A NASA Perspective on
Quantum Computing, with Ties to Operator Algebras

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NASA QuAIL mandate: ***Determine the potential for quantum computation to enable more ambitious and safer NASA missions in the future***

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+ active year-round intern program

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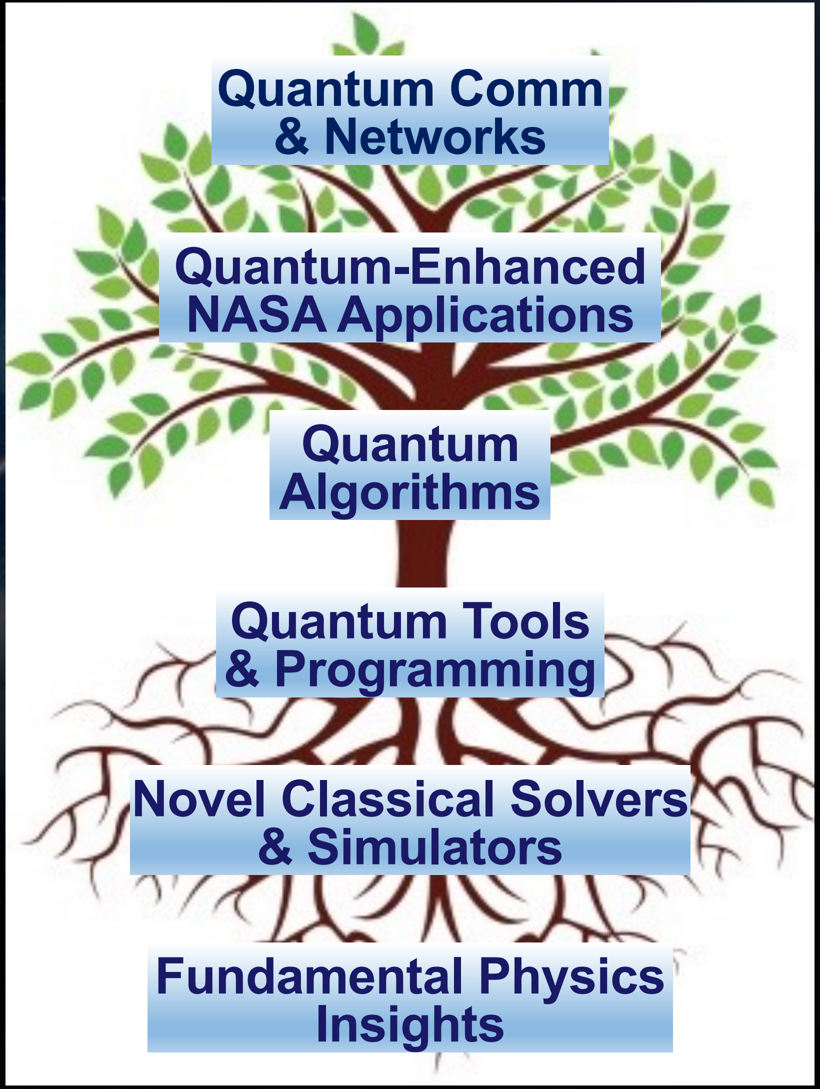
Quantum Computing R&D at NASA Ames



Quantum Comm & Networks

Quantum-Enhanced NASA Applications

Quantum Algorithms

Quantum Tools & Programming

Novel Classical Solvers & Simulators

Fundamental Physics Insights

Communication & Networks

Quantum networking
Distributed QC

Application Focus Areas

Planning and scheduling
Material science
Fault diagnosis
Machine learning
Recently: Computational fluid dynamics (CFDs)

Software Tools & Algorithms

Quantum algorithm design
Compiling to hardware
Mapping, parameter setting, error mitigation
Hybrid quantum-classical approaches

Solvers & Simulators

Physics-inspired classical solvers
HPC quantum circuit simulators

Physics Insights

Co-design quantum hardware

E. Rieffel *et al.* (2019), From Ansätze to Z-gates: A NASA view of quantum computing, *Adv. in Parallel Computing* **34**, 133–160
R. Biswas *et al.* (2017), A NASA perspective on quantum computing: Opportunities and challenges, *Parallel Computing* **64**, 81–98

Outline

Topics in quantum error correction

- Brief overview of quantum error correction fundamentals
- Review of Bacon-Shor subsystem codes
- Very brief glimpse of Honeycomb Floquet code
- Description of new family of Floquet codes

Brief glimpses of other topics

- Wigner friend inequalities & non-local games
- T-designs

Quantum Error Correction: Rough Beginnings (1990s)

Is Shor’s factoring algorithm more than a theoretical curiosity?

- If nature doesn’t allow robust quantum computing, then no one will be able to ever run it

“unless some unforeseen new physics is discovered, the implementation of error-correcting codes will become exceedingly difficult as soon as one has to deal with more than a few gates”

-Haroche & Raimond

“if error correction is needed, this is inevitably dissipative and incoherent, and prevents quantum parallelism”

“error correction in quantum computation cannot follow the recipes we learned for classical digital computers. ... we cannot, in general, tell whether two arbitrary quantum states differ, or not. Even if we were able to recognize errors, we cannot throw away the description of the error”

- Landauer

Qubits

State picture

Qubits modeled by 2-dimensional vector space

- Vectors are equivalent up to factors
- Complex projective space

Conventions
- Standard basis: {|0>, |1>}
- Alternative basis {|+>, |1>}

Qubit states: complex unit vectors

Operator picture

Pauli Z operator |0><0| - |1><1|

- Eigenstates define standard basis
- Pauli X operator |1><0| +|0><1|
- Eigenstates define alternative basis
- Pauli group

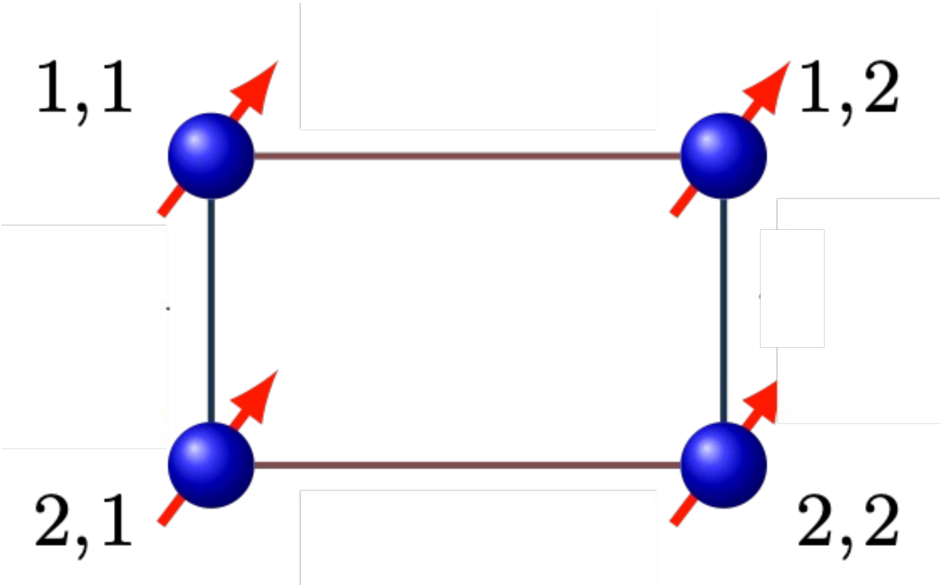
n-qubit systems

- Tensor product of n single qubit systems

[[4,2,2]] stabilizer error-detecting code

$$S^X = X_{1,1}X_{1,2}X_{2,1}X_{2,2}$$
$$S^Z = Z_{1,1}Z_{2,1}Z_{1,2}Z_{2,2}.$$

- Code space is the +1 eigenspace of the stabilizer operators
- Measurement of the stabilizers gives error syndrome
  - Fed to a decoding algorithm running on a classical (non-quantum) computer, which determines likely error and passes it to the controller, also running on a classical computer, of the quantum computer

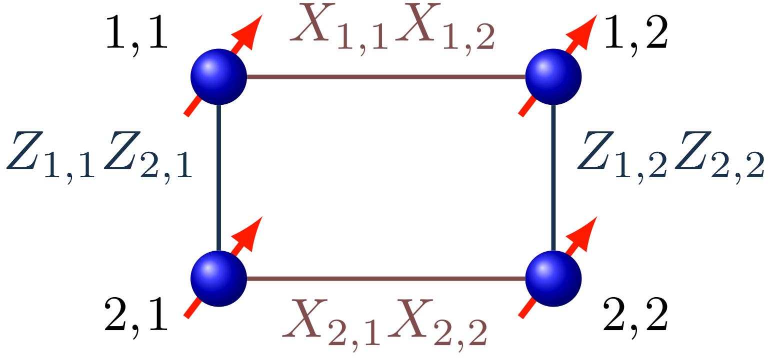


$$\tilde{X}_{L1} = X_{1,1}X_{2,1}\ , \quad \tilde{Z}_{L1} = Z_{1,1}Z_{1,2}$$
$$\tilde{X}_{L2} = X_{1,1}X_{1,2}\ , \quad \tilde{Z}_{L2} = Z_{1,1}Z_{2,1}$$

[[4,1,2]] Bacon-Shor Subsystem code

Stabilizer code logical operators

$$\tilde{X}_{L1} = X_{1,1}X_{2,1}\,, \quad \tilde{Z}_{L1} = Z_{1,1}Z_{1,2}$$
$$\tilde{X}_{L2} = X_{1,1}X_{1,2}\,, \quad \tilde{Z}_{L2} = Z_{1,1}Z_{2,1}$$



$$\mathcal{G} = \langle G_1^X,\, G_2^X,\, G_1^Z,\, G_2^Z \rangle$$

$$G_1^X = X_{1,1}X_{1,2}\,, \quad G_2^X = X_{2,1}X_{2,2}\,,$$
$$G_1^Z = Z_{1,1}Z_{2,1}\,, \quad G_2^Z = Z_{1,2}Z_{2,2}\,,$$

$$S^X = X_{1,1}X_{1,2}X_{2,1}X_{2,2}$$
$$S^Z = Z_{1,1}Z_{2,1}Z_{1,2}Z_{2,2}\,.$$

These stabilizers define a [[4,2,2]] stabilizer code.

Lower weight measurements at the expense of fewer logical qubits

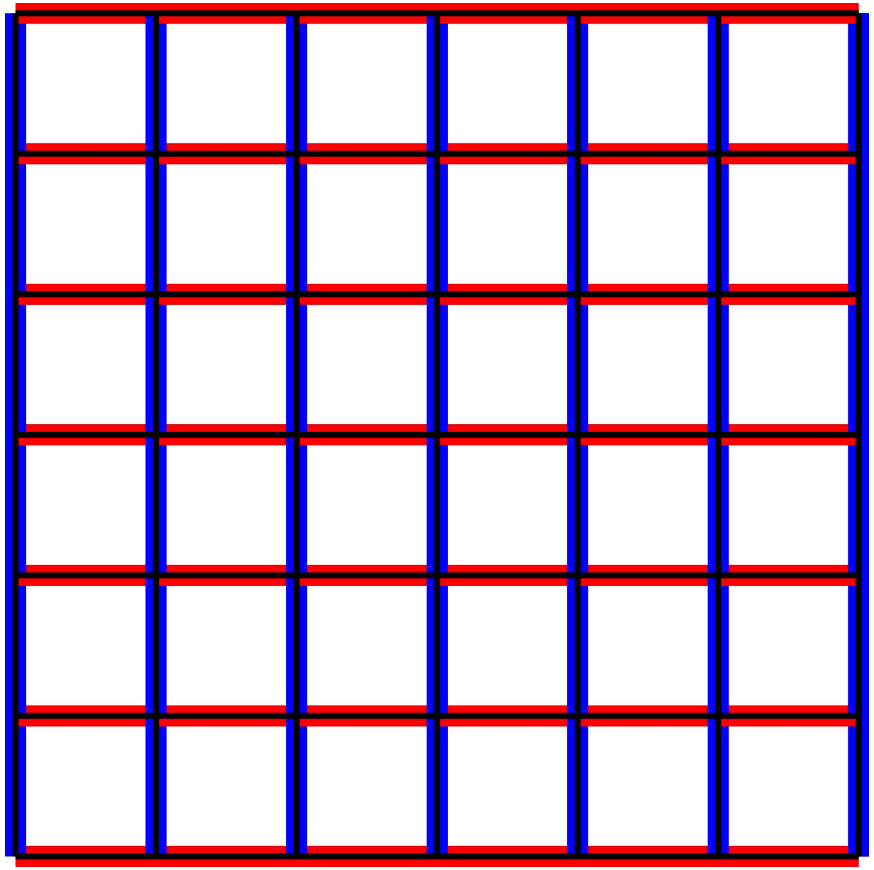
$$X_L = X_{1,1}X_{2,1}\,, \quad Z_L = Z_{1,1}Z_{1,2}$$

Side note: Jiang & Rieffel (2017) "Non-commuting two-local Hamiltonians for quantum error suppression" used B-S codes to circumvent a no-go theorem for stabilizer codes, enabling more practically implementable error suppression in AQC

BRIEF REVIEW OF BACON-SHOR SUBSYSTEM CODES

Bacon-Shor code

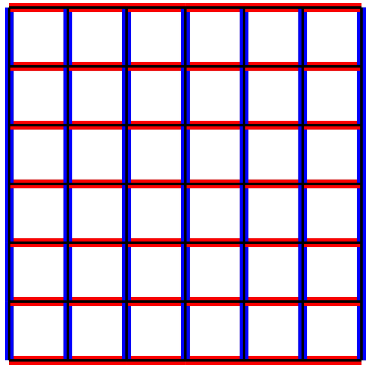
Special case of subsystem codes, which are a subclass of operator codes

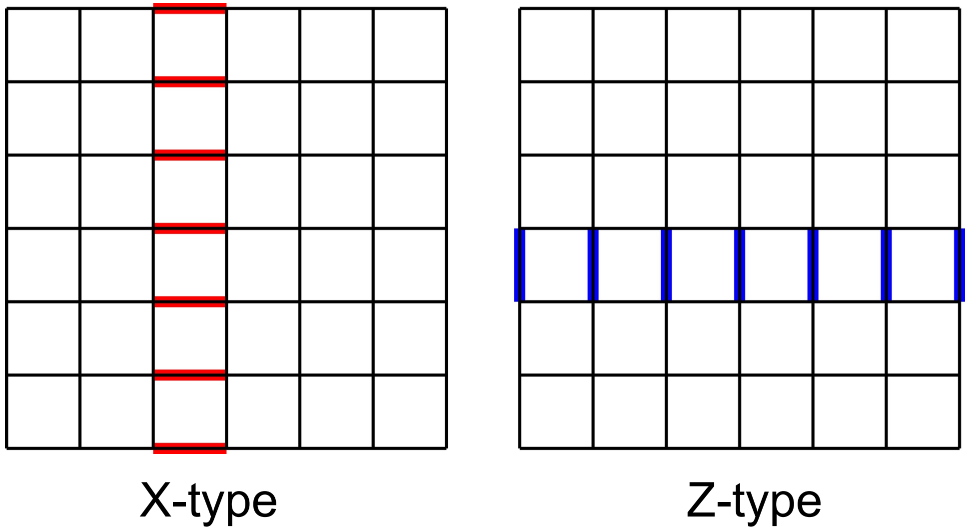


Gauge group generated by all nearest-neighbor 2-body

- XX checks (red, horizontal)
- ZZ checks (blue, vertical)

Bacon-Shor code

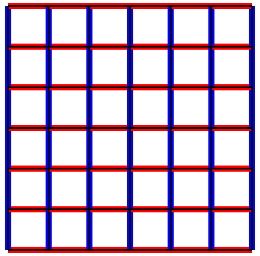




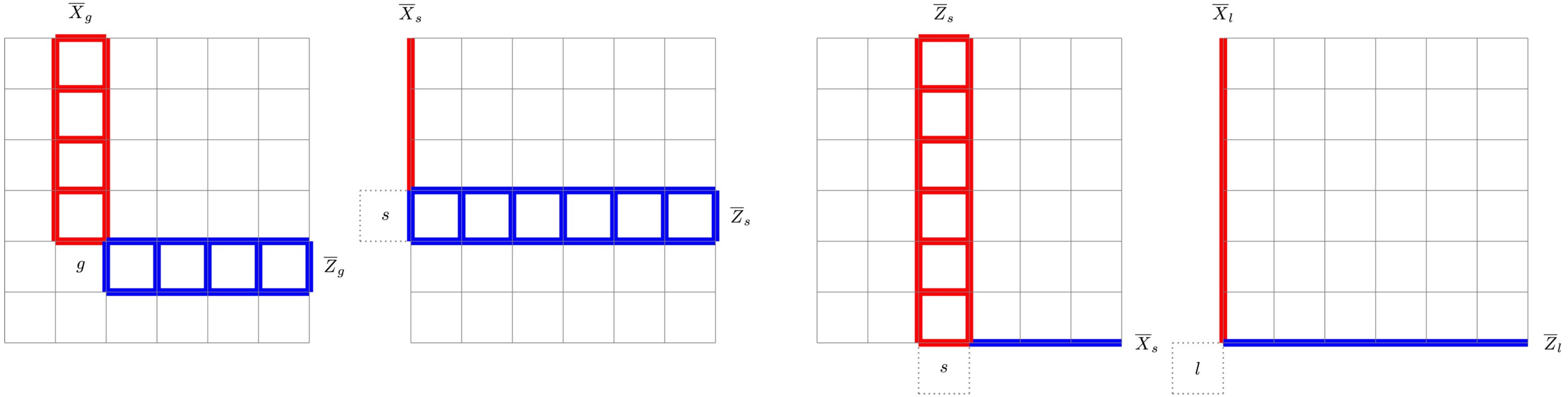
Stabilizer group *S*
 - Center of gauge group (elements of *S* that commute with all elements of *S*)

One stabilizer generating set
 - columns of XX operators, and
 - rows of ZZ operators

Bacon-Shor code



Virtual qubit operators



Gauge qubits, in one-to-one correspondence with plaquettes

Horizontal Stabilizer Qubit defined by a horizonal $\bar{Z}$ stabilizer (product of single qubit Z operators) and a vertical $\bar{X}$ operator (product of X ops)

Vertical Stabilizer Qubit defined by a vertical $\bar{Z}$ stabilizer (product of X ops) and a horizontal $\bar{X}$ operator (product of Z ops)

Logical qubit for B-S code logical $\bar{X}$ and $\bar{Z}$ ops

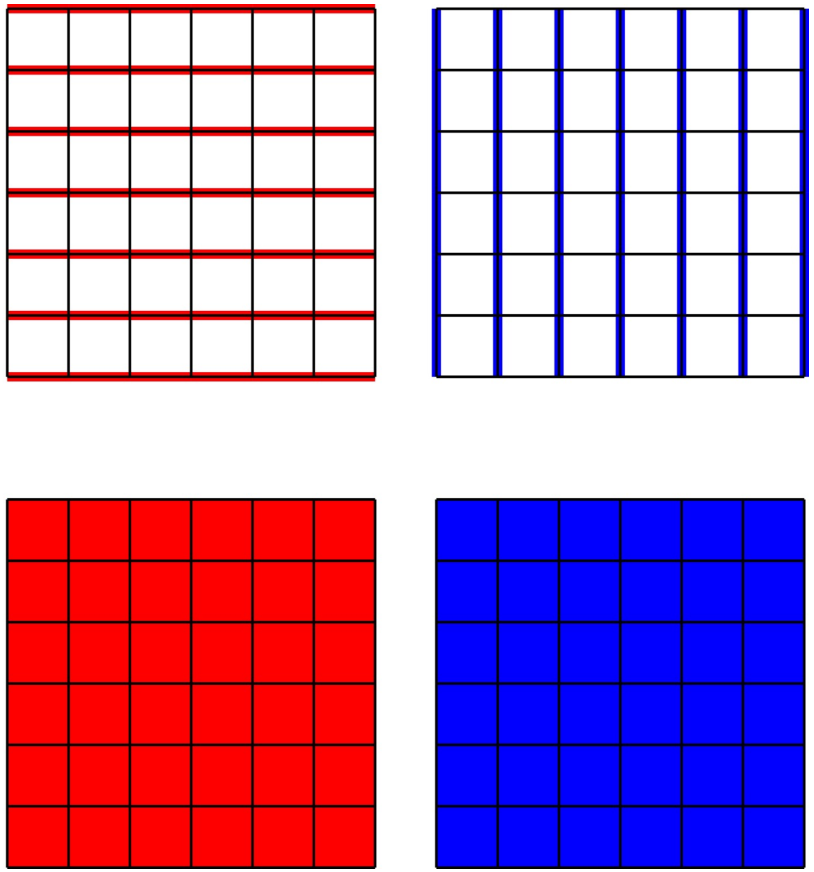
Bacon-Shor code

Two rounds of measurement
- Round 1: All XX checks (horizontal)
- Round 2: All ZZ checks (vertical)

We are periodically moving between two different **gauge fixings**
- one where the X operators of all gauge qubits have been fixed, and
- one where the Z operators of all gauge qubits have been fixed

Instantaneous Stabilizer Groups (ISGs)
- Bacon-Shor stabilizers plus all X gauge qubit operators
- Bacon-Shor stabilizers plus all Z gauge qubit operators

Measurement schedule

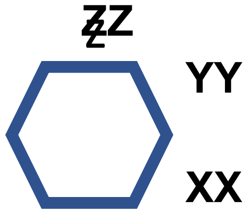


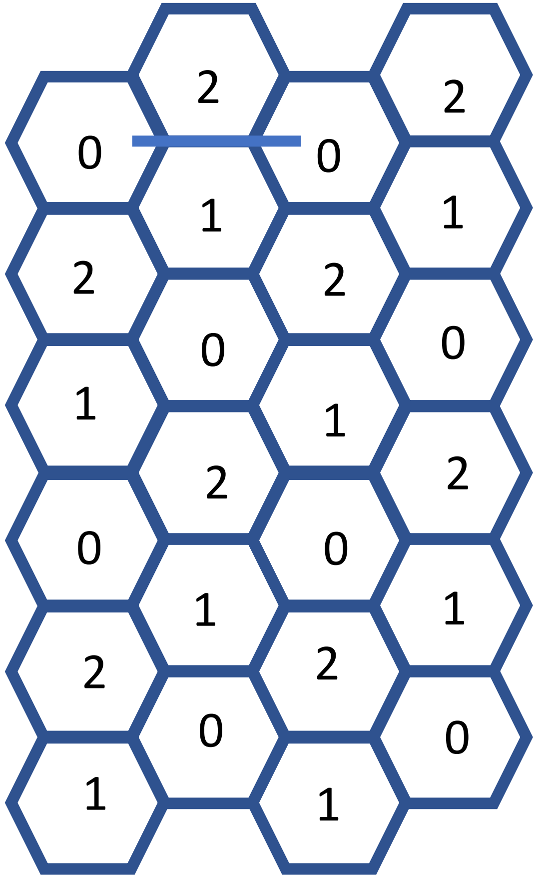
Instantaneous Stabilizer Groups (ISGs)

BRIEF GLIMPSE OF THE HONEYCOMB FLOQUET CODE

M. B. Hastings and J. Haah, Dynamically Generated Logical Qubits,
Quantum 5, 564 (2021)

Honeycomb code





M. B. Hastings and J. Haah, "Dynamically generated logical qubits." Quantum 5 (2021)

- Measure 2-qubit checks in 3 rounds
- At each round, there is an instantaneous stabilizer group
- Logical information is carried safely from one round to the next
- -uses equivalence of logical operator representations when multiplying by stabilizers
- Periodic (Floquet) structure
- General construction for 3-valent graphs
- Are there other ways to construct Floquet codes?

M. Sohaib Alam, Eleanor Rieffel

arXiv:2403.03291

DYNAMICAL LOGICAL QUBITS IN THE BACON-SHOR CODE

New family of Floquet codes

- On a square lattice rather than a trivalent graph
- Add defects to Bacon- Shor codes to obtain dynamical logical qubits
- Four rounds of measurement, alternating between X measurements and Z measurements

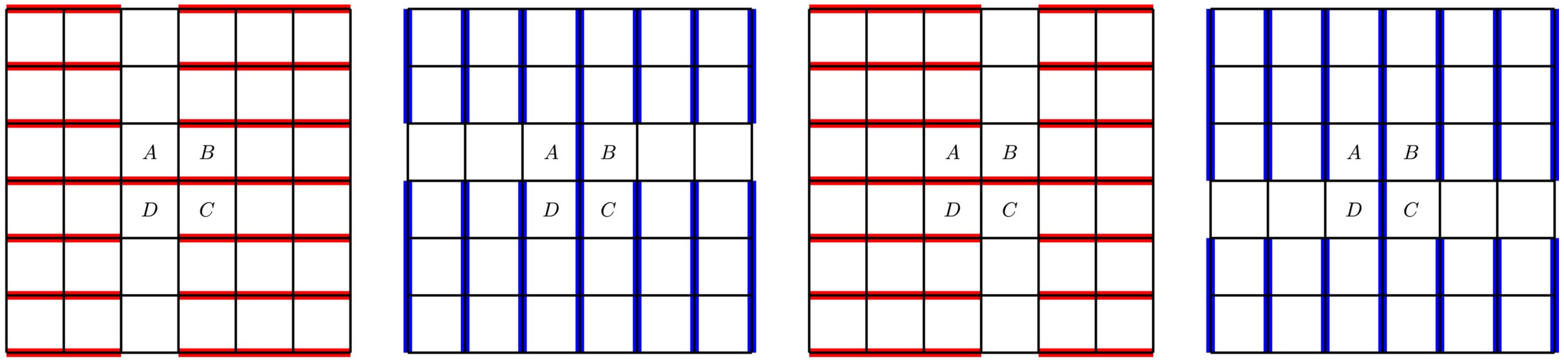
This work is part of a larger program trying to understand when one can define Floquet codes, when it is useful to do so, and subtleties with regard to defining their distance

Open question: General framework for constructing, and understanding, Floquet codes

Floquet-Bacon-Shor code

- To free up space for an additional logical qubit, we refrain from fixing one of the gauge degrees of freedom
  - We introduce a “gauge defect”
- We need to do so carefully to ensure all Bacon-Shor stabilizers get measured

Measurement schedule



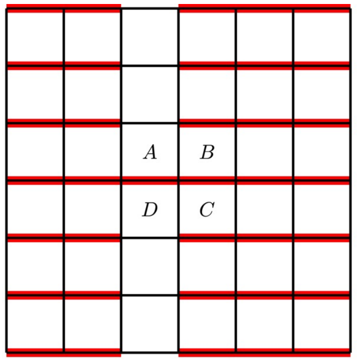
Subset of horizontal XX checks

Subset of vertical ZZ checks

Subset of horizontal XX checks

Subset of vertical ZZ checks

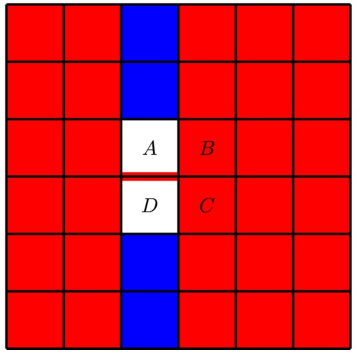
A Floquet-Bacon-Shor code: one of the Instantaneous Stabilizer Group (ISG)



$(\overline{X}_A, \overline{Z}_A \overline{Z}_D)$

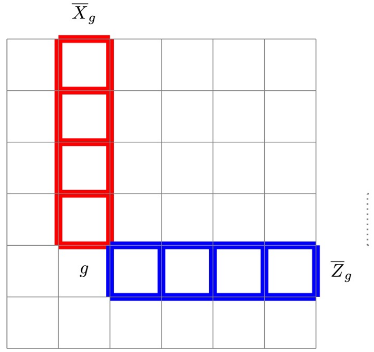
At any round, the ISG consists of

- the usual Bacon-Shor stabilizers,
- the just measured check operators
- all elements from previous ISGs that commute with the currently measured checks



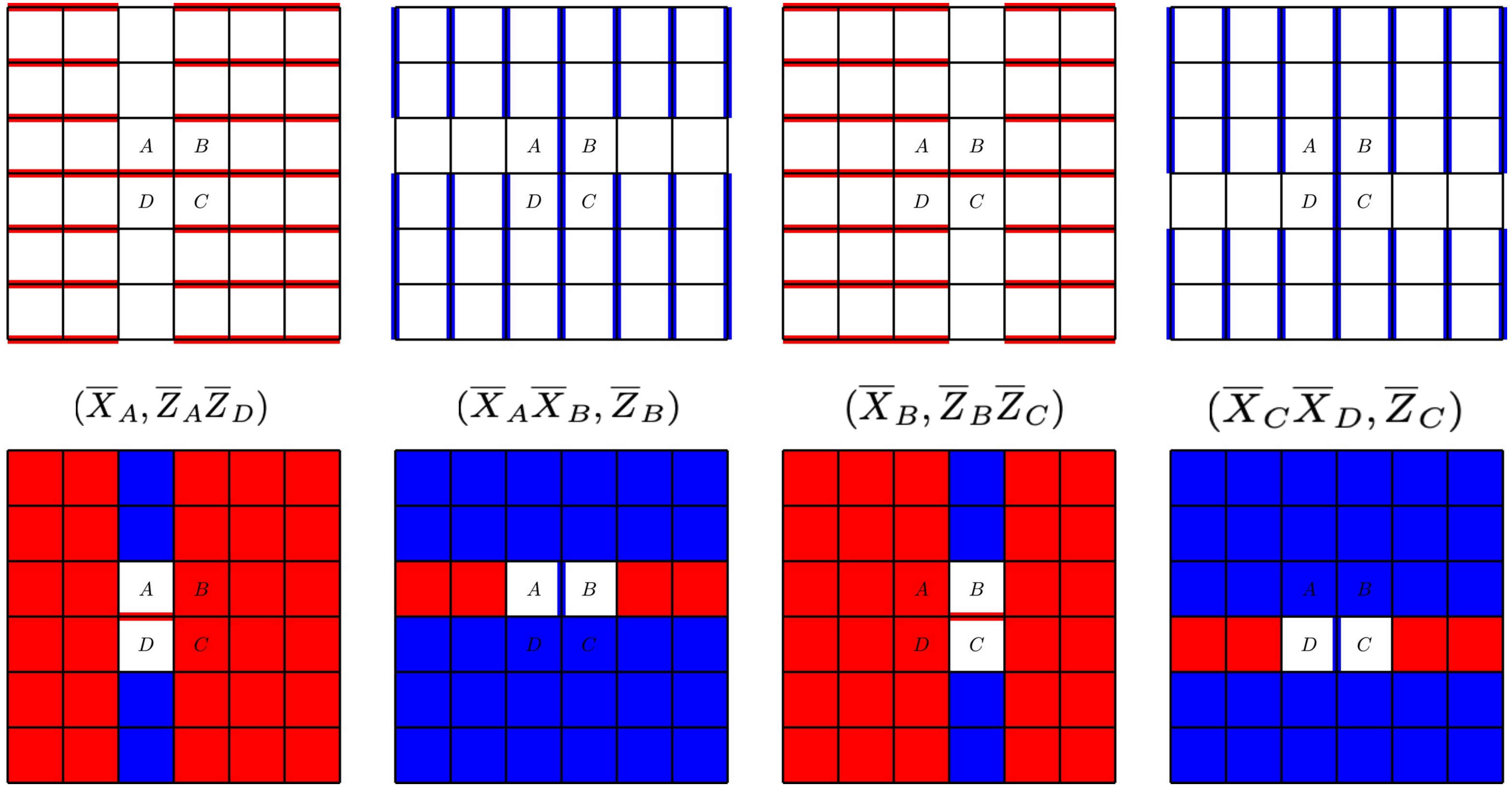
Any cell (or equivalently gauge qubit) colored either red or blue corresponds to gauge fixing its $\bar{X}$ or $\bar{Z}$ operator respectively

Gauge qubits, in one-to-one correspondence with plaquettes



A Floquet-Bacon-Shor code: ISGs

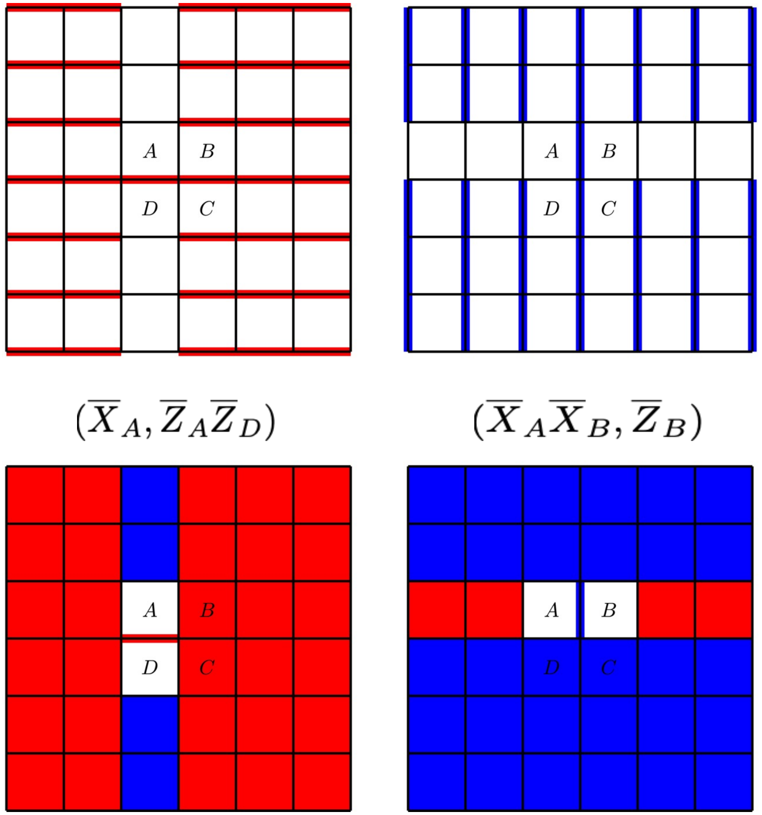
Measurement schedule



$(\overline{X}_A, \overline{Z}_A\overline{Z}_D)$ $(\overline{X}_A\overline{X}_B, \overline{Z}_B)$ $(\overline{X}_B, \overline{Z}_B\overline{Z}_C)$ $(\overline{X}_C\overline{X}_D, \overline{Z}_C)$

Instantaneous Stabilizer Groups
(ISGs)

A Floquet-Bacon-Shor code: preserving logical info



$(\overline{X}_A, \overline{Z}_A\overline{Z}_D)$ $(\overline{X}_A\overline{X}_B, \overline{Z}_B)$

Round 0 Round 1

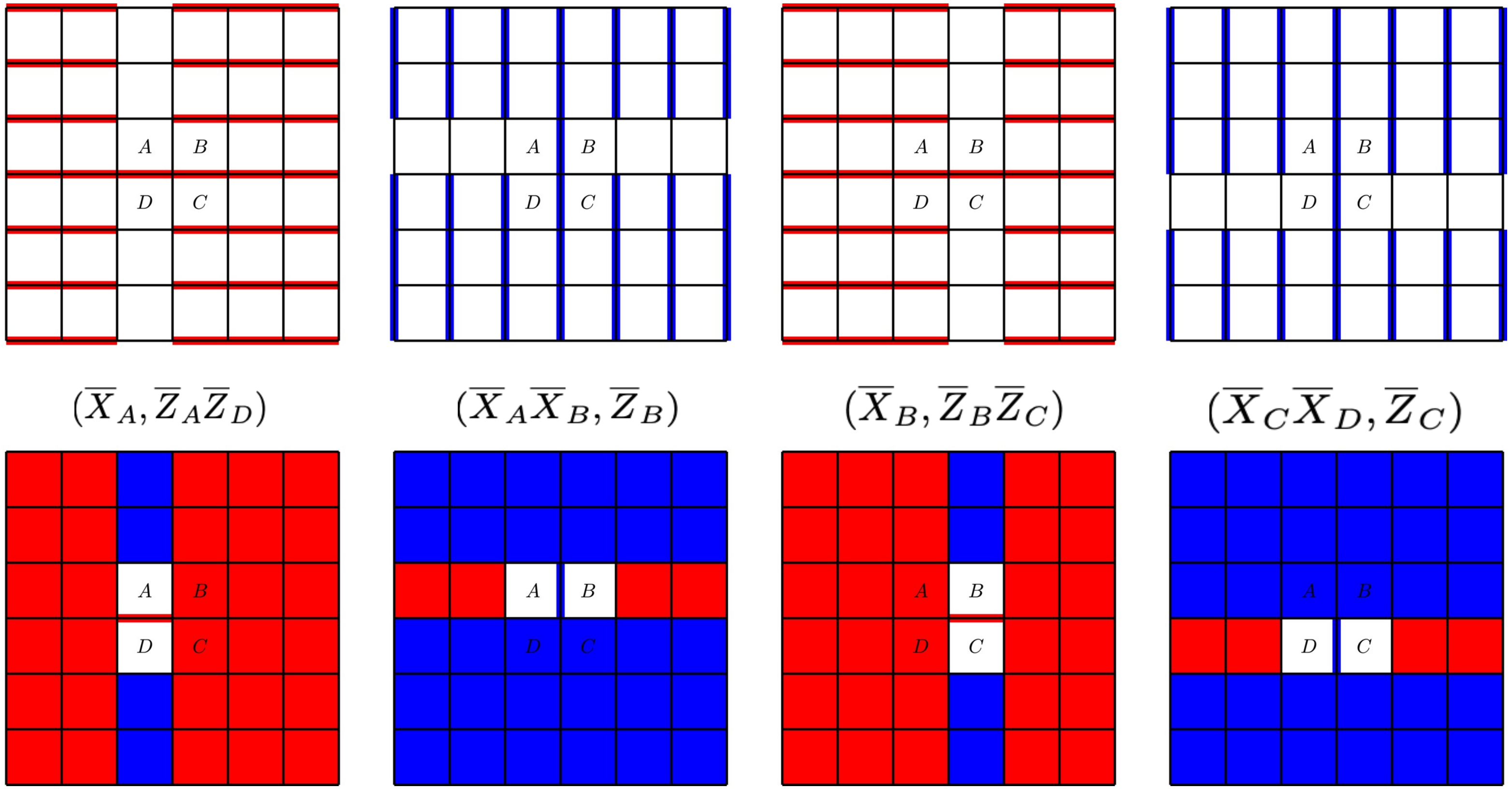
Preserving logical information from round to round

- $\overline{X_B}$ is in the ISG of round 0
- $\overline{Z_A Z_B}$ is in the ISG of round 1 (it is the single vertical blue bar in the picture)
- $\overline{Z_D}$ is in the ISG of round 1

Each pair of successive rounds define a (generalized) Bacon-Shor code

A Floquet-Bacon-Shor code: relation between logical ops

Measurement schedule



$(\overline{X}_A, \overline{Z}_A\overline{Z}_D)$ $(\overline{X}_A\overline{X}_B, \overline{Z}_B)$ $(\overline{X}_B, \overline{Z}_B\overline{Z}_C)$ $(\overline{X}_C\overline{X}_D, \overline{Z}_C)$

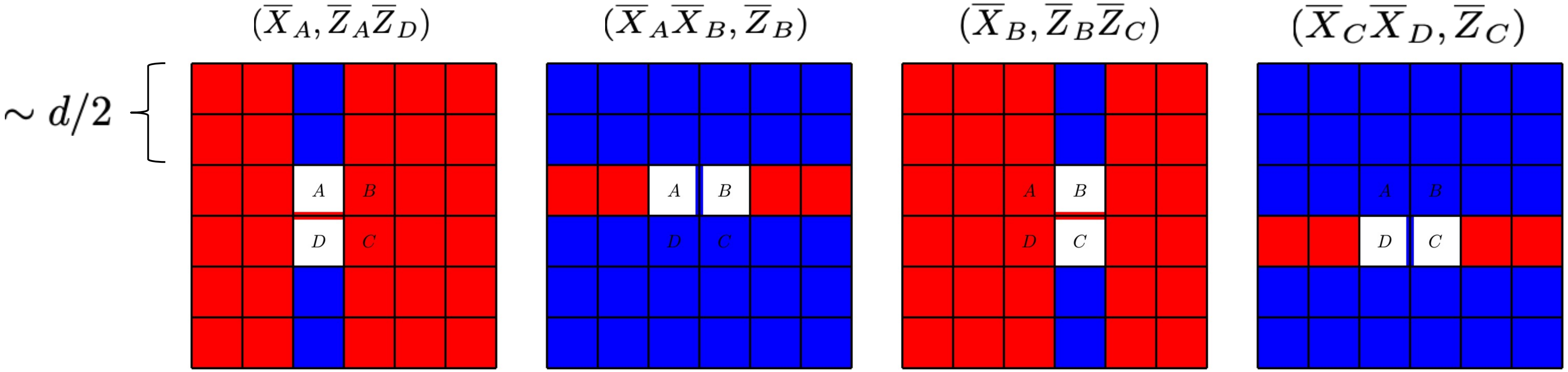
Instantaneous Stabilizer Groups
(ISGs)

Floquet-Bacon-Shor codes: distance

- Distance of each ISG is ~d if we place the defect in the center
  - ~d/2 from hole to edge, and have a product of weight 2 check operators, so ~d
  - Warning: That does not mean that the entire code has distance d.

- Better is the distance of the Bacon-Shor codes defined by two rounds, but still not sufficient

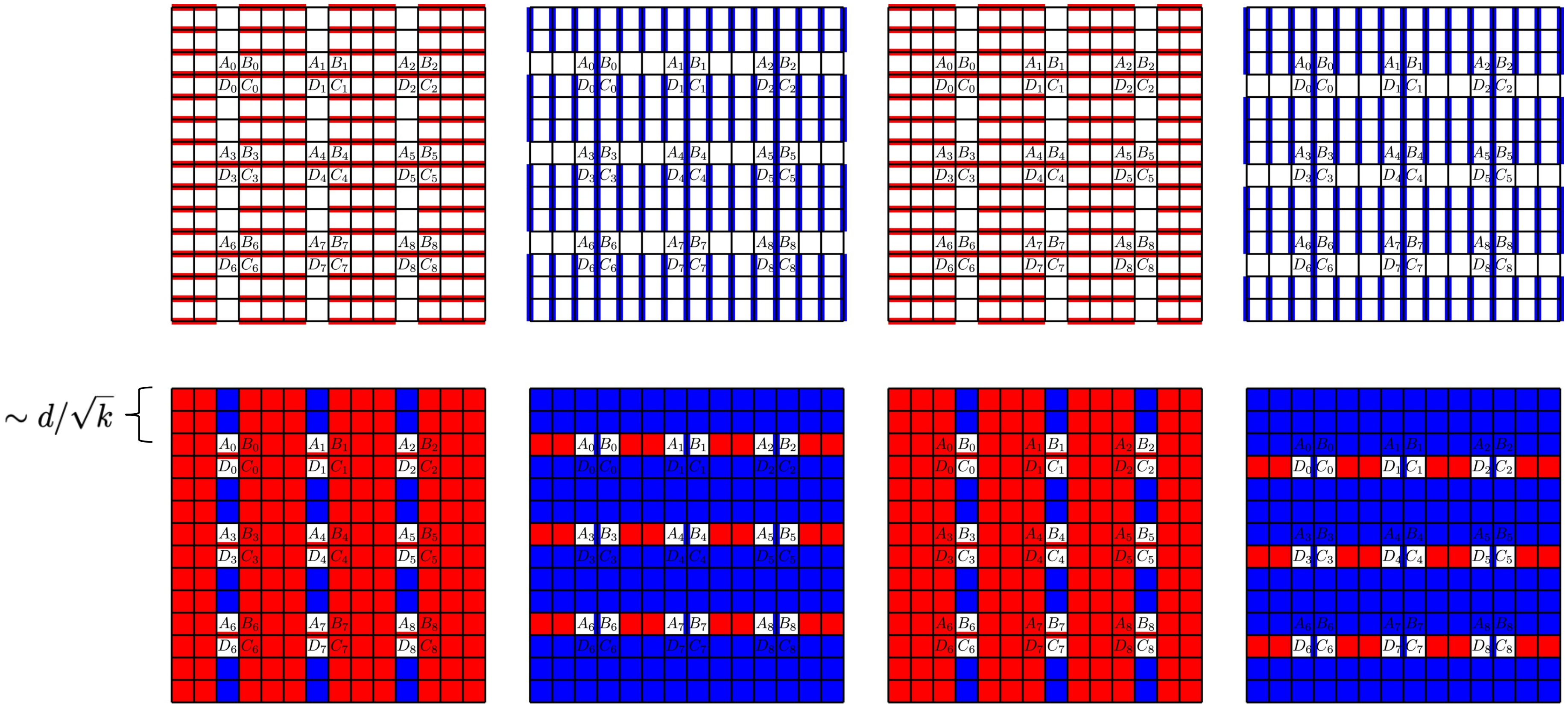
- New paper by Fu & Gottesman on distances for Floquet codes: arXiv:2403.04163



Instantaneous Stabilizer Groups (ISGs)

More Floquet-Bacon-Shor codes

Measurement schedule



Instantaneous Stabilizer Groups
(ISGs)

Further remarks on Floquet code constructions

- Floquet codes can be constructed from subsystem codes by introducing gauge defects
- Some errors can be self-corrected purely by measurement schedule
- Decoding hypergraph may require beyond standard MWPM/UF decodingtechniques
- Like the parent Bacon-Shor code, the Floquet-Bacon-Shor family of codes does not possess a threshold

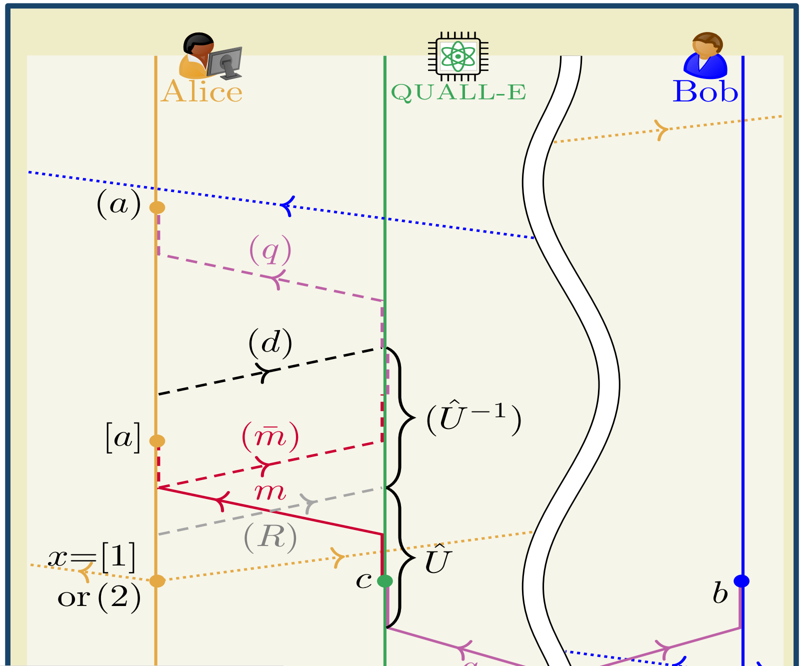
Open questions

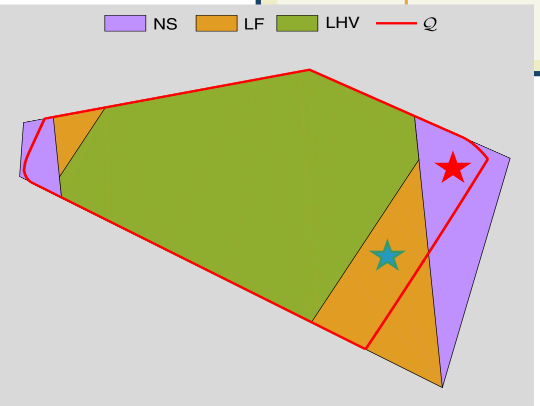
- **What are general schemes for constructing Floquet codes? When are there barriers? Is there a general framework?**
- Can we construct Floquet LDPC codes?
- Remaining open questions with respect to distance of Floquet codes

BRIEF GLIMPSES OF OTHER TOPICS

Wigner's friend inequalities & Experiments

- Wigner friend scenario recent work
  - new inequalities, with weaker assumptions than Bell's inequalities
  - Proof-of-principle experiments have been done: Single photon as friend
- Full experiment would combine AI and Quantum Computing
  - QUALL-E
- Open research directions for intermediate experiments
- Ties with non-local games
  - Connections with deep results in mathematics
  - Applications to device/algorithm benchmarking





Polytope with 96 extreme points and 932 facets

Bong, Kok-Wei, Aníbal Utreras-Alarcón, Farzad Ghafari, Yeong-Cherng Liang, Nora Tischler, Eric G. Cavalcanti, Geoff J. Pryde, and Howard M. Wiseman. **"A strong no-go theorem on the Wigner's friend paradox." Nature Physics 16, 12 (2020)**
H.M. Wiseman, E.G. Cavalcanti, E.G. Rieffel, **A "thoughtful" Local Friendliness no-go theorem: a prospective experiment with new assumptions to suit,** *arXiv:2209.08491 (Accepted to Quantum)*

Unitary Designs

Saied et al., Advancing quantum networking: some tools and protocols for ideal and noisy photonic systems, Quantum Computing, Communication, and Simulation IV 12911, 37-65, 2024 & Saied et al (in preparation)

- Unitary designs approximate Haar-random unitaries
  - Uses(i) benchmarking quantum devices, (ii) reconstruction of quantum states/processes, (iii) quantum ML, (iv) subroutines for oracular QC,
  - t-design = order t approx. of Haar-random states/unitaries
  - Qubits: Pauli group => 1-design. Clifford group => 2-design and 3-design
- Can we construct them via a "tabletop" linear optical experiment? No.
  - Linear optical unitaries can only generate a 1-design => not sufficient for most applications of interest. E.g, no Clifford randomized benchmarking
- Result (translated from math literature): Any continuous (infinite closed) 2-design must be universal
  - For m>2 modes: adding single gate to linear optics => universality => all t-designs.
  - We have 1-designs in linear optics or universality, nothing in between

Operator entanglement

Select related references from QuAIL team and collaborators:
Anand et al., Quantum coherence as a signature of chaos. Phys. Rev. Research, 2021
Andreadaki et al., Scrambling of Algebras in Open Quantum Systems PRA, 2023
Anand et al, Eigenstate phase transitions, operator space entanglement, and nonstabilizerness. (In preparation)

- Entanglement is generally with respect to a tensor decomposition
  - Makes use of the partial trace
- In many physical settings, there are symmetries that mean that the appropriate model is not a tensor decomposition
  - Example: photons (or bosons, more generally)
  - Controversies as to how entanglement is best defined in these settings
  - Partial trace not defined
- Operator entanglement
  - Based on Gel'fand- Naimark-Segal (GNS) construction
  - Partial trace is replaced by restriction to a subalgebra
- Applications to resource theories, integrals of motion, scrambling, quantum chaos, out-of-time-correlators (OTOCs)

Thank you for your attention!

QUESTIONS?

A Historical Perspective

NASA Ames director Hans Mark brought Illiac IV to NASA Ames in 1972

Illiac IV – first massively parallel computer

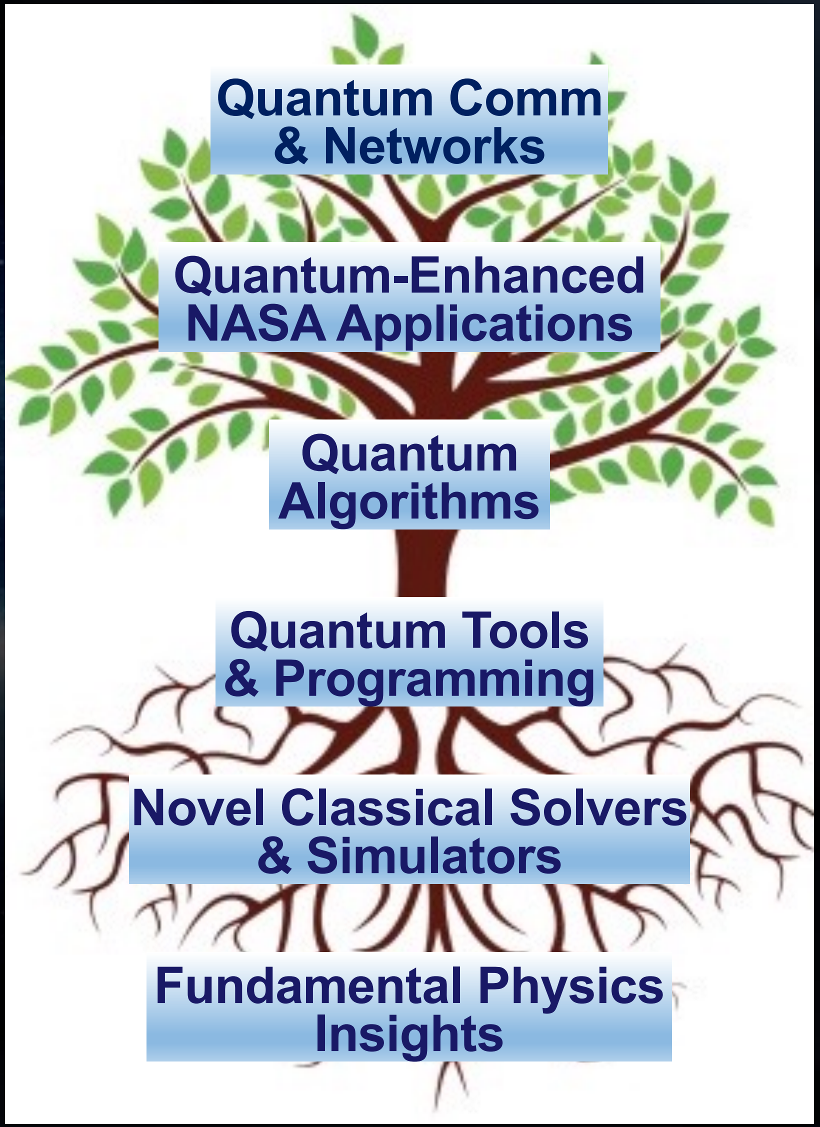
- 64 64-bit FPUs and a single CPU
- 50 MFLOP peak, fastest computer at the time

Finding good problems and algorithms was challenging

Questions at the time:

- How broad will the applications be of massively parallel computing?
- Will computers ever be able to compete with wind tunnels?

Quantum Computing R&D at NASA Ames



Quantum Comm & Networks

Quantum-Enhanced NASA Applications

Quantum Algorithms

Quantum Tools & Programming

Novel Classical Solvers & Simulators

Fundamental Physics Insights

Communication & Networks

Quantum networking
Distributed QC

Application Focus Areas

Planning and scheduling
Material science
Fault diagnosis
Machine learning

Software Tools & Algorithms

Quantum algorithm design
Compiling to hardware
Mapping, parameter setting, error mitigation
Hybrid quantum-classical approaches

Solvers & Simulators

Physics-inspired classical solvers
HPC quantum circuit simulators

Physics Insights

Co-design quantum hardware

QuAIL team has published 100+ peer-reviewed papers since 2012

E. Rieffel *et al.* (2019), From Ansätze to Z-gates: A NASA view of quantum computing, *Adv. in Parallel Computing* **34**, 133–160
R. Biswas *et al.* (2017), A NASA perspective on quantum computing: Opportunities and challenges, *Parallel Computing* **64**, 81–98