could be a difference in the flowfield itself caused by the difference in model or facilities. A possible contributor to the streak amplitude would be a step at the junction between the nose (steel) and the IR surface (PEEK), though the measurements presented in Section III suggest this should be small.

Increasing Re_L results in more prevalent transition lobes in the outboard regions (Figs. 8c and 8d). The two campaigns show nearly identical transition onset locations at approximately $x/L = 0.55$ and $z/L = \pm 0.125$, evidenced by a streamwise increase in St . A more quantitative picture of the transition location is given in Section VI where the experimental St distributions have been compared to the laminar predictions. The high degree of symmetry in the transition pattern confirms the correct 0°-yaw mounting of the models in their respective test sections. A transition front characterized by outboard lobes is consistent with previous Mach 6 experimental campaigns in conventional tunnels [18, 47]. When approaching the rear end of the models, the transition lobes broaden in spanwise extent to cover most of the model acreage. However, the state of the centerline boundary layer remains ambiguous due to the relatively low heat transfer rates observed. Again, the laminar CFD predictions included in this study provide additional insight. Finally, at the highest Re_L investigated, the two lobes have clearly enveloped the centerline by $x/L = 1$ (Fig. 8e). The onset of transition has also moved forward substantially to approximately $x/L = 0.3$. It seems likely that the centerline transition is caused by contamination from the turbulent lobes and not due to some independent centerline instability.

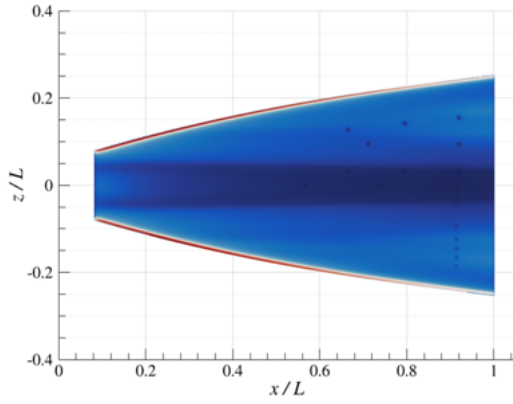
Spanwise heat transfer comparisons at two x/L locations are shown in Figs. 9 and 10. This juxtaposition of ONERA and DLR measurements allows a better quantification of the differences in the development of transition and vortical structures over the model, which are evidenced by localized peaks in the spanwise distributions. Close to the nose of the body, at $x/L = 0.231$, the overall St distributions between facilities are in generally good agreement at both Reynolds numbers (Fig. 9). Heat transfer values are consistent for the two Re_L considered, particularly in the near-centerline region bounded by $z/L = \pm 0.05$. The main difference appears around $z/L = \pm 0.065 - 0.085$ where heat flux striation is present over the ONERA model but not in the DLR case.

When moving further downstream, at $x/L = 0.693$, St distributions from the two facilities at $Re_L \approx 1.4 \times 10^6$ (Fig. 10a) are still matching well, apart from the numerous streaks present in the outboard region of the ONERA measurements. Increasing Re_L causes the transition/turbulent lobes to appear in the St evolution (Fig. 10b) with detection of two considerably increased heating plateaus from $z/L = \pm 0.1$ up to the edges of the models. Their shape and intensity are comparable from one wind tunnel to the other. Both ONERA and DLR measurements display some oscillations in heat flux between the centerline and $z/L = \pm 0.05$, though they are increased in intensity in R2Ch. In addition, a discrepancy is noted between the two Re_L conditions at $x/L = 0.693$; the narrowing of the central low heat flux region with increased Re_L .

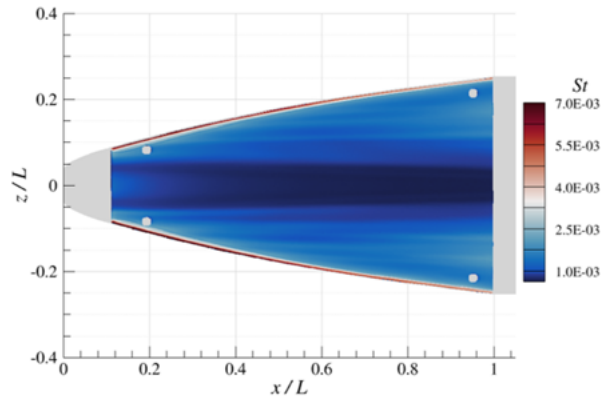
2. Surface pressure fluctuation spectra

PSDs obtained from PCBs are illustrated in Fig. 11 for $Re_L \approx 1.4 \times 10^6$. The low Re_L condition is of great interest here since it corresponds to an early transitional stage and displays instability measurements in several regions of the model acreage. The reader is again referred to Fig. 6 for an overview of the different sensor layouts used in the two campaigns. For clarity, the plot only shows the spectra that indicate a potential presence of instability.

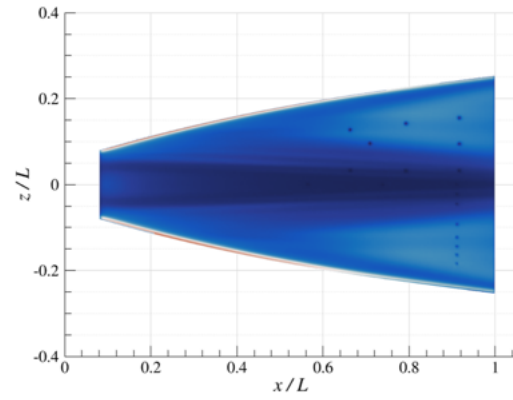
By examining PSDs from the DLR campaign (Fig. 11a), a prominent peak is measured by the outboard PCBs (PCB1, PCB4, and PCB6) around 160 – 180 kHz. Further inboard of the model, PCB7 measures a comparable peak at a slightly lower frequency. The transition front first appears in this region where traveling and stationary CF instabilities and MMs are expected to coexist. High-energy content in this frequency range could correspond to the MM instability [48]. Indeed, this is shown to be the case in the following computational sections. As previously stated, no analogous instability measurements were recorded during the BOLT-1 preflight ground-testing campaign. Given that an increase of freestream and thus of boundary-layer-edge Mach number (from below $Me = 4$ to above) causes this instability to appear, they are likely to be MM type waves [49]. As is characteristic for MM instabilities, the peak frequency is



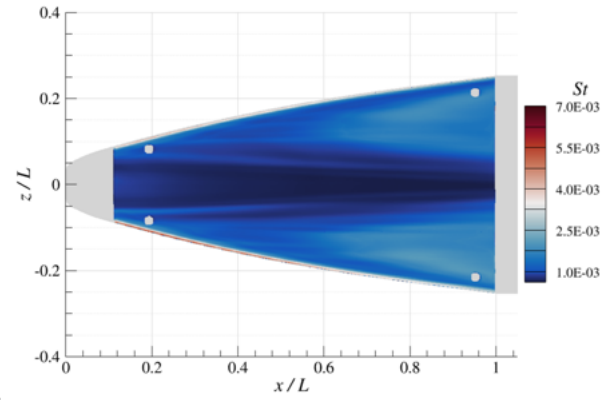
(a) Run 23: $Re_L = 1.41 \times 10^6$ (DLR)



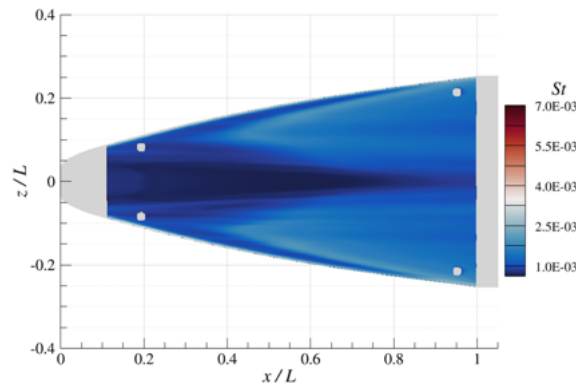
(b) Run 5314: $Re_L = 1.29 \times 10^6$ (ONERA)



(c) Run 24: $Re_L = 2.72 \times 10^6$ (DLR)



(d) Run 5299: $Re_L = 2.63 \times 10^6$ (ONERA)



(e) Run 5296: $Re_L = 5.43 \times 10^6$ (ONERA)

Fig. 8 IR heat transfer comparison for various Re_L .

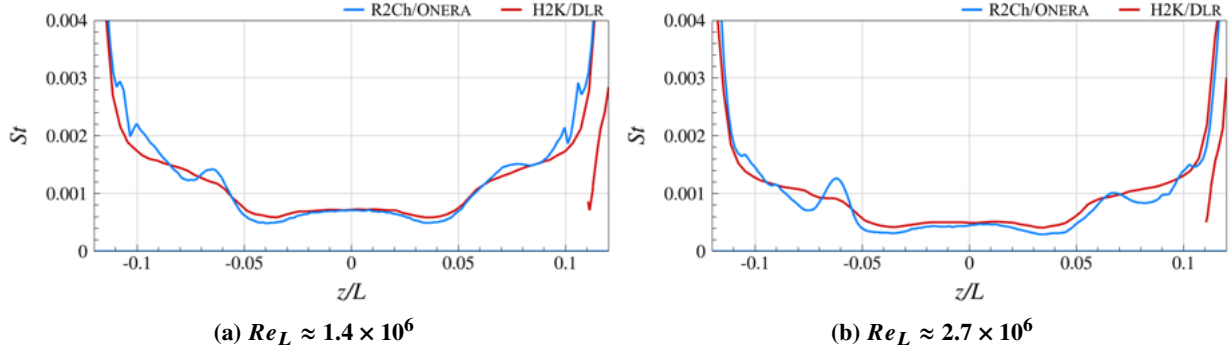


Fig. 9 Spanwise IR comparison at $x/L = 0.231$.

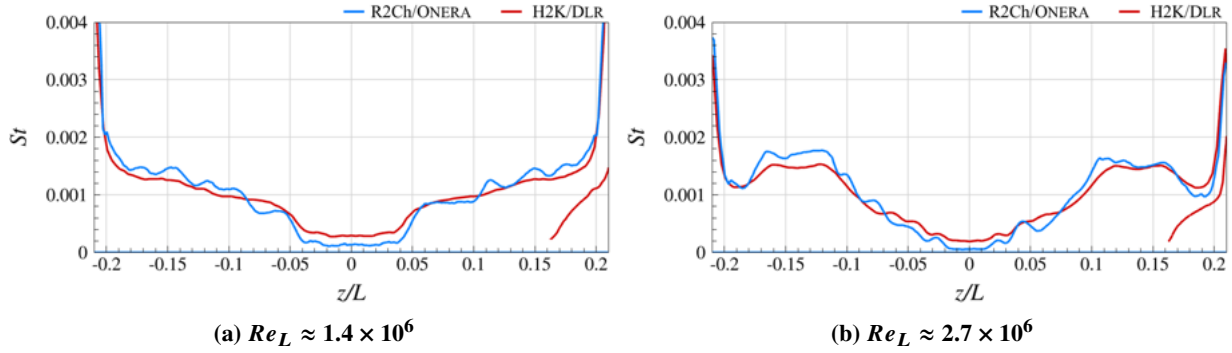


Fig. 10 Spanwise IR comparison at $x/L = 0.693$.

reduced as the instabilities propagate downstream. This is due to the dependency of the instability on the boundary-layer thickness along with the known facts that the boundary layer thickens downstream and that the MM frequency scales as U_e/δ . For instance, comparing maximal amplified frequencies between PCB4 and PCB6, one can notice a shift of the peak from 180 kHz to 160 kHz. It is notable, however, that a comparable shift is not observed between PCB1 and PCB4. Computational results shown in Section VII suggest this is because PCB1 is measuring a different family of MM instabilities, which amplify along the acreage. At more inboard locations (PCB5 and PCB8), there is no strong amplification in the spectra. Nevertheless, a tiny hump in the spectrum of PCB5 around 70 kHz could indicate the presence of an instability. High amplitude content is noticeable at frequencies below 50 kHz for PCB1, but it is unclear whether it comes from aerodynamic effects or not. Strong content displayed above 250 kHz is believed to result from the environmental noise.

The ONERA measurements are plotted in Fig. 11b. Some spectra display numerous sharp peaks that actually come from the facility environment. PCB06, PCB08, and PCB11 display a considerable amplification in their spectrum around 200 kHz, seemingly similar to the DLR measurements. The present investigation seeks to determine whether or not the measured instability is consistent with the DLR measurements. The most amplified frequency shifts to lower frequencies further downstream on the model and there is considerable broadening of the frequency content contained in the spectrum. This spectral broadening corresponds to an imperative stage toward turbulent breakdown: redistribution of the energy content to a broader band of frequencies through nonlinear interactions. Along the centerline, another distinct instability measurement is made: spectra from PCB05, PCB09, and PCB10 display amplifications around 50 kHz. A low-frequency shift is also associated with this instability as the reference moves closer to the back of the model.

Given the different model sizes employed in the two campaigns (ONERA 1/3 scale and DLR 1/2 scale) and that measured instabilities are highly dependent on the boundary-layer thickness, it becomes necessary to properly scale their frequencies with respect to the flow developing over the model. The scaling used to compare both wind tunnel measurements is fL/U_∞ , i.e., the Strouhal number. This scaling considers the effects of different model scales and freestream temperatures but is affected by the different sensor layout. An improved scaling would be to use the local

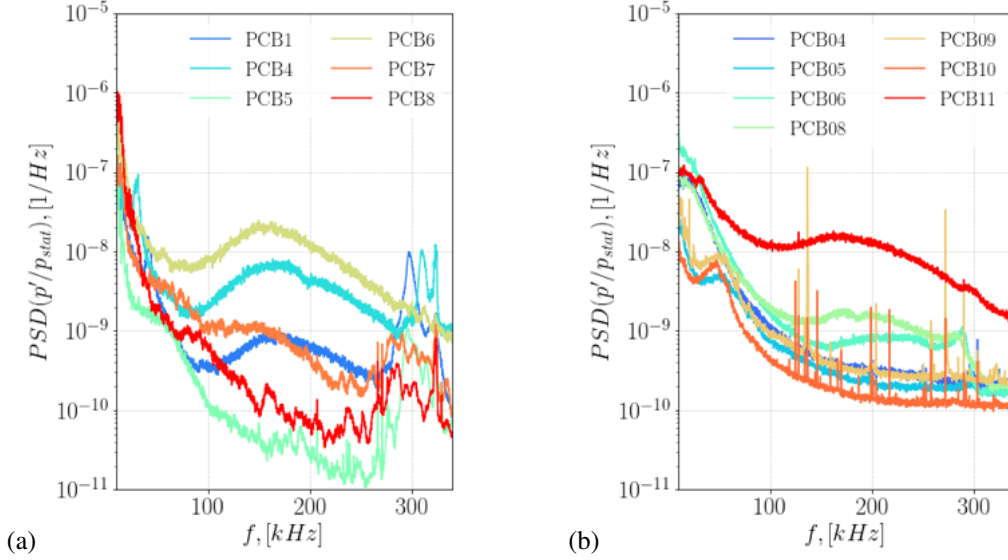


Fig. 11 PCB measurements at Mach 7 for $Re_L \approx 1.4 \times 10^6$ (a) in the DLR H2K Campaign (run 23) and (b) in the ONERA R2Ch Campaign (run 5348).

boundary-layer thickness and edge velocity. Figure 12a presents rescaled spectra from both ONERA and DLR. While they appear to be qualitatively similar, instabilities appear to be at a lower nondimensional frequency for the ONERA measurements. This could be explained by the placement of the ONERA sensors, which are closer to the center where the boundary-layer is thicker, as opposed to the DLR sensors that are closer to the highly-swept leading edges. The ONERA sensors would be expected to register instabilities at lower frequencies, given the already mentioned effect of boundary layer thickening on the instability frequency.

By focusing on the most downstream row of PCBs, Fig. 12b sheds additional light on the various instabilities present over the BOLT geometry. Along the centerline (PCB10), an instability develops at $fL/U_\infty = 20$. This instability is not measured by its closest neighbor (PCB8), probably located below one of the central region vortices. Although those vortices are presumed to support numerous instabilities [20], no clear spectral amplification is detected by this sensor at this Re_L condition. Then, as sensors get closer to the leading edges (in order: PCB7, PCB11, and, finally, PCB6), the outboard region instability appears at fL/U_∞ between 50 and 70. The greatest amplification is displayed by the sensor located the farthest from the centerline. Maximum N-factors for this instability should therefore be found near the swept leading edges. The three sensors show a consequent frequency shift as the boundary layer gets thinner. The reduction of the frequency between PCB7 and PCB11 could be explained either by the further downstream location of PCB11 or the presence of numerous vortices developing over the body.

The Re_L sweep illustrated in Fig. 13a clarifies the good agreement in the outboard PCB measurements across facilities. An abrupt modulation of the fluctuation spectrum is observed as the transition process advances in this region; the instability peak observed at the low Re_L condition has been entirely masked by the developing turbulent spectra for higher Re_L . This emphasizes the need to investigate both lower Reynolds numbers (to capture the beginning of the transition process) and Reynolds numbers between $Re_L \approx 1.4 \times 10^6$ and $Re_L \approx 2.6 \times 10^6$ (to measure the process up to the turbulent breakdown in this region).

The second Re_L sweep (Fig. 13b) focuses on the centerline region. ONERA PCB09 is located exactly along the centerline, while the DLR PCB5 sensor is offset by $z/L = 0.032$ (near a central region vortex). PCB09 exhibits a pronounced amplification at fL/U_∞ between 15 and 20 for the low and intermediate Reynolds numbers. There is then a dramatic broadband spectral amplification with a notable shift toward higher frequencies at the highest studied Reynolds number. This broadband amplification is potentially indicative of the presence of nonlinear interactions (and redistribution of energy). However, a fully turbulent spectrum is not realized even at this high-Re condition, emphasizing the slow transition process over the centerline in comparison with the outboard region. The DLR sensor at high Re_L , however, measures a largely broadband spectrum. Even though non obvious, a slight amplification is noticeable for PCB5 around $fL/U_\infty = 23$ between the low and intermediate Re_L cases, which could be consistent with the measurement of another instability type in the central region.

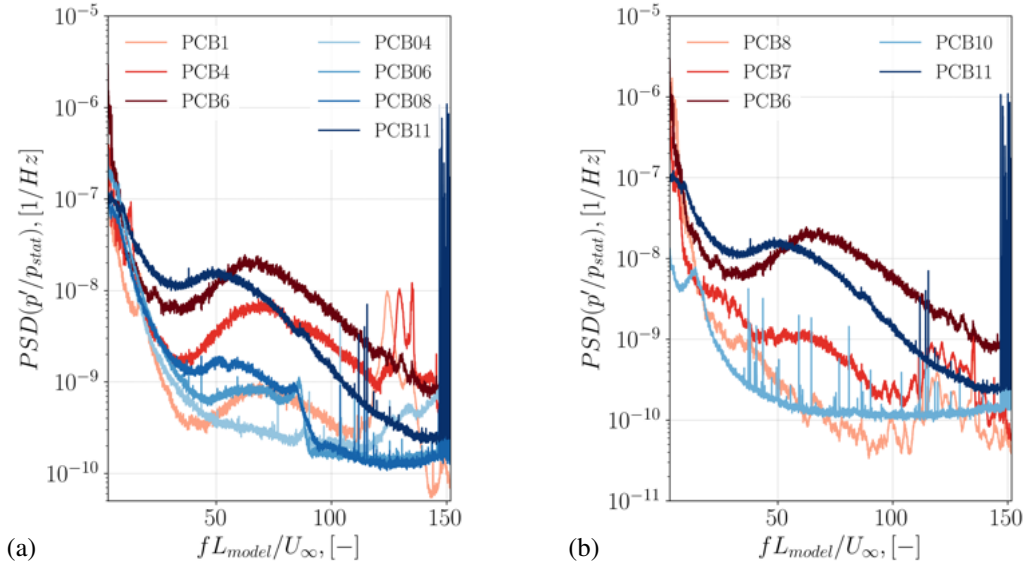


Fig. 12 Comparison of PCB measurements for $Re_L \approx 1.4 \times 10^6$ (a) in the outboard region and (b) for the most downstream row of PCB's. Red and blue lines corresponds to DLR H2K and ONERA R2Ch experiments, respectively.

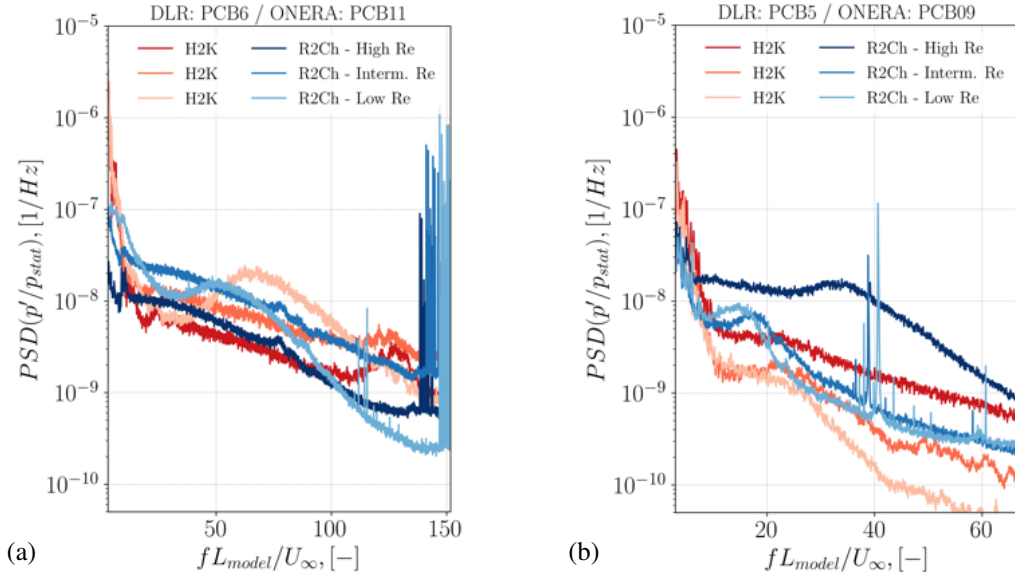


Fig. 13 Re sweep comparison of PCB measurements at Mach 7 for the (a) outboard region and (b) central region.

VI. Laminar Computations

A side-by-side visualization of the laminar base flows computed with US3D for both DLR H2K Run 23 and Run 24 conditions is presented in Figure 14. The model surface is colored by heat flux and crossplane streamwise velocity contours are shown at the downstream end of the model. The DLR and ONERA PCB locations are indicated by red and blue dots along the surface, respectively. Note that these locations are mirrored in the image for clarity.

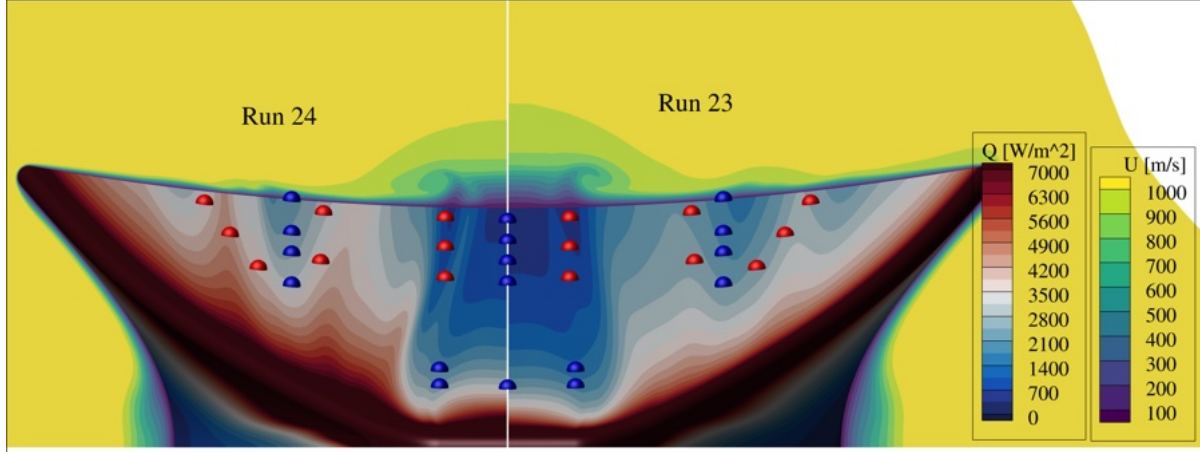


Fig. 14 Visualization of the laminar base flows for each condition showing surface heat flux and a slice of streamwise velocity at the outflow plane.

The near-centerline flow topology is dominated by a large vortex structure typical of the BOLT flowfield. The vortex has achieved a greater degree of roll-up for Run 24 conditions, enhancing the amplitude of the attendant heating streak. The midspan and outboard regions of the flow topology are characterized by some spanwise waviness in the boundary-layer thickness. This waviness results in a pair of streaks with increased heat flux which roughly overlap with the locations of the PCB arrays. Increasing Re also increases the waviness, resulting in more prominent streaks for Run 24 conditions. In fact, the midspan boundary layer appears to have reached a state of incipient roll-up by the end of the computational domain for Run 24.

The laminar flow states computed using US3D and DNS-WENO are compared in Figure 15, where the surface contours correspond to heat flux and the crossplane contours (shown at $x = 0.4$ m) represent streamwise velocity. This comparison demonstrates that both computational methodologies produce near identical flow topologies for this condition. It is thus implicit that stability analyses will be agnostic to the selection of base flow. The experimental comparisons made throughout the remainder of this section are based on the US3D solutions.

A comparison between computational and experimentally observed Stanton number distributions is presented in Figure 16 for several axial stations where St has been computed in the same manner as for the experimental comparison.

At Run 23 conditions, the laminar CFD predicts a somewhat elevated St relative to the IR measurements across the model span at an axial station of 0.1 m, though the agreement improves substantially at 0.2 m. At $x = 0.3$ m, the experimental value is seen to increase above the laminar prediction around $z = 0.05$ m, signaling transition onset. At $x = 0.4$ m, this transitional/turbulent region is seen to encompass the span from $z = 0.04 - 0.09$ m. The experimental centerline region, however, still demonstrates laminar heating levels.

Similar trends may be observed in the bottom image of Figure 16 for Run 24 conditions with some notable differences. The experimental deviation from laminar heating levels may now be observed at the $x = 0.2$ m station in the vicinity of $z = 0.05$ m. At $x = 0.4$ m, the magnitude of the discrepancy in St between computation and experiment is much greater than for Run 23, implying the boundary layer has developed closer toward a fully turbulent state. Additionally, the centerline region shows some evidence of transition at this furthest downstream station.

The Stanton number values may be compared between the computations and experiments in a global sense to identify the approximate experimental transition fronts. The percentage difference in the experimentally measured St over the laminar prediction is presented in Figure 17 for each condition.

Regions of elevated experimental heat flux may be interpreted as transitional or turbulent. For both cases, the primary transition front is clearly located in the outboard regions of the model and moves forward substantially going from Run 23 to Run 24. This transition front appears to consist of two lobes, which merge due to turbulent spreading.

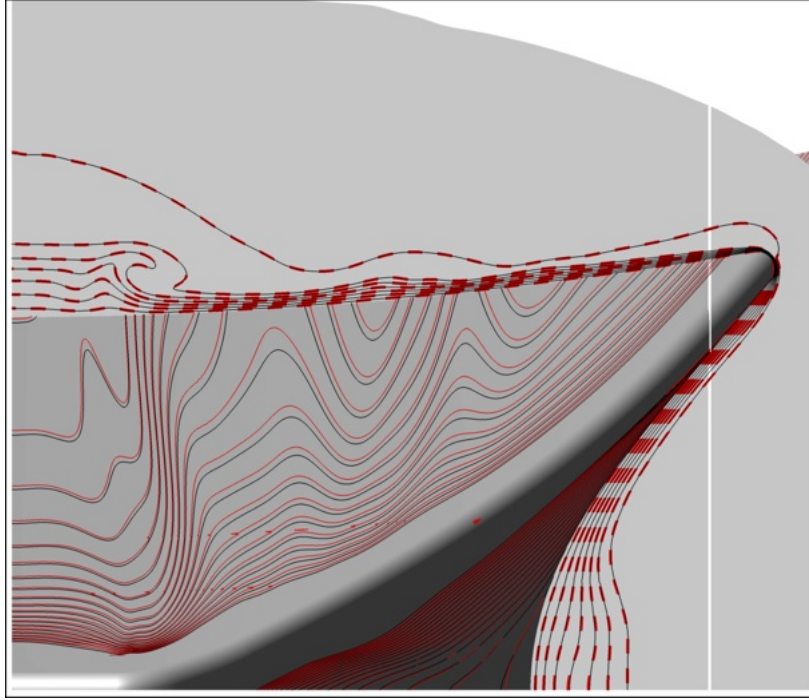


Fig. 15 Comparison of the flow topology from the APL (black) and NASA LaRC (red) laminar base flows for Run 23 conditions.

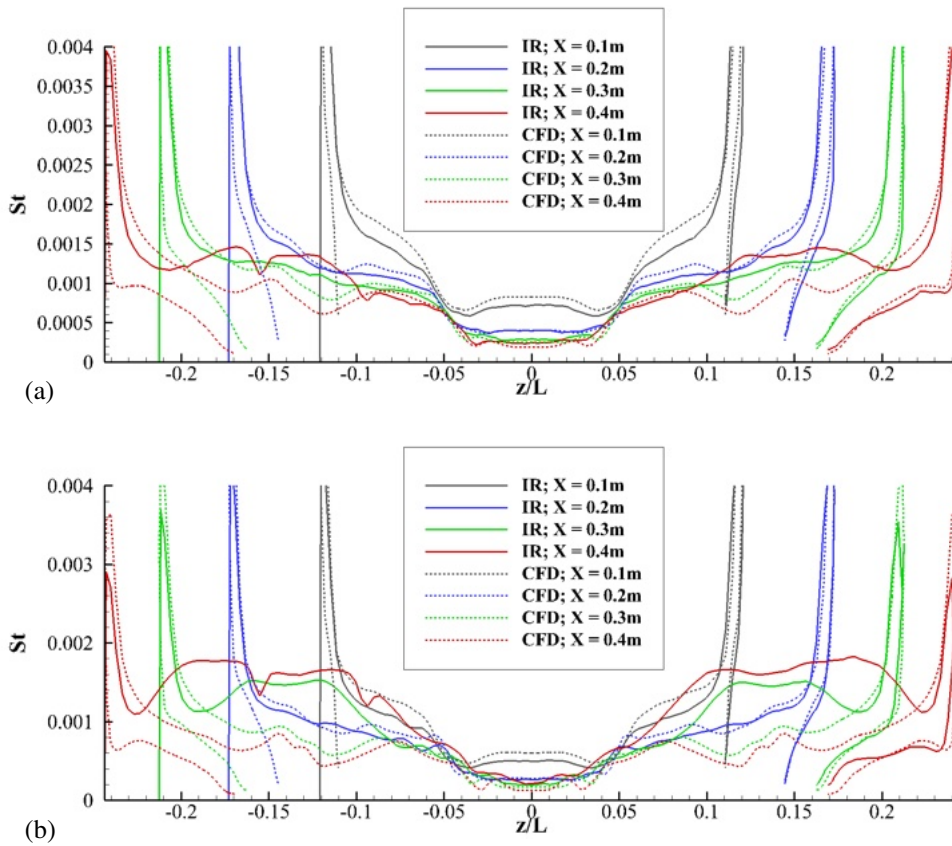


Fig. 16 Comparison of computed and measured Stanton number for (a) Run 23 and (b) Run 24 conditions.

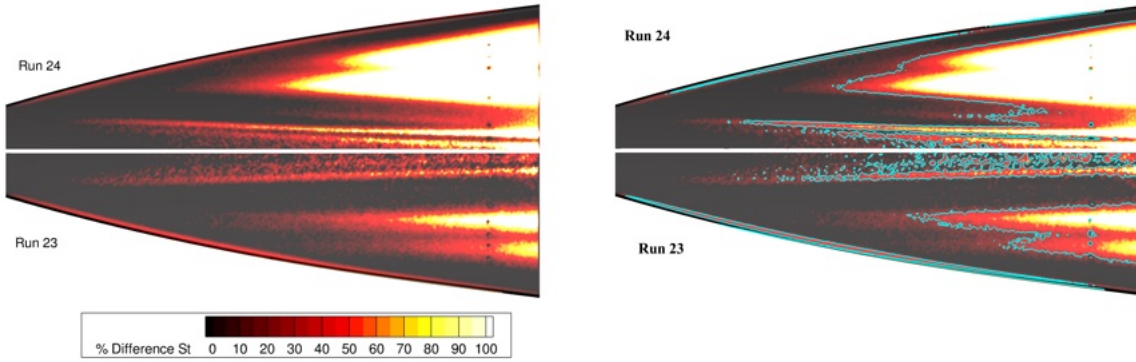


Fig. 17 Percentage increase in experimentally observed heat flux over laminar computations for Run 23 and Run 24. Teal line in right image corresponds to 20 percent cutoff.

The approximate location of the experimental transition front may be extracted from this comparison by assigning a threshold value to the St variation. The right image of Figure 17 includes a teal line, which follows the front resulting for a cutoff threshold of 20 percent. This front will be correlated with the predicted amplification of boundary-layer instabilities in the following section.

This method of comparison in Figure 17 also reveals near-centerline streaks of elevated heat flux over the laminar prediction. However, due to the lack of sufficiently detailed unsteady measurements to supplement the heat-flux data, it is unclear whether this is indicative of transition or is instead artificial due to underresolution of the surface heating profile in the IR imagery.

VII. Stability Analysis

A. Run 23

1. Line-marching LST/PSE

An initial stability analysis was performed by APL with the LASTRAC code on a relatively coarse mean flow. This stability analysis was intended to be representative of typical methods that do not consider the spanwise variation of the mean flow. Figure 18 shows the N-factors computed at Run 23 conditions for MM and stationary CF instabilities. Relatively little N-factor growth was predicted at these conditions, with maximum N-factors near two at the aft end of the body. It is important to note that similar to previous stability calculations on the BOLT geometry, both stationary CF and MM growth are predicted to occur within the same region of the geometry.

A modified procedure was adopted for the subsequent stability analyses based on the laminar basic states described in the previous section. Three different types of analyses were performed at the NASA Langley Research Center for planar MMs, stationary CF vortices, and traveling CF disturbances, respectively. Each of these analyses employed the line-marching form of the linear, parabolized stability equations (LMPSE). Similar to the preliminary LST analysis mentioned above, the LMPSE involve certain approximations related to the spanwise variations of the basic state as well as the disturbance field. However, the LMPSE include the nonparallel effects along the trajectory as well as the effects of surface curvatures. Two different sets of trajectories are employed for the LMPSE computations. Whereas inviscid streamlines were used for the MM instabilities, the CF disturbances were tracked along the inflection lines [50, 51] and the diffidence of those lines was used to capture the axial variation in the spanwise wavenumber of the CF disturbance [32]. Both sets of trajectories were used for the analysis of traveling CF instability. However, for brevity, only the results corresponding to the inflection lines will be presented in this paper. Because the envelope N-factors for the traveling CF disturbances are always larger than those of the stationary CF disturbances, they are denoted as simply CF in this paper, to emphasize that they denote the N-factor envelopes for CF instability of either kind.

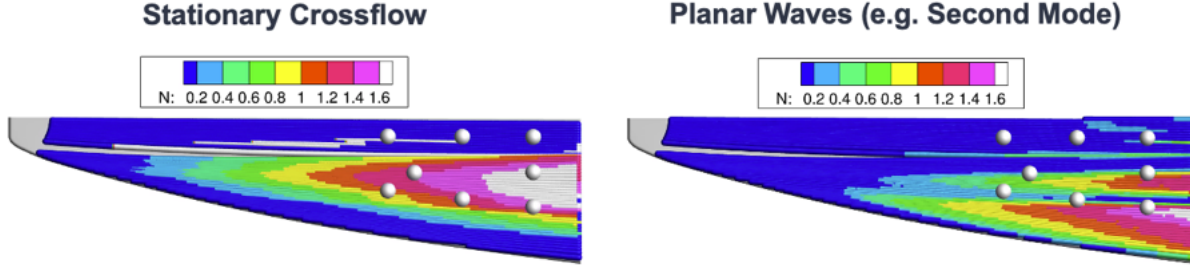


Fig. 18 LSTRAC LST results for the Run 23 H2K condition show N-factors for both the stationary CF and MM occurring in the same region of the geometry.

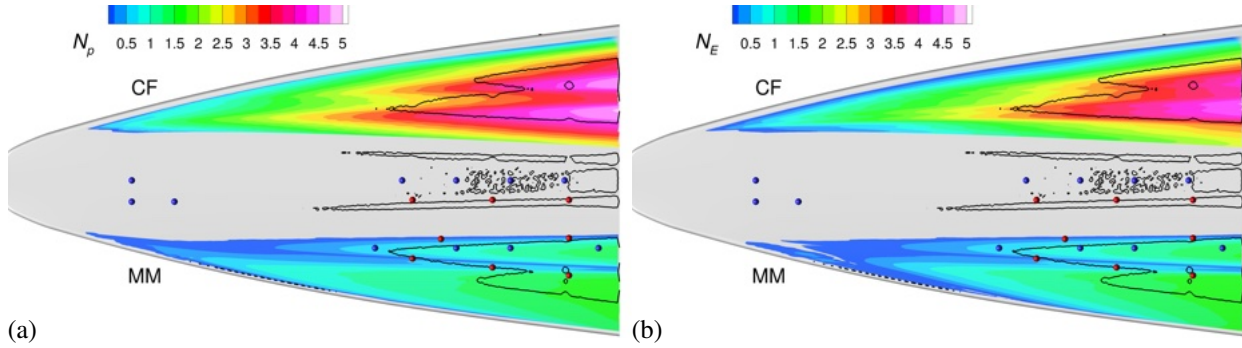


Fig. 19 Contours of LMPSE-based N-factor envelopes based on (a) pressure fluctuations at the wall, N_p , and (b) disturbance energy norm, N_E , corresponding to both CF and MM disturbances for the Run 23 H2K condition. The black lines denote the meaasured transition front.

Figure 19 shows the LMPSE results for Run 23, plotted as contours of the N-factors based on surface pressure fluctuation (part (a) of Fig. 19) and disturbance energy integrated across the wall normal (part (b)). In each figure, we have utilized the spanwise symmetry of the computed basic state to plot the contours of envelope N-factors for CF instabilities in the top half and the N-factors for MMs in the bottom half. We recall that the experimental transition fronts on both starboard and port sides show two lobes each in the region away from the spanwise center of the test surface. The inboard lobe indicates the earliest onset of transition near $x/L = 0.62$, whereas the outer lobe indicates a downstream apex near $x/L = 0.76$. The predicted N-factor contours for both types of instabilities indicate a bilobed structure. Indeed, the locus of a minimum in the MM N-factors aligns rather well with the (outer) boundary of the inboard lobe in the measured transition front. However, the energy based N-factors at the apices of the inboard and outboard lobes in the measured transition front are only about 1 and 2, respectively. With such low levels of amplification, it seems unlikely that the MM modes would be able to induce transition on their own. On the other hand, the CF N-factors are significantly higher and they can reach values of approximately 3 in the vicinity of the above apices. While $N_E = 3$ still represents a somewhat low N-factor, N-factors between 3 and 4 have been found to correlate to the onset of transition due to (high-frequency) MM instabilities over simple axisymmetric cones in analogous conventional facilities [52]. Given the higher amplitudes of freestream noise in the (lower) frequency range of traveling CF disturbances (and the possibility of nonlinear interactions with the modestly amplified MMs in the same azimuthal regions), it seems plausible that the CF instabilities played an important role in inducing transition during Run 23 in the H2K.

2. 2-D Eigenvalue Analysis and Planar-Marching PSE

Normalized eigenfunction contours for the instabilities predicted away from the centerline at an axial station of $x/L = 0.577$ are presented in Figs. 20 and 21 for the LSTPACK and MAFIA codes, respectively. The differences in growth rate spectra between codes is due to the different definitions of the homogenous streamwise direction. The dominant instability families identified at this location consist of spanwise modulated forms of MM and CF instabilities. The CF instabilities appear at frequencies below 50 kHz outboard of the boundary-layer hump between

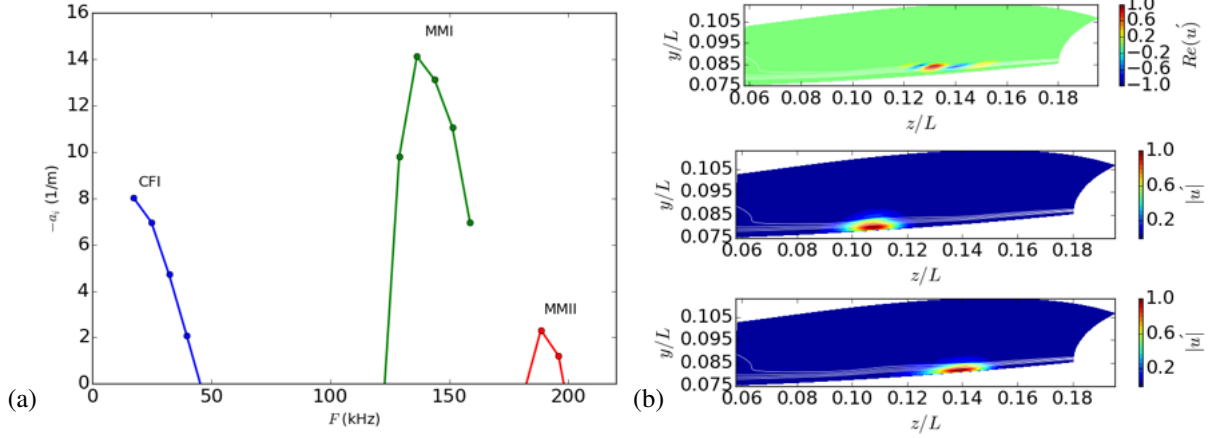


Fig. 20 (a) Amplification rate spectra and (b) mode shapes for select families of instabilities at $x/L = 0.577$ from Sandia LSTPACK.

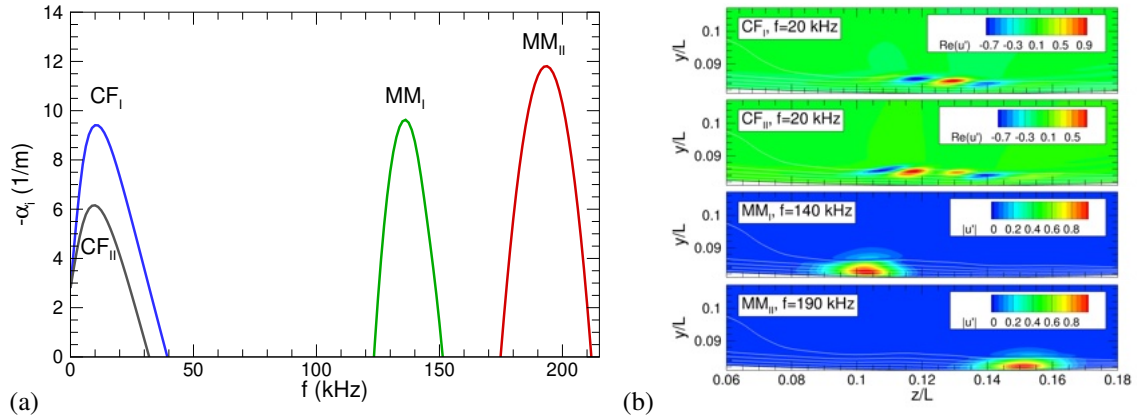


Fig. 21 (a) Amplification rate spectra and (b) mode shapes for select families of instabilities at $x = 0.577$ from MAFIA. CF mode shapes are plotted in terms of $Re(u')$ to illustrate the details of the disturbance structure. Numbering of the MM and CF modes is arbitrary.

$z/L = 0.115 - 0.162$ ($z = 50 - 70$ mm). There exist at least two unstable families of CF modes, which correspond to different spanwise length scales. Note that only a single CF family was identified in the Sandia LSTPACK analysis, but is qualitatively equivalent to the CF_I instability family predicted by MAFIA. The MM disturbances also include two families of unstable modes, which are generally consistent across codes in terms of mode shape. One of them is more aligned with the inboard heating streak and the outboard ONERA PCBs, $z/L = 0.0924 - 0.115$ ($z = 40 - 50$ mm), and spans approximately 140 – 180 kHz. Without any loss of generality, we denote it as the MM_I family. The MM_{II} instabilities appear at higher frequencies (175 – 200 kHz) and are situated in the same spanwise region as the CF family. Although this MM family is predicted to have a much higher amplification rate at $x/L = 0.577$ by the MAFIA computations, it will be shown that both codes produce similar integrated amplification magnitudes over the length of the model. Note that the MM_{II} family best correlates with the locations of PCB sensors in the DLR experiment and its frequency range also aligns with the peaks in the measured PCB spectra.

As discussed in the preceding section, the CF disturbances have the highest overall amplification over the test surface according to the LMPSE computations. To assess whether the effect of spanwise modulation would contribute to a significant increase in the MM N-factors, PMPSE computations were performed to determine the MM N-factors after accounting for the spanwise variations in the basic state. The results of these computations with the LSTPACK code

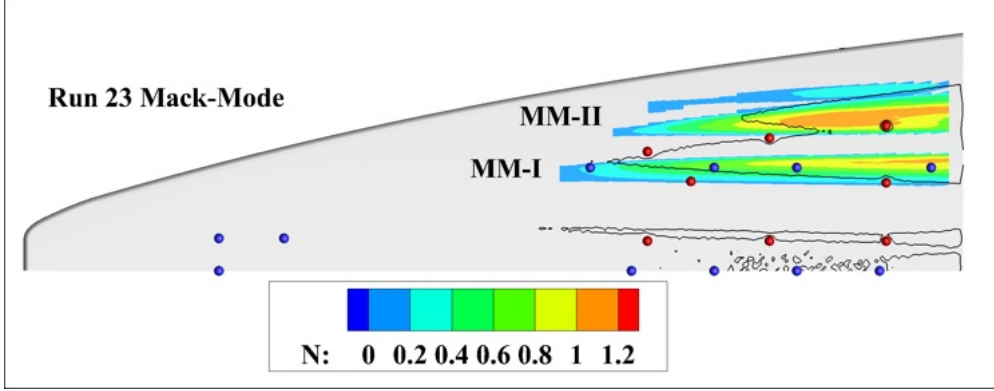


Fig. 22 Maximum N-factors predicted by LSTPACK PMPSE for MM families at Run 23 conditions.

are shown in Fig. 22 while the MAFIA results are overlaid on the LMPSE results in Fig. 24. It may be seen that the resulting N-factors for MM disturbances are still rather low to cause transition on their own (less than 1.5 for both codes). The bottom half of Fig. 24 represents analogous MAFIA results for Run 24, and it is discussed in the following subsection. We note that N-factors computed by LSTPACK are based on the wall pressure fluctuation while MAFIA N-factors are defined by the disturbance energy norm. One observation to be made from Fig. 22 is that the outboard PCB array in the ONERA computations (blue dots) clearly follows the inner MM instability (MM_I), while the DLR PCB array (red dots) cover the amplification of both, MM_I and MM_{II} , amplification regions. DLR PCB1 is situated close to MM_I , while PCB4 and PCB6 are located along the amplification of MM_{II} . This may account for some of the discrepancies observed across facilities in the maximal nondimensional frequency of the measured instabilities.

We note that the streaks at the outboard end of the thicker boundary-layer region near the center are also known to support instabilities that require PMPSE analysis [20, 53]. However, for the Run 23 condition, the PMPSE-based N-factors for those streak instabilities were barely detectable (less than 0.1) for both codes. Even at the higher Reynolds number condition of Run 24, PMPSE predicted relatively low N-factors of approximately 2 at the back end of the model. The frequency range of these instabilities was relatively broad, extending from about 20 kHz to approximately 100 kHz, depending on the location of interest. Further analysis is necessary to determine whether or not the PCB sensors in the experiment would allow a meaningful comparison between these predictions and the measured surface pressure spectra.

B. Run 24

1. Line-marching LST/PSE

LMPSE results for the higher Reynolds number case (Run 24) are shown in Fig. 23. As discussed previously and similarly to the Run 23, the experimentally inferred transition front indicates two prominent lobes in the outboard region, with apices near $x/L \approx 0.48$ (inboard lobe) and $x/L \approx 0.60$ (outboard lobe), both of which indicate an appreciable upstream shift with respect to the lower Reynolds number case. The measured transition front also shows multiple additional lobes closer to the center region. These are typically rather narrow in spanwise extent and, again, it is not clear whether or not they represent a true onset of transition. As expected, the higher Reynolds number also leads to increased amplification rates for both CF and MM instabilities. Thus, despite the upstream shift in measured transition front, the CF N-factors reach values of slightly larger than 3 in the neighborhood of the apices of the two (primary) lobes in the measured transition front. This range of N-factors is close to that observed earlier for Run 23. The MM N-factors at the locations of interest are again rather low (nearly 1 or 2, depending on the lobe). The PMPSE results for the MM modes (lower half of Fig. 24) indicate that the MM N-factors remain small even after accounting for the effects of spanwise modulation associated with the basic state.

2. Adaptive-Mesh-Refinement Wavepacket-Tracking Immersed-Boundary Method

The computational domain used for the disturbance flow simulations extended over $[0.185, 0.5]$ m ($x/L = [0.427, 1.15]$) in the x -direction, $[0, 0.020]$ m in the y -direction and $[0, 0.068]$ m in the z -direction. The octree-based block-structured Cartesian mesh used for the AMR-WPT-IBM approach was organized with a total of 6 refinement levels

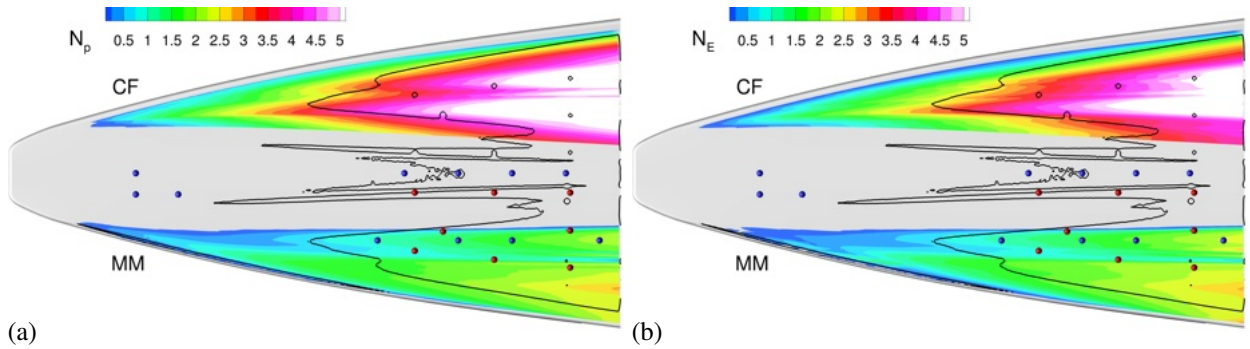


Fig. 23 Contours of LMPSE-based N-factor envelopes based on (a) pressure fluctuations at the wall, N_p , and (b) disturbance energy norm, N_E , corresponding to both CF and MM disturbances for the Run 24 H2K condition.

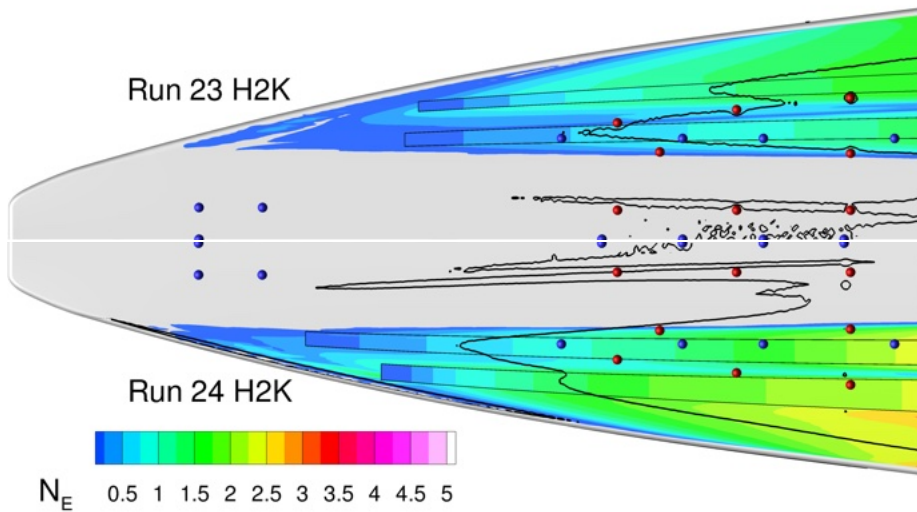


Fig. 24 Contours of N-factor envelopes based on disturbance energy norm, N_E , for MM disturbances at both H2K conditions. The LMPSE-based N-factors from Fig. 23 are overlaid with PMPSE-based N-factors in the vicinity of selected streamline trajectories in the vicinity of prominent streaks outboard of the center region.

where the finest mesh levels are located inside the boundary layer. Each box contains $16 \times 16 \times 16$ grid points with an aspect ratio of $AR_{\Delta y/\Delta x} = 0.48$ and $AR_{\Delta z/\Delta x} = 1.30$. In order to reduce the computational cost, only one quarter of the vehicle was simulated and extrapolation boundary conditions were assumed on the z -plane for the disturbance flow.

Figure 25 shows instantaneous contours of the disturbance wall pressure field visualizing the evolution of the disturbance flow field after the pulse disturbance has been introduced in the flow field for Run 24. Note that this visualization focuses on the near-centerline region due to the lack of amplification predicted by the AMR-WPT analysis in the outboard regions. Shortly downstream of the location where the pulse was introduced, the disturbances spread out radially outward until the initial transient (largely due to the numerical receptivity process) has passed. Initially, the disturbance flow field appears to separate into two wavepackets. The wavepacket propagating closer to the centerline appears to be dominated by (fast) acoustic disturbances traveling faster than the second wavepacket, but the pressure signatures quickly weakens when propagating downstream. The slower second wavepacket is more dominant but also seems to not experience significant amplification.

To quantify the disturbance amplitude development along the wall we performed FFT of the wall pressure signal in a frequency range of 0 – 240 kHz. Figure 26 gives an overview of the amplitude development in different frequency ranges. It should be noted that the initial rapid decay of disturbance amplitude in the vicinity of the forcing location is expected due to the numerical receptivity process. Some weak amplification can be observed for frequencies between 80 and 160 kHz from $x/L = 0.485 - 0.577$ ($x = 0.21 - 0.25$ m). This would correspond to a scaled frequency (fL/U_∞) of 33 – 66, which is notably higher than the instabilities measured along the centerline in the ONERA experiments ($fL/U_\infty < 20$). That said, the limited amplification predicted along the centerline by this analysis correlates with the experimental conclusions that centerline transition is driven by contamination from the outboard transition lobes.

VIII. Summary and Conclusions

In support of the NATO AVT-346 working group, this paper has detailed experimental measurements obtained on subscale BOLT-1 models in the DLR H2K and ONERA R2Ch facilities along with computational analyses of the laminar flow and boundary-layer transition phenomena at the experimental conditions. The laminar flow over the BOLT configuration is highly complex, comprised of intrinsic stationary streaks associated with the body geometry as well as with the stationary crossflow (CF) instability of the 3D boundary layer. Despite that, the low-Re measurements obtained by IR thermography in two conventional facilities exhibit reasonable agreement in the overall distribution of the Stanton number contours. Furthermore, the laminar basic state computations without any surface roughness are able to match the measured heatflux patterns fairly well upstream of transition, providing some confidence that the computed basic states can be used to correlate the measured transition patterns with the amplification characteristics of multiple instabilities.

Additionally, the measurements obtained in both facilities demonstrate consistent development of the transition front as a function of Reynolds numbers. At these experimental conditions, the transition front is characterized by outboard lobes that eventually contaminate the centerline. Despite differing sensor locations, high-frequency surface pressure fluctuation data from both campaigns demonstrate significant amplification of instabilities in the outboard regions of the acreage. PSDs of the pressure fluctuations characterize these as Mack's second-mode (MM) instabilities with peak frequencies greater than 150 kHz.

These measurements are compared to stability analyses of varying fidelity. The LMPSE computations for both MM and CF instabilities mimic the (primarily) two-lobed structure of the transition front inferred from the heat flux measurements during Runs 23 and 24 from the H2K experiments. However, the envelope N-factors for the MM instabilities are rather low (near 1 for the upstream and inboard lobe and about 2 for the outboard lobe). On the other hand, the (traveling) CF N-factor values reach approximately 3 for Run 23 and slightly larger than 3 near the apexes of these lobes.

While 2-D eigenvalue analysis was found to predict MM instabilities which correlate well with the experimental measurements, PMPSE computations showed no additional amplification of the instabilities due to the spanwise modulation included in a planar analysis. The AMR-WPT computations were unable to predict significant amplification for instabilities in the outboard regions of the acreage but identified weak amplification in the near-centerline region at higher Reynolds numbers.

Unlike the BOLT configuration at either flight conditions or the conditions of a quiet tunnel experiment, where the stationary streaks in different azimuthal subregions of the test surface played a crucial role in determining the onset of transition, the present work examines the BOLT configuration in a conventional tunnel. The findings indicate that transition fronts inferred from surface heat flux measurements align fairly well with LMPSE-based CF N-factors

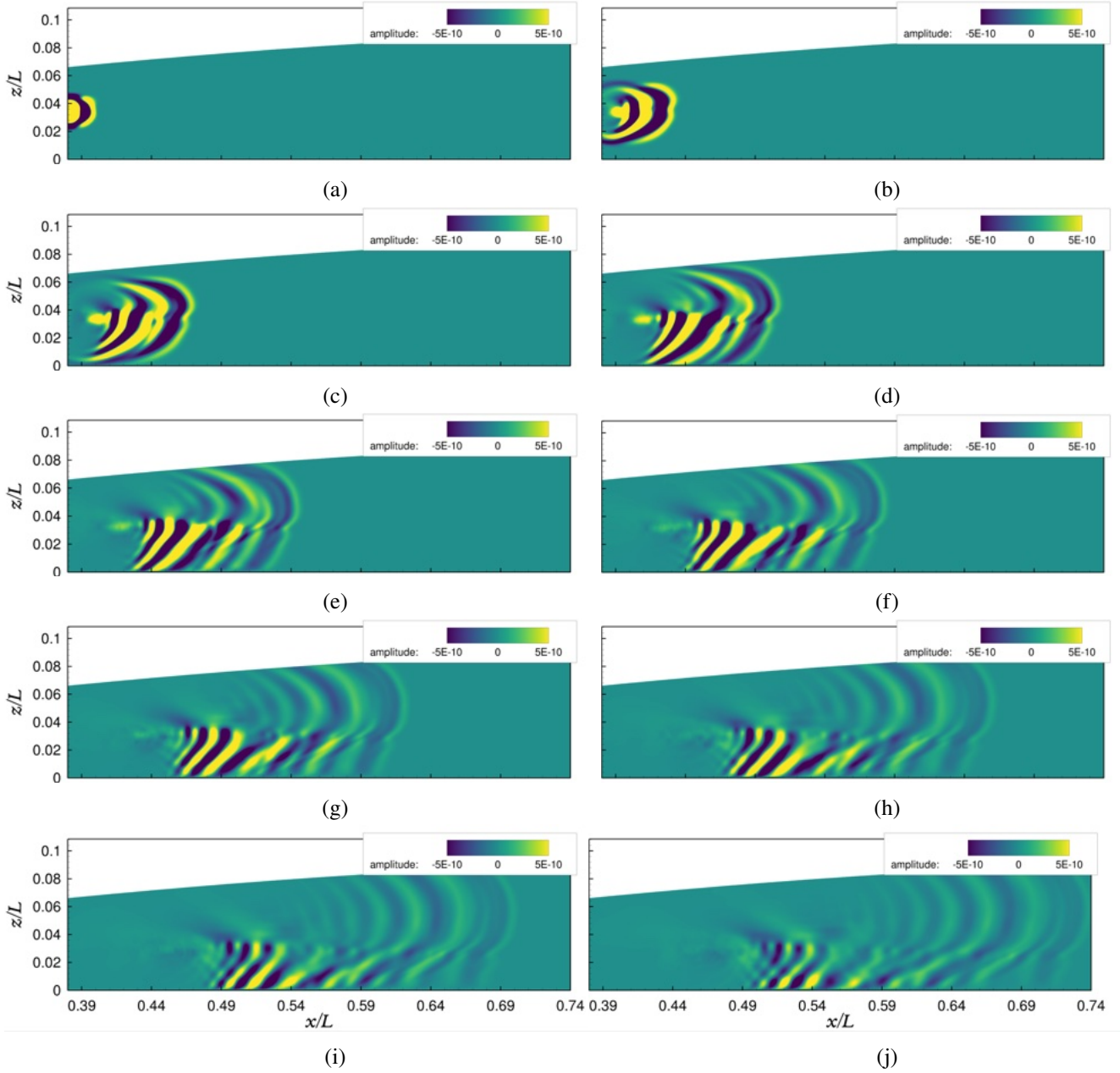


Fig. 25 Development of the disturbance wall pressure signature taken at different times: (a) $t_1 = 16.1 \mu\text{s}$, (b) $t_2 = 32.2 \mu\text{s}$, (c) $t_3 = 48.3 \mu\text{s}$, (d) $t_4 = 64.4 \mu\text{s}$, (e) $t_5 = 80.5 \mu\text{s}$, (f) $t_6 = 96.6 \mu\text{s}$, (g) $t_7 = 112.7 \mu\text{s}$, (h) $t_8 = 128.8 \mu\text{s}$, (i) $t_9 = 144.9 \mu\text{s}$, (j) $t_{10} = 160.9 \mu\text{s}$.

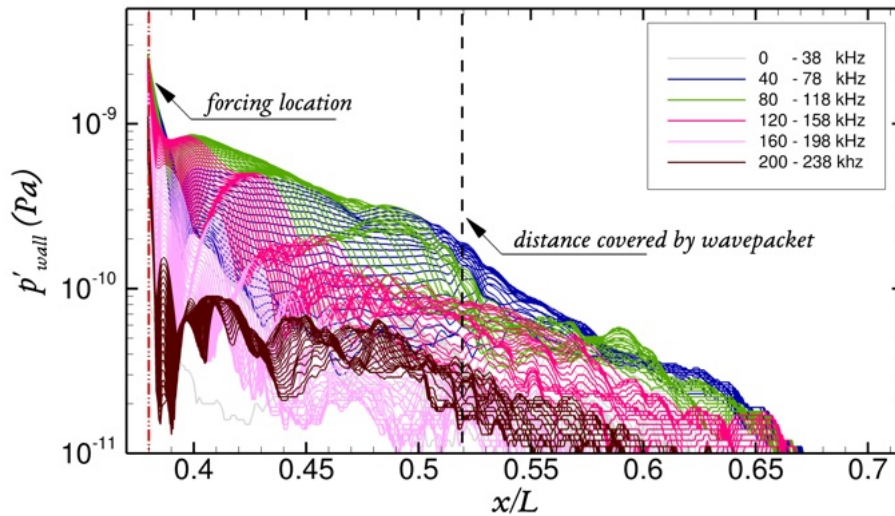


Fig. 26 Amplitude curves obtained by extracting the maximum value of the disturbance pressure tracked at the wall in the spanwise direction, computed at each streamwise position.

of around 3–3.5. This correlation holds, at least when the transition begins sufficiently downstream of the nose, approximately within the middle third of the body length. Even though the computed laminar base flows in the present cases include a pattern of stationary streaks, they are significantly weaker than those observed in quiet facilities. Thus, even though the possibility of these streaks exerting a measurable influence on the disturbance amplification cannot be discounted entirely, our analyses have shown that the transition fronts can be well approximated by the LMPSE analysis, without requiring the substantially more complex analysis based on the PMPSE. We believe this to be a rather novel result that was made possible by the collaborative effort under the NATO STO AVT-346 Research Task Group.

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