M. Sohaib Alam

arXiv:2403.0329

(work with Eleanor Rieffel)

DYNAMICAL LOGICAL QUBITS IN THE BACON-SHOR CODE

BRIEF REVIEW OF BACON-SHOR SUBSYSTEM CODES

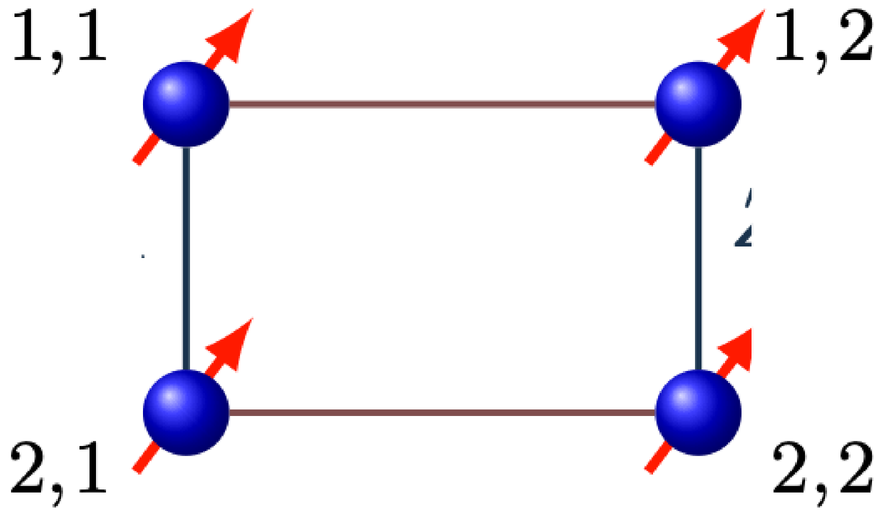
D. Bacon, Operator quantum error-correcting subsystems for self-correcting quantum memories, Phys. Rev. A 73, 012340 (2006

[[4,2,2]] stabilizer error-detecting code

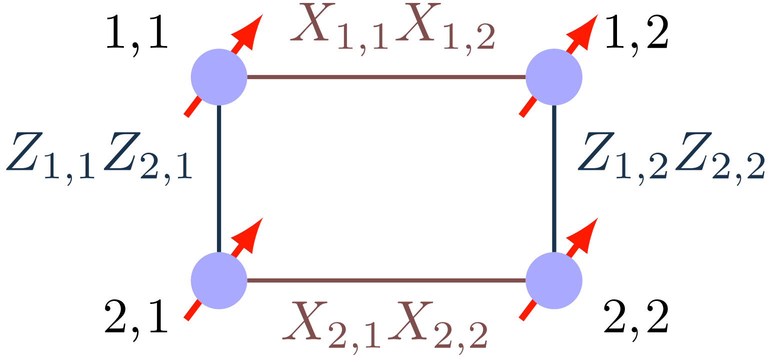
$$S^X = X_{1,1}X_{1,2}X_{2,1}X_{2,2}$$
$$S^Z = Z_{1,1}Z_{2,1}Z_{1,2}Z_{2,2}.$$

Code space is the +1 eigenspace of the stabilizer operators

$$\tilde{X}_{L1} = X_{1,1}X_{2,1}\ , \quad \tilde{Z}_{L1} = Z_{1,1}Z_{1,2}$$
$$\tilde{X}_{L2} = X_{1,1}X_{1,2}\ , \quad \tilde{Z}_{L2} = Z_{1,1}Z_{2,1}$$



[[4,1,2]] Bacon-Shor Subsystem code



$$\mathcal{G} = \langle G_1^X, G_2^X, G_1^Z, G_2^Z \rangle$$

$$G_1^X = X_{1,1}X_{1,2}\,, \quad G_2^X = X_{2,1}X_{2,2}\,,$$
$$G_1^Z = Z_{1,1}Z_{2,1}\,, \quad G_2^Z = Z_{1,2}Z_{2,2}\,,$$

$$S^X = X_{1,1}X_{1,2}X_{2,1}X_{2,2}$$
$$S^Z = Z_{1,1}Z_{2,1}Z_{1,2}Z_{2,2}\,.$$

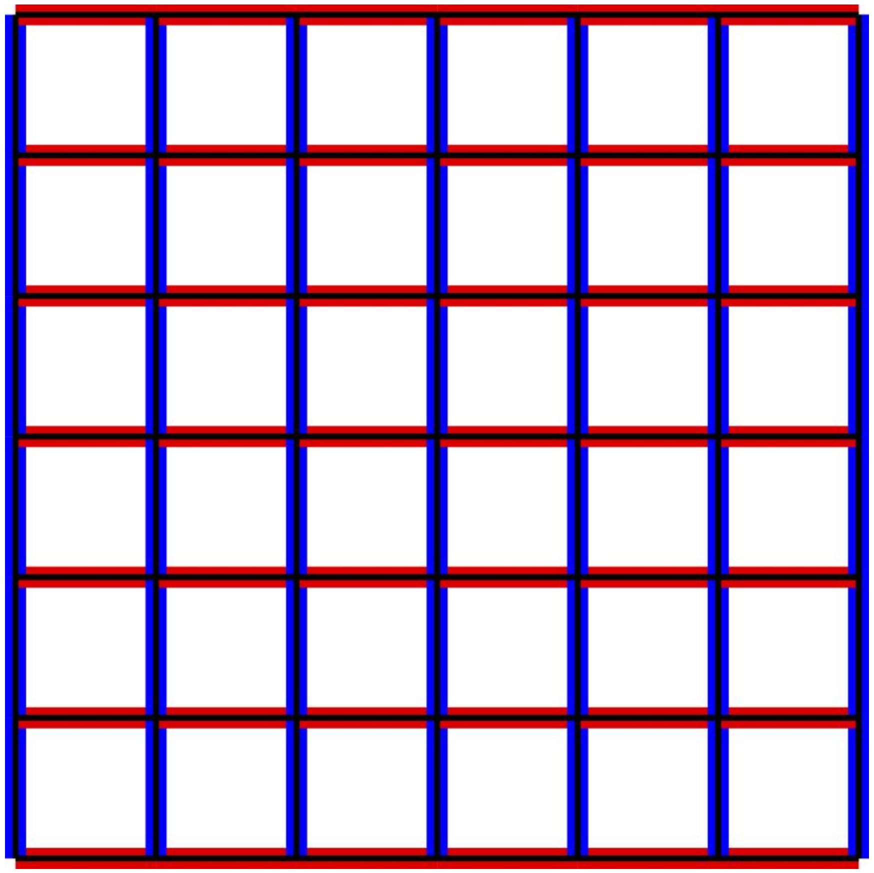
These stabilizers define a [[4,2,2]] stabilizer code.

Lower weight measurements at the expense of fewer qubits

$$X_L = X_{1,1}X_{2,1}\,, \quad Z_L = Z_{1,1}Z_{1,2}$$

Side note: Jiang & Rieffel (2017) "Non-commuting two-local Hamiltonians for quantum error suppression" used B-S codes to circumvent a no-go theorem for stabilizer codes, enabling more practically implementable error suppression in AQC

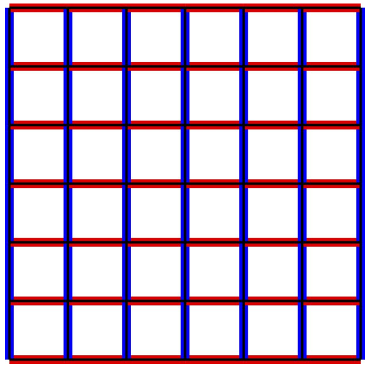
Bacon-Shor code

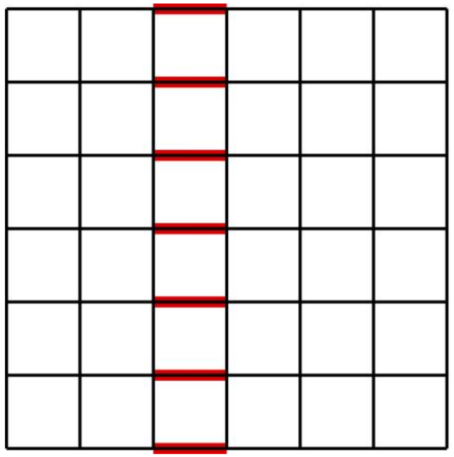


Gauge group generated by all nearest-neighbor 2-body

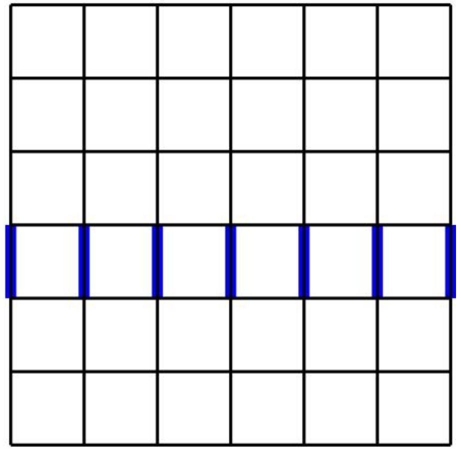
- XX checks (red, horizontal)
- ZZ checks (blue, vertical)

Bacon-Shor code





X-type

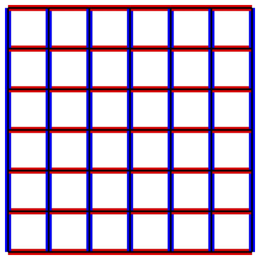


Z-type

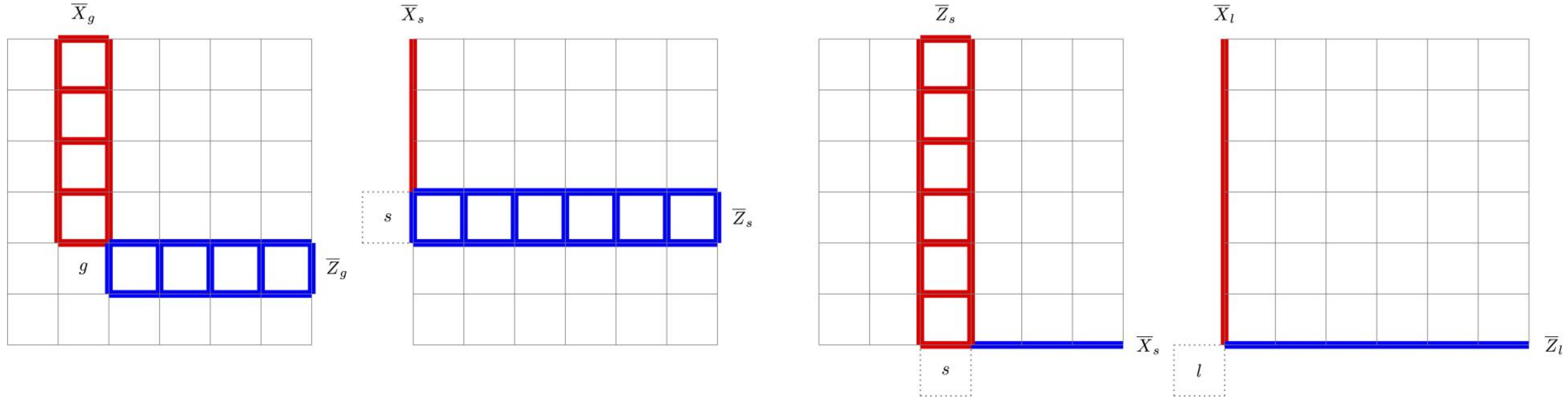
Stabilizer group *S*
 - Center of gauge group (elements of *S* that commute with all elements of *S*)

One stabilizer generating set
 - columns of XX operators, and rows of ZZ operators

Bacon-Shor code



Virtual qubit operators



Gauge qubits, in one-to-one correspondence with plaquettes

Horizontal Stabilizer Qubit defined by a horizonal $\bar{Z}$ stabilizer (product of single qubit Z operators) and a vertical $\bar{X}$ operator (product of X ops)

Vertical Stabilizer Qubit defined by a vertical $\bar{Z}$ stabilizer (product of X ops) and a horizontal $\bar{X}$ operator (product of Z ops)

Logical qubit for B-S code logical $\bar{X}$ and $\bar{Z}$ ops

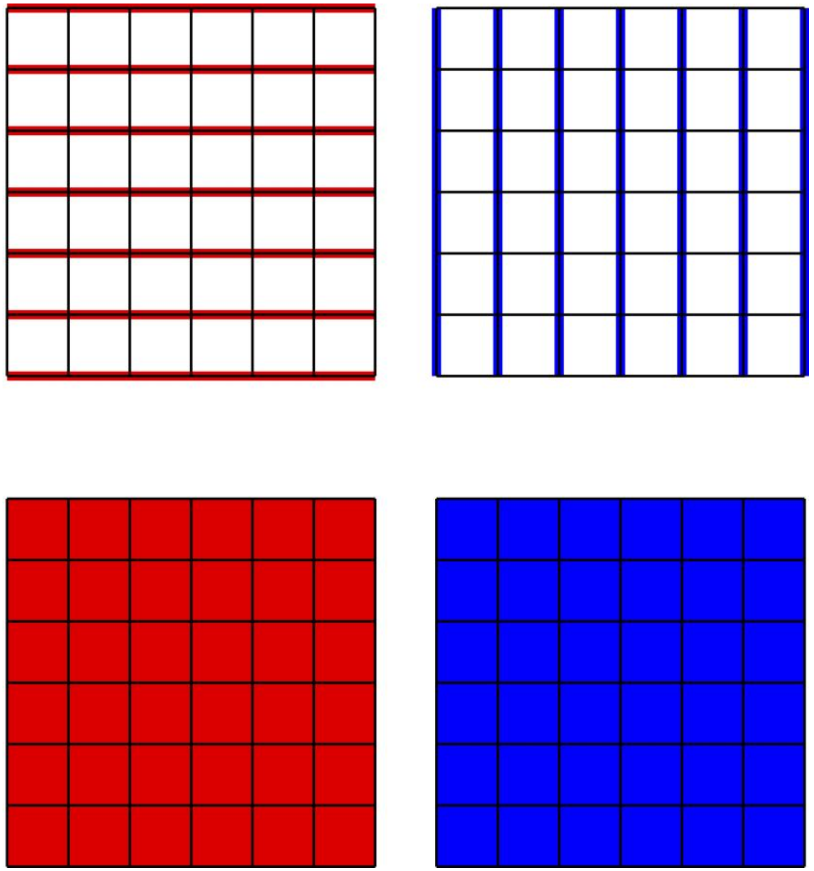
Bacon-Shor code

Two rounds of measurement
- Round 1: All XX checks (horizontal)
- Round 2: All ZZ checks (vertical)

We are periodically moving between two different gauge fixings
- one where the X operators of all gauge qubits have been fixed, and
- one where the Z operators of all gauge qubits have been fixed

Instantaneous Stabilizer Groups (ISGs)
- Bacon-Shor stabilizers plus all X gauge qubit operators
- Bacon-Shor stabilizers plus all Z gauge qubit operators

Measurement schedule

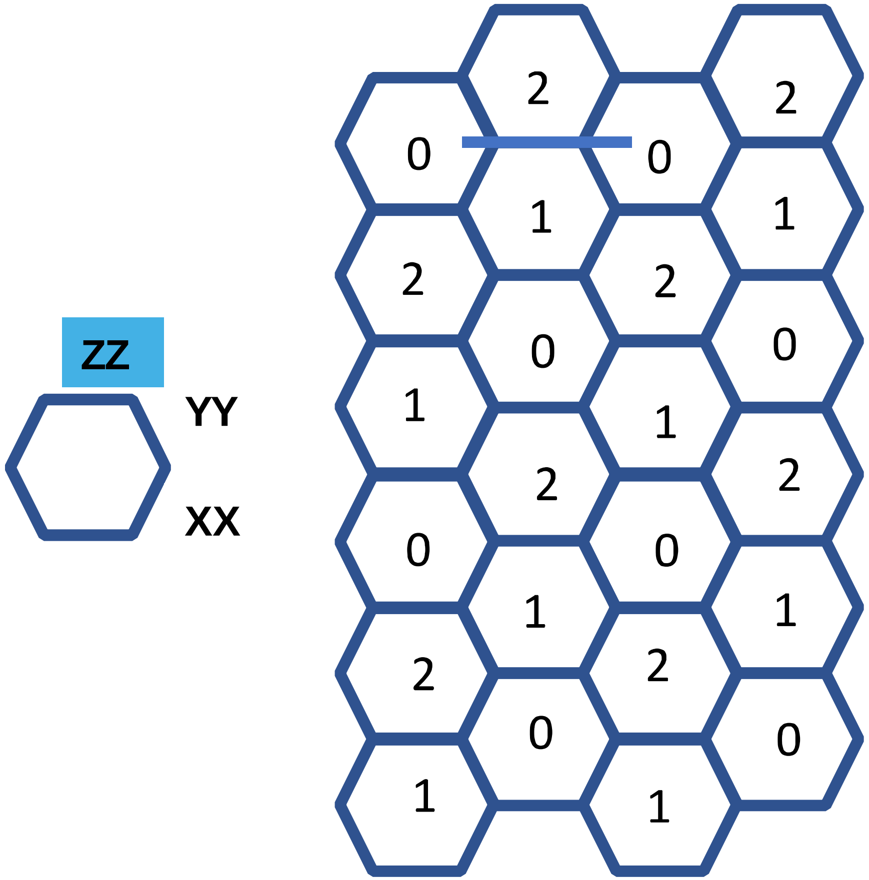


Instantaneous Stabilizer Groups (ISGs)

BRIEF REVIEW OF HONEYCOMB FLOQUET CODE

M. B. Hastings and J. Haah, Dynamically Generated Logical Qubits, Quantum 5, 564 (2021)

Honeycomb code



M. B. Hastings and J. Haah, "Dynamically generated logical qubits." Quantum 5 (2021)

- Measure 2-qubit checks in 3 rounds
- At each round, there is an instantaneous stabilizer group
- Logical information is carried safely from one round to the next
- -uses equivalence of logical operator representations when multiplying by stabilizers
- Periodic (Floquet) structure
- General construction for 3-valent graphs
- Are there other ways to construct Floquet codes?

M. Sohaib Alam, Eleanor Rieffel

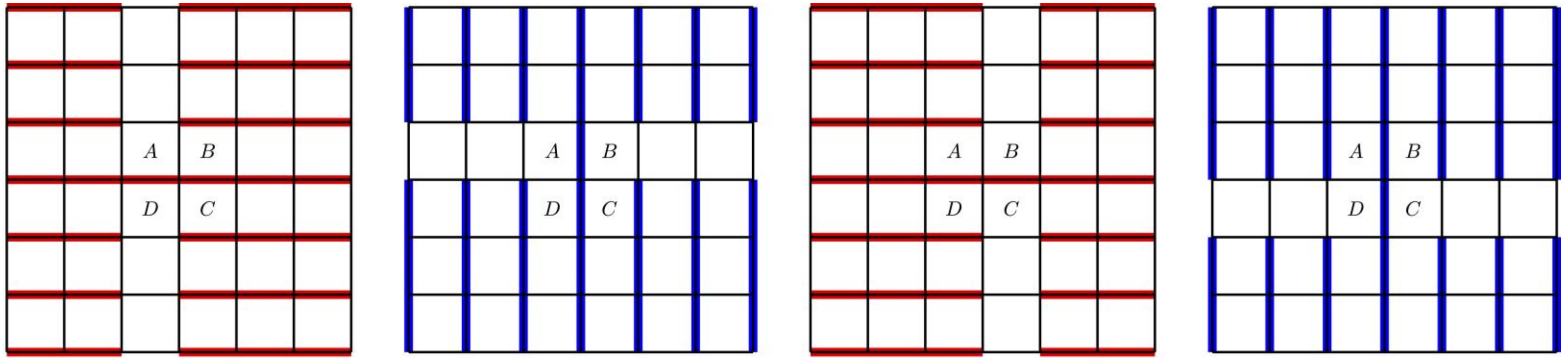
arXiv:2403.0329

DYNAMICAL LOGICAL QUBITS IN THE BACON-SHOR CODE

Floquet-Bacon-Shor code

- To free up space for an additional logical qubit, we refrain from fixing one of the gauge degrees of freedom
  - We introduce a “gauge defect”
- We need to do so carefully to ensure all the Bacon-Shor stabilizers also get measured

Measurement schedule



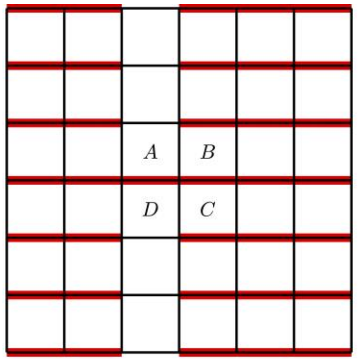
Subset of horizontal XX checks

Subset of vertical ZZ checks

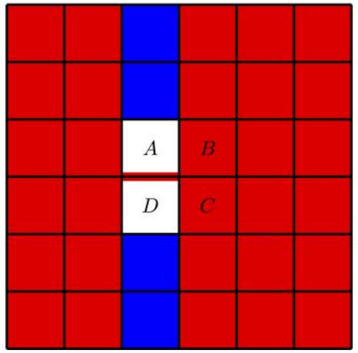
Subset of horizontal XX checks

Subset of vertical ZZ checks

A Floquet-Bacon-Shor code: an Instantaneous Stabilizer Group (ISG)



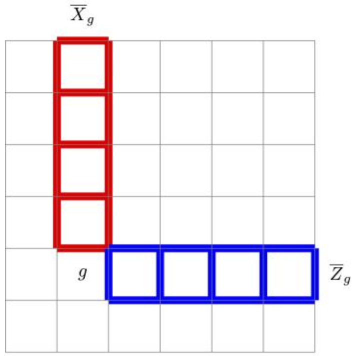
$(\overline{X}_A, \overline{Z}_A \overline{Z}_D)$



At any round, the ISG consists of

- the usual Bacon-Shor stabilizers,
- the just measured check operators
- all elements from previous ISGs that commute with the currently measured checks

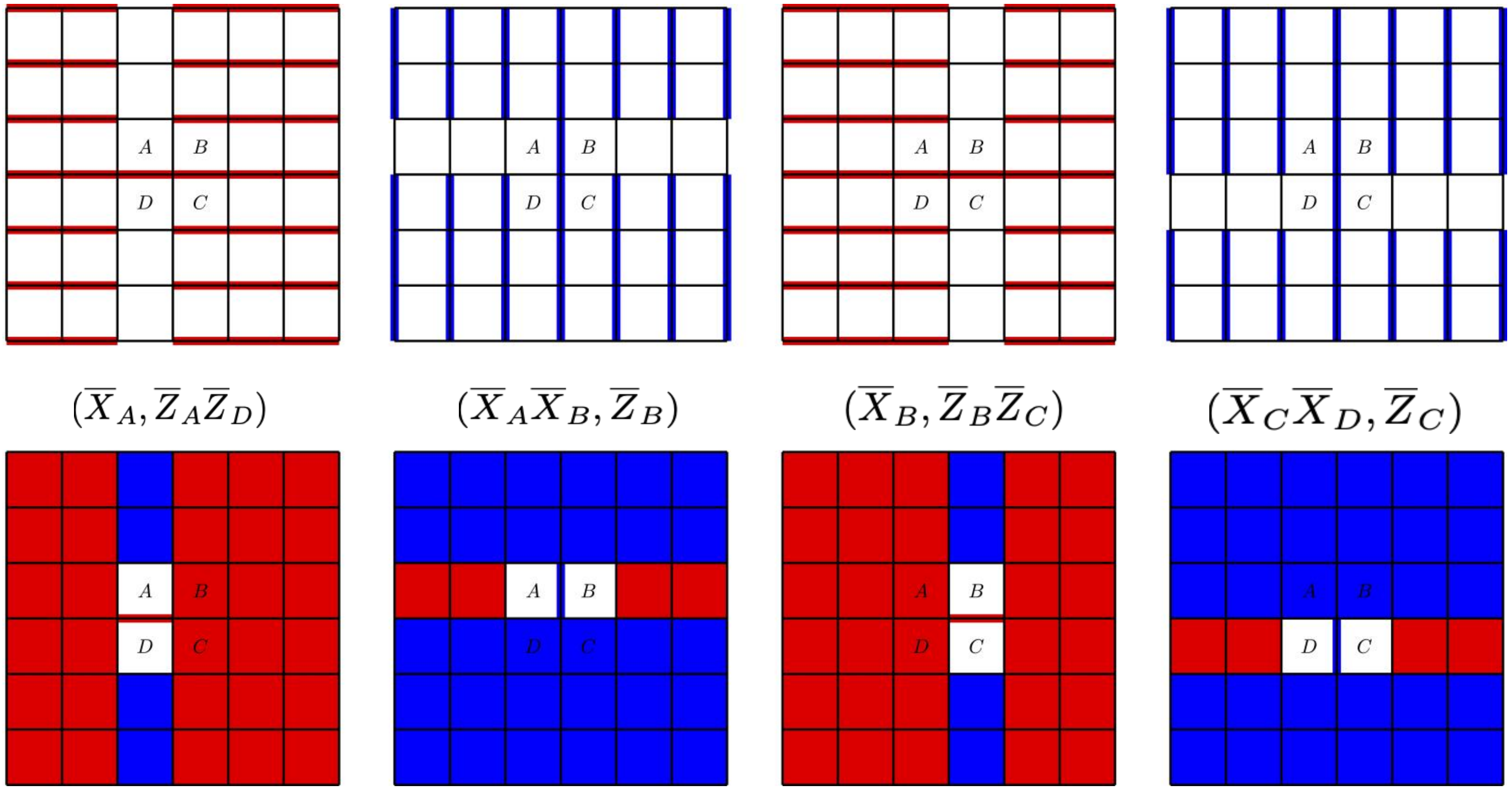
Any cell (or equivalently gauge qubit) colored either red or blue corresponds to gauge fixing its $\bar{X}$ or $\bar{Z}$ operator respectively



Gauge qubits, in one-to-one correspondence with plaquettes

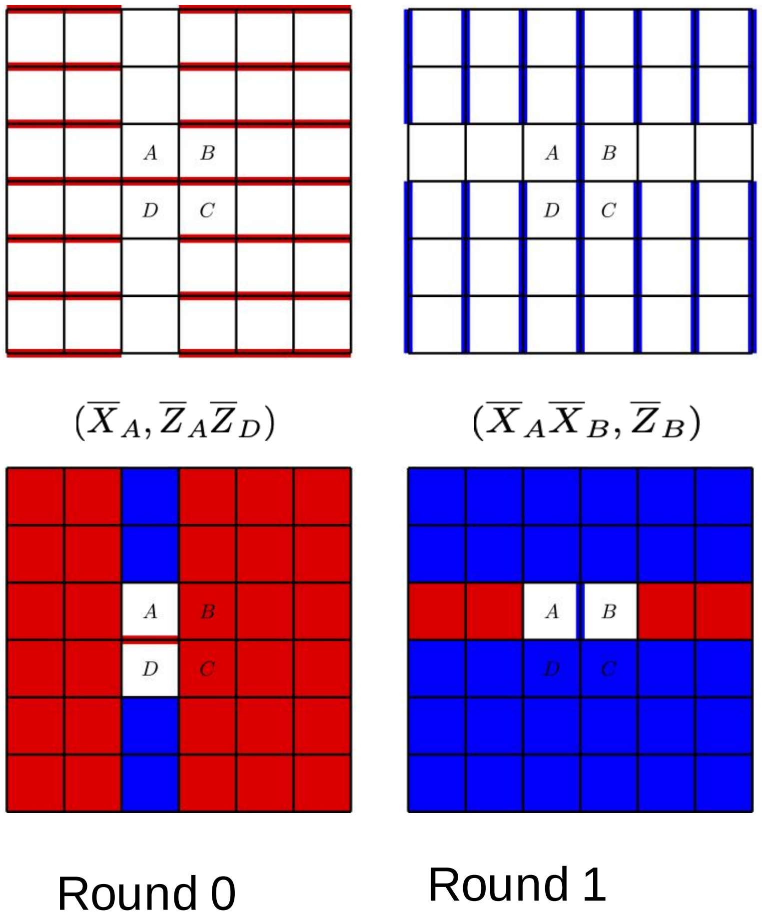
A Floquet-Bacon-Shor code: ISGs

Measurement schedule



Instantaneous Stabilizer Groups (ISGs)

A Floquet-Bacon-Shor code: preserving logical info



Preserving logical information from round to round

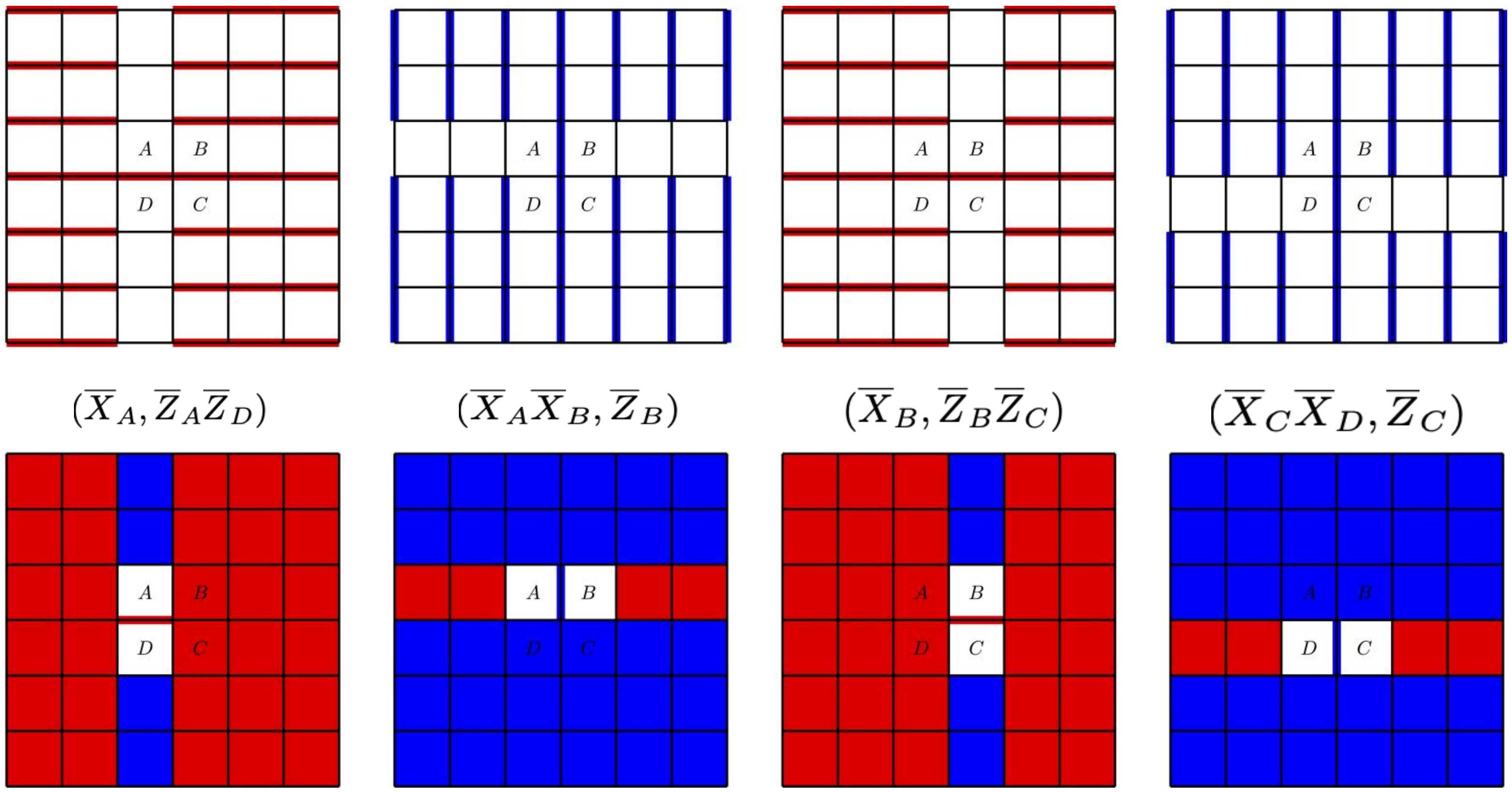
- $\overline{X_B}$ is in the ISG of round 0

- $\overline{Z_A Z_B}$ is in the ISG of round 1 (it is the single vertical blue bar in the picture)

Each pair of successive rounds define a (generalized) Bacon-Shor code

A Floquet-Bacon-Shor code: relation between logical ops

Measurement schedule



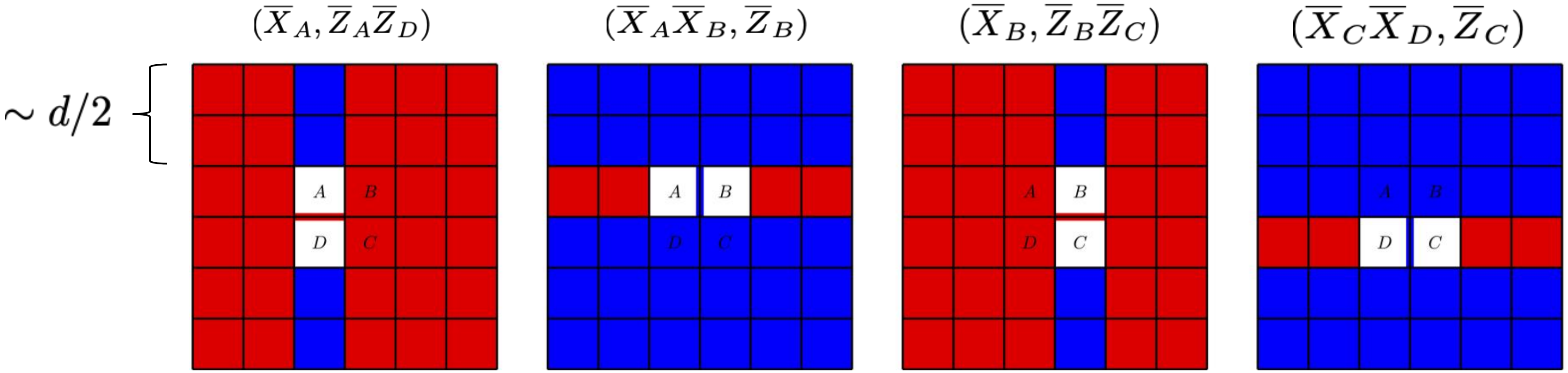
Instantaneous Stabilizer Groups (ISGs)

Floquet-Bacon-Shor codes: distance

- Distance of each ISG is ~d if we place the defect in the center
  - ~d/2 from hole to edge, and have a product of weight 2 check operators, so ~d
  - Warning: That does not mean that the entire code has distance d.

- Better is the distance of the Bacon-Shor codes defined by two rounds, but still not sufficient

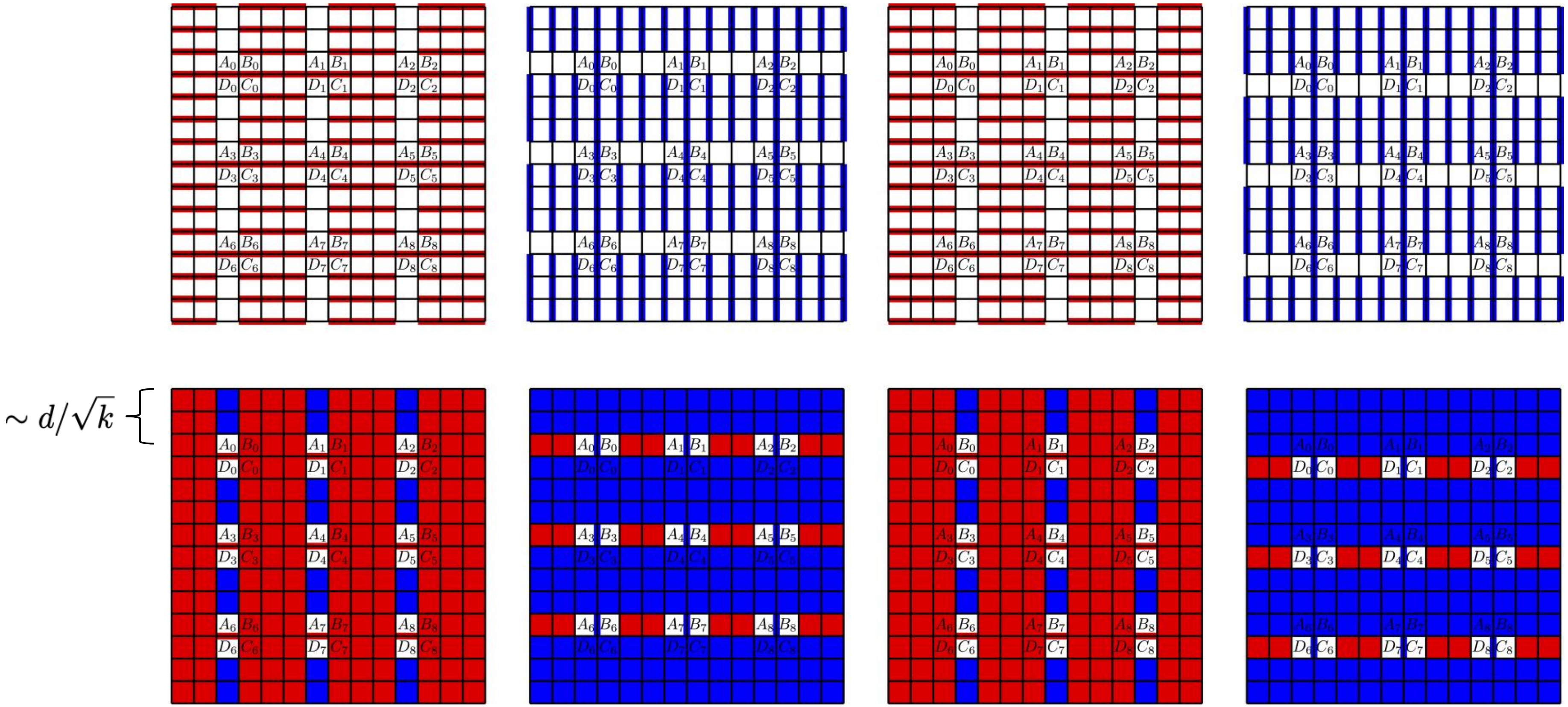
- New paper by Fu & Gottesman on distances for Floquet codes: arXiv:2403.04163



Instantaneous Stabilizer Groups (ISGs)

More Floquet-Bacon-Shor codes

Measurement schedule



Instantaneous Stabilizer Groups
(ISGs)

Further remarks on Floquet code constructions

- Floquet codes can be constructed from subsystem codes by introducing gauge defects
- Some errors can be self-corrected purely by measurement schedule
- Decoding hypergraph may require beyond MWPM/UF techniques
- Like the parent Bacon-Shor code, the Floquet-Bacon-Shor family of codes does not possess a threshold

Open questions

- What are general schemes for constructing Floquet codes? When are there barriers?
- Still open questions with respect to distance of Floquet codes

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