Use of beacon satellites for efficient uplink transmission for free-space quantum entanglement distribution

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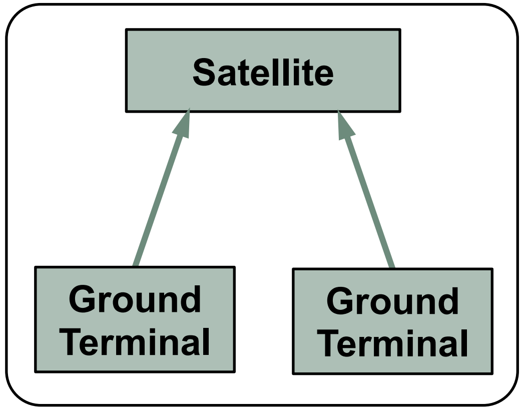
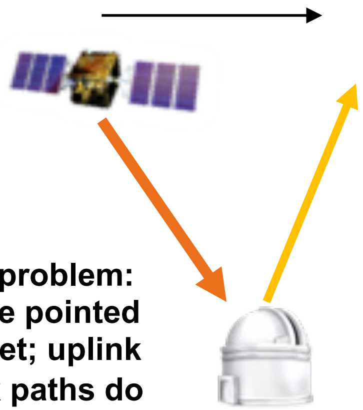
Dual Uplink Entanglement Swap

Dual Uplink

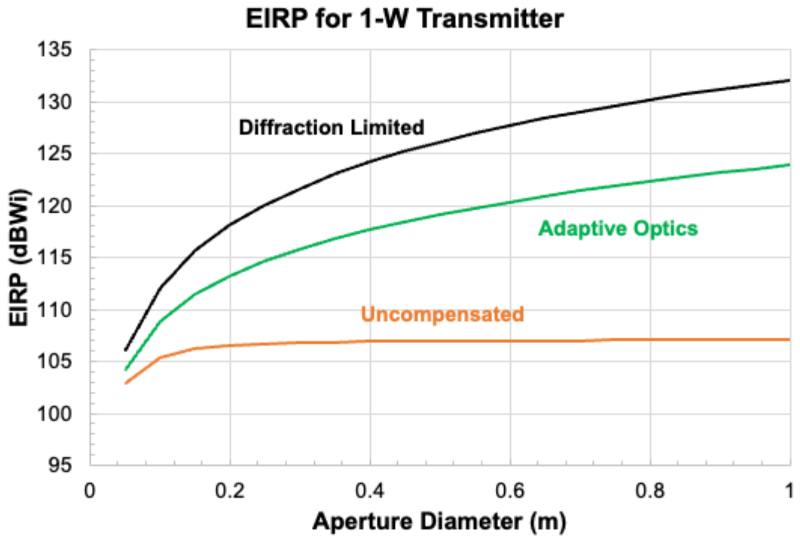


- Entanglement sources at both ground stations send qubits to satellite
- Optical Bell state measurement on satellite entangles remaining qubits at ground stations

- Entanglement swap rate is sensitive to loss over ground-to-space path
  - Proportional to product of irradiances of two uplink beams at satellite
- Atmospheric turbulence introduces wavefront aberration that spreads the uplink beam and reduces irradiance at the quantum satellite
- Adaptive-optics is effective way to compensate turbulence-induced loss, but performance can be limited by point-ahead angular offset



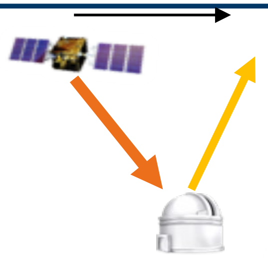
Point-ahead problem: Laser must be pointed ahead of target; uplink and downlink paths do not overlap



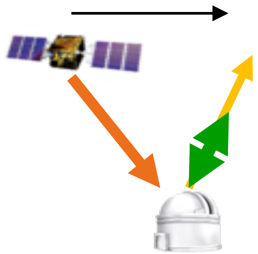
EIRP = Equivalent Isotropic Radiated Power (Irradiance * $4\pi \text{Range}^2$)

Uplink Adaptive Optics Approaches

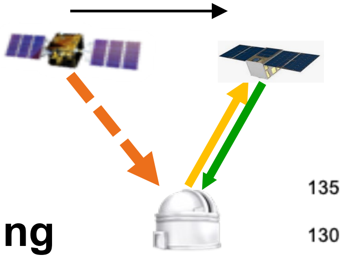
- **Use the downlink beam from target satellite as the adaptive-optics**
  - **Regret: Large anisoplanatism loss**



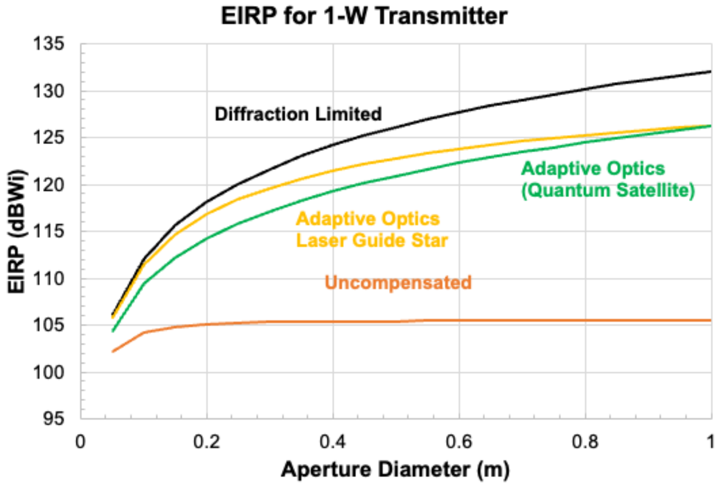
- **Project a synthetic beacon (laser guide star) in the point-ahead direction and use the return as the adaptive-optics beacon**
  - **Regret: Loss caused by tilt anisoplanatism**
    Weak laser-guide star-return inadequate for high-performance compensation



- **Place a satellite (cubesat) in the point-ahead direction to provide adaptive-optics beacon**
  - **Regret: Requires additional satellite and accurate satellite positioning**

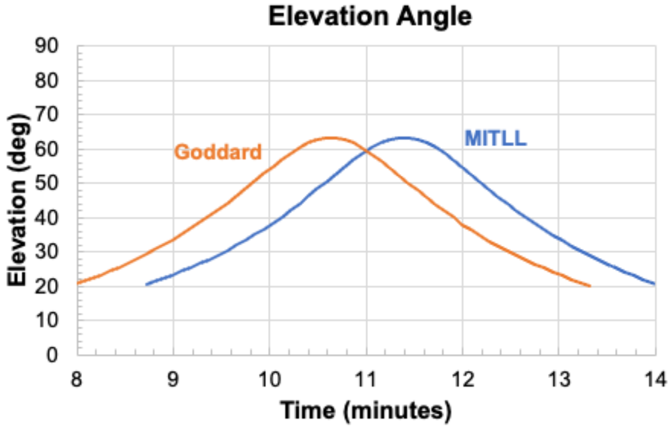


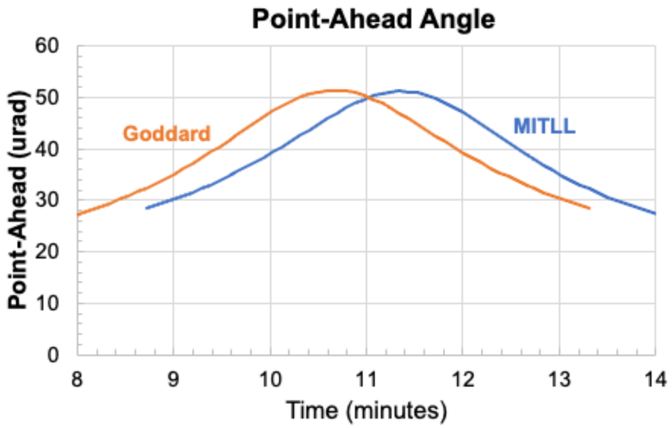
- **A-O performance with satellite beacon(s) addressed in following charts**
  - **Multiple combinations of beacon satellite number and position analyzed**

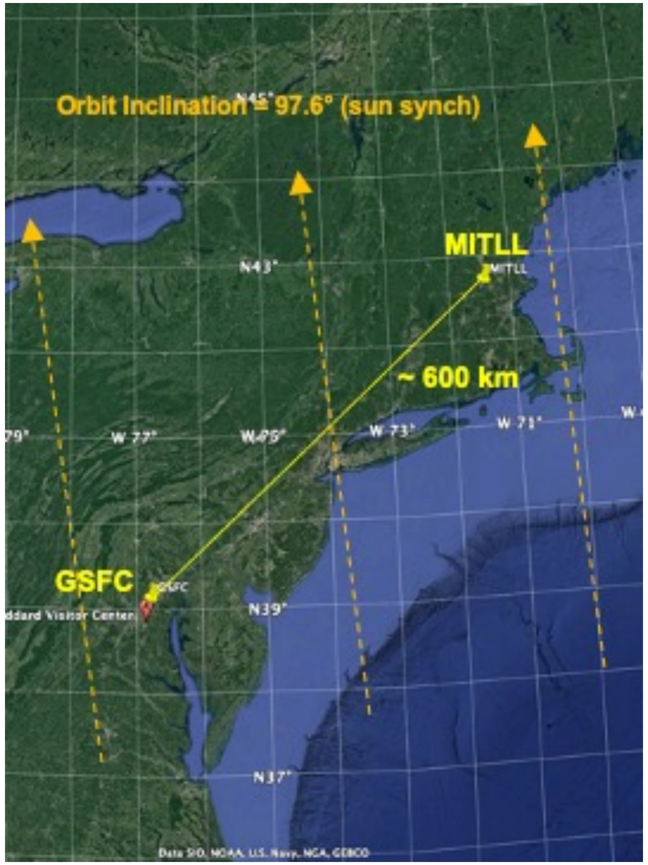


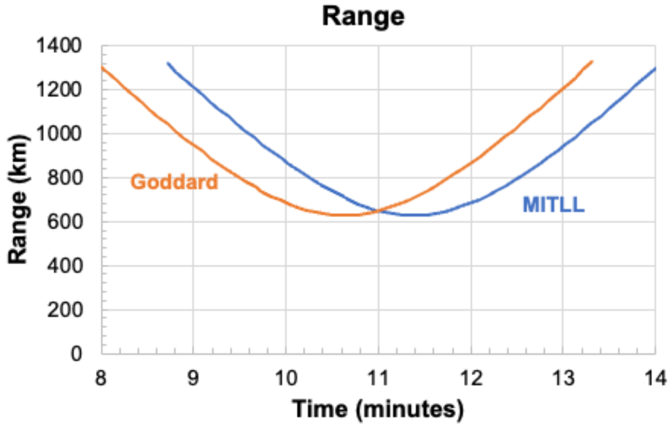
Satellite Orbits

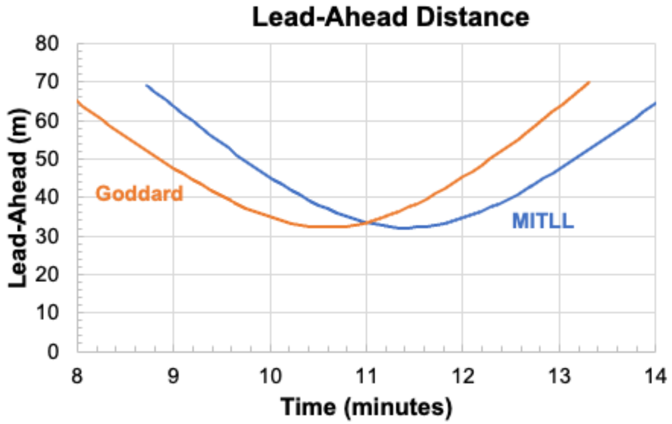
- Analysis performed for sun-synchronous orbit for notional ground sites at NASA Goddard Space Flight Center and MIT Lincoln Laboratory
  - Orbit altitude = 567 km, Inclination = 97.6 deg





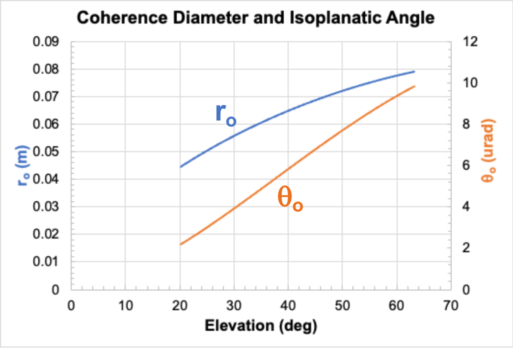


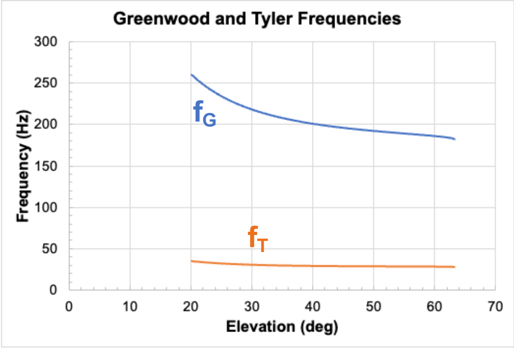




Analysis Methodology

- **Combination of theoretical and empirical models used to calculate impact of individual effects that limit adaptive-optics compensation of atmospheric turbulence; individual effects then combined to estimate overall Strehl ratio* for uplink beam**





<u>Terms modeled in analysis include:</u>

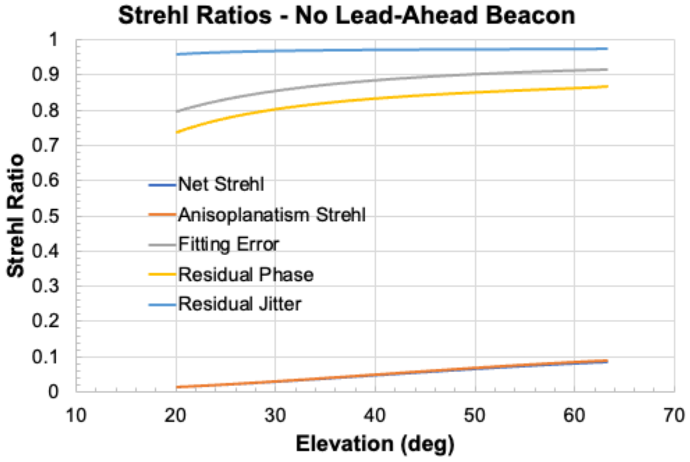
- **Deformable mirror (DM) fitting error: $\sigma_\phi^2 \propto (d/r_o)^{5/3}$**
- **Wavefront sensor sampling error: $\sigma_\phi^2 \propto (d/r_o)^{5/3}$**
- **Residual phase error (finite correction bandwidth): $\sigma_\phi^2 = F(f_G, f_{BW})$**
- **Residual jitter (finite tracking bandwidth): $\sigma_\phi^2 = F(f_T, f_{BW})$**
- **Wavefront sensor measurement noise: $\sigma_\phi^2 = F(S, n_e)$**
- **Angular anisoplanatism: Strehl = $F(\theta/\theta_o, D/r_o, C_n^2)$**

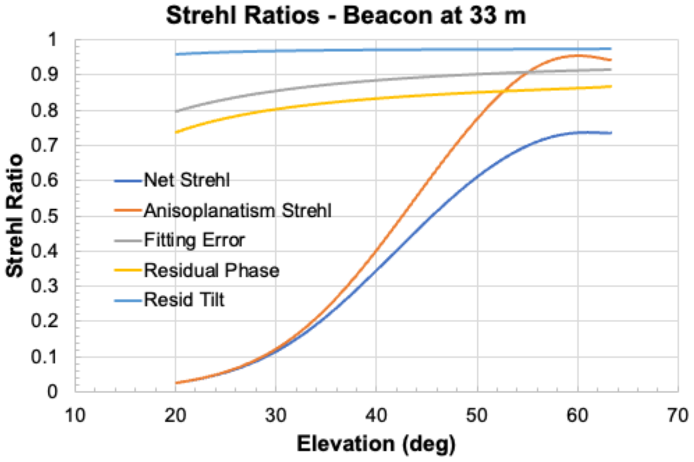
System Parameters

- **D = telescope aperture**
- **d = WFS subaperture diameter**
- **S = signal in WFS subaperture**
- **n_e = WFS camera read noise**
- **f_{BW} = correction bandwidth of a-o system**
- **λ = 0.78 μm**

Channel Parameters

- **r_o = atmospheric coherence diameter (m)**
- **θ_o = isoplanatic angle (rad)**
- **f_G = Greenwood (phase) frequency (Hz)**
- **f_T = Tyler (tracking) frequency (Hz)**





*** Strehl Ratio = on-axis intensity / diffraction-limited on-axis intensity**

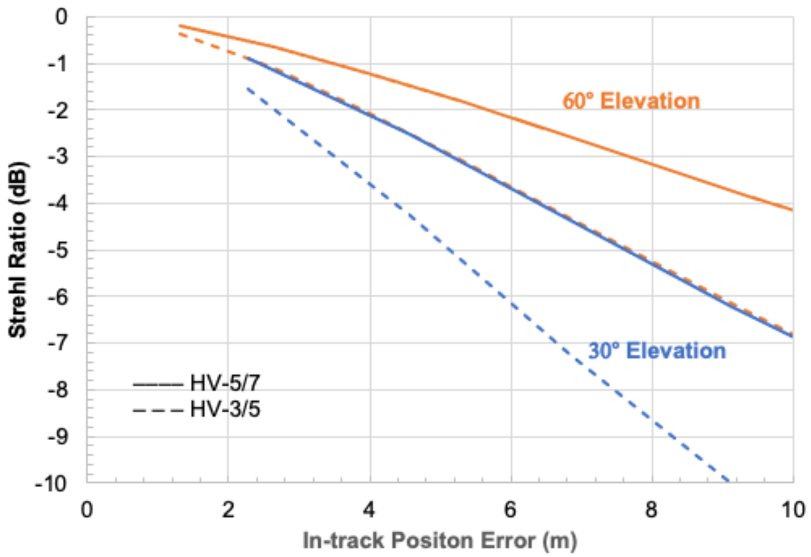
Point-Ahead Anisoplanatism

- **Isoplanatic error is the wavefront difference between the measured and corrected paths**
  - **Point-ahead isoplanatic error limits the quality of the correction (Strehl ratio) of the uplink beam**
- **Anisoplanatic error is a function of (θ_{pa}/θ_o) and (D/r_o)**
  - **Isoplanatic angle, θ_o, and the coherence diameter, r_o, depend on the strength and distribution of turbulence along the path**
- **Typical values for λ = 0.78 μm:**
  - **HV-5/7 C_n^2 (moderate strength)**

	30°	60° elevation
r_o =	5.63	7.82 cm
θ_o =	3.94	9.48 μrad

  - **HV-3/5 C_n^2 (stressing)**

	30°	60° elevation
r_o =	3.37	4.69 cm
θ_o =	2.81	6.77 μrad



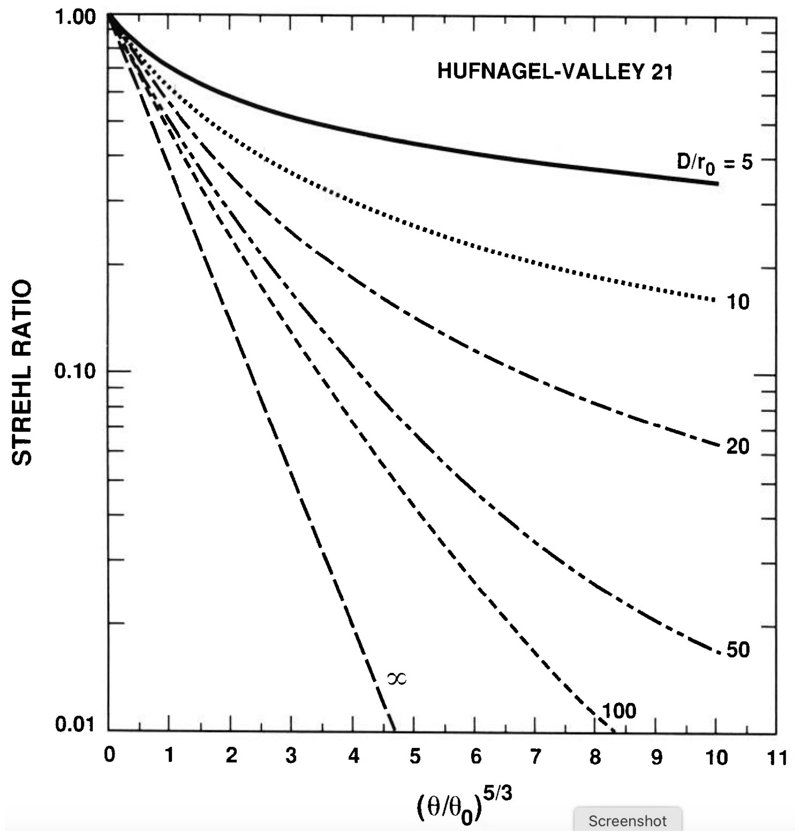
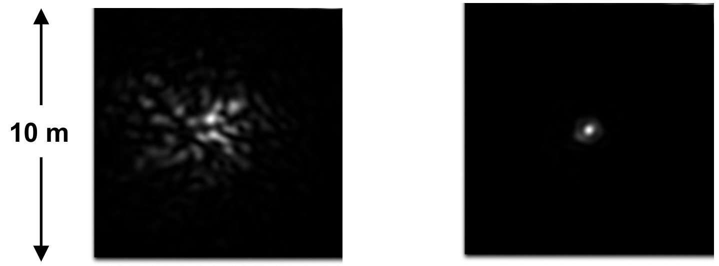


Figure from J. Herrmann, Point-Ahead Anisoplanatism, MITLL Project Memorandum 54PM-SWP-0014, Dec. 1990

Propagation-Code Simulation

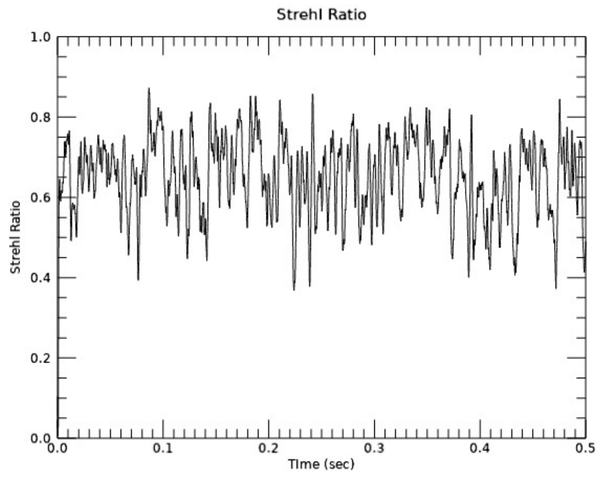
- **Propagation code simulates a-o correction of uplink beam**
  - Performs numerical simulation of laser beam propagation through turbulence
  - Includes numerical simulation of adaptive-optics system
  - Higher fidelity than scaling-law analysis because all degrading effects included together, but very time-consuming to run; typically simulate specific point(s) in a pass

Uplink Beam at Target



Uncompensated *Compensated*

- **Simulation steps include:**
  - Propagate beacon beam down through atmosphere
  - Measure wavefront of beacon in wavefront sensor
  - Apply wavefront correction to outgoing uplink beam
  - Propagate uplink beam back through atmosphere with point-ahead offset
    - Calculate Strehl ratio from time-averaged beam profile at target

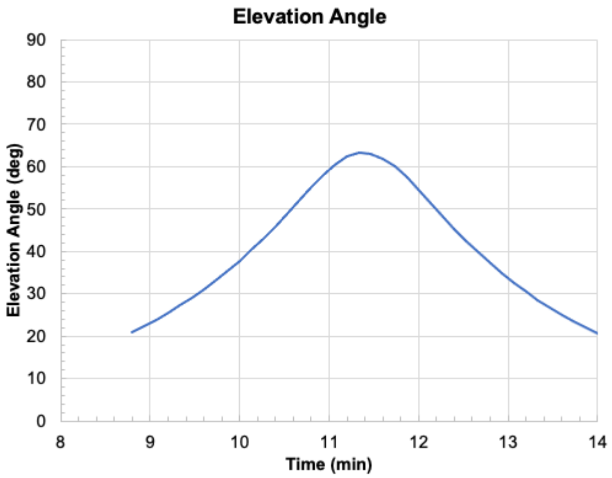


- **Code includes:**
  - Shack-Hartmann wavefront sensor w noise
  - MEMS deformable mirror
  - Slew-driven dynamics
  - Finite phase and tilt control-loop bandwidth
  - Kolmogorov phase screens (turbulence)

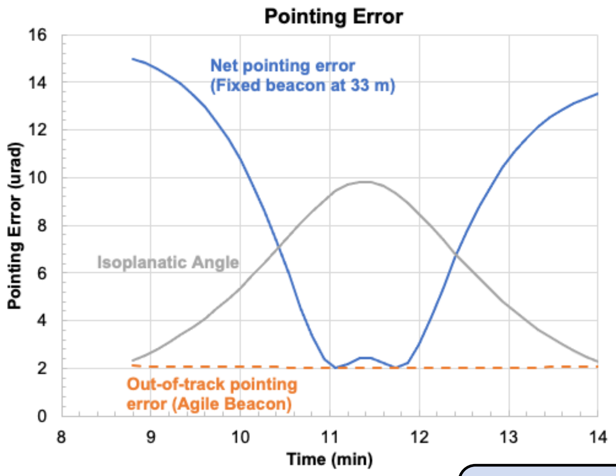
- **Conditions:**
  - 10-kHz WFS sample rate
  - HV-5/7 turbulence model
  - D = 1 m, 33 DM actuators across aperture
  - beacon and uplink wavelength = 0.78 μm
  - 0.5-sec duration of simulation (5000 time steps)

Comparison of Propagation Code and Models

- Propagation-code simulations run for discrete points in pass for uplink from MITLL site
  - Considered sun-synchronous orbit used in theory/model calculations (97.6° inclination)
  - Simulated operation with fixed beacon and agile beacon (no in-track pointing error)

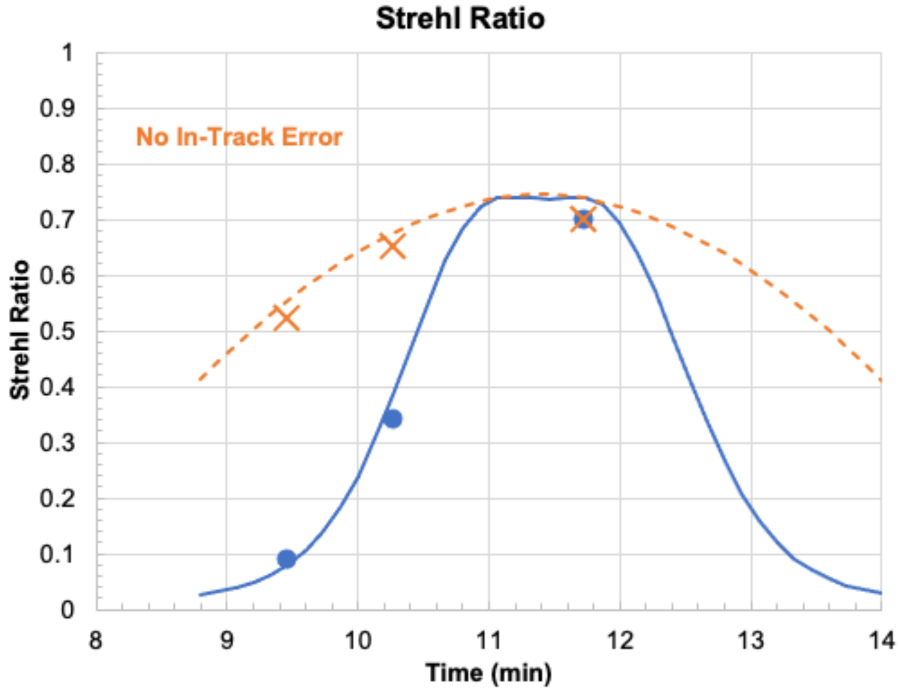


Pass achieves maximum elevation = 63 deg



Net pointing error applies to case with fixed beacon

No out-of-track error applies to agile beacon maintained at in-track point-ahead position

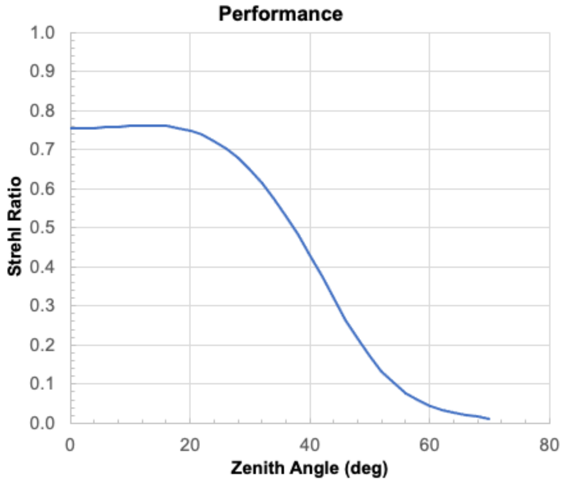


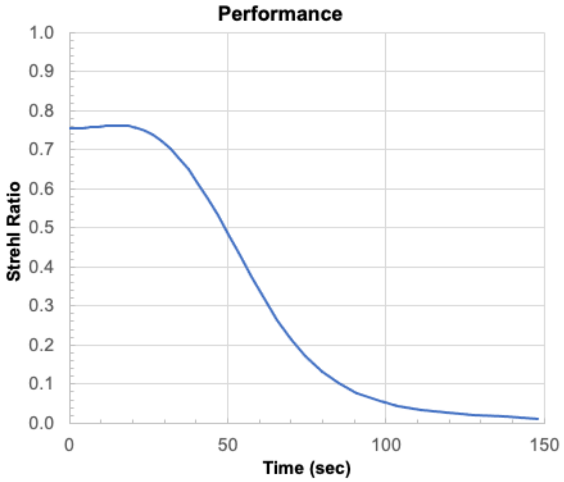
Discrete points are simulation results

Very good agreement between semi-empirical model and prop code

Uplink Performance and Link Figure of Merit

- Uplink Strehl ratio calculated as function of zenith angle for variety of a-o system design parameters, atmospheric conditions, orbit inclinations, and beacon position

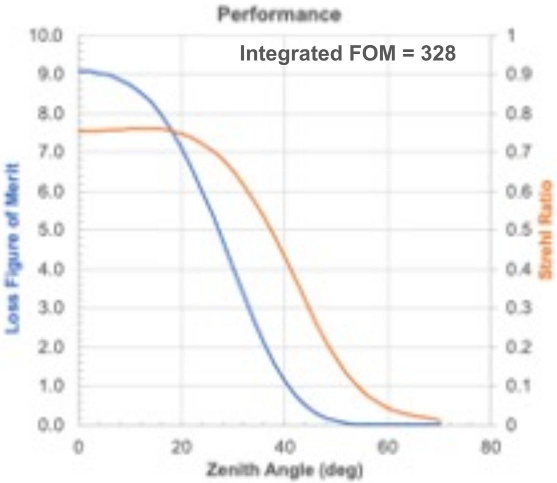




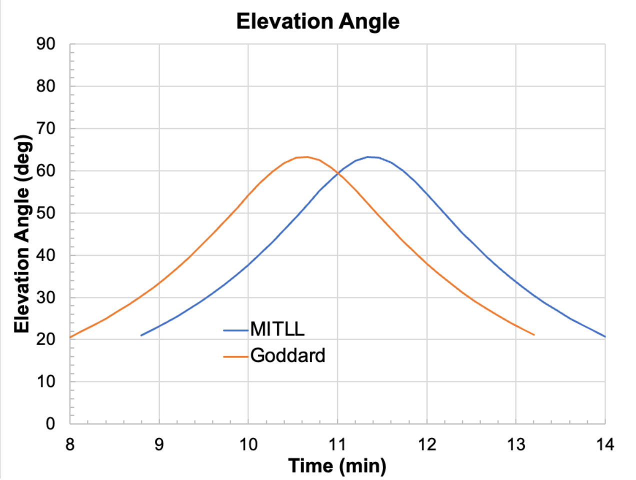
Assumed conditions:
- HV-5/7 turbulence models
- Bufton wind with 25 mph ground wind
- Wavelength = 0.78 μm (beacon and uplink)
- 500-km circular orbit, overhead pass
- D = 1 m (telescope aperture diameter)
- 10 kframes/sec WFS frame rate
- 33 actuators across aperture (d_{subap} ~ 3 cm)
- Orbit inclination = 30 deg
- Beacon position = 25 m

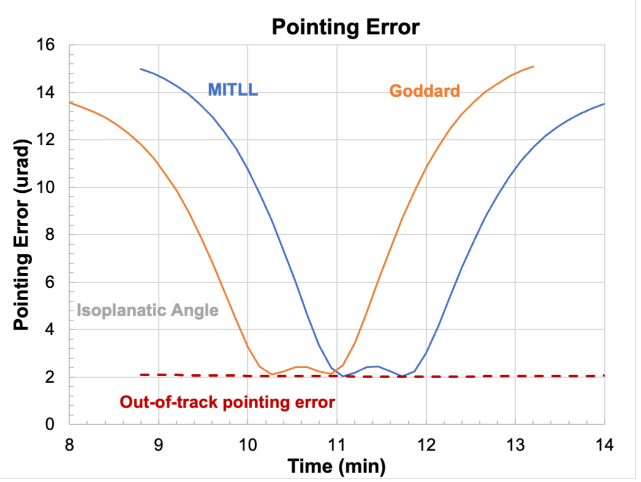
- Entanglement swap rate scales as product of uplink irradiance from each site
  - Irradiance of single uplink scales as Strehl * $(D/Range)^2$,
  - Define Figure of Merit for quantum performance:

    Figure of Merit = $[(Strehl*(D/Range)^2]_{Site\ 1}$ * $[(Strehl*(D/Range)^2]_{Site\ 2}$
  - Figure of Merit integrated over pass allows comparison of different orbits and beacon configurations

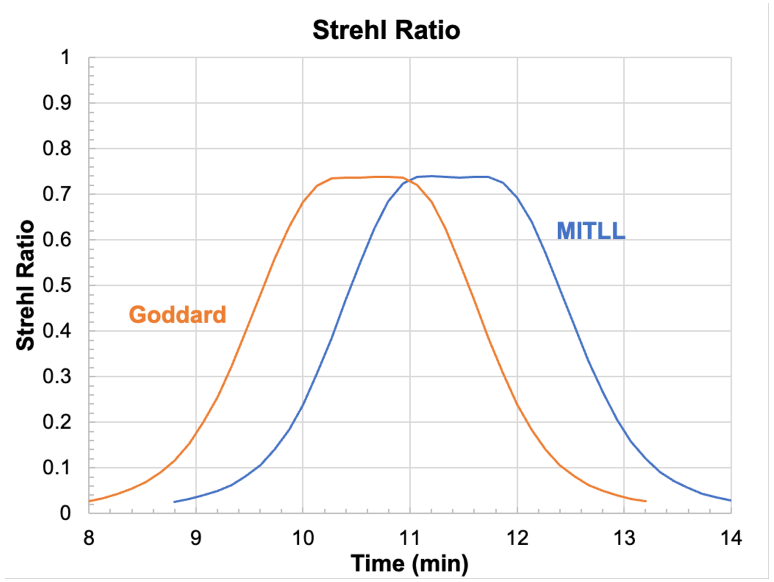


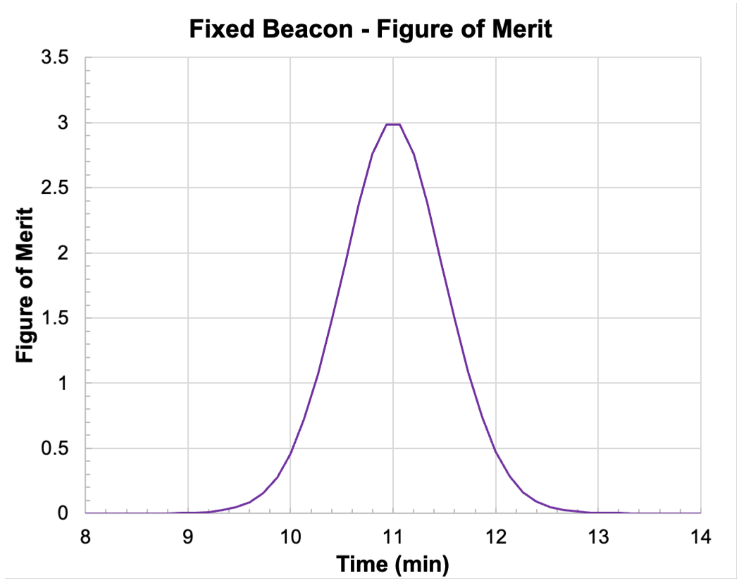
Case 1: Single Fixed Beacon





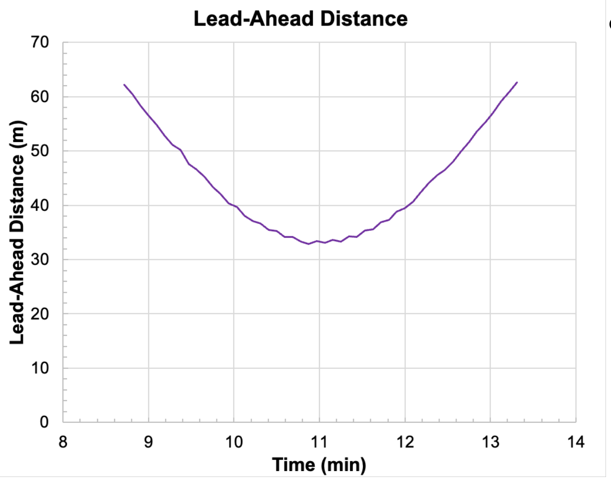
- **Uplink beams propagated from two ground sites to quantum satellite with single lead-ahead beacon satellite**
  - **Lead-ahead beacon position fixed at 33 m ahead of quantum satellite throughout pass**
  - **Orbit passes mid-way between ground sites (longitude -67.65°)**

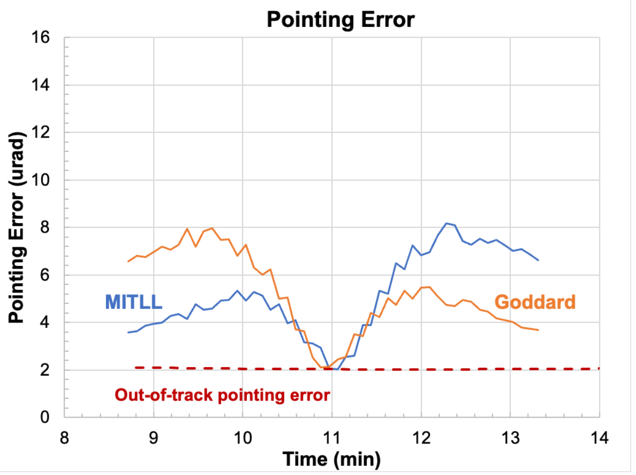




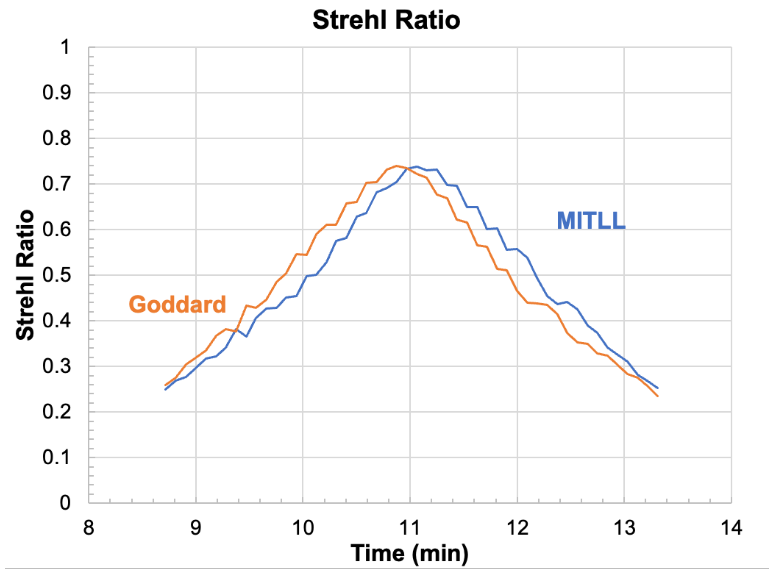
Integrated Figure of Merit = 231.1 sec

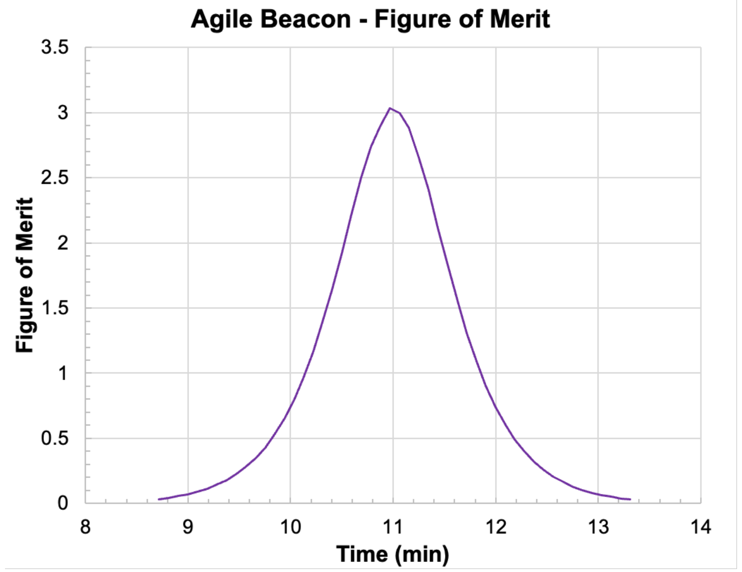
Case 2: Single Agile Beacon





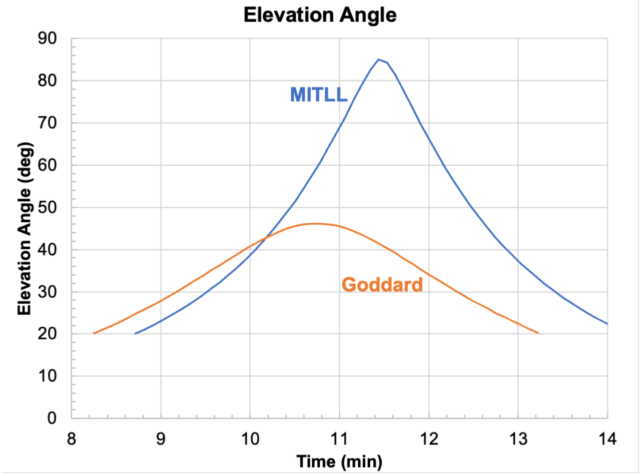
- Uplink beams propagated from two ground sites to quantum satellite with single lead-ahead beacon satellite
  - Lead-ahead beacon position continuously adjusted during pass to maximize figure of merit
  - Orbit passes mid-way between ground sites (longitude -67.65°)

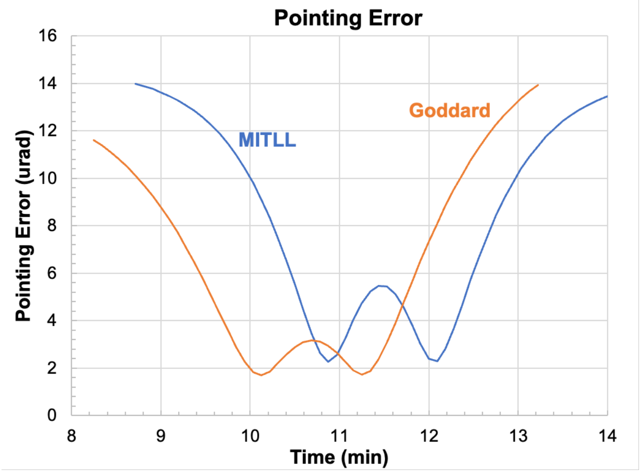




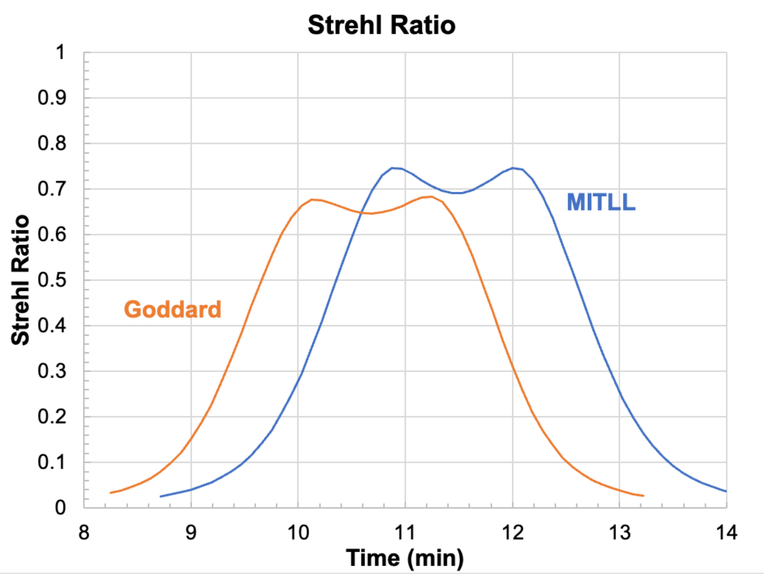
Integrated Figure of Merit = 269.1 sec

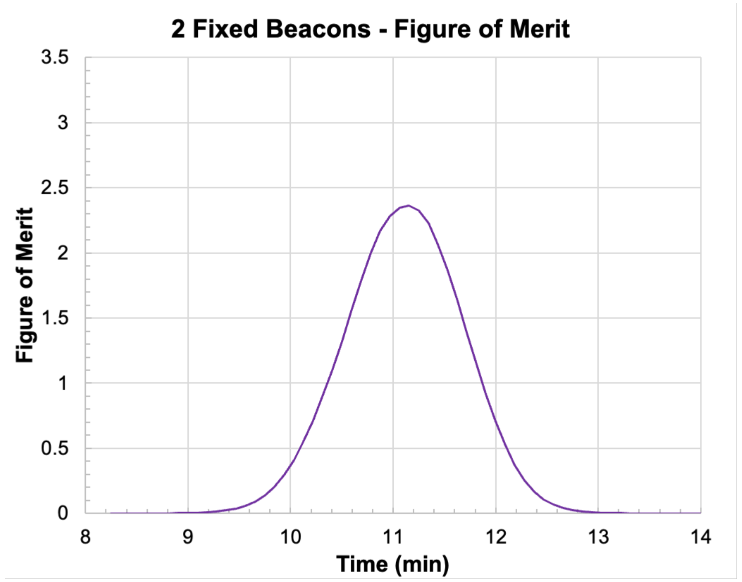
Case 3: Dual Fixed Beacons





- **Uplink beams propagated from two ground sites to quantum satellite with two lead-ahead beacon satellites**
  - **Lead-ahead beacon positions fixed at 32 m (MITLL) and 41 m (Goddard) ahead of quantum satellite**
  - **Orbit longitude shifted to pass closer to MITLL (longitude -65°)**

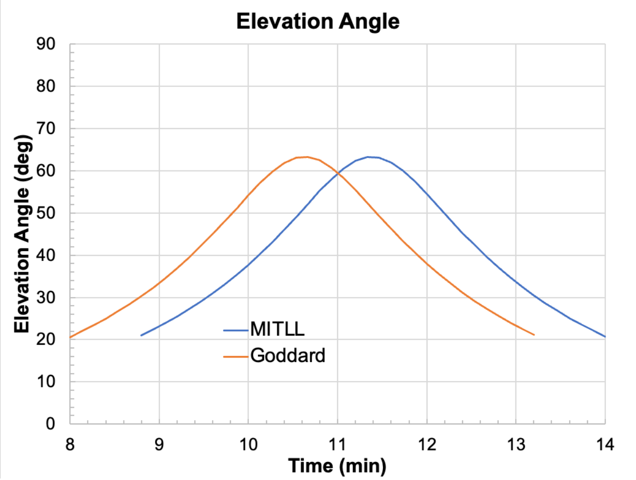


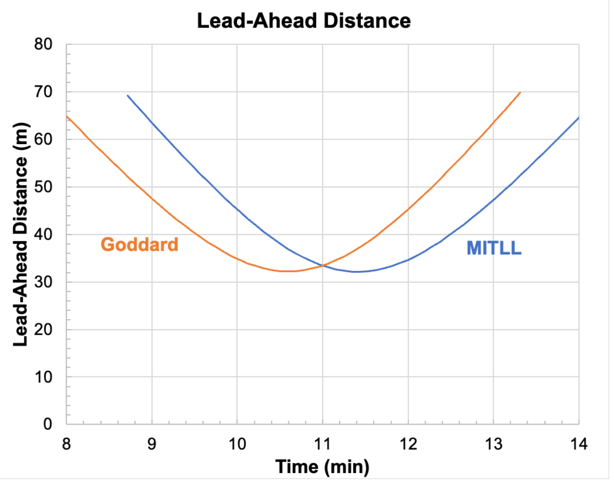


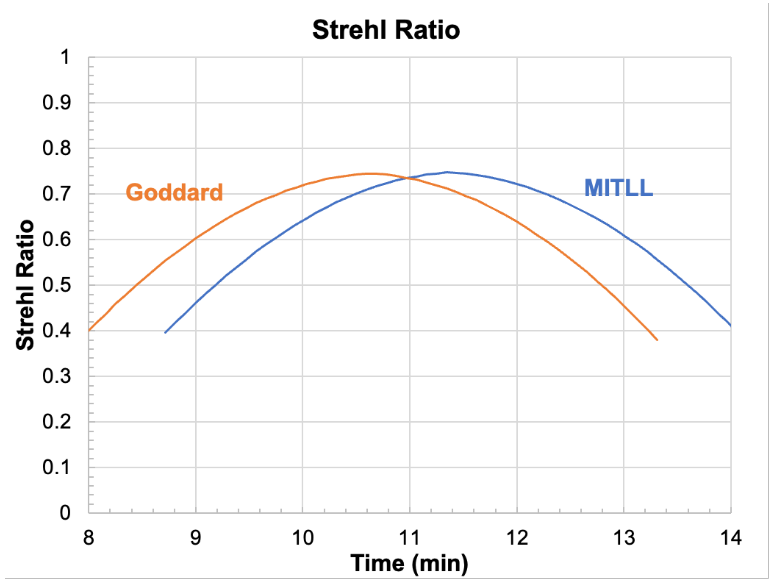
Integrated Figure of Merit = 204.1 sec

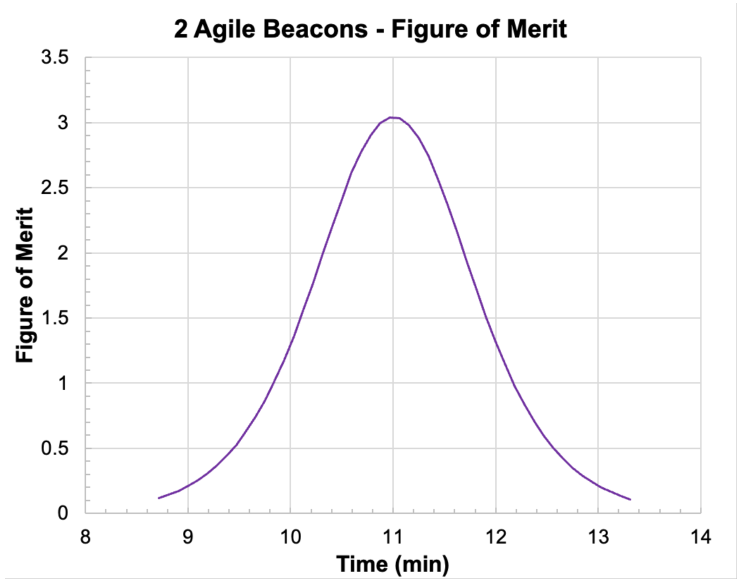
Case 4: Dual Agile Beacons

- **Uplink beams propagated from two ground sites to quantum satellite with two lead-ahead beacon satellites**
  - **Lead-ahead beacons continuously adjusted to be in correct lead-ahead positions**
  - **Orbit passes mid-way between ground sites (longitude -67.65°)**









Integrated Figure of Merit = 362.8 sec

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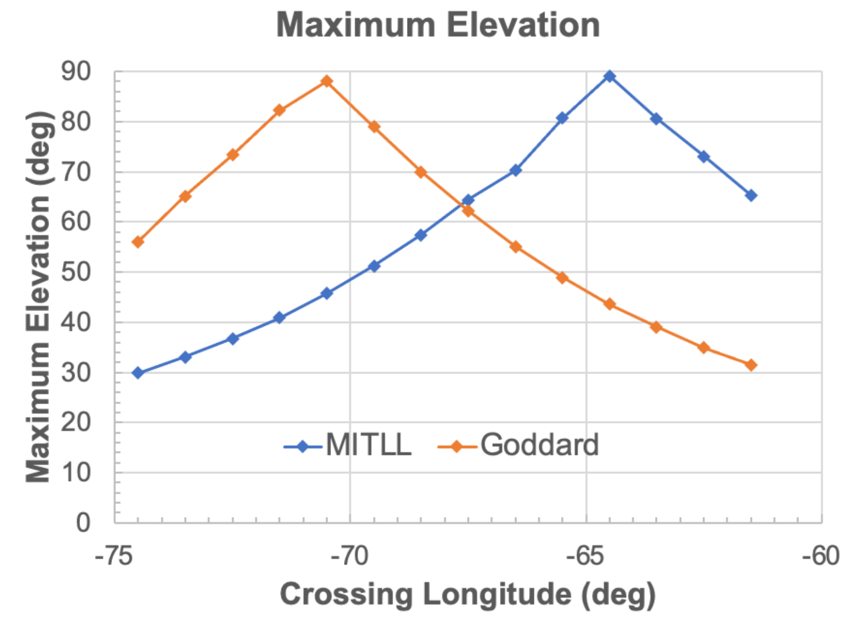
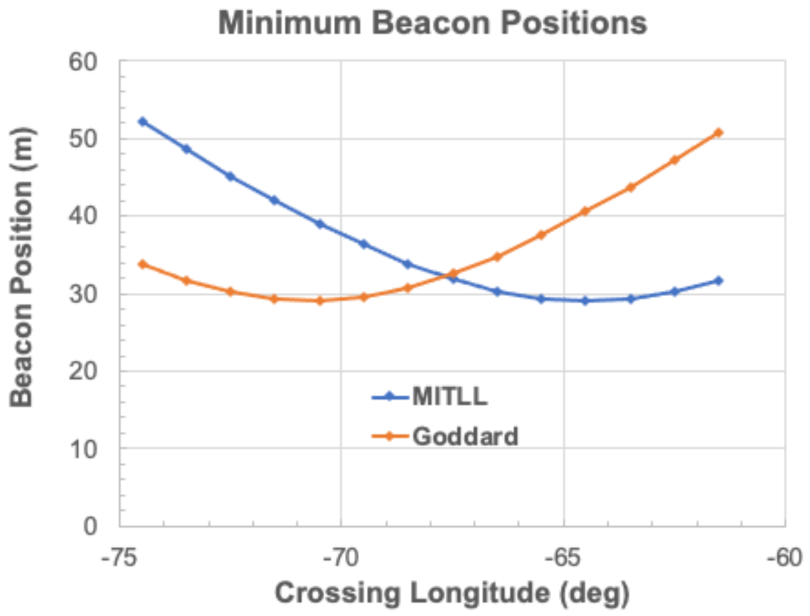
Performance v. Orbit Longitude

- As plane of orbit rotates around Earth, sites will see different maximum elevations
  - Potential for entanglement swap ends when maximum elevation from either site drops below horizon

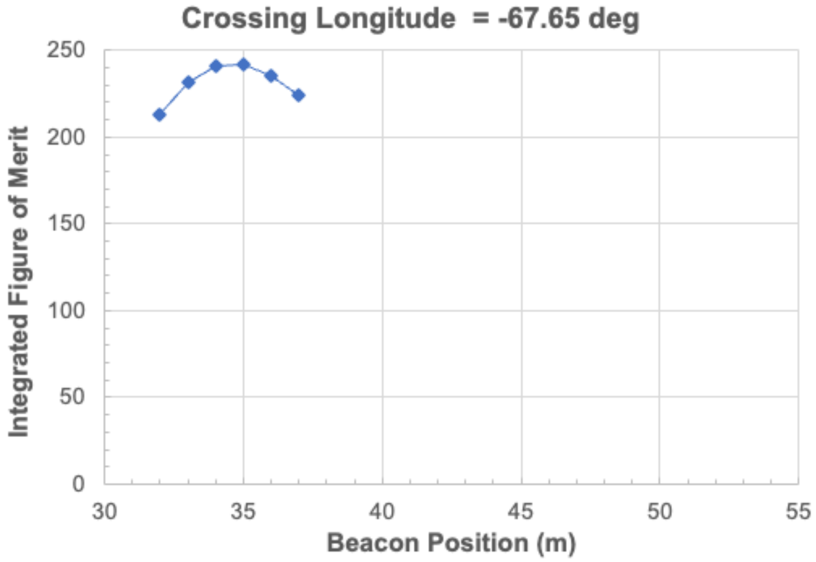
- Integrated Figure of Merit for uplinks calculated for orbits spanning useful range of longitudes where maximum elevation for both sites is at least 30 deg
  - Approximate ranges of equator-crossing longitudes: - 74.5 to -61.6 deg
  - Calculations performed for several beacon configurations (1 or 2, fixed or agile)



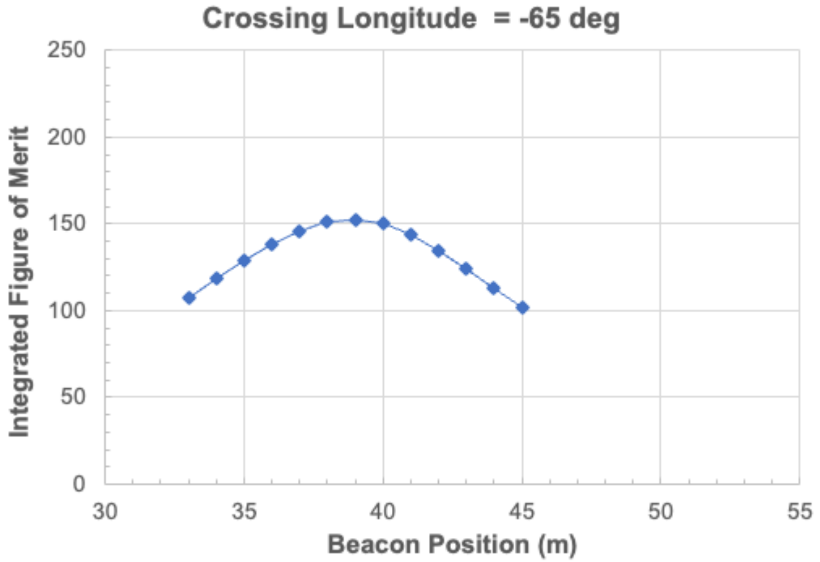


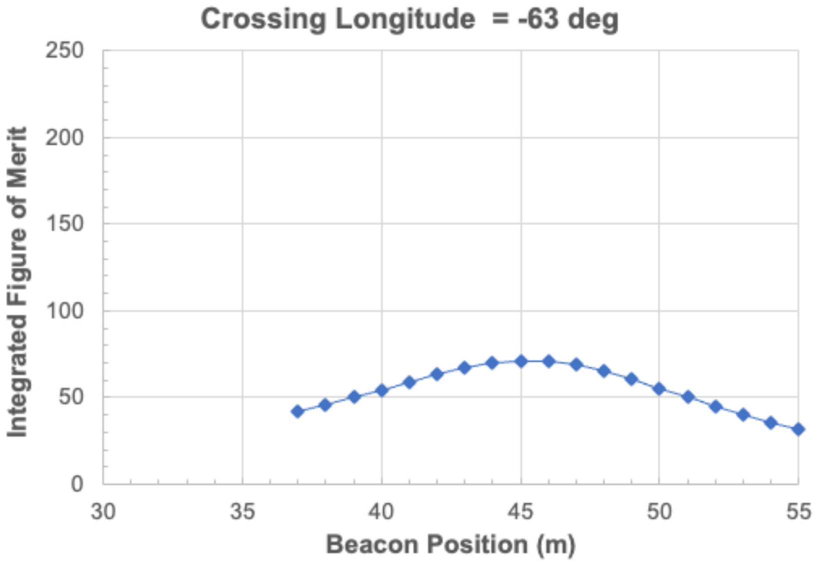
- Curves show minimum beacon distances from quantum satellite as function of orbit position
- *Optimum* beacon position varies over course of pass

Integrated Figure of Merit – Single Beacon



- Single satellite provides beacon for both ground sites
- Integrated Figure of Merit calculated as function of single-beacon position for 3 orbits in order to find best position for fixed beacon

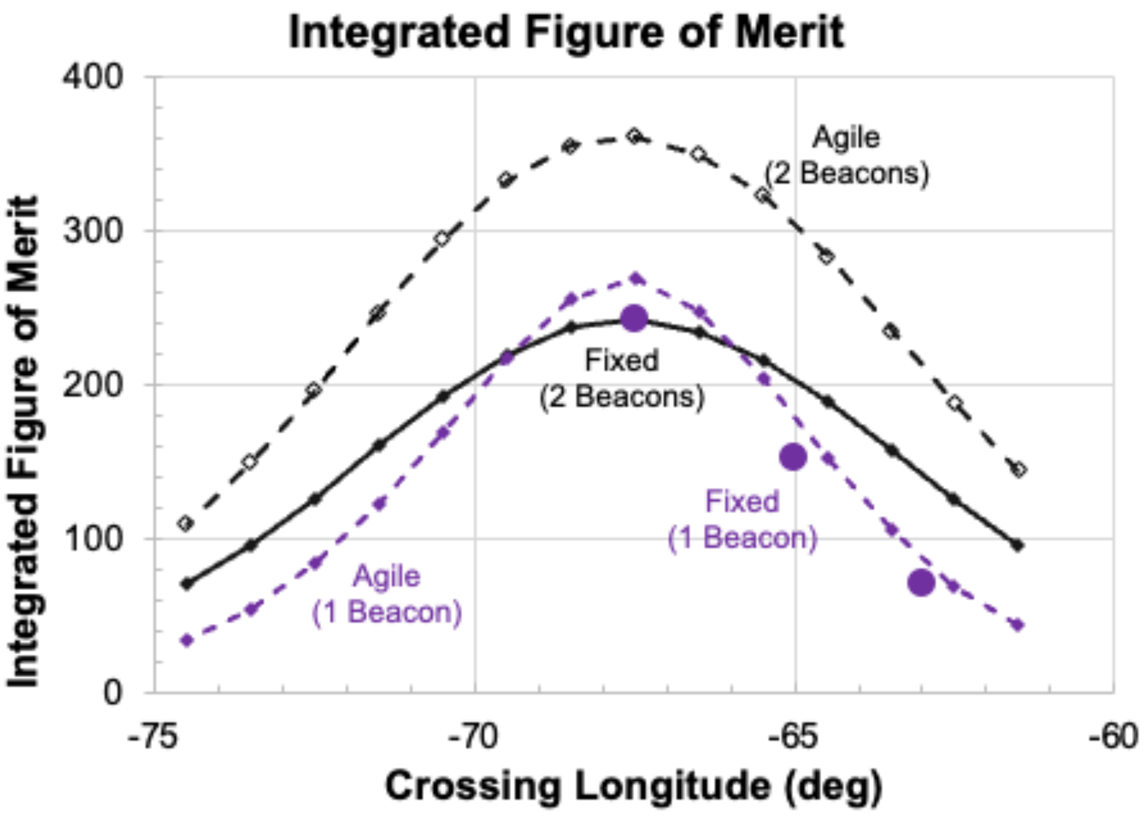




Comparison of Beacon Configurations

- Figure of merit integrated over entire pass (above 20° elevation) as function of longitude of 567-km sun-synchronous orbit

- Four beacon configurations modeled:
  - Single beacon satellite for both uplink sites, positioned ahead of pass at best position; not moved during pass (1 Fixed Beacon)
  - Single beacon satellite for both uplink sites, continually adjusted throughout pass to be in best position (1 Agile Beacon)
  - Two beacon satellites, one for each uplink sites, each positioned ahead of pass at best position; not moved during pass (2 Fixed Beacons)
  - Two beacon satellites, each continually adjusted during pass to be in correct point-ahead position (2 Agile Beacons)



Summary

- **Dual-uplink entanglement swap rate is sensitive to loss over ground-to-space channel**
  - Loss can be minimized with large transmission apertures if adaptive-optics is employed to compensate atmospheric-turbulence-induced wavefront aberration

- **Point-ahead problem for uplink compensation can be overcome by positioning beacon satellite ahead of the quantum satellite**
  - Dedicated beacon can provide strong signal for wavefront sensor as well as minimize angular anisoplanatism

- **Multiple options for configuring satellite beacons examined to determine impact on entanglement-swap rate for a pair of ground-based transmitters**
  - Examined use of single beacon and two beacons, with position fixed throughout pass or continually adjusted to optimize entanglement swap rate
  - Results intended to inform system architecture of future dual-uplink entanglement swap demonstration