STRING POTENTIOMETERS FOR SUPERSONIC SPLIT-LINE THRUST VECTOR AMPLIFICATION FACTOR VERIFICATION

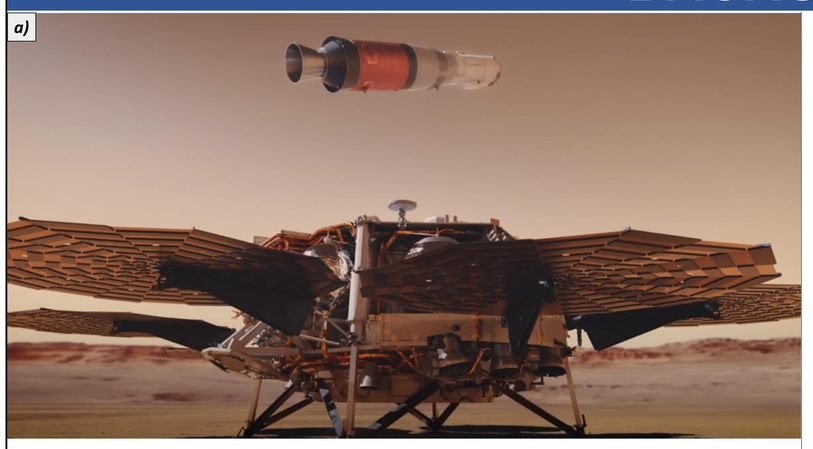
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ABSTRACT: ***The static fire of NASA's Mars Ascent Vehicle (MAV) first stage Solid Rocket Motor (SRM1) Demonstration Motor-1 (DM-1) occurred at Edwards Air Force Base (AFB) on April 7, 2023. Among other objectives, this test successfully demonstrated the Thrust Vector Control (TVC) system, including the Technology Readiness Level (TRL) of the Supersonic Split-Line (SSSL) nozzle. An essential characteristic of the SSSL is the thrust vector amplification factor (TVAF) it exhibits; however, empirically quantifying this behavior is difficult due to the limitations of measuring the internal nozzle gas dynamics of a Solid Rocket Motor (SRM) during burn. Using four string potentiometers mounted inside the test chamber connected to the nozzle and motor case and a thrust measurement system (TMS), the TVAF can be derived from the known nozzle and motor geometry. Increasing the accuracy of the TVAF is essential for certain groups working on the MAV including the Guidance, Navigation, and Control (GN&C) group, who prior to DM-1, solely relied on Computational Fluid Dynamics (CFD) data in their Monte Carlo simulations for TVAF. The methodology shown in this paper can be generally applied to any test article implementing a SSSL nozzle, presuming string potentiometers are a viable option and the facility is conducive of their implementation.***

Figure Legend:
- ***a) Rendering of MAV being thrown from Martian lander; one of the unique challenges of the MSR campaign and to TVC specifically***
- ***b) SSSL nozzle internal aerodynamics CFD analysis, illustrating variation in Mach number throughout [3, 6, 7]***
- ***c) Simple illustration of string potentiometer construction***
- ***d) Free body diagram of the test article used to calculate thrust vector angle, θ_{TV}.***
- ***e) Visualization of Euler transformations using the DM-1 kinematic math model.***
- ***f) Gimbal center reference location motion derived from kinematic math model of DM-1 test.***
- ***g) Graph of the DM-1 reconstructed gimbal angles derived from the kinematic math model.***

BACKGROUND

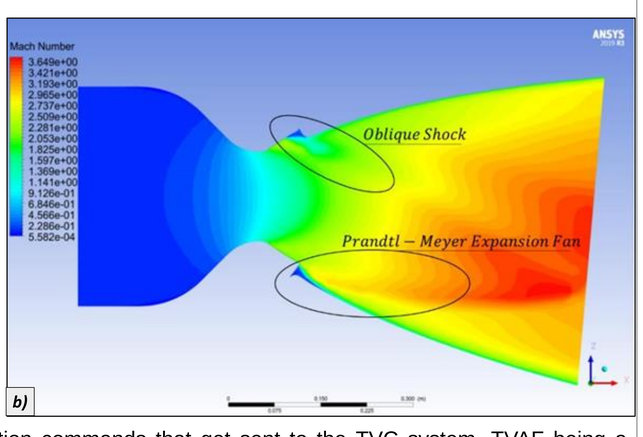
a)



The Mars Sample Return (MSR) campaign is a series of missions lead by NASA with the European Space Agency (ESA), targeting the objective of collecting Martian samples and returning them to Earth for scientific research. For the task of getting samples off the surface of Mars to a retrievable location, NASA's Marshall Space Flight Center (MSFC) in collaboration with Northrop Grumman (NG) is building MAV, with Lockheed Martin (LM) as vehicle integrator. Designated as a Class A project, it will launch from the Jezero Crater on Mars and deliver its payload of samples to Martian orbit for capture by a separate Earth-return spacecraft. The vehicle configuration is in two stages: a guided first stage featuring TVC, and an unguided second stage that is spin-stabilized. This Solid-Solid Guided-Unguided (SSGU) architecture is primarily a result of mass constraints, which lead to design choices such as the inclusion of the SSSL nozzle on the first stage. The SSSL is a mass conscious choice as its intrinsic characteristic of supplying the TVAF to a commanded gimbal angle provides greater thrust vectoring capability with smaller electromechanical actuators (EMAs). The trade-off being that the SSSL has lower TRL and additional complexity relative to more commonly implemented nozzles (vectorable unsubmerged/submerged, flex bearing, etc.). One of the primary objectives of the MAV DM-1 static fire test was to empirically define the TVAF of the vehicle's nozzle and use this data to refine flight profile models. Measuring the TVAF is a technical challenge as: there is only one opportunity to measure it per each conducted test given the SRM is single-use, the environmental effects from test cell and motor (e.g. vacuum, cold temperature), and the nozzle measurement needs to be referenced independently from vehicle body due to the dynamic geometry of the motor as a function of pressure. The aim of this work is to show how string potentiometers were successfully implemented to overcome these challenges, detail the test data provided by them for TVAF analysis, and propose how this methodology represents an industry standard for test of SSSL nozzles and mechanical gimbal angle for any nozzle architecture.

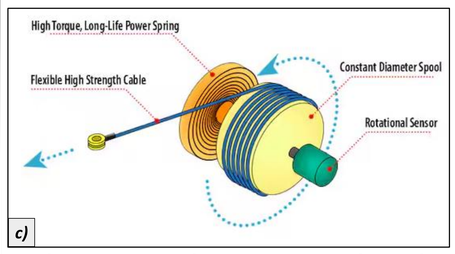
Overview of SSSL Nozzle and Characteristic TVAF

The TVAF of an SSSL nozzle is integral to the successful deposition of Martian samples to orbit. This fact alone makes empirically bounding it necessary. GN&C must compensate for TVAF in the nozzle position commands that get sent to the TVC system. TVAF being a function of thrust, compensation is also a function of thrust, which then GN&C estimates from inertial measurement unit (IMU) acceleration data. This compensation is also part of GN&C's Monte Carlo simulations. To summarize:



b)

- SSSL chosen for MAV because inherently lower mass required to gimbal the thrust on the guided first stage.
- Shockwaves reflect off the nose tip of the exit cone as it is inserted into the supersonic portion of the flow during gimbal events.
- Characteristic shockwaves in the SSSL nozzle are the result of producing a Prandtl-Meyer expansion fan and an oblique shock, which promotes an increased thrust vectoring capability – TVAF.
- TVAF results in effective mechanical nozzle deflection by a factor of 1.1 – 1.6 and decreases the time constant and packaging volume of the TVC system.



c)

String Potentiometer Rationale

String potentiometers were necessary for DM-1 because they allow for accurate measurement of nozzle position that on-motor sensors miss due to their inertial frame of reference being the same as the nozzle's. Specifically, the string potentiometers account for contributions to nozzle mechanical gimbal angle from **loop stiffness**, **hot null bias**, and **cross-talk**. By being independent of nozzle position, the string potentiometers accurately measure nozzle position, thus facilitating TVAF determination.

Loop Stiffness: The complete test article (motor, actuators, vacuum chamber mounting points) form the load path, which can be considered an arrangement of very stiff springs. This is an obstacle because geometric nozzle deflection measurement as a commanded gimbal angle is indistinguishable in the vehicle inertial frame from structural deformation induced angular deflections. In fact, if accelerated back and forth, at or near the structural resonance frequency, it is possible to move the nozzle at a relatively large sinusoidal amplitude with actuator feedback registering little motion. Actuator feedback is agnostic to structural deflection induced nozzle angles due to their shared frame of reference, so string potentiometers compensate for this by the loop stiffness contribution to gimbal angle being captured in their sensed displacement.

Hot Null-Bias: This is a compensation angle commanded by the vehicle to offset the mechanical angle induced by motor case and nozzle deformation under pressure. When a rocket motor is pressurized, the case and nozzle can be observed to change Vehicle instrumentation includes this bias in its gimbal angle commands, so it is captured by its own telemeters, thus, the independent string potentiometers are needed to understand the true nozzle position.

Cross-Talk: The pitch and yaw actuators control the same nozzle; thus, they are dynamically linked. When actuators in this control scheme are both influencing the mechanical gimbal angle, their commands invariably impact each other. While this is normal occurrence, some error can be introduced in measurements of nozzle position in the circumstances where both actuators are influencing gimbal angle. This effect is known as cross talk, and therefore it is necessary to independently verify nozzle angle, since the on-motor position measurement sensors like the linear variable.

String Potentiometer Functionality

Relative simplicity, along with its reliability, availability, and resolution, has made it a popular option for gathering test data in the aerospace industry since at least the Apollo era. Can be thought of as simple Voltage Divider:

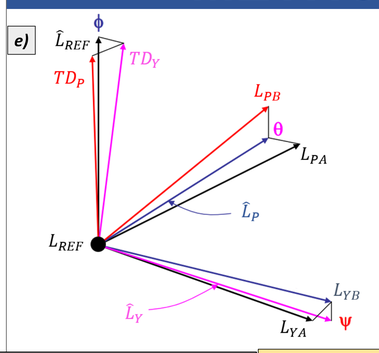
$$V_{out} = \frac{Z_2}{Z_1+Z_2} V_{in},$$

Converts linear displacement into a proportional voltage.
Example:

$$V_{out} = 4\ V$$
$$V_{exc} = 10\ V \text{ and } FS = 30\ in$$
$$x_L = \frac{4V}{10\ V} 30\ in = 12\ in$$

METHODOLOGY

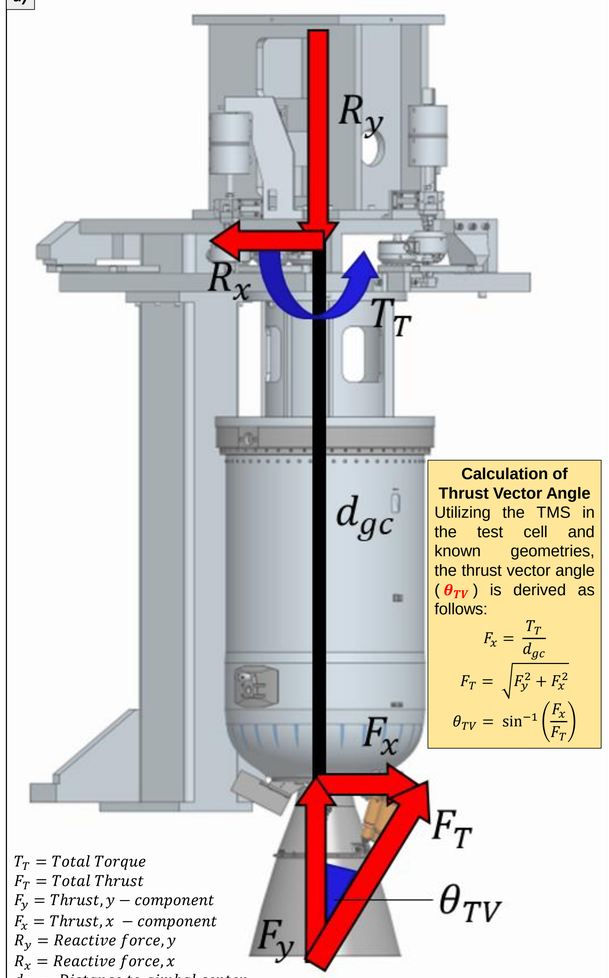
e)



GREY BOX MODEL

Data Collection → Interpolation → Level Shifting → Filtering → Solving Nonlinear System → Mechanical Gimbal Angle

d)



$T_T = Total\ Torque$
$F_T = Total\ Thrust$
$F_y = Thrust, y - component$
$F_x = Thrust, x\ - component$
$R_y = Reactive\ force, y$
$R_x = Reactive\ force, x$
$d_{gc} = Distance\ to\ gimbal\ center$
$\theta_{TV} = Thrust\ Vector\ angle$

Calculation of Thrust Vector Angle

Utilizing the TMS in the test cell and known geometries, the thrust vector angle ($\boldsymbol{\theta_{TV}}$) is derived as follows:

$$F_x = \frac{T_T}{d_{gc}}$$
$$F_T = \sqrt{F_y^2 + F_x^2}$$
$$\theta_{TV} = \sin^{-1}\left(\frac{F_x}{F_T}\right)$$

Kinematic Math Model: Under the Hood

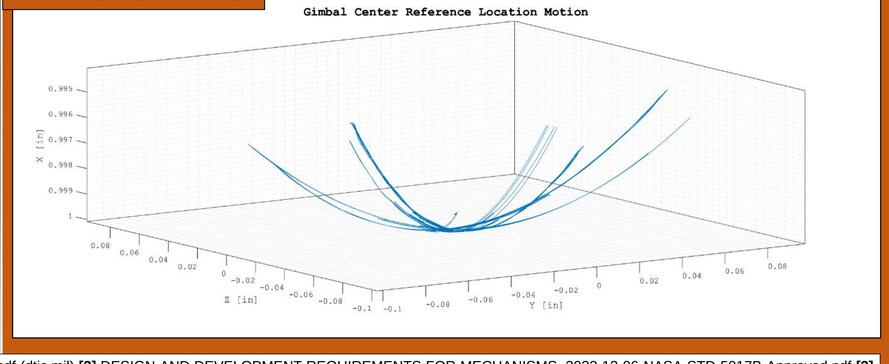
The model utilizes the fundamentals of the kinematic of moving frames, assuming a 2-DOF rigid pendulum model. First, the frames of reference are established from the nozzle gimbal center and the string potentiometer mounting locations, which set the position of the test article. These geometric parameters are expressed as: $L_{REF} = [x_{L_{REF}} \quad y_{L_{REF}} \quad z_{L_{REF}}]$ where L_{REF} is the gimbal nozzle reference position vector, attached to the gimbal center-of-rotation (COR), and pointing along the longitudinal center line of the nozzle, orthogonal to the nozzle exit-plane. The nozzle pitch string potentiometer (L_{PA}), nozzle yaw string potentiometer (L_{YA}), motor pitch string potentiometer (L_{PB}), and motor yaw string potentiometer (L_{YB}) position vector attached locations are defined in the same way as L_{REF}. These coordinates are then unit vectorized to be parsed by the model. Before any rotation calculations can be performed, the gimbal center coordinates have to match the inertial frame of reference. The unit vectors of the dynamic nozzle gimbal center and static test article geometric center are calculated to achieve this via: $\hat{L}_{REF} = \frac{L_{REF}}{\|L_{REF}\|}$,This is done for the string potentiometer coordinates as well $\hat{L}_P = \|L_{PB} - L_{PA}\|$, $\hat{L}_Y = \|L_{YB} - L_{YA}\|$ where $\hat{L}_P$ and $\hat{L}_Y$ are the unit vectors orthonormal to the $\hat{L}_{REF}$ unit vector. A visualization of Euler transformations using the DM-1 kinematic math model is visualized in figure 9. In addition to the unit vectors $\hat{L}_{REF}$, $\hat{L}_P$, and $\hat{L}_Y$, the Euler angles ϕ, θ, and ψ, pitch and yaw data from the start (TD_P and TD_Y, respectively) are utilized to inform the nozzle position relative to L_{REF}. With the unit vector of the reference location position vector with respect the inertial-frame defined, the rotation transformation matrix about the gimbal-center from the body-frame to the inertial-frame is needed. This is defined as: $T = \left[e^{\left(\begin{bmatrix} 0 & \delta\phi & -\delta\theta \\ -\delta\phi & 0 & \delta\psi \\ \delta\theta & -\delta\psi & 0 \end{bmatrix} * \alpha \right)} \right]^T$ where α is the angle net-angular deflection between the unit reference position vector ($\hat{L}_{REF}$) and the unit position vector ($\hat{Z}$). This equation is an alternate form of Rodrigues' Rotation Formula. To determine the instantaneous final angular position of $\hat{L}_{REF}$, rotation matrices are applied successively, in a yaw-pitch-roll / Z-Y-X rotation order. ${\hat{L}_{REF}}^3$ is the final position of the reference vector after the Z-Y-X rotation order application of the Euler angles. Finally, the mechanical gimbal position can be derived using a non-linear spherical geometry solver on the following system of equations:

$$F_1 = \|\hat{Z}\| - \|\hat{L}_{REF}\|$$
$$F_2 = \|\hat{Z} - \hat{L}_{PB} + T(L_{PA} - L_{REF})\| - \|L_{PB} - L_{PA}\| + TD_P$$
$$F_3 = \|\hat{Z} - \hat{L}_{YB} + T(L_{YA} - L_{REF})\| - \|L_{YB} - L_{YA}\| + TD_Y$$

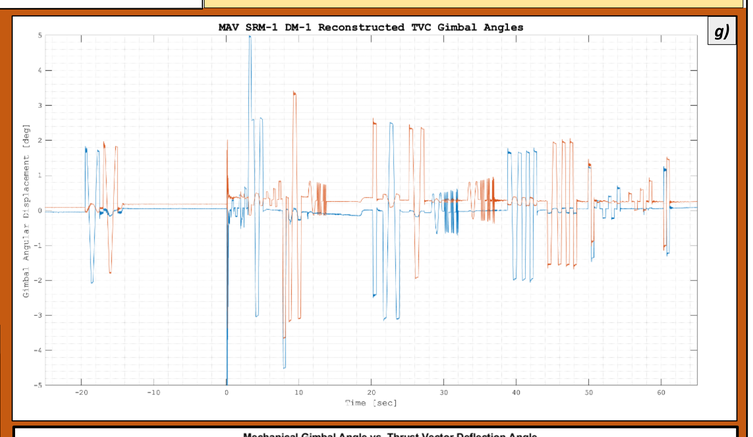
where F_1, F_2, and F_1 are the solutions to the vector equations used by the nonlinear solver to calculate gimbal angles at every time step. The nonlinear solver function can be studied in greater detail in [15], which describes the method as a compromise between the Newton-Raphson algorithm and the method of steepest descents and is applied to minimize a function to combine fast convergence with steady progress. The output of the nonlinear solver (Figure ***f***, ***g***) provides the mechanical gimbal angle ($\boldsymbol{\theta_G}$), which will be used to determine TVAF.

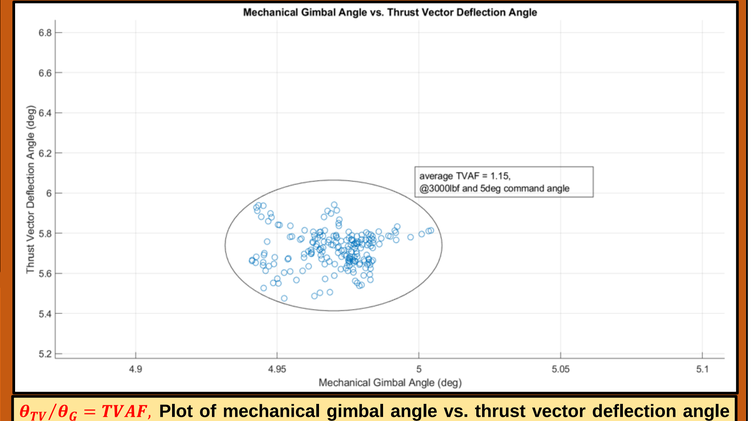
RESULTS

f)



g)





$\theta_{TV}/\theta_G = TVAF$, Plot of mechanical gimbal angle vs. thrust vector deflection angle at 3000lbf of thrust and 5 degrees of commanded gimbal angle in DM-1 duty cycle shows results of model; average TVAF = 1.15, a result consistent with CFD.

[1] Strome, R. K., *Test Firing of a Supersonic Split-Line Nozzle,* Technical Report AFRPL-TR-69-203 (October 1969) AD0865977.pdf (dtic.mil) **[2]** DESIGN AND DEVELOPMENT REQUIREMENTS FOR MECHANISMS, 2022-12-06-NASA-STD-5017B-Approved.pdf **[3]** Ravnan, S., *Advantages of a Supersonic Split-Line Nozzle over a Movable Nozzle,* TU Delft, (March 2021) thesis_sigurd_ravnan_4159160_final_version.pdf] **[4]** TE Connectivity, MT2A Cable Actuated Sensor Test Applications, last verified March 2024, accessed March 2024, available at https://www.te.com/commerce/DocumentDelivery/DDEController [miniature cable-extension position transducers] **[5]** Dougherty, N. Sam. "Liquid Rocket Engine Testing - Historical Lecture: Simulated Altitude Testing at AEDC." (2010). **[6]** NASA-GRC, "Prandtl-Meyer Expansion Fan," last verified January 2011, accessed March 2024 [Prandtl-Meyer Expansion (nasa.gov)] **[7]** NASA-GRC, "Oblique Shock on a 15 degree Wedge at Mach 2.5," last verified February 2021, accessed March 2024, available at https://www.grc.nasa.gov/WWW/wind/valid/wedge/wedge.html Oblique Shock on a 15 Degree Wedge at Mach 2.5 (nasa.gov) **[8]** R. Ellis, M. Berdoyes, R. Ellis and M. Berdoyes. "Supersonic splitline (SSSL) flexseal nozzle technology evaluation program," AIAA 1997-2721. 33rd Joint Propulsion Conference and Exhibit. July 1997. **[9]** NASA-GRC, "Isentropic Flow," last verified May 2021, accessed March 2024, available at https://www.grc.nasa.gov/WWW/K-12/airplane/isentrop.html Isentropic Flow Equations (nasa.gov) **[10]** NASA-GSFC, "Mars Fact Sheet," last verified January 2024, accessed March 2024, available at https://www.grc.nasa.gov/WWW/K-12/airplane/isentrop.html Mars Fact Sheet (nasa.gov). **[11]** Perov, P., "The Physics of Guitar String Vibrations," *American Journal of Physics*, 84, 38–43 (2016) The physics of guitar string vibrations | American Journal of Physics | AIP Publishing **[12]** Hauth, J., *Grey-Box Modelling for Nonlinear Systems*, Technical University of Kaiserslautern, December 2008. D:/Promotion/Dissertation/diss.dvi (rptu.de) **[13]** Hiroshi Akima. 1970. A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures. J. ACM 17, 4 (Oct. 1970), 589–602. https://doi.org/10.1145/321607.321609 A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures (acm.org) **[14]** MIT, "Kinematics of Moving Frames," Design of Electromechanical Robotic Systems, (Fall 2009). **[15]** Powell, M. J.D.. A Fortran subroutine for solving systems of nonlinear algebraic equations. United Kingdom: N. p., 1968. Web.