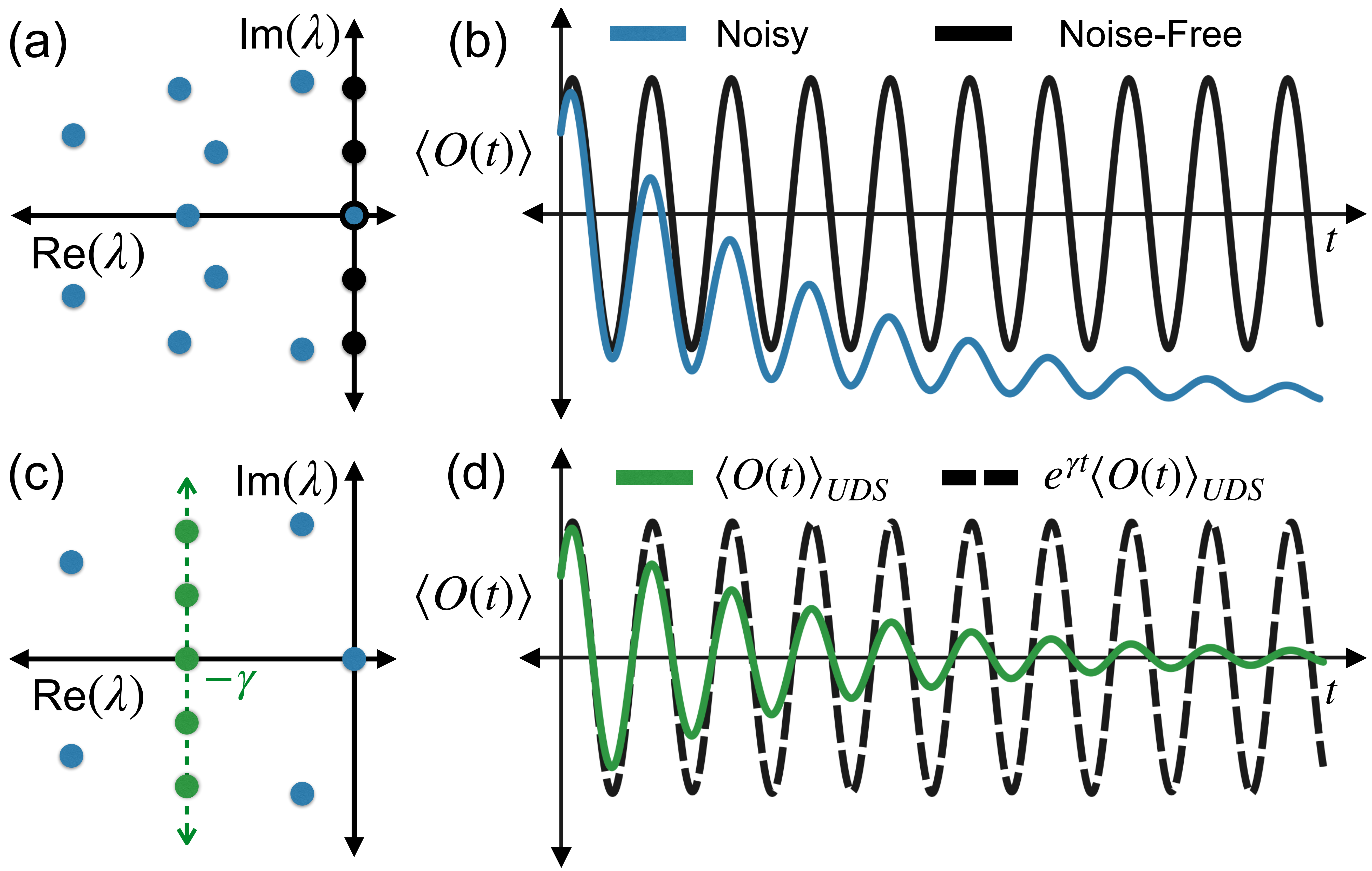
Uniformly Decaying Subspaces for Error Mitigated Quantum Computation



Nishchay Suri, Jason Saied and Davide Venturelli
(arXiv:)

Error Suppression

Decoherence Free Subspaces

- Passive protection from decoherence
- Requires high symmetry in noise

Dynamical Decoupling

- Active protection by decoupling
- Limited by fast timescales of noise

Error Mitigation

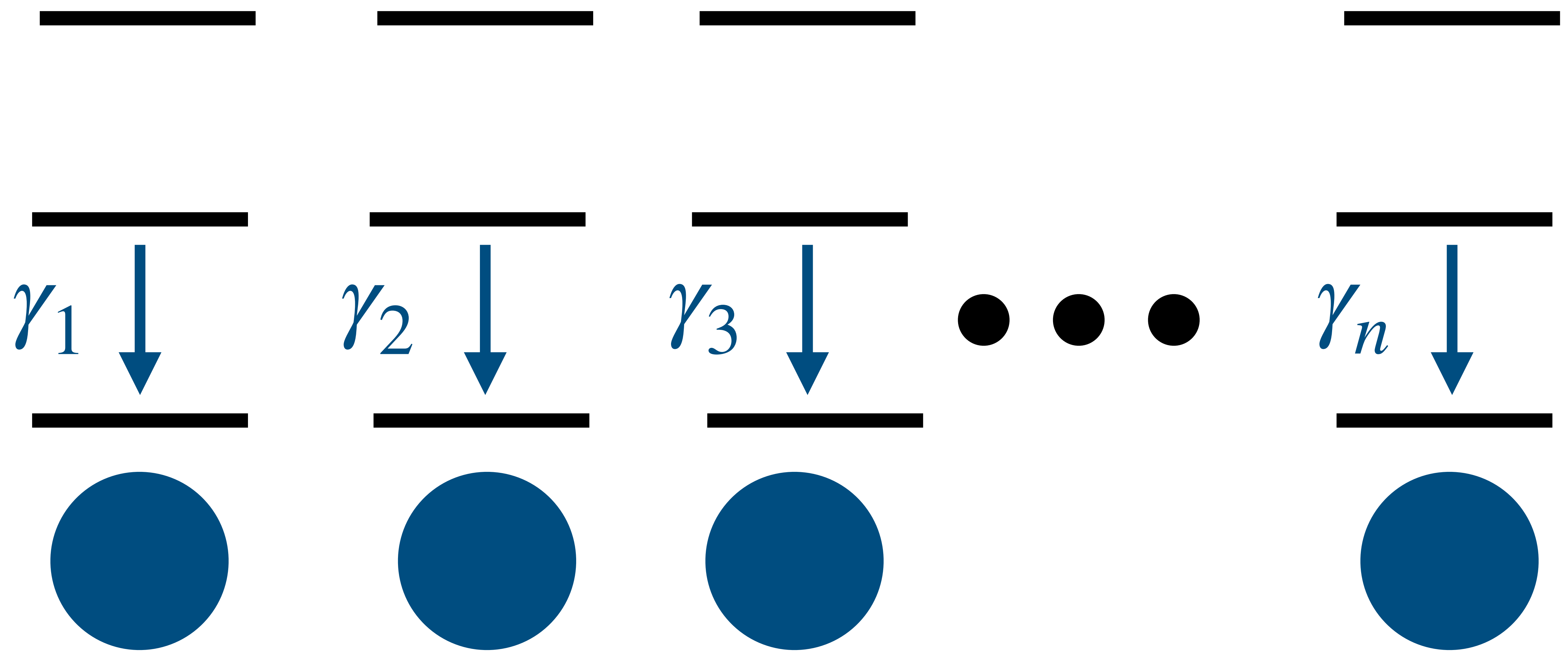
Symmetry Verification

- Protection from Noise that breaks symmetry
- Cannot remove symmetry preserving errors

Probabilistic Error Cancellation

- Works well, can obtain zero bias
- Requires full knowledge of noise

Our Results



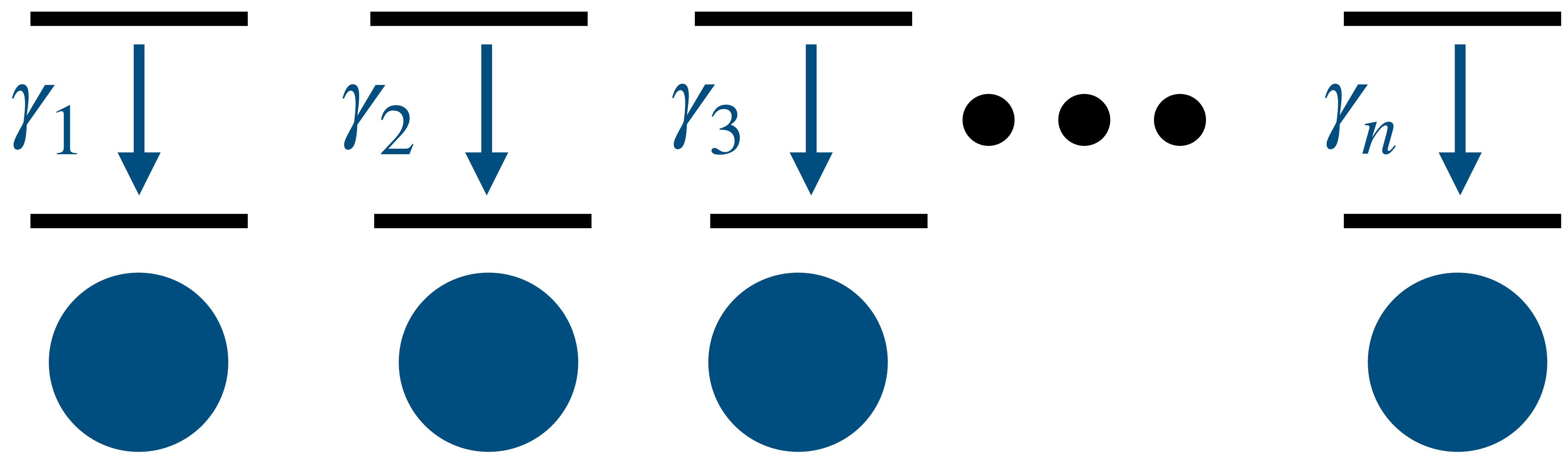
- Passive Error Mitigation scheme for Qubits/Qudits with inhomogeneous relaxation
- Corrects bias undetectable by Symmetry Verification
- Works with partial knowledge of noise (can outperform Probabilistic Error Cancellation)

System of n qubits and qudits with individual decay rates

$$\dot{\rho} = \mathcal{L}\rho = -i[H, \rho] + \sum_i \gamma_i \mathcal{D}[a_i]\rho$$

$$\dot{\rho} = \mathcal{L}\rho = -i[H, \rho] + \sum_i \gamma_i \mathcal{D}[\sigma_i^-]\rho$$

$$\mathcal{D}[\sigma_i^-]\rho = \sigma_i^- \rho \sigma_i^+ - \frac{1}{2}\{\sigma_i^+ \sigma_i^-, \rho\}$$



$$\sigma_i^- = |0\rangle\langle 1|_i$$

- Photon Loss is Dominant for Photonic Qubits
- cQED circuits
- Transmons can be operated at resonance (Dual Rail Qubits)
- Dynamical Decoupling
- Other systems: NV Center & Ionic Traps

$$T_1 \ll T_\phi$$

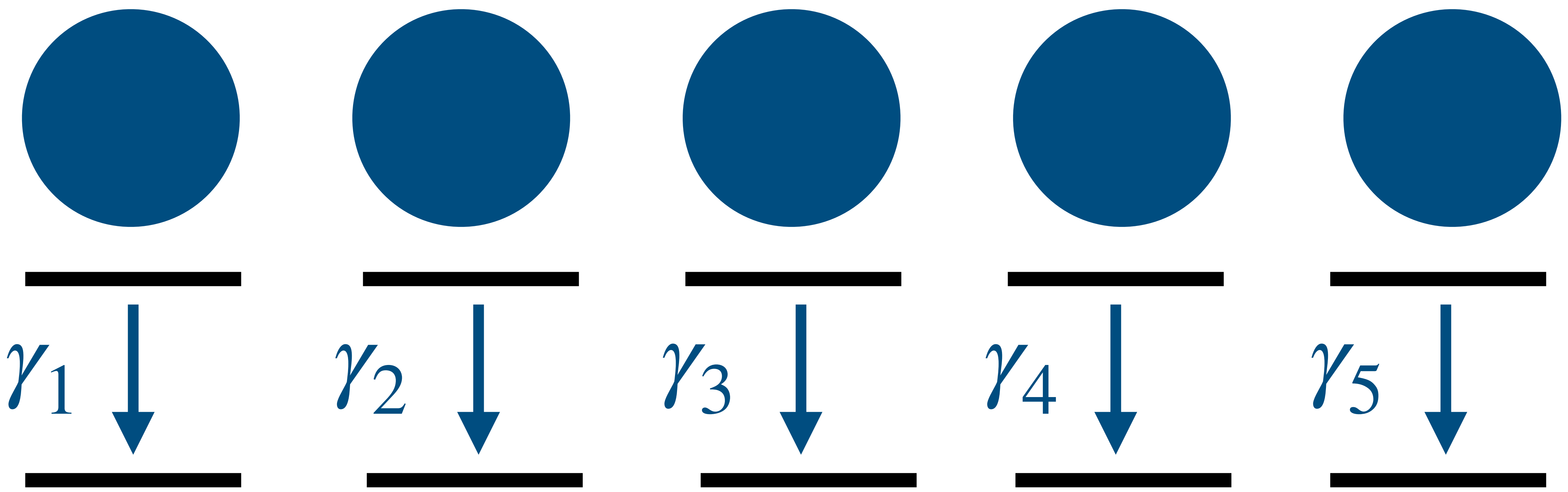
System of n qubits with individual decay rates

$$\dot{\rho} = \mathcal{L}\rho = -i[H,\rho] + \sum_i \gamma_i \mathcal{D}[\sigma_i^-]\rho$$

$$\gamma_i = \bar{\gamma} + \Delta\gamma_i$$

Can be Large (Known) — $\bar{\gamma}$

Small (Unknown) — $\Delta\gamma_i$



Experimentally - Align the decay rates of all qubits with the worst one.
(By tuning operating frequencies)

$$\mathscr{L}\rho = \underbrace{-\,i[H,\rho] + \bar{\gamma}\sum_{i=1}^{n} \mathscr{D}[\sigma_i^-]\rho}_{\mathscr{L}_0} + \underbrace{\sum_i \Delta\gamma_i \mathscr{D}[\sigma_i^-]\rho}_{\mathscr{L}_1}$$

Unperturbed Part

$$\dot{\rho} = \mathscr{L}_0\rho = -i[H,\rho] + \bar{\gamma}\sum_{i=1}^{n}\mathscr{D}[\sigma_i^-]\rho$$

$N = \mathbf{dim}(\mathcal{S}_k) = \binom{n}{k}$

Hilbert space

$\mathcal{S}_k$: k-hot Encoding Subspace spanned by $\{P_\alpha\}_{\alpha=1}^{N}$

$$\bar{\gamma}\sum_{i=1}^{n}\mathscr{D}[\sigma_i^-]P_\alpha = -\bar{\gamma}P_\alpha + P^\perp$$

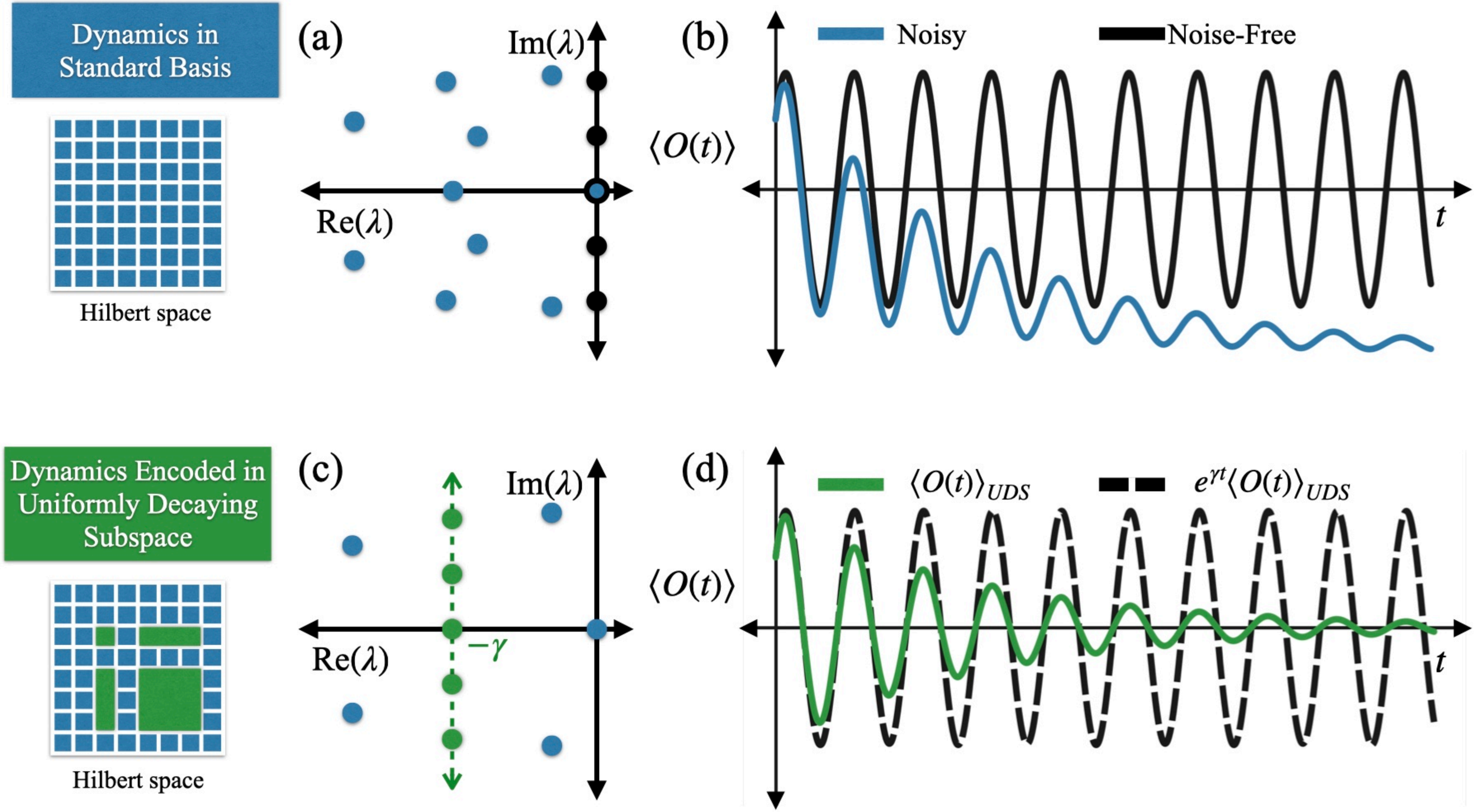
$$\boxed{\langle O\rangle_{UDS} = e^{-k\bar{\gamma}t}\langle O\rangle_0}$$

Coherent/Noise Free

- The dissipation term factors out into a scaling
- It gives exact results at the cost of exponentially more shots in time

Unperturbed Part

$$\dot{\rho} = \mathscr{L}_0\rho = -i[H,\rho] + \bar{\gamma}\sum_{i=1}^{n}\mathscr{D}[\sigma_i^-]\rho$$



System of n qubits with individual decay rates

$$\mathscr{L}\rho = -i[H,\rho] + \bar{\gamma}\sum_{i=1}^{n}\mathscr{D}[\sigma_i^-]\rho + \sum_i \Delta\gamma_i \mathscr{D}[\sigma_i^-]\rho$$

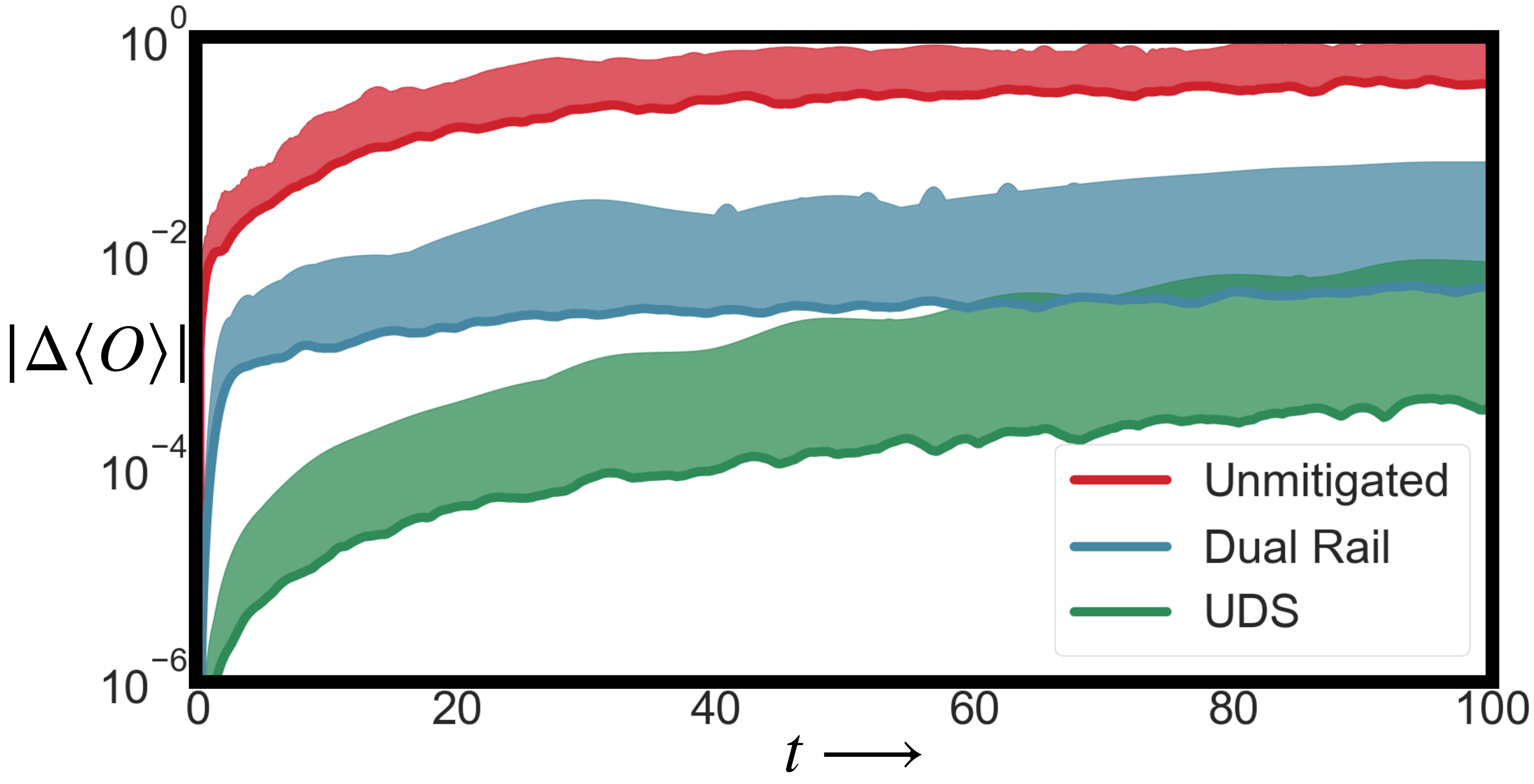
In $\mathscr{S}_k$: k-hot Encoding Subspace/Constant Hamming Weight Subspace

$$\langle O\rangle_{UDS} - e^{-k\bar{\gamma}t}\langle O\rangle_0 = O(\Delta\gamma_{\max}t)) + \mathcal{O}(\Delta\gamma_{\max}^2 t^2))$$

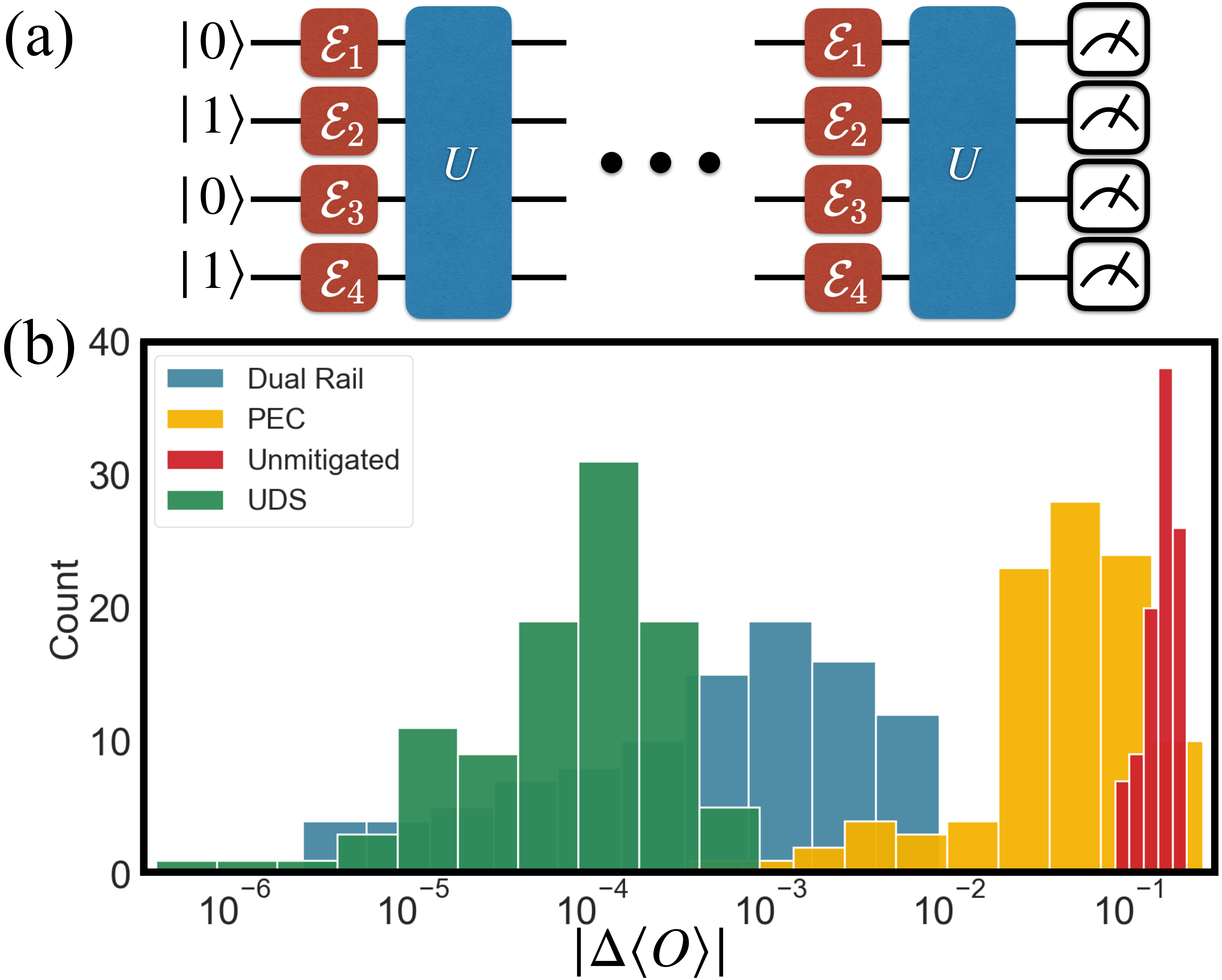
Evolve system n times, by shifting encoded basis

$$|P_{\boldsymbol{\alpha}}^{(1)}\rangle = |\alpha_1 \ldots \alpha_n\rangle \quad |P_{\boldsymbol{\alpha}}^{(2)}\rangle = |\alpha_n \ldots \alpha_{n-1}\rangle \quad \cdots \quad |P_{\boldsymbol{\alpha}}^{(n)}\rangle = |\alpha_2 \ldots \alpha_1\rangle$$

$$\frac{1}{n}\sum_{p=1}^{n}\left(\langle O\rangle_{UDS}^{(p)} - e^{-k\bar{\gamma}t}\langle O\rangle_0\right) = \cancel{\mathcal{O}(\Delta\gamma_{\max}t)} + \mathcal{O}(\Delta\gamma_{\max}^2 t^2))$$

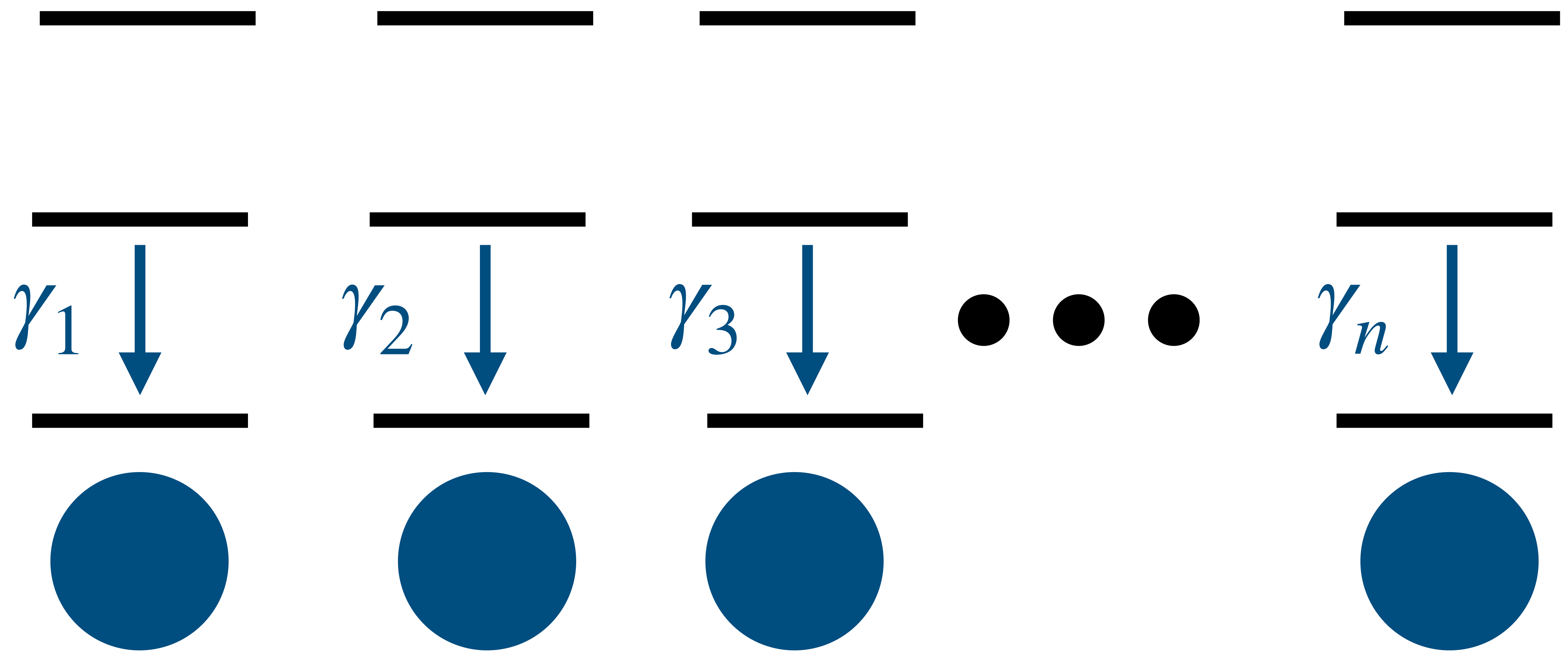


10^0
10^{-2}
10^{-4}
10^{-6}
$|\Delta\langle O\rangle|$
0
20
40
60
80
100
$t \longrightarrow$
Unmitigated
Dual Rail
UDS



(a)
$|0\rangle$
$|1\rangle$
$|0\rangle$
$|1\rangle$
$\mathcal{E}_1$
$\mathcal{E}_2$
$\mathcal{E}_3$
$\mathcal{E}_4$
U
(b)
Dual Rail
PEC
Unmitigated
UDS
Count
0
10
20
30
40
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$|\Delta\langle O\rangle|$

Summary



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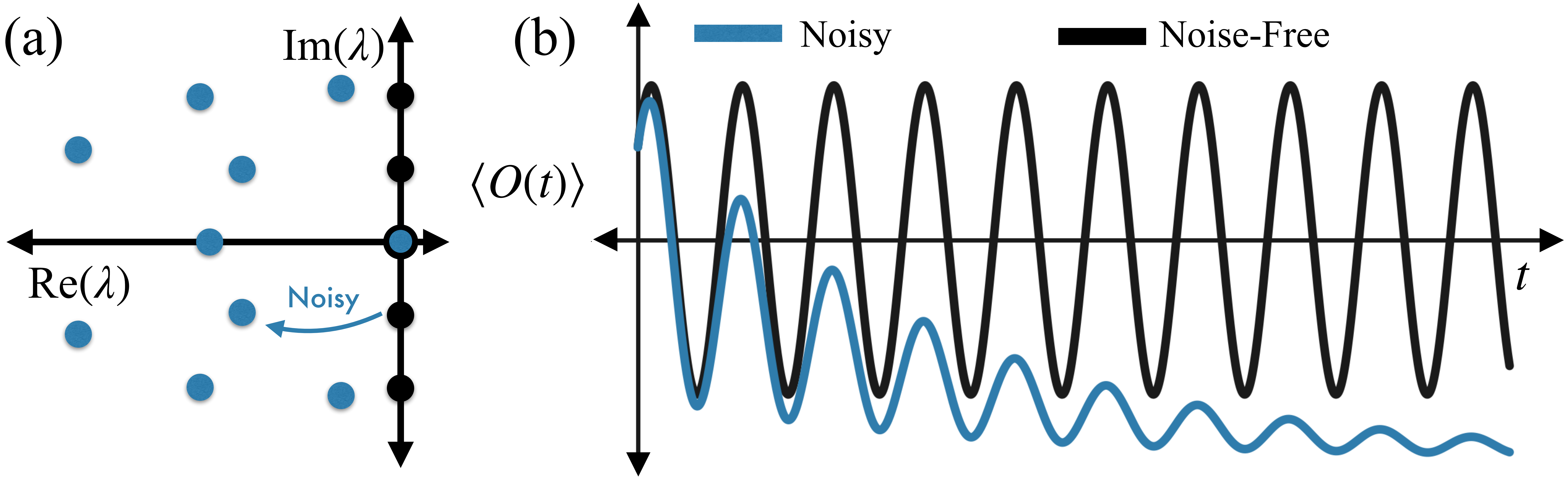
(a)
Im(λ)
Re(λ)
Noisy
(b)
Noisy
Noise-Free
$\langle O(t) \rangle$
t