

# Comparison of Mars Entry Performance Using Bank-Angle and Alpha- Beta Steering

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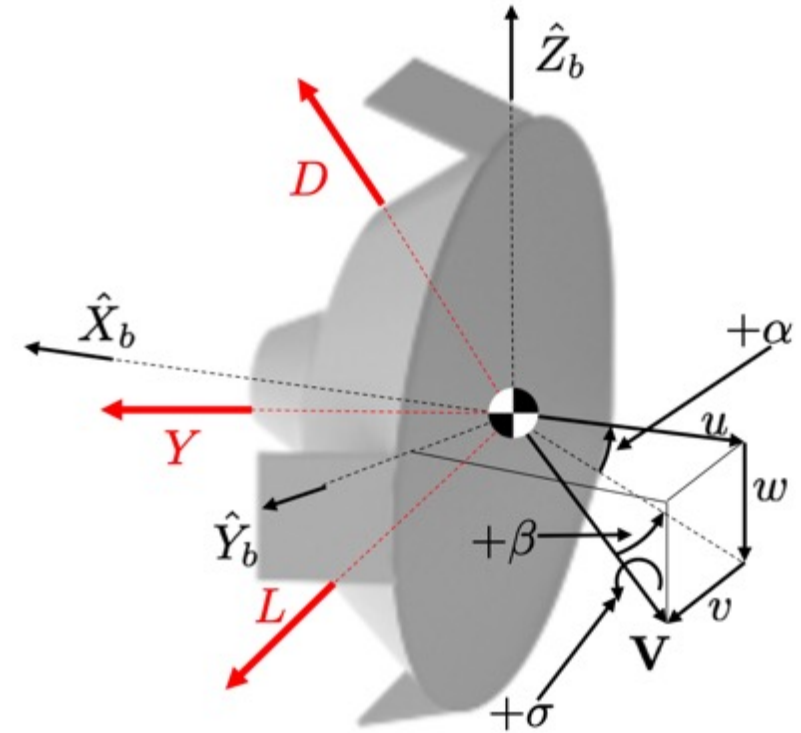
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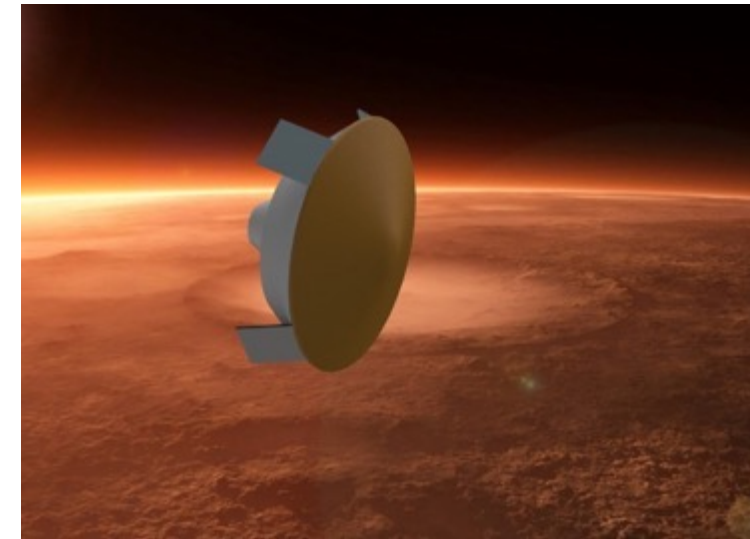
# Hypersonic Steering for Mars Entry

- Bank-angle steering is state-of-the-art for hypersonic trajectory control
- Future Mars entry missions may demand higher entry performance (accuracy, altitude, mass)
- Alpha-beta ( $\alpha$ - $\beta$ ) steering or direct force control (DFC) is a proposed alternative hypersonic steering scheme
  - Morphing structure
  - Center-of-gravity movement system
  - Aerodynamic flaps



# Motivation

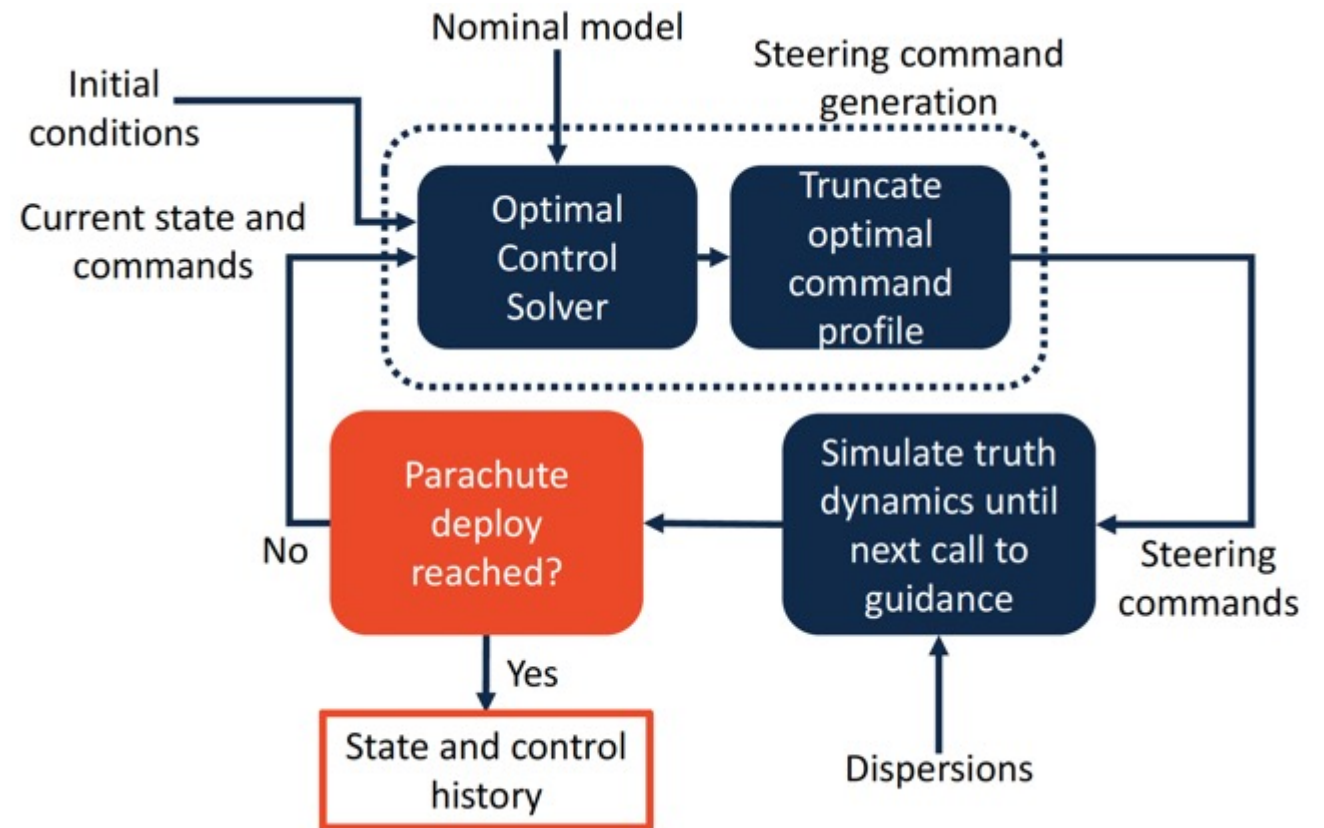
- $\alpha$ - $\beta$  steering may improve flight performance over bank-angle steering, but magnitude of performance differences not fully understood
- Design of guidance/steering commands may influence performance comparison
  - Tuning parameters
  - Guidance structure
  - Use of bank reversals
- Optimal control can be used to provide more equal comparison over several trajectory objectives



# Optimal Control Solutions

Maximize altitude at final time

- Trajectory planning optimal control problems
- Use of optimal control to generate steering commands
- All problems solved using the commercial pseudospectral solver GPOPS-II



# Optimal Control Models

- Three-degree-of-freedom (3DOF) motion over spherical, non-rotating Mars
- **Atmosphere model:** exponential
- **Aerodynamic model:** fit to Mars Science Laboratory aerodynamics database
  - Drag: quadratic in  $\alpha$  and  $\beta$
  - Lift: linear in  $\alpha$
  - Side force: linear in  $\beta$

$$\dot{z} = V \sin \gamma$$

$$\dot{\theta} = \frac{V \cos \gamma \sin \psi}{r \cos \phi}$$

$$\dot{\phi} = \frac{V \cos \gamma \cos \psi}{r}$$

$$\dot{V} = -D - g \sin \gamma$$

$$\dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + \left( \frac{V^2}{r} - g \right) \cos \gamma \right]$$

$$\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma + Y}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi \right]$$

$$\text{Bank} \begin{cases} \mathbf{x}_\sigma = [z \quad \theta \quad \phi \quad V \quad \gamma \quad \psi \quad \sigma \quad \dot{\sigma}]^T \\ u_\sigma = \ddot{\sigma} \end{cases}$$

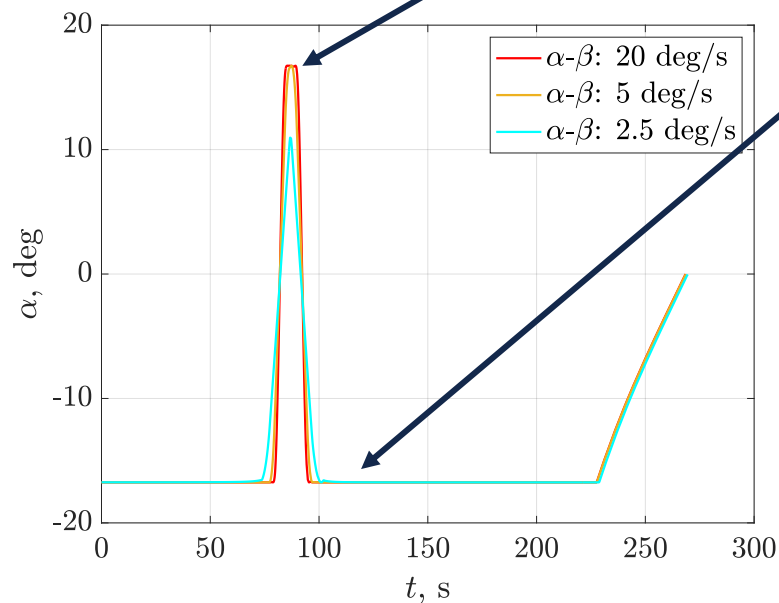
$$\alpha\text{-}\beta \begin{cases} \mathbf{x}_{\alpha\text{-}\beta} = [z \quad \theta \quad \phi \quad V \quad \gamma \quad \psi \quad \alpha \quad \beta \quad \dot{\alpha} \quad \dot{\beta}]^T \\ \mathbf{u}_{\alpha\text{-}\beta} = [\ddot{\alpha} \quad \ddot{\beta}]^T \end{cases}$$

# Maximum Terminal Altitude

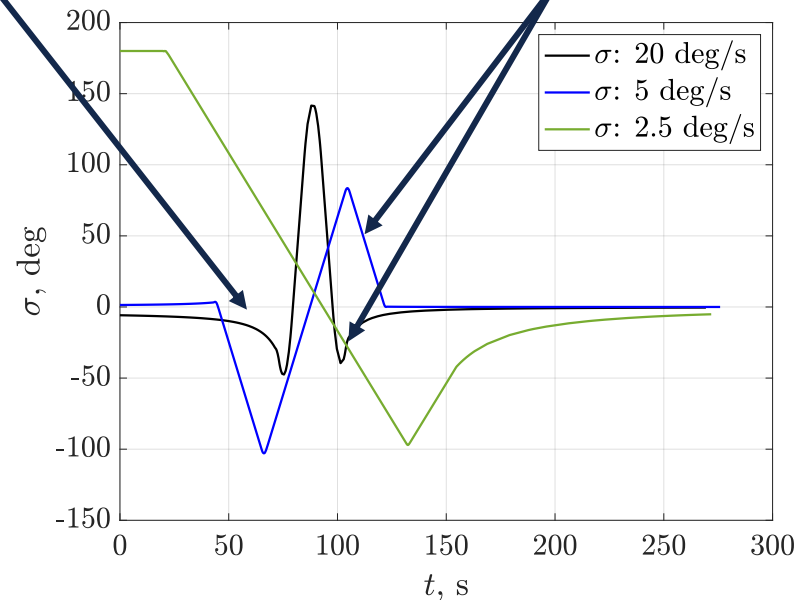
$\alpha$ - $\beta$  steering through

Both steering of near bank

Significant coupling to maximize altitude and hit zero deg latitude target



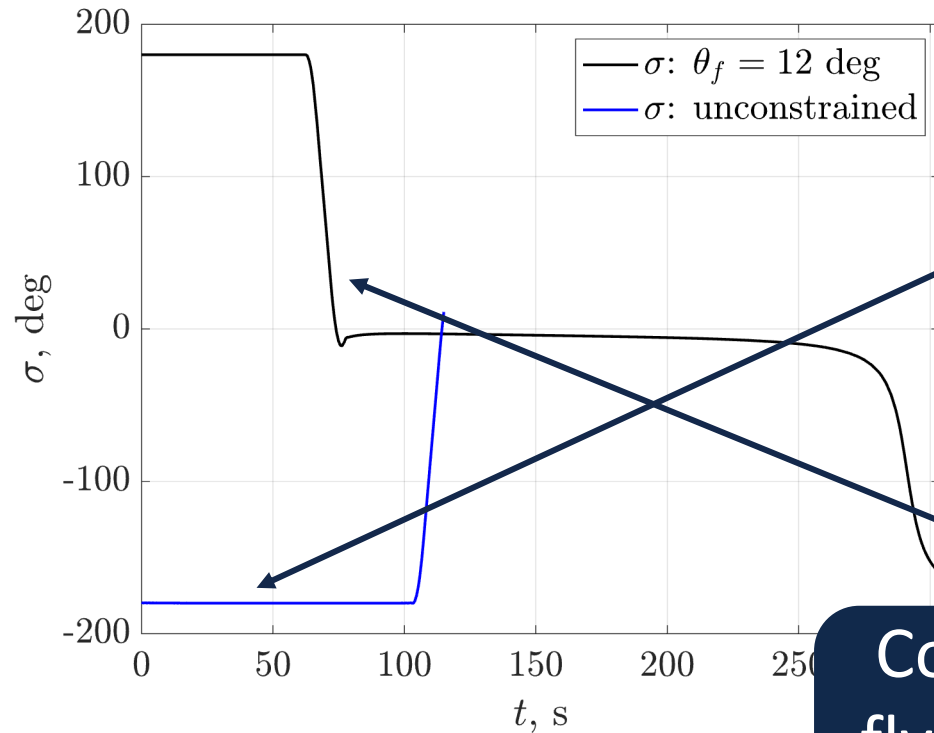
$\alpha$ - $\beta$



Bank

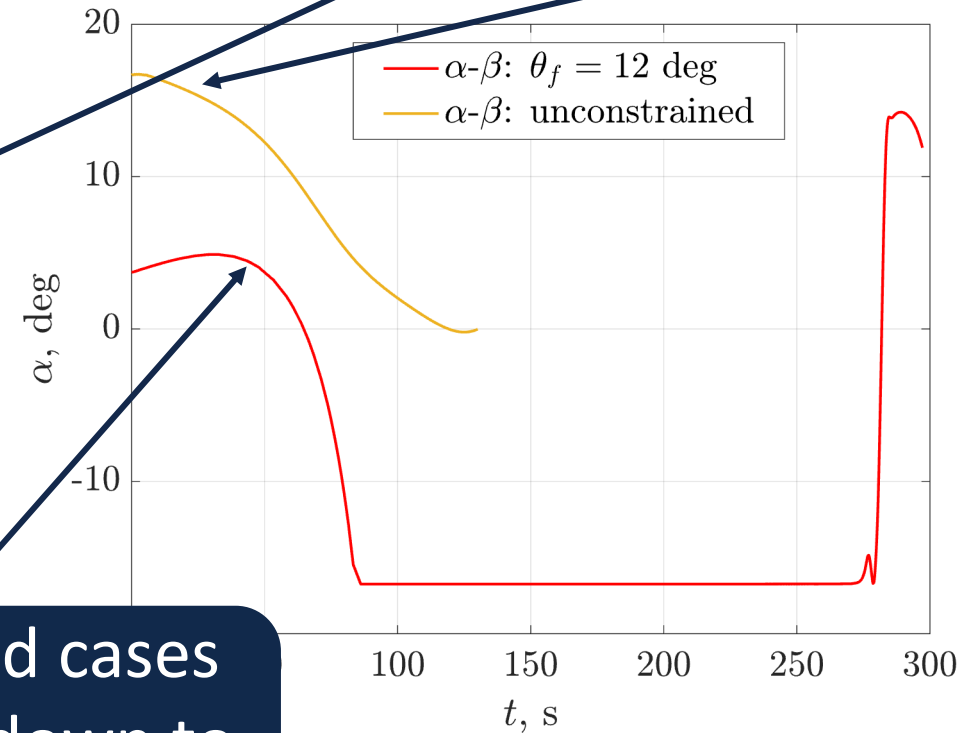
# Minimum Heat Load

$\alpha$ - $\beta$  uses lift-down flight with higher drag to reduce heat load



Bank

Constrained cases fly less lift-down to extend range

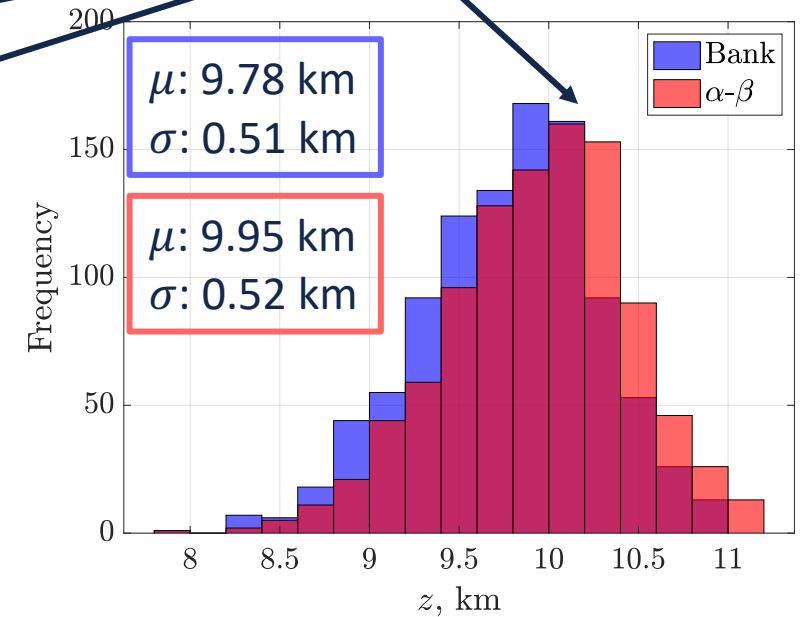
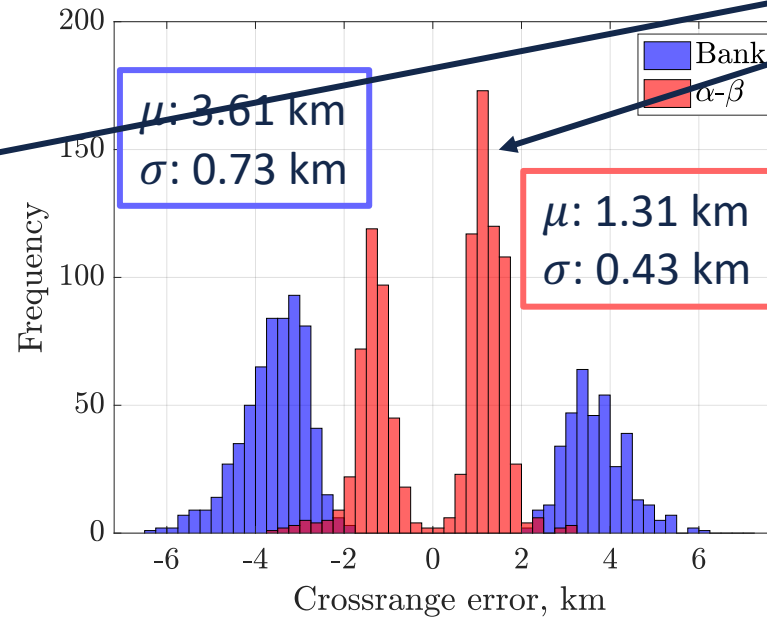
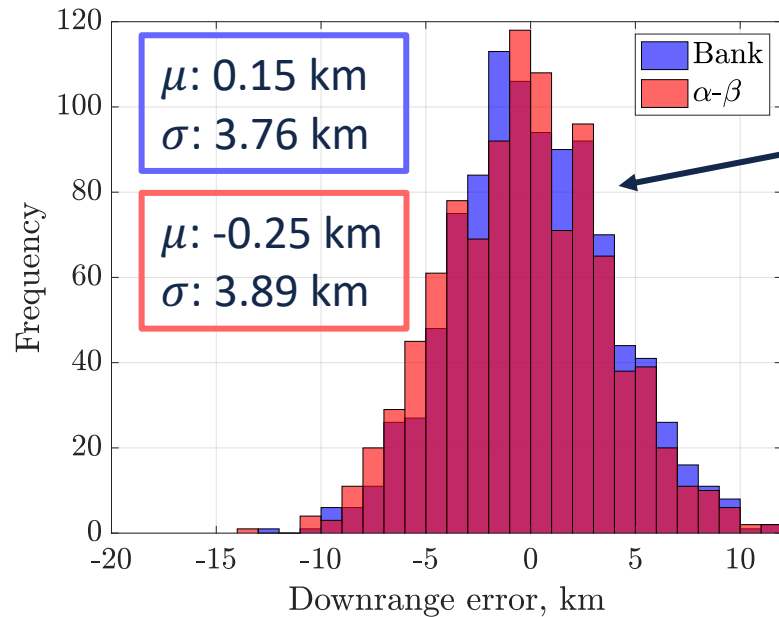


$\alpha$ - $\beta$

# Monte Carlo: Maximum Altitude

**Goal:** hit desired longitude, latitude while maximizing terminal altitude

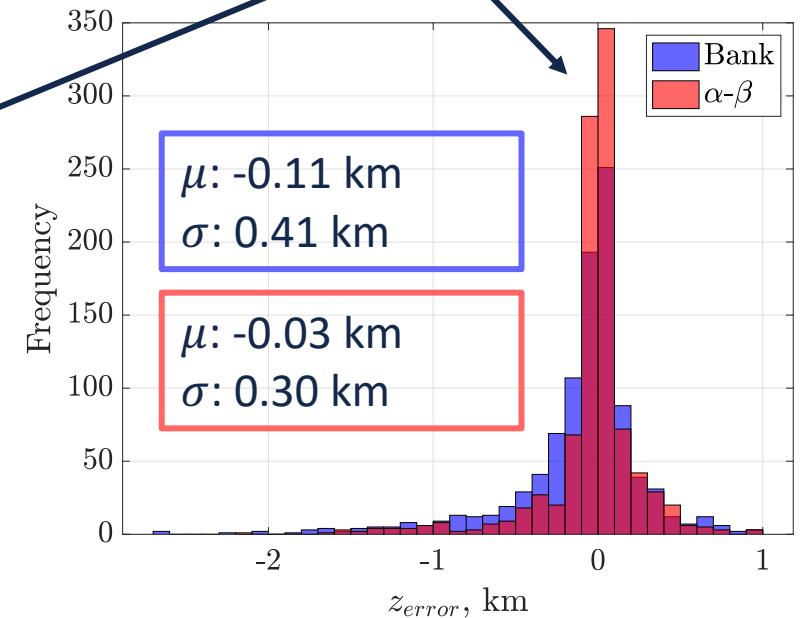
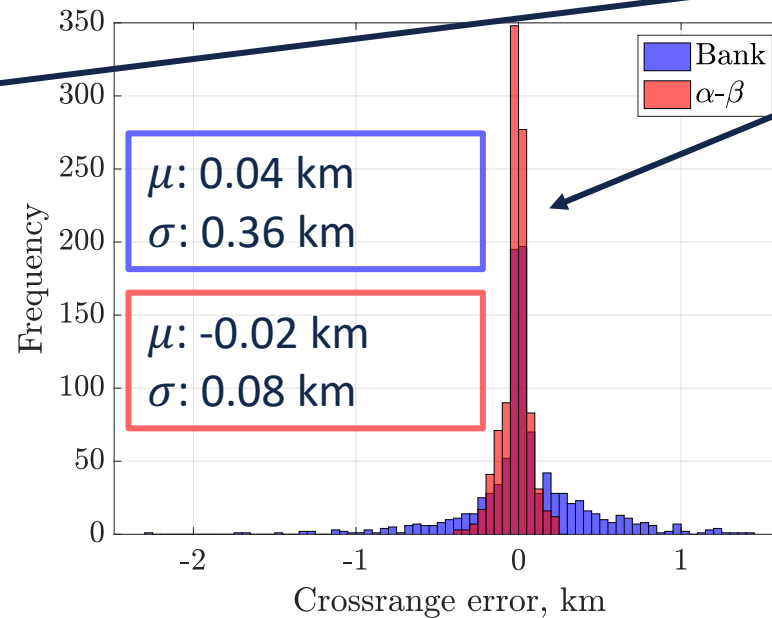
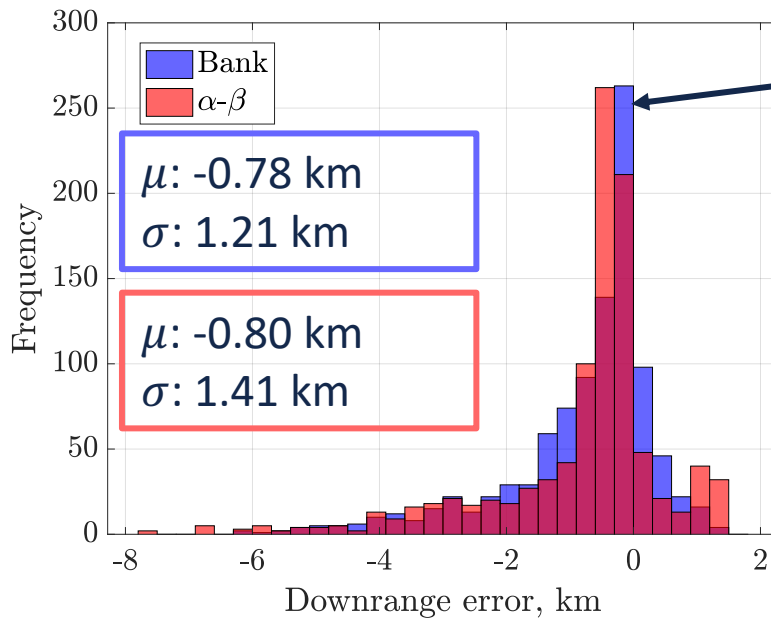
Both options have similar downrange targeting



# Monte Carlo: Minimum Control Effort

Goal: hit desired longitude, latitude, and altitude target while minimizing vehicle rates

$\alpha\text{-}\beta$  has lower crossrange error



# Conclusions

- Optimal control problems solved for bank-angle and  $\alpha$ - $\beta$  steering
- $\alpha$ - $\beta$  steering results in improved performance over bank-angle steering
  - Maximum altitude (especially as rates are lowered), minimum heat load, and lateral targeting
  - Differences are due to coupling, response time, and control over drag
- $\alpha$ - $\beta$  steering may be useful when bank-angle steering vehicle cannot move quickly, if marginal performance gains are needed, or if using simple guidance algorithms

## Acknowledgements

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## Backup

# Selected References

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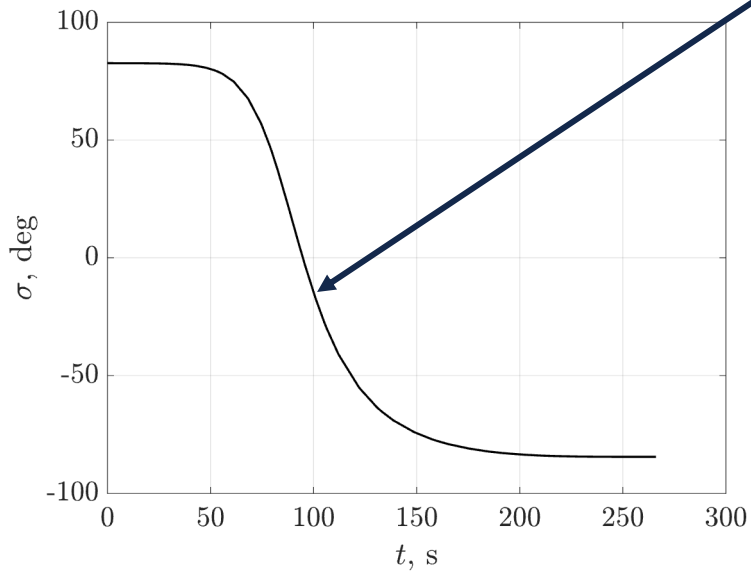
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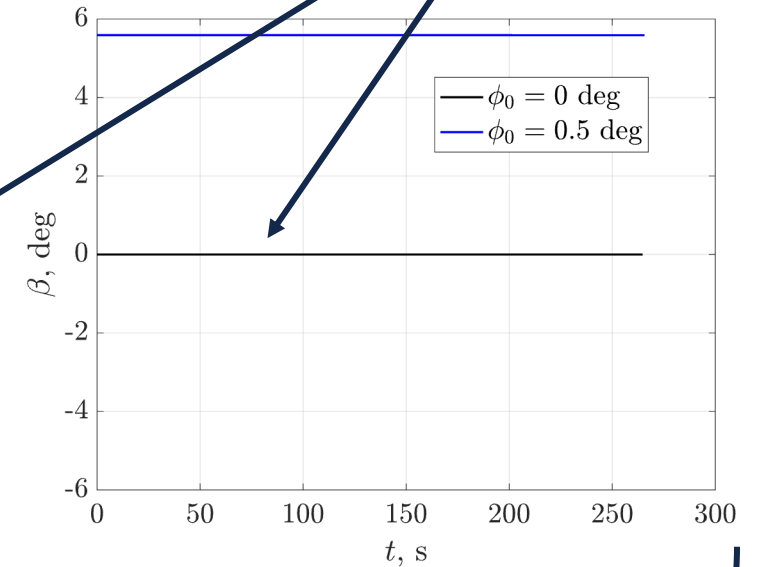
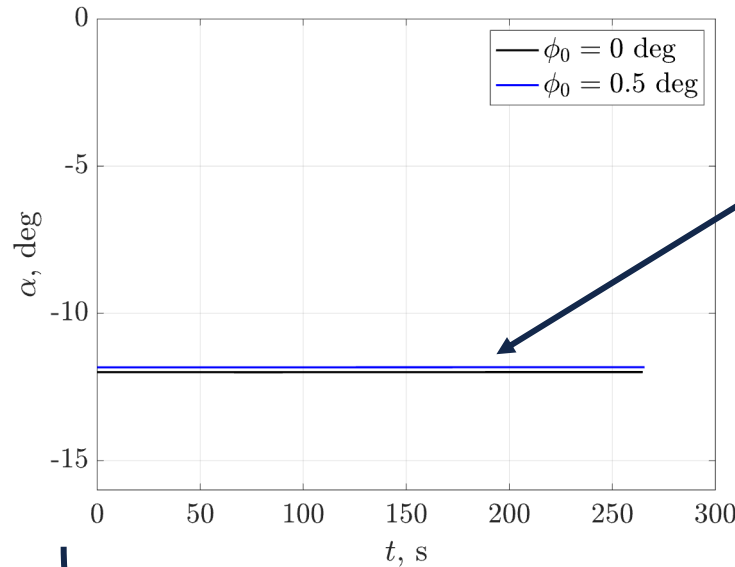
# Minimum Control Effort

Single bank reversal

Constant profiles



Bank

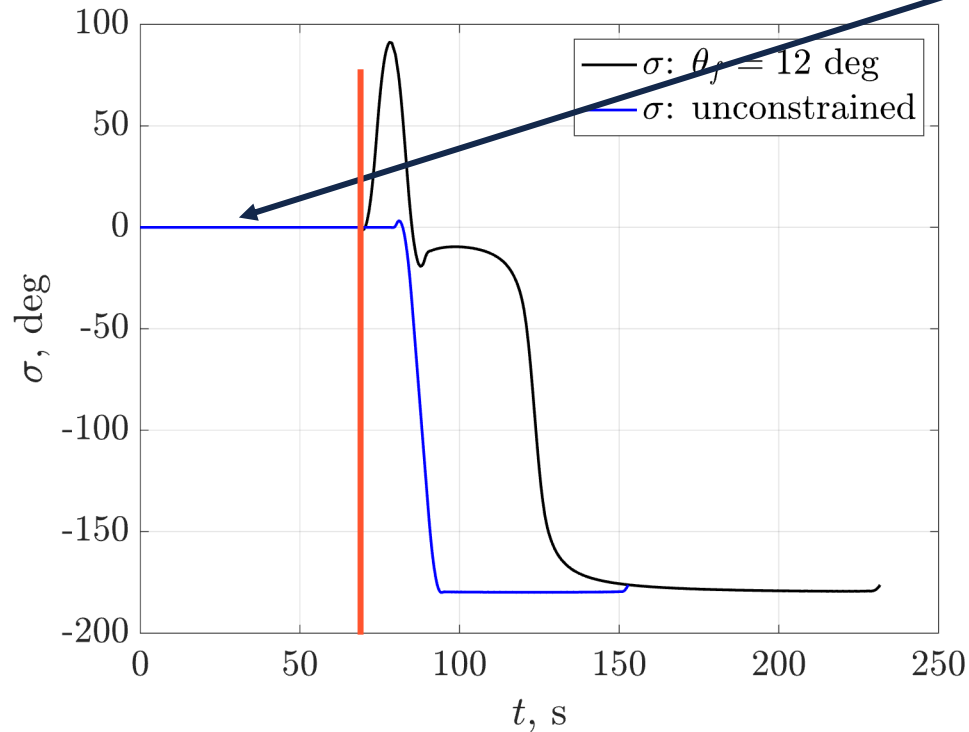


$\alpha$ - $\beta$

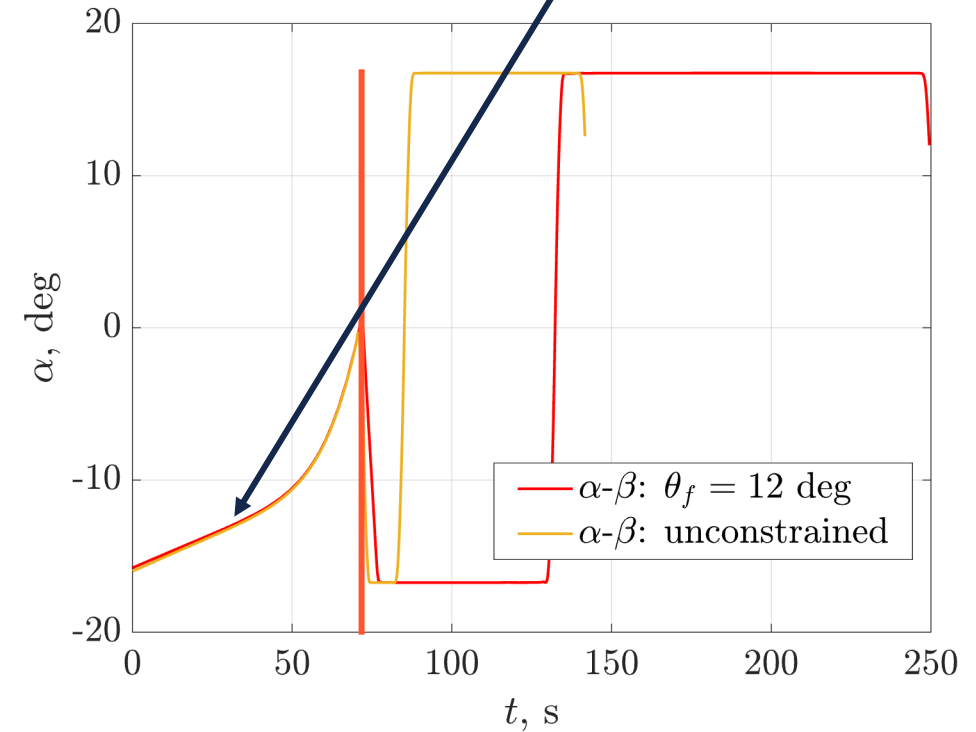
# Minimum Peak Heat Rate

$\alpha-f$   
high

Bank-angle steering  
vehicle flies lift up



Bank



$\alpha-\beta$

# Models: Trajectory Planning

| Parameter      | Value                  |
|----------------|------------------------|
| $m$            | 3300 kg                |
| $S_{ref}$      | 15.90 m <sup>2</sup>   |
| $C_{D,0}$      | 1.67                   |
| $C_{D,\alpha}$ | -2.86/rad <sup>2</sup> |
| $C_{D,\beta}$  | -2.86/rad <sup>2</sup> |
| $C_{L,\alpha}$ | -1.31/rad              |
| $C_{Y,\beta}$  | 1.29/rad               |

| Parameter                | Value                  |
|--------------------------|------------------------|
| Surface Density          | 0.02 kg/m <sup>3</sup> |
| Atmospheric Scale Height | 11.10 km               |
| Surface Gravity          | 3.71 m/s <sup>2</sup>  |
| Mars Equatorial Radius   | 3396.20 km             |

$$C_D = C_{D,0} + C_{D,\alpha}\alpha^2 + C_{D,\beta}\beta^2$$

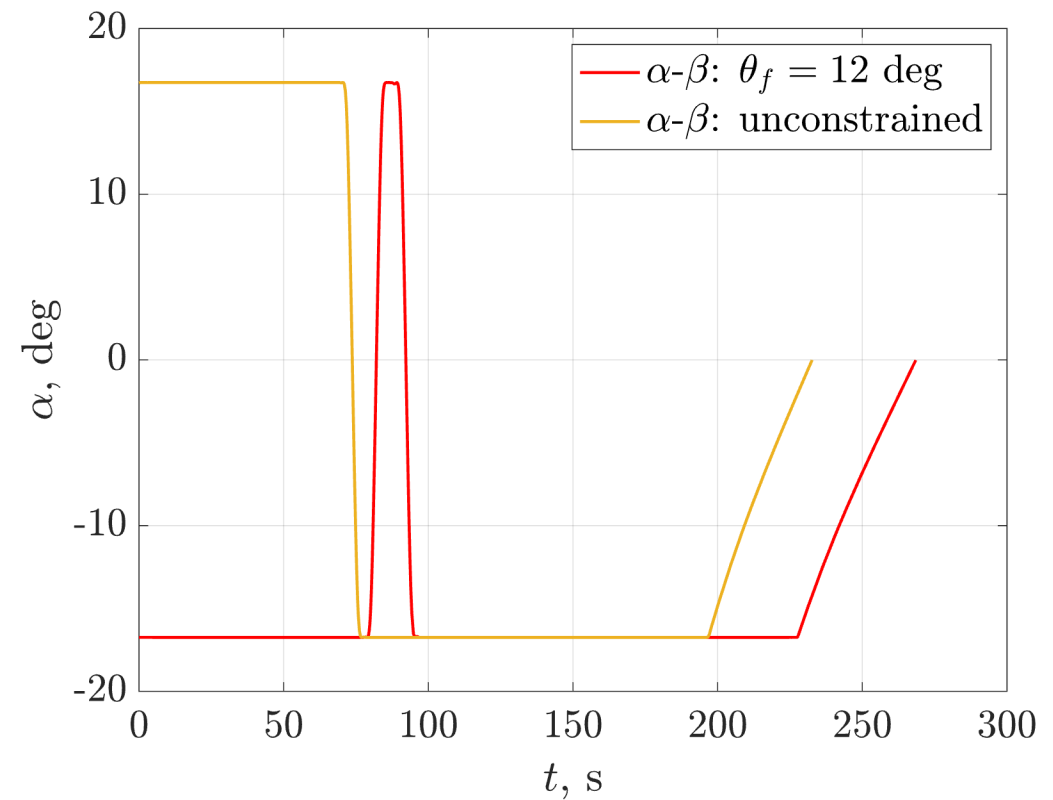
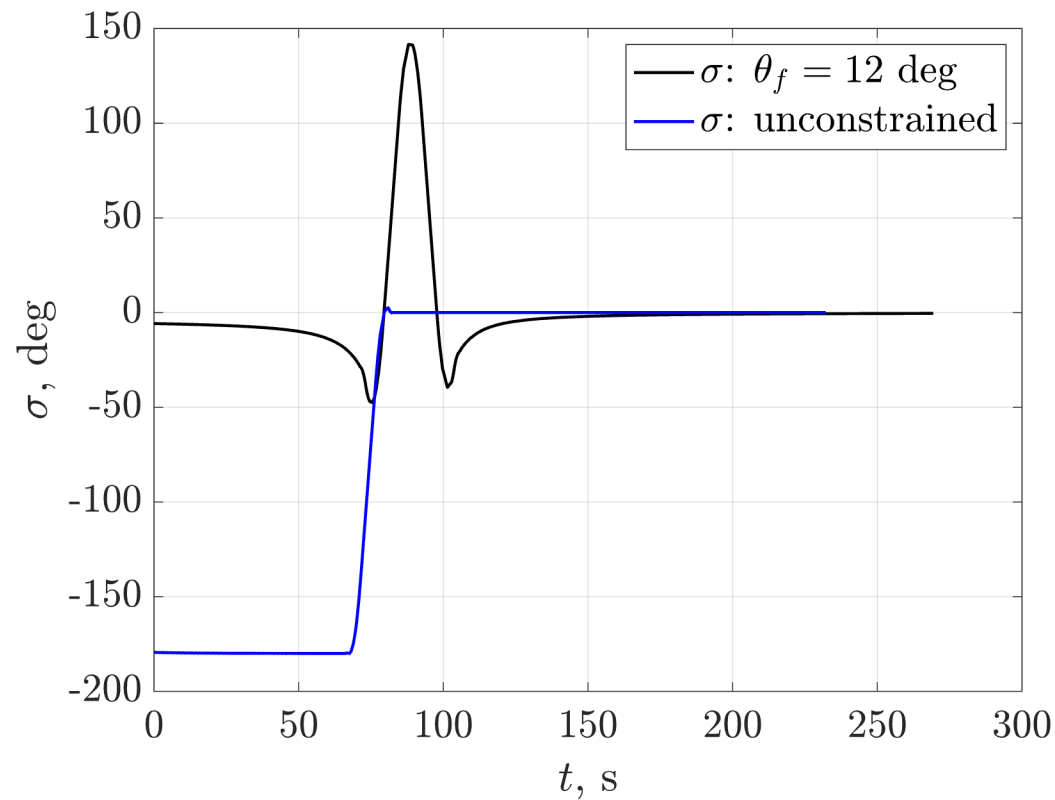
$$C_L = C_{L,\alpha}\alpha$$

$$C_Y = C_{Y,\beta}\beta$$

# Control Constraints-Trajectory Planning

| Parameter             | Value                |
|-----------------------|----------------------|
| $\alpha_{max}$        | 16.74 deg            |
| $\beta_{max}$         | 16.74 deg            |
| $\dot{\alpha}_{max}$  | 20 deg/s             |
| $\dot{\beta}_{max}$   | 20 deg/s             |
| $\ddot{\alpha}_{max}$ | 5 deg/s <sup>2</sup> |
| $\ddot{\beta}_{max}$  | 5 deg/s <sup>2</sup> |
| $\dot{\sigma}_{max}$  | 20 deg/s             |
| $\ddot{\sigma}_{max}$ | 5 deg/s <sup>2</sup> |
| $\alpha_{bank}$       | -16.74 deg           |

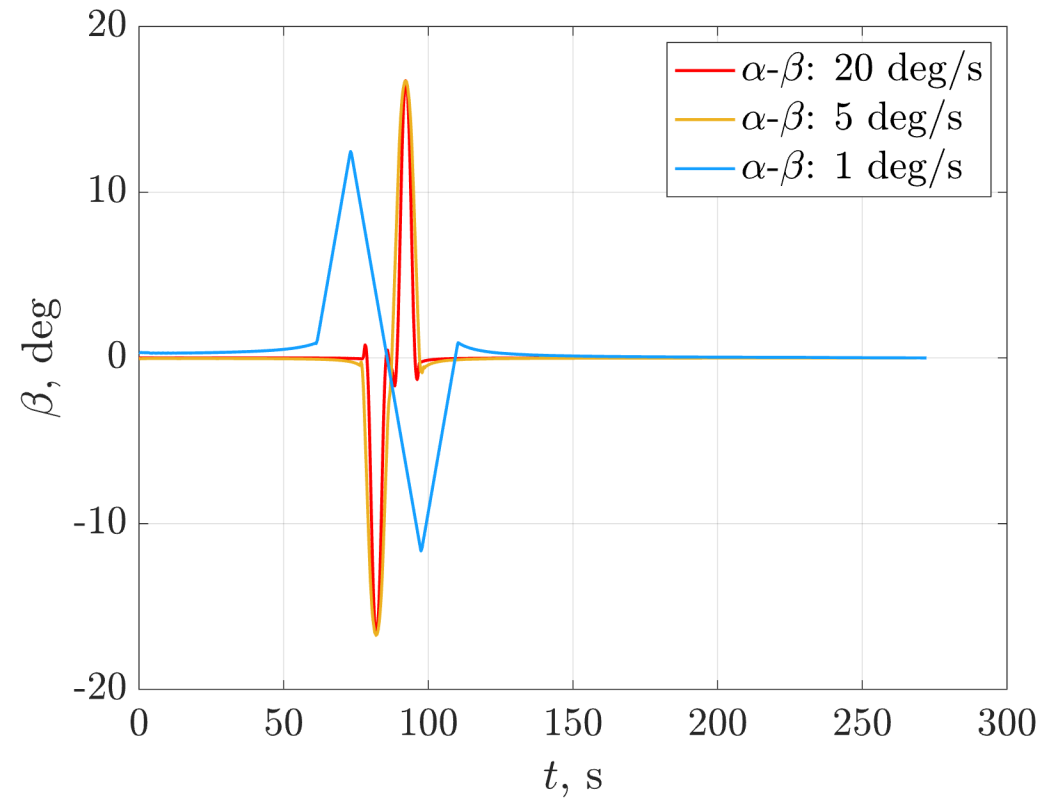
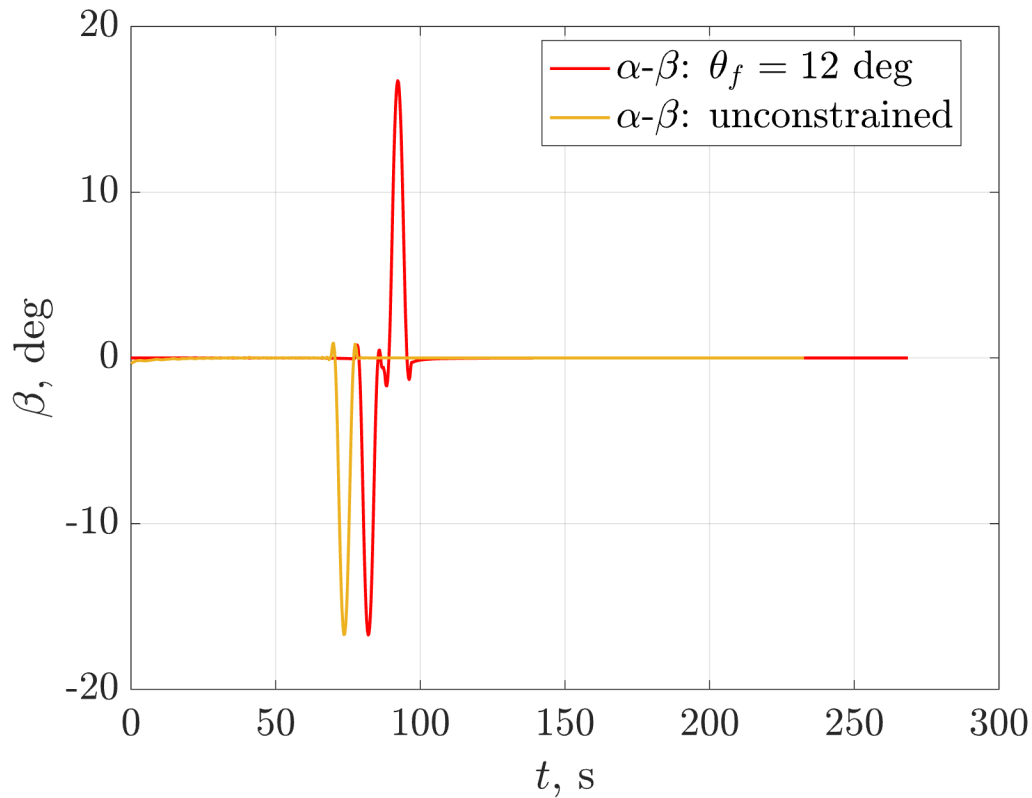
# Maximum Altitude-Constrained and Unconstrained



# Maximum Altitude-Constrained and Unconstrained

| Case                                              | Terminal Altitude, km |
|---------------------------------------------------|-----------------------|
| $\sigma : \theta_f = 10 \text{ deg}$              | 18.05                 |
| $\alpha\text{-}\beta : \theta_f = 10 \text{ deg}$ | 18.35                 |
| $\sigma : \theta_f = 12 \text{ deg}$              | 18.33                 |
| $\alpha\text{-}\beta : \theta_f = 12 \text{ deg}$ | 18.81                 |
| $\sigma : \theta_f = 14 \text{ deg}$              | 14.76                 |
| $\alpha\text{-}\beta : \theta_f = 14 \text{ deg}$ | 15.22                 |
| $\sigma : \text{unconstrained}$                   | 19.75                 |
| $\alpha\text{-}\beta : \text{unconstrained}$      | 20.04                 |

# Sideslip Angle Impact on Altitude Performance

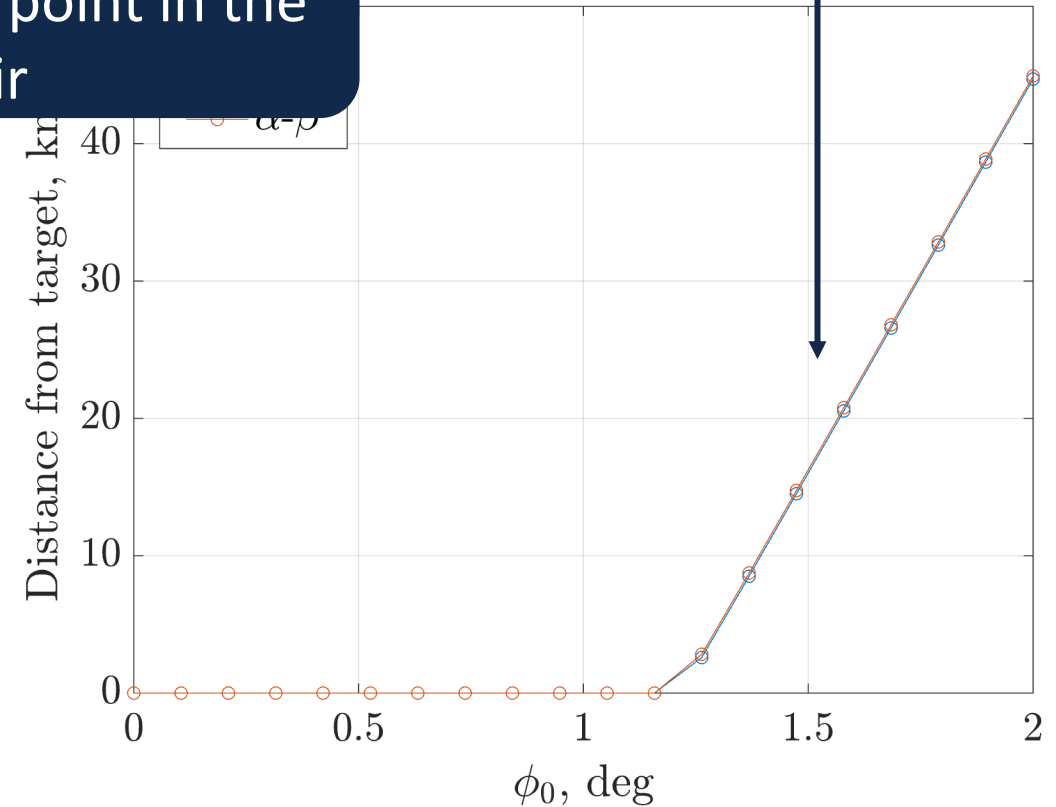


# Minimum Terminal Error

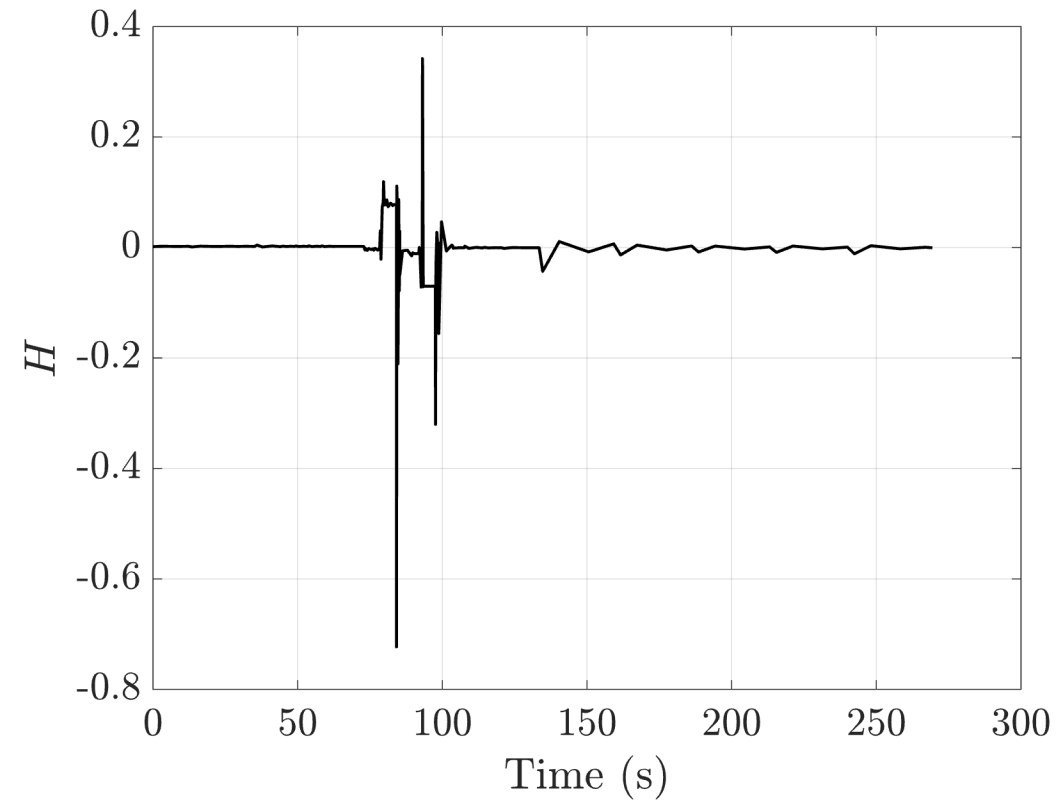
$$\begin{aligned}
 \min_{\mathbf{u}} \quad & J = x_e^2 + y_e^2 + z_e^2 \\
 \text{s.t.} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\
 & V(t_f) = V_f \\
 \alpha\text{-}\beta : \quad & \alpha \leq \alpha_{max} \quad -\alpha \leq \alpha_{max} \\
 & \beta \leq \beta_{max} \quad -\beta \leq \beta_{max} \\
 & \alpha_{total} \leq |\alpha_{bank}| \\
 & \dot{\alpha} \leq \dot{\alpha}_{max} \quad -\dot{\alpha} \leq \dot{\alpha}_{max} \\
 & \ddot{\alpha} \leq \ddot{\alpha}_{max} \quad -\ddot{\alpha} \leq \ddot{\alpha}_{max} \\
 & \dot{\beta} \leq \dot{\beta}_{max} \quad -\dot{\beta} \leq \dot{\beta}_{max} \\
 & \ddot{\beta} \leq \ddot{\beta}_{max} \quad -\ddot{\beta} \leq \ddot{\beta}_{max} \\
 \sigma : \quad & \dot{\sigma} \leq \dot{\sigma}_{max} \quad -\dot{\sigma} \leq \dot{\sigma}_{max} \\
 & \ddot{\sigma} \leq \ddot{\sigma}_{max} \quad -\ddot{\sigma} \leq \ddot{\sigma}_{max}
 \end{aligned}$$

Minimize miss distance to Cartesian point in the air

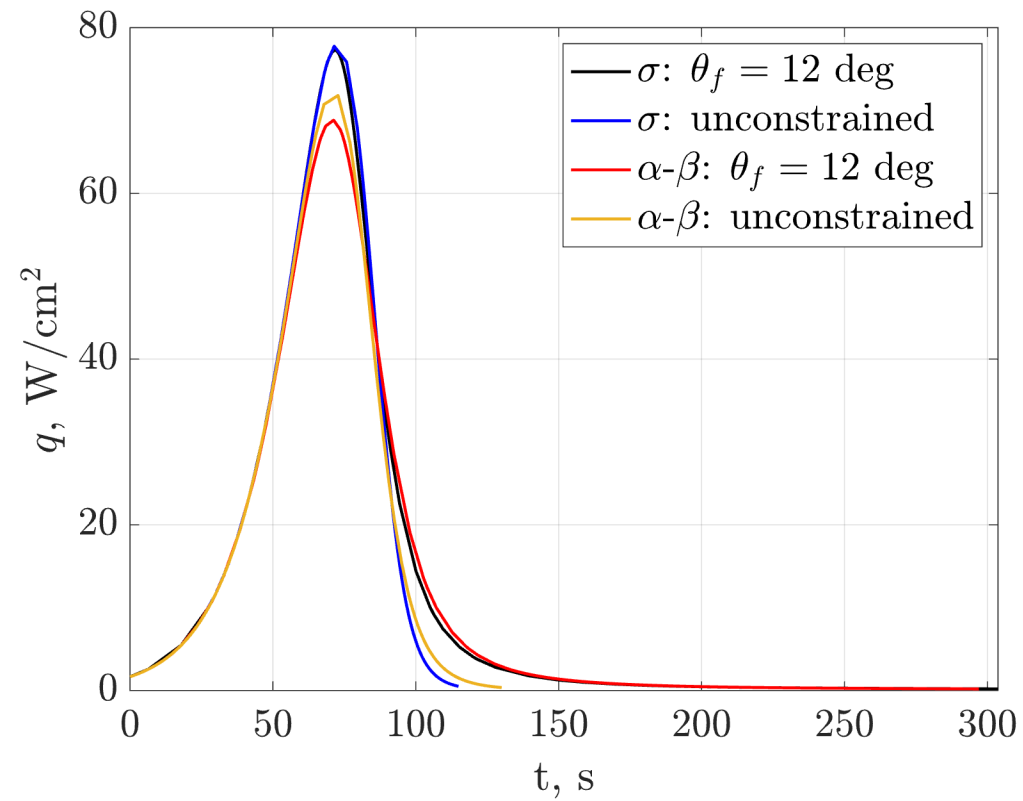
Steering options perform essentially identically as control authorities are the same



# Hamiltonian

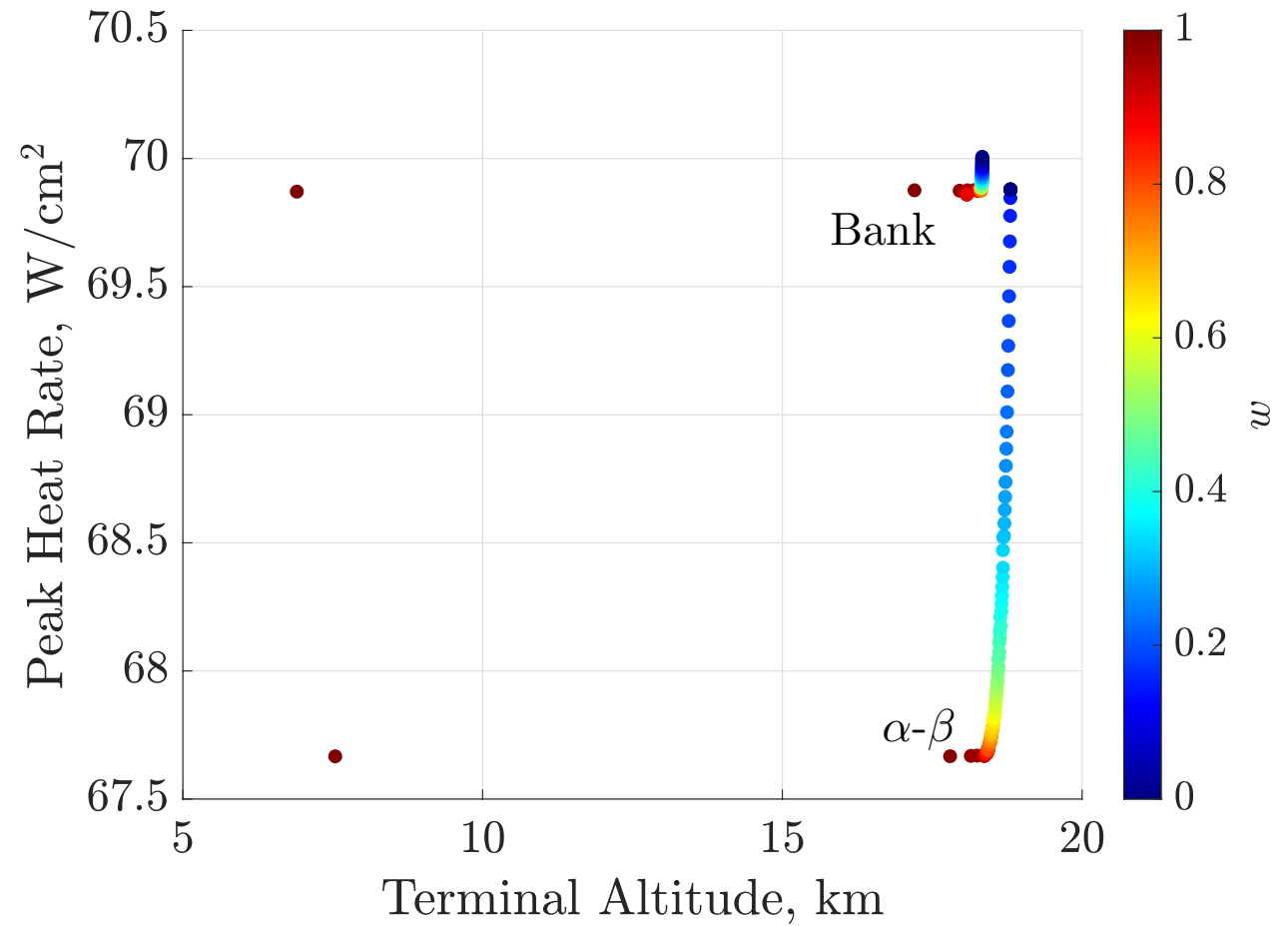


# Minimum Heat Load Curves



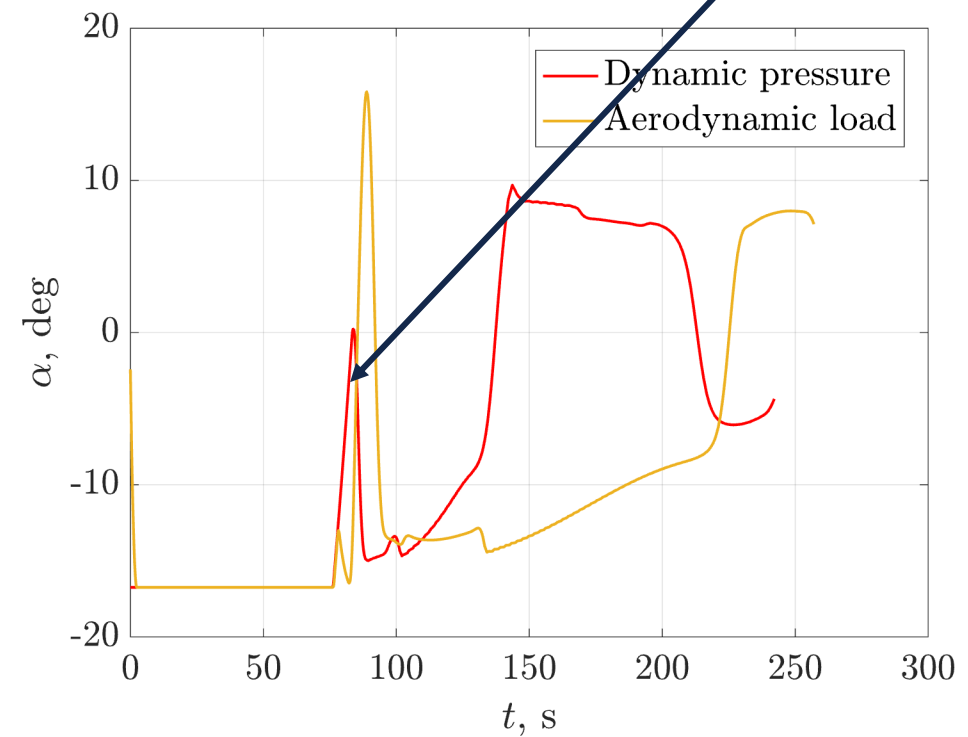
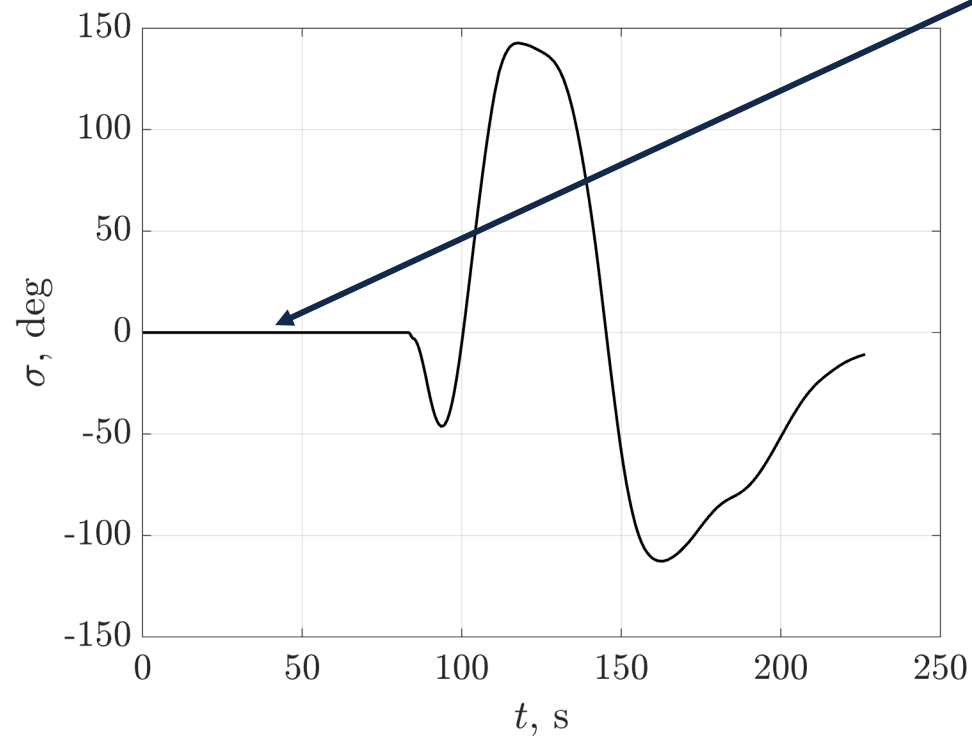
# Heat Rate Pareto Fronts

$$J = w \frac{g(t_f)}{q_{scale}} - (1 - w) \frac{z(t_f)}{z_{scale}}$$

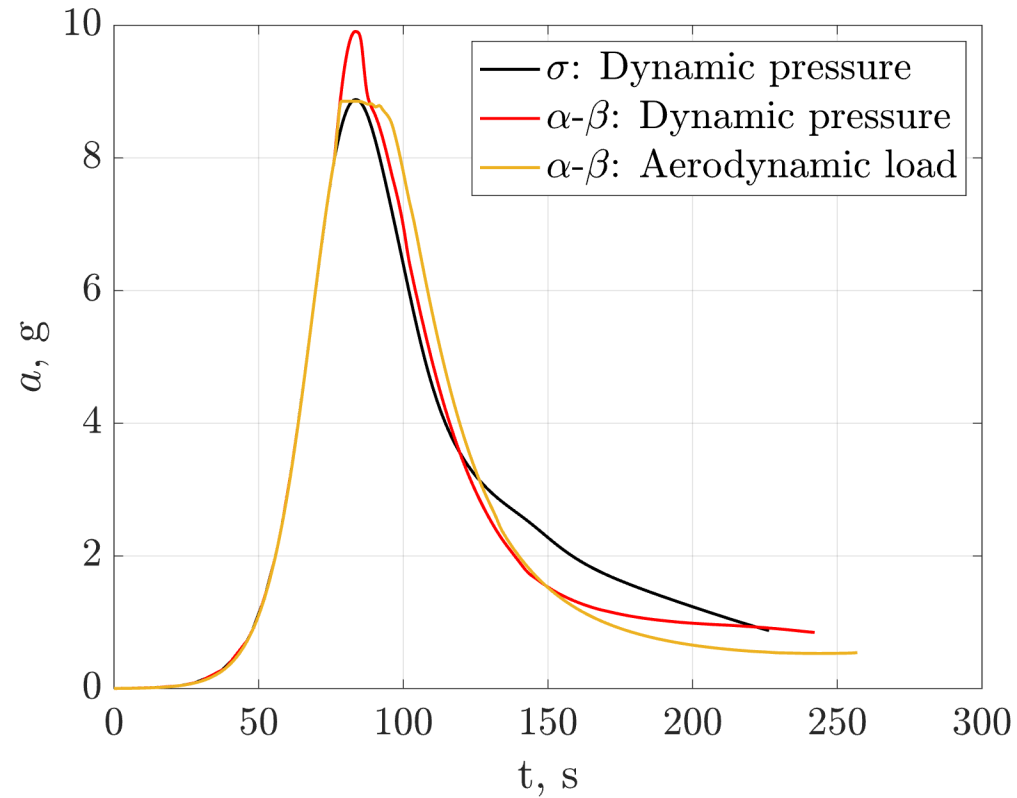


# Minimum Peak Dynamic Pressure and Aerodynamic Load

Bank-angle steering  
vehicle flies lift up  
with peak  
dynamic pressure



# Minimum Peak Aerodynamic Load



# Models and Equations of Motion: Closed-Loop Steering Commands

Truth dynamics  
consider 3DOF  
translational motion of  
a future large entry  
vehicle

- **Atmosphere:** Mars  
GRAM
- **Planet:** oblate,  
rotating
- **Aerodynamics:** Mars  
Science Laboratory  
aerodynamics  
database

| Parameter       | Value                |
|-----------------|----------------------|
| $m$             | 5600 kg              |
| $S_{ref}$       | 15.90 m <sup>2</sup> |
| $\alpha_{max}$  | 18 deg               |
| $\beta_{max}$   | 18 deg               |
| $\alpha_{bank}$ | -18 deg              |

# Models Used in Optimal Control Solver

$$C_D = C_{D,0} + C_{D,\alpha}\alpha^2 + C_{D,\beta}\beta^2$$

$$C_L = C_{L,\alpha}\alpha$$

$$C_Y = C_{Y,\beta}\beta$$

| Parameter                | Value                   |
|--------------------------|-------------------------|
| Surface Density          | 0.012 kg/m <sup>3</sup> |
| Atmospheric Scale Height | 11.888 km               |
| Surface Gravity          | 3.71 m/s <sup>2</sup>   |
| Mars Equatorial Radius   | 3396.20 km              |

| Parameter      | Value                  |
|----------------|------------------------|
| $C_{D,0}$      | 1.67                   |
| $C_{D,\alpha}$ | -2.86/rad <sup>2</sup> |
| $C_{D,\beta}$  | -2.86/rad <sup>2</sup> |
| $C_{L,\alpha}$ | -1.31/rad              |
| $C_{Y,\beta}$  | 1.29/rad               |

# Optimal Control Problems

$$\begin{cases} L = \dot{\sigma}^2 \\ L = \dot{\alpha}^2 + \dot{\beta}^2 \end{cases}$$

## Problem 1: Maximum Altitude

$$\begin{aligned} \min_{\mathbf{u}} \quad & J = -w_1 \frac{z(t_f)}{R_m} + \theta_{error}^2 + \phi_{error}^2 \\ \text{s.t.} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ & V(t_f) = V_f \\ \alpha\text{-}\beta : & \alpha \leq \alpha_{max} \quad -\alpha \leq -\alpha_{max} \\ & \beta \leq \beta_{max} \quad -\beta \leq -\beta_{max} \\ & \alpha_{total} \leq |\alpha_{bank}| \\ & \dot{\alpha} \leq \dot{\alpha}_{max} \quad -\dot{\alpha} \leq -\dot{\alpha}_{max} \\ & \ddot{\alpha} \leq \ddot{\alpha}_{max} \quad -\ddot{\alpha} \leq -\ddot{\alpha}_{max} \\ & \dot{\beta} \leq \dot{\beta}_{max} \quad -\dot{\beta} \leq -\dot{\beta}_{max} \\ & \ddot{\beta} \leq \ddot{\beta}_{max} \quad -\ddot{\beta} \leq -\ddot{\beta}_{max} \\ \sigma : & \dot{\sigma} \leq \dot{\sigma}_{max} \quad -\dot{\sigma} \leq -\dot{\sigma}_{max} \\ & \ddot{\sigma} \leq \ddot{\sigma}_{max} \quad -\ddot{\sigma} \leq -\ddot{\sigma}_{max} \end{aligned}$$

## Problem 2: Minimum Control Effort

“Hit target latitude/longitude while maximizing altitude”

Dynamics

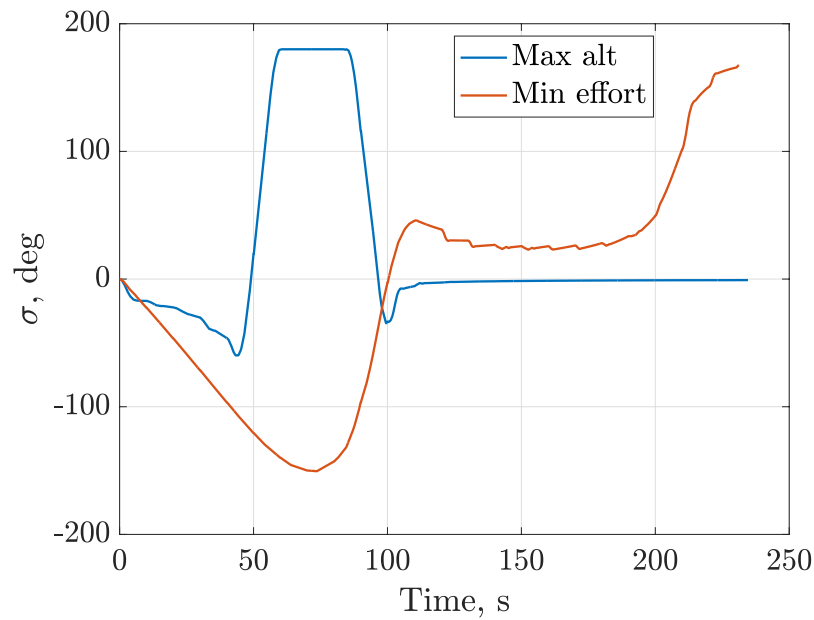
Terminal velocity

Control constraints

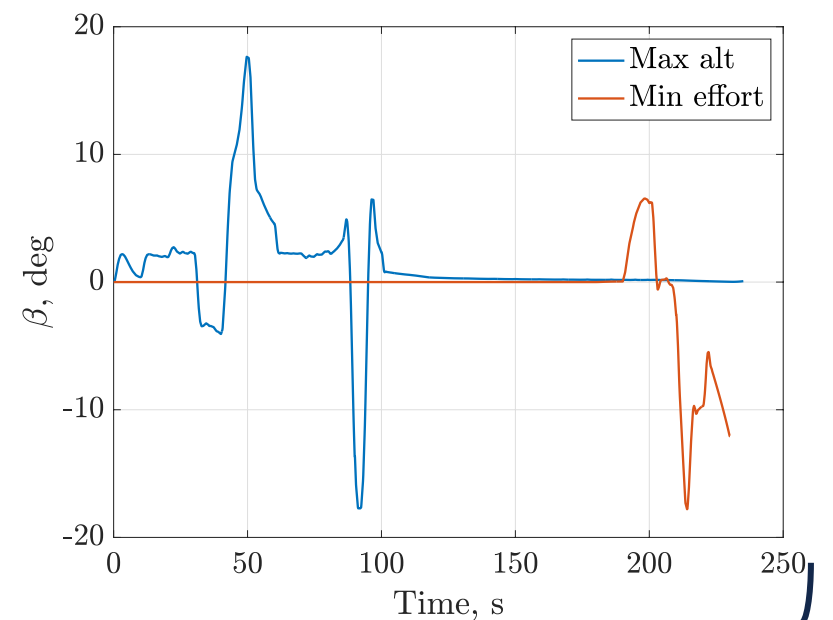
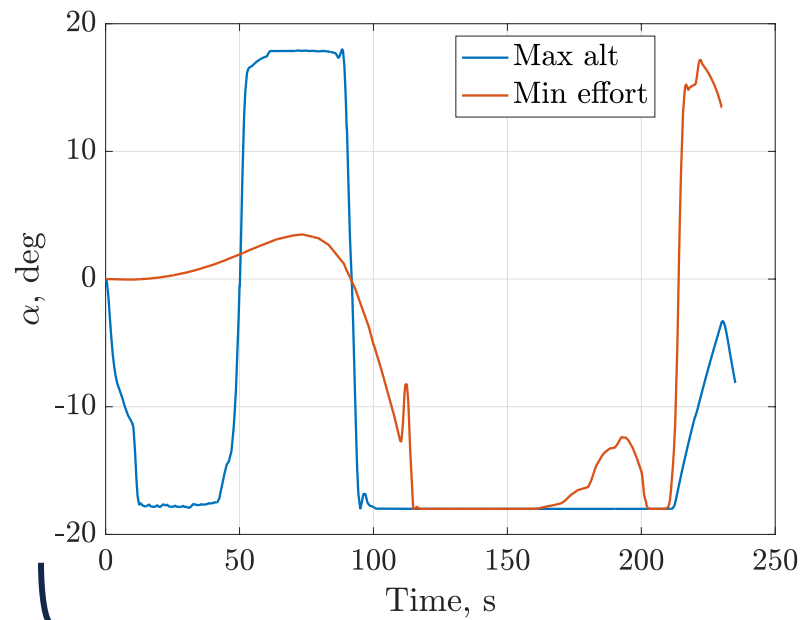
“Hit target point while minimizing control effort”

$$\begin{aligned} \min_{\mathbf{u}} \quad & J = \int_{t_0}^{t_f} L(\mathbf{x}, \mathbf{u}) dt + Z_e^2 \\ \text{s.t.} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ & V(t_f) = V_f \\ \alpha\text{-}\beta : & \alpha \leq \alpha_{max} \quad -\alpha \leq -\alpha_{max} \\ & \beta \leq \beta_{max} \quad -\beta \leq -\beta_{max} \\ & \alpha_{total} \leq |\alpha_{bank}| \\ & \dot{\alpha} \leq \dot{\alpha}_{max} \quad -\dot{\alpha} \leq -\dot{\alpha}_{max} \\ & \ddot{\alpha} \leq \ddot{\alpha}_{max} \quad -\ddot{\alpha} \leq -\ddot{\alpha}_{max} \\ & \dot{\beta} \leq \dot{\beta}_{max} \quad -\dot{\beta} \leq -\dot{\beta}_{max} \\ & \ddot{\beta} \leq \ddot{\beta}_{max} \quad -\ddot{\beta} \leq -\ddot{\beta}_{max} \\ \sigma : & \dot{\sigma} \leq \dot{\sigma}_{max} \quad -\dot{\sigma} \leq -\dot{\sigma}_{max} \\ & \ddot{\sigma} \leq \ddot{\sigma}_{max} \quad -\ddot{\sigma} \leq -\ddot{\sigma}_{max} \end{aligned}$$

# Nominal Results: Closed-Loop Steering Commands



Bank



$\alpha$ - $\beta$

# Monte Carlo Simulation

Uncertainty  
analysis  
performed using  
Monte Carlo  
simulations with  
1000 samples

| Variable                  | Dispersion                       | Distribution |
|---------------------------|----------------------------------|--------------|
| Initial altitude          | $\pm 2.8 \text{ km } 3\sigma$    | Gaussian     |
| Initial longitude         | $\pm 0.017 \text{ deg } 3\sigma$ | Gaussian     |
| Initial latitude          | $\pm 0.013 \text{ deg } 3\sigma$ | Gaussian     |
| Initial velocity          | $\pm 2 \text{ m/s } 3\sigma$     | Gaussian     |
| Initial flight-path angle | $\pm 0.2 \text{ deg } 3\sigma$   | Gaussian     |
| Initial azimuth           | $\pm 0.006 \text{ deg } 3\sigma$ | Gaussian     |
| Atmosphere profile        | 1000 Mars GRAM atmospheres       | -            |
| Hypersonic $C_A$          | $\pm 3\% 3\sigma$                | Gaussian     |
| Hypersonic $C_N$          | $\pm 5\% 3\sigma$                | Gaussian     |
| Supersonic $C_A$          | $\pm 10\% 3\sigma$               | Gaussian     |
| Supersonic $C_N$          | $\pm 8\% 3\sigma$                | Gaussian     |
| Mass                      | $\pm 2 \text{ kg } 3\sigma$      | Gaussian     |

# Impact of Vehicle Rate Limits

20 deg/s

| Parameter                | Downrange, km | Crossrange, km | Altitude, km |
|--------------------------|---------------|----------------|--------------|
| Mean: bank               | 0.15          | 3.61           | 9.78         |
| SD: bank                 | 3.76          | 0.73           | 0.51         |
| Mean: $\alpha$ - $\beta$ | -0.25         | 1.31           | 9.94         |
| SD: $\alpha$ - $\beta$   | 3.89          | 0.43           | 0.52         |

$\alpha$ - $\beta$  has slightly improved performance

5 deg/s

| Parameter                | Downrange, km | Crossrange, km | Altitude, km |
|--------------------------|---------------|----------------|--------------|
| Mean: bank               | 4.22          | 19.09          | 9.02         |
| SD: bank                 | 3.58          | 1.17           | 0.54         |
| Mean: $\alpha$ - $\beta$ | 0.09          | 2.90           | 9.89         |
| SD: $\alpha$ - $\beta$   | 3.51          | 0.58           | 0.52         |

Reduced performance for bank-angle steering

1 deg/s

| Parameter                | Downrange, km | Crossrange, km | Altitude, km |
|--------------------------|---------------|----------------|--------------|
| Mean: bank               | 3.45          | 56.44          | 4.72         |
| SD: bank                 | 5.74          | 0.91           | 0.52         |
| Mean: $\alpha$ - $\beta$ | -1.90         | 1.01           | 8.76         |
| SD: $\alpha$ - $\beta$   | 4.07          | 0.37           | 0.57         |

Severe performance loss for bank-angle steering