

Improvements to Calibration Modeling of a Tri-Axial Accelerometer Using Response Surface Methods

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Abstract—Aerodynamic ground test facilities require precision attitude measurements to support fundamental research and aerodynamic vehicle characterization. Accelerometers are used as one solution for measuring test article attitude. Several wind-tunnel facilities experience elevated temperatures during sustained testing of approximately 38°C (100°F) and extremes up to 149°C (300°F). Accelerometer thermal corrections are required to maintain attitude measurement accuracy during elevated temperature tunnel conditions. This manuscript details characterization of non-linear and thermal effects of the Mini/MEMs Tri-Axis Sensor System (MTASS) developed at NASA. Non-linear effects are characterized using a two-axis index head, and temperature effects are characterized using a metallic cube in a thermal chamber. The results show a significant decrease in back-computed residuals for room temperature and elevated temperatures experiments.

Index Terms—Accelerometers, Attitude Measurement, Temperature Calibration, Response Surface Methods

I. INTRODUCTION

Aerodynamic ground test facilities use accelerometers to provide attitude, or angle of attack, measurements. The energy used to sustain high-speed operations can lead to elevated fluid temperatures. During normal testing, temperatures of approximately 38°C (100°F) can be realized, and some facilities may produce fluid temperatures up to 149°C (300°F). Moreover, several hyper-velocity facilities can reach total temperatures of 694°C (1,280°F). To provide accurate attitude measurements within $\pm 0.01^\circ$, temperature effects on sensor outputs need to be characterized.

Demonstration testing of the NASA Mini/MEMs Tri-Axis Sensor System (MTASS) at NASA Ames Unitary Plan Wind-Tunnel recorded a maximum sensor temperature of 38°C (100°F) [1]. This temperature translates to a rise of approximately 17°C (30°F) from the calibration laboratory environment. The MTASS uses commercial-off-the-shelf $\pm 5g$ accelerometers with a nominal bias temperature coefficient of $187 \mu g/^\circ C$ and a scale factor temperature coefficient of $120 \text{ ppm}/^\circ C$. An angular error of up to 0.20° can occur for a 10° tilt with the stated specification and temperature delta, which exceeds the required $\pm 0.01^\circ$ tolerance.

This paper details improvements in calibration modeling of a tri-axis accelerometer package by reducing non-linear and temperature effects. This work will expand the calibration model and experimental results of the MTASS at room temperature and multiple elevated temperatures. A full tri-axis calibration combining room temperature and elevated temperature experiments were completed.

II. CALIBRATION MODELING

Previous calibration models used at NASA included complex math routines, or linear models for limited temperature ranges [2, 3]. Previous work reported at Transducers'23 explored using a simple linear calibration model of the MTASS units at room temperature. However, gravity vector residuals of the previous work demonstrated non-linear behavior. Several of the accelerometers displayed two inflection points in residual structure, suggesting the need for 3rd order sensitivity terms. In this paper we propose expanding the model for acceleration components, g_i , and temperature, T , to include the higher order behavior:

$$v_i = \beta_0 + \beta_{0T}T + \beta_{0TT}T^2 + \sum_{i=1}^3 \beta_{iT}g_iT + \sum_{i=1}^3 \beta_{iTT}g_iT^2 + \sum_{i=1}^3 \beta_{ii}g_i + \sum_{i=1}^3 \sum_{j=i}^3 \beta_{ij}g_i g_j + \sum_{i=1}^3 \beta_{iii}g_i^3 + \epsilon \quad (1)$$

where v_i is the i^{th} accelerometer voltage, β are the calibration coefficients, and ϵ is the random error. This equation is repeated for each of the three (3) accelerometer signals, where each accelerometer component could potentially have 21 calibration coefficients. However, due to the constraints of calibrating in a 1g environment, the root-sum-squared of the acceleration input must equal 1. This constraint requires the removal of either the intercept or one pure quadratic term. Previous knowledge of the sensor's behavior suggests that the pure quadratic term is only required for its respective component, for example, v_x would only have the g_x^2 term. This

is also extended to the cubic terms due to model hierarchy. Similarly, only the temperature terms for a given component are kept, such as, v_x component only having β_{xT} and β_{xTT} . After removing terms that are not significant, only 11 terms remain for each component. The final calibration model is shown later in this paper in Table II.

III. RESULTS

This section discusses the results from the room temperature, elevated temperature, and validation experiments on a $\pm 5g$ MTASS unit.

A. Room Temperature Calibration

Room temperature calibration of the MTASS is conducted on a precision two-axis index table with 360° pitch and roll capability. The system can repeat positioning within $\pm 2''$. A spherical-central-composite design (SCCD) is used to support solving Equation 1 for the non-temperature calibration coefficients [4]. The SCCD is well suited for use in the 1g environment since all the data points are on the surface of a sphere with radius of 1. The design can be rotated using a rotation matrix to generate models higher than 3rd order, but is not required in this work.

The MTASS was allowed to stabilize over 1 hour after power up to minimize start up drift. After the initial calibration, a repeated, independent, calibration was performed with additional points to represent operational use. Residuals for each accelerometer component are shown in Figure 1 after applying a linear and nonlinear model. It is shown that the nonlinear model removes the higher-order behavior seen in the linear model residuals.

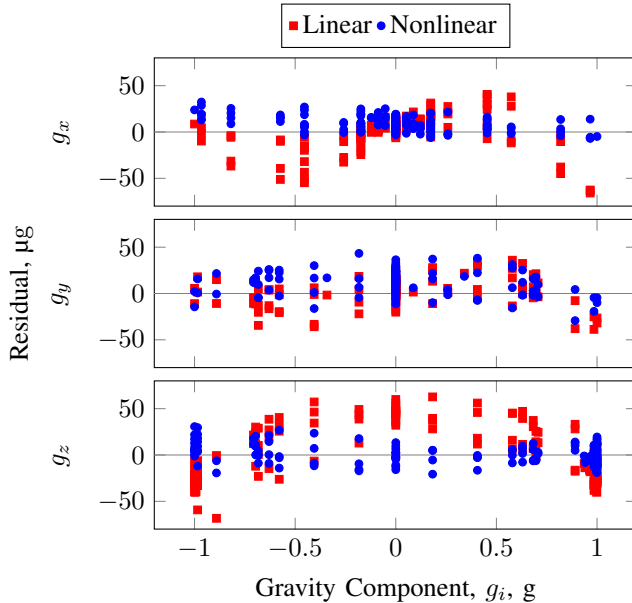


Fig. 1: Accelerometer residuals from room temperature calibration experiment using linear and nonlinear models. Each plot is respective to their own inputs, i.e., g_i residual versus g_i applied acceleration.

Standard deviations of the room temperature residuals are shown in Table I for each accelerometer component. It is shown that the nonlinear model significantly reduces the residual errors of the g_x and g_z outputs. The g_y component does not improve significantly with the higher model terms, thus a simpler linear model could be used to model the g_y output.

TABLE I: Room Temperature Residual Standard Deviations

Component	g_x	g_y	g_z
Linear, μg	22.1	13.7	29.9
Nonlinear, μg	8.3	12.5	11.0
Percent Diff.	-63%	-9%	-63%

B. Elevated Temperature Calibration

The temperature calibration of the MTASS package was accomplished in a laboratory oven as shown in Figure 2. The sensor under test is installed in a near-orthogonal cube to align each channel axis at three levels (+1g, 0g, and -1g) to solve Equation 1 for the temperature coefficients, β_0 , and β_i . Since the sensor is not placed in any factorial combinations, i.e. where each channel is set close to $1/\sqrt{3}g$, the β_{ij} and β_{iii} can not be estimated. During elevated temperature runs, the cube is rotated 180° about the gravity direction to remove offsets in the oven platform as recommend by standard accelerometer testing procedures [5]. Additionally, two temperature compensated Q-Flex¹ accelerometers were used to monitor platform tilt and drift during testing.

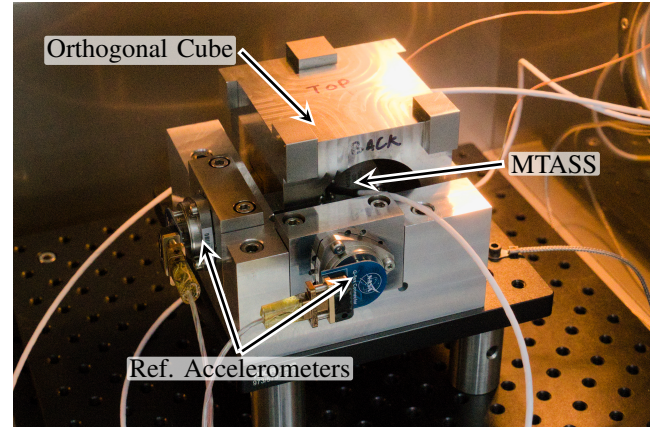


Fig. 2: MTASS package in an oven for temperature testing with two reference accelerometers to monitor oven attitude.

The oven was set to three temperature levels (21 °C (70 °F), 54 °C (130 °F), and 82 °C (180 °F)) to support the estimation of second-order temperature terms. Three thermocouples were installed to ensure the cube, oven, and MTASS was fully heat soaked; one on the oven platform, one on the outer surface of the cube, and one inside the cube near the MTASS. All

¹Trade names and trademarks are used in this report for identification only. Their usage does not constitute an official endorsement, either expressed or implied, by the National Aeronautics and Space Administration.

three temperature measurements were within 0.55°C (1.0°F) for all data recordings.

Prior to the oven runs, the cube with an MTASS unit installed was evaluated on a leveled granite table. The reference accelerometer fence was installed to the granite table to provide repeatable positioning of the cube. A total of 12 points were taken, two points for each face with 180° yaw rotations at each face. The two points for each face were averaged to reduce the influence of granite table angular offsets. Gravity vectors for each face were estimated using the MTASS outputs and the nonlinear room temperature model from Section III-A. The gravity vector estimates were adjusted for local gravity differences between NASA Langley and NASA Glenn Research Centers, where the room and elevated temperature evaluates were completed, respectively. These estimates were used as inputs for the temperature calibration modeling and accounts for the cube not being perfectly orthogonal.

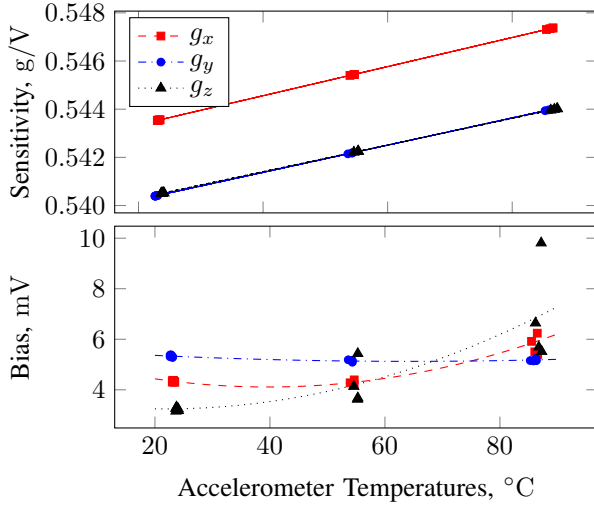


Fig. 3: Sensitivity and bias estimates for each temperature run.

For each temperature level, estimates were made for the linear terms, β_0 and β_{iT} . These estimates for each temperature level are shown in Figure 3 with a second order fit. From the results, sensitivities of the sensor are linear over the temperature range. This allows for removing the second order temperature term, β_{iTT} , from the calibration model.

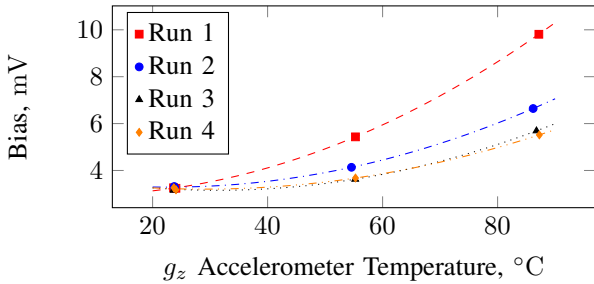


Fig. 4: Bias estimates for the g_z component during each temperature run.

Nonlinear behavior is seen in the bias for both g_x and g_z components, whereas, g_y is linear and has lower overall shifts. Larger spread of the bias estimates is observed in the g_z component than the other two components. Bias temperature shifts for four consecutive oven entries are shown in Figure 4 for the g_z component. It is shown that the g_z bias shift changes between consecutive temperature cycles, with the difference between the first two cycles being the largest change at maximum test temperature. The g_z bias curve remains similar between runs 3 and 4, suggesting that the consecutive temperature cycling may have stabilized the accelerometer. Further evaluation of the test setup and thermal testing is required to better understand if this behavior is due to the test method or the accelerometer.

Since the room and elevated temperature experiments each only produce a limited portion of the calibration coefficients, results from both experiments are combined to create a full model. The room temperature experiment is considered to be more accurate due to the precision of the two-axis index table. All non-temperature related terms from the room temperature calibration are used as the base of the full calibration model. Only the temperature based coefficients are used from the elevated temperature experiments. The room temperature bias and sensitivity are adjusted to reflect the inclusion of the temperature terms by:

$$\beta_i^{Adjusted} = \beta_i^{RT} - \beta_{iT}T_{RT} - \beta_{iTT}T_{RT}^2 \quad (2)$$

$$\beta_0^{Adjusted} = \beta_0^{RT} - \beta_{0T}T_{RT} - \beta_{0TT}T_{RT}^2 \quad (3)$$

where T_{RT} is the average sensor temperature during the room temperature calibration from Section III-A. This modification would not be required if a reference temperature is subtracted from the temperature estimates so that the terms have zero effect at the calibration temperature. The final combined calibration model is shown in Table II.

TABLE II: Combined calibration matrix for the $\pm 5g$ MTASS.

Term	v_x	v_y	v_z
β_0	6.56E-03	5.80E-03	5.29E-03
β_x	5.41E-01	9.89E-04	5.07E-03
β_y	1.04E-03	5.38E-01	-1.99E-03
β_z	1.43E-03	-5.58E-03	-5.38E-01
β_{xx}	4.20E-05		
β_{xy}	2.54E-05	-1.01E-05	
β_{xz}	1.02E-05		-6.40E-05
β_{yy}		1.82E-06	
β_{yz}		-1.05E-05	-1.07E-05
β_{zz}			-5.04E-05
β_{xxx}	3.43E-05		
β_{yyy}		3.13E-05	
β_{zzz}			2.43E-05
β_{0xT}	-4.71E-05		
β_{0xTT}	2.21E-07		
β_{Tx}	3.42E-05		
β_{0yT}		-9.10E-06	
β_{0yTT}		2.89E-08	
β_{Ty}		3.16E-05	
β_{0zT}			-4.59E-05
β_{0zTT}			2.56E-07
β_{Tz}			-3.05E-05

The combined model estimated from the room temperature calibration and temperature run 3 was used to back calculate accelerations of the elevated temperature runs 3 and 4. Residuals for each acceleration component during runs 3 and 4 are shown in Figure 5 over the temperature range. It is demonstrated that the full temperature model significantly reduces the residual errors at elevated temperatures.

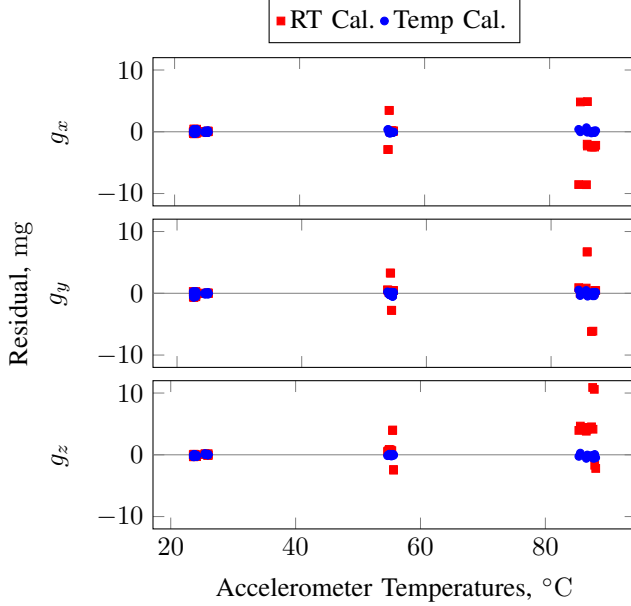


Fig. 5: Accelerometer residual errors for runs 3 and 4 after applying calibration models from the room and elevated temperature run 3 calibrations.

Standard deviations of the elevated temperature residuals show a large reduction for each acceleration component, and these values are listed in Table III. Each acceleration component has an order of magnitude improvement by including the temperature modeling. However, when compared against Table I, the temperature calibration standard deviations are at least 20 times larger. This is potentially due to the larger variability in the elevated temperature test method. Since only the temperature corrections are being estimated, the affects of larger test variances may not be important to the overall performance of the sensor.

TABLE III: Standard Deviation of Temperature Calibration

Component	g_x	g_y	g_z
RT Model, mg	2.17	1.91	2.57
Full Model, mg	0.21	0.27	0.18

IV. CONCLUSIONS

It was shown that expanding the calibration model to include higher order terms significantly reduces the residual errors for the MTASS. The use of a spherical-central-composite design allowed for generating test points to support the third-order model. After calibration at room temperature, the ± 5 g

sensor provided a standard deviation of up to $12.5 \mu\text{g}$ for a fully independent data set including validation points at room temperature.

It was shown that temperature coefficients can be estimated using a nearly orthogonal cube in a laboratory oven. This approach worked by using a precise room temperature calibration to model the cube surface offsets. Results from the thermal testing showed that two of the three accelerometers provided repeatable thermal shifts. The g_y accelerometer showed desirable thermal characteristics and low non-linearity. However, the g_z accelerometer demonstrated changes to the bias over consecutive thermal cycles. After 3 thermal cycles the g_z bias was found to be repeatable. More studies are required to better understand if the affect was due to test setup or the accelerometer. Further improvements can be made to the test method by incorporating temperature compensated accelerometers into the cube to reduce testing variations.

The overall results show that individual accelerometer temperature characterization may be required prior to assembly to reduce temperature issues. Binning of the best sensors could allow for lower order models if all three accelerometers perform similar to the g_y component shown in this paper. In this study, a linear model could be sufficient for this particular accelerometer.

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