

# How to Train Your Jacobian: Adaptive algorithms for high-contrast wavefront sensing & control

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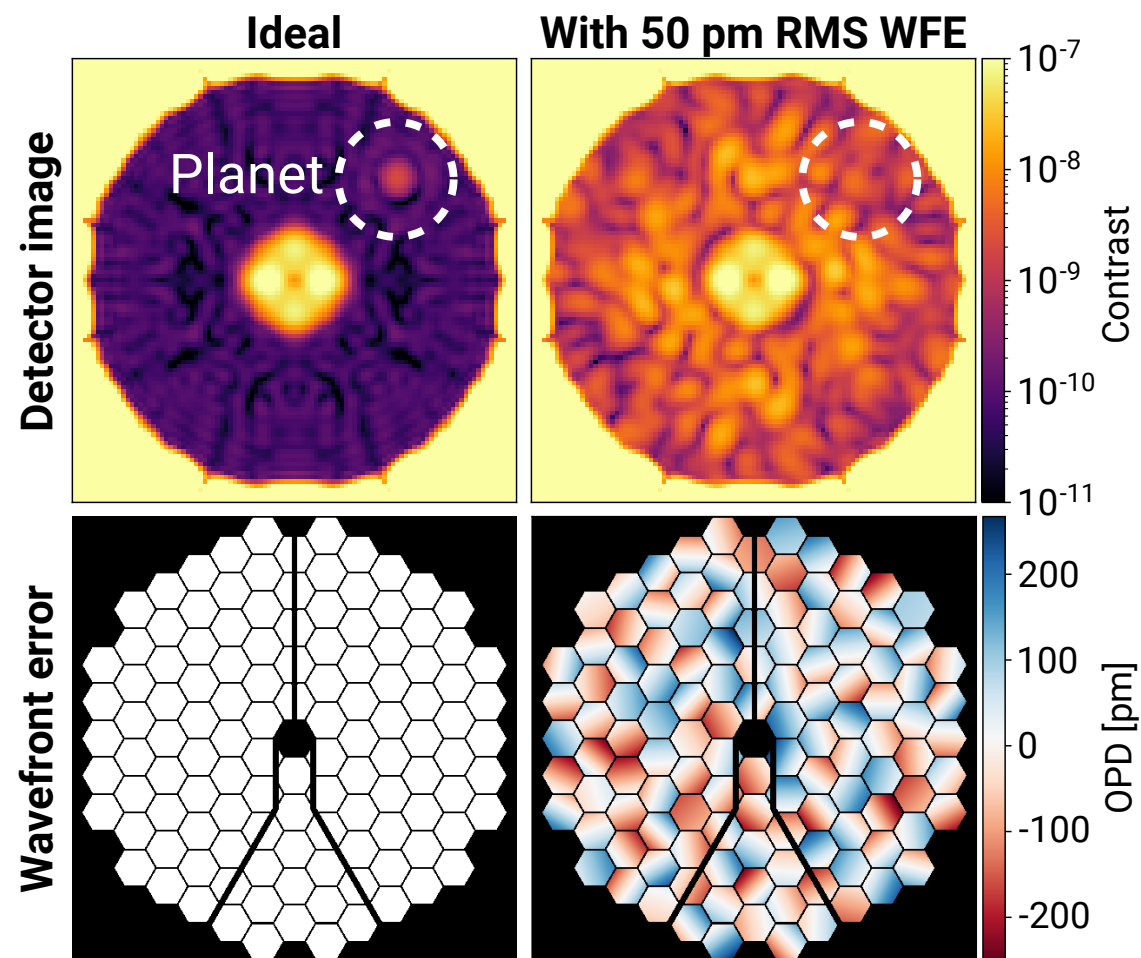
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SPIE Astronomical Telescopes + Instrumentation

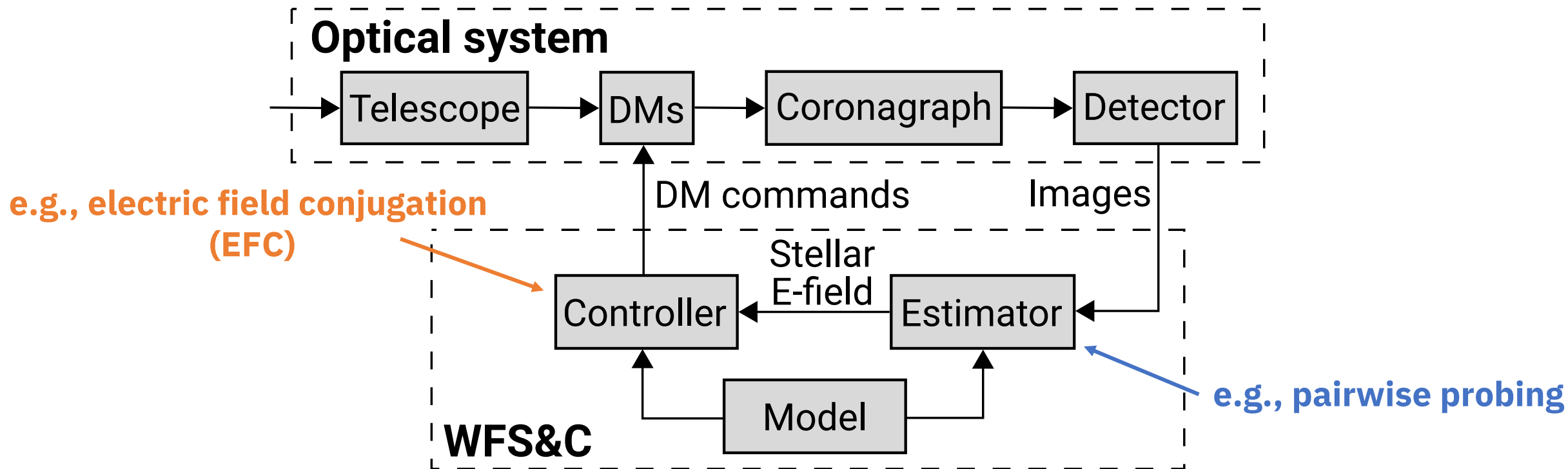
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# Coronagraphy: the effects of wavefront error

- Space-based coronagraphs searching for Earth-like exoplanets will need to suppress starlight to  $\lesssim 10^{-10}$  of its peak value at separation values  $\lesssim 0.1$  arcsec
- **But:** speckle noise generated by aberrations intrinsic to coronagraph optics is brighter than planetary image
- **The solution:** closed-loop wavefront sensing and control with deformable mirrors (DMs)



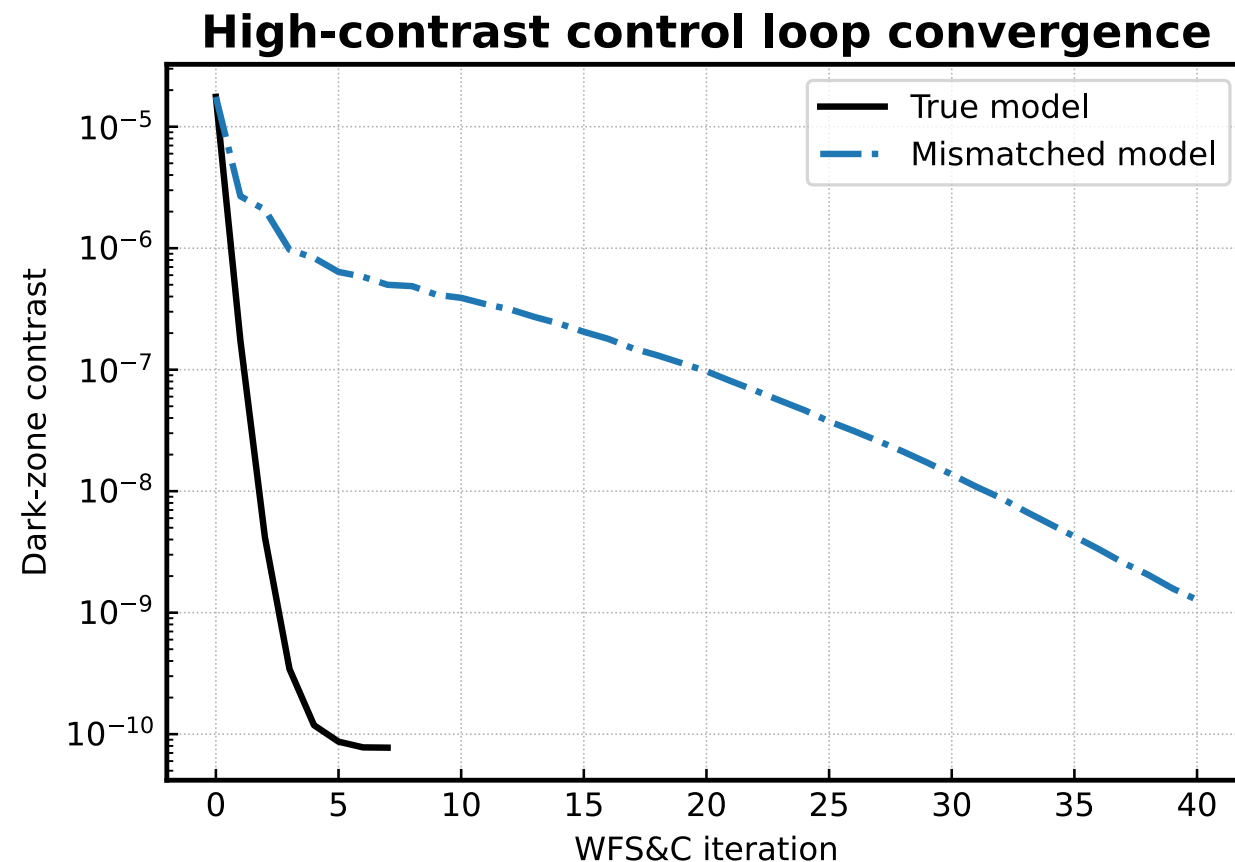
# Model-based wavefront sensing & control



- Alternate between two basic steps:
  - 1. Estimate** electric field from star inside dark zone (**estimation step**)
  - 2. Update** DM commands to minimize E-field (**control step**)
- With **model-based algorithms** (e.g., pairwise probing, EFC), an *a priori* Fourier optics model of coronagraph is used to calculate E-field estimate/optimal DM correction

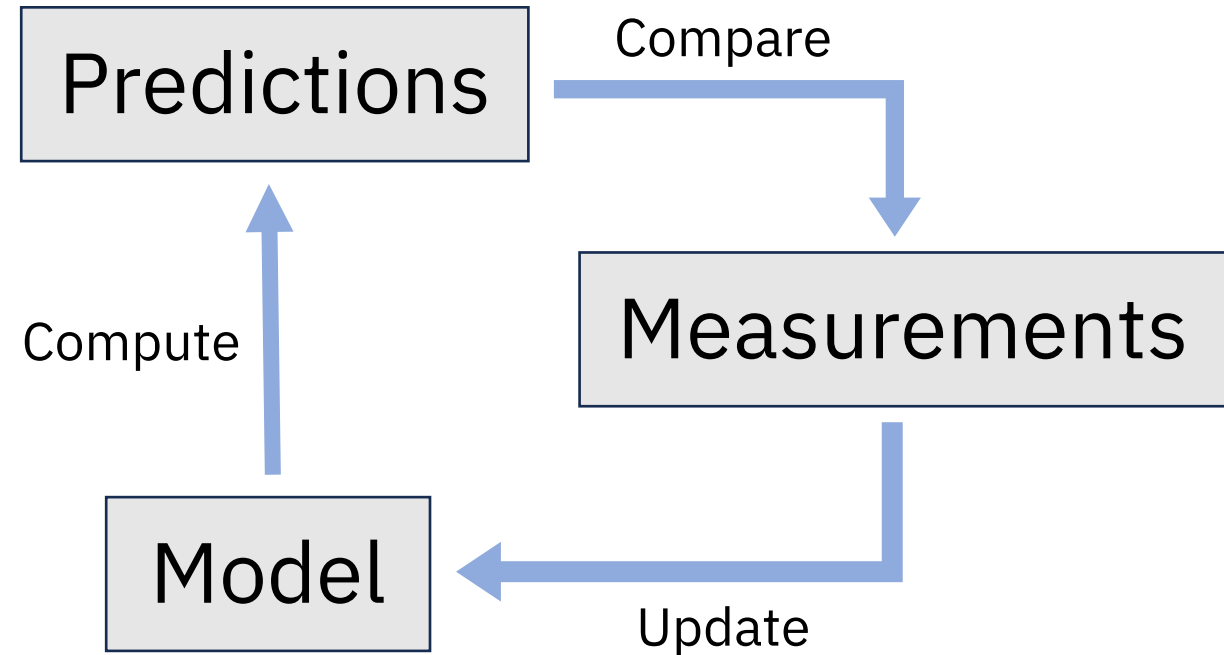
# Model error

- Computer models are imperfect descriptions of reality. The optimal DM correction (according to model)  $\neq$  true optimal correction.
- The total number of time steps required to reach  $10^{-10}$  contrast increases
- **Any additional time spent digging dark zones reduces the time available for science observations. Over the lifetime of a mission, this adds up!**
- **Can we use focal-plane measurements to improve the coronagraph model in situ?**



# System identification for coronagraphy

- Discrepancies between **predictions** (based on the coronagraph model) and **measured data** are due to a combination of model error and random noise
- By systematically adjusting the coronagraph model to reduce these discrepancies, the coronagraph model can be improved (**adapted**) to match the true system



# System identification for coronagraphy

- The focal-plane electric field evolves according to a linear state-space model at each pixel:



Electric field

Jacobian

Control update  
(from EFC)

$$\mathbf{E}_k = \mathbf{E}_{k-1} + \mathbf{\Gamma} \mathbf{u}_k + \mathbf{w}_k \quad (1)$$

$$\Delta \mathbf{I}_k = \mathbf{H} \mathbf{E}_k + \mathbf{n}_k \quad (2)$$

Random noise

Pairwise probe  
images

Pairwise probe observation matrix,  $\mathbf{H} = 4(\mathbf{\Gamma} \mathbf{\Psi})^T$

Matrix of probe commands

- Interpretation:** the electric field changes in response to DM commands and random perturbations. We observe it indirectly through noisy images.

# System identification for coronagraphy



$$\mathbf{E}_k = \mathbf{E}_{k-1} + \mathbf{\Gamma} \mathbf{u}_k + \mathbf{w}_k \quad (1)$$

$$\Delta \mathbf{I}_k = \mathbf{H} \mathbf{E}_k + \mathbf{n}_k \quad (2)$$

- **Problem:** given **inputs** ( $\mathbf{u}_k$ ) and **outputs** ( $\Delta \mathbf{I}_k$ ) for a series of time steps, estimate **responses** ( $\mathbf{\Gamma}$ )
- **Prior work:** H. Sun et al., using expectation-maximization (EM) algorithm<sup>1</sup>
- **This work:** a simple least-squares approach
  - Straightforward to implement, few tuning parameters
  - Less computationally demanding than EM

<sup>1</sup>Sun, Kasdin, and Vanderbei, JATIS 4(4), 049006 (2018)

# System identification for coronagraphy

$$\mathbf{E}_k = \mathbf{E}_{k-1} + \mathbf{\Gamma} \mathbf{u}_k + \mathbf{w}_k \quad (1)$$

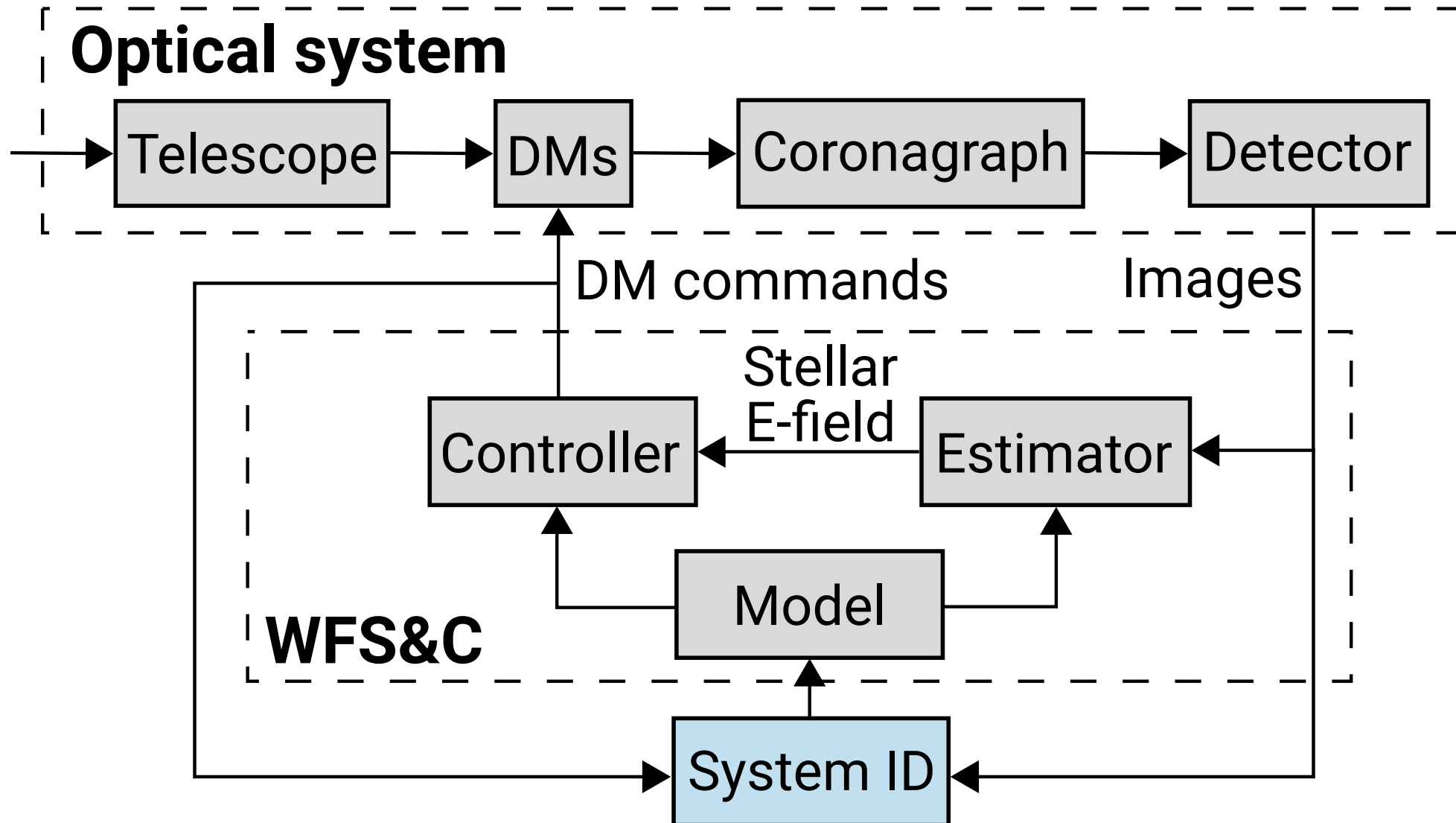
$$\Delta \mathbf{I}_k = \mathbf{H} \mathbf{E}_k + \mathbf{n}_k \quad (2)$$

- **Intuition:** given a change  $\mathbf{u}_k$  to the DM commands,  $\mathbf{\Gamma}$  should be able to predict the resulting change in the electric field  $\mathbf{E}_k$ . We can't measure  $\mathbf{E}_k$  directly (the estimate depends on  $\mathbf{\Gamma}$ ), but we **can** measure the corresponding pairwise difference images  $\Delta \mathbf{I}_k$ .

- Rearrange (1):
$$\mathbf{E}_k - \mathbf{E}_{k-1} = \mathbf{\Gamma} \mathbf{u}_k + \text{noise}$$
- Multiply by  $\mathbf{H}$ :
$$\mathbf{H} \mathbf{E}_k - \mathbf{H} \mathbf{E}_{k-1} = \mathbf{H} \mathbf{\Gamma} \mathbf{u}_k + \text{noise}$$
- Insert (2):
$$\Delta \mathbf{I}_k - \Delta \mathbf{I}_{k-1} = \mathbf{H} \mathbf{\Gamma} \mathbf{u}_k + \text{noise}$$
- Minimize over many time steps to estimate  $\mathbf{\Gamma}$ :

$$\widehat{\mathbf{G}} = \arg \min_{\mathbf{\Gamma}} \sum_k \|\Delta \mathbf{I}_k - \Delta \mathbf{I}_{k-1} - \mathbf{H} \mathbf{\Gamma} \mathbf{u}_k\|^2 = \arg \min_{\mathbf{\Gamma}} \sum_k \left\| \Delta \mathbf{I}_k - \Delta \mathbf{I}_{k-1} - \underbrace{4 \mathbf{\Psi}^T \mathbf{\Gamma}^T}_{\mathbf{H}} \mathbf{\Gamma} \mathbf{u}_k \right\|^2$$

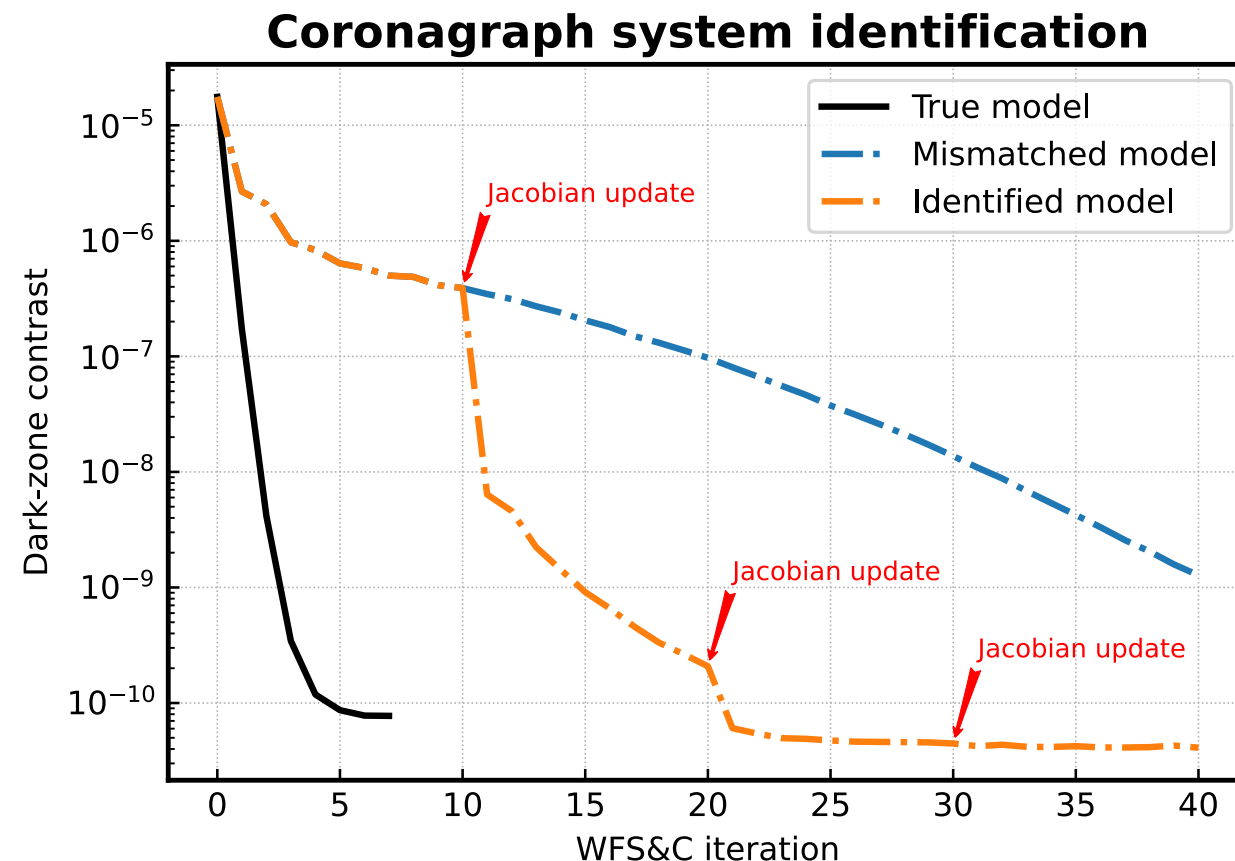
# System identification for coronagraphy



- 1. Open-loop:** apply many random commands to the DMs, take pairwise probing images for each random command
  - The number of random inputs required to fully characterize the Jacobian is proportional to  $\min\{N_{\text{actuator}}, N_{\text{pixel}}\}$
- 2. Closed-loop:** estimate Jacobian from closed-loop control data (e.g., EFC + pairwise probing)
  - Prior work and simulations suggest that Jacobian can be improved using many fewer time steps than degrees of freedom
  - By updating the Jacobian periodically, the WFS&C loop can adapt to temporal drifts and disturbances

# Simulations: closed-loop identification

- LUVUOIR-A<sup>†</sup> APLC<sup>‡</sup> design
- Two 64×64 DMs, out-of-pupil DM at flight-like separation ( $\Delta z = 800$  mm)
- 10 nm RMS wavefront error in coronagraph entrance pupil
- Model mismatch represented by 20% knowledge error on DM actuator gain coefficients
- Peak signal-to-noise of probe images  $\sim 55$  in dark zone

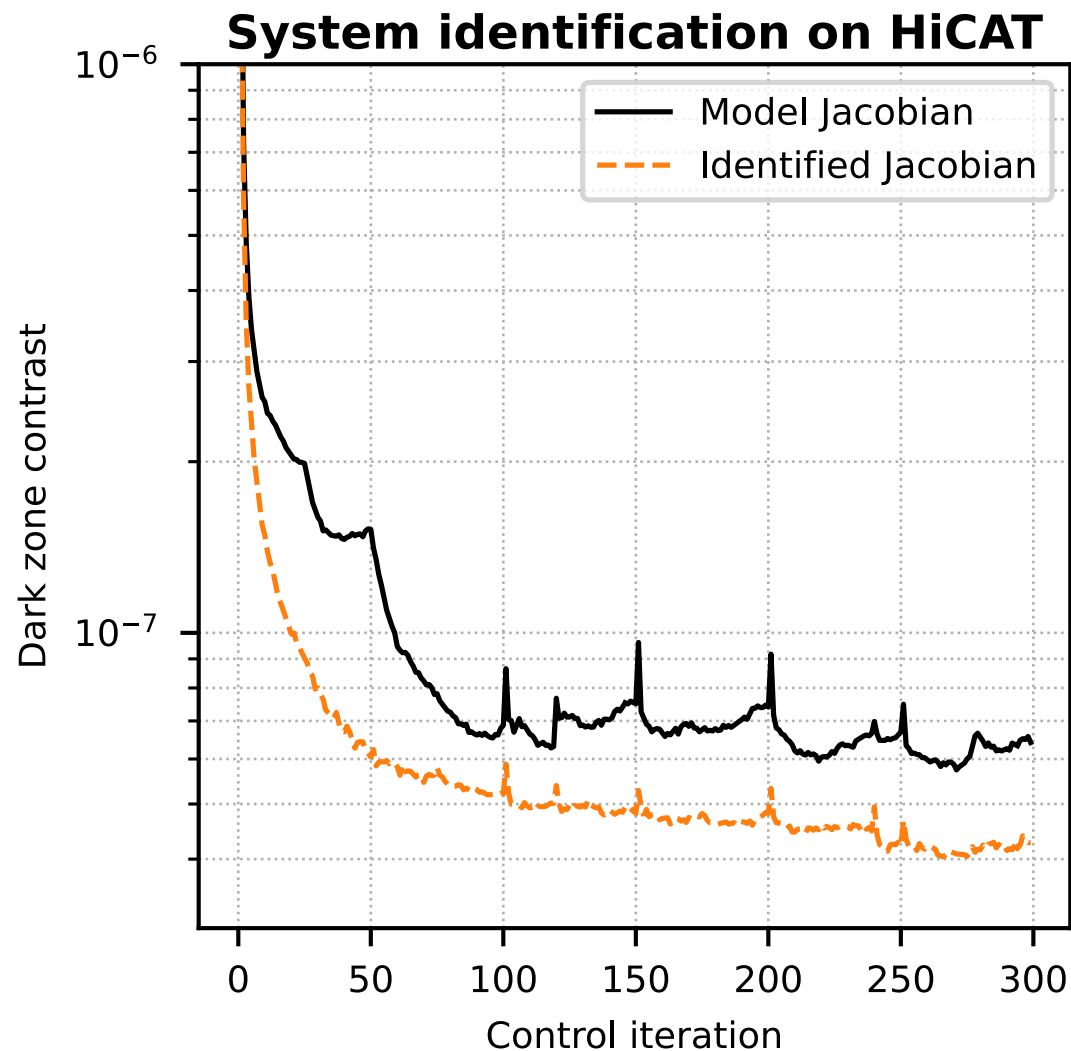


<sup>†</sup> Large UV/Optical/Infrared Surveyor (LUVUOIR), Architecture “A”

<sup>‡</sup> Apodized Pupil Lyot Coronagraph

# Experiments: open-loop identification

- High Contrast Imager for Complex Aperture Telescopes (HiCAT) at Space Telescope Science Institute
- Two 34x34 Boston Micromachines kilo-DMs (1904 actuators total)
- $\mathbf{u}_k$ : Gaussian white noise (5,000 time steps total)
- $\Psi$ : single-actuator probe commands (4 per time step)
- $\Delta \mathbf{I}_k$ : monochromatic at 638 nm
- Compared EFC + pairwise probing with model-based Jacobian vs. identified Jacobian
- Final contrast improved by ~50% ( $6.4 \times 10^{-8}$  vs.  $4.3 \times 10^{-8}$ )



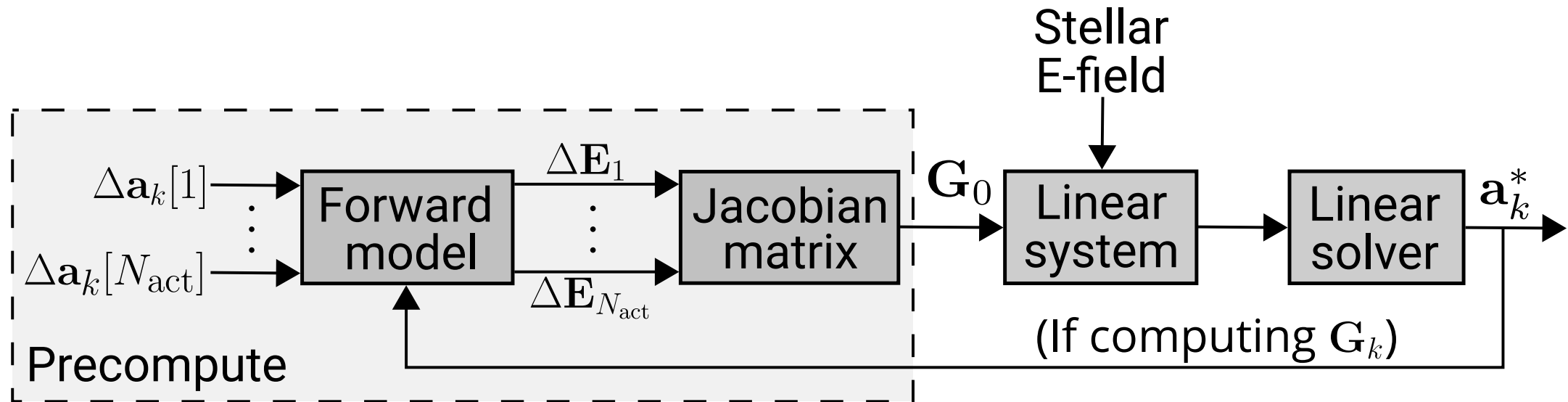
# Conclusions and next steps



- System identification using simple least-squares approaches have the potential to enhance the starlight suppression performance of space-based coronagraphs
- Short-medium term priorities:
  - **Continued experiments** on HiCAT with both open-loop and closed-loop identification
  - **Jacobian-free system identification:** learn coronagraph model parameters instead of Jacobian matrix to improve performance of Jacobian-free control algorithms
- Areas for future study:
  - Performance with low-photon data
  - Dimensionality reduction techniques
  - Studies for Habitable Worlds Observatory: performance simulations, update frequency/drift tracking

# Backup slides

# Wavefront control with Jacobian matrices



- **Using Fourier optics coronagraph model:** perturb each DM actuator individually and ***predict*** the resulting change in E-field at detector plane
- Collect responses into a matrix (one column per actuator)  $\Rightarrow$  **Jacobian matrix**
- Given Jacobian and E-field estimate, calculate DM correction that optimally conjugates E-field  $\Rightarrow$  **electric field conjugation (EFC)**