Dose error correction using simulation extrapolation for community noise dose-response modeling

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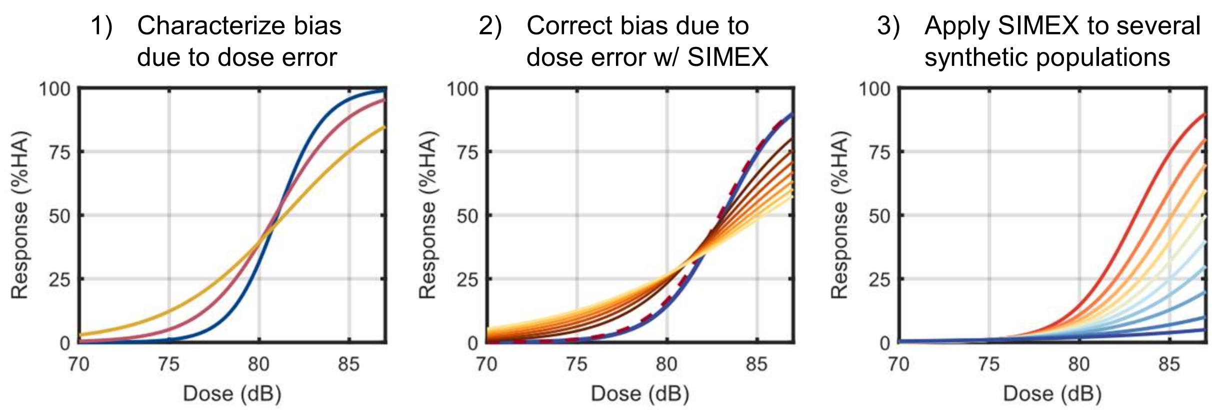
This work supports the NASA Quesst mission with the quiet supersonic X-59 aircraft under the Commercial Supersonic Technology project

- Annoyance to sonic booms contributed to the ban on overland commercial supersonic flight
- NASA is building the X-59 to demonstrate quiet supersonic flight technology
- Collect community response data
  - 5 US communities + possibly 1 international
- Data to be provided to regulators to inform their decision in potentially replacing the supersonic speed limit with permissible noise standards

Objective: Examine simulation extrapolation (SIMEX) to counteract the biases in dose-response relationship due to dose error



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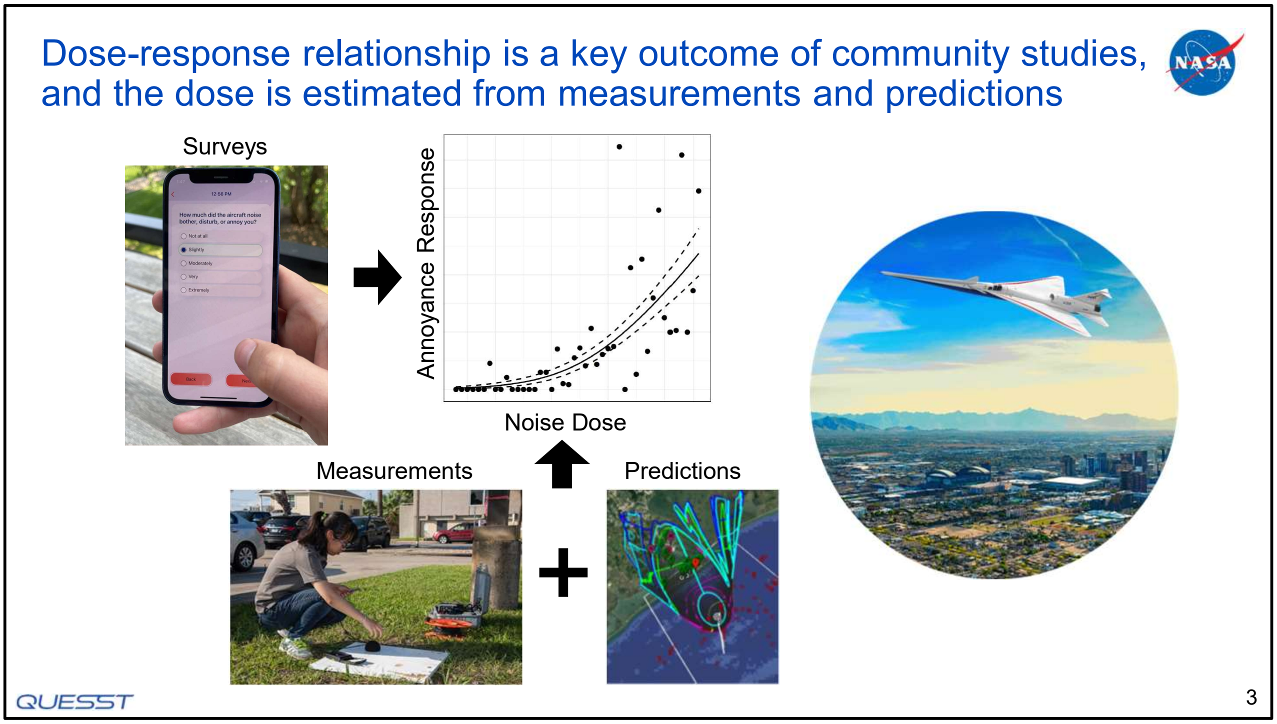
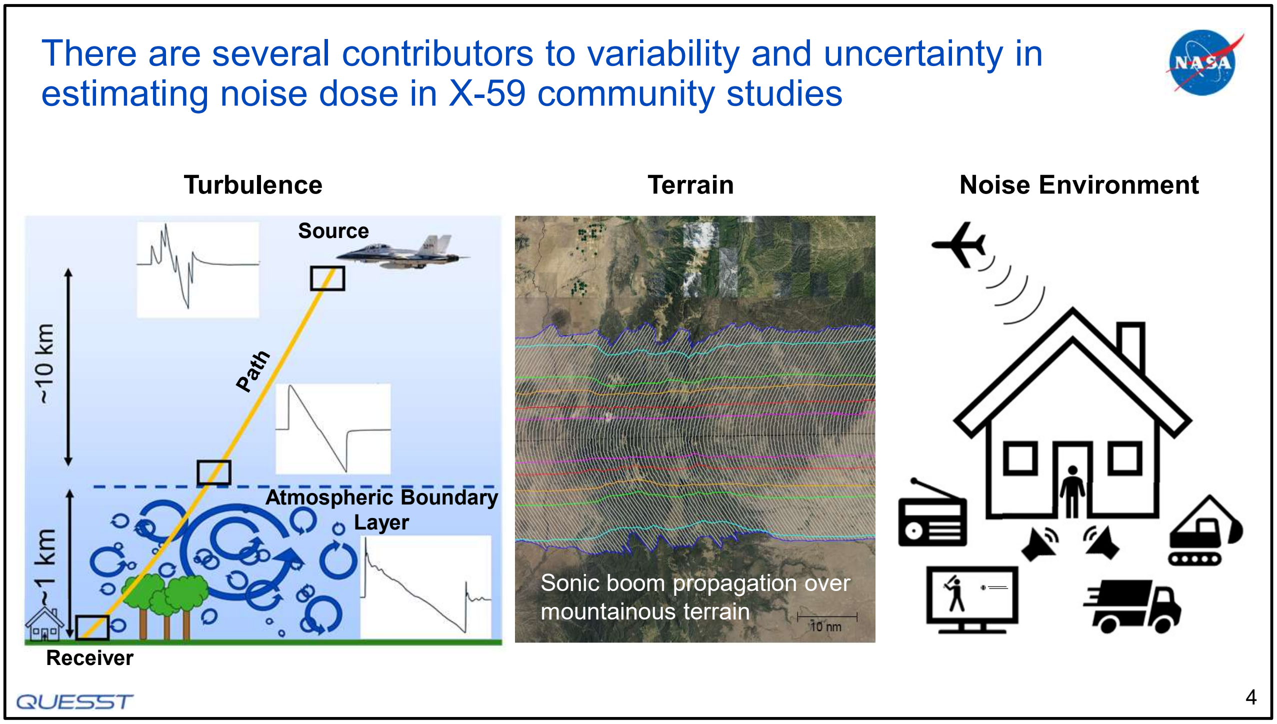
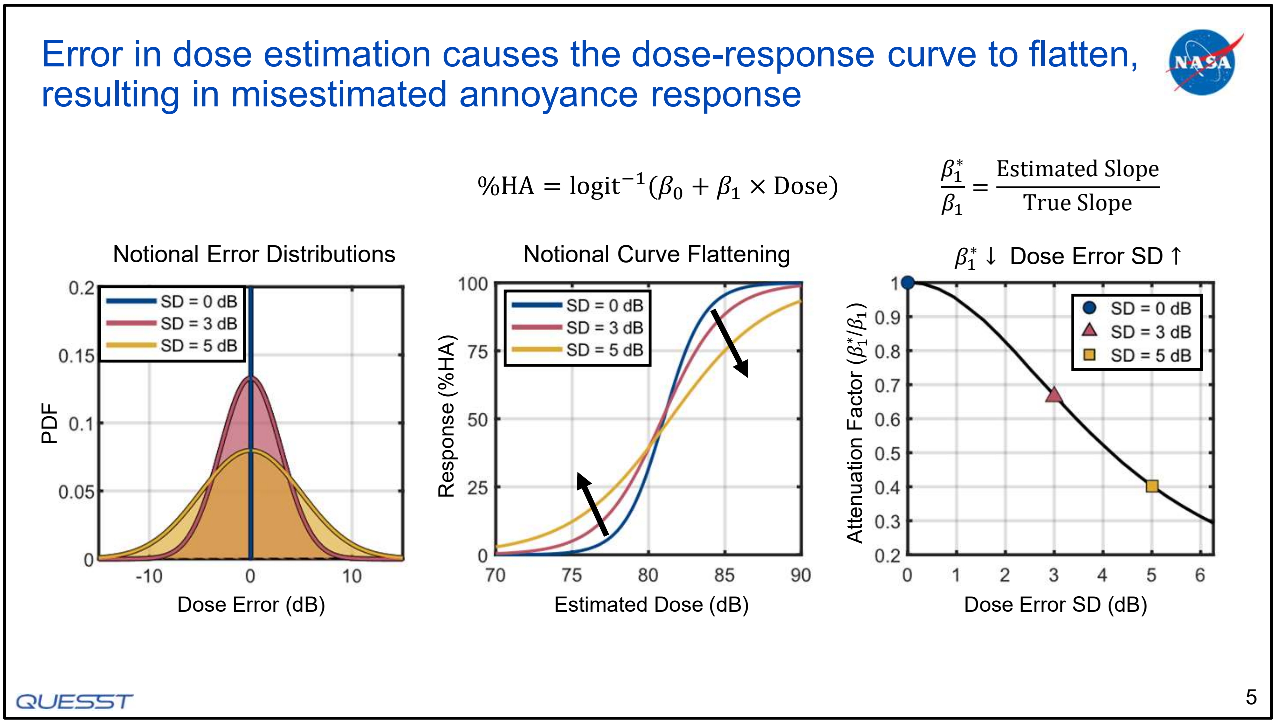
- Objective: Examine SIMEX to correct for bias due to dose error
  1. **Characterize bias due to dose error**
     - What impact does dose error have on the dose-response curve?
     - How is response under/over estimated as a result?
  2. **Correct bias due to dose error with SIMEX**
     - While there are several methods for dose error correction, focusing here on SIMEX
  3. **Apply SIMEX to several synthetic populations**
     - Does SIMEX work well on all populations?
     - We do not know how sensitive the populations will be in our future community noise studies
     - Testing on synthetically generated populations helps develop an intuition for what to expect with SIMEX



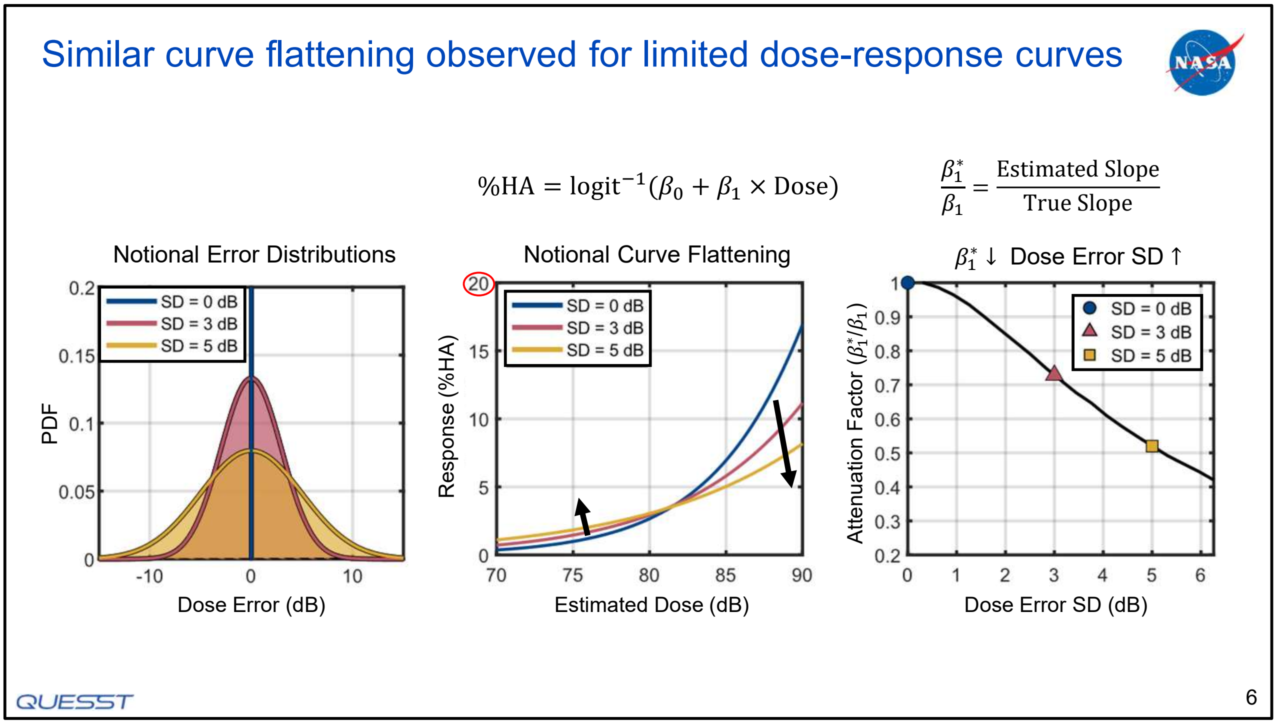
- The data to be collected in X-59 community studies consist of noise dose and annoyance response
    - The X-59 aircraft will supersonically overfly a community multiple times a day for several weeks
    - Annoyance responses collected via survey after each overflight and at the end of day
    - Noise dose values are estimated from a combination of measurements and predictions and assigned per event for each participant based on their location
        - Measurements use a sparse array of ground recording (GRS) systems (100 units and 1000 participants in a community)



- Beyond having relatively few GRS units, dose estimation is challenging
    - Noise level receiver receives vs what can be estimated
- Many factors that contribute to variability and uncertainty, such as
    - Turbulence:
        - Atmospheric boundary layer causes significant variation, even for nearby measurements
    - Terrain:
        - Both natural and urban terrain can impact the levels
    - Noise environment:
        - Estimating outdoor noise dose, though people will likely be indoors
        - Not measuring all acoustic factors in their vicinity
- Refer to the misestimation of dose as “dose error”
- After making the best estimation of dose, we apply a post-processing correction to account for bias due to dose error



- Using simulated data to observe notional trends
- Notional Error Distributions:
  - Simply assuming dose error is homogenous, same amount at any dose (though alternative assumptions can be made)
  - Normal distribution with mean 0 and some standard deviation (SD) – Example on left
  - "SD = 0" is "true" or no dose error case
  - Random additive error
- Notional Curve Flattening:
  - Logistic in shape
    - $\%\text{HA} = \text{logit}^{-1}(\beta_0 + \beta_1 \times \text{Dose})$
  - With increased dose error, dose-response curve flattens
  - <u>Over/underestimate response at lower/higher doses</u>
- Attenuation Factor (AF):
  - Ratio of estimated to true slope coefficient (β_1)
  - Estimated magnitude of model slope coefficient decreases as dose error increases

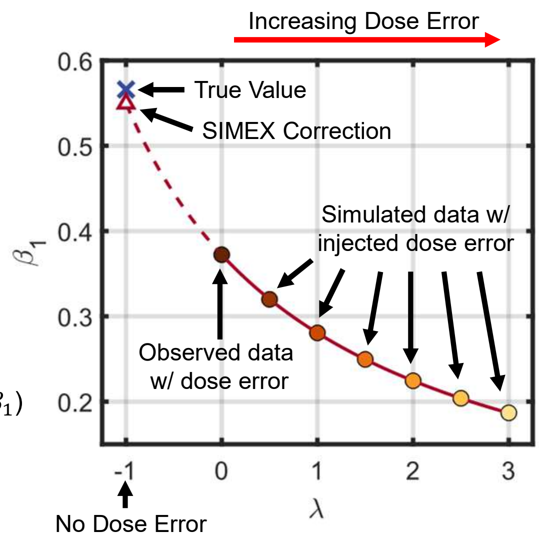


- Consider a shallow or limited dose-response curve (0-16%HA) instead of full curve (0-100%HA)
  - May be a more realistic scenario, as it is impractical to attempt to solicit 100% highly annoyed responses
  - Note that the curve is still logistic (%HA < 20%)
  - Same dose error SD
  - Similar trend of slope flattening and similar AF
  - Magnitude of misestimation at any point along the curve is less than the previous example
- **Takeaway: Dose error causes slope flattening**
  - <u>Over/underestimate response at lower/higher doses</u>
- If we can correct biases in estimated model coefficients due to dose error, we can produce better data (better dose-responses relationship) to inform regulators.

Simulation extrapolation (SIMEX) is a method to correct for bias due to dose error

<u>SIMEX Method:</u>

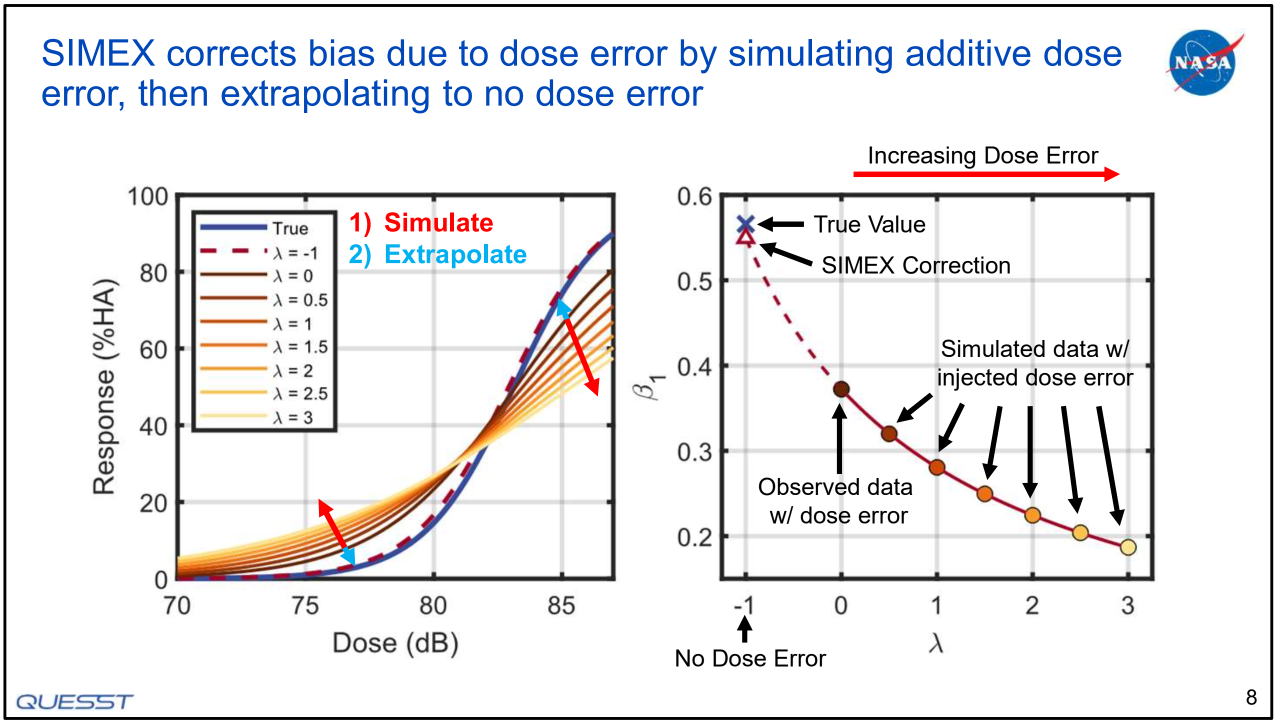
1. Model fit to observed data
   - No dose error at $\lambda = -1$
   - Estimated dose at $\lambda = 0$
     - Dose error SD 3 dB
2. **Simulate** data and fit to the model
   - Inject additive error: $(1 + \lambda) \cdot \widehat{\mathrm{SD}}^2_{\mathrm{dose}}$
   - Repeat many times (e.g., 100x) at each λ value
3. Fit curve to each model parameter (e.g., β_1)
4. **Extrapolate** to no dose error
   - No dose error at $\lambda = -1$



- Simulation extrapolation is one method to correct for dose error

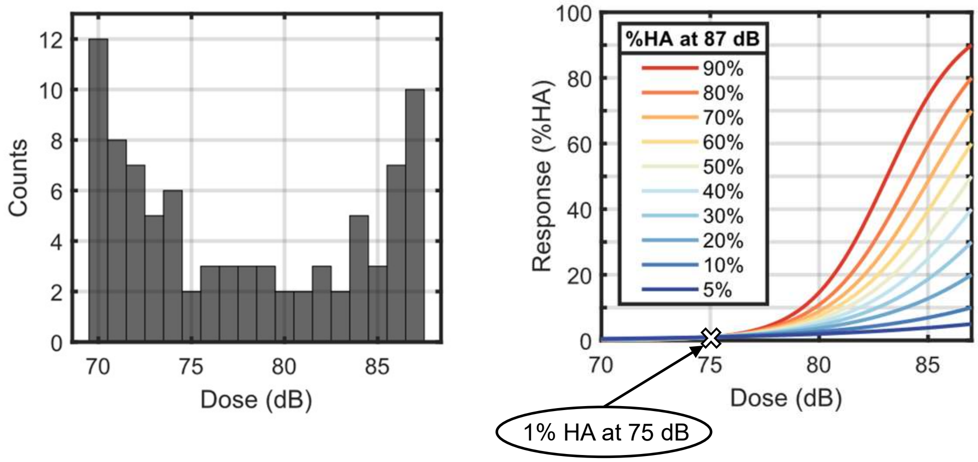
1. Model fit to observed data
   - λ is a multiplicative factor that increases the level of simulated dose error
   - Goal is to get to the no dose error case at $\lambda = -1$
   - Start with observed data with estimated dose at $\lambda = 0$
     - 3 dB dose error present in this example
     - Need this value, either a reasonable estimate or assumed known, to conduct SIMEX as described
2. Simulate data and fit to the model
   - Simulate data with additive dose error
   - Take average over 100 simulations at each λ
   - $\lambda = 0.5$ is 50% increase of variability
3. Curve fit β_1 values from the initial observed and simulated data
4. Extrapolate curve fit to no dose error case at $\lambda = -1$

- We can compare with true β_1 value because we are using a simulated dataset



- What's going on with the dose-response curve during SIMEX?
- Graphically, the process can be depicted simply in 2 steps:
  1. **Simulate** data with additive dose error
     - Start at observed data (brown curve; $\lambda = 0$)
     - Observe flattening with simulated data (brown to orange to yellow curves; $\lambda > 0$)
  2. **Extrapolate** β_1 slope estimates to the no dose error case
     - Taking the information from the simulation, extrapolate to SIMEX corrected curve (dashed red; $\lambda = -1$)
     - This approaches the true curve (blue)

Synthetic datasets generated based on X-59 preliminary dose design and populations with varying annoyance response



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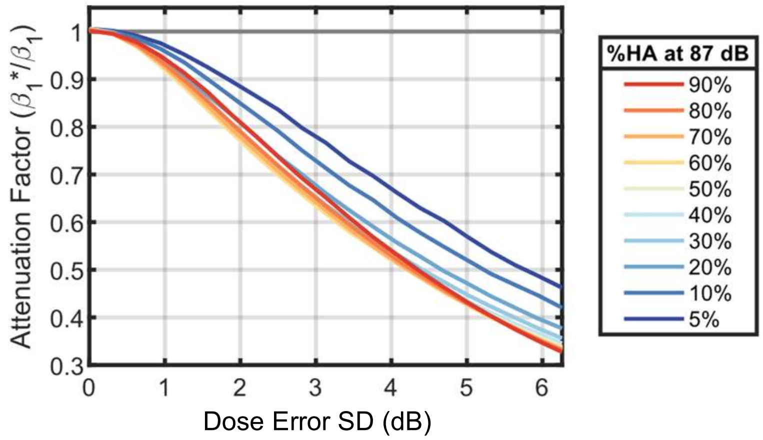
- Now we generate several synthetic populations to apply SIMEX
  - We do not know how sensitive the populations will be in our future community noise studies
  - Does SIMEX work as well on less-sensitive populations with more limited dose-response curves?
- Preliminary dose design distribution
  - Dose range (70 to 87 dB) set by aircraft constraints
  - 86 total flights over several weeks
- Generating 10 synthetic populations
  - 1000 participants each (86,000 total dose-response pairs)
  - Percent highly annoyed (%HA) at 75 dB set to 1% based on previous quiet supersonic community studies
  - %HA at 87 dB varied
    - Red = more sensitive
    - Blue = less sensitive
- Next, add incremental homogenous dose error (mean zero, normally distributed) to each population to observe bias due to dose error

Attenuation curves depict similar decrease in naïve estimates of slope (β_1^*) with increased dose error SD for all synthetic populations



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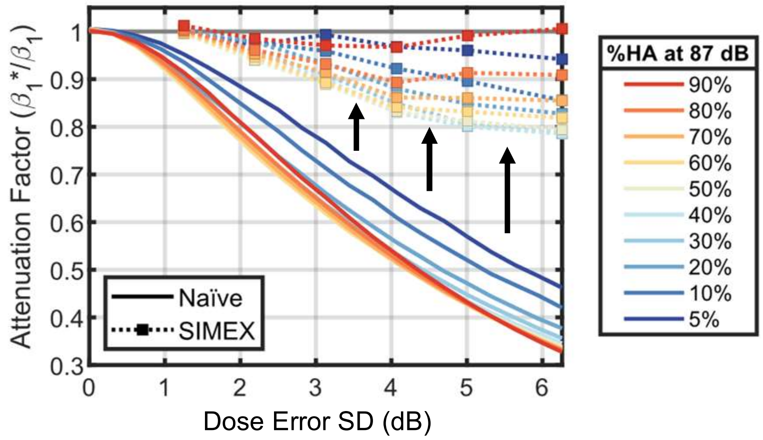
- Attenuation curves shown here (not dose-response curves)
- Naïve refers to data with dose error and no correction applied
- All 10 synthetic populations have similar attenuation curve
    - Attenuation factor (estimated slope β_1^*) decreases with increasing dose error
    - Least annoyed populations in blue (5% and 10%HA) are slightly shallower
    - At 3 dB, 20 to 35% relative bias in slope
- Takeaway: Similar attenuation curve for all populations

Applying SIMEX mitigates bias as demonstrated by improved attenuation factor ($\beta_1^*/\beta_1 \rightarrow 1$) for all populations



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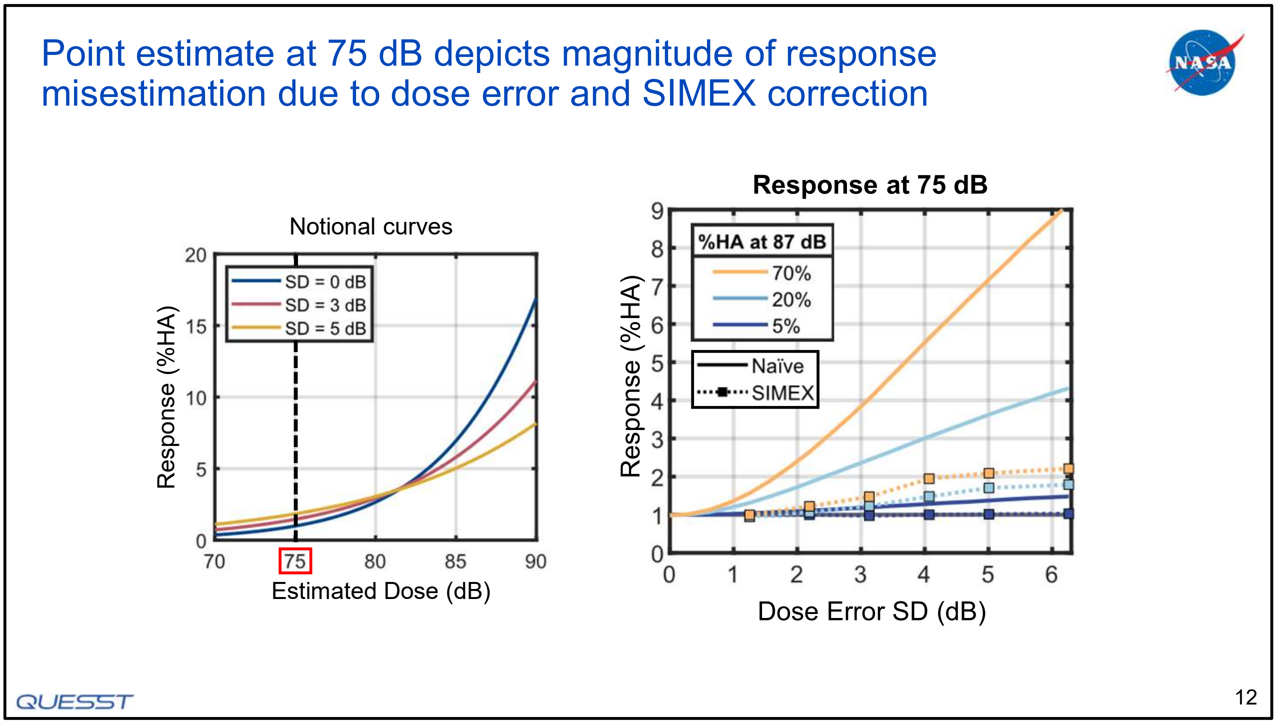
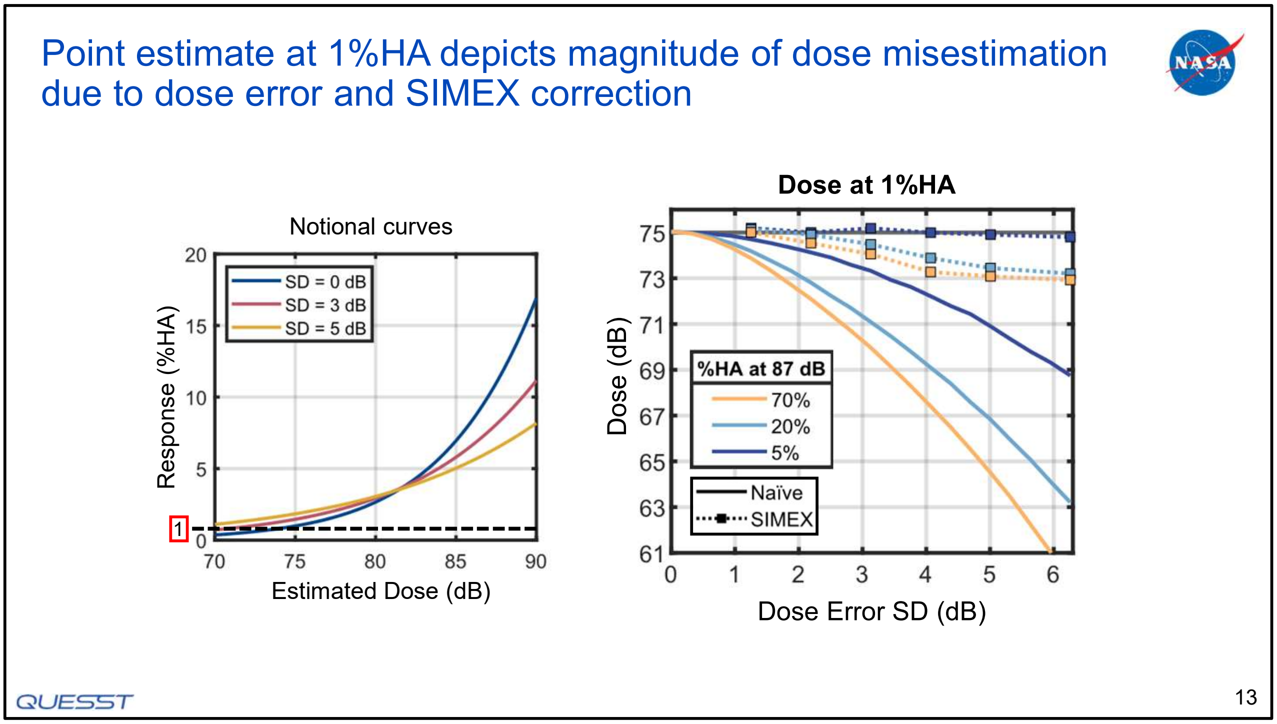
- Attenuation curves corrected using SIMEX
  - Applied SIMEX with known estimated dose error
  - 6 test cases per population at ~1 dB increments of SD
- Significant improvement to estimated slope ($\beta_1^*/\beta_1 \rightarrow 1$)
  - At 3 dB, less than 10% relative bias in slope
  - At 6 dB, less than 20% relative bias in slope instead of 55 to 65%
- This represents an idealized scenario where the standard deviation of the distribution of additive dose errors is known or at least reliably estimated
- Correcting bias due to dose error improves the dose-response relationship to more accurately portray the underlying truth



- Point estimates provide further insight on the bias due to dose error by giving a sense of the magnitude of misestimation
    - Consider a vertical slice
        - Left plot is notional (colors are unrelated to right plot)
        - Depicts dose-response curves and over/under predicted annoyance
        - At 75 dB, annoyance becomes overpredicted with increased dose error SD
- Right plot shows predicted annoyance (%HA) at 75 dB as a function of dose error SD for 3 populations of varying noise sensitivities (5%, 20%, and 70%HA at 87 dB)
    - These are not dose-response curves, just a point on the curve
    - 75 dB is set to be 1%HA for the no dose error case in the simulations for all populations
    - Lines show the naïve estimates (data with dose error)
    - Dotted lines with squares show the 6 test cases where SIMEX was applied
    - Difference in magnitude for differing population sensitivities
    - Over-prediction of annoyance is mitigated for all population sensitivities for the amount of dose error SD shown



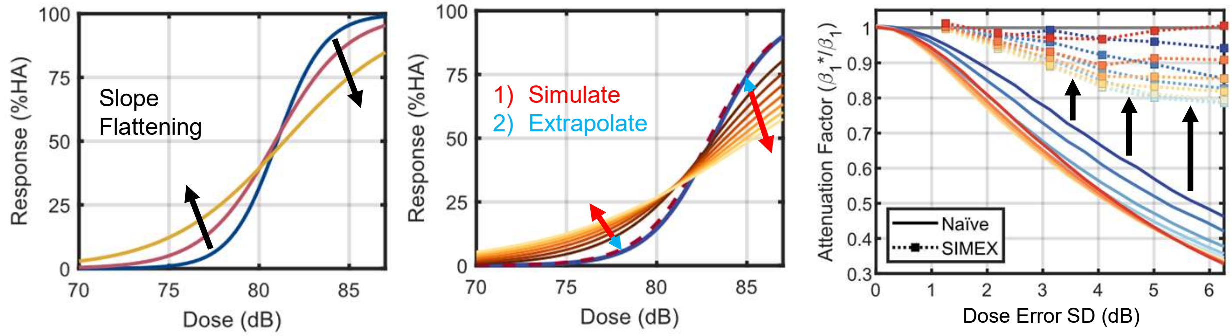
- This is now an inverse prediction: find the dose given a fixed level of annoyance
    - Consider a horizontal slice
        - Left plot is notional (colors are unrelated to right plot)
        - Depicts dose-response curves and over/under predicted annoyance
        - Dose that elicits 1%HA shifts left, below 75 dB due in the presence of increasing dose error SD
- Right plot shows predicted dose that elicits 1%HA as a function of dose error SD for 3 populations of varying noise sensitivities (5%, 20%, and 70%HA at 87 dB)
    - These are not dose-response curves, just a point on the curve
    - 1%HA is set to 75 dB for the no dose error case in the simulations for all populations
    - Lines show the naïve estimates (data with dose error)
    - Dotted lines with squares show the 6 test cases where SIMEX was applied
    - Difference in magnitude for differing population sensitivities
    - SIMEX brings inverse predictions (dose that elicits 1%HA) close to the true reference point of 75 dB for cases shown and all combinations considered

In summary, SIMEX can improve dose-response modeling by mitigating bias due to dose error in community noise studies

1) Characterized bias due to dose error

2) Corrected bias due to dose error w/ SIMEX

3) Applied SIMEX to several synthetic populations



QUESSTIONS?

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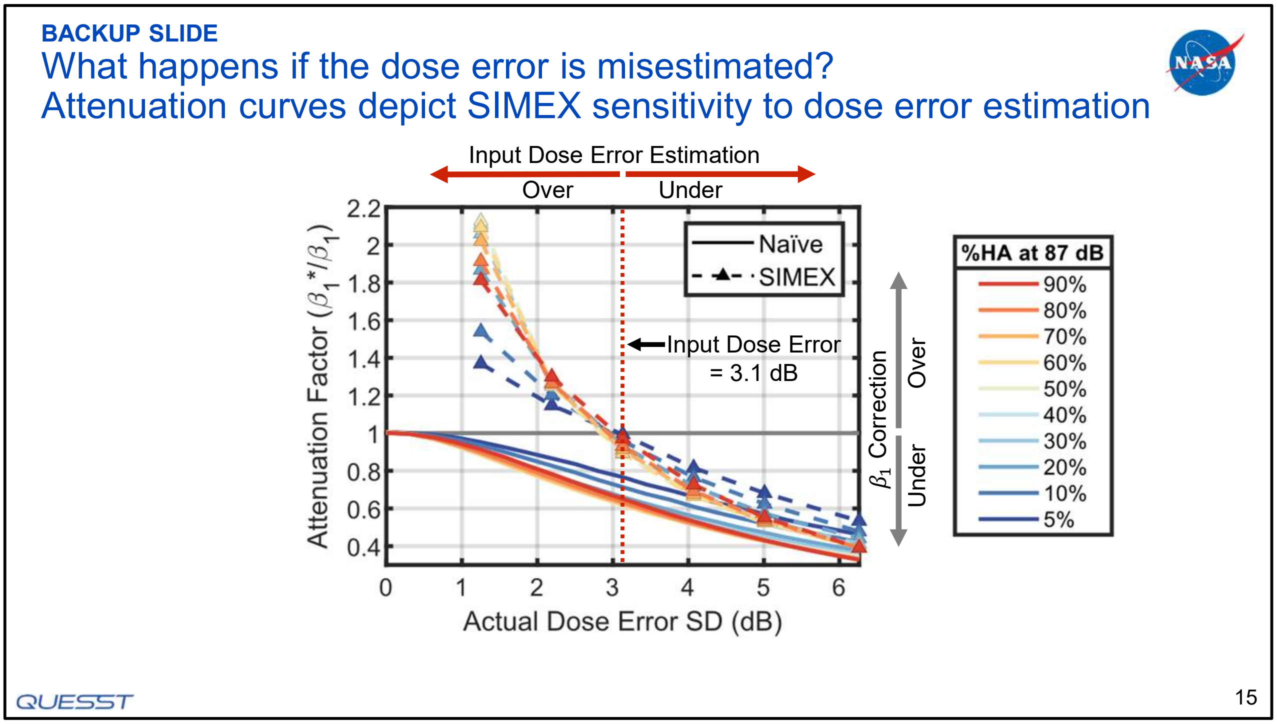
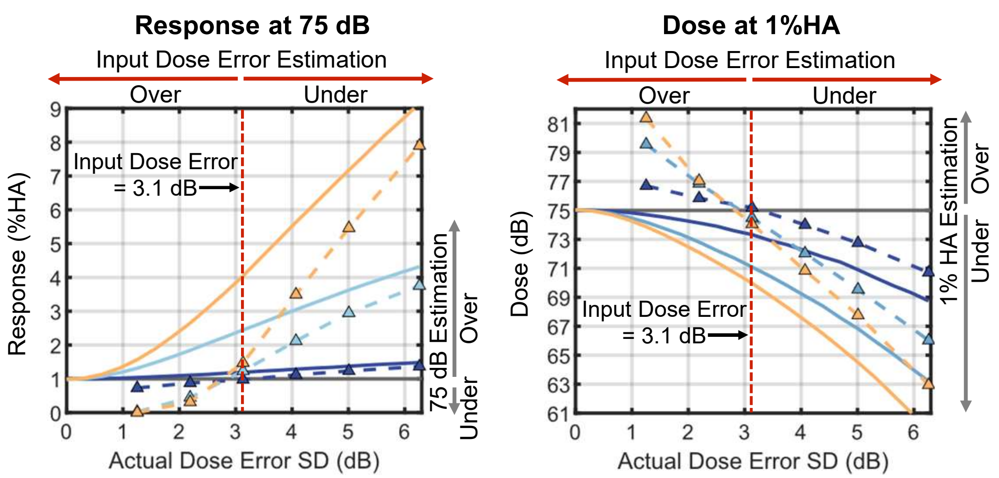
- SIMEX is one method to mitigate bias due to dose error
1. Characterized bias due to dose error
   - Dose error causes dose-response curve to **flatten**
   - **Over/underestimate response at lower/higher doses**
2. Corrected bias due to dose error with SIMEX
   - Two main steps in SIMEX process:
     1. **Simulate** additive dose error
     2. **Extrapolate** to the "no error" case
3. Applied SIMEX to several simulated populations
   - SIMEX mitigates bias for all synthetic populations
- Mitigating bias due to dose error with SIMEX can be applied to community studies with the X-59 as well as other community noise studies



- What happens when the dose error is misestimated?
- Applying SIMEX with misestimated dose error
  - 6 test cases per population at ~1 dB increments of SD
  - Input 3.1 dB SD dose error to SIMEX model for all cases
  - Performs well at 3.1 dB (input dose error = actual dose error)
  - Left side, input dose error > actual dose error
    - Overestimated dose error, overcorrected slope
    - Too much correction
  - Right side, input dose error < actual dose error
    - Underestimated dose error, underestimated slope
    - Some correction, but not much
- This demonstrates the potential sensitivity of SIMEX to dose error estimation

BACKUP SLIDE

What happens if the dose error is misestimated?
Point estimates depict SIMEX sensitivity to dose error estimation



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- What happens when the dose error is misestimated?
- These figures depict point estimates for SIMEX corrections made with misestimated dose error
    - 6 test cases per population at ~1 dB increments of SD
    - Input 3.1 dB SD dose error to SIMEX model for all cases
    - Performs well at 3.1 dB (input dose error = actual dose error)
    - Left side, input dose error > actual dose error
        - Overestimated dose error, overcorrected slope
        - Too much correction
    - Right side, input dose error < actual dose error
        - Underestimated dose error, underestimated slope
        - Some correction, but not much
- This demonstrates the magnitude of misestimation due to the sensitivity of SIMEX to dose error estimation