



Prediction of Inlet Tone Noise from Two-Stage Fans

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Prediction of noise from two-stage fans that would likely power some of the proposed civil supersonic passenger aircraft concepts under development is an important element of the design of turbofan engines that could meet both the landing and takeoff noise regulations while providing efficient performance at the supersonic cruise. Existing noise databases, and most of the available empirical fan noise prediction models, were developed with a focus on the subsonic engine fan noise and as such have large uncertainties in predicting the noise from two-stage fans designed for supersonic applications. The problem is further complicated because such fans operate in the presence of inflow distortions caused by the complex inlet systems with which they are coupled. To address these complexities, the NASA Commercial Supersonic Technology Project has focused on the use of physics-based noise prediction tools for supersonic-relevant fan-inlet systems. This paper and a companion one, summarize this effort, which is currently focused on the fan inlet tone noise though it is also applicable to the fan exhaust tone noise. The methodology adopted for this work along with some of the recent results will be presented in this paper.

I. Introduction

To power some of the planned civil supersonic passenger aircraft, medium bypass ratio turbofans with two-stage fans have been proposed as a possible compromise between acceptable landing and takeoff (LTO) noise levels, and efficient performance in supersonic cruise. The engine operability also requires that the inlet provide sufficient mass flow to the engine at the low flight speeds while also providing subsonic flow at the engine face at the high flight speeds. These twin requirements lead to inlet designs that have auxiliary doors for the LTO operability and an inlet spike for the cruise operability. As a result, the supersonic fan-inlet systems are more complex than their subsonic counterparts as depicted in Fig. 1.

Since there are virtually no publicly available noise databases for the supersonic-relevant fan-inlet systems, aircraft system level noise assessments of candidate civil supersonic aircraft concepts have tended to rely on the existing tools and databases that are rooted in the subsonic experience base. This limitation leads to large uncertainties in the system level noise prediction for the civil supersonic aircraft [1]. In order to address this shortcoming, the NASA Commercial Supersonic Technology (CST) Project has focused on the development of a physics-based high-fidelity noise prediction approach for the supersonic-relevant fan-inlet systems. The approach is being used for investigating the noise impact of the various fan and inlet design parameters as well as supporting the development of a virtual noise database to reduce the uncertainties in the system level noise predictions for the civil supersonic passenger aircraft.

It is well known that both the fan and inlet influence the character of the farfield radiated noise. However, due to the large disparity between the aerodynamic and acoustic scales, the fan and inlet systems are often decoupled and treated separately to make modeling more tractable. This is justifiable if the fan and inlet decoupling is done judiciously. In this paper, the decoupling is done only in the acoustic sense. The aerodynamic coupling is effectively retained by imposing the inflow distortion into the fan from the inlet as a boundary condition on the fan. The implied assumption is that the inlet-induced inflow distortion can influence the fan aerodynamic and acoustic performance, but the acoustic and potential field responses of the fan do not alter the inflow distortion profiles. This is a reasonable assumption given the disparity in the relative strengths of the aerodynamic and acoustic perturbations present in the fan-inlet systems.

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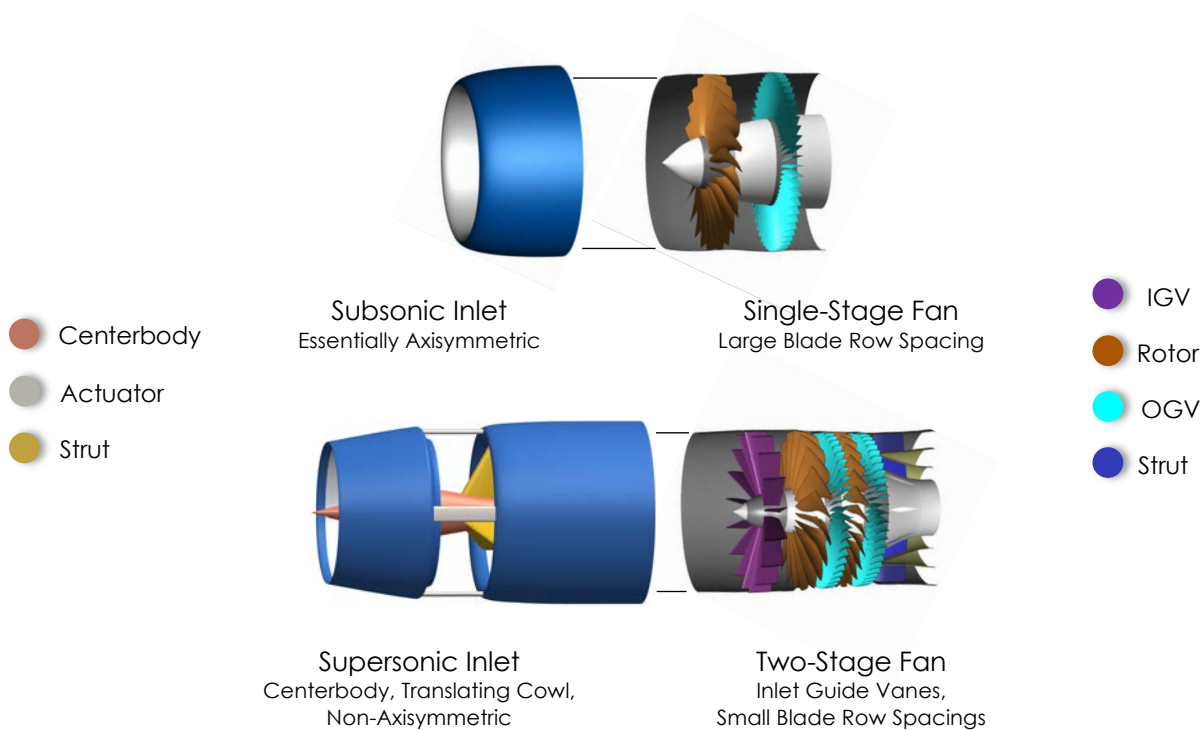


Fig. 1 Fan-inlet systems: Subsonic (top) and Supersonic (bottom).

The modeling approach adopted here is a synthesis of high-resolution Unsteady Reynolds Averaged Navier-Stokes (URANS) simulations for the fan source noise prediction, and computational aeroacoustic (CAA) simulations for the inlet noise propagation to the farfield. This paper addresses the source element while a companion paper [2] addresses the inlet noise propagation element. The initial focus is on the prediction of fan inlet tone noise but, in principle, this methodology is also applicable to the fan exhaust tone noise provided that the impact of the mixed flow exhaust system on the fan noise propagating through the exhaust duct is properly modeled. Fan broadband noise prediction for multistage fans will be addressed at a later time.

In this paper the rationale for choosing this approach over the other candidate approaches will be discussed first. This will then be followed by a description of the approach and the steps involved in extracting the acoustic content from the simulations. Finally, the application of this approach to the prediction of the inlet distortion noise generated by a supersonic fan-inlet system will be presented. The paper will also address an intrinsic limitation of modeling approaches that assume perfect blade row geometries and presents a method for estimating the ‘extraneous’ noise generated by realistic blade row geometries that is above and beyond that generated by the idealized geometries.

II. Fan Noise Modeling Considerations

Though the mechanisms of fan noise generation are basically the same for the single- and two-stage fans, there are notable differences in the relative importance of the various contributing sources. For example, the potential field interaction, i.e., the scattering of the static pressure perturbations of the rotor by the stator, and those of the stator by the rotor, is weak for the single stage fans due to the typically large rotor-stator spacings and large blade/vane counts and is often ignored. In contrast, for the two-stage fans, because of the smaller blade row spacings, the potential field interaction is much stronger and cannot be ignored. For single-stage fans, the inflow distortion is mainly caused by the angle of attack of the engine at the LTO conditions. For two-stage fans, the inflow distortion is always present at the LTO conditions due to the flow coming through the auxiliary inlet doors even at the zero angle of attack. Acoustic transmission through the blade rows is present for both the single- and two-stage fans but is likely more important for the two-stage fans due to the smaller blade row spacings and stronger acoustic coupling between the blade rows. Similarly, multiple-pure-tone (i.e., buzzsaw) noise is present for both types of fans at the supersonic rotor relative tip speeds but is likely weaker for the two-stage fans since the rotor-locked shock wave system must pass through the inlet guide vanes (IGV) before reaching the inlet auxiliary doors and inlet termination. Finally, the presence of the IGV for the two-stage fans means that there are additional blade row interactions that are not typically present in the single-stage fans.

III. Modeling Approaches

Fan noise models cover a wide range of fidelities and may be classified as follows; empirical, analytical, acoustic analogy based, linearized Euler, and high-fidelity aero-acoustic approaches such as the large eddy simulations. The experience at NASA has shown that for single-stage fan noise prediction, the linearized Euler and the high-fidelity simulation approaches have consistently yielded better results.

For example, the linearized Euler approach was used to predict fan tone noise for the Advanced Ducted Propulsor (ADP) fan. The details of the application and the associated results [3] are summarized in Fig. 2. The viscous wakes of the rotor were computed via RANS simulations and their circumferential Fourier harmonics were used as vortical excitations for computing the acoustic response of the stator. The acoustic field downstream of the stator is the fan exhaust noise. The upstream acoustic field was in turn used as an acoustic excitation for computing the acoustic response of the rotor. The acoustic field upstream of the rotor is the fan inlet noise. In theory, one would repeat the whole process using the rotor downstream acoustic field as the initial excitation for the stator response computation and continue to iterate until the inlet and exhaust acoustic levels do not change. However, the data-theory comparisons in Fig. 2 indicate that one iteration was sufficient for predicting the tone acoustic power levels and that the levels were in good agreement with the wind tunnel data. In carrying out these computations, which were run in the frequency domain, only one passage for each blade row was necessary resulting in very modest grid sizes (i.e., three hundred thousand grid points for the stator and one million grid points for the rotor). The computations required roughly 20 hours per simulation on a vintage 2010 single processor desktop with four cores. While the linearized Euler approach is sufficiently accurate and robust and can be run at very modest computing costs, it is unwieldy for the two-stage fan applications. That is because it requires more simulations per iteration given the larger number of blade rows involved. It is also likely that more than one iteration of the process would be needed given the stronger coupling between the blade rows. Lastly, in addition to the viscous wake interactions, the potential field interactions must also be considered. For all of these reasons, the linearized Euler approach is not practical for the prediction of noise from two-stage fans.

In a separate effort, a high-fidelity simulation approach was used to predict the fan noise spectra for the Source Diagnostic Test (SDT) fan. The approach relied on an implementation of the Lattice Boltzmann Method (LBM) as embodied in the commercial code PowerFLOW. The details of the simulation and the associated results [4] are summarized in Fig. 3. The unsteady flowfield of the fan stage was simulated over an extensive range of scales. A permeable Ffowcs Williams-Hawkins (FW-H) surface enclosing the fan stage was used to compute the narrowband spectrum of the noise generated by the fan on a sideline parallel to the rotational axis of the fan. The data-theory agreement proved to be remarkably good up to the frequencies associated with the smallest resolved scales of turbulence in the flowfield. The simulations were run for three speeds and three fan stage configurations and generally showed consistently good agreement with the wind tunnel data. At the finest level, the simulations were run on a grid with roughly 950 million voxels and with a turnaround time of 8 days per 1000 cores. This type of approach, while clearly robust and accurate, is too computer resource intensive to be practical for large parametric studies of the type that were required in the present effort.

In view of the challenges associated with the use of either the linearized Euler or the high-fidelity simulations, an alternative prediction approach was sought. This led to the choice of high-resolution URANS for computing the periodic unsteady flowfield of the fan stage on the premise that the acoustic perturbations extracted from such a simulation would be reasonably accurate given sufficient grid resolution. Of course, a full URANS simulation that is adequately resolved to provide the necessary accuracy of the acoustic perturbations in the vicinity of the fan stage would itself be computer resource intensive and not a practical approach for the purposes of this study. Therefore, it was decided to rely on an approximation of URANS known as the non-linear harmonic (NLH) method. In the NLH approximation, only a pre-selected set of frequencies are tracked in the simulation which results in significant reductions in the computing resource requirements. Briefly, in the NLH approximation, all the flow variables $U(\vec{x}, t)$ are split into time-averaged $\bar{U}(\vec{x})$ and periodic components $u(\vec{x}, t)$, namely,

$$U(\vec{x}, t) = \bar{U}(\vec{x}) + u(\vec{x}, t) \quad (1)$$

where $U(\vec{x}, t)$ denotes any flow variable. The periodic component in Eq. (1) is approximated by a truncated temporal Fourier series as follows,

$$u(\vec{x}, t) = \sum_{n=1}^N \tilde{u}_n(\vec{x}) e^{in\omega t} + \tilde{u}_{-n}(\vec{x}) e^{-in\omega t} \quad (2)$$

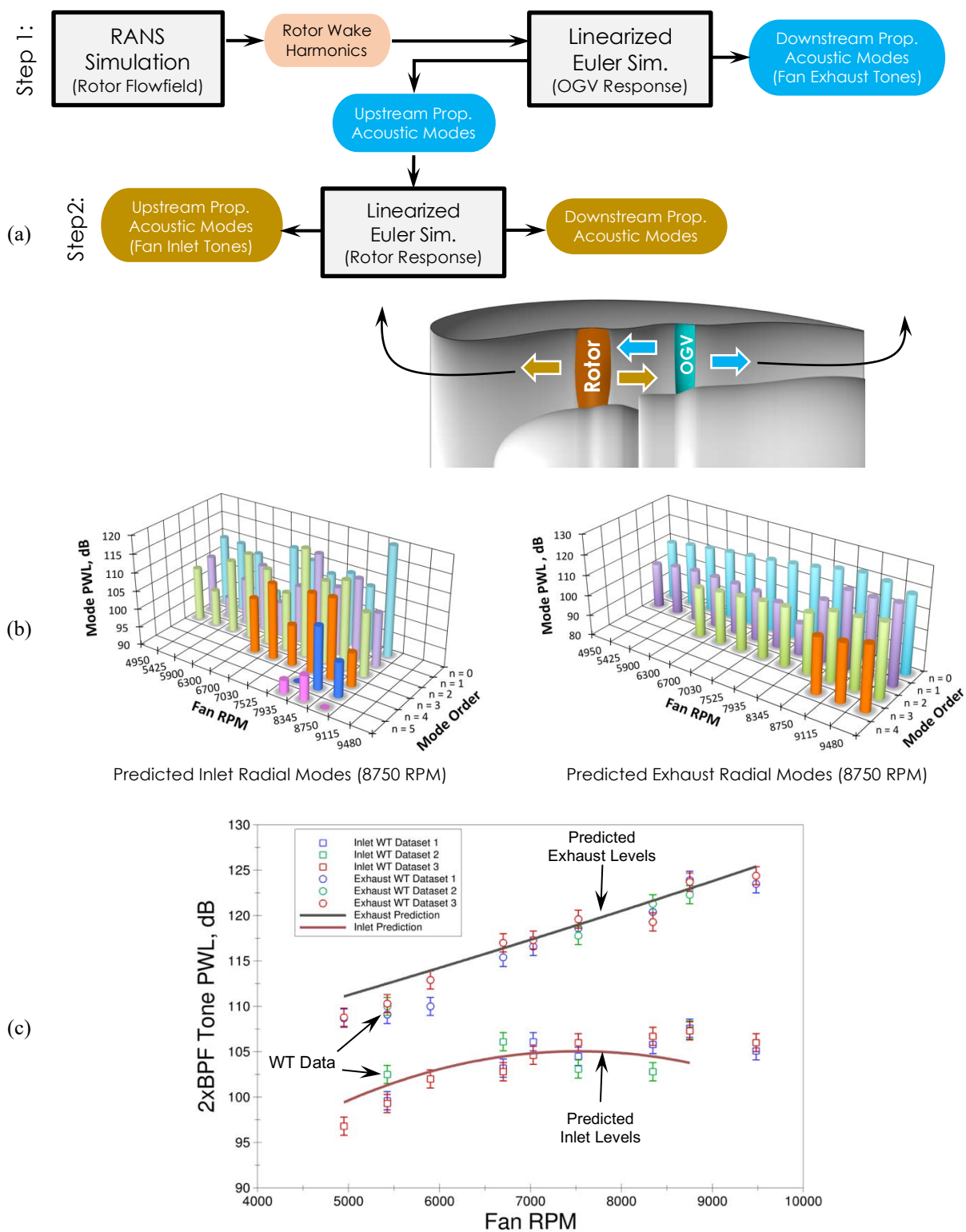


Fig. 2 Linearized Euler approach applied to the fan tone noise prediction. (a) Simulation steps involved. (b) Inlet and exhaust radial mode power levels. (c) 2×BPF tone inlet and exhaust power levels.

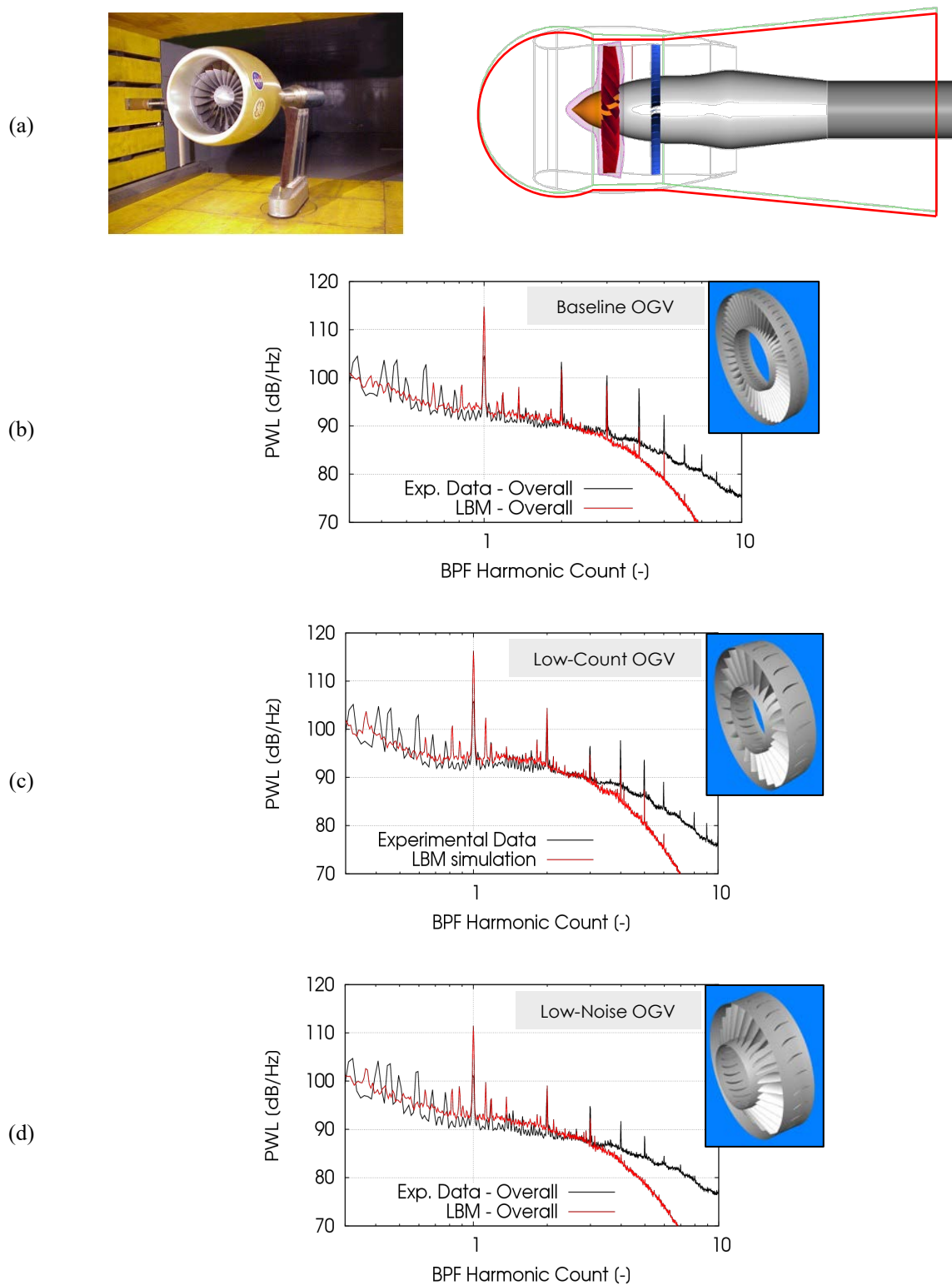


Fig. 3 High-fidelity simulations applied to the fan narrowband noise prediction. (a) SDT fan stage and FW-H surface. (b), (c), and (d) Sideline noise spectra for three different stator vane packs.

where N represents the number of blade-passing-frequency (BPF) or vane-passing-frequency (VPF) harmonics retained for each blade row. For most applications of interest, $N = 3$ is sufficient to provide a good approximation of the periodic flowfield. Substituting these approximations in the Navier-Stokes equations and combining terms lead to a set of equations for the time-averaged flow variables $\bar{U}(\vec{x})$ and the harmonic flowfield coefficients $\tilde{u}_n(\vec{x})$ and $\tilde{u}_{-n}(\vec{x})$. The computational time required for solving this frequency-domain set of equations is substantially less than that needed for solving the full unsteady flow equations in the time domain. Furthermore, since the computational domain for an NLH simulation is comprised of only one passage of each blade row, there is also significant computer memory savings too. Together, the time and memory savings result in substantially reduced computer resource requirements which can be up to two orders of magnitude smaller compared with the full URANS depending on the domain size and grid resolution. This means that an NLH simulation for realistic fan stage geometries and operating conditions can be run on a modern multi-core desktop computer instead of a large computer cluster or a supercomputer. The particular implementation of the NLH that has been used in this study is that incorporated into the commercial turbomachinery aerodynamic code FINE/Turbo developed by Numeca now part of Cadence.[†]

The accuracy of the NLH approach was assessed using the SDT fan stage in an earlier effort [5] and will not be revisited here. Instead, the focus is on describing how the acoustic content are extracted from the NLH simulations and analyzed. The SDT fan is 22 inches (0.56m) in diameter and has 22 rotor blades and 54 stators for the baseline vane pack. Fig. 4 shows a sketch of the SDT fan stage and the associated NLH domain used in the simulation presented here which contains slightly more than 17 million grid points. The equivalent URANS domain, which is half of the full wheel owing to the symmetry of the geometry, would have required roughly 374 million grid points. The rotating and stationary block zones are color-coded for clarity. In what follows the acoustic modes are extracted at two axial planes of the flowfield shown in Fig. 4, one in the inlet in the rotating blocks zone, and one in the exhaust in the stationary blocks zone. Since the NLH domain spans, azimuthally only one passage of for each blade row, the full-wheel solution is first constructed. The construction can be done since the phase relationship between the flow variables in one passage and flow variables in any other passage is deterministic and is known a priori through the knowledge of the inter-blade phase angle, which is $2\pi V/B$ for the rotating blocks zone and $2\pi B/V$ for the stationary blocks zone, where B and V are the rotor blade and stator vane counts, respectively. The simulation shown here was carried out at the approach condition for the SDT fan at an rpm of 7808.

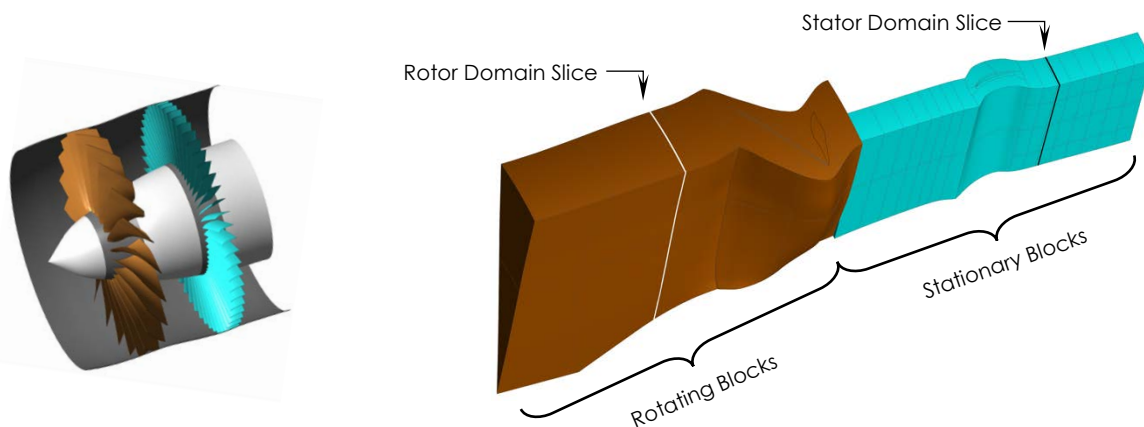


Fig. 4 SDT fan stage (left) and the corresponding NLH computational domain (right).

IV. Acoustic Mode Analysis

Once the full-wheel solution is constructed, the azimuthal content of the flowfield can be extracted by the application of a circumferential Fourier decomposition. The results are shown in Fig. 5 for the inlet axial slice from the rotating blocks zone. The constructed full-wheel solution is shown on the left and the associated circumferential mode spectra for the first two harmonics of VPF, as seen in the rotating frame, are shown on the right. The levels for

[†] FINE/Turbo is a density-based URANS solver which incorporates a number of spatial numerical schemes including the finite-difference and upwind, and temporal schemes such as the explicit time-marching Runge-Kutta. It features a number of convergence acceleration strategies including the local time-stepping, multigrid, and implicit residual smoothing. It offers a menu of optimized turbulence models and is an MPI-parallelized code.

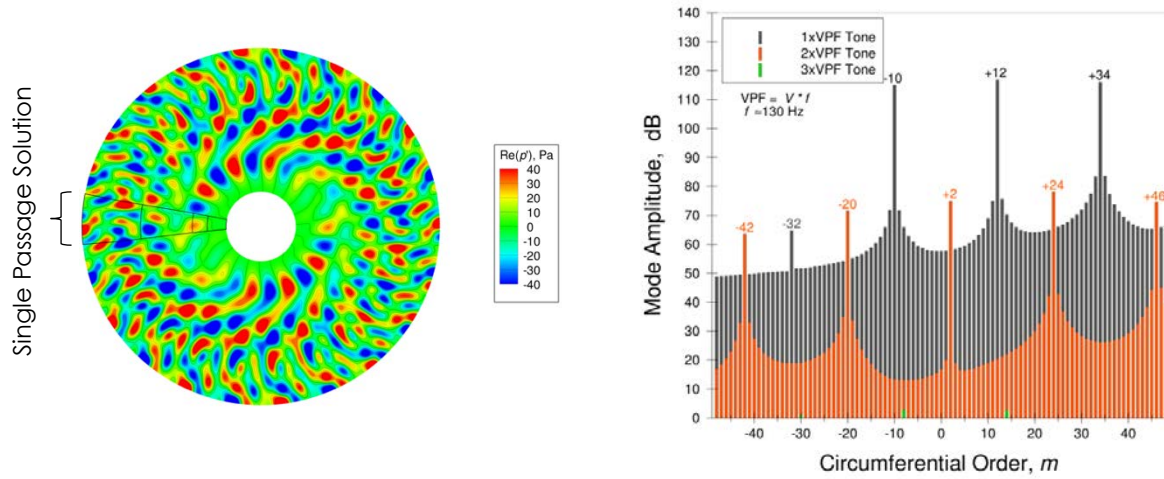


Fig. 5 The full-wheel solution for the rotating frame axial slice shown in Fig. 4. Left: Contours of the real part of the pressure perturbations for 1×VPF tone. Right: Subset of the circumferential modes for 1×VPF through 3×VPF tones.

the circumferential modes at 3×VPF are barely visible in this plot since they are quite low. All the modes shown in this plot are associated with the rotor-stator interaction and are consistent with the Tyler-Sofrin rule which, for the rotating frame, is given by the expression $m = kB - nV$ where n is the frequency harmonic order and k is any integer. To properly interpret the frequencies of rotating frame modes, their phases should be expressed in the stationary frame through the coordinate transformation $\theta_r = \theta_s - \Omega t$ which relates the rotating frame angular position variable θ_r to the stationary frame angular position variable θ_s . The transformation for the mode phase is explicitly given by

$$p'_m(\theta_r) = A_m e^{i\phi_m(\theta_r)}, \quad \phi_m = m\theta_r - nV\Omega t = m(\theta_s - \Omega t) - nV\Omega t = m\theta_s - (m + nV)\Omega t. \quad (3)$$

As a result of the coordinate transformation, the rotating frame frequency of a mode, i.e., $nV\Omega$, is Doppler-shifted to the frequency $(m + nV)\Omega$ in the stationary frame. The frequency shift is mode order dependent which means that every mode with the rotating frame frequency of $n \times \text{VPF}$ is Doppler-shifted to a different frequency in the stationary frame. Table 1 shows the frequency shifts for a subset of modes for the first three VPF harmonics. Note that, every mode in the rotating frame has a stationary frame frequency equal to a particular multiple of the blade passing frequency. For example, the mode $m = -10$ has a frequency of 1×VPF in the rotating frame and a frequency of 2×BPF in the stationary frame. The modes $m = -54, -108, -162$ are the harmonics of the stator potential field having frequency of zero in the stationary frame reflecting the fact that they are non-rotating spatial circumferential patterns in the multiples of the stator vane count as would be expected.

Table 1. Relationship between the frequencies of circumferential modes in the rotating and stationary frames of reference.

		Rotating Frame Frequencies		
		1xVPF	2xVPF	3xVPF
Stationary Frame Frequencies	⋮	⋮	⋮	⋮
	-3xBPF	-120	-174	-228
	-2xBPF	-98	-152	-106
	-1xBPF	-76	-130	-184
	0xBPF	-54	-108	-162
	1xBPF	-32	-86	-140
	2xBPF	-10	-64	-118
	3xBPF	+12	-42	-96
	⋮	⋮	⋮	⋮

For the slice in the stationary blocks zone, the mode extraction follows the same procedure as before except that there is no need to Doppler-shift the mode frequencies. The results are shown in Fig. 6. As before, all of the modes are consistent with the Tyler-Sofrin rule which, for the stationary frame, is given by the expression $m = nB - kV$. Note that the mode $m = -10$ has the frequency of $2 \times \text{BPF}$ consistent with the results of the Doppler-shifted rotating frame slice. The modes $m = 22$ and $m = 44$ are the first and second harmonics of the rotor potential field with the frequencies $1 \times \text{BPF}$ and $2 \times \text{BPF}$, respectively, indicating that they rotate past the stator at the blade passing rate and twice the blade passing rate, respectively, as would be expected.

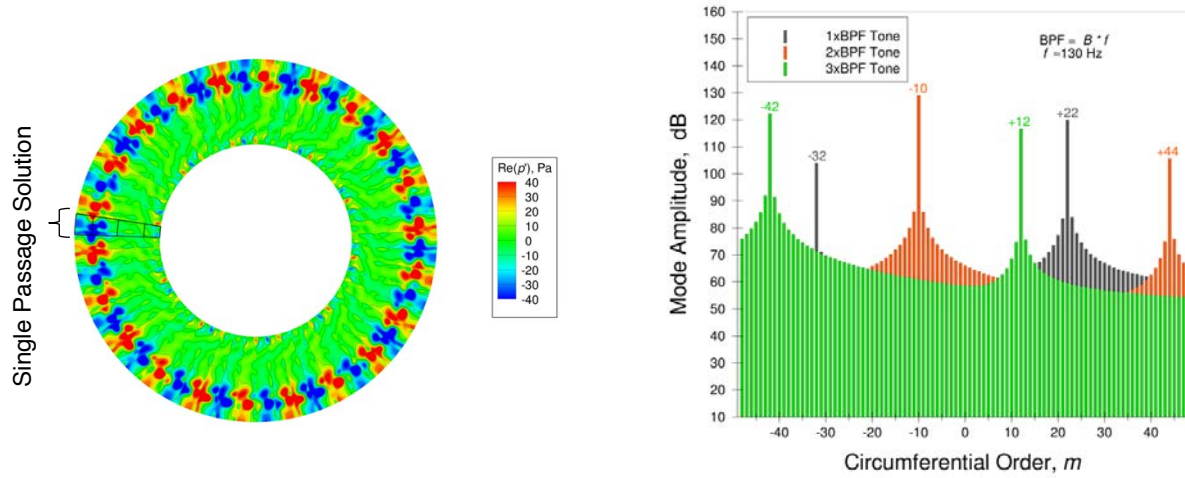


Fig. 6 The full-wheel solution for the stationary frame axial slice shown in Fig. 4. Left: Contours of the real part of the pressure perturbations for $1 \times \text{BPF}$ tone. Right: Subset of the circumferential modes for $1 \times \text{BPF}$ through $3 \times \text{BPF}$ tones.

The next step is to extract the radial mode content for each of the circumferential modes to determine which ones are propagating. This is done in the usual way by an eigenfunction expansion of the circumferential mode amplitude in the radial direction. For the purposes of this work the eigenfunction basis is assumed to be a combination of the Bessel functions of the first and second kinds tailored for a hard-walled duct, namely,

$$\Psi_{m,n}(r) = \Pi^{-1} \left(J_m(\kappa_{mn}r) - \frac{J'_m(\kappa_{mn})}{Y'_m(\kappa_{mn})} Y_m(\kappa_{mn}r) \right), \quad \Pi = \sqrt{a-b}$$

$$a = \left(1 - \frac{m^2}{\kappa_{mn}^2} \right) \left(J_m(\kappa_{mn}) - \frac{J'_m(\kappa_{mn})}{Y'_m(\kappa_{mn})} Y_m(\kappa_{mn}) \right)$$

$$b = \left(\sigma^2 - \frac{m^2}{\kappa_{mn}^2} \right) \left(J_m(\kappa_{mn}\sigma) - \frac{J'_m(\kappa_{mn})}{Y'_m(\kappa_{mn})} Y_m(\kappa_{mn}\sigma) \right)$$
(4)

where κ_{mn} are the radial eigenvalues of the duct and σ is the duct hub-to-tip radius ratio. The expansion is carried out using the least squares fit of the unknown amplitudes of the basis functions $\Psi_{m,n}$ to the known spanwise profiles of the real and imaginary parts of the circumferential mode's amplitude. The solution is obtained using the singular value decomposition method. Given the fine resolution of the underlying NLH solution, the basis function expansion can be carried out using a large number of radial modes if necessary. However, the mode cut-off condition means that typically only the first few radial modes are propagating, and the rest are evanescent (i.e., decaying). The radial mode numbering scheme used in this work follows the classical one which equates the radial mode index to the number of zero crossings of the mode shape. So, the radial mode index counter starts at "0" in the results shown here. The steps involved in the expansion are pictorially depicted in Fig. 7 which shows the expansion as applied to inlet slice in Fig. 5. The slice is for the $1 \times \text{VPF}$ harmonic perturbation pressure in the rotating frame; see the left portion of the part (a)

of Fig. 7. Since, to the linear order, the amplitudes of the flow perturbations do not depend on the frame of reference, the rotating frame amplitudes can be used directly in the analysis. The predicted circumferential mode content of the inlet slice indicates that the mode $m = -10$ is the dominant one at $2 \times \text{BPF}$ in agreement with the data obtained using the rotating rake [6] for the SDT fan. The extracted contours of the mode $m = -10$ are shown in the part (b) of the figure. Note that the number of the circumferentially interlaced positive and negative pressure perturbation peaks (i.e., the blue and red patches) is 10 consistent with the mode identity.

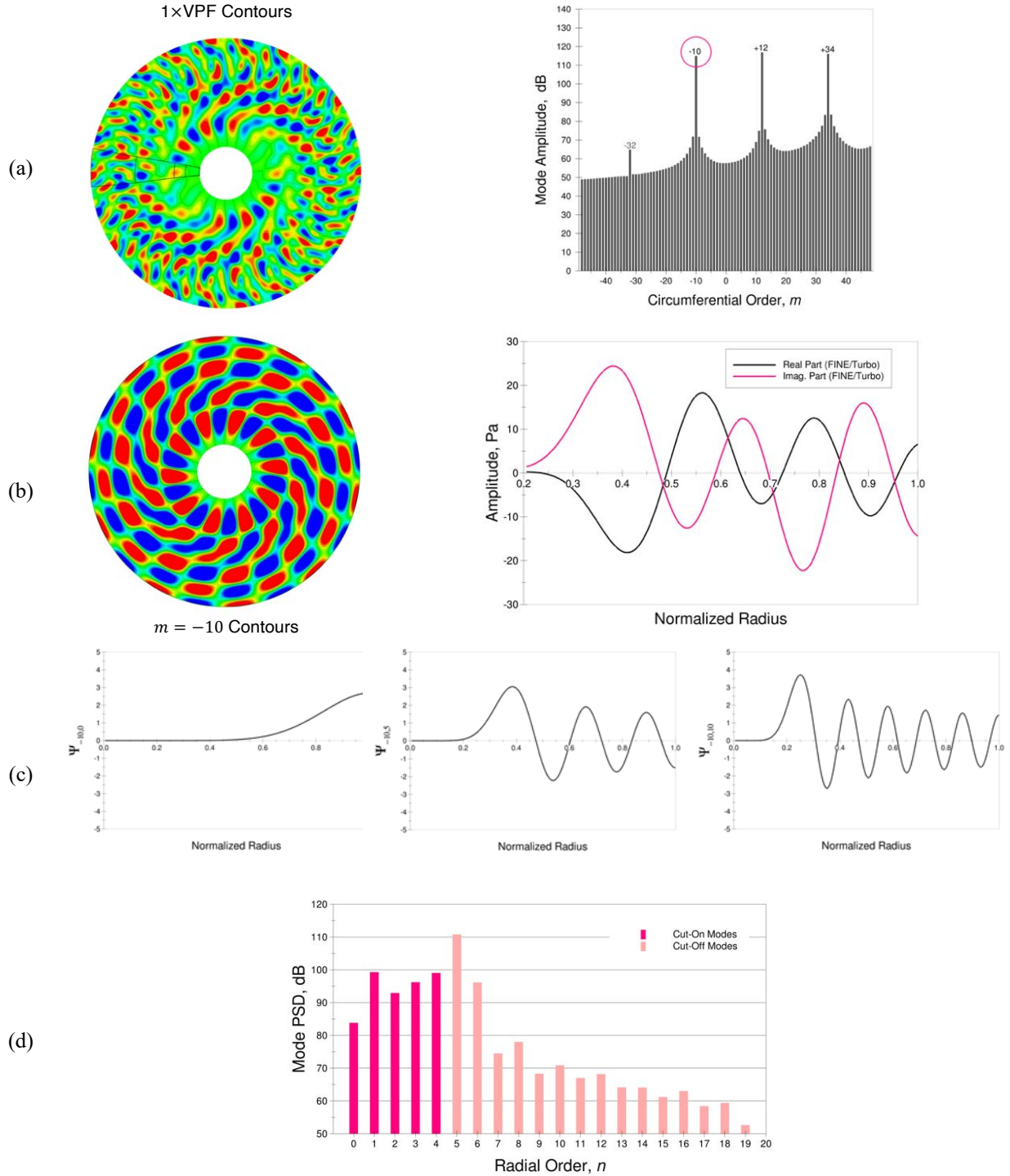


Fig. 7 Radial mode extraction steps for the mode $m = -10$ in the SDT fan inlet.

The spanwise variations of the real and imaginary parts of the mode $m = -10$ are plotted in the part (b) of the figure. In the part (c), three of the basis Bessel functions used in the expansion are plotted, namely, $\Psi_{-10,0}$, $\Psi_{-10,5}$, and $\Psi_{-10,10}$. In the part (d), the radial mode spectrum resulting from that eigenfunction expansion is shown for the orders up to $n = 19$. The cut-off condition indicates that only modes up to order $n = 4$ (i.e., a total of five modes) are propagating and that the rest are evanescent. The evanescent modes are shown by the lighter colored bars. It should be noted that the number of propagating inlet radial modes for the mode $m = -10$ as measured by the rotating rake for the SDT fan is also five at the approach condition. The predicted dominance of the mode $m = -10$ and the predicted number of its constituent propagating radial modes are both in agreement with the measured data suggesting that a sufficiently resolved NLH simulations can provide realistic acoustic predictions.

V. Extraneous Circumferential Modes

There is an intrinsic limitation in the prediction schemes that assume all blades and all vanes in their respective blade rows are identical. In reality, due to small variations in the way blades and vanes are installed in the hub, there are small, but non-negligible, variations in the airfoils' stagger angles around the wheel. For the real-world fan stages, these differences lead to the generation of shaft orders in the frequency spectra and 'extraneous' circumferential modes in the mode spectra of fans.[‡] By extraneous modes we mean circumferential mode orders that are generated in addition to the expected Tyler-Sofrin modes for given blade and vane counts according to the $m = nB - kV$ rule.

As an example, the measured power levels of the circumferential modes that comprise the 2×BPF inlet tone for the SDT fan at the approach condition are plotted in Fig. 8. These modes were measured using the rotating rake. Note the preponderance of the extraneous modes shown by the black bars in addition to the expected Tyler-Sofrin mode $m = -10$ shown by the magenta bar. For this case, the measured extraneous circumferential modes are in the range $-26 \leq m \leq 26$. While the power level of any individual extraneous mode is substantially lower than that of the Tyler-Sofrin mode $m = -10$, collectively their contributions to the radiated acoustic power from the inlet is sufficient to modify the sideline directivity of the tone as shown in Fig. 9. In this figure the predicted 89.3-inch (2.27 m) sideline directivities of the duct modes[§] shown in Fig. 8 are plotted for the measured mode $m = -10$ only and for all the measured modes. The measured 89.3-inch sideline directivity of the tone is also plotted for comparison. Clearly the contribution of the extraneous modes to the directivity of the 2×BPF tone is substantial and cannot be ignored.

The generation mechanism of these modes can be explained as follows. For identical blades and identical vanes, there are deterministic phase relationships between the sound fields generated by the individual blades and vanes. These relationships are governed by the inter-blade phase angle. When the acoustic fields of the individual blades or vanes combine, only the circumferential modes given by the expression $m = nB - kV$ survive due to the perfect constructive reinforcement of the constituent sound fields, and all other possible circumferential modes cancel out due to the complete destructive interference.

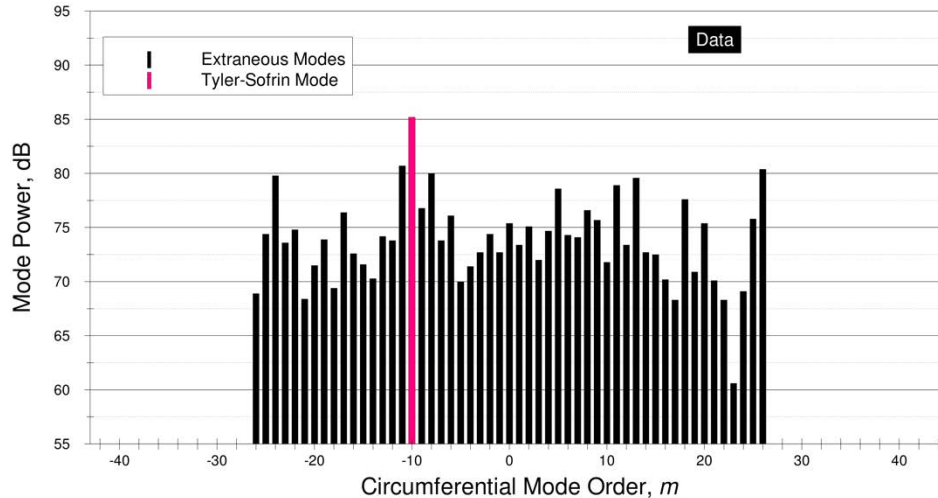


Fig. 8 Power levels of all the constituent circumferential mode of the 2×BPF tone measured using the rotating rake system in the SDT fan inlet.

[‡] This is true for all fan stages unless their rotor or stator packs are built as blisks.

[§] The in-duct modes were propagated to the sideline using the Eversman inlet radiation code [7].

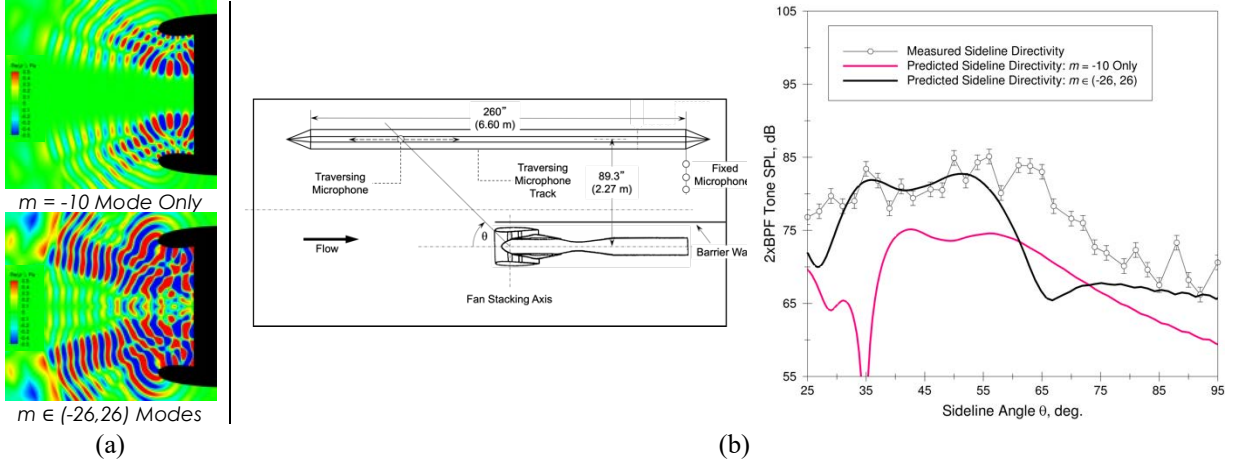


Fig. 9 Impact of the extraneous circumferential modes on the sideline directivity of the 2×BPF tone for the SDT fan. (a) Predicted nearfield radiation patterns, (b) Predicted 89.3-inch sideline directivities.

Consider the case of V identical vanes interacting with the wakes of B identical blades. The modal acoustic field (at a given frequency) inside the fan duct generated as a result of this interaction can be expressed as the sum of the acoustic fields generated by the individual vanes. Using the normal mode representation, the mode acoustic pressure p_{mn} in the duct can be expressed as shown in the Eqs. (5a-5d). The coefficient A_{mn} is the acoustic mode amplitude which is the sum over the contributions from the individual vanes. Each contribution in turn is given by the integral of the aerodynamic pressure loading f_j on the j^{th} vane surface weighted by the gradient of the duct Green's function, G_{mn} , normal to the vane surface.** Since the contributions of the individual vanes are related through the inter-blade phase angle, the expression for A_{mn} simplifies to an integration over the 0^{th} vane and a sum over the phase terms $e^{2\pi i j(m-nB)/V}$ as follows,

$$p_{mn}(x, r, \theta) = A_{mn} \Psi_{mn}(r) e^{i(k_{mn}x + m\theta)} \quad (5a)$$

$$A_{mn} = \sum_{j=0}^{V-1} \iint_{S_j} f_j (\hat{n} \cdot \nabla G_{mn}) dS = \left[\iint_{S_0} f_0 (\hat{n} \cdot \nabla G_{mn}) dS \right] \sum_{j=0}^{V-1} e^{2\pi i j \left(\frac{m-nB}{V} \right)} \quad (5b)$$

$$G_{mn}(x, r, \theta | x', r', \theta') = \frac{i}{2\pi k_{mn} A_D} \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{+\infty} \Psi_{mn}(r) \Psi_{mn}(r') e^{i[k_{mn}(x-x') + m(\theta-\theta')]} \quad (5c)$$

$$\sum_{j=0}^{V-1} e^{2\pi i j \left(\frac{m-nB}{V} \right)} = \begin{cases} V & m = nB - kV \\ 0 & m \neq nB - kV \end{cases} \quad (5d)$$

where k_{mn} is the axial wavenumber, A_D the duct cross-sectional area, and (x, r, θ) and (x', r', θ') denote the observer and source coordinates, respectively. The sum in the Eq. (5b) is non-zero only for the modes obeying the relationship $m = nB - kV$; see Eq. (5d). If, however, the stagger angles of the vanes are not identical but instead have small randomly distributed variations around wheel, then there will be small random perturbations in the phases of the individual vanes contributions. These phase perturbations will modify the sum in Eq. (5b) as follows,

$$\tilde{A}_{mn} = \left[\iint_{S_0} f_0 (\hat{n} \cdot \nabla G_{mn}) dS \right] \sum_{j=0}^{V-1} e^{2\pi i j \left(\frac{m-nB}{V} + \epsilon g(j) \right)} \quad (6)$$

where ϵ is assumed to be a small parameter and g a random function. Small variations in the stagger angle also introduce small variations in the amplitudes of the contributions from the individual vanes (i.e., the term inside the

** See, for example, Chapter 4 in Ref. [8].

square bracket in the Eq. (6)), but these variations are smaller than those due to the phase variations and are ignored here. Furthermore, as a first order approximation, we also ignore the weak dependence of the value of the surface integral in the Eq. (6) on the index m . If ε is not zero, Eq. (6) produces not just a single mode $m = nB - kV$, as does Eq. (5b), but also all the circumferential modes in the range $-nB \leq m \leq nB - 1$. Effectively, Eq. (6) represent a “mistuning” of the constituent acoustic fields so that the perfect reinforcements and cancellations do not occur.

Assuming reasonable values for ε and g , Eq. (6) provides a means for estimating the levels of extraneous modes without the need for simulations that model the variations in the stagger angles of the blades and vanes. Such simulations would have to be run in the full URANS mode resulting in order(s) of magnitude increases in the computational resource requirements. While the distribution of the stagger angles of the SDT vane pack are not known, it is reasonable to assume that they follow a normal random distribution. We assume that the distribution has zero mean, a standard deviation of 0.1, and set $\varepsilon = 0.1$. The results are plotted for the $2\times\text{BPF}$ tone at the approach condition in Fig. 10 where the levels are anchored to the measured power level of the mode $m = -10$ (see Fig. 8). At this operating condition, not all the modes in the range $-44 \leq m \leq 43$ are propagating, only the ones that are in the range $-26 \leq m \leq 26$, which is consistent with the measured data shown in Fig. 8. In Fig. 10, the predicted evanescent modes are grayed out.^{††} It should be noted that the levels plotted in Fig. 10 are the result of a Monte Carlo simulation involving 2,000 realizations for which ε was randomly perturbed by a maximum of 5%. While this analysis is preliminary, the comparison of the results shown in Figs. 8 and 10 suggests that Eq. (6) could provide reasonable estimates of the levels of the extraneous modes by anchoring them to the predicted level of the Tyler-Sofrin mode $m = nB - kV$ from the NLH simulations. Further work is needed to fine-tune the model especially as regards to the specifics of the random distribution and the size of the phase perturbations. This could follow from an analysis of the effects of introducing small perturbations in the vane pack stagger angles and investigating the resulting impact on the aerodynamic and acoustic responses of the vanes following the approach described in Chapter 3 of Ref. [8].

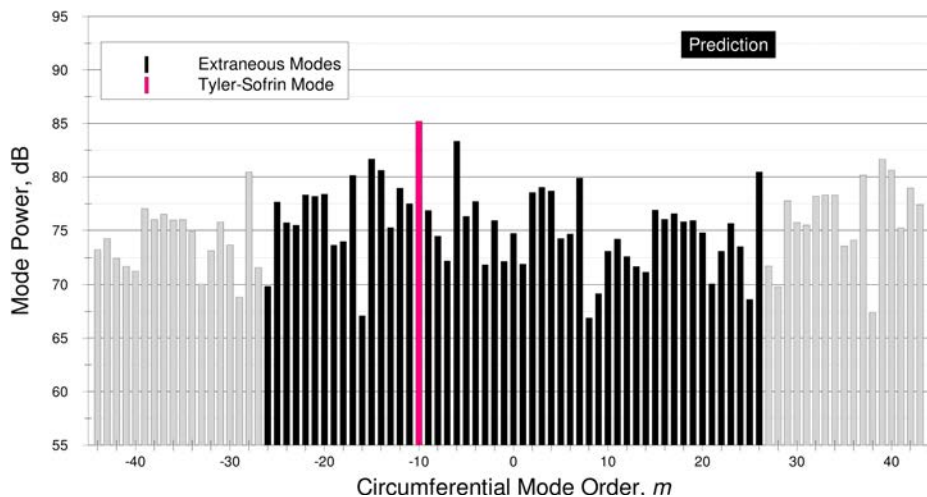


Fig. 10 Extraneous circumferential modes predicted using Eq. (6) in the SDT fan inlet for the $2\times\text{BPF}$ tone.

VI. Importance of Potential Field Interactions

Finally, as was mentioned in the introduction, for fans with small blade row spacings, the contribution of the blade row potential field to the amplitude of the rotor-stator interaction modes cannot be ignored. Since, an NLH simulation includes the combined effect of both the viscous and potential field interactions, it is not possible to directly separate their relative contributions to the expected Tyler-Sofrin mode. However, it is possible to infer the influence of the potential field using an indirect approach. This is done by comparing the levels of an expected Tyler-Sofrin mode with those of the contributing potential field modes for two fan stages. We use the SDT fan as an example of a single-stage fan with large blade row spacing, and a NASA internally designed two-stage fan as an example of a fan with closely spaced blade rows. The comparisons of the results are shown in Fig. 11.

The dominant Tyler-Sofrin mode amplitudes are shown as the blue bars in this figure. For the SDT fan this is the mode $m = -10$ at $2\times\text{BPF}$, and for the NASA fan it is the mode $m = -4$ at $1\times\text{BPF}$. The relevant potential field modes

^{††} The rotating rake measurements did not report the levels of the evanescent modes (i.e., $m > |26|$).

are shown as the black bars for the rotor-associated modes and as the maroon bars for the stator/IGV-associated modes. For the SDT fan, the relevant potential field modes are $m = 44$ and $m = 54$, respectively, and for the NASA fan they are $m = 32$ and $m = 36$, respectively. Note that both potential fields contribute to the Tyler-Sofrin mode whether the mechanism is the potential field of the rotor scattered by the stator/IGV, or that of the stator/IGV scattered by the rotor. Generally speaking, the potential field of the rotor tends to be significantly stronger than that for the stator/IGV due to the higher loading on the blades, lower blade counts, and longer blade chords all of which reduce the rate of decay of the potential field. Taking the rotor potential field as the dominant contributor, it is clear that for the single stage fan, the dominant potential field is about 10 dB weaker than the Tyler-Sofrin mode $m = -10$, implying that most of the contribution to it must be coming from the rotor-stator viscous interaction. In contrast, for the NASA fan, the dominant potential field is only 2 dB weaker than the Tyler-Sofrin mode $m = -4$, implying that the rotor potential field is contributing to the amplitude of mode $m = -4$ nearly on par with the viscous interaction between the IGV and rotor.

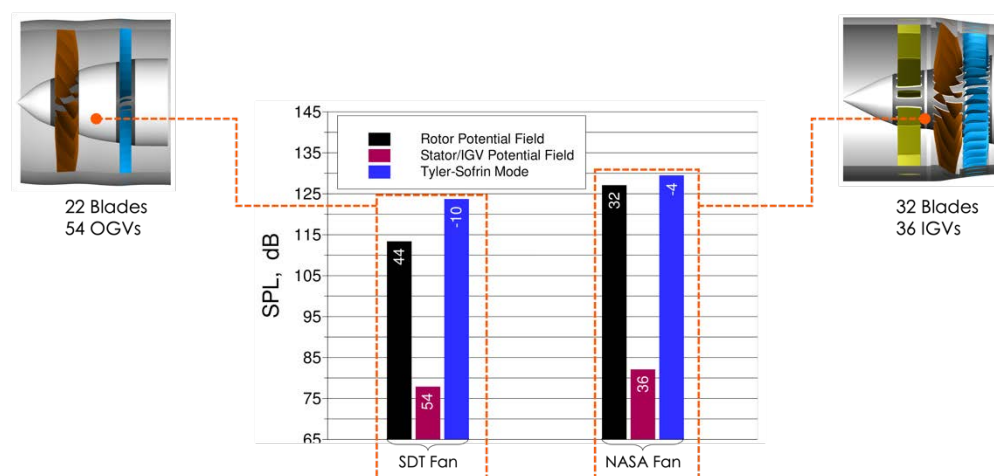


Fig. 11 Comparisons of the potential field mode levels and Tyler-Sofrin mode levels for a fan with large blade row spacing (left) and a fan with closely spaced blade rows (right).

VII. NLH Simulations of a Two-Stage Fan

A modern two-stage fan and a supersonic inlet with a translating cowl were designed by General Electric to serve as the conceptual testbeds for the fan noise studies as part of the NASA CST Project's Airport Noise Tech Challenge. Sketches of the fan stage and inlet are shown in Fig. 12. At the design point,^{††} the pressure ratio of this fan is 2.155, its bypass ratio is 2.812, and its first stage rotor tip speed is 1417 ft/s (431.9 m/s). With the translating cowl in the open position, additional mass flow is supplied to the fan during the LTO operations. The mass flow rate split between the inlet and the auxiliary doors is 62.1% and 37.9% at takeoff, 81.7% and 18.3% at sideline, 92.2% and 7.8% at approach, and 93.6% and 6.4% at cutback.

As was discussed in the introduction section, the aerodynamic impact of the distortion caused by the flow through the inlet and the auxiliary doors is modeled as a set of boundary conditions in terms of the total pressure and temperature for the fan NLH simulations. Using the SolidWorks flow simulation software, RANS-based inlet flowfields were computed for multiple inlet configurations [9]. These configurations will be discussed in more detail in the next section. From these flowfields, the total pressure and temperature profiles were extracted at the inflow plane to the fan domain indicated by the red dashed line in Fig. 13. Examples of the total pressure contours at this plane are shown in this figure for the 0° and 5.2° angles of attack at the approach condition (4534 rpm). The wakes of the three inlet struts are clearly visible in both contours as are the subtle circumferential distortions caused by the four actuators of the translating cowl. For the angle-of-attack case, additional circumferential distortion features are also visible around the circumference of the plot. Radial profiles of total pressure and temperature were extracted at one degree circumferential angle increments and were used as the inflow boundary conditions for the NLH fan simulations.

^{††} The aero design point for this fan is the supersonic cruise at the flight Mach number of 1.4.

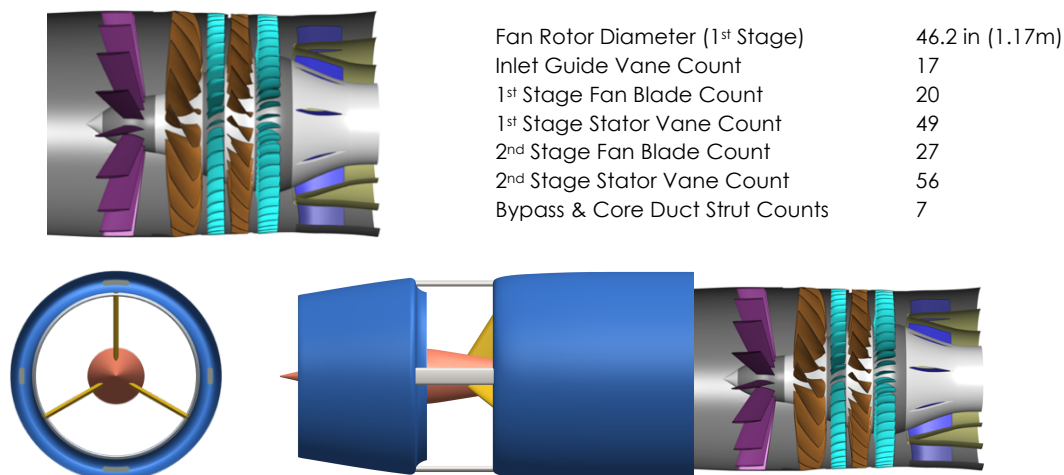


Fig. 12 Two-stage fan and supersonic inlet investigated in this paper.

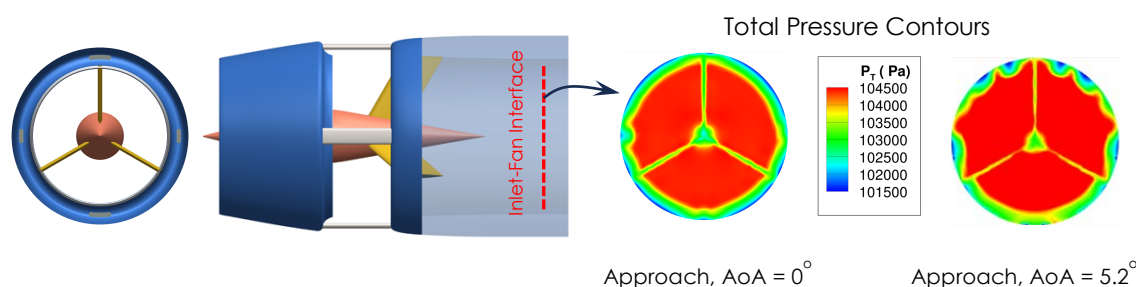


Fig. 13 Predicted contours of total pressure in the distorted inflow upstream of the two-stage fan.

The NLH computational domain for the two-stage fan is shown in Fig. 14 in which the blocks associated with each blade row are shown in different color for clarity. Change of color from one set of blocks to the other represents the transition from one frame of reference to another starting with the stationary blocks for the IGV (i.e., the purple blocks). This high-resolution grid, which contains one passage of each blade row, is comprised of roughly 19.8 million points. The equivalent full wheel grid for a URANS simulation would have required 413 million grid points. The NLH simulations for this fan included the first three harmonics of the blade/vane passing frequency for each blade row. For each operating condition, the bypass and core duct exit plane average static pressure levels were set in order to achieve the total target mass flow rate for the fan at that condition. The resulting predicted mass flow split as well as the fan pressure ratio for each simulation were consistent with those supplied by General Electric.

The parallelized NLH simulations were run using a vintage 2022 desktop computer in double precision on 32 cores using 170 GB of memory. The simulations took 76 hours each to run for 4000 iterations to achieve convergence. The flow solutions include a large set of the mean flow and thermodynamic variables as well as the real and imaginary parts of the harmonic perturbations of the flow variables. An example of the simulation results the real part of the harmonic pressure perturbations are shown in Fig. 15 for an axial slice of the computational domain just upstream of the spinner nose at the sideline operating condition (6673 rpm) and an angle of attack 6.7° . The $1 \times \text{BPF}$ harmonic pressure perturbations are clearly dominant upstream of the IGV and are in the range between -500 Pa to 500 Pa. The corresponding $2 \times \text{BPF}$ harmonic levels are an order of magnitude smaller, and the $3 \times \text{BPF}$ harmonic levels are much smaller yet and between -1 Pa and 1 Pa. The dominant propagating Tyler-Sofrin circumferential modes at $1 \times \text{BPF}$ are $m = 3$ and $m = -14$. These modes are clearly discernable in the contour plots for $1 \times \text{BPF}$ showing a 14-interlaced pattern in the outer region of the contour surrounding a 3-interlaced pattern in the inner region. There are, of course, several other propagating circumferential modes at this frequency, but they are much weaker and do not contribute significantly to the total sound pressure level at $1 \times \text{BPF}$.

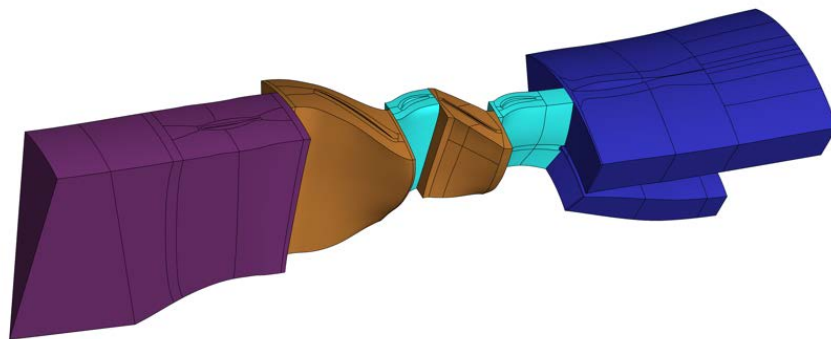


Fig. 14 Two-stage fan NLH computational domain with various blade row blocks highlighted.

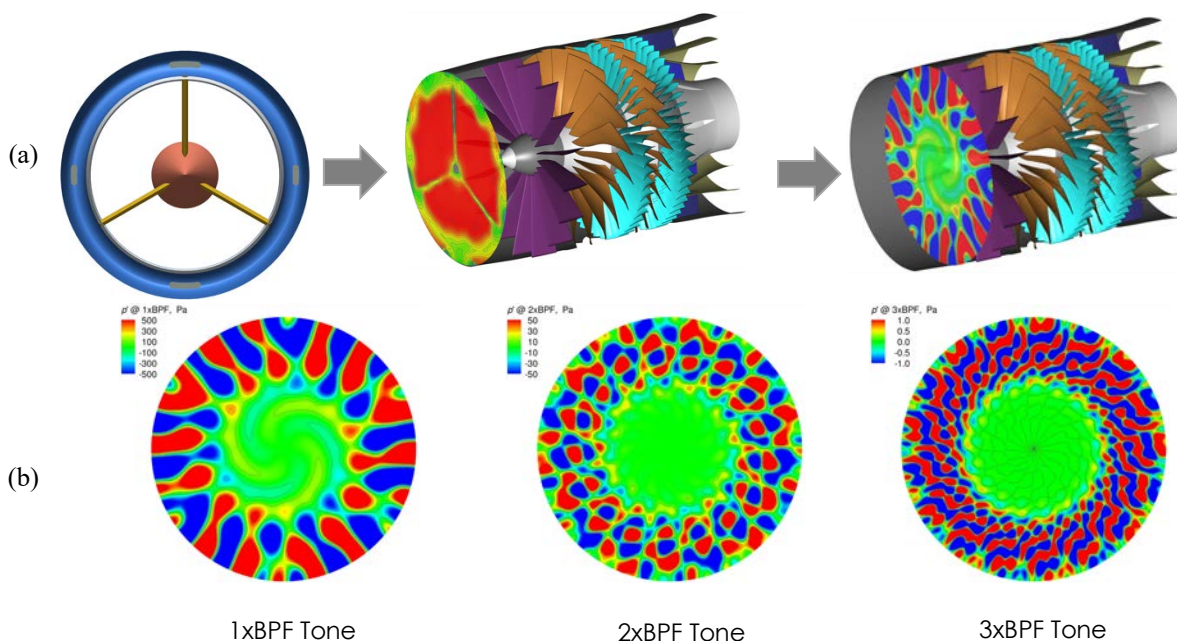


Fig. 15 Inlet flow distortion noise simulation steps. (a) The inflow distortion computed using RANS and the harmonic pressure perturbations upstream of the IGV computed using NLH. (b) Contours of the real part of the pressure perturbations for the first three harmonics of BPF.

VIII. Inlet Distortion Noise Study

To investigate the impact of the inlet flow and inlet hardware components on the inflow distortion noise levels generated by the fan, a parametric study was carried out by systematically removing the various inlet hardware components and running the NLH simulations with the corresponding inflow distortion pattern. The configurations studied are depicted in Fig. 16. In part (a) the baseline configuration is shown for which the translating cowl is removed making the inlet essentially a conventional axisymmetric inlet which causes no inflow distortion at the zero angle of attack. Part (b) depicts the complete inlet with the translating cowl in the forward position with the auxiliary inlet doors open. This configuration represents the most realistic inlet configuration for the LTO operability. Part (c) shows the same as the full inlet, but with the inlet centerbody struts removed. Part (d) depicts the inlet without either the centerbody struts or the inlet actuators. Simulations were run with and without an angle of attack for each configuration except for the baseline configuration which was run only at the zero angle of attack to serve as the reference noise level. Two fan operating conditions were considered, approach and sideline. The inlet angle of attack for approach is 5.2° and that for sideline is 6.7° . As before, the first three harmonics of the blade/vane passing frequency of each blade row were included in the simulations. In each case, the inlet flowfield was computed and total pressure and total

temperature profiles were extracted at the interface plane between the inlet and the fan domains (See Fig. 13). In Fig. 17 the total pressure contours for all simulated cases for the inlet configurations (b) through (d) are shown. The total pressure and temperature are uniform for the baseline case. The acoustic contents from the corresponding NLH simulations were extracted at an axial location upstream of the fan spinner nose in order to estimate the impact of the inflow distortion caused by the various inlet configurations on the inlet tone levels. A more detailed picture can be obtained by propagating the extracted acoustic fields through the inlet and its auxiliary doors to the farfield.

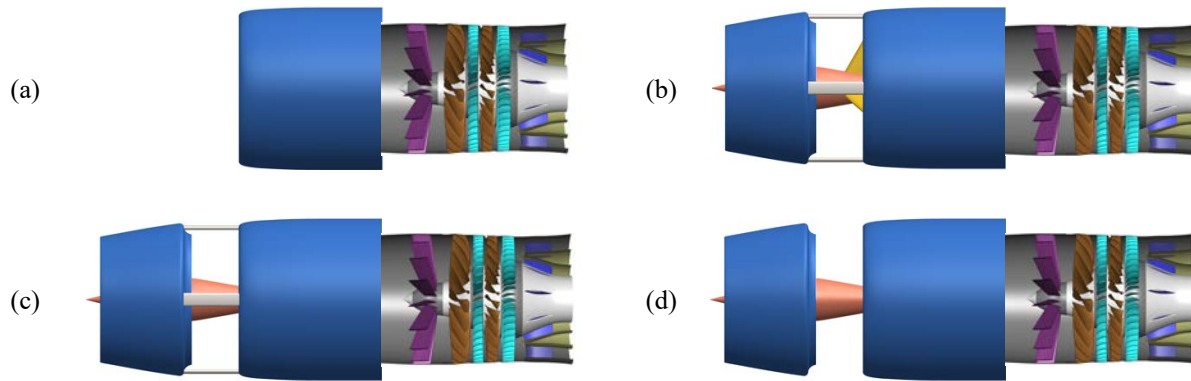


Fig. 16 The inlet configurations investigated. (a) baseline axisymmetric inlet with all hardware removed. (b) Complete inlet with the inlet cowl in the forward position. (c) Same as (b) but with the centerbody struts removed. (d) Same as (c) but with translating cowl actuators removed.

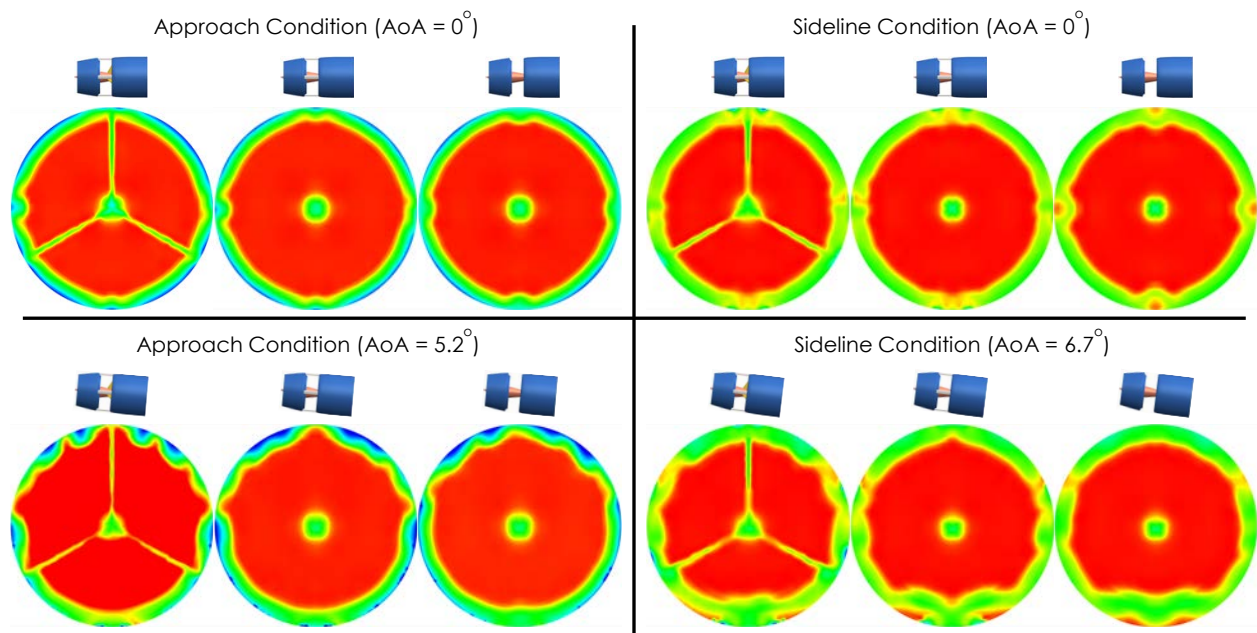


Fig. 17 The inlet distortion cases investigated. Total pressure contours are plotted.

Given the relatively large number of propagating radial modes present, acoustic power level was deemed as the best metric for judging the impact of the various inlet configurations on the noise levels for the purposes of this study. In order to account for the acoustic power $\wp$ inclusive of all of the inlet acoustic modes (propagating and evanescent) the computations were carried out using Eq. (7a), which accounts for the axial flux of the acoustic intensity I normal

to the cross section S of the inlet duct integrated over the cross section with $\hat{\mathbf{n}}$ denoting the unit normal vector to S [10].^{§§} The axial intensity is denoted by I_z and defined by Eq. (7b). In these equations, the variables with the subscript 0 denote the mean flow components and the rest denote the acoustic variables. The acoustic perturbations are the various harmonic perturbations computed in the NLH simulations. The mean flow density is denoted by ρ_0 , the mean flow velocity vector by $\mathbf{V}_0$, and its axial component by V_{0z} where z is the rotational axis of the fan. The acoustic pressure, density, and velocity vector are denoted by p' , ρ' , and $\mathbf{u}$, respectively. The three components of the acoustic velocity are denoted by u_x , u_y , and u_z , respectively.

$$\wp = \int_S \mathbf{I} \cdot \hat{\mathbf{n}} dS \rightarrow \int_S I_z dS, \quad (7a)$$

$$\mathbf{I} \equiv \left(\frac{p'}{\rho_0} + \mathbf{u} \cdot \mathbf{V}_0 \right) (\rho_0 \mathbf{u} + \rho' \mathbf{V}_0) \rightarrow I_z \equiv \left(\frac{p'}{\rho_0} + \mathbf{u} \cdot \mathbf{V}_0 \right) (\rho_0 u_z + \rho' V_{0z}) \quad (7b)$$

Expanding the expression for I_z in Eq. (7b) and arranging the resulting terms in four groups for the sake of the brevity of the notation, we arrive at the expressions in Eq. (8a) with the constituent terms I_{z1} through I_{z4} , defined in Eqs. (8b) through (8e).

$$I_{z1} = p'(u_z)^*, \quad I_{z2} = \frac{V_{0z}}{\rho_0} p'(\rho')^* \quad (8a)$$

$$I_{z3} = \rho_0 (u_x V_{0x} + u_y V_{0y} + u_z V_{0z}) (u_z)^*, \quad I_{z4} = V_{0z} (u_x V_{0x} + u_y V_{0y} + u_z V_{0z}) (\rho')^*$$

$$I_{z1} = (p'_r u_{zr} + p'_i u_{zi}) - i (p'_r u_{zi} - p'_i u_{zr}) \quad (8b)$$

$$I_{z2} = \frac{V_{0z}}{\rho_0} (p'_r \rho'_r + p'_i \rho'_i) - i \frac{V_{0z}}{\rho_0} (p'_r \rho'_i - p'_i \rho'_r) \quad (8c)$$

$$I_{z3} = \begin{aligned} & \rho_0 V_{0x} (u_{xr} u_{zr} + u_{xi} u_{zi}) - i \rho_0 V_{0x} (u_{xr} u_{zi} - u_{xi} u_{zr}) + \\ & \rho_0 V_{0y} (u_{yr} u_{zr} + u_{yi} u_{zi}) - i \rho_0 V_{0y} (u_{yr} u_{zi} - u_{yi} u_{zr}) + \\ & \rho_0 V_{0z} (u_{zr} u_{zr} + u_{zi} u_{zi}) - i \rho_0 V_{0z} (u_{zr} u_{zi} - u_{zi} u_{zr}) \end{aligned} \quad (8d)$$

$$I_{z4} = \begin{aligned} & V_{0z} V_{0x} (u_{xr} \rho'_r + u_{xi} \rho'_i) - i V_{0z} V_{0x} (u_{xr} \rho'_i - u_{xi} \rho'_r) + \\ & V_{0z} V_{0y} (u_{yr} \rho'_r + u_{yi} \rho'_i) - i V_{0z} V_{0y} (u_{yr} \rho'_i - u_{yi} \rho'_r) + \\ & V_{0z} V_{0z} (u_{zr} \rho'_r + u_{zi} \rho'_i) - i V_{0z} V_{0z} (u_{zr} \rho'_i - u_{zi} \rho'_r) \end{aligned} \quad (8e)$$

In Eq. (8a), the asterisks denote complex conjugate quantities. Since the acoustic intensity is a second order quantity, all terms involve the products of the real and imaginary parts of the perturbation quantities. Finally, adding together all four components of the axial intensity flux and integrating the resulting sum over the cross section of the inlet duct provides the acoustic power $\wp$ in the inlet. It should be noted that all of the variables in Eqs. (8b-8e) are explicitly available from the NLH simulations. To account for the contribution of the complex conjugate part of the axial intensity flux to the acoustic power, the real part of the axial intensity flux is doubled in computing $\wp$ as shown in the equation below,

$$\wp = \int_S 2 \operatorname{Re}(I_{z1} + I_{z2} + I_{z3} + I_{z4}) dS. \quad (9)$$

^{§§} The implied assumption here is that flow components (mean flow and perturbations) can be approximated as irrotational and isentropic. Since the inflow distortion was introduced as total pressure and temperature profiles at the inflow boundary of the fan simulation, the flow upstream of the fan stage remains effectively irrotational except within the inlet duct boundary layer. It is likely that flow upstream of the fan is not strictly isentropic and as such the power calculations presented here are estimates only.

Using the acoustic power level metric $\wp$ the influence of the various contributors to the inflow distortion noise can be explicitly analyzed. The results are summarized in Fig. 18 where the baseline acoustic power levels are shown as the black bars. The acoustic power levels for the inlet configurations (b), (c), and (d) are shown as the colored bars. For the zero angle of attack cases, they are shown as the yellow, orange, and brown bars. The corresponding cases for the angle of attack are shown as the light, medium, and dark blue bars. The results for the approach condition are plotted in the part (a) of the figure and those for the sideline condition in the part (b). For each operating condition, the power levels for the blade passing frequency harmonics 1×BPF through 3×BPF are plotted. Note that for the sideline condition, the baseline 3×BPF acoustic power level, which is approximately 85 dB, is not visible in the plot.

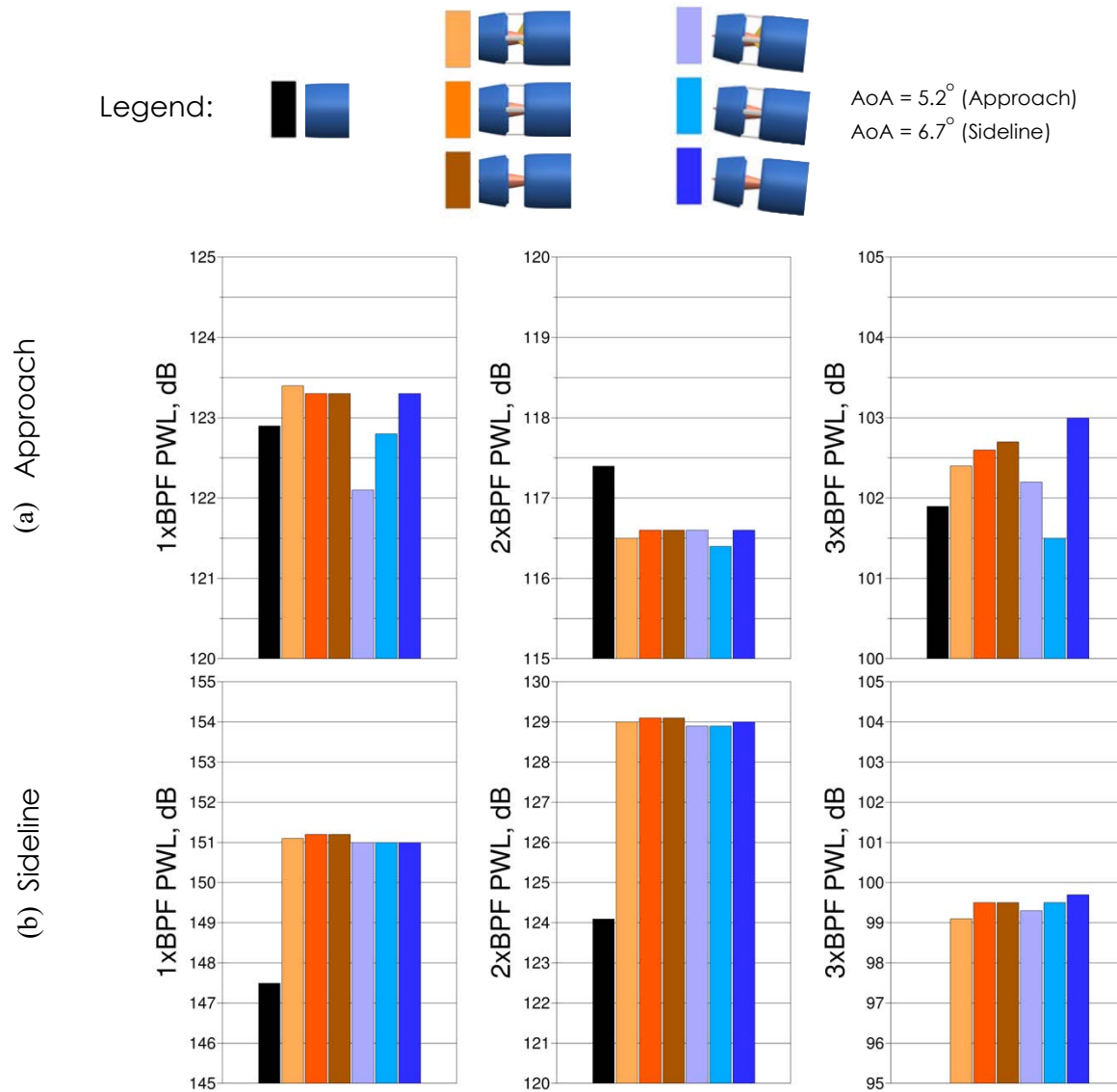


Fig. 18 Impact of the inlet distortion on tone acoustic power levels. The ranges are not the same in these plots.

Generally speaking, the introduction of the inflow distortion increases the inlet noise levels in nearly all case, the exception being the 2×BPF tone at the approach condition for which the power levels decrease for the distortion cases. The power level increases are significantly larger for the sideline condition than for the approach condition. The peak increases for the sideline case are 3.7dB at 1×BPF, 5.0 dB at 2×BPF, and 14.8 dB at 3×BPF. For the approach condition the peak increases are 0.5 dB at 1×BPF and 1.1 dB at 3×BPF. The maximum power level reduction for the approach condition at 2×BPF is 1.0 dB. For the sideline condition, an inspection of the power levels indicates that the level increases are dominated by the inflow distortion caused by the open auxiliary doors but are otherwise essentially

insensitive to the details of the inlet hardware or the angle of attack. This conclusion follows from the fact that the scatter of the power level increases for the sideline condition for the configurations (b) through (d) is less than 0.2 dB for all three harmonics with or without an angle of attack. The picture is a bit more nuanced for the approach condition in that there is some sensitivity to the inlet hardware details and angle of attack. The approach power levels for 1×BPF and 3×BPF indicate that the angle of attack influences the distortion noise level for the different inlet configurations. The increase in the power level is the largest at 1.5 dB for the full inlet at the angle of attack compared with the zero angle of attack. For 3×BPF, the angle of attack has the largest effect for the inlet configuration (c) for which the introduction of the angle of attack increases the distortion noise level by 1.2 dB. The 2×BPF power level reductions when inflow distortion is introduced are not very sensitive to the angle of attack or the inlet hardware details. Considering the relative changes in the power levels for both operating conditions suggests that the principal source of inlet distortion noise is the inflow through the auxiliary doors with the inlet hardware and angle of attack playing minor roles. It should be noted that, since the interest in this study was on the power levels changes with and without the inlet distortion, the extraneous mode contributions were not considered.

IX. Summary

Two-stage fans are among the candidate designs that are being considered for the proposed civil supersonic passenger aircraft. The prediction of fan noise from such fans is crucial in designing turbofan engines that could meet the LTO noise regulations and provide efficient performance at the supersonic cruise. In this paper, the use of a URANS approximation known as the non-linear harmonic method for computing the acoustic tone source levels for a two-stage fan is demonstrated. The NLH simulations, which keep track of a small number of pre-selected frequencies of the unsteady flow, achieve a good balance between fidelity and computing resource requirements. The NLH method enables extensive parametric studies to be carried out for exploring the design space of two-stage fans on a desktop computer. The NLH simulations model all the important mechanisms of fan noise generation including the blade row viscous interaction, potential field interaction, and acoustic transmission. It is shown that the extracted acoustic mode contents from such simulations are consistent with the measured data for the SDT fan stage lending credence to the predictions.

The NLH approach has been used to investigate the inlet noise source for a representative modern two-stage fan coupled to a supersonic inlet with a translating inlet cowl to allow additional airflow through the inlet at the LTO conditions. A parametric study has been carried out for this fan-inlet system to investigate the impact of the inflow distortion on the fan inlet tone noise and to study the influence of the various inlet hardware components on the noise levels. The predicted acoustic tone power levels indicate that inflow distortion increases the inlet noise level for the fan, and that the increase is more significant at the sideline condition than it is for the approach condition. The results also indicate that the principal source of inlet distortion noise is the flow through the open auxiliary inlet doors and that the noise is relatively insensitive to the inlet hardware components and the inlet angle of attack.

Finally, to address an intrinsic limitation of the prediction schemes that assume all blades and all vanes in their respective blade rows are identical, a probabilistic model has been introduced to account for the levels of extraneous circumferential modes. These modes are generated by the real fan hardware due to the small variations in the blade and vane stagger angles around the wheel. The inclusion of these extraneous modes is crucial in modeling the directivities and the absolute levels of the farfield tone levels.

Acknowledgement

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References

- [1] Bridges, J.E., Stephens, D.B., and Berton, J.J., “Quantifying Uncertainty of Landing and Takeoff Noise for Commercial Supersonic Aircraft,” AIAA Paper 2022-3051, 28th AIAA/CEAS Aeroacoustics Conference, June 14–17, 2022, Southampton, U.K.
- [2] Stephens, D.B. and Svetgoff, A.A., “Flow and Acoustics Simulations of a Supersonic Engine Inlet,” to be presented at the 30th Aeroacoustics Conference, Rome, Italy, June 4–7, 2024.
- [3] Envia, E., “Acoustic Power Transmission Loss Through a Ducted Fan,” AIAA Paper 206-3064, 22nd AIAA/CEAS Aeroacoustics Conference, May 30 – June 1, 2016, Lyon, France.
- [4] Gonzalez-Martino, I. and Casalino, D., “Fan Tonal and Broadband Noise Simulations at Transonic Operating Conditions Using Lattice-Boltzmann Methods, AIAA Paper 2018-3919, 24th AIAA/CEAS Aeroacoustics Conference, June 25–29, 2018, Atlanta, Georgia.

- [5] Svetgoff, A.A., Stephens, D.B., and Envia, E., “Inlet Radiated Noise Predictions for the NASA Source Diagnostic Test Fan Using Physics-Based Simulations,” AIAA Paper 2022-2941, 28th AIAA/CEAS Aeroacoustics Conference, June 14-17, 2022, Southampton, U.K.
- [6] Heidelberg, L.J., and Hall, D.G., “Inlet Acoustic Mode Measurements Using a Continuously Rotating Rake,” *J. of Aircraft*, Vol. 32, No.4, July – August 1995.
- [7] Eversman, W., “Mapped Infinite Wave Envelope Elements for Acoustic Radiation in a Uniformly Moving Medium,” *J. of Sound and Vibration*, 224(4), pp. 665-687, 1999.
- [8] Meyer, H.D. and Envia, E., “Aeroacoustic Analysis of Turbofan Noise Generation,” NASA Contractor Report 4715, 1996.
- [9] Bridges, J.E., “RTAPS SuperFan inlet distortions,” Internal summary report, private communication, July 2022.
- [10] Goldstein, M.E., *Aeroacoustics*, McGraw-Hill, New York, 1976, pp. 41.