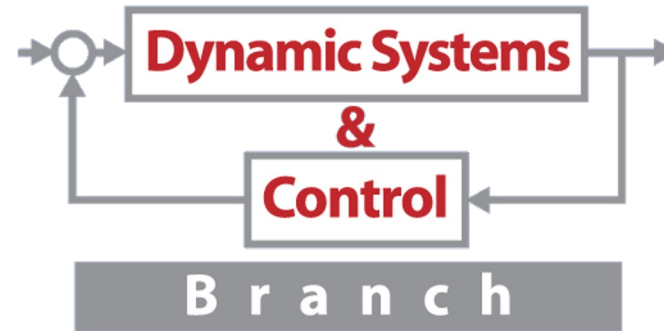


A Scenario-Based Approach to Robust Control Design



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Motivation

- Automated control tuning
- Improving control robustness
- How to fix a system that fails a Monte Carlo test?



Introduction

$$\begin{array}{ll} \min_{\theta \in \Theta} & J(\theta) \\ \text{subject to:} & r_k(\theta, \delta) \leq 0 \text{ for } k = 1, \dots, n_r, \end{array}$$

- Decision variable: θ
- Uncertainty parameters/operating conditions: δ



Introduction

$$\begin{array}{ll} \min_{\theta \in \Theta} & J(\theta) \\ \text{subject to:} & r_k(\theta, \delta) \leq 0 \text{ for } k = 1, \dots, n_r, \end{array}$$

- Decision variable: θ
- Uncertainty parameters/operating conditions: δ
- Example:

Find the Q and R matrices for an open-loop plant fully prescribed by δ that minimizes the cost $\int x^T Q x + u^T R u \, dt$, while being stable, having a bounded control effort and settling time.



Introduction

$$\begin{array}{ll} \min_{\theta \in \Theta} & J(\theta) \\ \text{subject to:} & r_k(\theta, \delta) \leq 0 \text{ for } k = 1, \dots, n_r, \end{array}$$

- Decision variable: θ
- Uncertainty parameters/operating conditions: δ
- δ characterized by a sequence of scenarios

$$\mathcal{D} = \left\{ \delta^{(i)} \right\}_{i=1}^n$$



Introduction

- Non-relaxed formulation

$$\begin{array}{ll} \min_{\theta \in \Theta} & J(\theta, \xi) \\ \text{subject to:} & r_k(\theta, \delta^{(i)}) \leq 0 \text{ for } k = 1, \dots, n_r, i = 1, \dots, n, \end{array}$$

- This program might be infeasible



Introduction

- Relaxed formulation

$$\begin{aligned} \min_{\theta \in \Theta, \xi_i \geq 0} \quad & J(\theta, \xi) + \rho \sum_{i=1}^n \xi_i \\ \text{subject to:} \quad & r_k(\theta, \delta^{(i)}) \leq \xi_i \text{ for } k = 1, \dots, n_r, i = 1, \dots, n \end{aligned}$$



Introduction

- Relaxed formulation

$$\begin{aligned} \min_{\theta \in \Theta, \xi_i \geq 0} \quad & J(\theta, \xi) + \rho \sum_{i=1}^n \xi_i \\ \text{subject to:} \quad & r_k(\theta, \delta^{(i)}) \leq \xi_i \text{ for } k = 1, \dots, n_r, i = 1, \dots, n \end{aligned}$$

- Find the most robust controller (the one satisfying the requirements for as many plants as possible)



Introduction

- Relaxed formulation

$$\begin{aligned} \min_{\theta \in \Theta, \xi_i \geq 0} \quad & J(\theta, \xi) + \rho \sum_{i=1}^n \xi_i \\ \text{subject to:} \quad & r_k(\theta, \delta^{(i)}) \leq \xi_i \text{ for } k = 1, \dots, n_r, i = 1, \dots, n \end{aligned}$$

- Generate a set of controllers that trade off robustness against performance by allowing some plants (outliers) to violate the requirements.



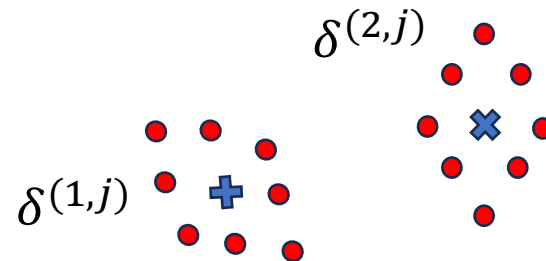
Introduction

- Robustness to the data



Introduction

- Robustness to the data
- Modeling the uncertainty in δ
 - Sets, distributions, cloud of points/scenarios



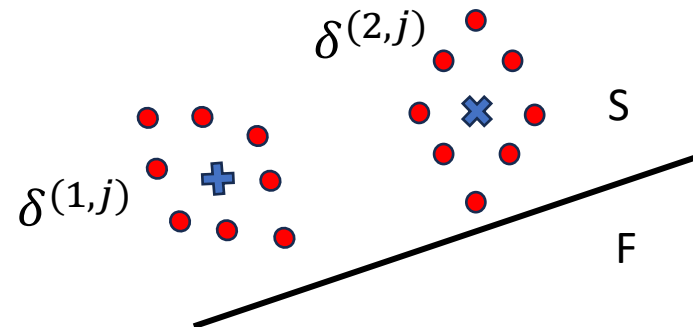


Formulations

- Worst-case design optimization

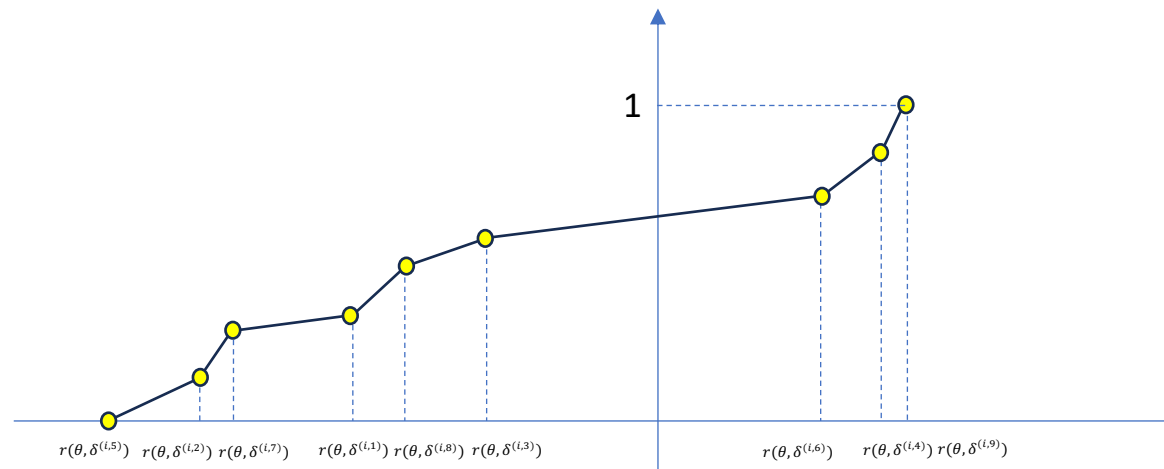
$$\min_{\theta \in \Theta, \xi_i \geq 0} J(\theta, \xi) + \rho \sum_{i=1}^n \xi_i$$

subject to: $r_k \left(\theta, \delta^{(i,j)} \right) \leq \xi_i$ for $k = 1, \dots, n_r$, $i = 1, \dots, n$ and $j = 1, \dots, m$,



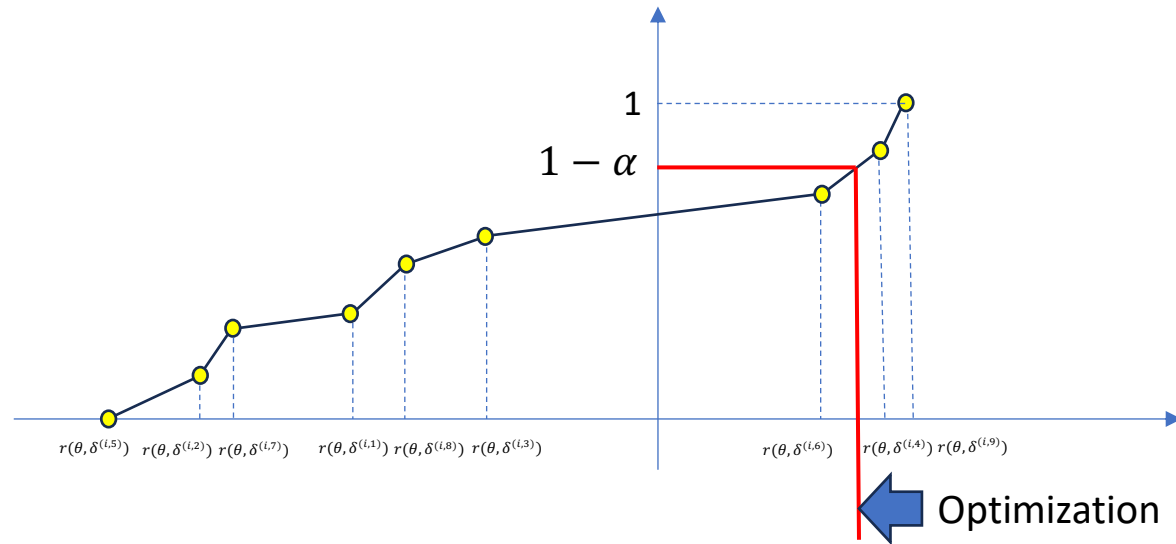
Formulations

- Chance-constrained design optimization



Formulations

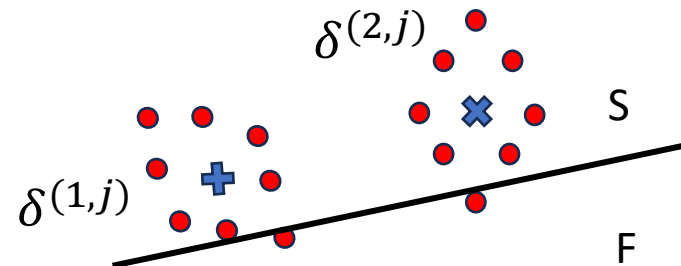
- Chance-constrained design optimization: risk



Formulations

- Chance-constrained design optimization

$$\begin{aligned} \min_{\theta \in \Theta, \xi_i \geq 0} \quad & J(\theta) + \rho \sum_{i=1}^n \xi_i \\ \text{subject to:} \quad & F_{\tilde{\mathcal{Z}}(\theta, \mathcal{D}_p, i, k)}^{-1}(\gamma_k) \leq \xi_i, \quad i = 1, \dots, n, \quad k = 1, \dots, n_r, \end{aligned}$$





Formulations

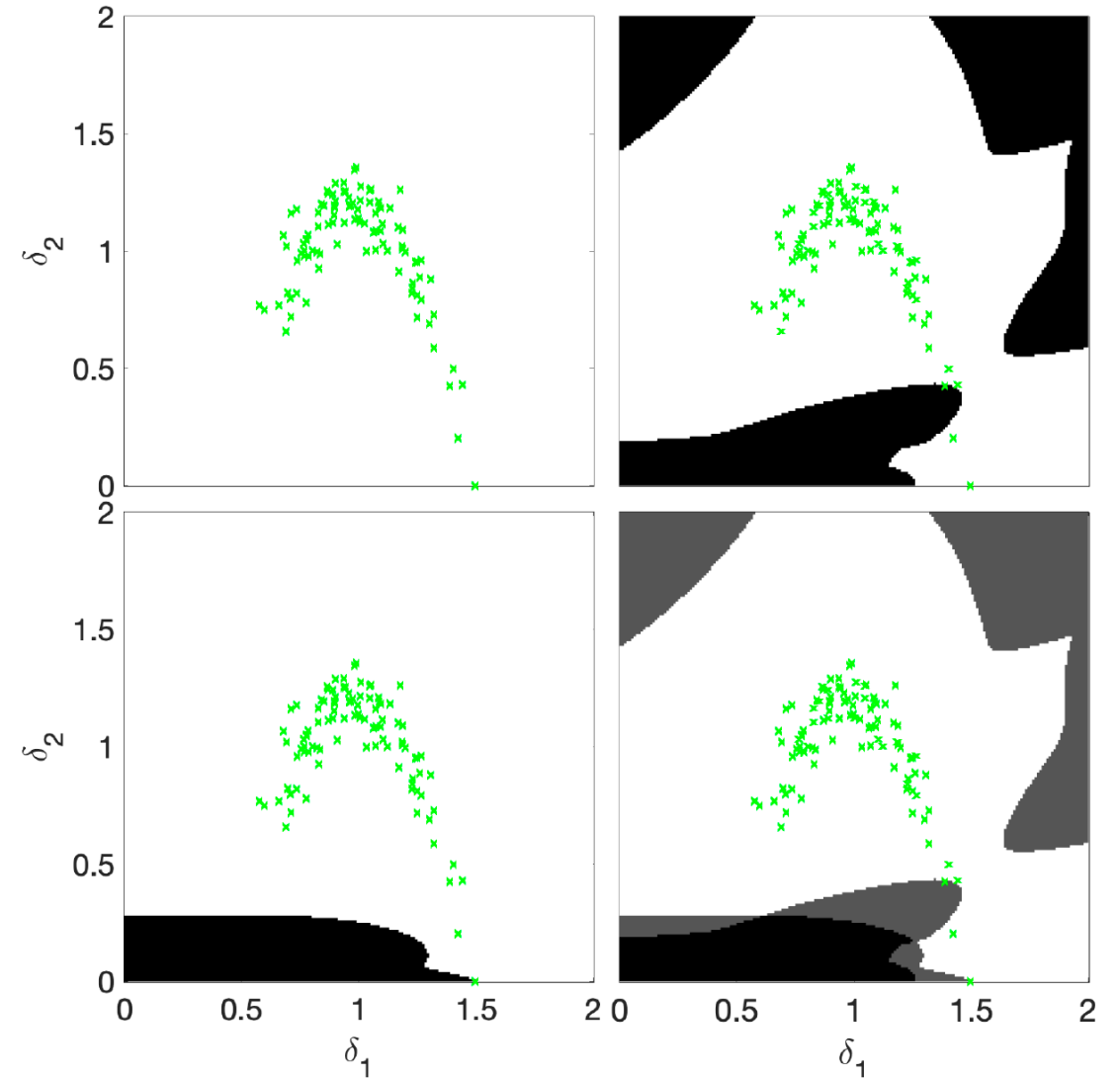
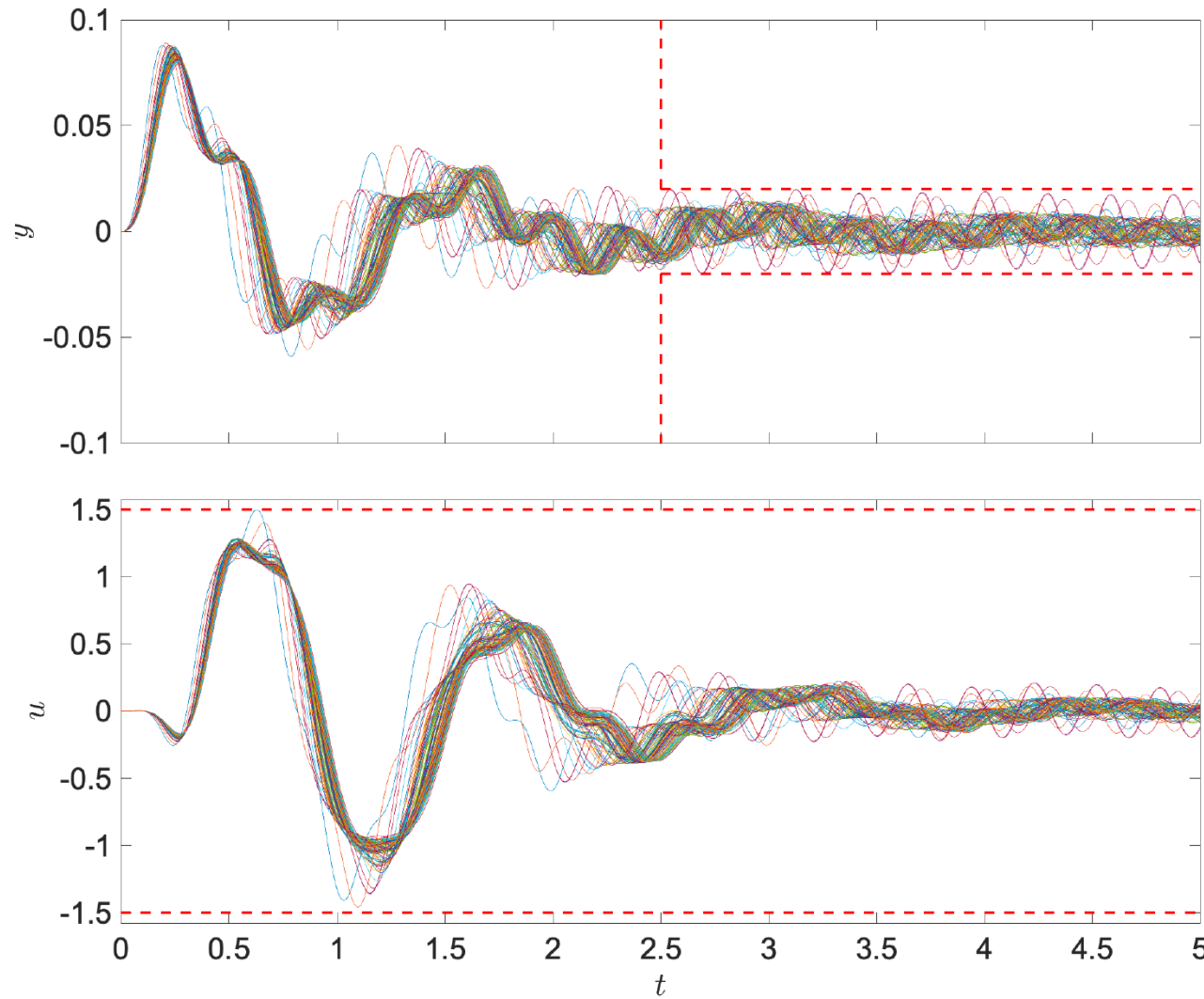
- Computational demands grow with n , not with n_δ



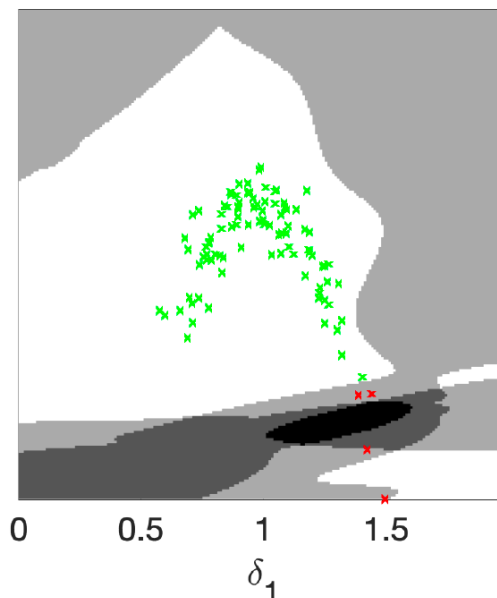
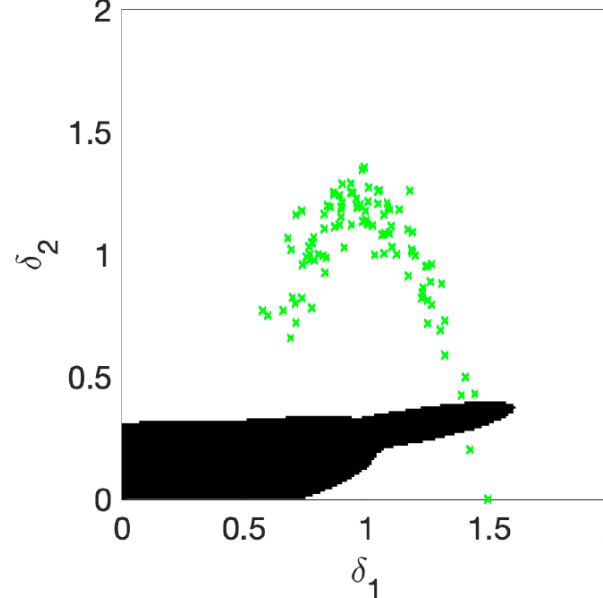
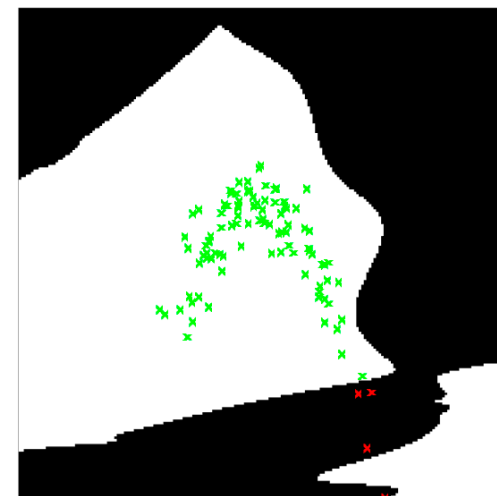
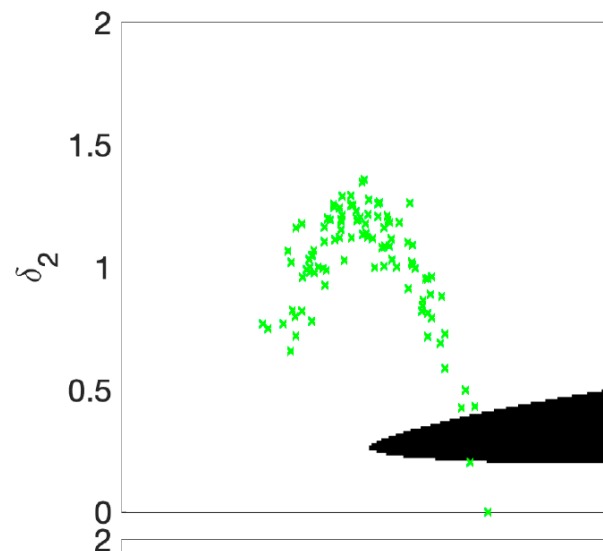
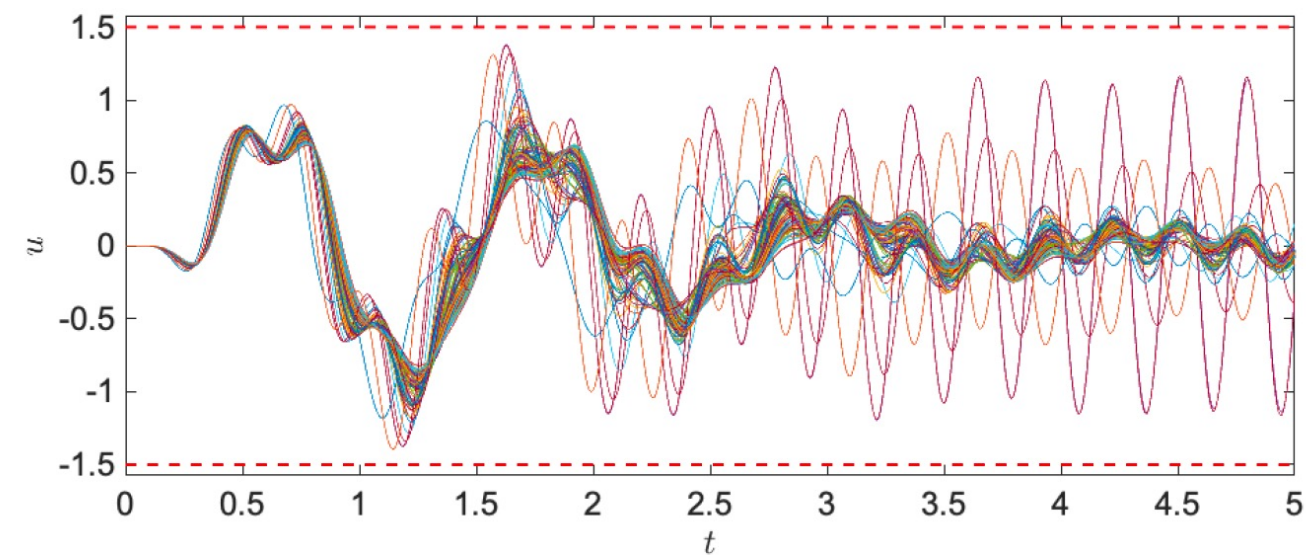
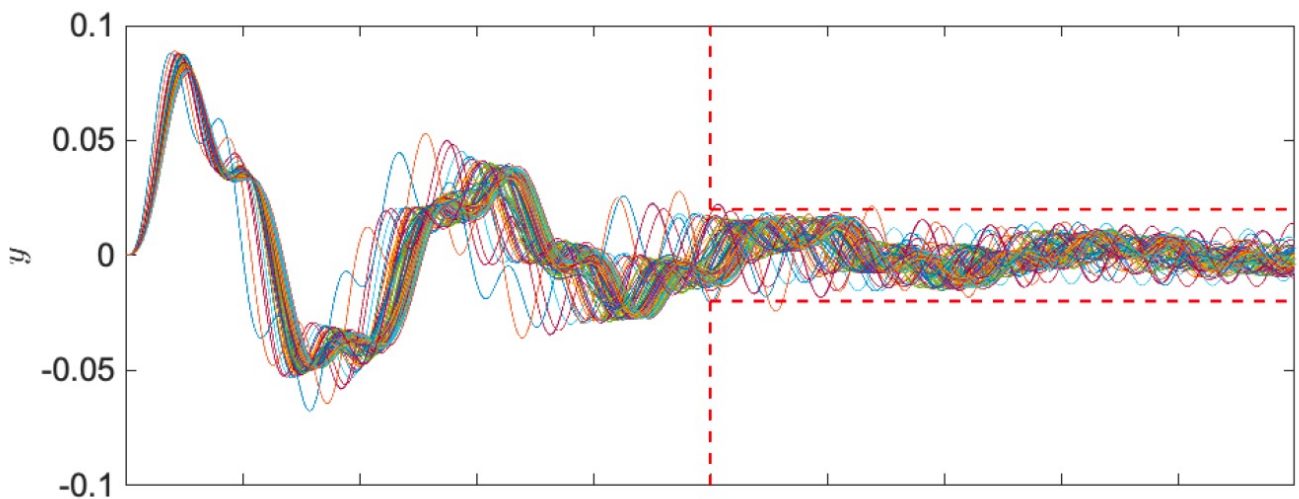
Example

- Control system: distance-separated sensor-actuator pair with 3 DOF
- θ is the set of controller gains
- δ prescribes the parameters of the open-loop plant
- The requirements
 - closed-loop stability
 - range-bounded control effort
 - settling time
- We consider $n=100$ open-loop plants

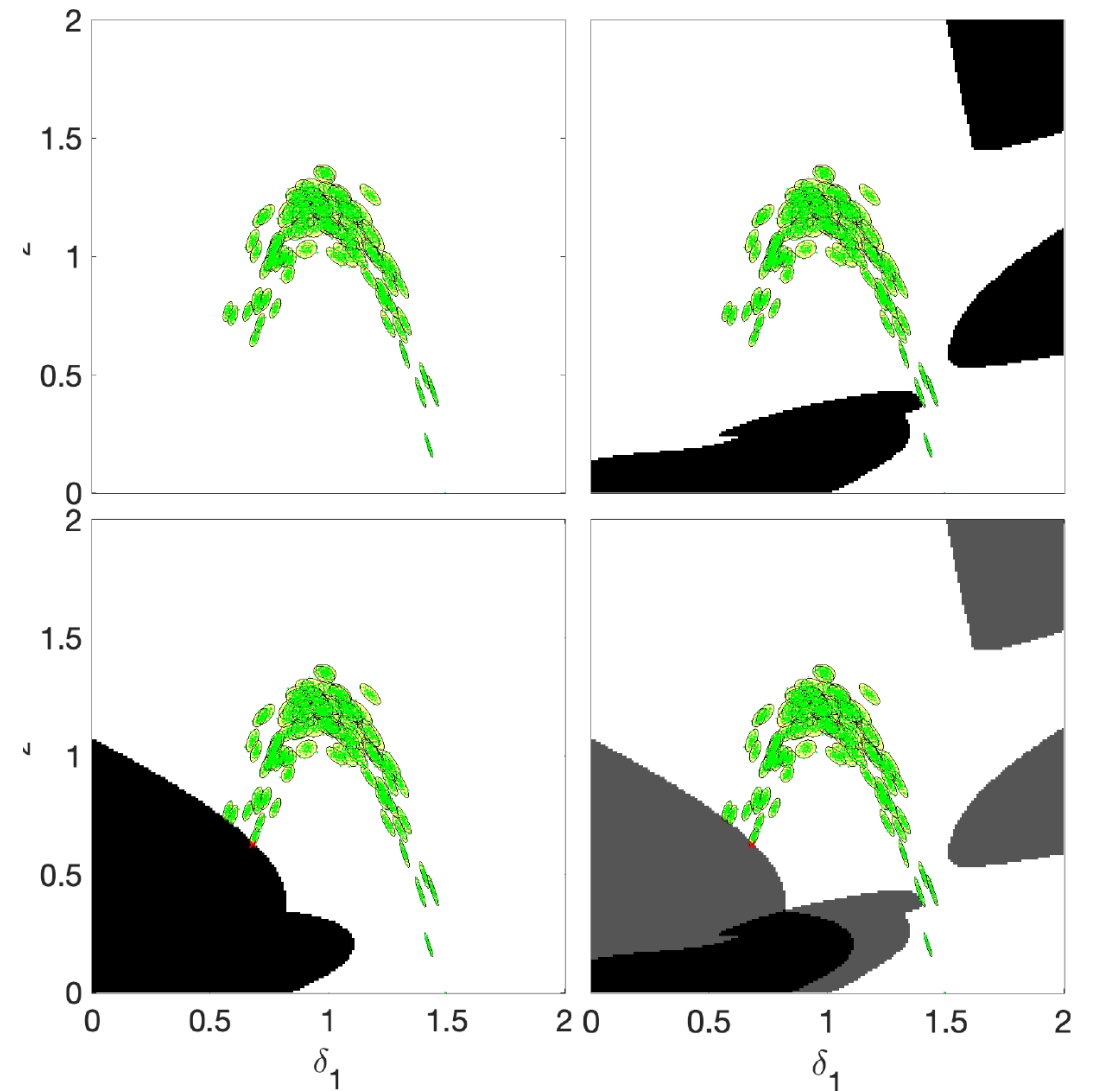
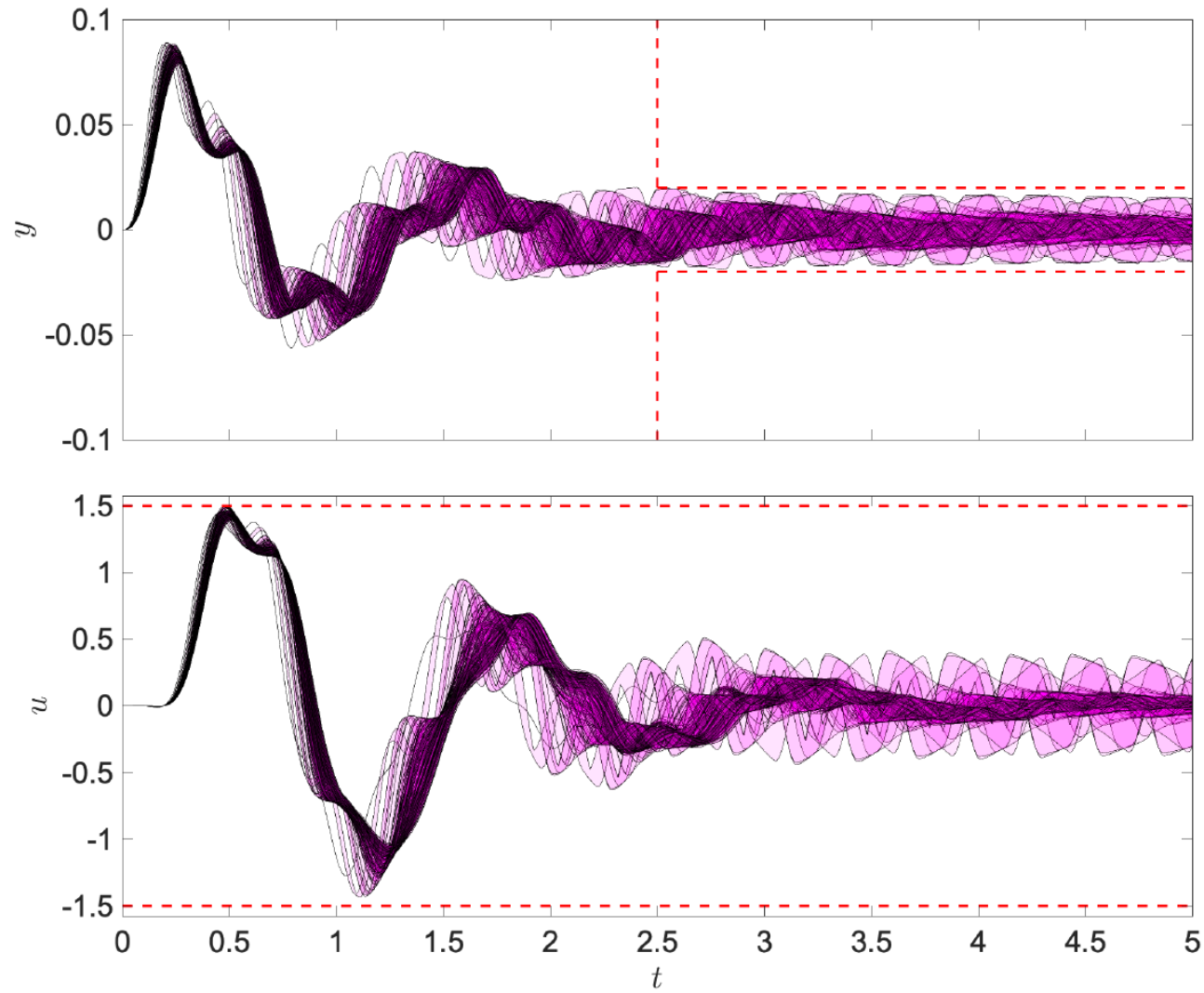
Control Example: θ_1 ($J = 3.34, n_o = 0$)



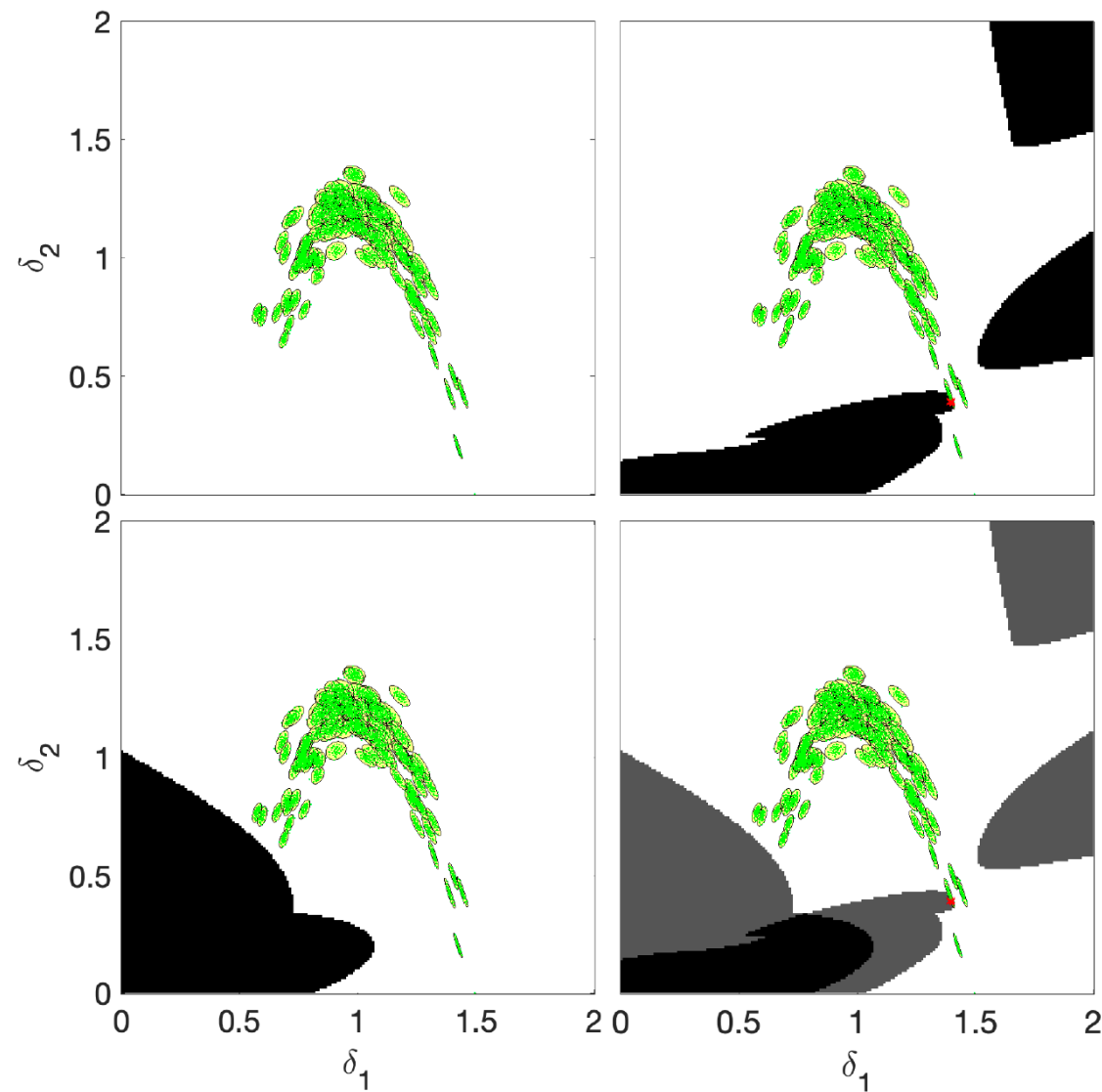
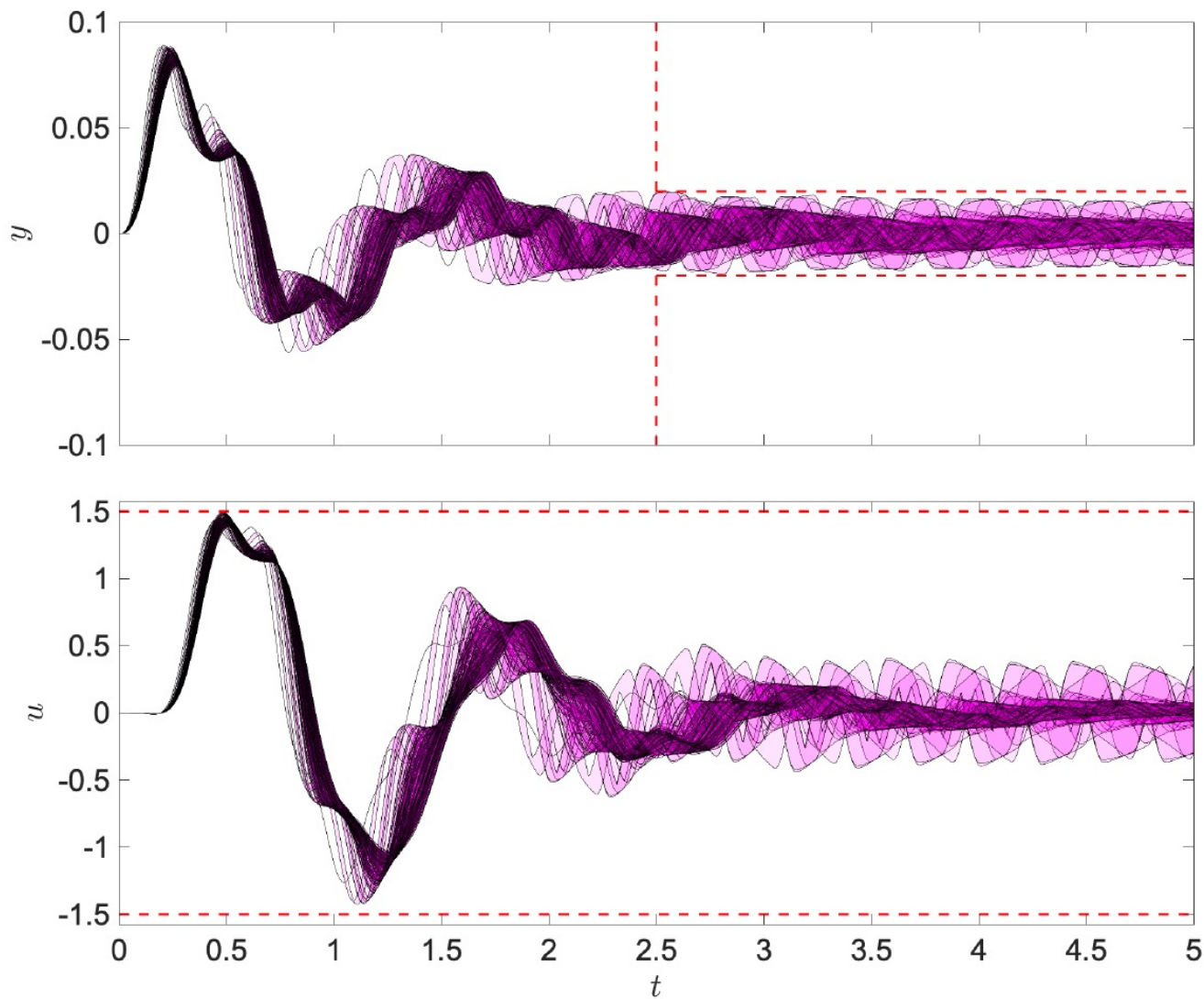
Control Example: θ_2 ($J = 1.72, n_o = 4$)



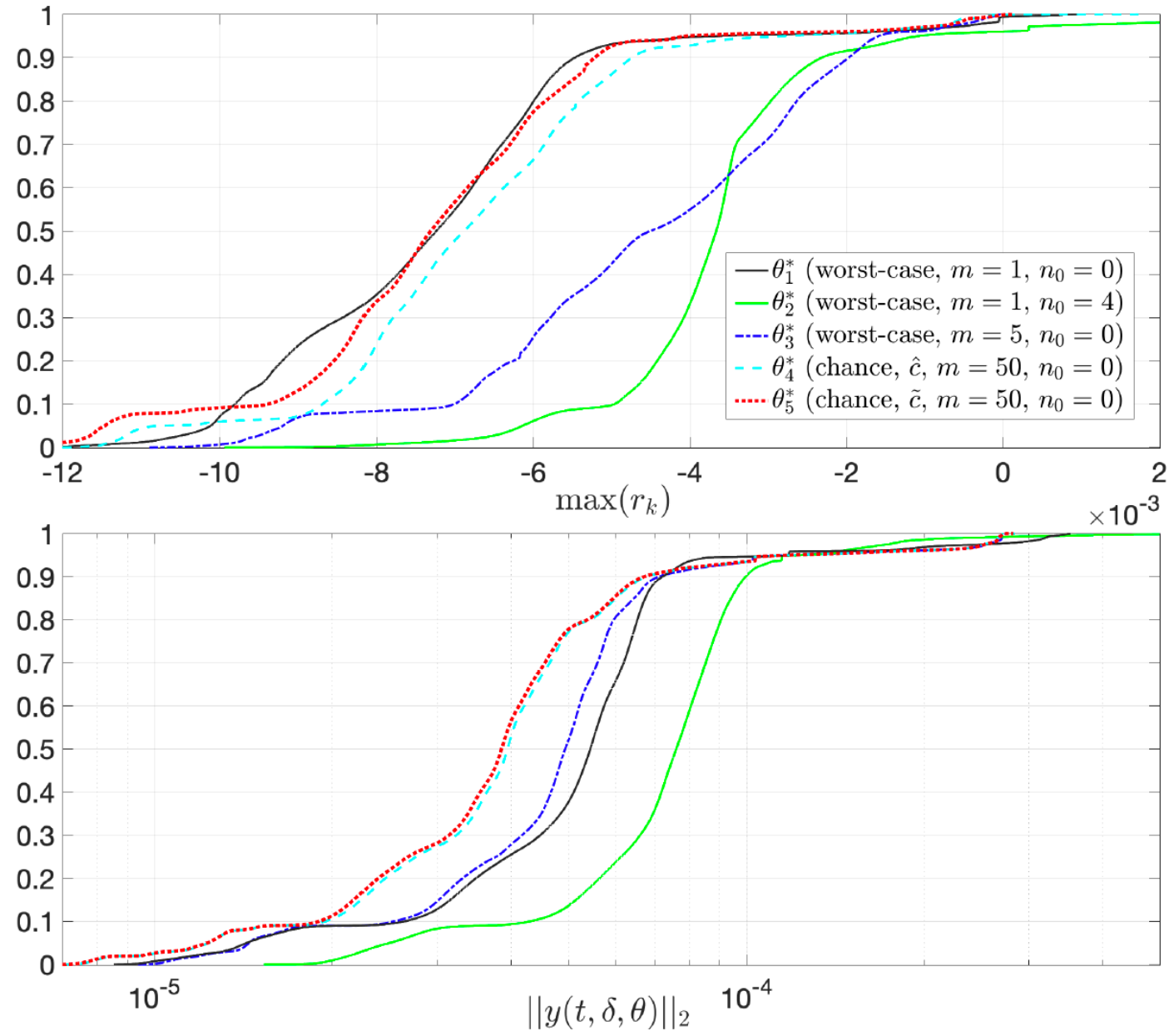
Control Example: θ_4 (u is critical, $n_o = 0$)



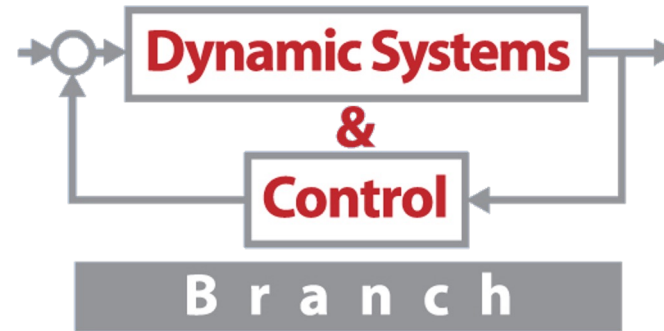
Control Example: θ_5 (y is critical, $n_o = 0$)



Control Example: robustness vs. performance



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