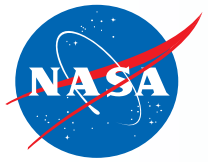

Acoustic Mode Decomposition in Rectangular Ducts with Sheared Flow

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Motivation

- Acoustic liner design and optimization becoming more important as bypass ratio increases
- Design, optimization, and verification process often relies upon Ingard-Myers BC (see [1, 2])
- Assumes infinitesimal thickness of boundary layer, maybe okay for large bypass duct, but not okay for small cross-section test rig (Grazing Flow Impedance Tube, GFIT, is 2 x 2.5 in)
- Flow in test rigs is highly 3D, e.g., the Curved Duct Test Rig (CDTR) cross-section (15 x 6 in) is 40% boundary layer
- Why is this important?
  - Always working to improve liner design and optimization
  - Desire to test up to 6 kHz in Liner Technology Facility (LTF); shear layer refraction will be increasingly important
  - Improve our physical understanding of impedance BC in shear flow and address questions regarding flow direction effects
- Thus, higher fidelity analysis methods that incorporate 3D shear flow refraction should be explored

Objectives

Challenges and Questions to Address

- Developing impedance eduction technique with 3D shear flow is a big challenge
- Natural first step: mode decomposition with 3D shear flow effects
- Are plane-wave or higher-order modes more sensitive to shear flow refraction effects? [3, 4]
- How does the acoustic propagation direction relative to flow direction affect acoustic modes? [5]

Objectives

- Develop mode decomposition procedure based upon the Pridmore-Brown Eqn. (PBE) for 3D sheared duct flow
- Investigate the effect of flow direction on the mode structure, propagation, and amplitudes
- Examine sensitivity of lower- and higher-order modes to shear flow

Process

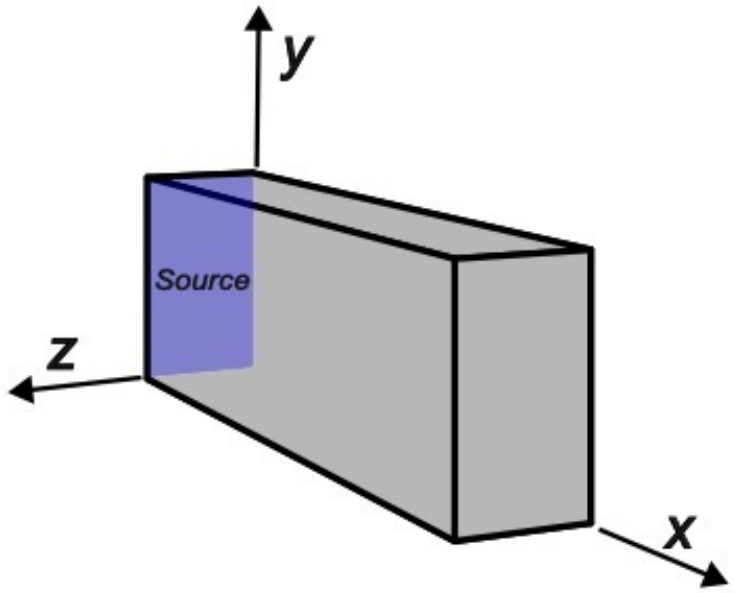
- CDTR already has Convective Helmholtz Eqn. (CHE) mode decomposition capabilities
- Ideal test bed for a higher-fidelity mode decomposition study
- Flow survey performed to extract flow profile
- Acoustic study with two different liner samples performed at three different centerline Mach numbers

Governing Equation

- Pridmore-Brown Equation (PBE) [6]

$$(k-\kappa M)^2\nabla\cdot\left(\frac{1}{(k-\kappa M)^2}\nabla\hat{p}\right)+[(k-\kappa M)^2-\kappa^2]\hat{p}=0 \qquad (1)$$

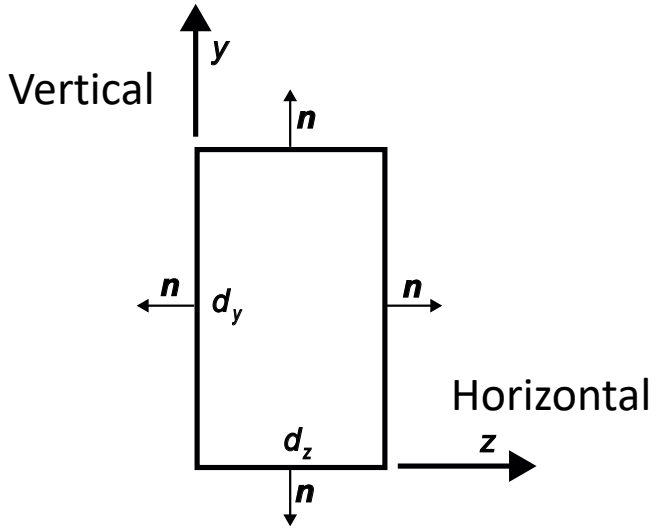
$$p(x,y,z,t)=\hat{p}(y,z;\kappa,\omega)\exp(-i\omega t+i\kappa x)$$



- Ingard-Myers boundary condition ([1, 2])

$$\nabla\hat{p}\cdot\boldsymbol{n}+\frac{(k-\kappa M)^2}{ik\zeta}\hat{p}=0 \qquad (2)$$

- No slip, $M=0$ at walls
- No straightforward dispersion relation for κ in nonuniform flow
- Set up an eigenvalue problem to determine κ (approach based upon Rienstra [7])
- Notation: CHE modes will be in (vertical, horizontal) format when discussed in presentation



Galerkin Projection

- Multiply Eqn. 1 by $\phi(k-\kappa M)^{-2}$ and integrate over cross-section (Assume hardwall BC for brevity)

$$\int_0^{d_z}\int_0^{d_y} \frac{(\nabla\phi\cdot\nabla\hat{p})}{(k-\kappa M)^2} - \left[1 - \frac{\kappa^2}{(k-\kappa M)^2}\right]\phi\hat{p}\,\mathrm{d}y\mathrm{d}z = 0 \tag{3}$$

- Assume a form of the solution

$$\hat{p} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} a_{mn}\psi_{mn}(y,z) \tag{4}$$

- Let $\psi_{mn}(y,z) = T_m(\tilde{y})T_n(\tilde{z})$, where $\tilde{y}$ and $\tilde{z}$ are normalized to the range -1 to +1
- $\phi = \psi_{m'n'}$
- Results in a matrix equation ($\mathcal{M}(\kappa)$ described in paper)

$$\mathcal{M}(\kappa)\boldsymbol{a} = 0 \tag{5}$$

Method of Successive Linear Problems

- Eqn. (5) is a nonlinear eigenvalue problem for $(\kappa, \boldsymbol{a})$
- Need to find all $(\kappa, \boldsymbol{a})$ pairs that satisfy the Galerkin projection
- Linearize about the point μ

$$\mathcal{M}(\kappa) = \mathcal{M}(\mu) + (\kappa - \mu)\left.\frac{d\mathcal{M}}{d\kappa}\right|_{\kappa=\mu} \qquad (6)$$

$\mathcal{M}'(\mu)$

- Approximate eigenvalue problem by the linearization (Method of Successive Linear Problems, MSLP, see [8, 9])

$$\mathcal{M}(\kappa)\boldsymbol{a} \approx [\mathcal{M}(\mu) + (\kappa - \mu)\mathcal{M}'(\mu)]\boldsymbol{a} = 0 \qquad (7)$$

- Process to find κ:
  1. Make initial guess, μ_0
  2. Iterate on the generalized eigenvalue problem, $\mathcal{M}(\mu)\boldsymbol{x} = -\lambda\mathcal{M}'(\mu)\boldsymbol{x}$
  3. Set $\mu = \mu_0 + \lambda$ and check convergence
  4. If not converged, use μ as new guess; if converged, set $\kappa^{(j)} = \mu$ and $\boldsymbol{a}^{(j)} = \boldsymbol{x}$

Modal Analysis

- Acoustic pressure field may be expressed as

$$P(x,y,z;\omega) = \sum_{j=1}^{J} \chi_j \overbrace{\sum_{n=0}^{\tilde{N}} \sum_{m=0}^{\tilde{M}} a_{mn}^{(j)} \psi_{mn}(y,z) e^{i\kappa^{(j)}x}}^{\hat{p}_j} \tag{8}$$

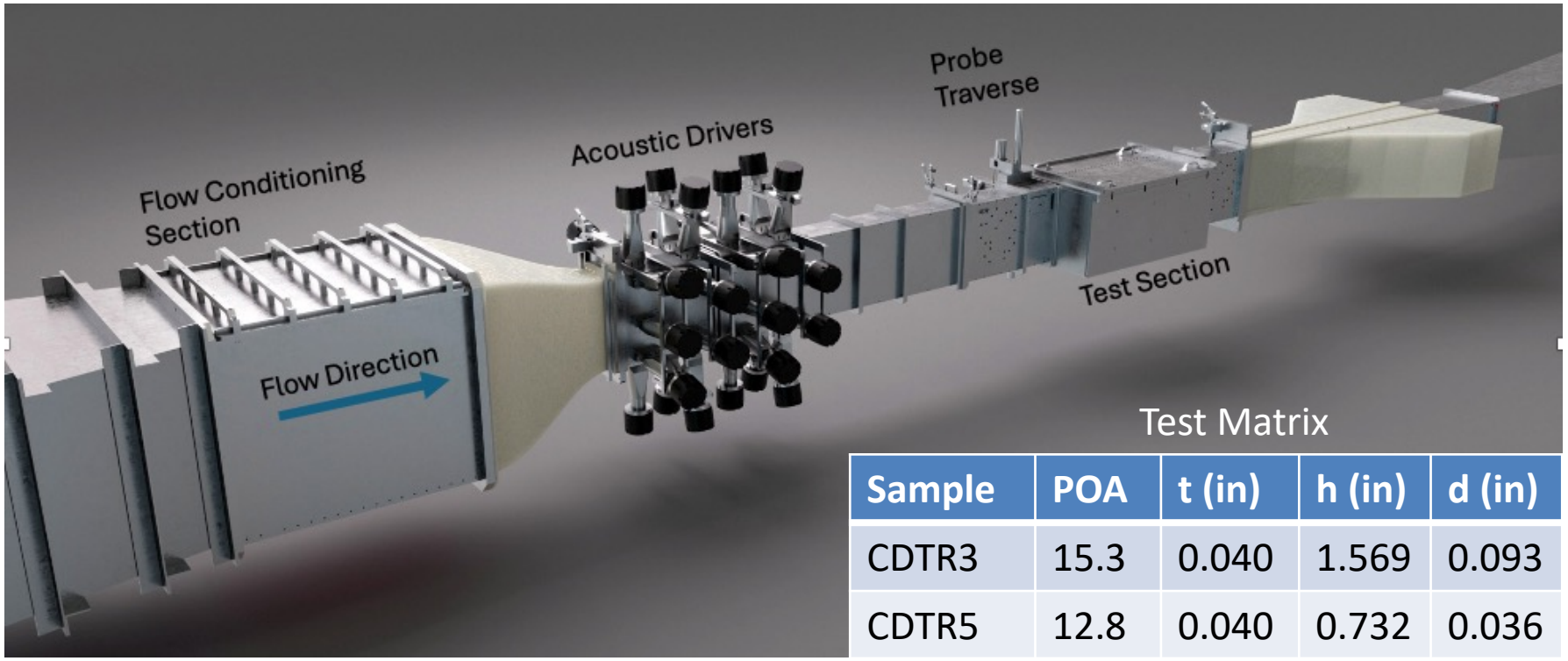
- χ_j are the complex mode coefficients
- Microphone measurements provide $P(x,y,z;\omega)$ at specific in-duct locations
- Then, linear least-squares regression may be used to determine χ_j
- Once χ_j are determined, the following quantities are of interest
  - Mode SPL, $\mathrm{SPL}_{\mathrm{mode},j} = 10\log\left(\frac{\chi_j \chi_j^*}{2p_{ref}^2}\right)$
  - Reconstruction error in SPL and phase

$$E_{\mathrm{SPL,PBE}} = \frac{\|\mathrm{SPL}_{\mathrm{PBE}} - \mathrm{SPL}_{\mathrm{measured}}\|_2}{\|\mathrm{SPL}_{\mathrm{measured}}\|_2} \qquad E_{\mathrm{phase,PBE}} = \frac{1}{N_{\mathrm{mics}}} \sum_{i=1}^{N_{\mathrm{mics}}} \frac{\min(s, s^c)}{\pi} \tag{9a,9b}$$

$$s = |\theta_{\mathrm{measured}} - \theta_{\mathrm{PBE}}|$$

Experimental Setup

- Curved Duct Test Rig (CDTR) – open-loop wind tunnel capable of $M_c = 0.5$ in test section
- 15 x 6 in test section
- 32 acoustic drivers – mode control
- Probe traverse for flow measurements along cross-section before and after test section
- Test matrix: CDTR3 and CDTR5 liner samples at $M_c = 0.1$, 0.3, and 0.5

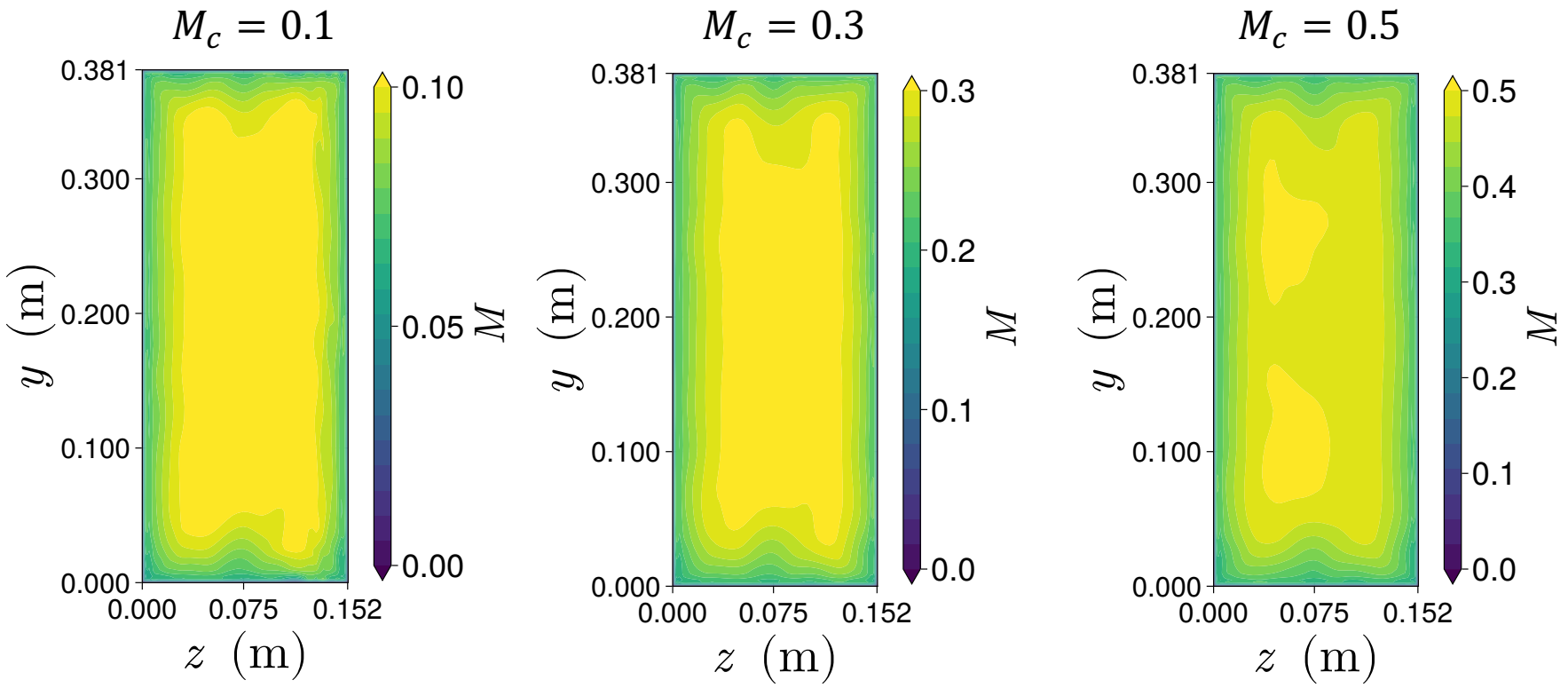


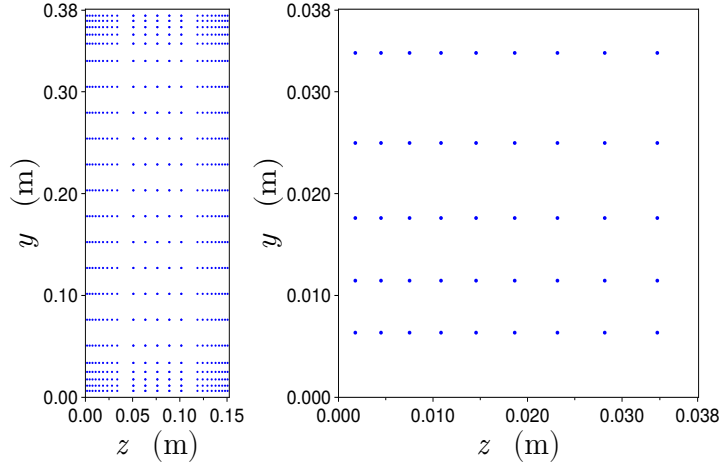
Test Matrix

Sample	POA	t (in)	h (in)	d (in)
CDTR3	15.3	0.040	1.569	0.093
CDTR5	12.8	0.040	0.732	0.036

Flow Measurements

- Total pressure probe to measure Mach number upstream of test section
- Downstream measurements could not be performed due to fan bearing seal failure
- NURBS surface constructed from measurements, then Mach number interpolated onto Legendre points for quadrature

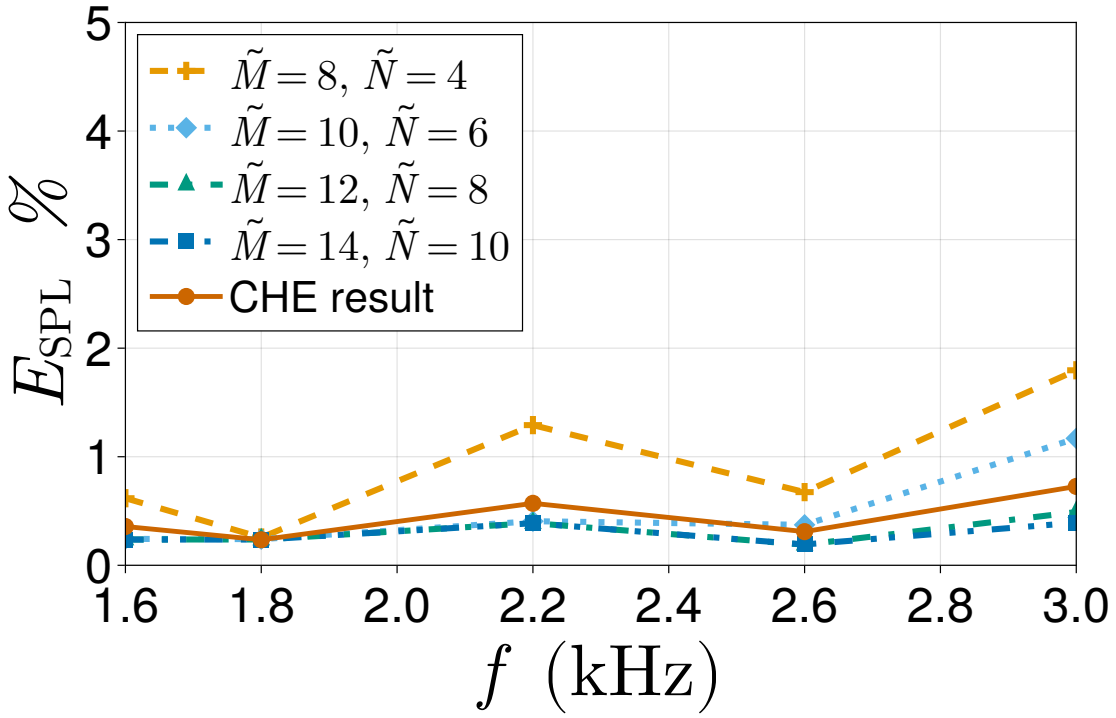




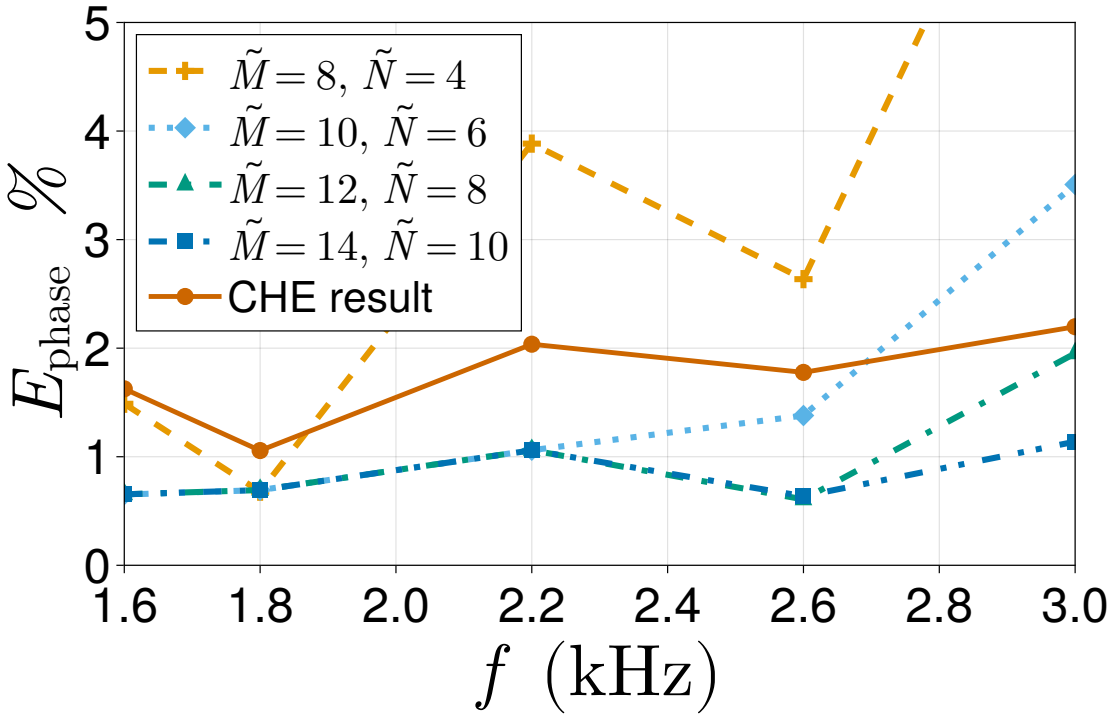
Reconstructed Sound Field

- With knowledge of Mach number at quadrature points, we may now solve $\mathcal{M}(\kappa)\boldsymbol{a} = 0$ and compute the PBE modes
- Validation: compare reconstruction error of PBE modes with traditional CHE modes
- Results show lower reconstruction error for PBE modes as $\tilde{M}$ and $\tilde{N}$ are increased

SPL Reconstruction Error, $M_c = 0.5$ and κ_4 mode



Phase Reconstruction Error, $M_c = 0.5$ and κ_4 mode



Wavenumber Deviations

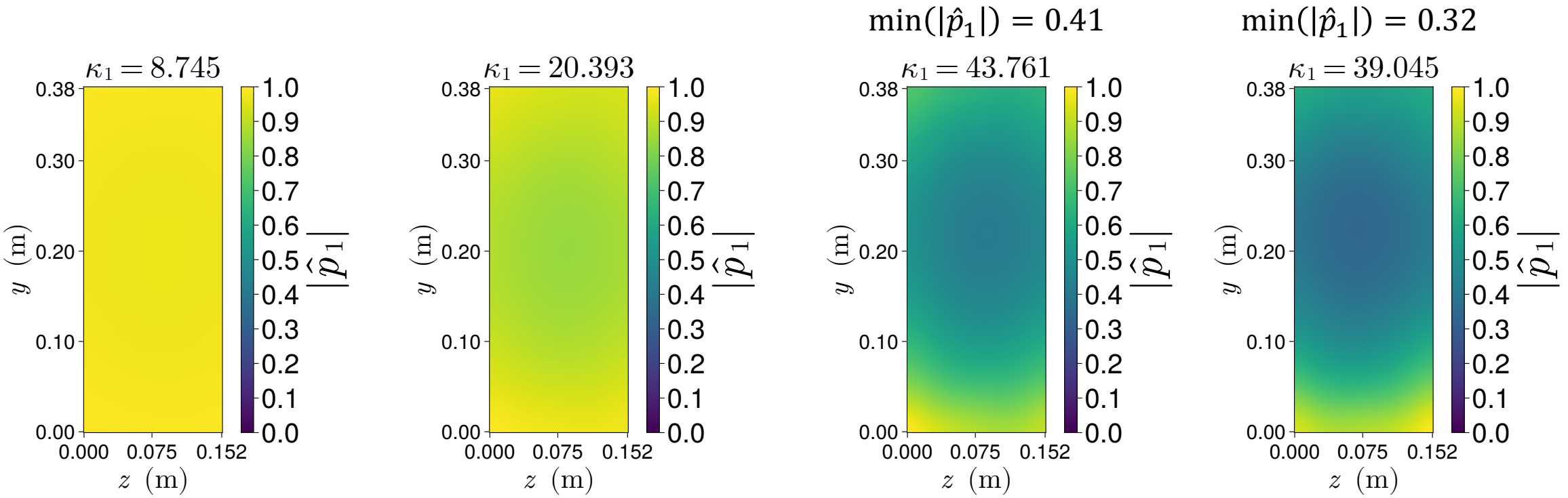
- Number of eigenvalues found for PBE modes is generally equal to number of wavenumbers for CHE modes
- No significant wavenumber deviations resulting from shear flow for $M_c = 0.1$ and 0.3
- For $M_c = 0.5$, negative modes are more significantly impacted than positive modes
- Absolute value of wavenumber is smaller for PBE modes propagating against the flow

$M_c = 0.5$

PBE mode #	CHE mode #	κ, PBE	κ, CHE
1	(0,0)	20.750	20.148
2	(1,0)	19.736	19.005
3	(2,0)	15.938	15.356
7	(2,1)	6.598	6.168
8	(3,1)	-5.638	-4.755
9	(4,0)	-8.511	-7.173
-9	-(4,0)	-25.340	-32.801
-8	-(3,1)	-25.965	-35.219
-7	-(2,1)	-37.798	-46.142
-3	-(2,0)	-48.780	-55.331
-2	-(1,0)	-53.020	-58.980
-1	-(0,0)	-55.658	-60.122

Modal Structure

- As frequency and Mach number increase, modal structure becomes more “distorted” relative to traditional CHE mode structure
- Lowest-order mode becomes increasingly non-planar as flow speed and frequency increase (shown below)
- Asymmetry between positive and negative mode structures also present (shown on next slide)
- Lowest-order mode is first mode to exhibit this asymmetry (discussed in paper)

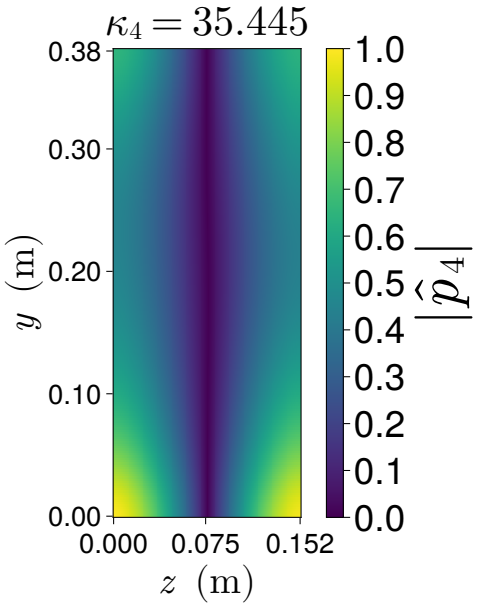


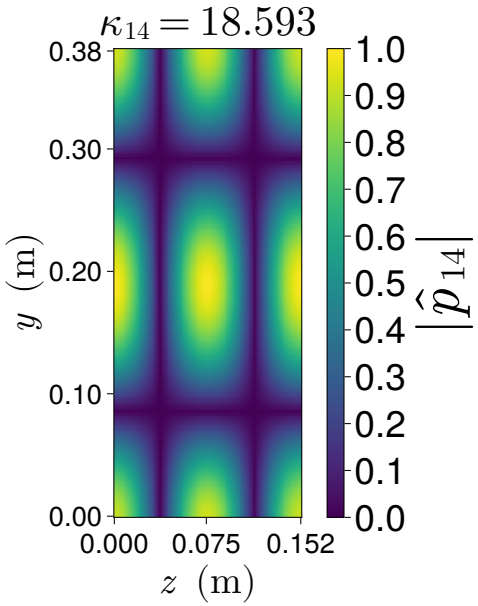
$f = 600$ Hz and $M_c = 0.3$ $f = 1400$ Hz and $M_c = 0.3$ $f = 3000$ Hz and $M_c = 0.3$ $f = 3000$ Hz and $M_c = 0.5$

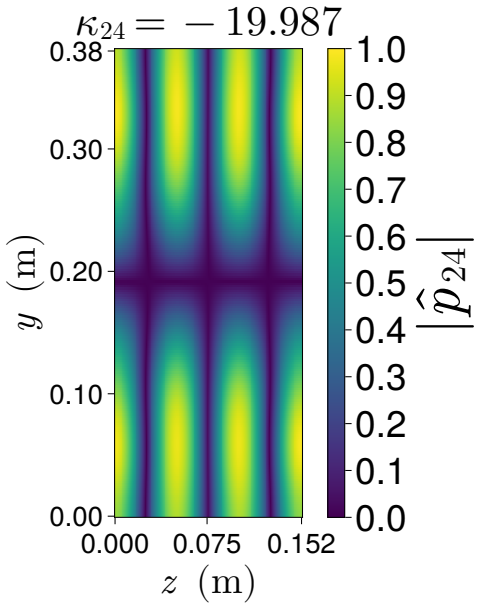
Modal Structure (Flow Direction Effects)

$f = 3000$ Hz and
$M_c = 0.5$

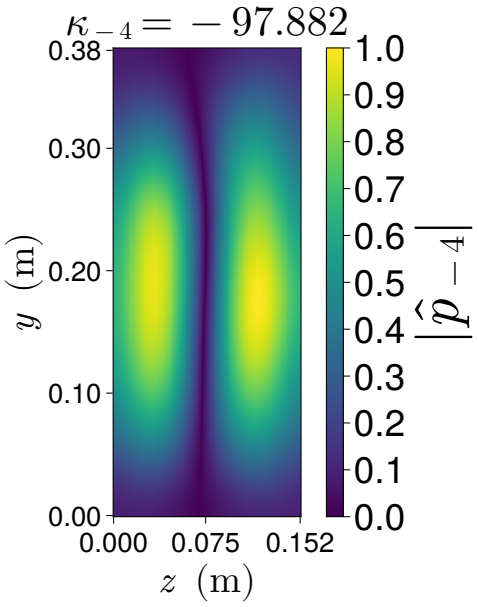
Positive Modes

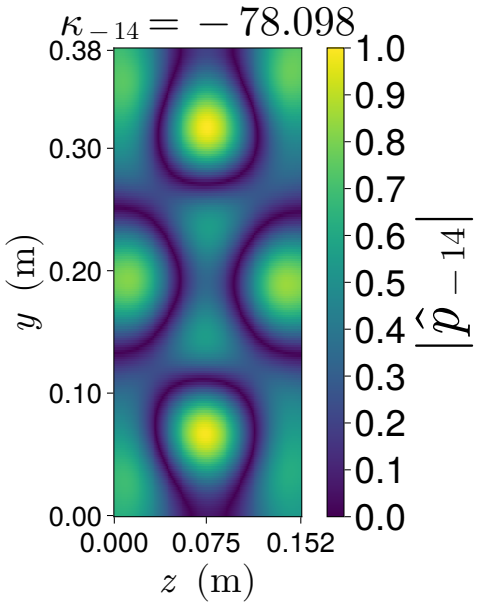


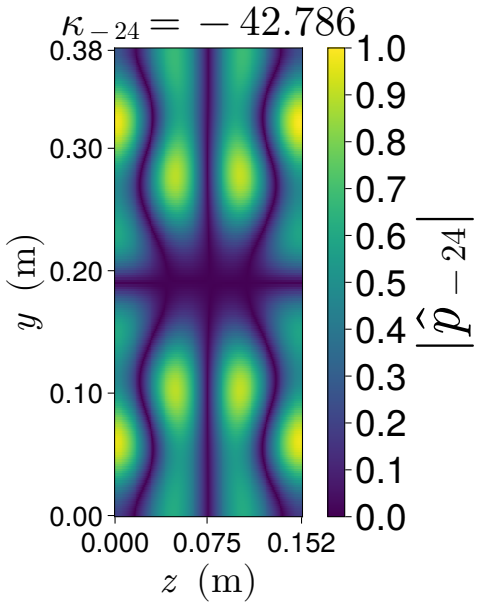




Negative Modes







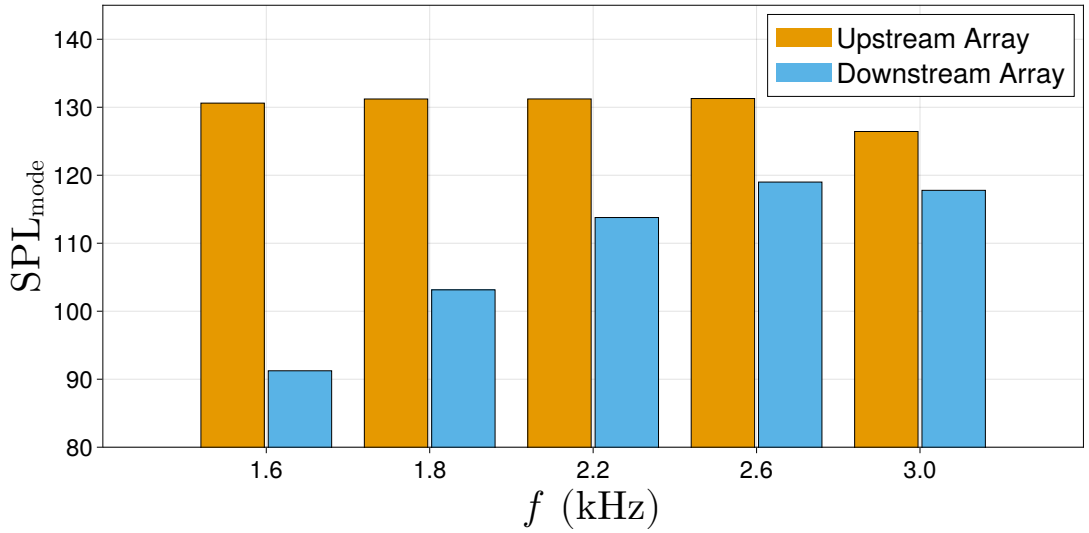
Mode SPL

- Mode SPL upstream and downstream of two liner samples computed using PBE and CHE modes
- Very good agreement for lowest-order mode
- More noticeable disagreement for higher-order modes, but CHE results may still be acceptable for design purposes
- Insensitive to Mach number, surprisingly

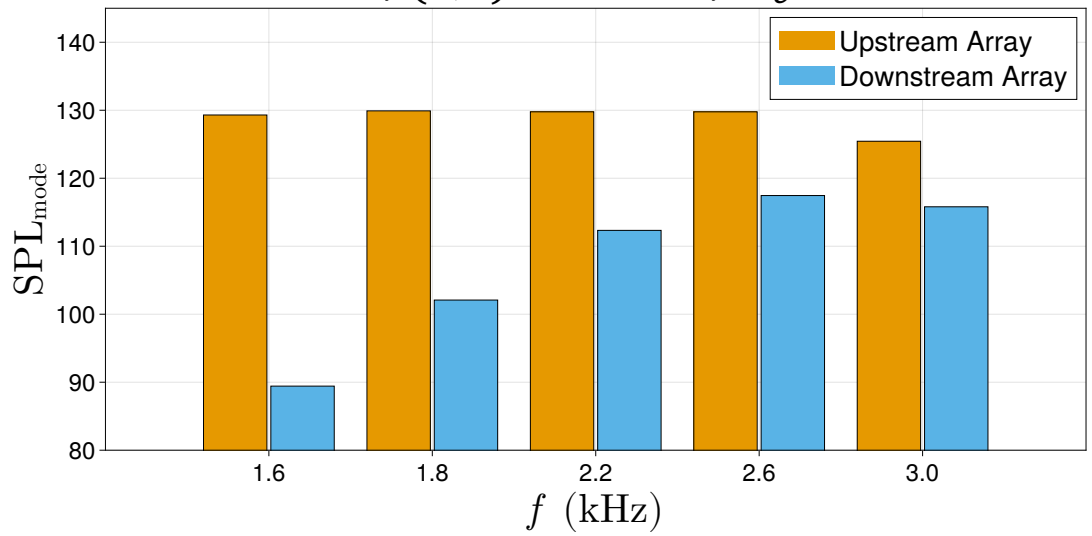
Avg. SPL difference in modes for CDTR3

PBE mode	CHE mode	$M_c = 0.1$	$M_c = 0.3$	$M_c = 0.5$
1	(0,0)	0.03	0.02	0.02
2	(1,0)	1.11	1.10	1.09
3	(2,0)	-0.26	-0.25	-0.12
4	(0,1)	1.23	1.38	1.44
5	(1,1)	2.27	2.35	2.32
6	(3,0)	-1.82	-1.83	-1.86

PBE, κ_4 dominant, $M_c = 0.5$



CHE, $(0,1)$ dominant, $M_c = 0.5$



Summary and Future Work

Summary (Modal Structure)

- Wavenumber deviation is only significant for negative modes at $M_c = 0.5$
- Lowest-order mode becomes increasingly non-planar as f and M_c increase
- Thus, a “plane-wave” component of the acoustic field may not exist under certain test conditions
- Asymmetry in positive and negative modes first appears for lowest-order mode
- This asymmetry can be drastic, as results at $f = 3000$ Hz and $M_c = 0.5$ show

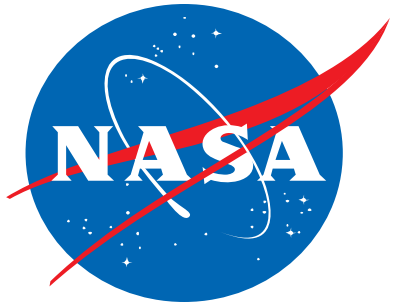
Summary (Mode SPL)

- Contrary to modal structure results, lowest-order mode appears least affected by shear flow (when looking purely at mode SPL)
- Insensitive to Mach number, not true for mode structure

Future Work

- Extend mode decomposition to GFIT to examine higher frequencies
- Develop an impedance eduction procedure that incorporates 3D shear flow effects
- Support further flow direction studies and improved impedance optimization tools

Questions?



References

[1] Ingard, U., “Influence of fluid motion past a plane boundary on sound reflection, absorption, and transmission,” The Journal of the Acoustical Society of America, Vol. 31, No. 7, 1959, pp. 1035–1036. https://doi.org/10.1121/1.1907805.

[2] Myers, M., “On the acoustic boundary condition in the presence of flow,” Journal of Sound and Vibration, Vol. 71, No. 3, 1980, pp. 429–434. https://doi.org/10.1016/0022-460X(80)90424-1.

[3] Salikuddin, M., and Ramakrishnan, R., “Acoustic power measurement for single and annular stream duct-nozzle systems utilizing a modal decomposition scheme,” Journal of Sound and Vibration, Vol. 113, No. 3, 1987, pp. 441–472. https://doi.org/10.1016/S0022-460X(87)80132-3.

[4] Savkar, S., “Propagation of sound in ducts with shear flow,” Journal of Sound and Vibration, Vol. 19, No. 3, 1971, pp. 355–372. https://doi.org/10.1016/0022-460X(71)90695-X.

[5] Nark, D. M., Jones, M. G., and Piot, E., “Assessment of axial wave number and mean flow uncertainty on acoustic liner impedance eduction,” 2018 AIAA/CEAS Aeroacoustics Conference, AIAA 2018-3444, Atlanta, Georgia, 2018. https://doi.org/10.2514/6.2018-3444.

[6] Pridmore-Brown, D. C., “Sound propagation in a fluid flowing through an attenuating duct,” Journal of Fluid Mechanics, Vol. 4, No. 4, 1958, pp. 393–406. https://doi.org/10.1017/S0022112058000537.

[7] Rienstra, S. W., “Numerical and asymptotic solutions of the Pridmore-Brown equation,” AIAA Journal, Vol. 58, No. 7, 2020, pp. 3001–3018. https://doi.org/10.2514/1.J059140.

[8] Mehrmann, V., and Voss, H., “Nonlinear eigenvalue problems: a challenge for modern eigenvalue methods,” GAMM-Mitteilungen, Vol. 27, No. 2, 2004, pp. 121–152. https://doi.org/10.1002/gamm.201490007.

References

[9] Ruhe, A., “Algorithms for the nonlinear eigenvalue problem,” SIAM Journal on Numerical Analysis, Vol. 10, No. 4, 1973, pp. 674–689. https://doi.org/https://doi.org/10.1137/0710059.

Backup Slides

Background: NASA LaRC Liner Technology Facility

High Intensity Modal Impedance Tube (HIMIT)

- Mach 0.0, SPL ≤ 170 dB, Freq ≤ **6 kHz**
- Tone and Broadband sources

Normal Incidence Tube (NIT)

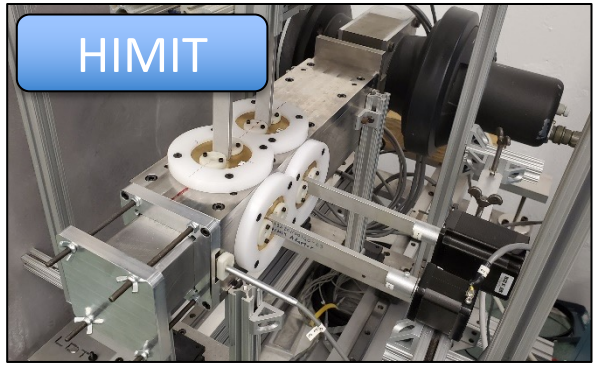
- Mach 0.0, SPL ≤ 155 dB, Freq ≤ 3 kHz
- Tone and Broadband sources

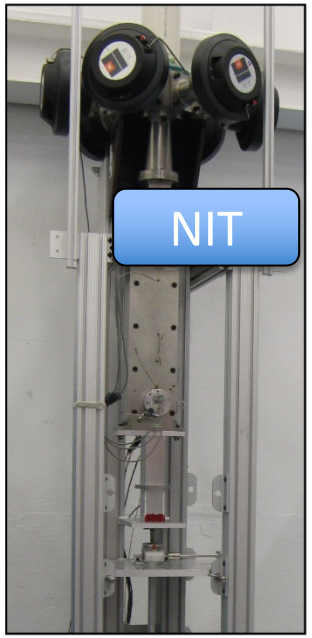
Grazing Flow Impedance Tube (GFIT)

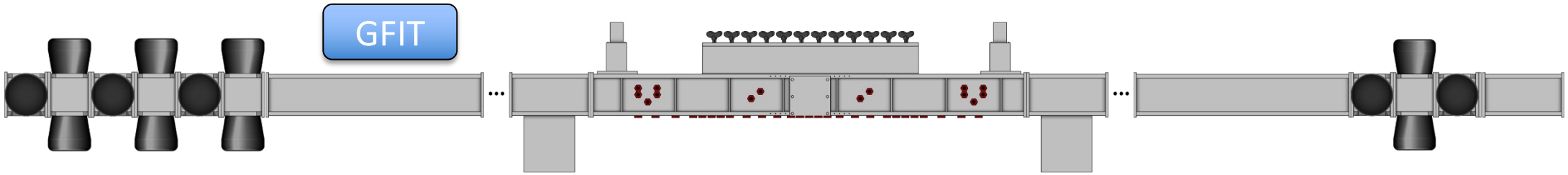
- Mach ≤ 0.6, SPL ≤ 155 dB, Freq ≤ **3 kHz**
- Tone source

Curved Duct Test Rig (CDTR)

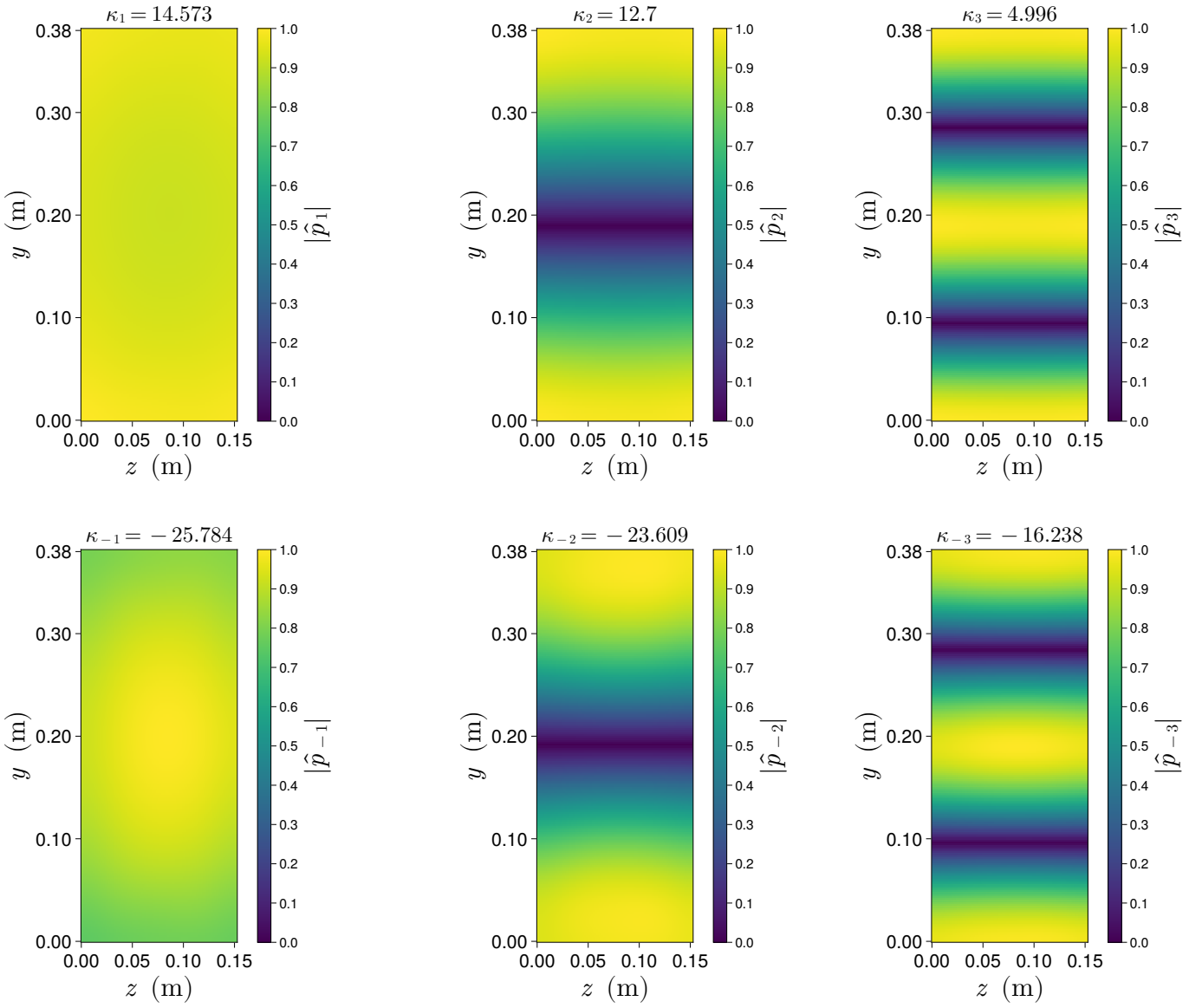
- Mach ≤ 0.5, SPL ≤ 135 dB, Freq ≤ 3 kHz
- *Controlled* Tonal mode and Broadband sources







1000 Hz and Centerline Mach 0.3



Wavenumber Deviation Additional Info

- Implies a higher phase speed relative to CHE counterpart
- Reduction in angle of incidence, θ_i
- Propagation simulations based on PBE may indicate increased effectiveness of liner as compared to CHE