Maximum-Likelihood Parameter Estimation for High-Contrast Wavefront Sensing & Control

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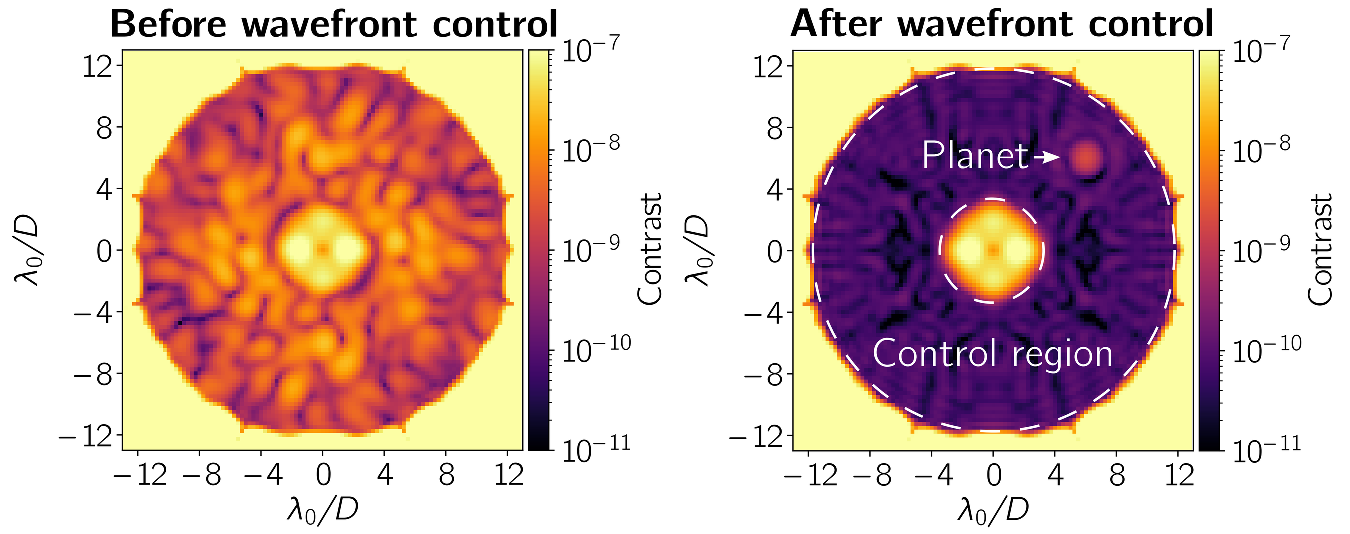
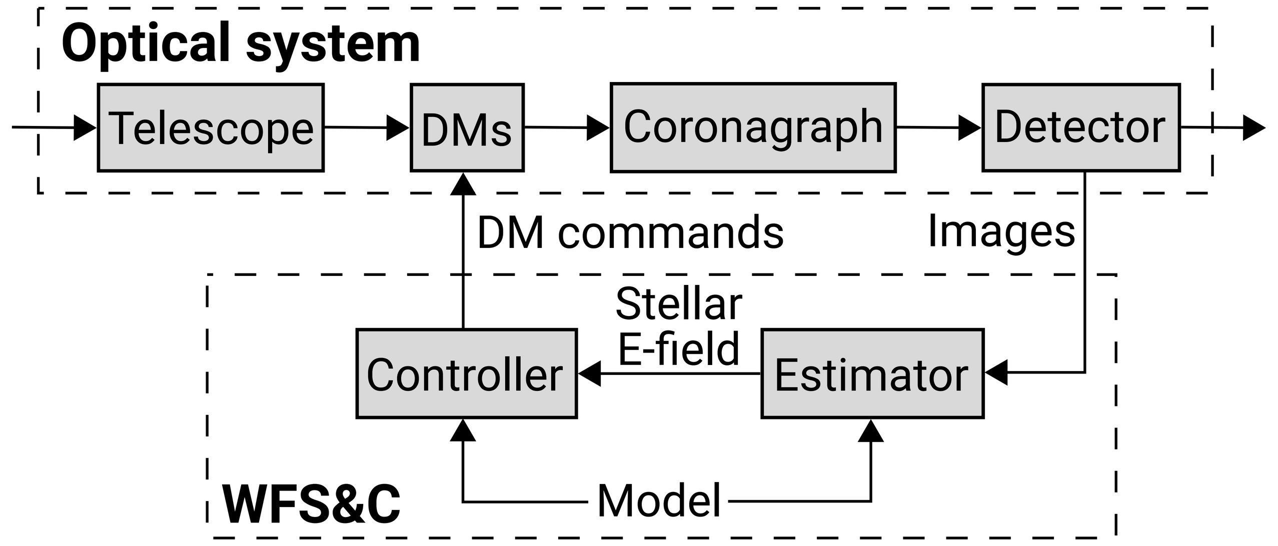
Overview

Exoplanet coronagraphs rely on **wavefront sensing & control (WFS&C)** with **deformable mirrors (DMs)** to create high-contrast dark zones



- In a **model-based** WFS&C framework, mismatch between the true and assumed coronagraph responses to DM commands reduces the effectiveness of the WFS&C loop (increases the time required to reach high contrast), and limits the deepest-achievable contrast.



- In a **model-free** framework, the coronagraph responses are assumed to be completely unknown *a priori*. To characterize the responses empirically requires prohibitive amounts of training data due to the extremely high dimensionality of the control problem (thousands of DM actuators).

By optimally combining *a priori* knowledge with empirical data, we can achieve the best of both model-based and model-free techniques: robustness to model error with minimal data overhead.

Mathematical background

The focal-plane electric field E_k evolves over time according to a linear state-space model:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{G}\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{z}_k &= \mathbf{H}\mathbf{x}_k + \mathbf{n}_k \end{aligned} \quad \text{where} \quad \begin{aligned} \mathbf{x}_k &= \left[\mathrm{Re}\{E_k\}\ \mathrm{Im}\{E_k\}\right]^T \\ \mathbf{H} &= 4(\mathbf{G}\boldsymbol{\Psi})^T \end{aligned} \quad \begin{aligned} \mathbf{w}_k &\sim \mathcal{N}\left(0, \mathbf{u}_k^T\mathbf{u}_k Q\mathbb{I}\right) \\ \mathbf{n}_k &\sim \mathcal{N}(0, R\mathbb{I}) \end{aligned}$$

Given DM commands $\mathbf{u}_k$, probe commands $\boldsymbol{\Psi}$, and measured probe images $\mathbf{z}_k$ for a series of WFS&C iterations, we can estimate $\mathbf{G}$, Q, and R empirically.

- **Open-loop identification**: apply a large number of DM commands to exercise all dimensions of control Jacobian matrix
- **Closed-loop identification**: use DM commands and images measured during closed-loop WFS&C to refine knowledge of Jacobian → **this study**

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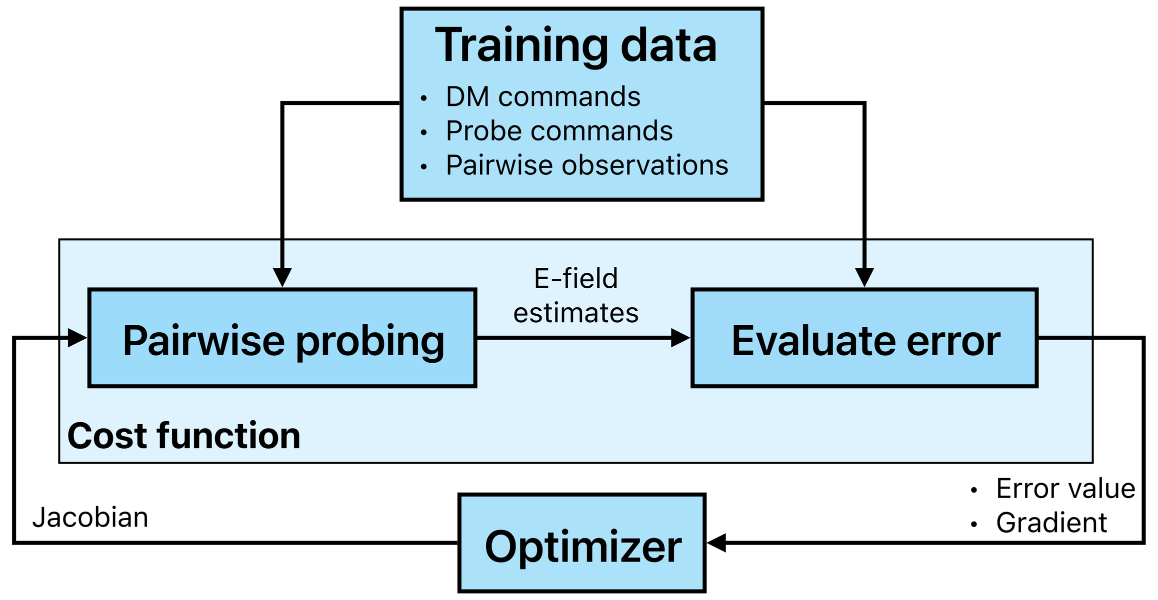
Algorithms for empirical estimation of control parameters

Prediction error minimization (state domain)

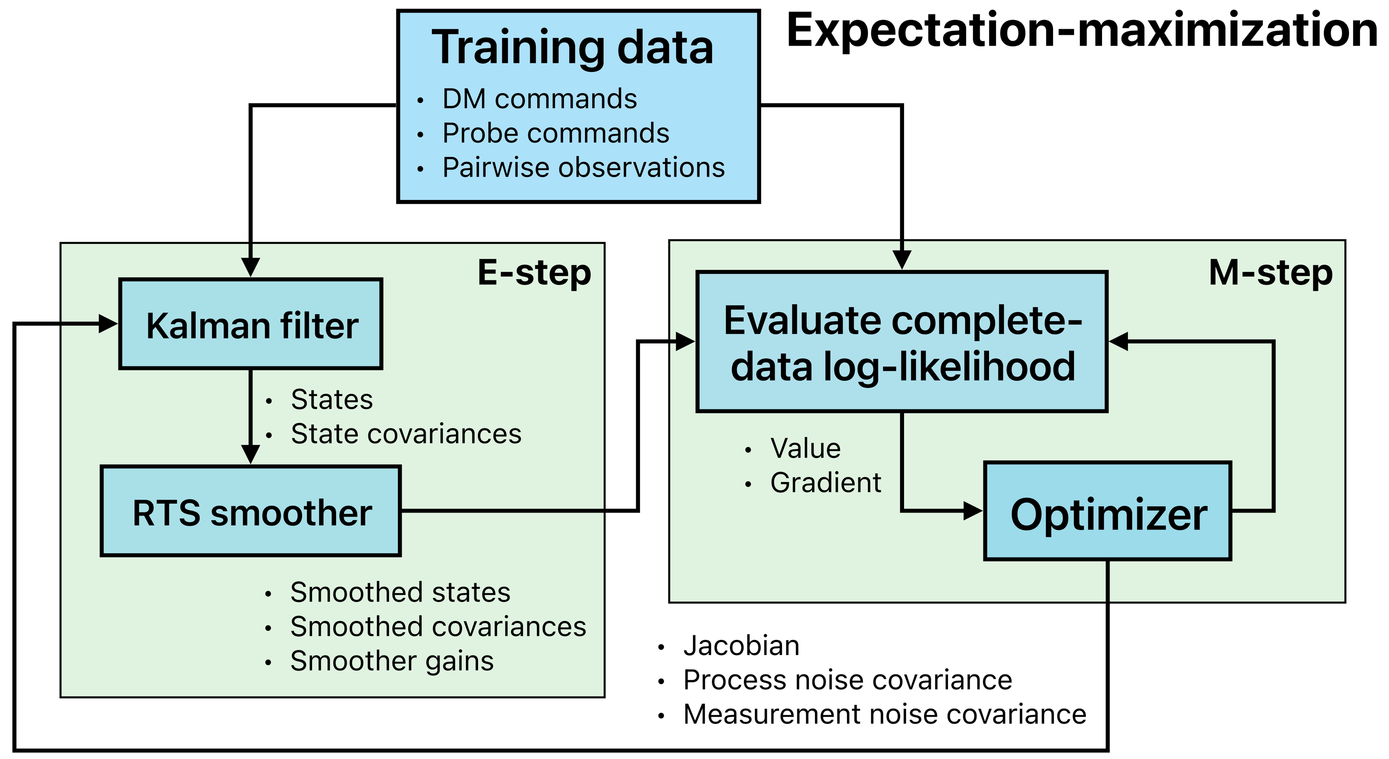


- Minimize the error between predicted changes in focal-plane electric field due to control ($\mathbf{G}\mathbf{u}_k$) and "actual" changes ($\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k-1}$):

$$\widehat{\mathbf{G}} = \underset{\mathbf{G}}{\arg\min} \sum_{k=1}^{K} \frac{1}{\mathbf{u}_k^T\mathbf{u}_k} \|\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k-1} - \mathbf{G}\mathbf{u}_k\|^2$$

- Alternate between using current $\mathbf{G}$ to estimate electric fields (e.g. with pairwise probing) and using electric field estimates to obtain new estimate of $\mathbf{G}$

Expectation-maximization

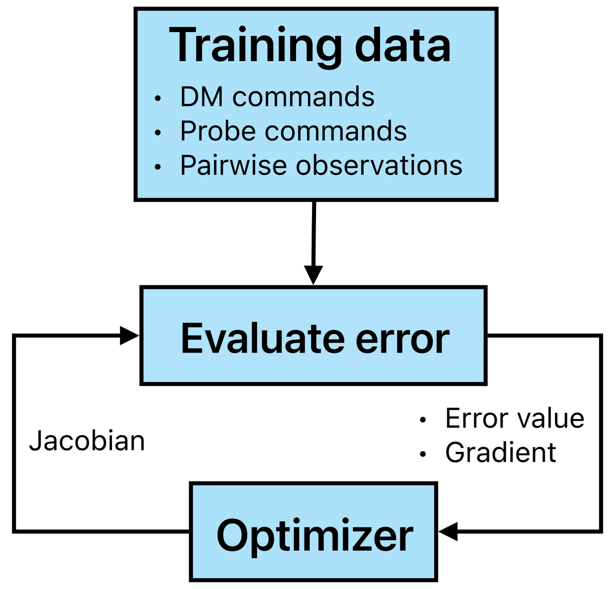


- Maximize the marginal log-likelihood of the data over all K time steps,

$$(\widehat{\mathbf{G}}, \widehat{Q}, \widehat{R}) = \underset{\mathbf{G}, Q, R}{\arg\min}\ \log p(\mathbf{z}_{1:K} \mid \mathbf{G}, Q, R)$$

by breaking into alternating subproblems:
  1. Estimate focal-plane electric field using current values of $(\widehat{\mathbf{G}}, \widehat{Q}, \widehat{R})$ (**E-step**)
  2. Maximize a surrogate for the marginal log-likelihood to obtain new values for $(\widehat{\mathbf{G}}, \widehat{Q}, \widehat{R})$ (**M-step**)[1]
- Repeat E-step and M-step until parameter estimates converge, or perform a fixed number of steps
- **Statistically optimal!**
- Equivalent to first-order optimization methods in convergence rate (e.g., steepest descent) → potentially slow
- Estimated value of Q can be used to optimally tune Tikhonov regularization for electric field conjugation (EFC)[2]
- Originally demonstrated for coronagraph system identification by Sun et al.[3,4]

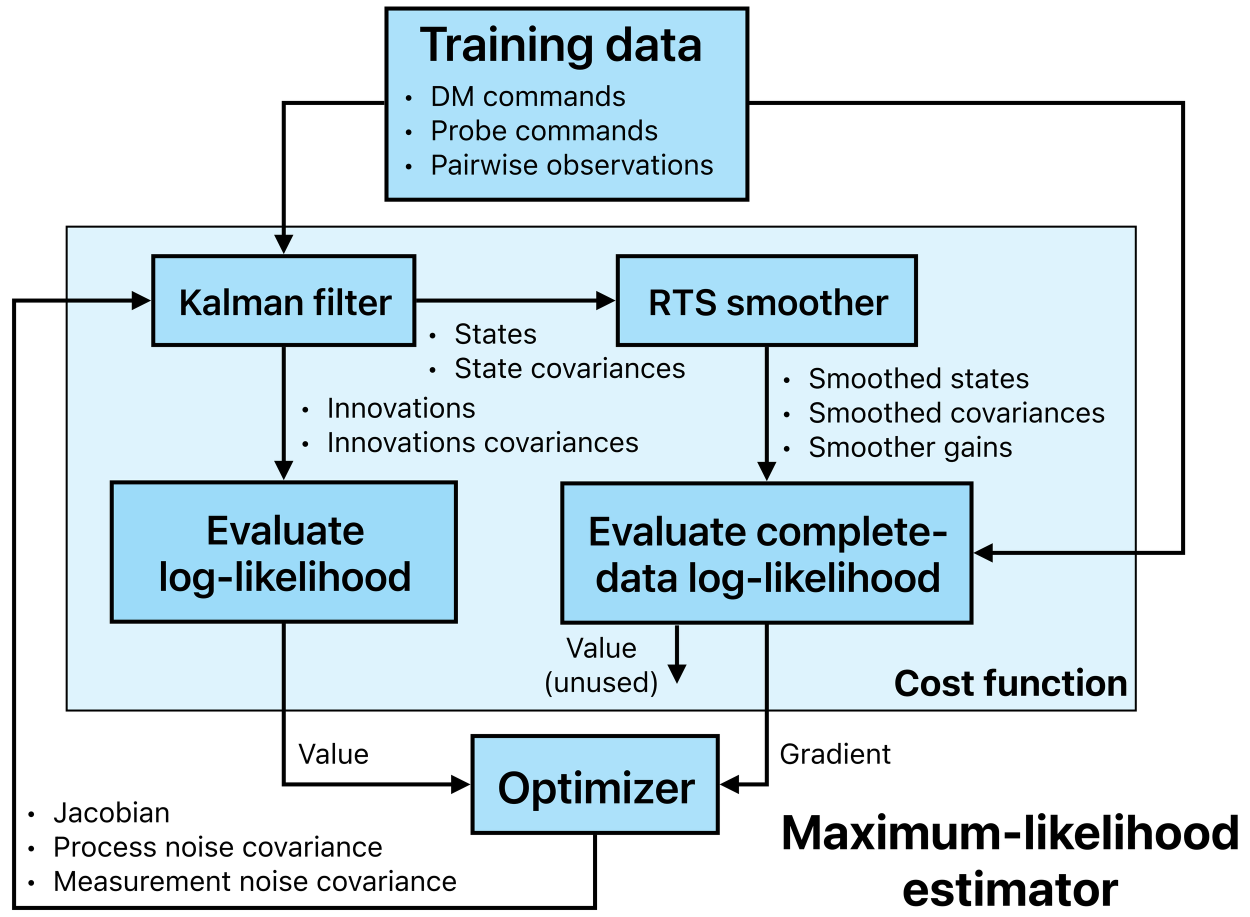
Prediction error minimization (data domain)



- Minimize the error between predicted changes in pairwise observations ($\mathbf{H}\mathbf{G}\mathbf{u}_k$) and actual changes in observations ($\mathbf{z}_k - \mathbf{z}_{k-1}$):

$$\widehat{\mathbf{G}} = \underset{\mathbf{G}}{\arg\min} \sum_{k=1}^{K} \frac{1}{\mathbf{u}_k^T\mathbf{u}_k} \left\|\mathbf{z}_k - \mathbf{z}_{k-1} - 4\mathbf{G}^T\boldsymbol{\Psi}^T\mathbf{G}\mathbf{u}_k\right\|^2$$

Maximum-likelihood estimator



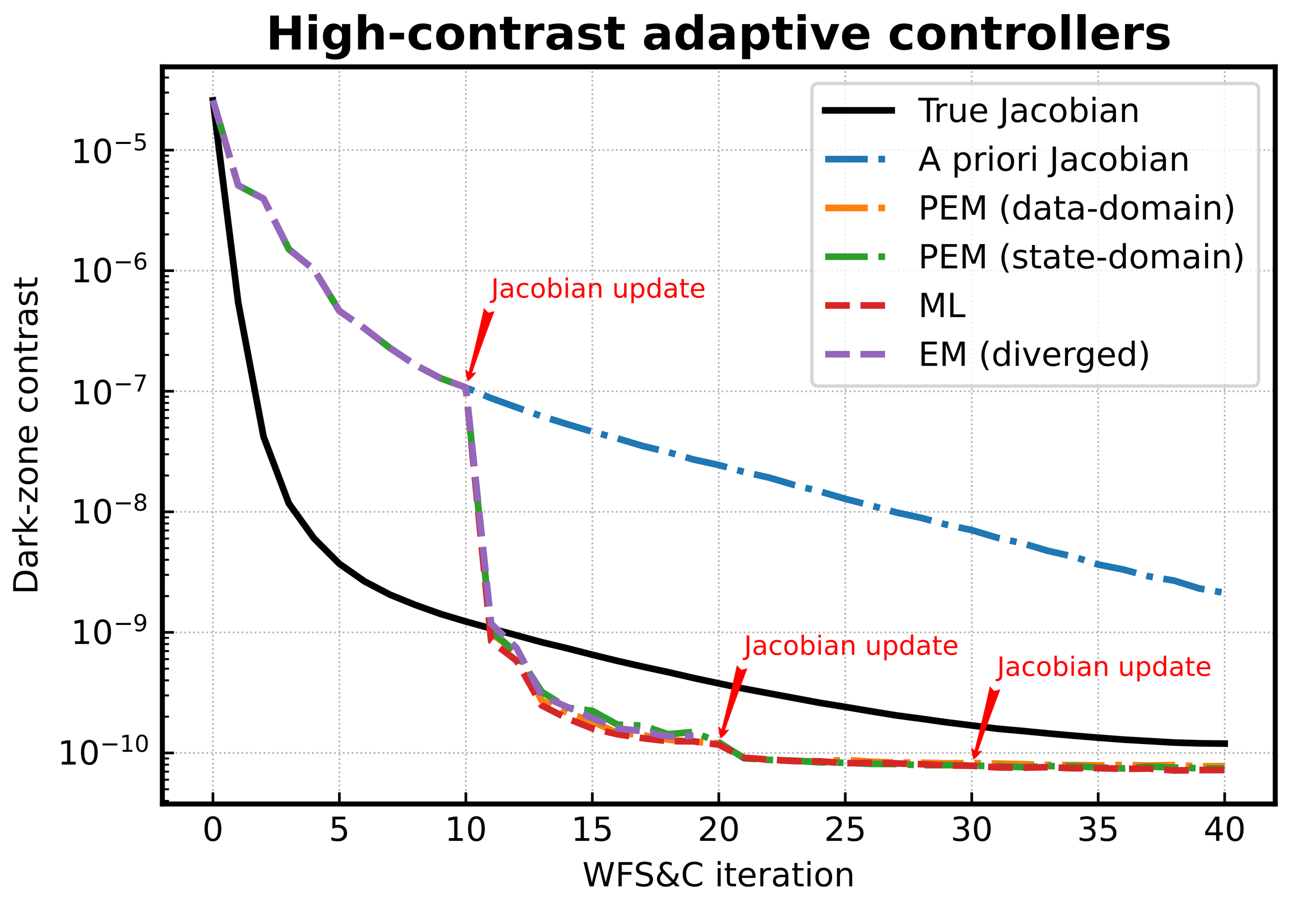
- Maximize the marginal log-likelihood of the data over all K time steps,

$$(\widehat{\mathbf{G}}, \widehat{Q}, \widehat{R}) = \underset{\mathbf{G}, Q, R}{\arg\min}\ \log p(\mathbf{z}_{1:K} \mid \mathbf{G}, Q, R)$$

using a second-order optimization algorithm (e.g., L-BFGS[5]) to converge to parameter estimates much more rapidly than EM.
- Use the fact that the gradient of the EM surrogate likelihood and the gradient of the marginal log-likelihood are the same immediately after the E-step (*Fisher's identity*[6])
- **Statistically optimal!**
- Maximizes same objective function as EM, but with better convergence

Results

- Large UV/Optical/Infrared (LUVOIR) apodized pupil Lyot coronagraph (APLC) design
- Two 34×34 DMs; out-of-pupil DM at Fresnel number $N_F = 550$
- 10 nm RMS random high-order wavefront error in entrance pupil
- 20% random gain errors applied to DM actuators to simulate model mismatch
- Simulated EFC + pairwise probing,[2] updating Jacobian every 10 iterations
- Implemented cost functions with Jax[7] and minimized using SciPy L-BFGS[8]



Conclusions and future work

- Jacobian-based system identification shows promise for improving control loop performance at high contrast
- **Next steps**:
  - Jacobian-free model parameter estimation
  - Experimental demonstrations (HiCAT testbed, ExoSpec Testbed)

References

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