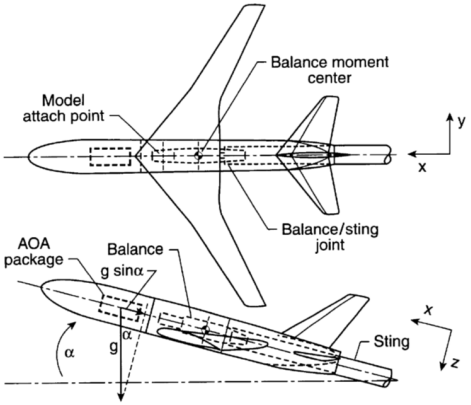



# Improvements to Calibration Modeling of a Tri-Axial Accelerometer Using Response Surface Methods

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March 28th, 2024
Hiroshima, Japan

# Introduction




Force and angle are two principal measurements used to evaluate aerodynamic performance.

Six-component force transducers (balances) provide direct model load measurement.

Angle is measured using a variety of methods:

- Onboard Accelerometers
- Support structure angle measurement (Accelerometers, potentiometers, LVDTs, etc.)
- Photogrammetry

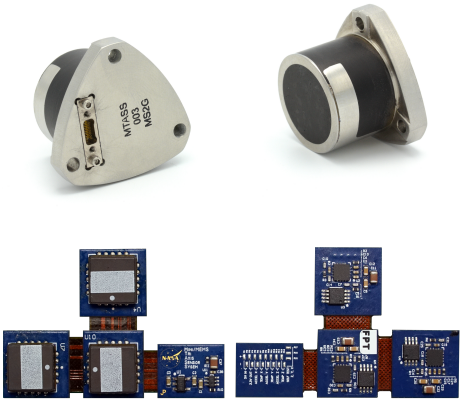
Aircraft performance testing typically requires a pitch accuracy of $\pm 0.01^{\circ}$.

# Mini/MEMs Tri-Axis Sensor System (MTASS)

High-speed facilities typically use small test articles, where current sensors cannot be accommodated.

MTASS was developed to create a small, accurate, single- or tri-axis accelerometer to close this technology gap.

Leveraged existing commercial-off-the-shelf (COTS) solutions with custom support electronics and printed circuit boards.




Two fabricated MTASS units with PCB assembly potted inside (Top). Internal PCB with COTS accelerometers (Bottom).

# Tunnel Temperature Conditions

During normal testing, temperatures of approximately 38 °C (100 °F) can be realized, and some facilities may produce fluid temperatures up to 149 °C (300 °F).

Moreover, several hyper-velocity facilities can reach total temperatures of 694 °C (1280 °F).

Sensor temperature of 38 °C (100 °F) was observed during previous demonstration.

Over a 10° tilt, temperature induced angular error of up to 0.20° can occur for high-performance accelerometers.

# Calibration Model

Previous calibration using linear modeling displayed two inflection points in residual structure.

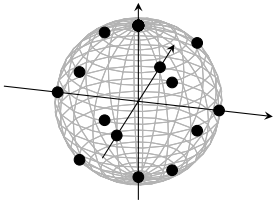
Model was expanded to include; third order terms, two-factor interaction and non-linear temperature effects.

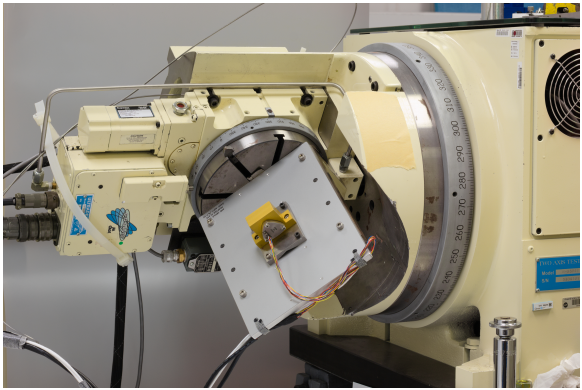
$$v_i = \beta_0 + \beta_{0T}\,T + \beta_{0TT}\,T^2 + \sum_{i=1}^{3} \beta_{iT} g_i\,T + \sum_{i=1}^{3} \beta_{iTT} g_i\,T^2 + \sum_{i=1}^{3} \beta_i g_i + \sum_{i=1}^{3}\sum_{j=i}^{3} \beta_{ij} g_i g_j + \sum_{i=1}^{3} \beta_{iii} g_i^3 + \epsilon$$

# Room Temperature Calibration

Calibrations are preformed on two-axis index table with 2 arcsecond repeatability.

Spherical-central-composite experimental design was used to support third-order model.





View of the MTASS on the two-axis index head during calibration.

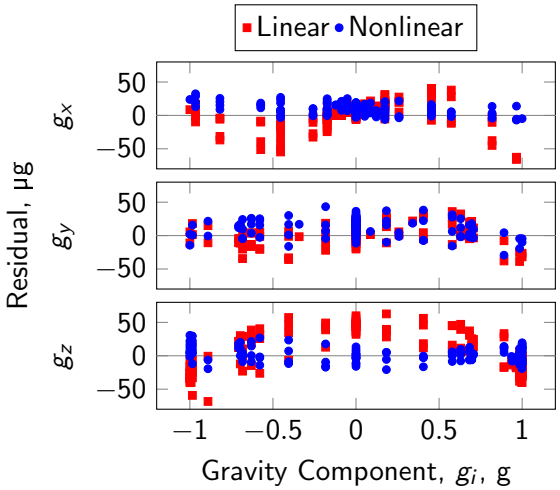
# Room Temperature Calibration Results

Repeated, independent, calibration was performed with additional points to represent operational use.

Residual errors for x- and z-accelerometers displayed approximately 60% reduction.

Room Temperature Residual Standard Deviations:

| **Component** | $g_x$ | $g_y$ | $g_z$ |
|---|---|---|---|
| **Linear,** µg | 22.1 | 13.7 | 29.9 |
| **Nonlinear,** µg | 8.3 | 12.5 | 11.0 |
| **Percent Diff.** | -63% | -9% | -63% |



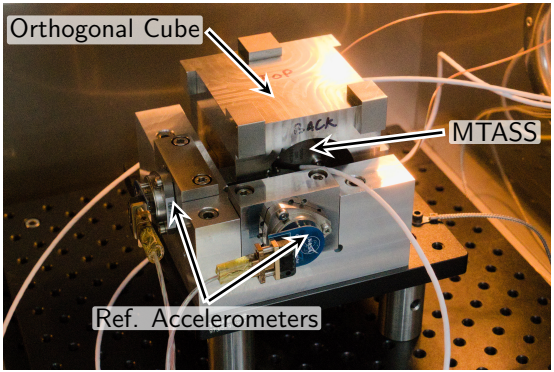
# Elevated Temperature Calibration

Near orthogonal cube, placed each axis at three levels (+1g, 0g, and -1g).

Three temperature levels (21 °C (70 °F), 54 °C (130 °F), and 82 °C (180 °F)) to support up to second-order terms:

- $\beta_{0T}$
- $\beta_{0TT}$
- $\beta_{iT}$
- $\beta_{iTT}$

Temperature compensated accelerometers were used to monitor oven attitude in both axes.




MTASS package in an oven for temperature testing with two reference accelerometers to monitor oven attitude.

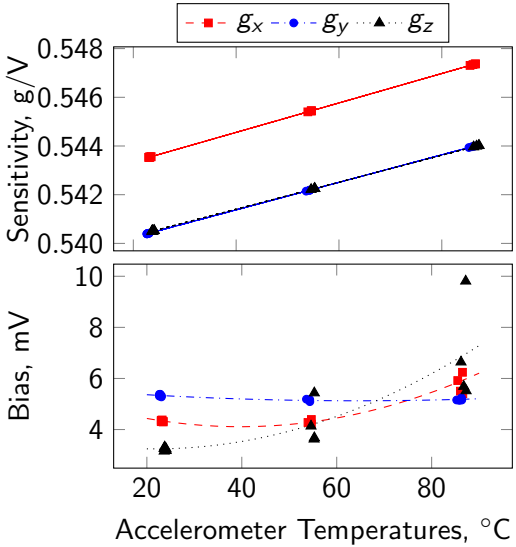
# Temperature Results

Four (4) consecutive elevated temperature runs were conducted.

Calibration coefficients estimated at each temperature.

Sensitivities were found to be linear, allows for removing $\beta_{iTT}$ term.

Sensor bias was found to be non-linear, with increased variance on z-axis sensor.



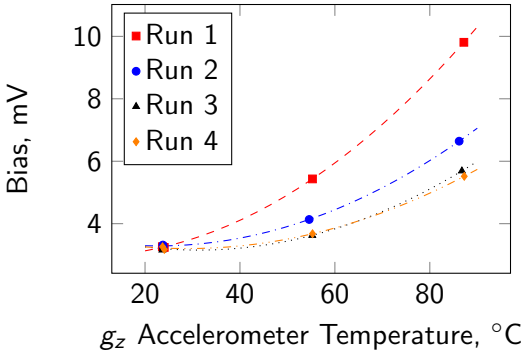
# Z-Axis Accelerometer Shifts

Z-axis accelerometer displayed shifts to the bias-temperature curve for each run.

After three (3) thermal cycles, changes to the bias-temperature curve stabilized.

Y and x axes displayed some changes, however, curve shifts were minimal.

The following analysis only uses runs 3 and 4.



# Full Calibration Model

Coefficient estimates from both room and elevated temperature experiments are combined into a full model:

$$v_i = \beta_0 + \beta_{0T} T + \beta_{0TT} T^2 + \sum_{i=1}^{3} \beta_{iT} g_i T + \sum_{i=1}^{3} \beta_i g_i + \sum_{i=1}^{3} \sum_{j=i}^{3} \beta_{ij} g_i g_j + \sum_{i=1}^{3} \beta_{iii} g_i^3 + \epsilon$$

$$\beta_i^{Adjusted} = \beta_i^{RT} - \beta_{iT} T_{RT}$$
$$\beta_0^{Adjusted} = \beta_0^{RT} - \beta_{0T} T_{RT} - \beta_{0TT} T_{RT}^2$$

| Term | $v_x$ | $v_y$ | $v_z$ |
|---|---|---|---|
| $\beta_0$ | 6.56E-03 | 5.80E-03 | 5.29E-03 |
| $\beta_x$ | 5.41E-01 | 9.89E-04 | 5.07E-03 |
| $\beta_y$ | 1.04E-03 | 5.38E-01 | -1.99E-03 |
| $\beta_z$ | 1.43E-03 | -5.58E-03 | -5.38E-01 |
| $\beta_{xx}$ | 4.20E-05 | | |
| $\beta_{xy}$ | 2.54E-05 | -1.01E-05 | |
| $\beta_{xz}$ | 1.02E-05 | | -6.40E-05 |
| $\beta_{yy}$ | | 1.82E-06 | |
| $\beta_{yz}$ | | -1.05E-05 | -1.07E-05 |
| $\beta_{zz}$ | | | -5.04E-05 |
| $\beta_{xxx}$ | 3.43E-05 | | |
| $\beta_{yyy}$ | | 3.13E-05 | |
| $\beta_{zzz}$ | | | 2.43E-05 |
| $\beta_{0_x T}$ | -4.71E-05 | | |
| $\beta_{0_x TT}$ | 2.21E-07 | | |
| $\beta_{Tx}$ | 3.42E-05 | | |
| $\beta_{0_y T}$ | | -9.10E-06 | |
| $\beta_{0_y TT}$ | | 2.89E-08 | |
| $\beta_{Ty}$ | | 3.16E-05 | |
| $\beta_{0_z T}$ | | | -4.59E-05 |
| $\beta_{0_z TT}$ | | | 2.56E-07 |
| $\beta_{Tz}$ | | | -3.05E-05 |

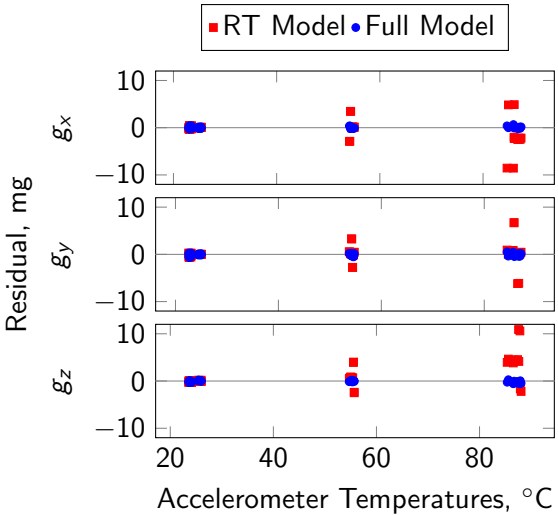
# Calibration Results

Significant improvement in full model over the room temperature only. One order of magnitude improvement.

Residual standard deviations from temperature data is 20 times larger than room temperature.

Standard deviations from temperature calibration:

| **Component** | $g_x$ | $g_y$ | $g_z$ |
|---|---|---|---|
| **RT Model,** mg | 2.17 | 1.91 | 2.57 |
| **Full Model,** mg | 0.21 | 0.27 | 0.18 |



# Conclusions

Higher order modeling improved residuals on two channels by 60%. Standard deviations of residuals found to be as high as 12.5 µg at room temperature.

Use of a spherical-central-composite experimental design allowed estimation up to third-order terms.

Elevated temperature cycling revealed change in bias shift on z-axis accelerometer. However, behavior stabilized after third thermal cycle.

Full model was developed to include temperature effects and residuals displayed one order of magnitude improvement.

Pre-integration accelerometer evaluation could be used if linear model is desired and to maximize performance.