AN ORBIT DETERMINATION COMPARISON STUDY AND DEMONSTRATION FOR RENDEZVOUS AND DOCKING IN A NEAR RECTILINEAR HALO ORBIT FROM THE LUNAR SURFACE

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For the upcoming NASA Artemis III mission and those that follow, both the Human Landing System (HLS) and Orion programs are invested in understanding the impacts of ground tracking performance in supporting rendezvous and docking in a Near Rectilinear Halo Orbit (NRHO). Several critical questions must be answered to ensure mission success and crew safety and an assortment of analysis tools are being incorporated to address them. Two of these tools, LINCOV and MONTE, are currently providing program decision making results through HLS Insight, HLS NASA-collaborations, and Orion/Gateway cross-program analysis. To ensure consistency in the orbit determination performance, a comparison trade-study is performed using a low-lunar orbit to NRHO rendezvous scenario anticipated for the upcoming Artemis missions. An overview of the two analysis tools is provided along with a detailed step-by-step evaluation of the core capabilities and models related to the orbit determination process. This incremental comparison effort reveals both tools produce consistent solutions for the criteria investigated. Given the confidence in the orbit determination process and solutions generated, these results are then applied to demonstrate an integrated, closed-loop system performance where the HLS lander ascends from the lunar surface and successfully inserts into the NRHO relative to the Orion spacecraft in preparation for the final rendezvous and docking phase.

INTRODUCTION

To support rendezvous and docking missions, it is not unreasonable to focus on the relative navigation sensors such as LiDARs, optical navigation cameras providing relative angle measurements, and communication systems that produce relative range and range-rate information vital in determining the relative motion between two spacecraft. Even though those types of relative measurements play a critical role, another subtle yet key element to support rendezvous and docking in a Near Rectilinear Halo Orbit (NRHO) is the orbit determination of the absolute state of both the active chaser vehicle and the passive target vehicle. By taking the state vector difference of the ground tracking solution of both spacecraft, the relative state can be determined. Then, it becomes possible to target the necessary relative constraints to support an assortment of rendezvous and proximity operations.

For the upcoming NASA Artemis III mission and those that follow, both the Human Landing System (HLS) and Orion programs are invested in understanding the impacts of ground tracking performance in supporting rendezvous and docking in a Near Rectilinear Halo Orbit (NRHO). Several critical questions must be answered to ensure mission success and crew safety and an assortment of analysis tools are being incorporated to address them. Two of these tools, LINCOV and MONTE, are currently providing program decision making results through HLS Insight, HLS NASA-collaborations, and Orion/Gateway cross-program analysis. To ensure consistency in the orbit determination performance, this paper performs a comparison trade-study using a low-lunar orbit to NRHO rendezvous scenario anticipated for the upcoming Artemis missions.

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The first section provides an overview of the orbit determination performance metrics and a description of the two analysis tools. Then the NRHO rendezvous trajectory and concept of operations is introduced followed by a detailed step-by-step evaluation of the core capabilities and models related to the orbit determination process. Once this incremental comparison effort establishes confidence in the orbit determination process and solutions generated, these techniques are then applied to demonstrate an integrated, closed-loop system performance where the HLS lander ascends from the lunar surface and successfully inserts into the NRHO relative to the Orion spacecraft in preparation for the final rendezvous and docking phase. A summary of the developments and core results is then captured in the concluding section.

ORBIT DETERMINATION ANALYSIS APPROACH AND TOOLS

To introduce the different techniques and analysis tools to perform the ground tracking orbit determination results, there are several performance metrics that must be defined which include the true trajectory dispersions $\delta\mathbf{x}$, the navigation dispersions $\delta\hat{\mathbf{x}}$, the true navigation error $\delta\mathbf{e}$, and the onboard navigation error $\delta\hat{\mathbf{e}}$ as depicted in Figure 1. The true dispersions $\delta\mathbf{x}$ are defined as the difference between the true state $\mathbf{x}$ and the

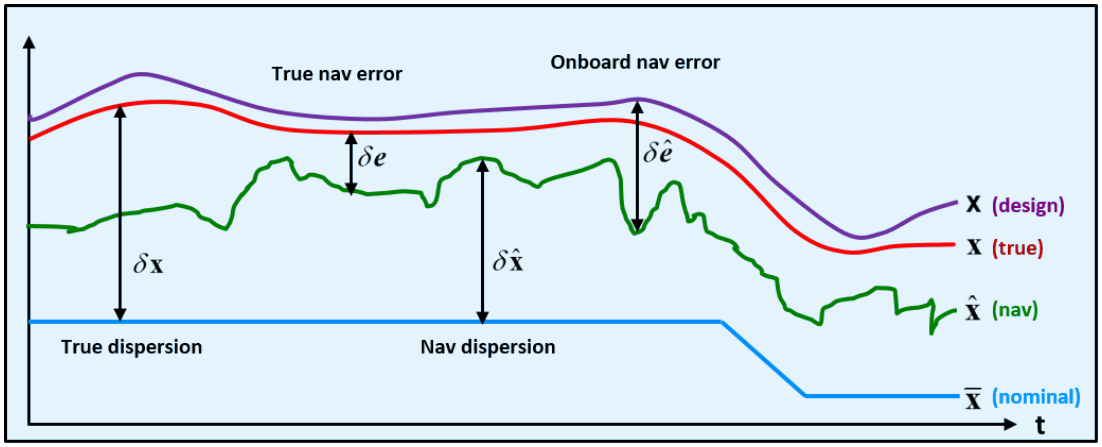


Figure 1. GN&C performance metric variables

nominal state $\bar{\mathbf{x}}$. The true state $\mathbf{x}$ is an n-dimensional vector that represents the *real world* environment or actual state.

$$\delta\mathbf{x} \triangleq \mathbf{x} - \bar{\mathbf{x}} \qquad\qquad \mathbf{D} = E\left[\delta\mathbf{x}\delta\mathbf{x}^{\mathrm{T}}\right] \tag{1}$$

The nominal state $\bar{\mathbf{x}}$ is also an n-dimensional vector that represents the desired or reference state. The covariance of the environment dispersions, $\mathbf{D}$, indicates how precisely the system can follow a desired trajectory.

The navigation dispersions $\delta\hat{\mathbf{x}}$ are defined as the difference between the navigation state $\hat{\mathbf{x}}$ and the nominal state $\bar{\mathbf{x}}$. The navigation state is an $\hat{n}$-dimensional vector ($\hat{n} < n$) that represents the filter's estimated state.

$$\delta\hat{\mathbf{x}} \triangleq \hat{\mathbf{x}} - \mathbf{M}_x\bar{\mathbf{x}} \qquad\qquad \hat{\mathbf{D}} = E\left[\delta\hat{\mathbf{x}}\delta\hat{\mathbf{x}}^{\mathrm{T}}\right] \tag{2}$$

The matrix $\mathbf{M}_x$ is an $(\hat{n} \times n)$ matrix that maps the estimated state in terms of the true and nominal state. The covariance of the navigation dispersions, $\hat{\mathbf{D}}$, reflect how precisely the onboard system thinks it can follow a prescribed reference trajectory.

The true navigation error $\delta\mathbf{e}$ is the difference between the environment and navigation states. It is also the difference between the environment and the navigation dispersions.

$$\delta\mathbf{e} \triangleq \mathbf{M}_x\mathbf{x} - \hat{\mathbf{x}} = \mathbf{M}_x\delta\mathbf{x} - \delta\hat{\mathbf{x}} \qquad\qquad \mathbf{P} = E\left[\delta\mathbf{e}\delta\mathbf{e}^{\mathrm{T}}\right] \tag{3}$$

The covariance of the true navigation error, $\mathbf{P}$, quantifies how precisely the onboard navigation system can estimate the actual state.

The onboard navigation error $\delta\hat{\mathrm{e}}$ itself is never computed, but it is used to develop the onboard navigation filter equations. It is defined as the difference between the design state, x, and the navigation state $\hat{\mathbf{x}}$.

$$\delta\hat{\mathbf{e}} \triangleq \mathrm{x} - \hat{\mathbf{x}} \qquad\qquad \hat{\mathbf{P}} = E\left[\delta\hat{\mathbf{e}}\delta\hat{\mathbf{e}}^{\mathrm{T}}\right] \tag{4}$$

The covariance of the onboard navigation error, $\hat{\mathbf{P}}$, quantifies how precisely the onboard navigation system expects it can determine the actual state. The performance of the onboard navigation system is determined by comparing $\hat{\mathbf{P}}$ to the actual navigation performance $\mathbf{P}$. If the *true* states and the *design* states are assumed to be the same, then the true navigation covariance will equal the onboard navigation covariance.

The covariances of the true dispersions, navigation dispersions, true navigation error, and the onboard navigation error are ultimately used to analyze and assess the performance of a proposed GN&C system. A common approach to obtain these performance metrics is to use a Monte Carlo simulation outlined in Figure 2, where the sample statistics of hundreds or thousands of runs, N, are used to numerically compute the desired covariance matrices.

$$\mathbf{D} = \frac{1}{N-1}\sum \delta\mathbf{x}\delta\mathbf{x}^{\mathrm{T}} \qquad \hat{\mathbf{D}} = \frac{1}{N-1}\sum \delta\hat{\mathbf{x}}\delta\hat{\mathbf{x}}^{\mathrm{T}} \qquad \mathbf{P} = \frac{1}{N-1}\sum \delta\mathbf{e}\delta\mathbf{e}^{\mathrm{T}} \tag{5}$$

The onboard navigation error covariance $\hat{\mathbf{P}}$ is the navigation filter covariance for each run. This same statistical information can be obtained using linear covariance analysis techniques.

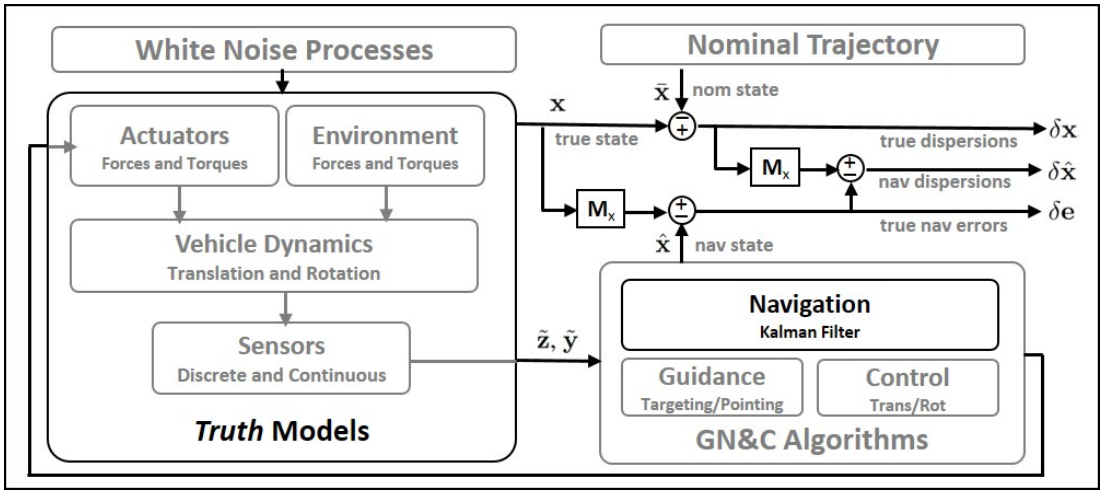


Figure 2. Extracting GN&C performance metrics using Monte Carlo techniques

LINCOV

LINCOV or the linear covariance analysis tool (LinCov) utilizes augmented and onboard state covariance matrices to produce the statistical performance metrics of a closed-loop GN&C system in a single-run, compared to hundreds or thousands of runs typically required for Monte Carlo analysis. Linear covariance analysis incorporates the non-linear system dynamics models and GN&C algorithms to generate a nominal reference trajectory $\bar{\mathbf{x}}$ which is then used to propagate, update, and correct an onboard navigation covariance matrix $\hat{\mathbf{P}}$ and an augmented state covariance matrix $\mathbf{C}$,

$$\mathbf{C} = E\left[\delta\mathbf{X}\delta\mathbf{X}^{\mathrm{T}}\right] \tag{6}$$

where the augmented state $\delta\mathbf{X}^{\mathrm{T}} = [\delta\mathbf{x}^{\mathrm{T}}\ \delta\hat{\mathbf{x}}^{\mathrm{T}}]$ consists of the true dispersions and the navigation dispersions. Pre- and post-multiplying the augmented state covariance matrix by the following mapping matrices, the

covariances for the trajectory dispersions, navigation dispersions, and the navigation error can be obtained.

$$\begin{aligned} \mathbf{D} &= [\,\mathbf{I}_{n\times n},\ \mathbf{0}_{n\times\hat{n}}\,]\,\mathbf{C}\,[\,\mathbf{I}_{n\times n},\ \mathbf{0}_{n\times\hat{n}}\,]^{\mathrm{T}} \\ \hat{\mathbf{D}} &= [\,\mathbf{0}_{\hat{n}\times n},\ \mathbf{I}_{\hat{n}\times\hat{n}}\,]\,\mathbf{C}\,[\,\mathbf{0}_{\hat{n}\times n},\ \mathbf{I}_{\hat{n}\times\hat{n}}\,]^{\mathrm{T}} \\ \mathbf{P} &= [\,\mathbf{I}_{\hat{n}\times n},\ -\mathbf{I}_{\hat{n}\times\hat{n}}\,]\,\mathbf{C}\,[\,\mathbf{I}_{\hat{n}\times n},\ -\mathbf{I}_{\hat{n}\times\hat{n}}\,]^{\mathrm{T}} \end{aligned} \tag{7}$$

For details regarding the development and implementation of the linear covariance simulation, see the following references [1–3].

The LinCov tool used for the HLS and Orion programs is developed and maintained by the Johnson Space Center community and has been utilized for multiple space programs for almost two decades for all critical flight phases. The LinCov ground tracking models [4] have been utilized to support several programs, including Artemis I analysis. The remainder of this subsection will contain the history and programs that have utilized this specific implementation of LinCov and provide an overview of the ground tracking models utilized, their origin, and heritage is depicted in Figure 3.

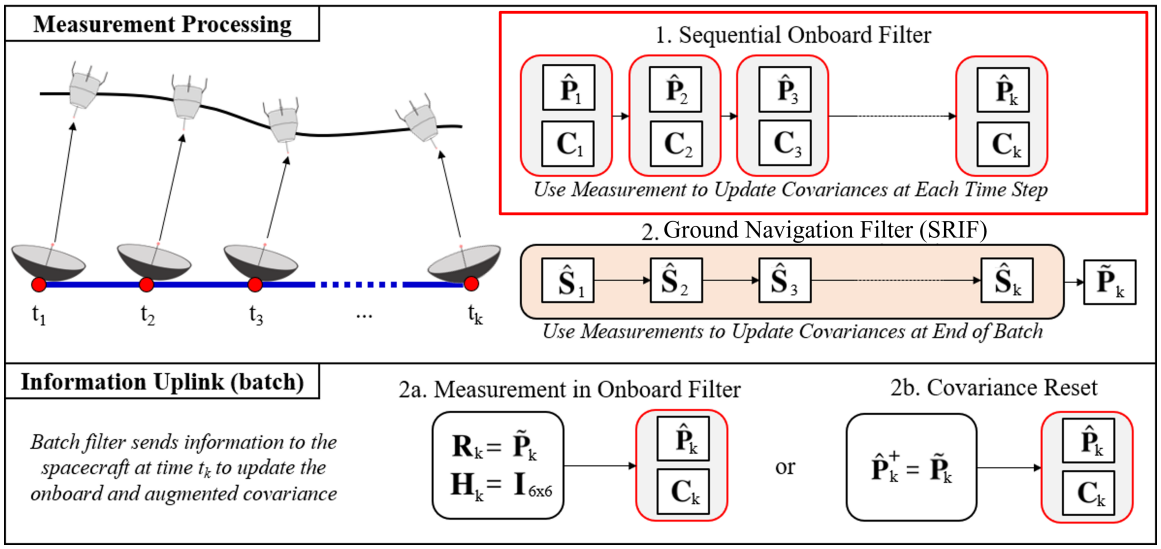


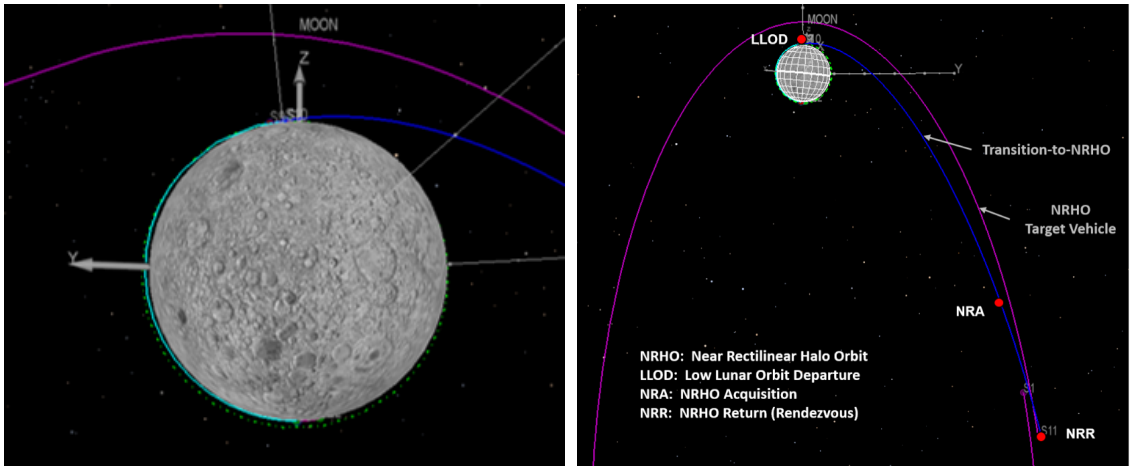
Figure 3. Notional ground tracking modeling options for the orbit determination problem

MONTE

MONTE is a state-of-the-art astrodynamics Python library produced by the Jet Propulsion Laboratory (JPL) [5,6]. It supports all phases of space mission development from early space design and analysis through flight navigation services and has over a decade of operational use on NASAs deep space robotic missions. It supplies an infrastructure for trajectory models, coordinate frames, timing, events, sensitivity analysis, numerical integrators, and optimization. The Johnson Space Center has determined MONTE will be used to support future Artemis II+ missions for orbit determination. The MONTE-based tool at the Johnson Space Center (JSC) utilized by the Flight Operations Directorate (FOD) is known as MANDO. This section will outline details related to MONTE, the Mission Analysis Operations and Navigation Toolkit Environment, and the orbit determination capabilities, historical usage, and modeling assumptions and capabilities.

NRHO RENDEZVOUS FROM LOW LUNAR ORBIT

This section will outline the basic concept of operations related to the NRHO rendezvous scenario from low lunar orbit that is utilized for the orbit determination comparison trade study as illustrated in Figure 4. For the comparison study, the Ascent-to-NRHO rendezvous trajectory starts with a 100 x 15 km orbit insertion from the lunar surface. A single revolution low-lunar orbit departure trajectory has been provided with four



(a) Low Lunar Orbit Departure

(b) Coasting Phase from LLO to NRHO

Figure 4. Notional HLS lunar surface to NRHO return rendezvous scenario for Artemis III

correction burns: a circularization burn to place the trajectory into a 100 km circular orbit, a Low Lunar Orbit Departure (LLOD) burn to depart from LLO, an NRA between LLOD and the NRHO return, and an NRHO return (NRR) burn. The LLOD and NRA burns target a relative off-set from the target vehicle at the NRHO return (NRR) point representing the transition location when the HLS vehicle becomes the passive target vehicle. The NRR burn nulls the relative velocity.

Ground Tracking Navigation Parameters

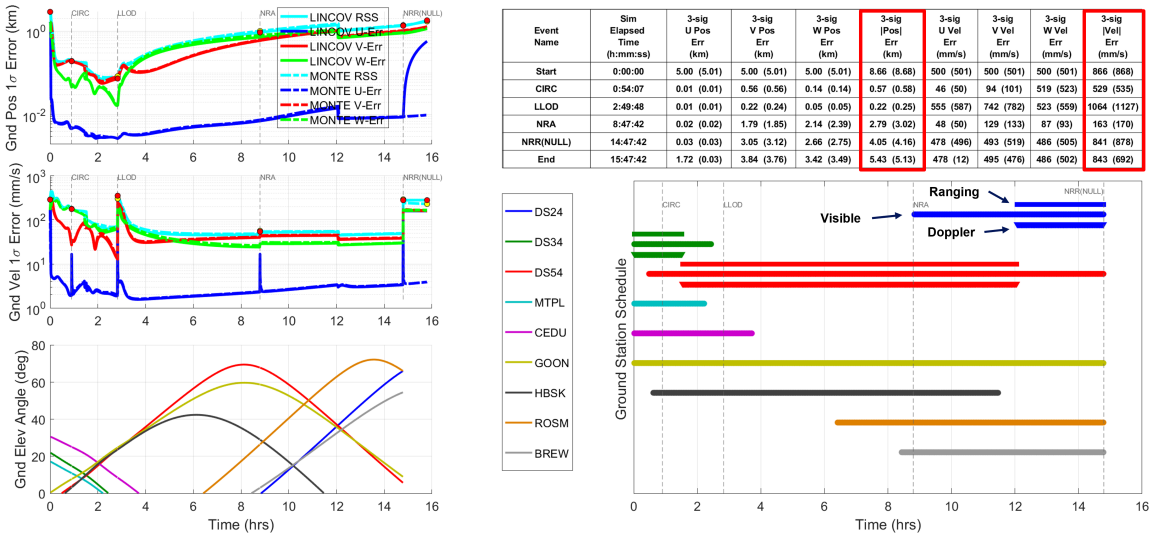
This subsection will have the performance specifications assumed for the HLS ground tracking. This includes the doppler and ranging noise and bias terms along with what specific ground stations are utilized in the study.

Vehicle Performance Assumptions

This subsection will contain a summary of the maneuver execution errors, disturbance accelerations, and initial condition assumptions made that are utilized in the comparison trade study.

MONTE / LINCOV COMPARISON TRADE STUDY RESULTS

This section will give a generic overview of the major objectives with the MONTE and LINCOV comparison trade study and the core capabilities of the ground tracking models that are activated in incremental steps to highlight the performance evaluation. These basic capabilities include: 1) ground station visibility, 2) disturbance accelerations, 3) initial navigation errors, 4) doppler noise and bias, 5) ranging noise and bias, 6) maneuver execution errors, 7) ground station location biases, etc. As shown in Figure 5, the time history of the absolute position and velocity navigation errors as predicted by both the LINCOV and MONTE ground based models are provided in the top left plots. The numerical comparison of these same performance metrics are provided in tabular format in the top right of the image where the values in parenthesis are those derived from the MONTE tool and those without are derived from LINCOV. The elevation angle from each ground station to the HLS vehicle is shown in the bottom left plot and the ground station availability and ground station selection is highlighted in the bottom right plot which indicates which doppler and ranging measurements are processed at which time from which ground station.

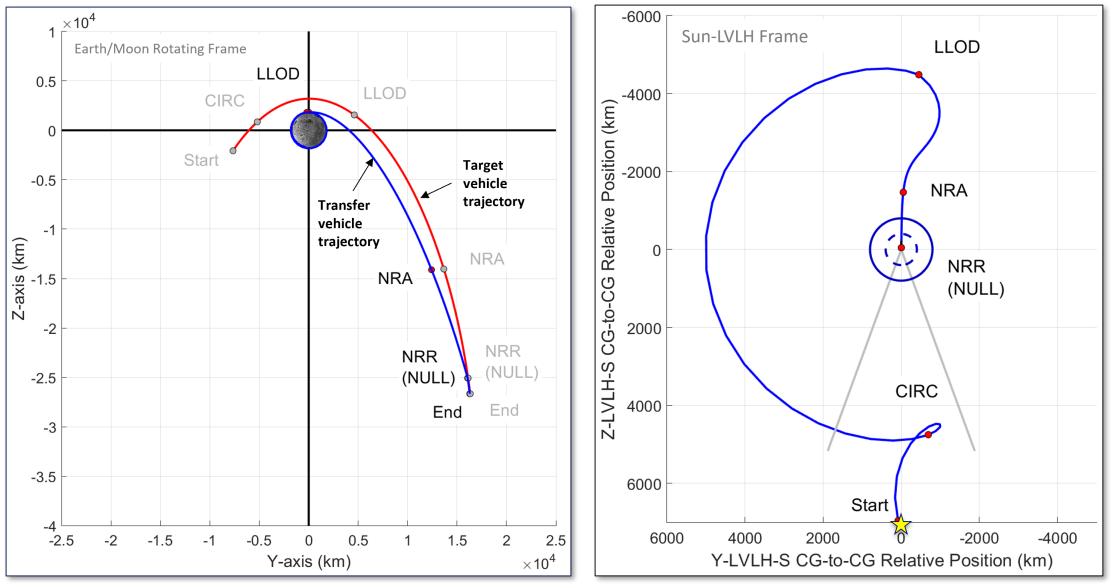


Event Name	Sim Elapsed Time (h:mm:ss)	3-sig U Pos Err (km)	3-sig V Pos Err (km)	3-sig W Pos Err (km)	3-sig |Pos| Err (km)	3-sig U Vel Err (mm/s)	3-sig V Vel Err (mm/s)	3-sig W Vel Err (mm/s)	3-sig |Vel| Err (mm/s)
Start	0:00:00	5.00 (5.01)	5.00 (5.01)	5.00 (5.01)	8.66 (8.68)	500 (501)	500 (501)	500 (501)	866 (868)
CIRC	0:54:07	0.01 (0.01)	0.56 (0.56)	0.14 (0.14)	0.57 (0.58)	46 (50)	94 (101)	519 (523)	529 (535)
LLOD	2:49:48	0.01 (0.01)	0.22 (0.24)	0.05 (0.05)	0.22 (0.25)	555 (587)	742 (782)	523 (559)	1064 (1127)
NRA	8:47:42	0.02 (0.02)	1.79 (1.85)	2.14 (2.39)	2.79 (3.02)	48 (50)	129 (133)	87 (93)	163 (170)
NRR(NULL)	14:47:42	0.03 (0.03)	3.05 (3.12)	2.66 (2.75)	4.05 (4.16)	478 (496)	493 (519)	486 (505)	841 (878)
End	15:47:42	1.72 (0.03)	3.84 (3.76)	3.42 (3.49)	5.43 (5.13)	478 (12)	495 (476)	486 (502)	843 (692)

Figure 5. MONTE and LINCOV comparison orbit determination performance results

LOW-LUNAR ORBIT TO NRHO RENDEZVOUS AND DOCKING PERFORMANCE

Using the ground tracking and orbit determination models introduced and compared in the previous sections, this portion of the paper will utilize these models to track both an active chaser vehicle returning from low-lunar orbit to rendezvous with a passive target vehicle in a Near Rectilinear Halo Orbit (NRHO). This demonstration will be expanded to not only highlight the orbit determination aspect of the problem, but analyze how accurate the relative state can be estimated along with how accurately the HLS chaser vehicle will be dispersed to the passive target vehicle in the NRHO. Delta-v dispersions and other integrated GN&C performance parameters are included to illustrate the predicted closed-loop GN&C performance when utilizing ground tracking orbit determination solutions.



(a) Nominal Trajectory Profile in the Earth/Moon Rotating Frame (b) Nominal Relative Trajectory Profile in the SLVLH

Figure 6. Simulated low-lunar orbit to NRHO rendezvous profile

The simulated low-lunar orbit to NRHO rendezvous trajectory is shown in Figure 6 where the profile presented in the Earth/Moon rotating frame depicting both the HLS vehicle ascending from the lunar surface (blue line) and the passive target vehicle in the NRHO (red line) is provided in Figure 6(a). The resulting relative trajectory profile in the sun-referenced local-vertical local-horizontal (LVLH) frame is in Figure 6(b).

CONCLUSION

This section will contain a summary of results and conclusion. Key observations will be briefly reviewed and general trends highlighted.

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