MULTICRITERION DESIGN METHODS FOR AEROSPACE SYSTEMS

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Multicriterion Design Methods

- **Outline / Introduction**
- Truss platform application
- Multiple design criteria
- Pareto optimality
- Develop approximations
- Develop bounding curves
- Summary / Proposed future work

Introduction

- Develop multicriterion methods for conceptual / preliminary-level design
- Apply Taguchi methods to identify key points on Pareto set
- Develop simple approximations for Pareto set
- Apply methods to representative problem
- Assess results

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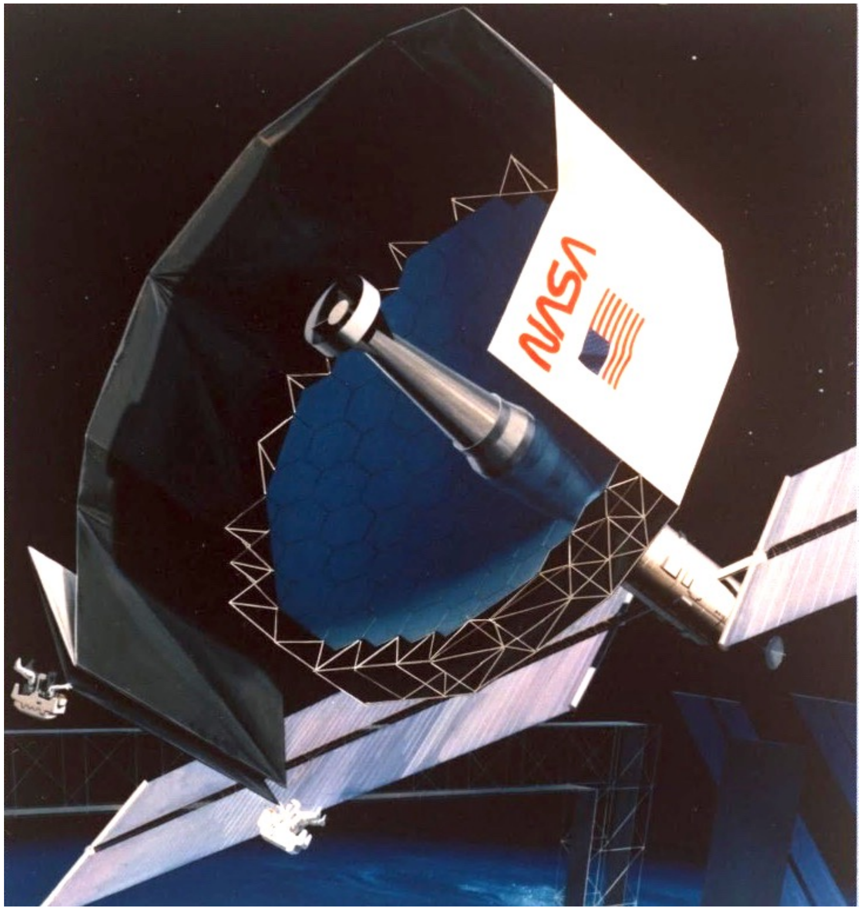
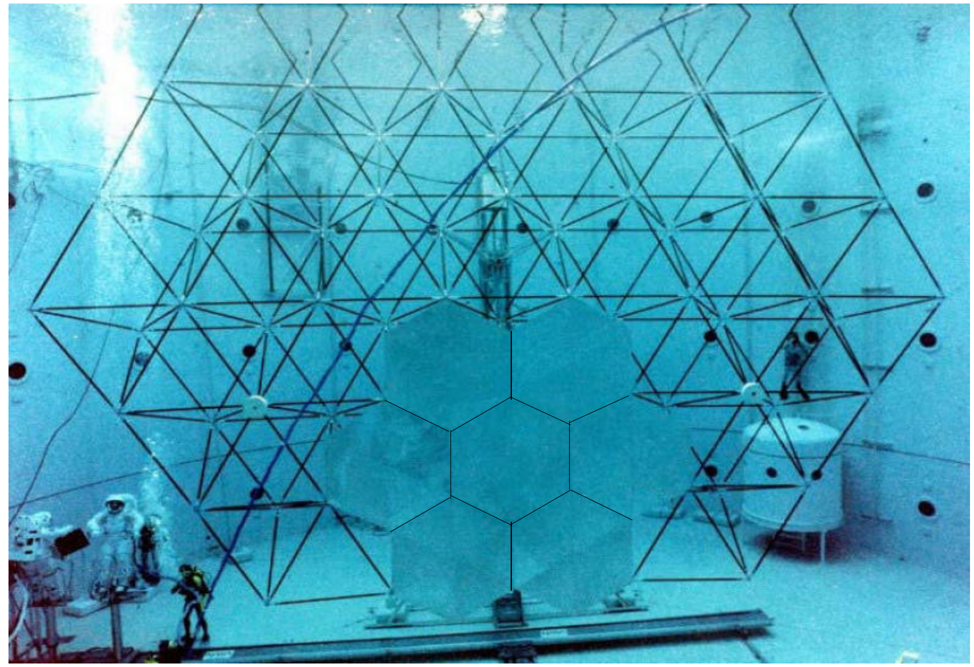
Application - Large Space Truss Platform



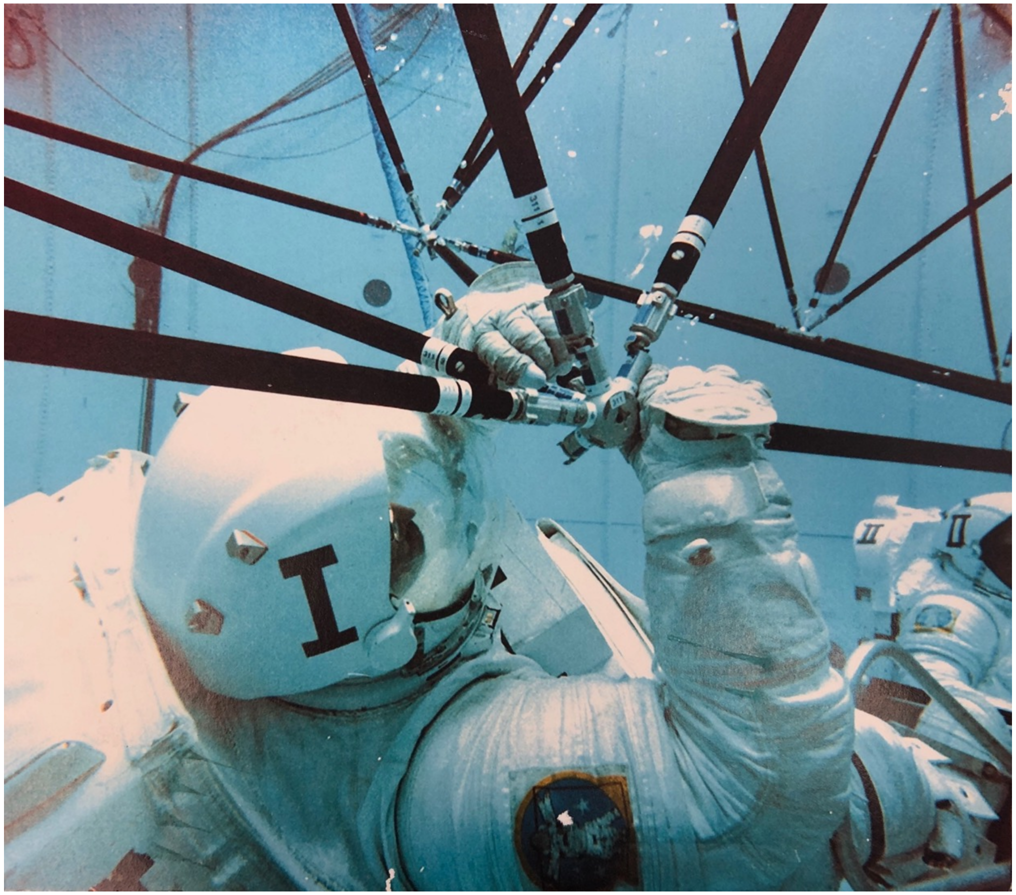
20m-diameter IR observatory



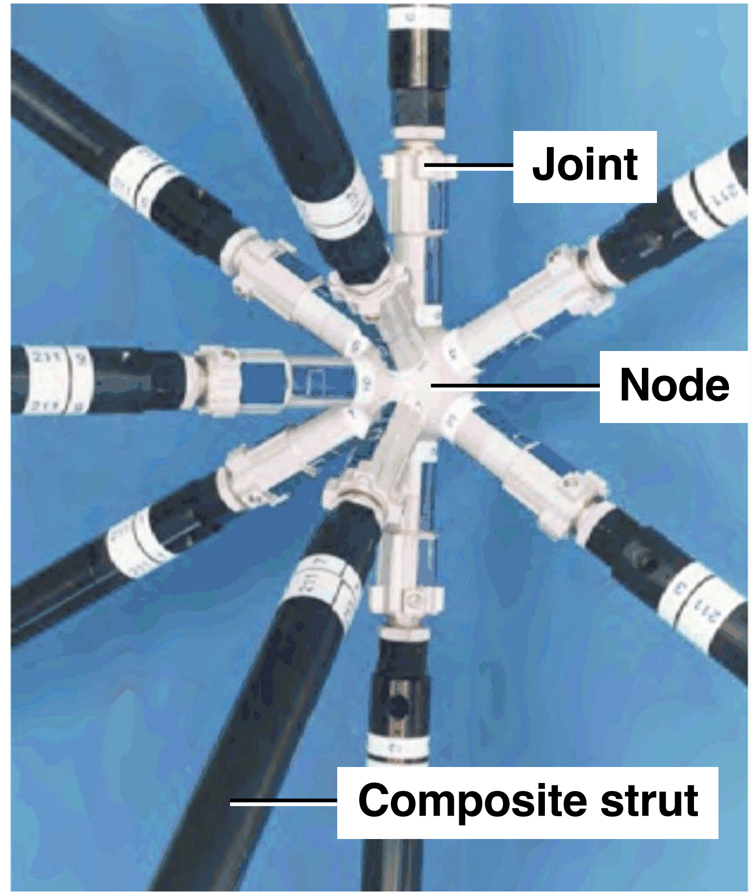
Truss assembly in simulated EVA

- 14m diam. doubly-curved truss
- 84 nodes; 315 struts, [1.98, 2.15]m
- SSF-derived joint hardware

Application - Large Space Truss Platform (2)

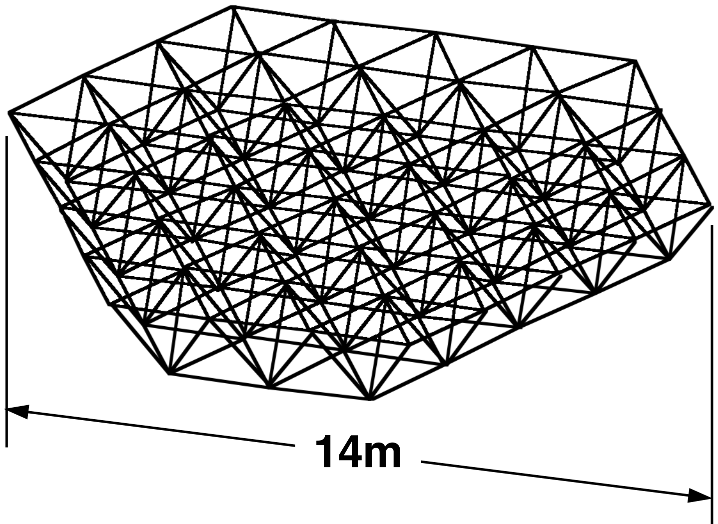


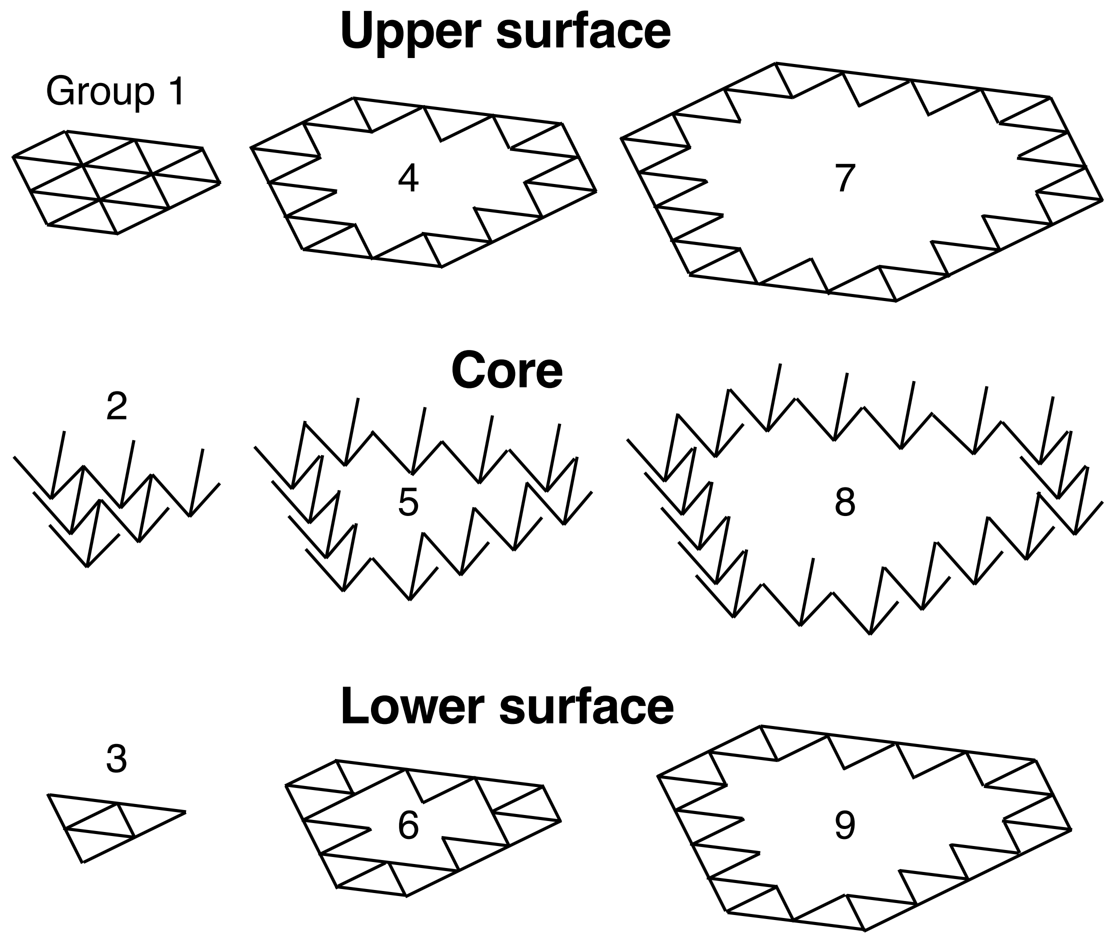
Truss assembly in simulated EVA



Truss node w/ struts/joints

Application - Large Space Truss Platform (3)





- **Flat 3-ring truss**
- **84 nodes, 315 struts (identical 2m length)**
- **9 member groups**
- **1, 3, or 5x nominal area**
- **3^9 = 19,683 combinations**

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Multiple Optimization Criteria

Well-defined analytical solutions / methods for single criterion (*i.e.*, first derivative = 0)

I have two objectives here; I want to simultaneously:
- **Minimize mass**
- **Maximize first natural frequency (*i.e.*, performance)**

Other possible criteria:
- **Dynamic amplitude (surface distortion)**
- **Manufacturing cost (number of unique struts)**

For multiple criteria (*i.e.*, n > 1), introduce concept of *Pareto optimality*

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Pareto Optimality

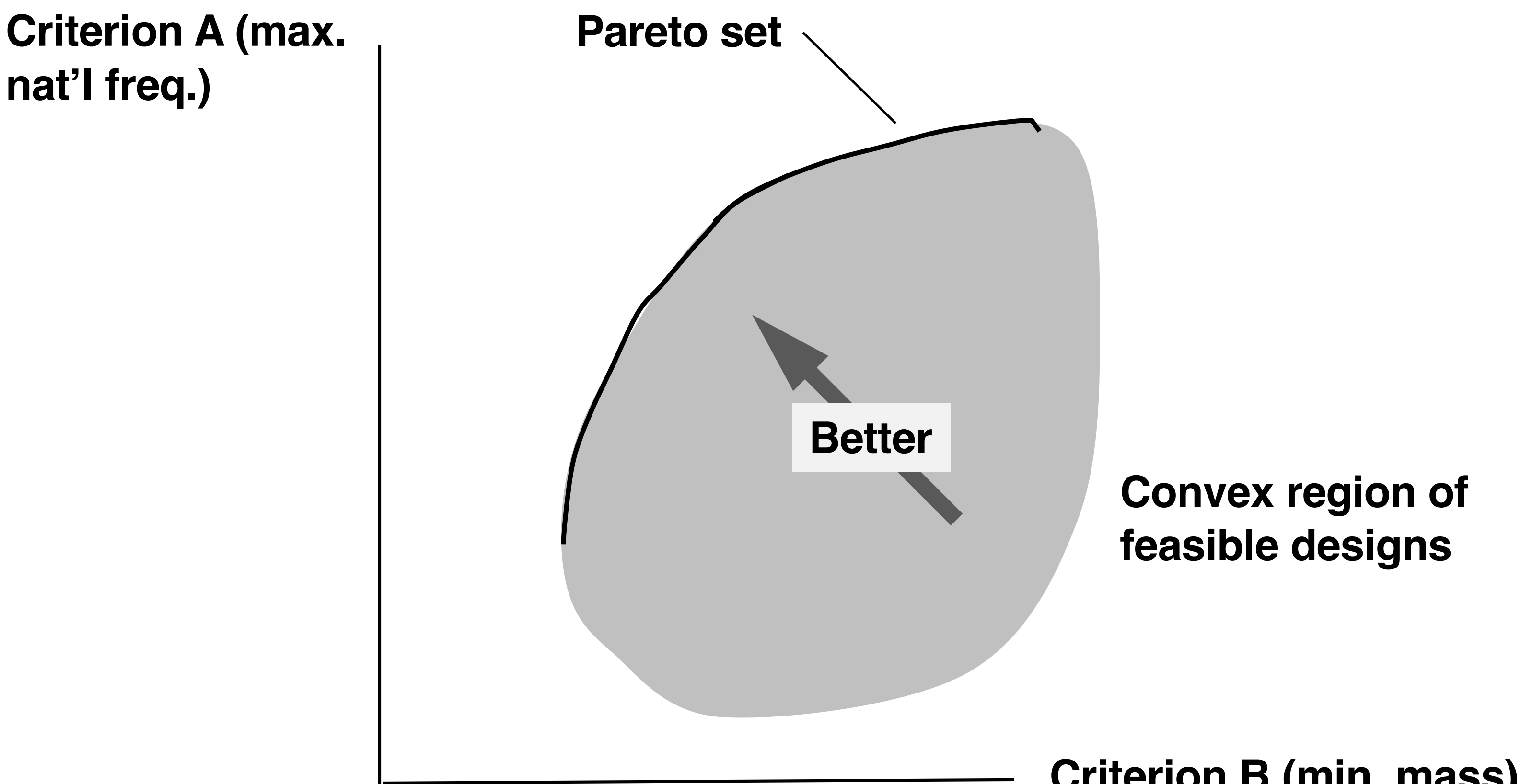
Named for Vilfredo Pareto (1848-1923), a French/Italian civil engineer, economist, and polymath, who also first noted the 80/20 Rule

Pareto optimality says you can't make Criterion A better, without simultaneously making Criterion B worse (or, at best, neutral)

In criterion space, the *Pareto set (front, frontier)* represents all possible trade-offs between Criterion A (e.g., freq.) and Criterion B (e.g., mass)

Can model Pareto set = $(1 - \alpha)$*A + α*B, with α = [0, 1]

Visualization of Pareto Set In Criterion Space



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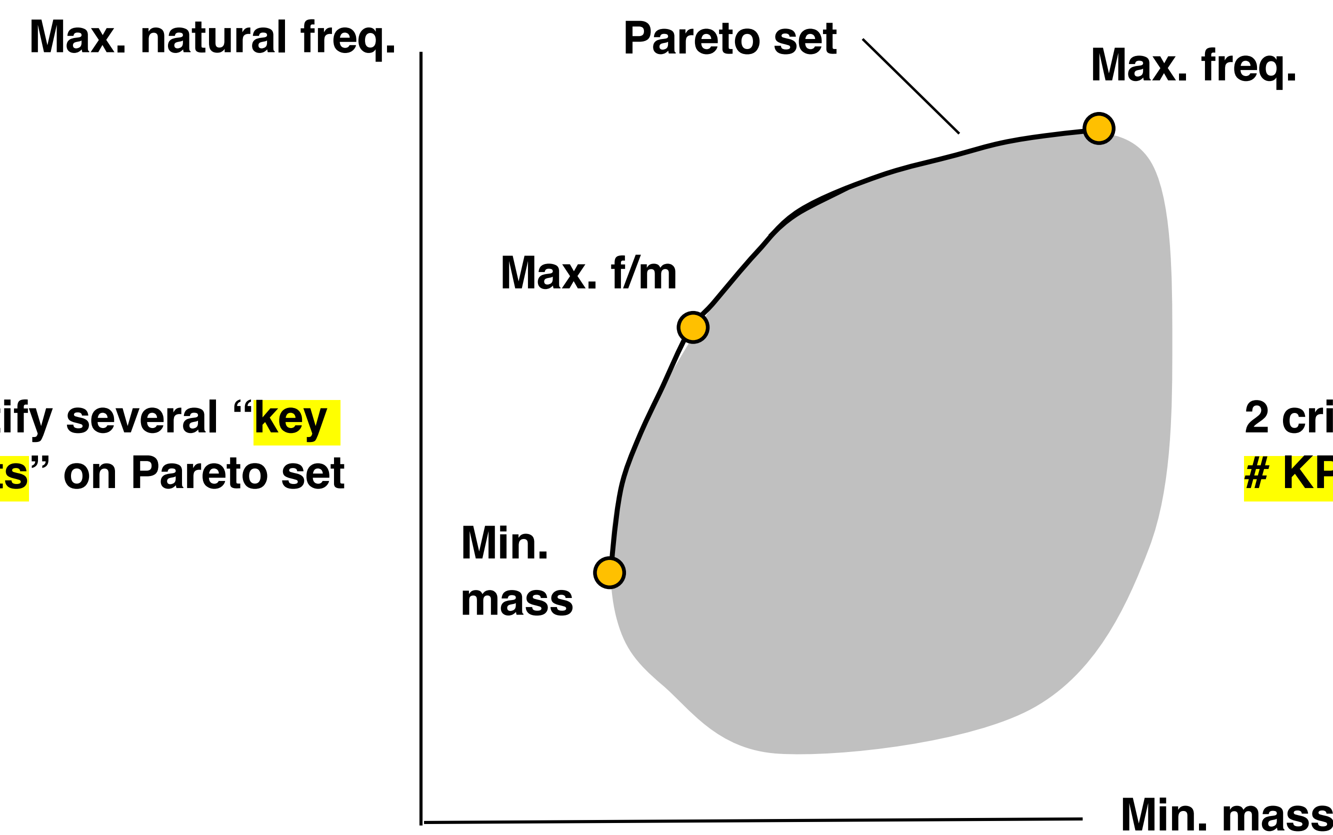
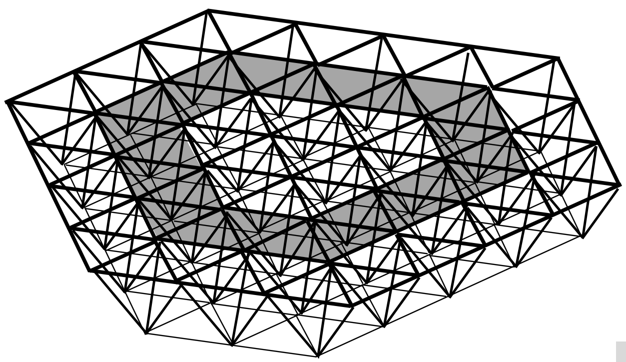
Definition of Pareto Set Key Points



Identify several "**key points**" on Pareto set

2 criteria => 3 key points
KP = $2^n - 1$

Taguchi Methods Used To Identify Key Points



- 3 possible areas (1, 3, 5x nominal)
- 9 strut groups
- 3^9 = 19,683 combos for full-factorial
- Run only 27 selected cases for “main effects” Taguchi analysis

Case	Normalized area A for truss member group: 1	2	3	4	5	6	7	8	9	f, Hz	m, kg	f/m, Hz/kg
1	1	1	1	1	1	1	1	1	1	29.93	217.20	0.1378
2	1	1	3	5	5	5	5	5	5	39.94	404.01	0.0989
3	1	1	5	3	3	3	3	3	3	38.13	315.30	0.1209
4	1	3	1	5	5	5	3	3	3	39.61	351.83	0.1126
5	1	3	3	3	3	3	1	1	1	36.19	263.12	0.1375
6	1	3	5	1	1	1	5	5	5	31.36	340.35	0.0921
7	1	5	1	3	3	3	5	5	5	36.84	376.87	0.0978
8	1	5	3	1	1	1	3	3	3	32.03	288.16	0.1112
9	1	5	5	5	5	5	1	1	1	38.20	309.04	0.1236
10	3	1	1	5	3	1	5	3	1	31.58	323.65	0.0976
11	3	1	3	3	1	5	3	1	5	39.31	316.34	0.1243
12	3	1	5	1	5	3	1	5	3	38.39	321.56	0.1194
13	3	3	1	3	1	5	1	5	3	42.72	320.52	0.1333
14	3	3	3	1	5	3	5	3	1	34.92	328.87	0.1062
15	3	3	5	5	3	1	3	1	5	36.97	330.95	0.1117
16	3	5	1	1	5	3	3	1	5	35.46	326.78	0.1085
17	3	5	3	5	3	1	1	5	3	40.00	335.13	0.1194
18	3	5	5	3	1	5	5	3	1	38.74	337.21	0.1149
19	5	1	1	3	5	1	3	5	1	32.48	329.91	0.0985
20	5	1	3	1	3	5	1	3	5	41.58	322.60	0.1289
21	5	1	5	5	1	3	5	1	3	39.06	334.08	0.1169
22	5	3	1	1	3	5	5	1	3	36.67	329.91	0.1112
23	5	3	3	5	1	3	3	5	1	39.71	338.26	0.1174
24	5	3	5	3	5	1	1	3	5	41.16	337.21	0.1221
25	5	5	1	5	1	3	1	3	5	44.20	336.17	0.1315
26	5	5	3	3	5	1	5	1	3	35.09	344.52	0.1019
27	5	5	5	1	3	5	3	5	1	39.50	343.48	0.1150

Taguchi Methods Used To Identify Key Points (2)

Analysis of L_{27} results for max-*f* design

A	1	2	3	4	5	6	7	8	9
	Average frequency, Hz, for truss member group								
1	35.80	36.71	36.61	35.54	37.45	34.51	**39.15**	36.32	35.69
3	37.56	37.70	37.64	37.85	**37.49**	38.10	37.02	**37.99**	37.97
5	**38.83**	**37.78**	**37.94**	**38.81**	37.25	**39.59**	36.02	37.88	**38.53**

Predicted max-*f* design has A = 555 535 135

Analysis of L_{27} results for min-*m* design

A	1	2	3	4	5	6	7	8	9
	Average mass, kg, for truss member group								
1	**318.43**	**320.52**	**323.65**	**313.21**	**314.25**	**316.34**	**306.95**	**307.99**	**310.08**
3	326.78	326.78	326.78	326.78	326.78	326.78	326.78	326.78	326.78
5	335.13	333.04	329.91	340.35	339.30	337.22	346.61	345.57	343.48

Predicted min-*m* design has all A = 1 => matches intuition!

Taguchi Methods Used To Identify Key Points (3)

Analysis of L_{27} results for max-*f/m* design

A	1	2	3	4	5	6	7	8	9
	Average frequency-to-mass ratio, Hz/kg, for truss member group								
1	0.1147	0.1159	0.1143	0.1145	**0.1199**	0.1102	**0.1282**	**0.1193**	**0.1165**
3	0.1150	**0.1160**	**0.1162**	**0.1168**	0.1155	0.1173	0.1133	0.1162	0.1163
5	**0.1159**	0.1137	0.1152	0.1144	0.1102	**0.1181**	0.1041	0.1102	0.1128

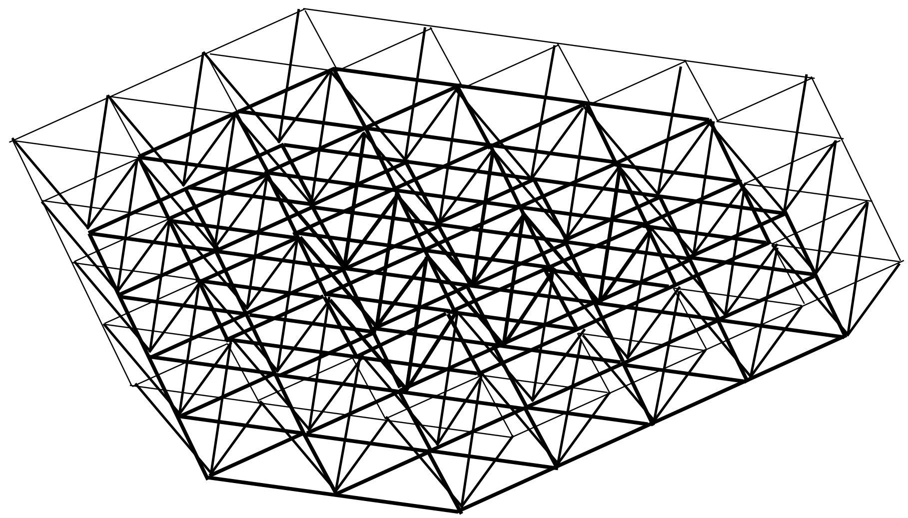
Predicted max-*f/m* design has A = 533 315 111

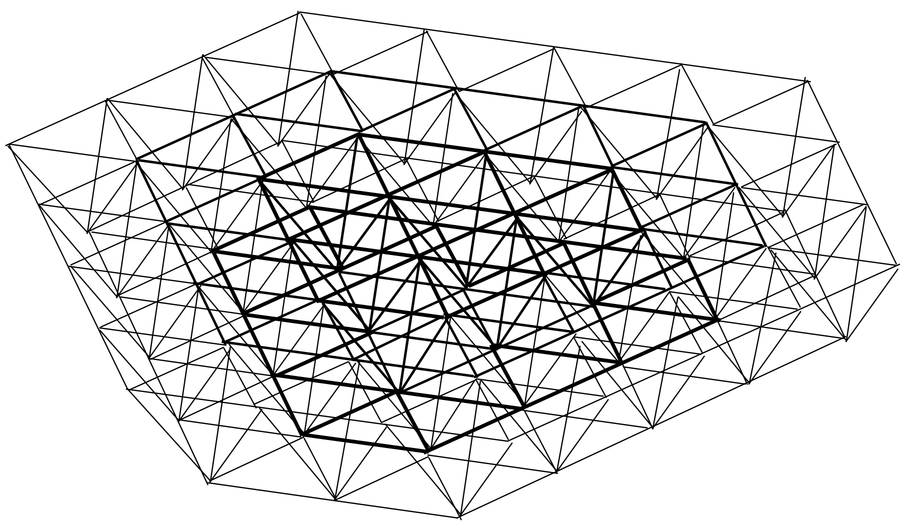
Predicted designs and performance

Description	Configuration	*f*, Hz	*m*, kg	*f*/*m*, Hz/kg
Baseline uniform, $A = 3$	333 333 333	42.96	326.78	0.1315
Maximum frequency,	555 535 135	50.73	365.39	0.1388
Percent from baseline	——	+18.07	+11.82	+5.55
Minimum mass, $A = 1$	111 111 111	29.93	217.20	0.1378
Percent from baseline	——	−30.33	−33.53	+4.80
Maximum frequency-to-mass ratio	533 315 111	42.77	277.73	0.1540
Percent from baseline	——	−0.45	−15.01	+17.11
Uniform, $A = 5$	555 555 555	48.40	436.36	0.1109

Taguchi Methods Used To Identify Key Points (4)

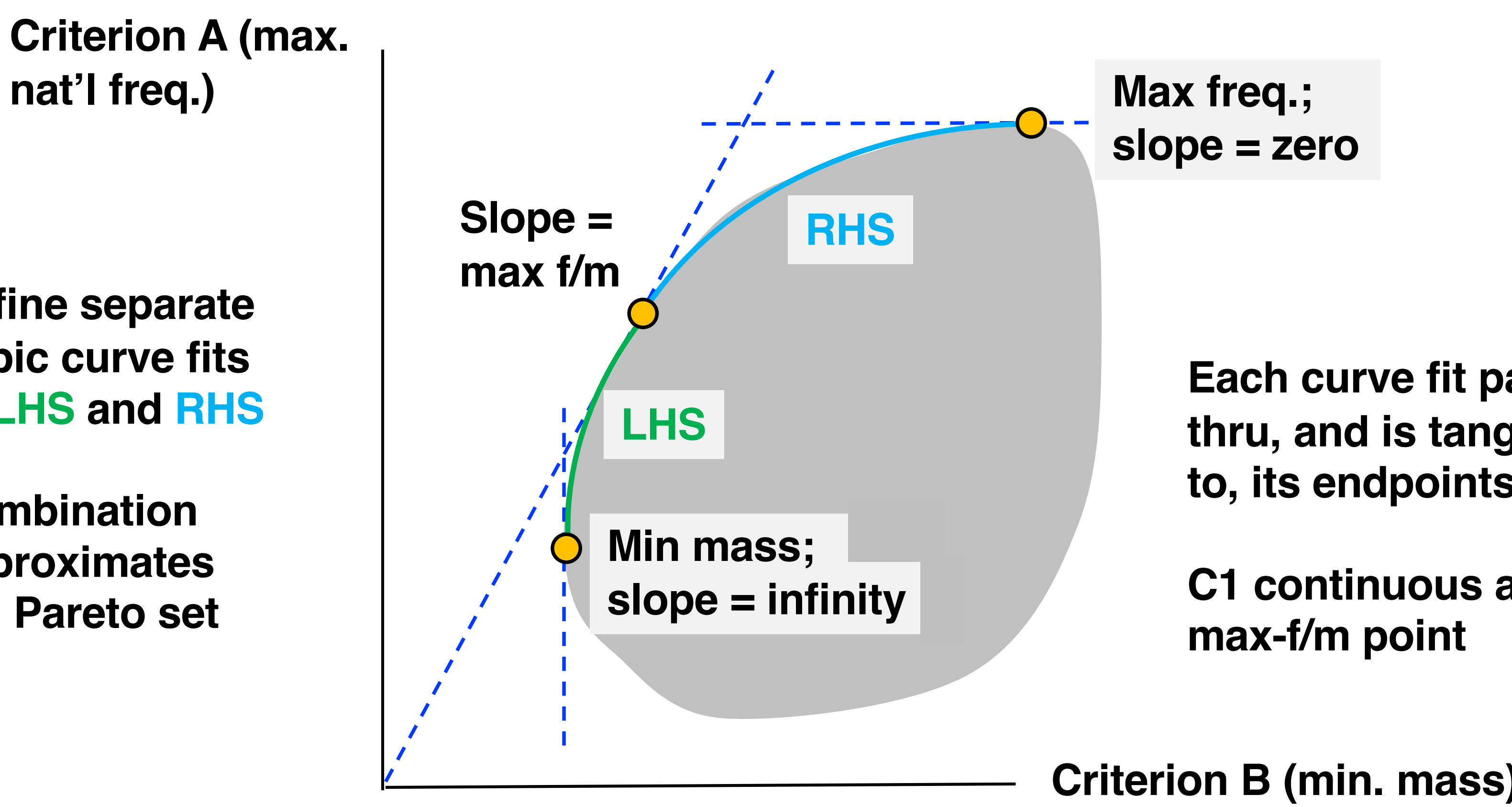
Max-*f* design
A = 555 535 135





Max-*f/m* design
A = 533 315 111

Approximation Of Pareto Set



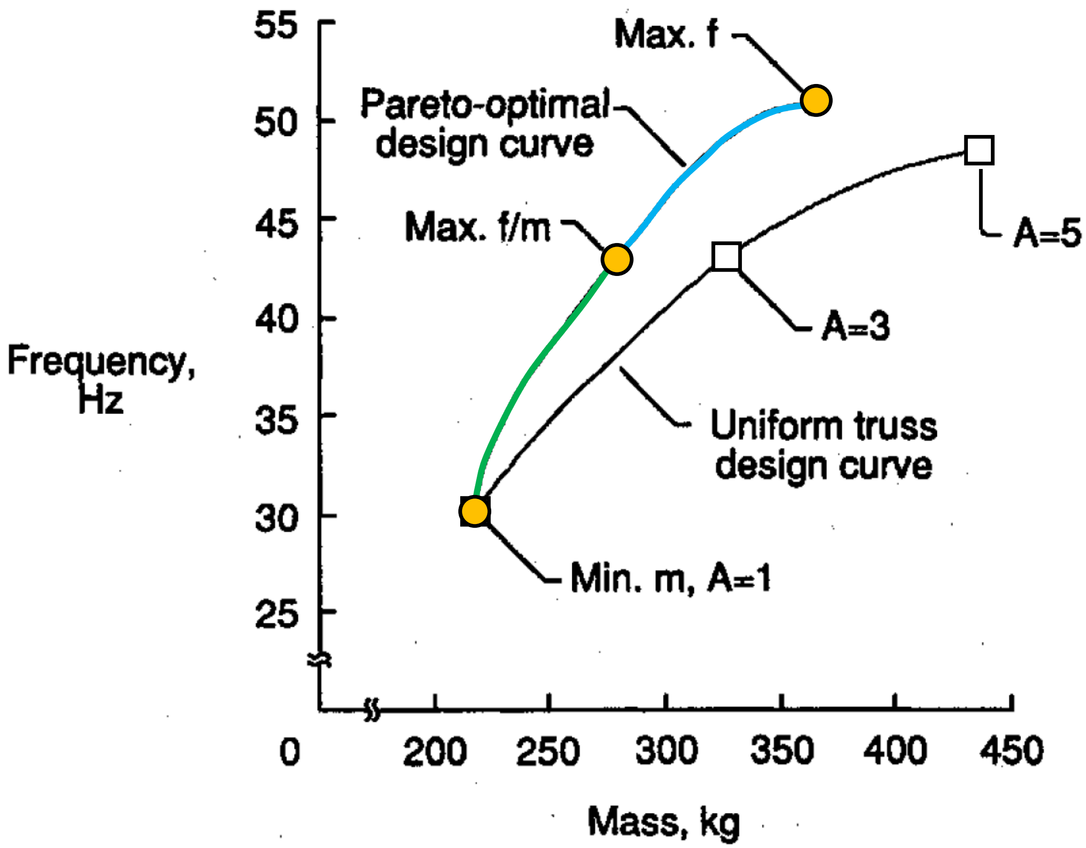
Define separate cubic curve fits to LHS and RHS

Combination approximates full Pareto set

Each curve fit passes thru, and is tangent to, its endpoints

C1 continuous at the max-f/m point

Key Points Define Pareto Design Curve



RHS between f and f/m

$$f = 45.6626 - 0.4520m + 0.002588m^2 - 0.000003593m^3$$

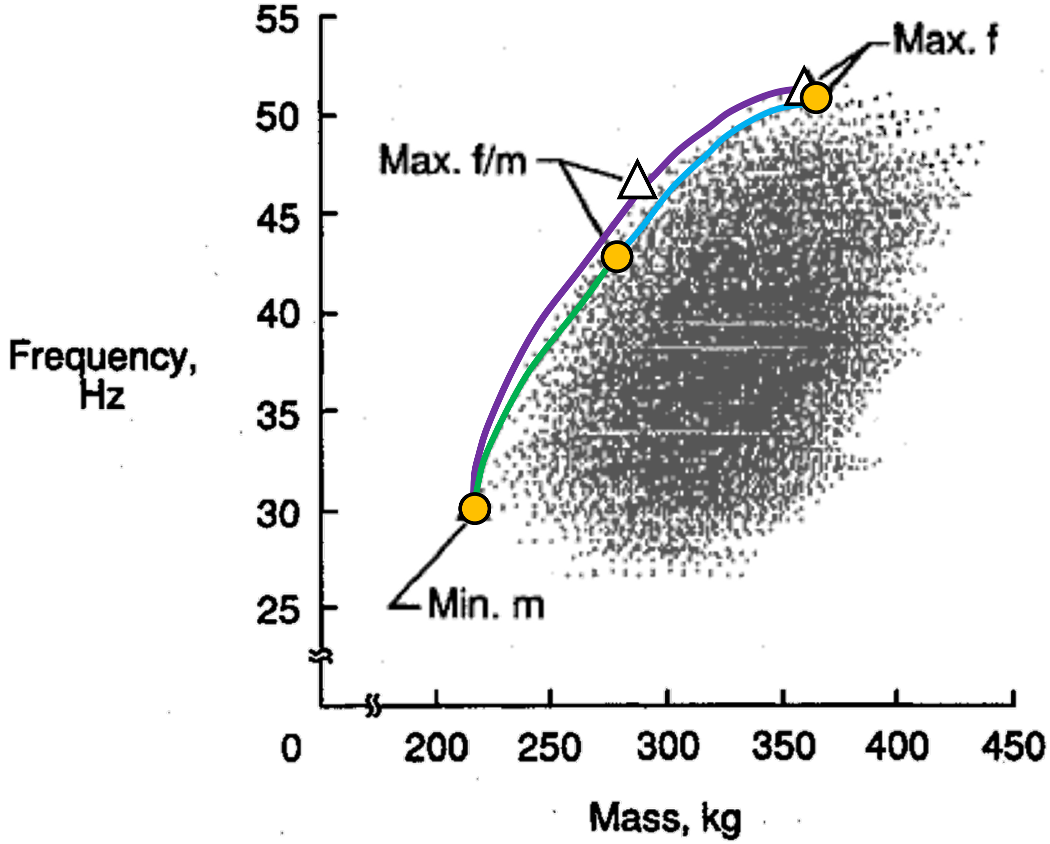
LHS between f/m and m

$$m = 1228.1303 - 83.4996f + 2.1941f^2 - 0.01780f^3$$

O(2) polynomial for uniform truss

$$f = -18.3359 + 0.2909m - 0.0003162m^2$$

Pareto Curve - Full Factorial Comparison



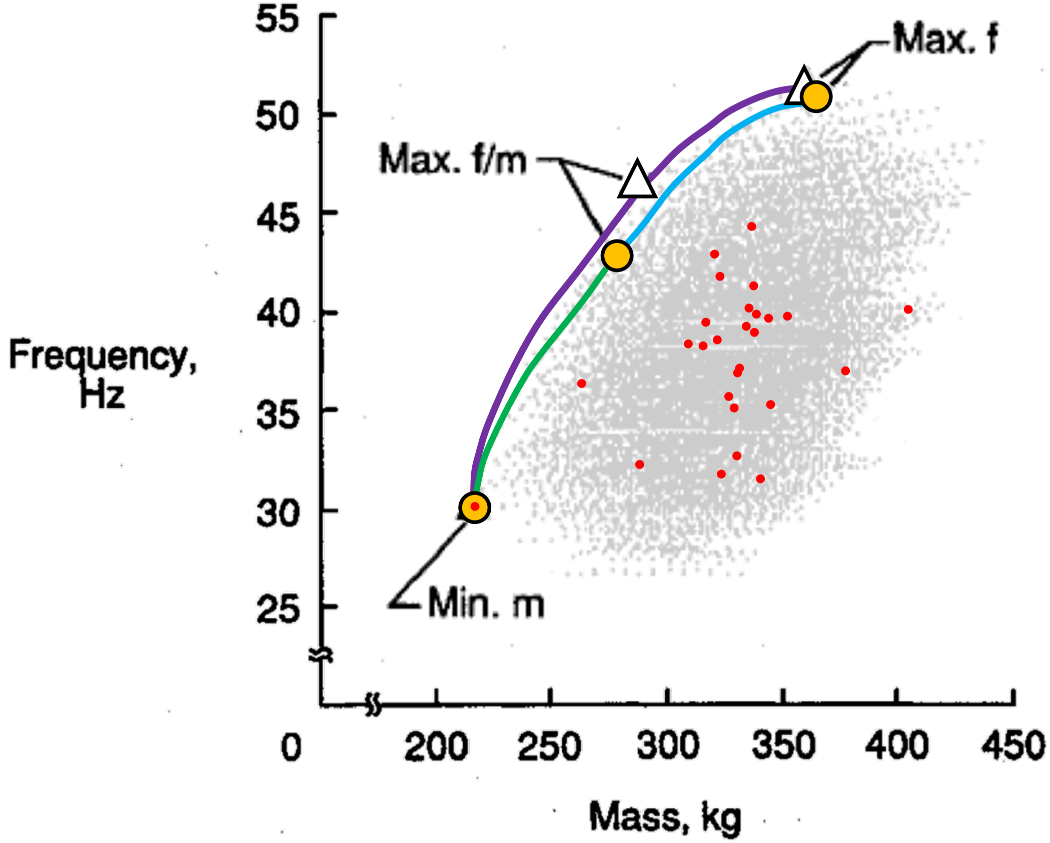
Run full factorial for validation
- Includes min-mass design
- Max-f, max-f/m are close (0.9, 4.4 percent, resp.)

Taguchi promises only “near optimal” solutions
=> *optimized*, not *optimal* …

99.5 percent of FF data within modeled Pareto design curve
- Only ~100 points missed

● Taguchi △ Full-factorial

Pareto Curve - Fractional Factorial Comparison

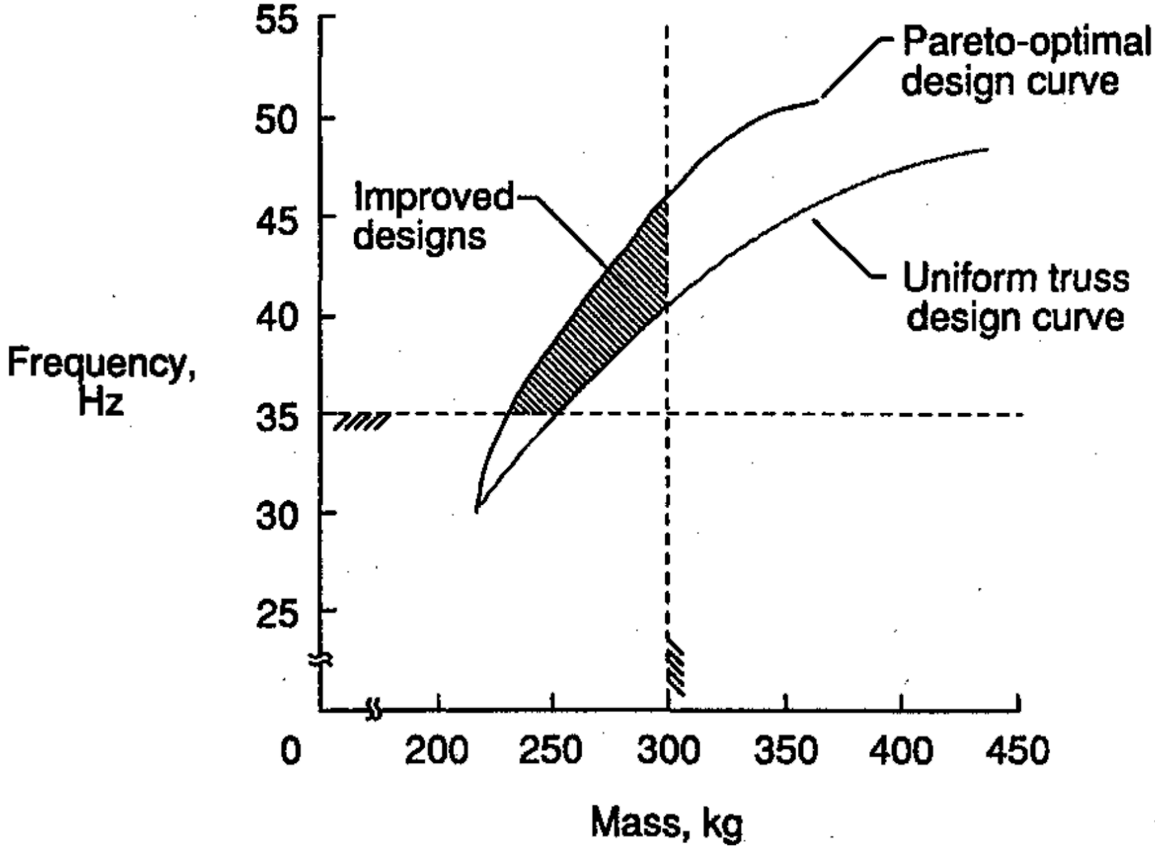


L_{27} fractional factorials
- Includes min-mass design
- (Randomly?) distributed over criterion space

Except for min-m, Taguchi solutions are far-removed from fractional factorials

Taguchi • Fractional factorial

Apply Pareto Curve For Design Trades



Now impose constraints
- 300 kg upper limit on mass
- 35 Hz lower limit on freq.

Won't tell you what optimized design looks like …
- May only know that viable solution(s) exist
- Useful for conceptual / preliminary design

Also assess sensitivity to constraint variations

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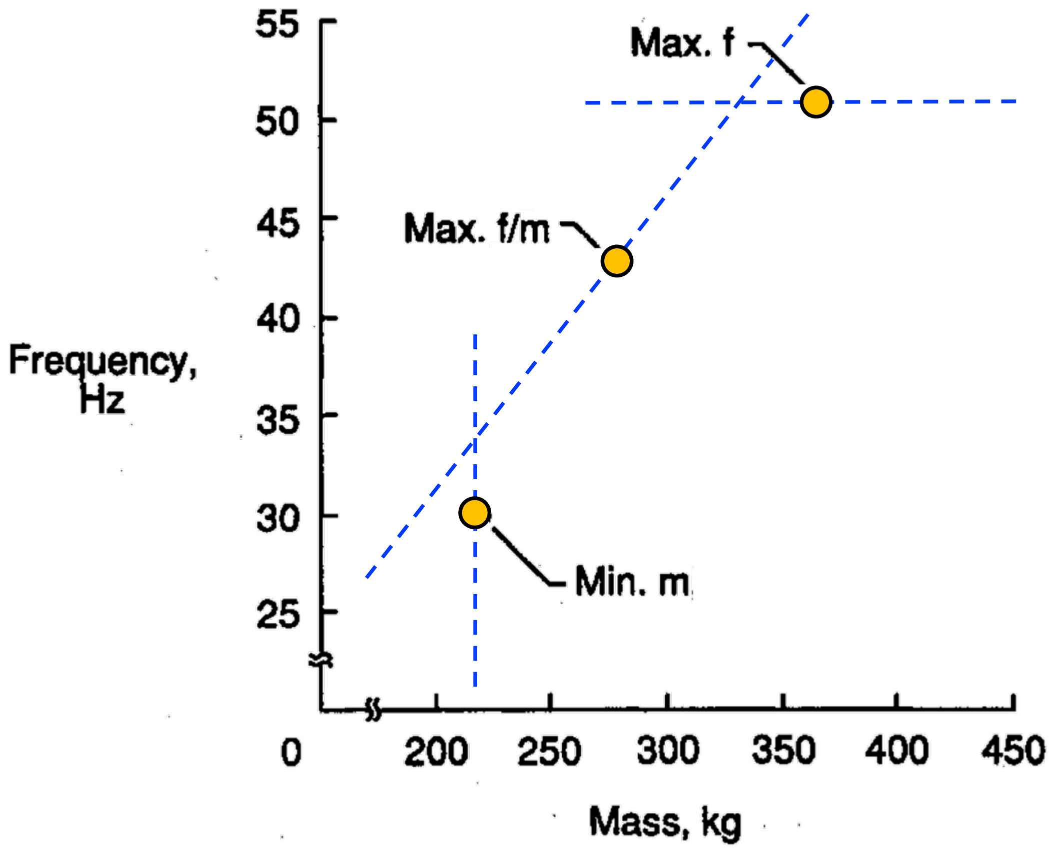
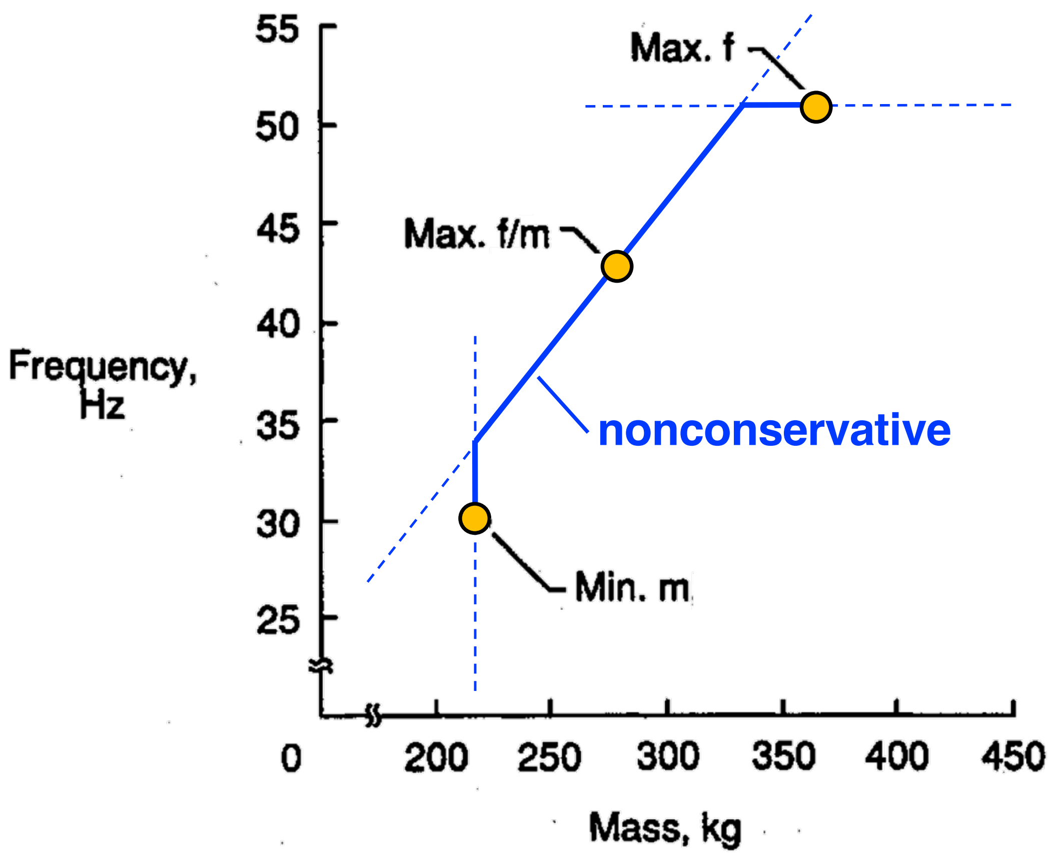
Bounding The Pareto Design Curve



Recall blue dashed lines used to define Pareto design curve

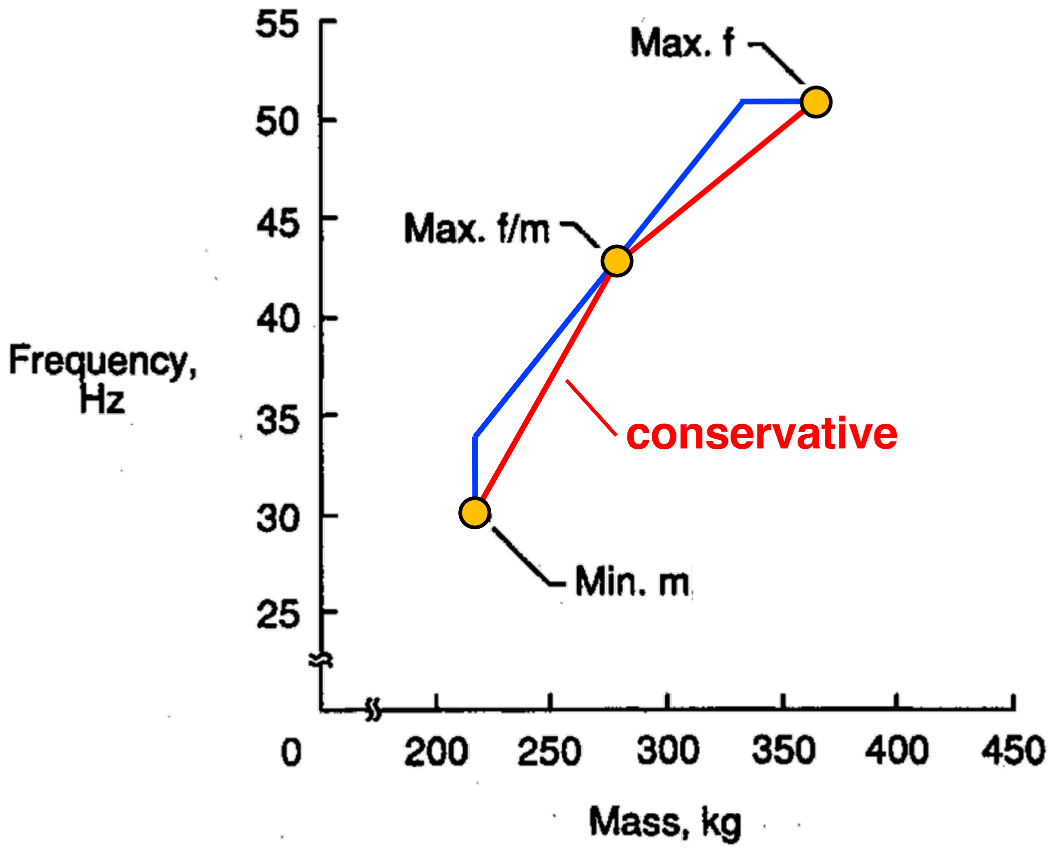
Bounding The Pareto Design Curve (2)



Recall blue dashed lines used to define Pareto design curve

Now define nonconservative outer surface between key points; # NC = $2^n - 1$

Bounding The Pareto Design Curve (3)

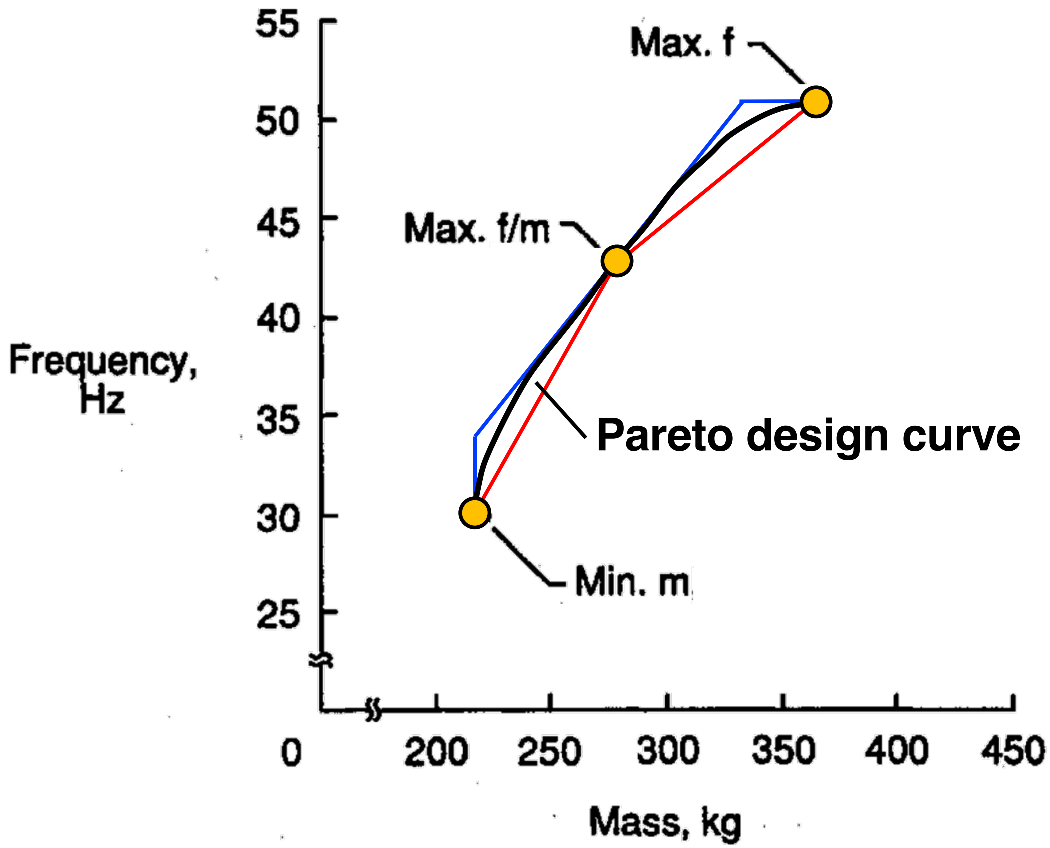


Recall blue dashed lines used to define Pareto design curve

Now define nonconservative outer surface between key points; # NC = $2^n - 1$

Now define conservative inner surface between key points; # C = $2^n - 2$

Bounding The Pareto Design Curve (4)



Recall blue dashed lines used to define Pareto design curve

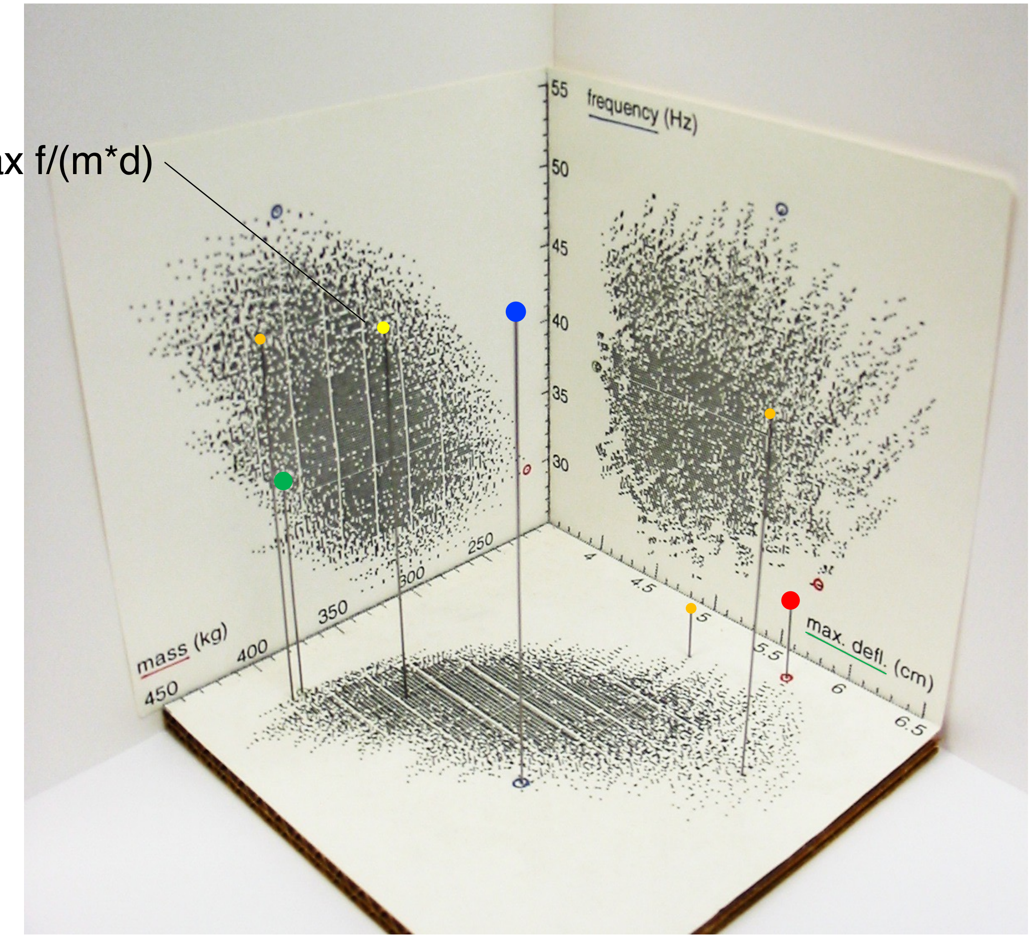
Now define nonconservative outer surface between key points; # NC = $2^n - 1$

Now define conservative inner surface between key points; # C = $2^n - 2$

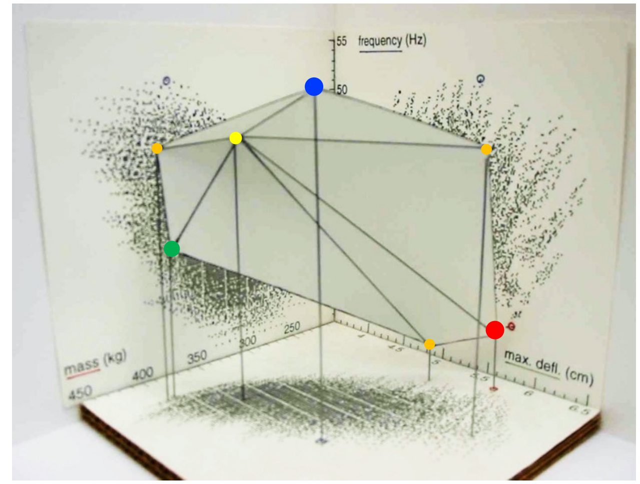
Know (assume) that Pareto design curve lies between these bounding curves

Extend Methodology Graphically To 3 Criteria

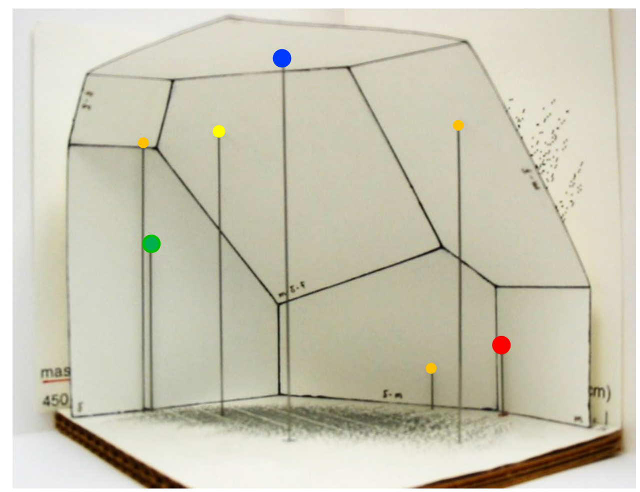
max f/(m*d)



Introduce deflection of truss platform under free vibration as criterion. 7 key points identified.



Conservative Pareto surface



Nonconservative Pareto surface

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Summary

- Identified space truss platform application
- Proposed relevant design criteria
- Used Taguchi methods to identify key points (could use other optimization methods)
- Developed analytical model for n = 2 Pareto surface
- Extended concept graphically to n = 3 criteria
    - Didn't develop supporting maths