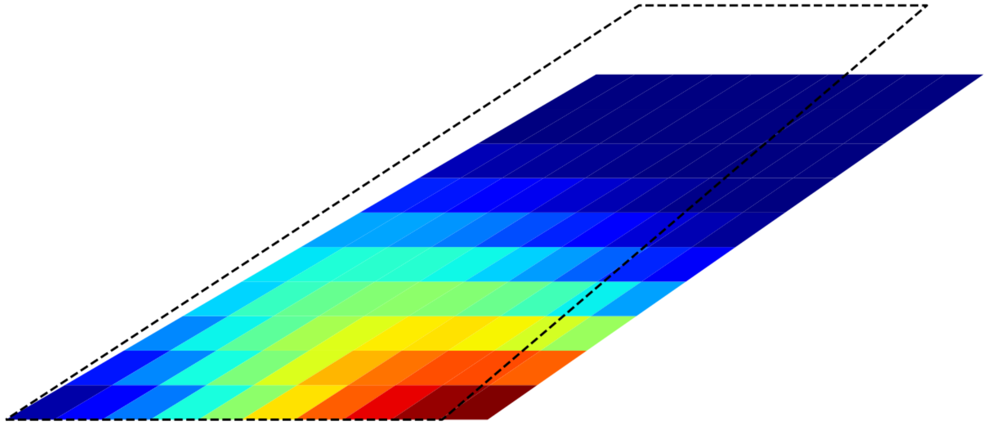

Linearized Frequency-Domain Gust Analysis and Adjoint-Based Sensitivities

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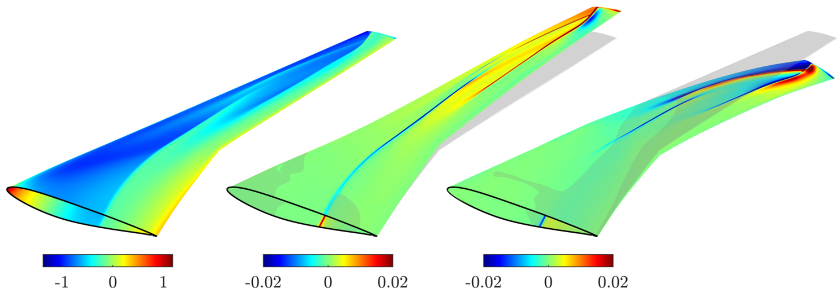
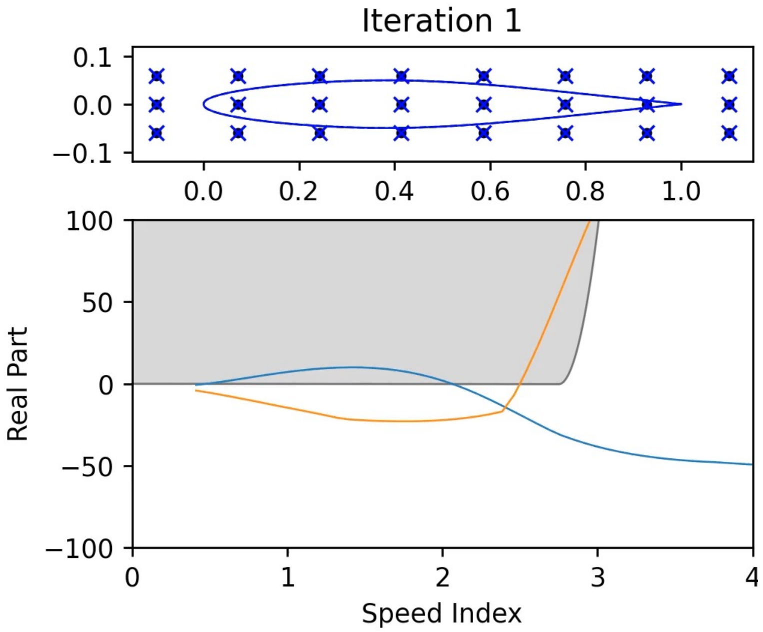
Aeroelasticity in Aircraft Design

Dynamic aeroelastic constraints in the transonic regime are critical for modern aircraft

- Linearized Frequency-Domain (LFD) analysis allows efficient CFD-based analysis
  - Appropriate for small perturbations about a nonlinear equilibrium
  - Computes Generalized Aerodynamic Forces (GAFs) that fit into typical industry processes
- Previously implemented LFD for flutter analysis and optimization
  - Adjoint-based sensitivities of flutter cost functions with respect to structural, aerodynamic, and geometry design variables
- Goal: extend LFD analysis and sensitivities to gust problems
  - Demonstrate with stochastic gust constraints





Analysis Toolset

Flow solver - FUN3D's Stabilized Finite Element solver

- Streamlined Upwind Petrov-Galerkin (SUPG)

Structural solver – in-house finite-element solver

- Linear shell elements + modal decomposition

Coupling and Optimization Framework – OpenMDAO+MPhys

- OpenMDAO: Automated coupling + adjoint-based sensitivities
- MPhys: standardized coupling for multiphysics ➜ modularity
  - High-fidelity tools (CFD+FEM)
  - Mid-fidelity tools (Doublet Lattice Method+FEM)

FUN3D
Fully Unstructured Navier-Stokes

Stochastic Gust Modeling

1. **Modal structural dynamics:**

- Solve over the frequency range of interest

Modal mass, damping, stiffness

Gust-on-structure GAFs

$$\left[-\omega^2 \bar{\mathbf{M}} + i\omega \bar{\mathbf{C}} + \bar{\mathbf{K}} - q_\infty \bar{\mathbf{Q}}_{ss}(i\omega)\right] \boldsymbol{\eta}(i\omega) = q_\infty \bar{\mathbf{Q}}_{gs}(i\omega)$$

Structural-on-structure GAFs

Generalized displacements

2. **Power Spectral Density (PSD) and Root Mean Square (RMS)**

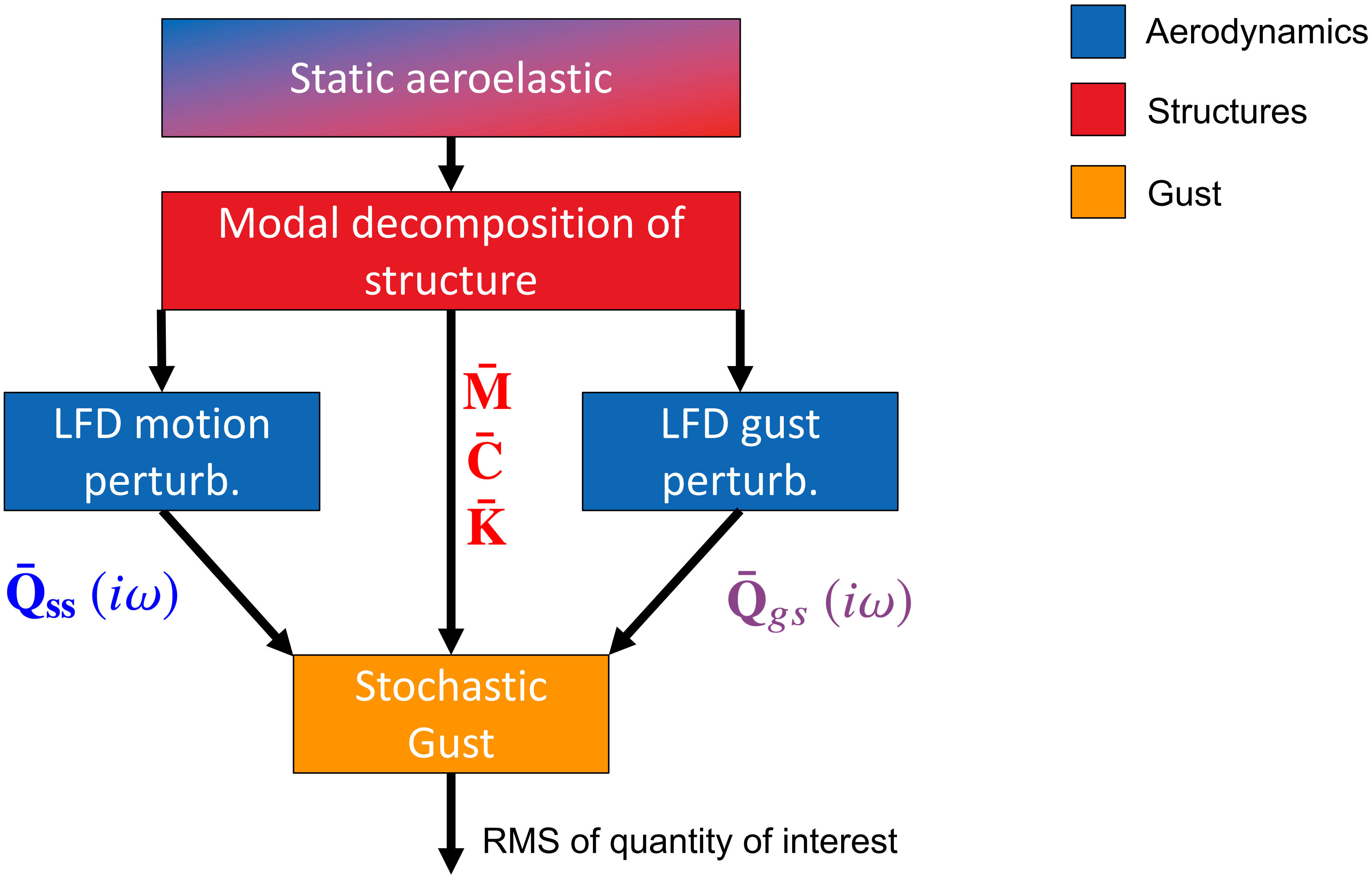
Dryden atmospheric turbulence model

$$PSD(i\omega) := \left|\mathbf{S}^T \boldsymbol{\eta}(i\omega)\right|^2 f_{atm}(i\omega)$$

Map from modal displacement to function of interest

$$RMS := \sqrt{\int_0^{\omega_{max}} PSD(i\omega) d\omega}$$

LFD-based Stochastic Gust Process



LFD – Modal Motion Perturbations

$$\mathbf{M}(\mathbf{q}, \mathbf{x}_G, \dot{\mathbf{x}}_G)\,\dot{\mathbf{q}} + \mathbf{R}(\mathbf{q}, \mathbf{x}_G, \dot{\mathbf{x}}_G) = 0$$

Exact linearization of governing residual about the equilibrium state with harmonic mode shapes perturbations

$$\left(i\omega \mathbf{M}\big|_{\mathbf{x}_{G0},\mathbf{q}_0} + \frac{\partial \mathbf{R}}{\partial \mathbf{q}}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0} \right) \hat{\mathbf{q}}_j = -\frac{\partial \mathbf{R}}{\partial \mathbf{x}_G}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0} \hat{\mathbf{x}}_{G,j} - i\omega \frac{\partial \mathbf{R}}{\partial \dot{\mathbf{x}}_G}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0} \hat{\mathbf{x}}_{G,j}$$

Inputs:

- $\mathbf{x}_{G0}, \mathbf{q}_0$ - static equilibrium volume mesh and flow state
- $\hat{\mathbf{x}}_{G,j}$ - the volume mesh motion due to structural mode *j*
- ω - the frequency at which the structural mode is oscillated

Output:

- $\hat{\mathbf{q}}_j$ - the frequency-domain state of the flow (Fourier coefficient)

LFD – Gust Perturbations

$$\mathbf{M}\left(\mathbf{q}, \mathbf{x}_G, \dot{\mathbf{x}}_G, \mathbf{u}_G\right)\dot{\mathbf{q}} + \mathbf{R}\left(\mathbf{q}, \mathbf{x}_G, \dot{\mathbf{x}}_G, \mathbf{u}_G\right) = 0$$

Gust applied with Field Velocity Method

Exact linearization of governing residual about the equilibrium state with gust perturbations

$$\left(i\omega \mathbf{M}\big|_{\mathbf{x}_{G0},\mathbf{q}_0} + \frac{\partial \mathbf{R}}{\partial \mathbf{q}}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0} \right)\hat{\mathbf{q}}_j = -\frac{\partial \mathbf{R}}{\partial \mathbf{u}_G}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0} \hat{\mathbf{u}}_G$$

Inputs:

- $\mathbf{x}_{G0}, \mathbf{q}_0$ - static equilibrium volume mesh and flow state
- $\hat{\mathbf{u}}_G$ - harmonic gust perturbation field
- ω - the gust frequency

$$\hat{\mathbf{u}}_G = e^{-j\omega x / M_\infty}$$

Sinusoidal gust convecting with freestream

Output:

- $\hat{\mathbf{q}}_j$ - the frequency-domain state of the flow (Fourier coefficient)

Verification of LFD Gust Implementation

- **NACA64A010 airfoil**
  - Mach 0.4, reduced frequency 0.5
  - Time-domain gust magnitude = 1e-5, then Fourier Transform (FT)

Variable	Value
LFD $\hat{\rho}$	$-\mathbf{0.7548}15161228179932 + \mathbf{0.545}927464962005615j$
FT $\hat{\rho}$	$-\mathbf{0.7548}08783531188965 + \mathbf{0.545}866072177886963j$
LFD $\hat{u}$	$\mathbf{1.7779}71386909484486 - \mathbf{1.5403}95259857177773j$
FT $\hat{u}$	$\mathbf{1.7779}30021286010174 - \mathbf{1.5403}35893630981450j$
LFD $\hat{w}$	$-\mathbf{0.3968}69659423828125 - \mathbf{0.4044}74884271621704j$
FT $\hat{w}$	$-\mathbf{0.3968}85335445404053 - \mathbf{0.4044}46154832839966j$
LFD $\hat{T}$	$-\mathbf{0.3458}17238092422485 + \mathbf{0.25134}9925994873047j$
FT $\hat{T}$	$-\mathbf{0.3458}05108547210693 + \mathbf{0.25134}1730356216431j$

LFD Adjoint Terms

- **Linearized forward problem ➜ 2nd derivatives of the flow governing equations for adjoint**

$$\left(i\omega \mathbf{M}\big|_{\mathbf{x}_{G0},\mathbf{q}_0} + \frac{\partial \mathbf{R}}{\partial \mathbf{q}}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0} \right) \hat{\mathbf{q}}_j = -\boxed{\frac{\partial \mathbf{R}}{\partial \mathbf{u}_G}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0} \hat{\mathbf{u}}_G}$$

- **Sensitivity of LFD gust with respect to steady flow state:**
  - 1st derivative: analytical, Automatic Differentiation (AD)
  - 2nd derivative: complex-step directional derivatives

$$\frac{\partial}{\partial \mathbf{q}_0}\left(\boxed{\frac{\partial \mathbf{R}}{\partial \mathbf{u}_G}\bigg|_{\mathbf{x}_{G0},\mathbf{q}_0,\mathbf{u}_G=0} \hat{\mathbf{u}}_{G,k}} \right)^T \psi_{A_{gust},k} = \frac{\operatorname{imag}\left(\frac{\partial \mathbf{R}}{\partial \mathbf{q}}\Big|^T_{\mathbf{x}_{G0},\mathbf{q}=\mathbf{q}_0,\mathbf{u}_G=ih\,\operatorname{real}(\hat{\mathbf{u}}_{G,k})} \operatorname{real}\left(\psi_{A_{gust},k}\right)\right)}{h}$$

$$- \frac{\operatorname{imag}\left(\frac{\partial \mathbf{R}}{\partial \mathbf{q}}\Big|^T_{\mathbf{x}_{G0},\mathbf{q}=\mathbf{q}_0,\mathbf{u}_G=ih\,\operatorname{imag}(\hat{\mathbf{u}}_{G,k})} \operatorname{imag}\left(\psi_{A_{gust},k}\right)\right)}{h}$$

Adjoint Verification

AGARD 445.6 wing with RMS of the tip displacement as the cost function

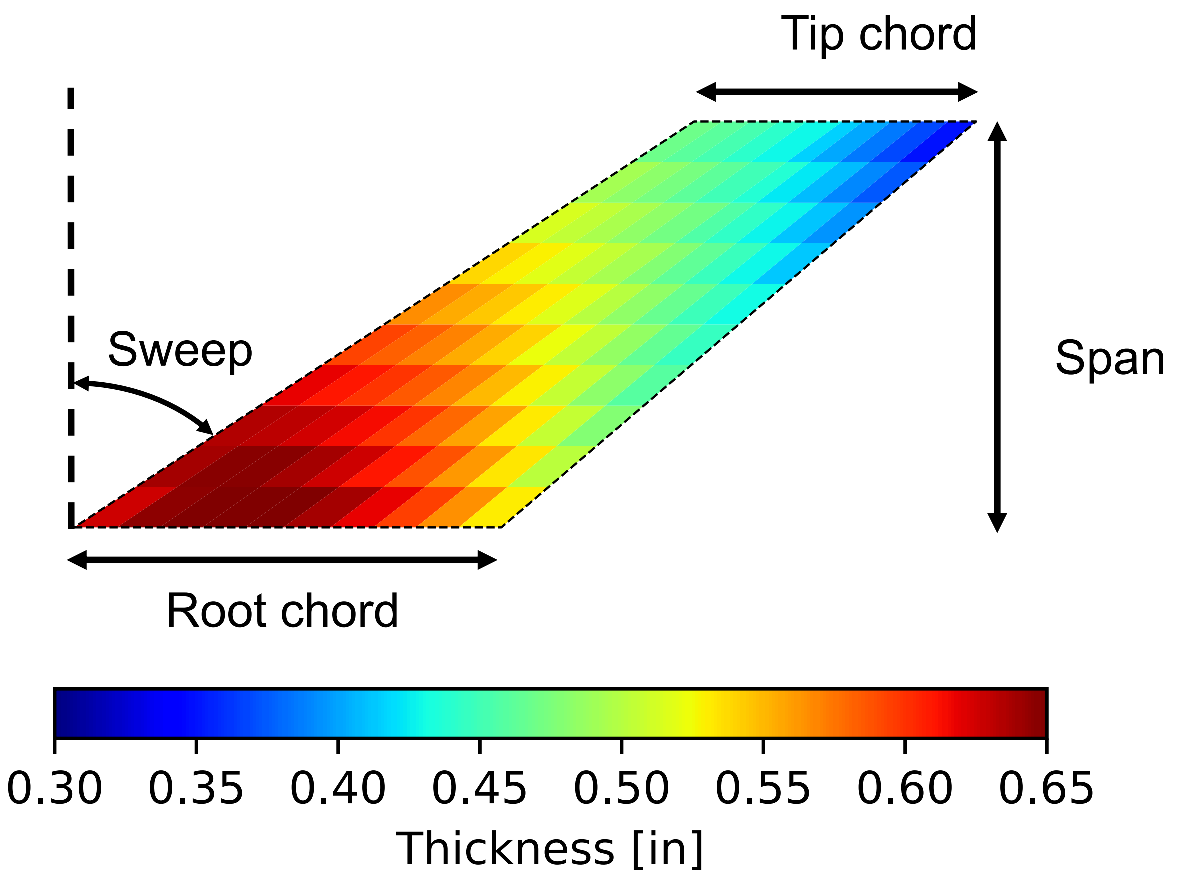
Method	Structural Thickness	Sweep
Analytical - Reverse	**−3.487636565298064**	**−0.028985445010219**
Finite Difference ($h = 10^{-2}$)	**−3.4**41091464667863	**−0.02895**1549078215
Finite Difference ($h = 10^{-3}$)	**−3.48**3042198356543	**−0.02898**206800478
Finite Difference ($h = 10^{-4}$)	**−3.487**177661110763	**−0.0289851**28032218
Finite Difference ($h = 10^{-5}$)	**−3.487**590761040337	$\boxed{\textbf{−0.0289857}85116674}$
Finite Difference ($h = 10^{-6}$)	$\boxed{\textbf{−3.48763}1555646658}$	**−0.02898**800582625
Finite Difference ($h = 10^{-7}$)	**−3.4876**25617533922	**−0.02898**9844024181
Finite Difference ($h = 10^{-8}$)	**−3.487**759321928024	**−0.02**909576892852 8

Stochastic Gust Constraint

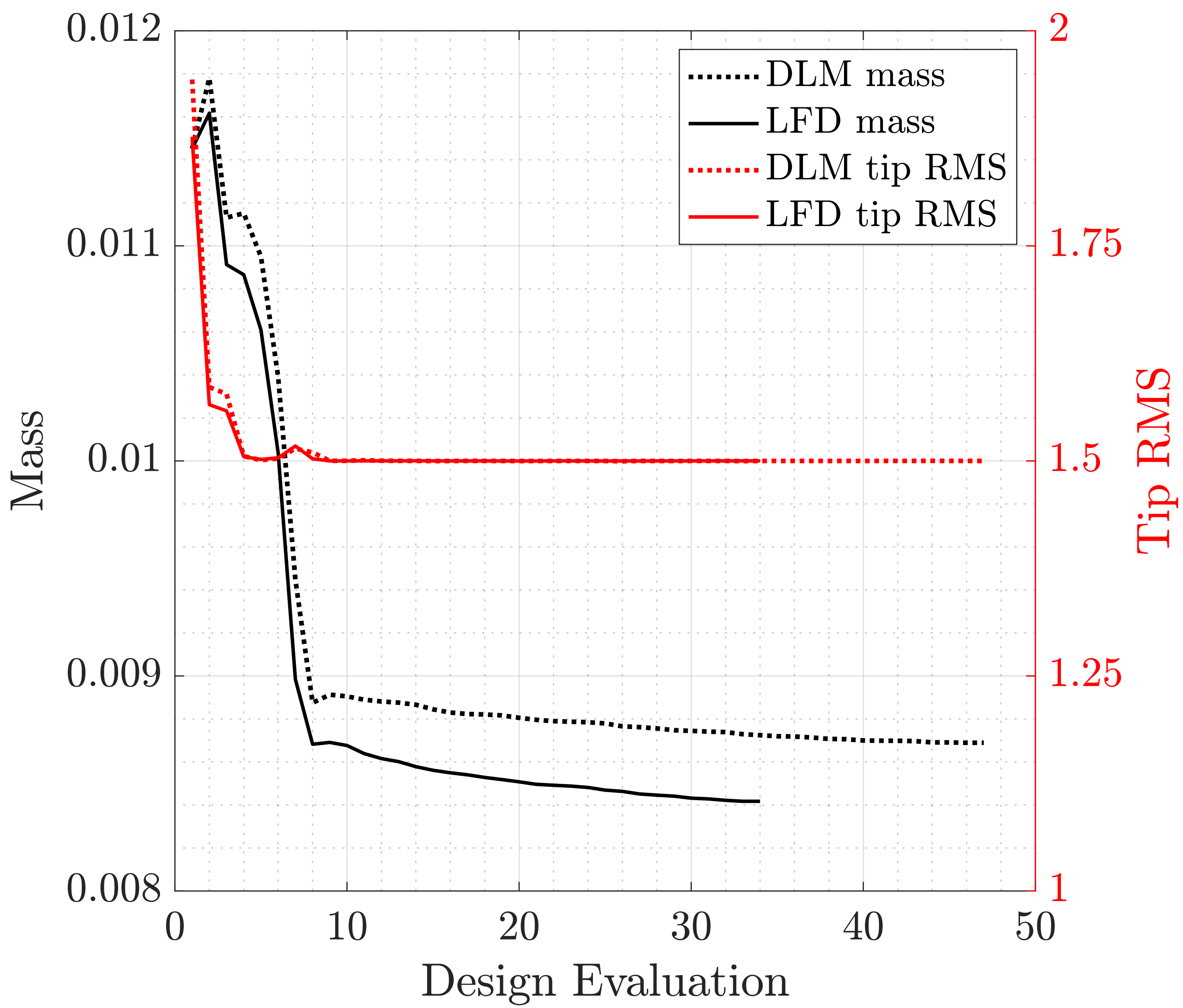
- **AGARD 445.6 wing at Mach 0.5**
  - Performed with LFD and DLM gust analysis
    - » LFD – inviscid analysis with 219,555 nodes in the mesh
- **Minimize the mass subject to a gust constraint**

$$\begin{aligned} \min \quad & mass \\ \text{subject to:} \quad & RMS_{tip} \leq 1.5 \\ & A = A_0 \end{aligned}$$

- **104 design variables**
  - 100 structural thicknesses
  - 4 geometric (affect aero. and struct. meshes)



Optimization History



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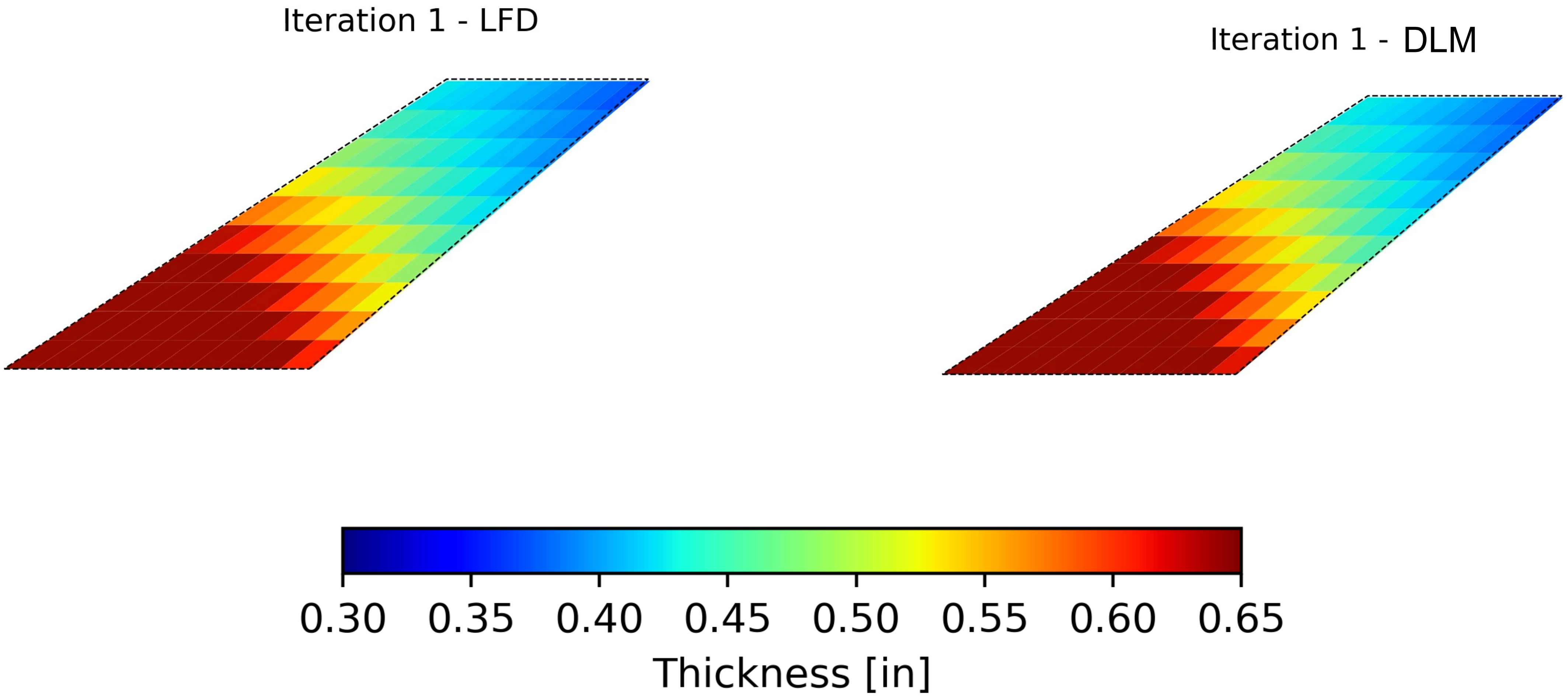
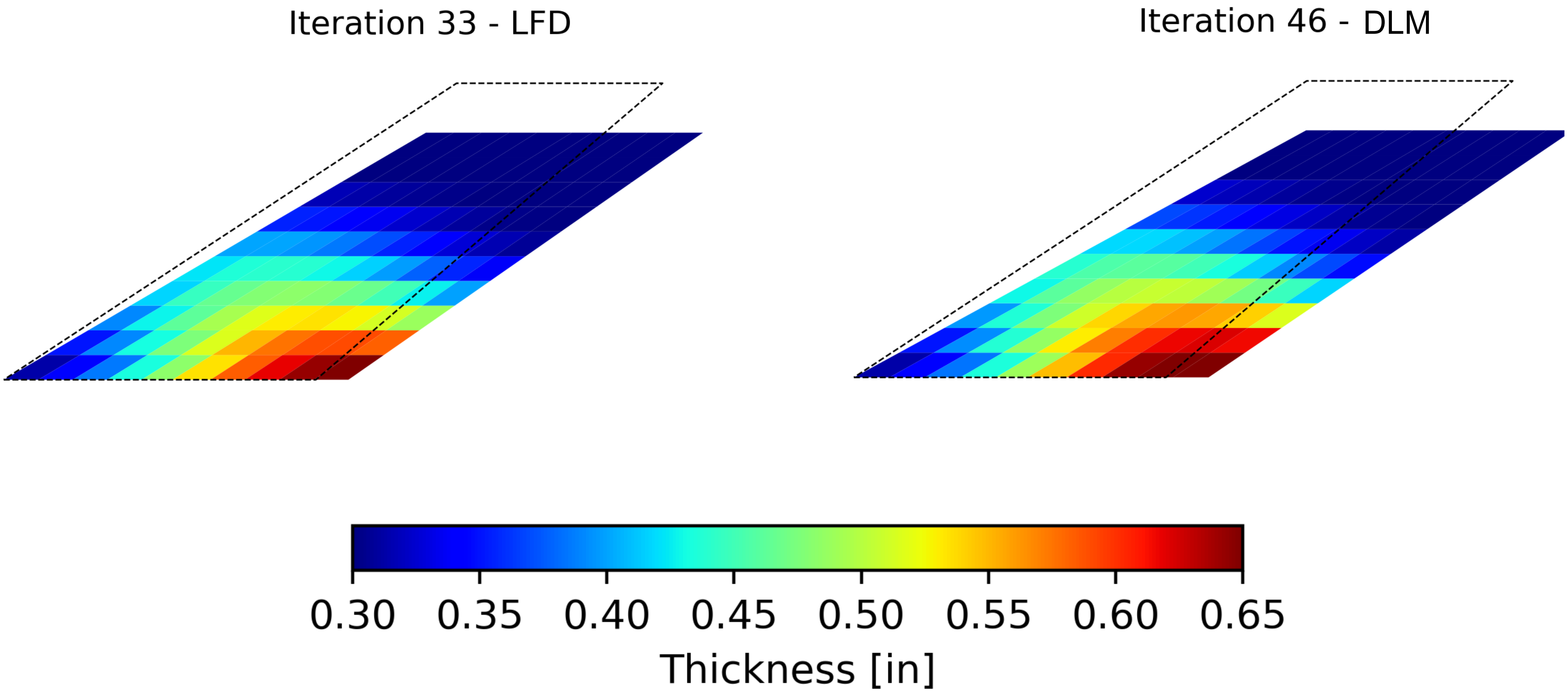


Final Designs



Conclusions

- **Added LFD gust modeling and sensitivities to FUN3D's stabilized finite-element solver**
  - Linearized and harmonic assumptions in LFD reduce cost of CFD-based aeroelastic analysis
  - Verified analysis and adjoint-based sensitivities
  - Demonstrated a stochastic gust constraint on the AGARD 445.6 wing
- **Future work:**
  - Apply to NASA Common Research Model (CRM) at transonic conditions
  - See Thelen et al. "Flutter and Stochastic Gust Constraint Studies of the Undeflected CRM using MPhys"
    - Today 1:20-1:40pm in Barrel Springs II