Dynamical Logical Qubits in the Bacon-Shor Code

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Google June 2024

Based on work with Eleanor Rieffel (NASA)
arXiv:2403.03291

General Idea

- Floquet codes identify the code space as a dynamically evolving subspace of the physical Hilbert space[1]

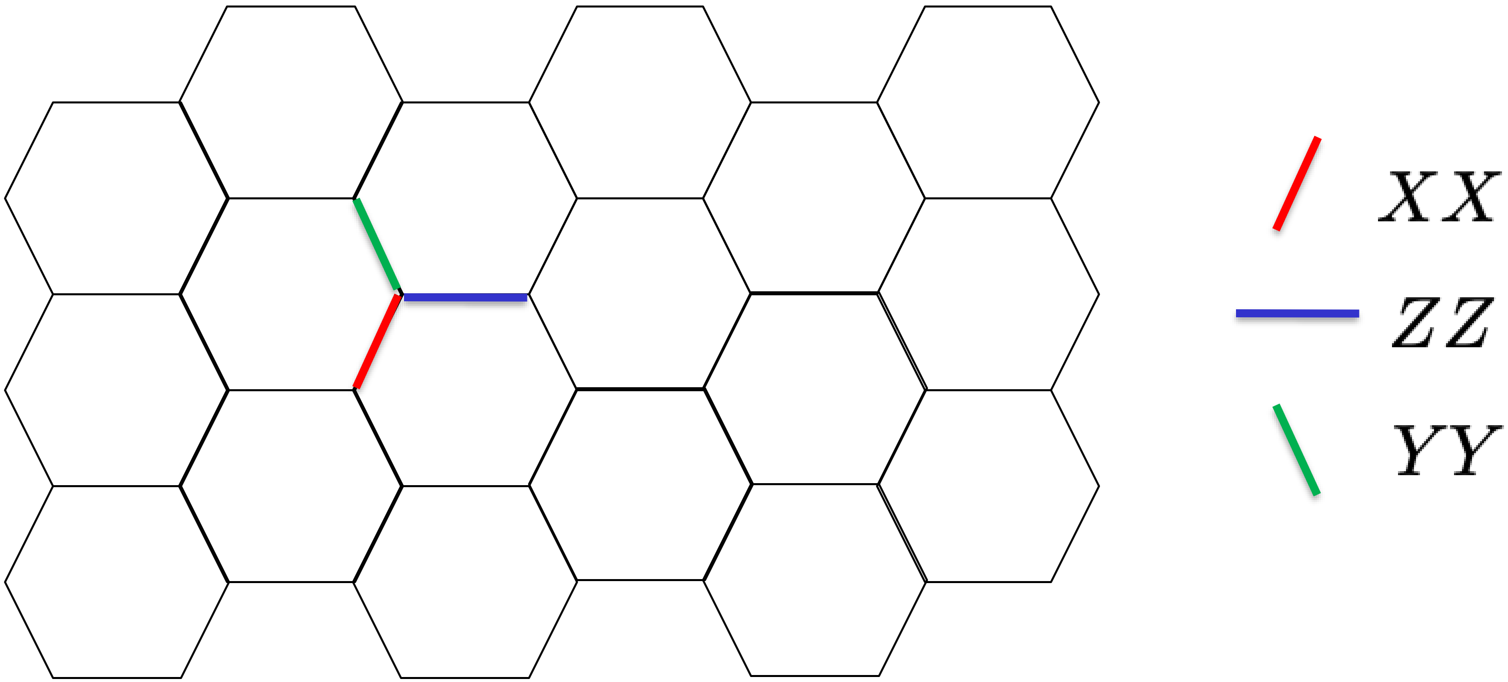
- Most examples involve a tri-valent, tri-colorable (typically hexagonal) lattice

- Usually involve descriptions of embedded toric codes, non-trivial automorphisms between electric and magnetic operators

- Floquetify-ing the Bacon-Shor subsystem code[2]
    - Simple square lattice architecture
    - Naturally equipped with a boundary
    - Using nothing more than *virtual* qubit operators
    - Add as many dynamical qubits as the lattice size, and target distance, allows

[1] M. B. Hastings and J. Haah, Dynamically Generated Logical Qubits, Quantum 5, 564 (2021)
[2] D. Bacon, Operator quantum error-correcting subsystems for self-correcting quantum memories, Phys. Rev. A 73, 012340 (2006)

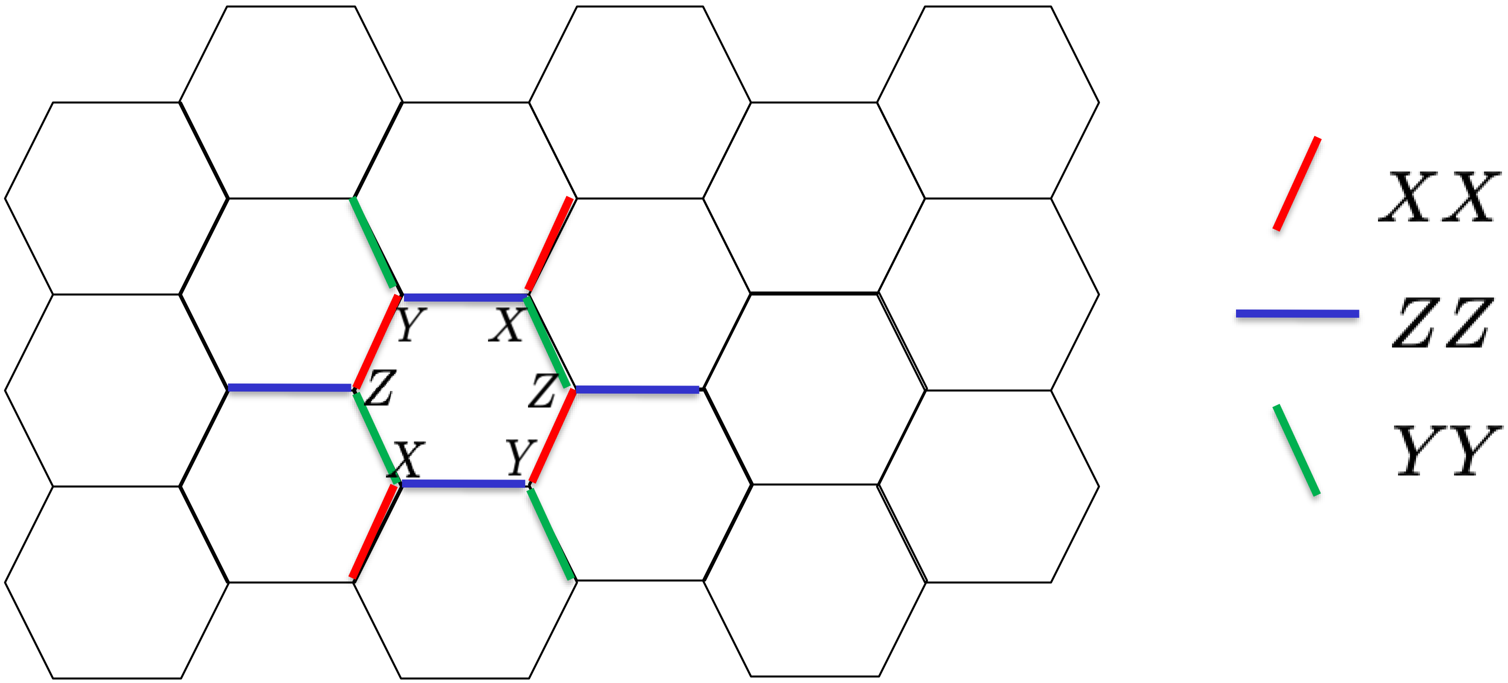
Honeycomb model



2-body check (gauge) operators

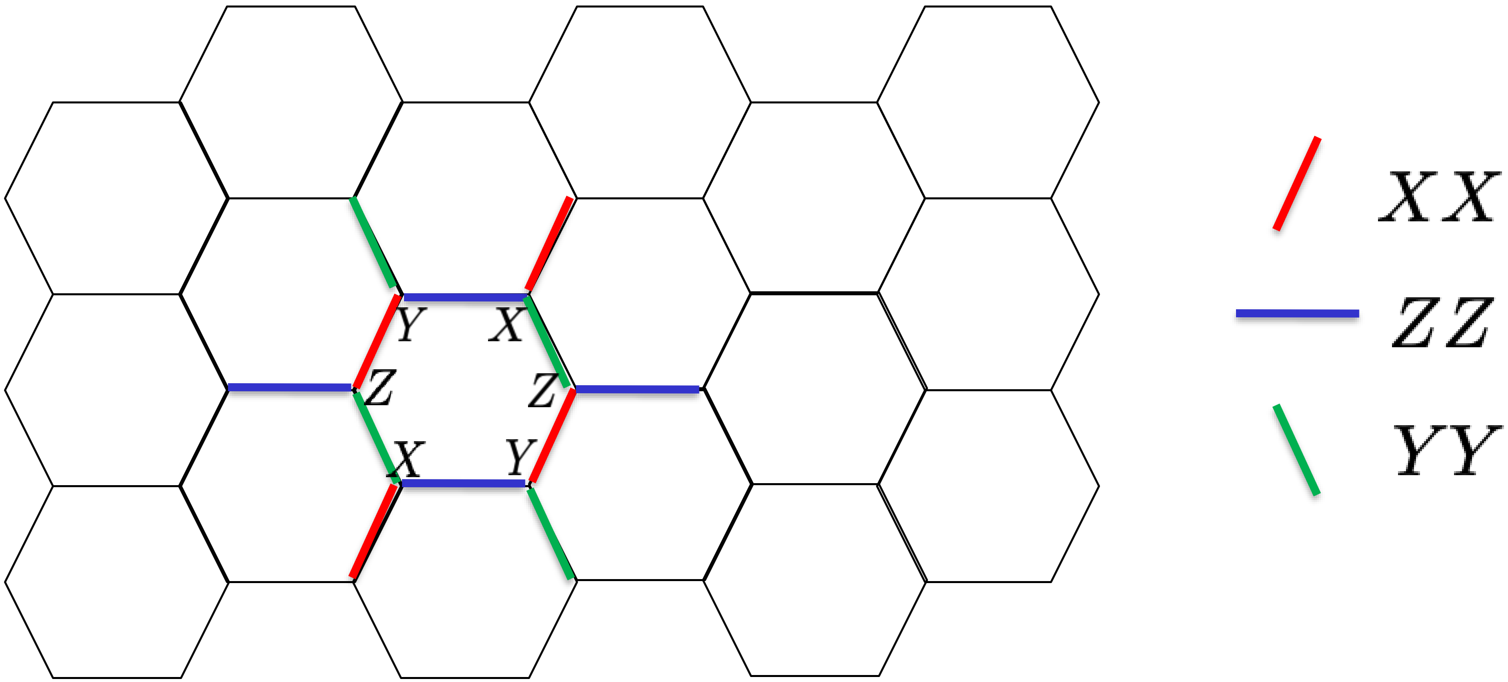
[1] A. Kitaev, Anyons in an exactly solved model and beyond, Annals of Physics 321 (2006) 2--111

Honeycomb model



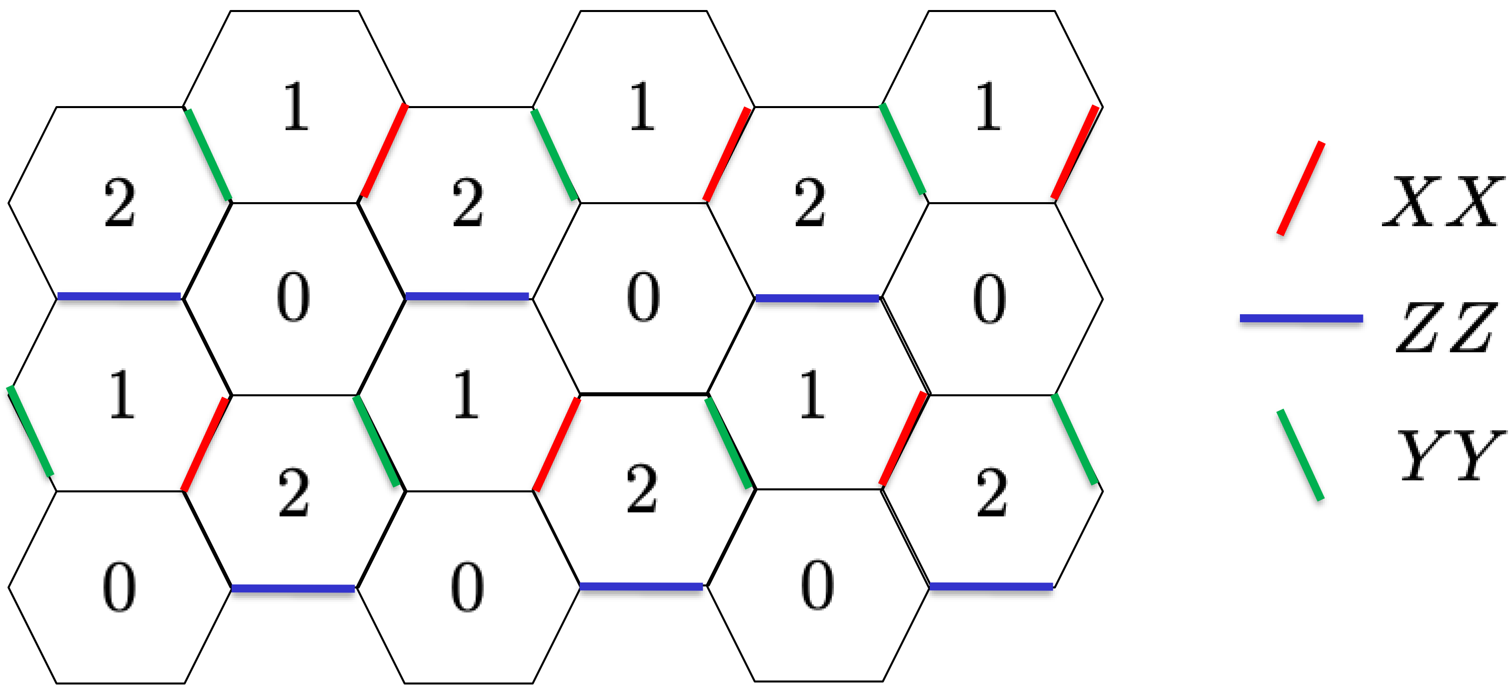
Plaquette (stabilizer) operators

Honeycomb model



Static (subsystem) code ==>
No space for any logical qubits!

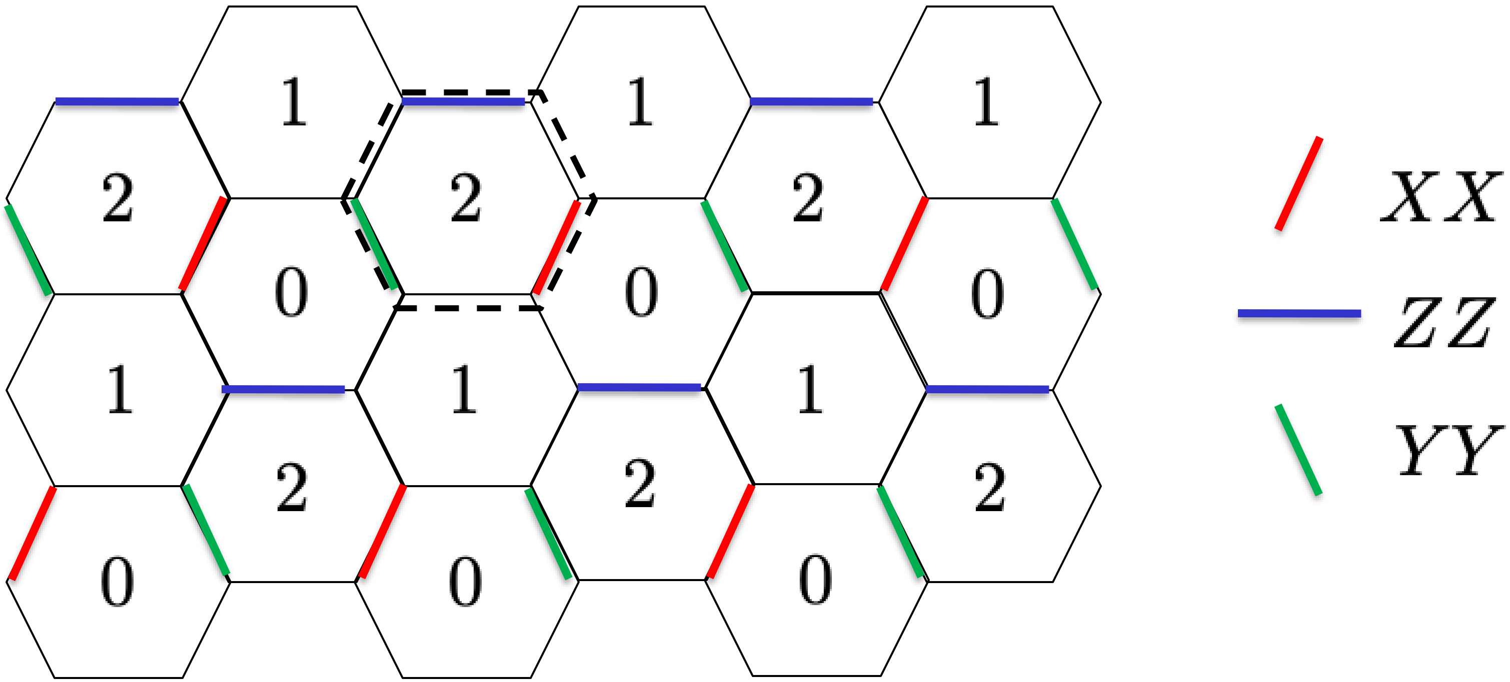
Honeycomb Floquet code



Measurement round 0

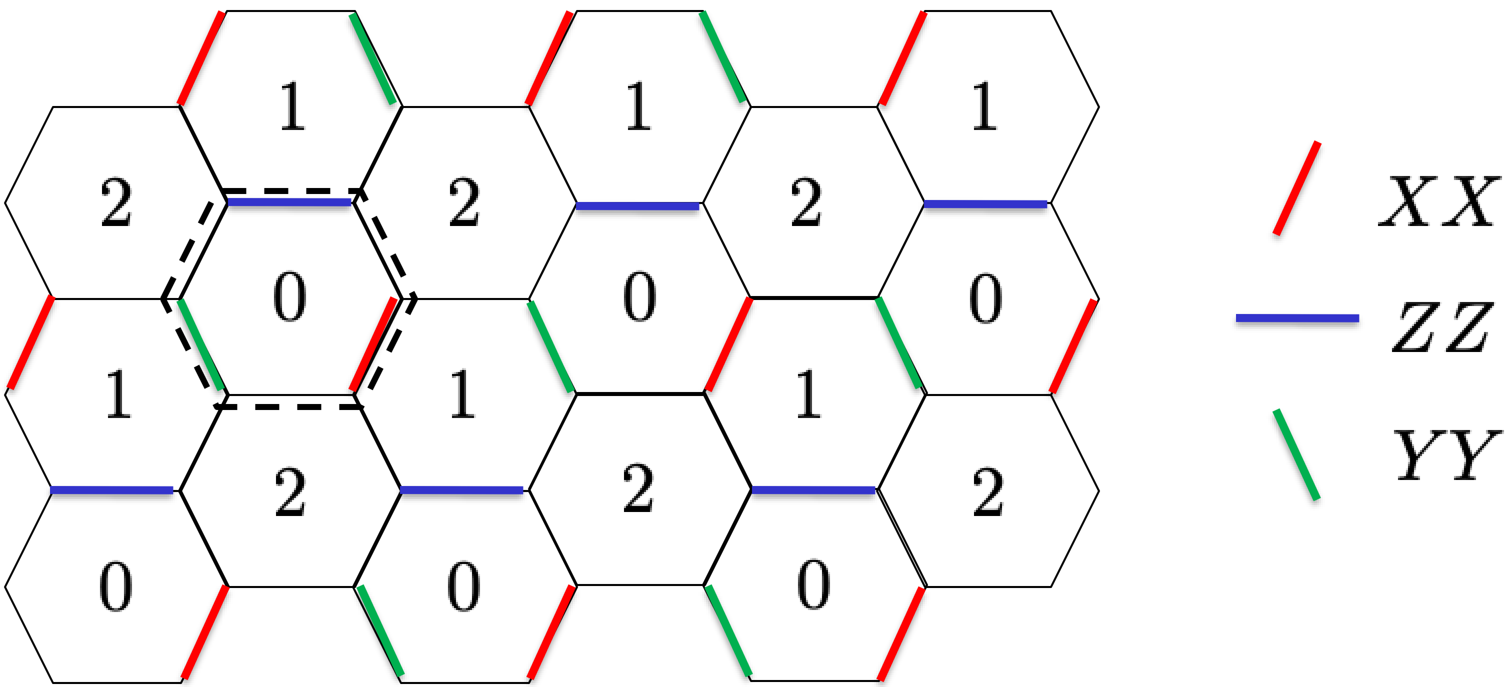
[1] M. B. Hastings and J. Haah, Dynamically Generated Logical Qubits, Quantum 5, 564 (2021)

Honeycomb Floquet code



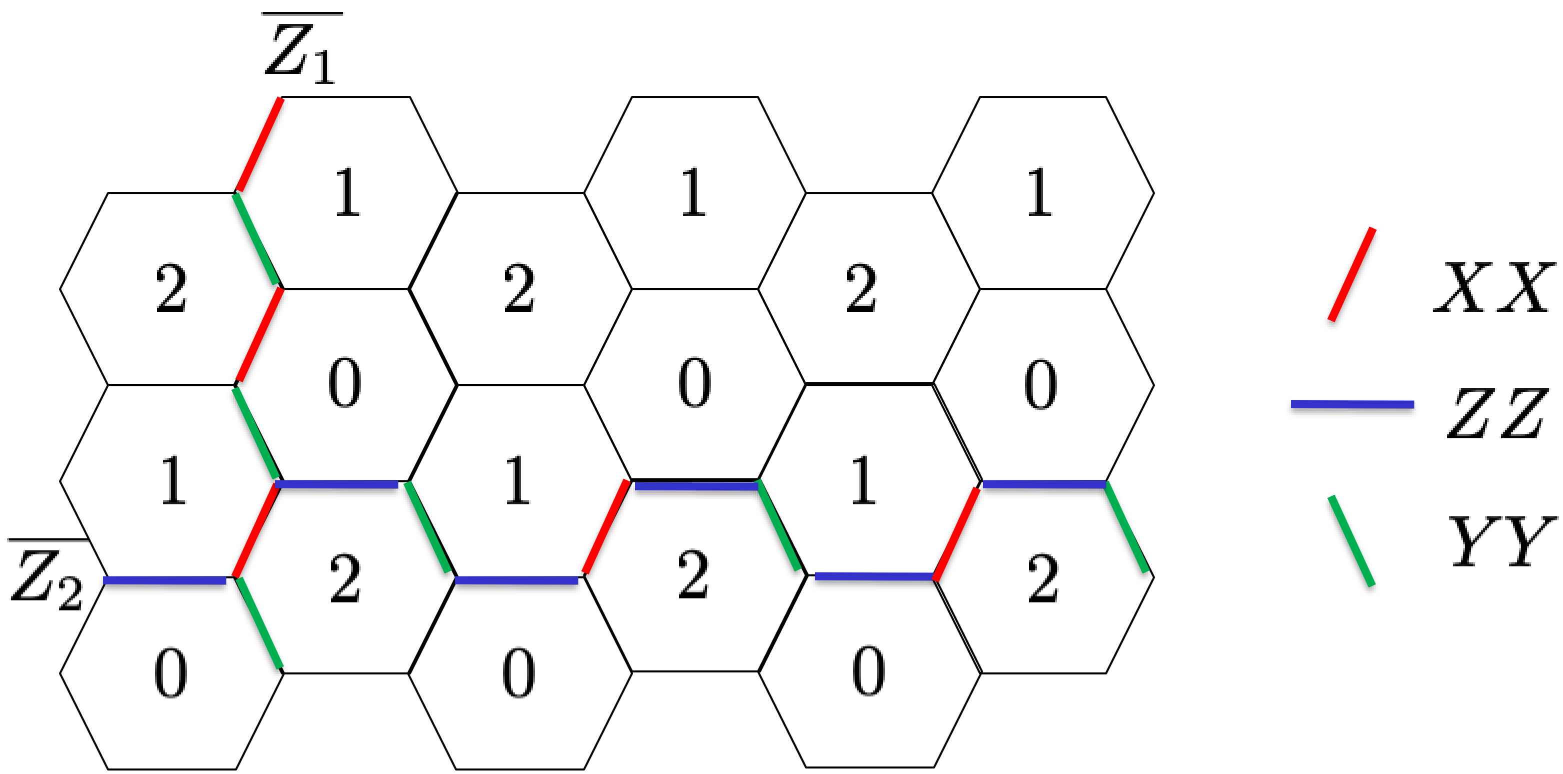
Measurement round 1

Honeycomb Floquet code

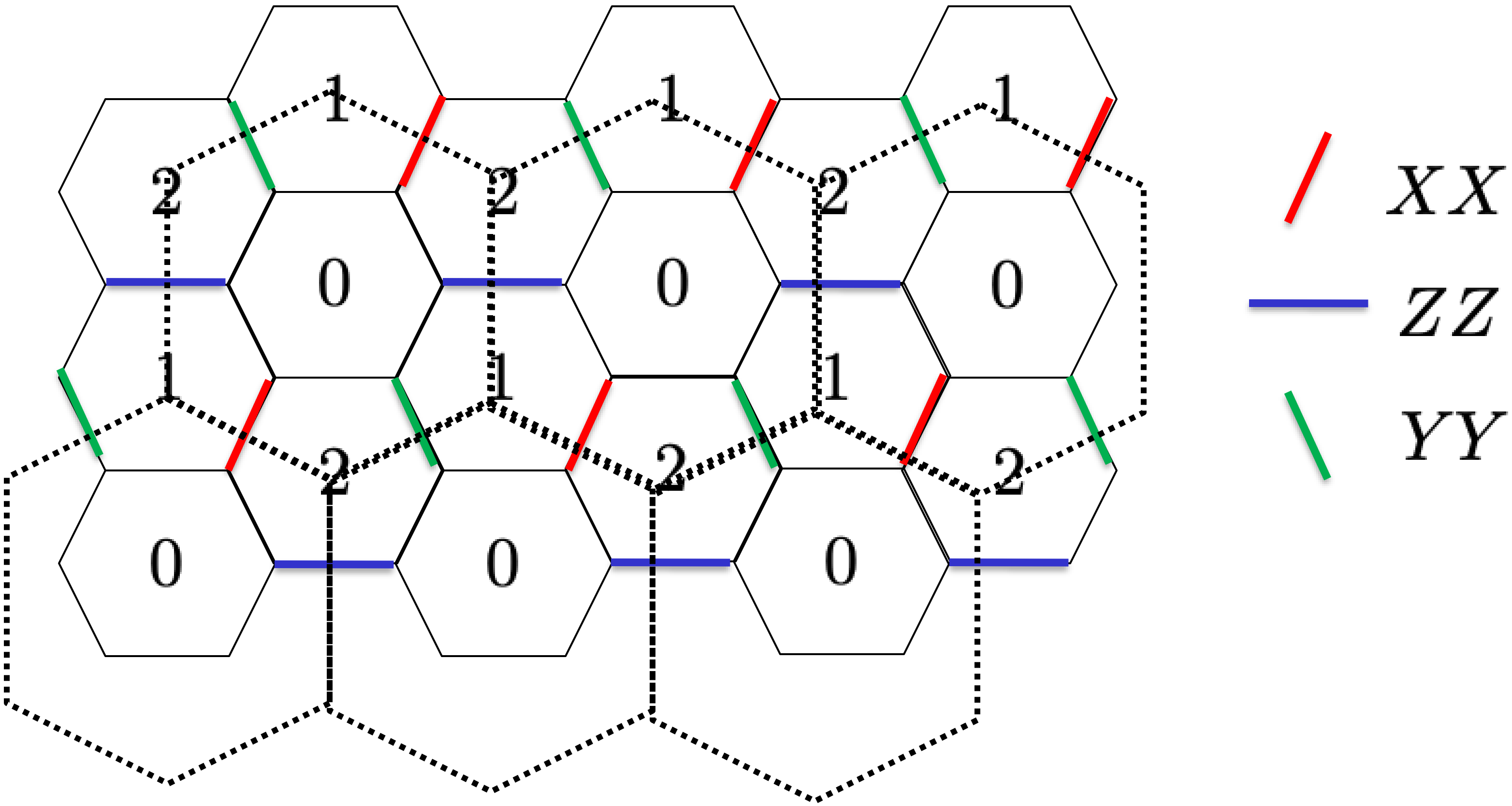


Measurement round 2

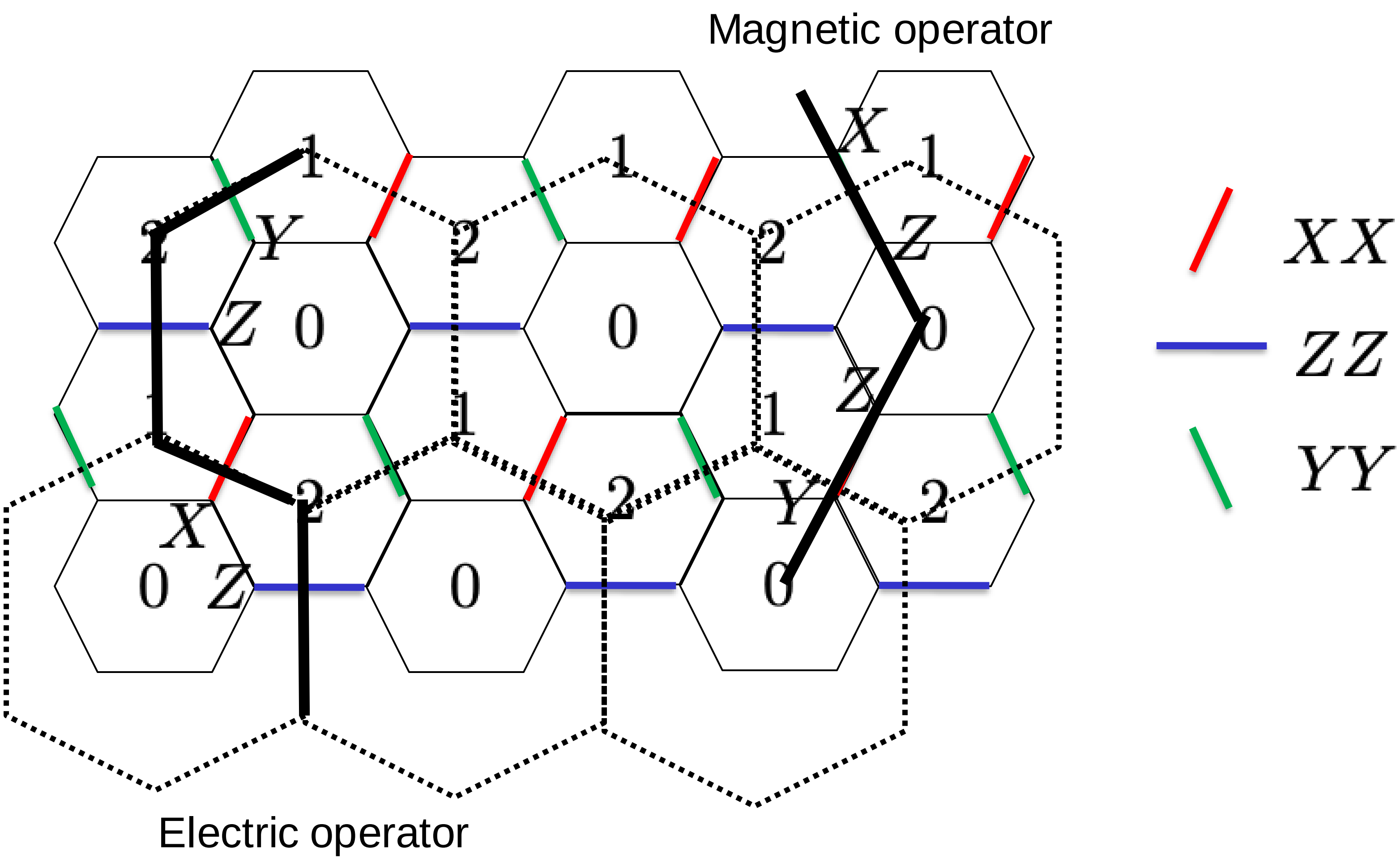
Inner logical operators



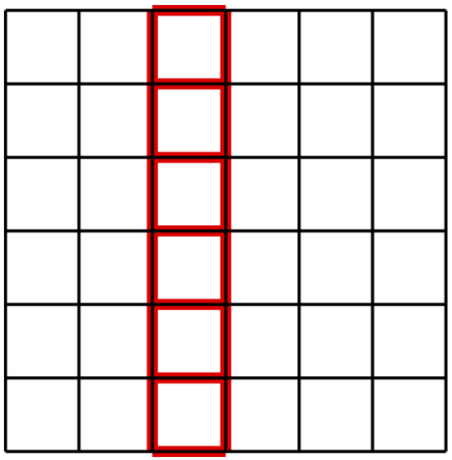
Outer (dynamical) logical operators



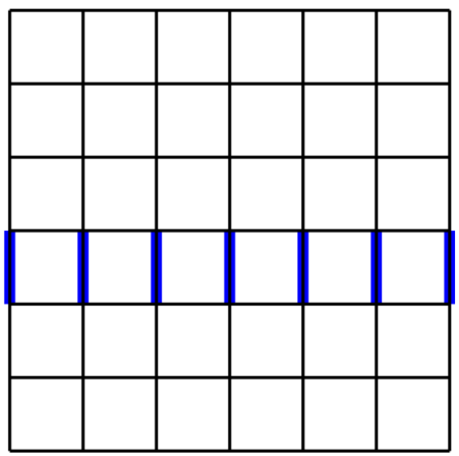
Outer (dynamical) logical operators



Shor code



X-type

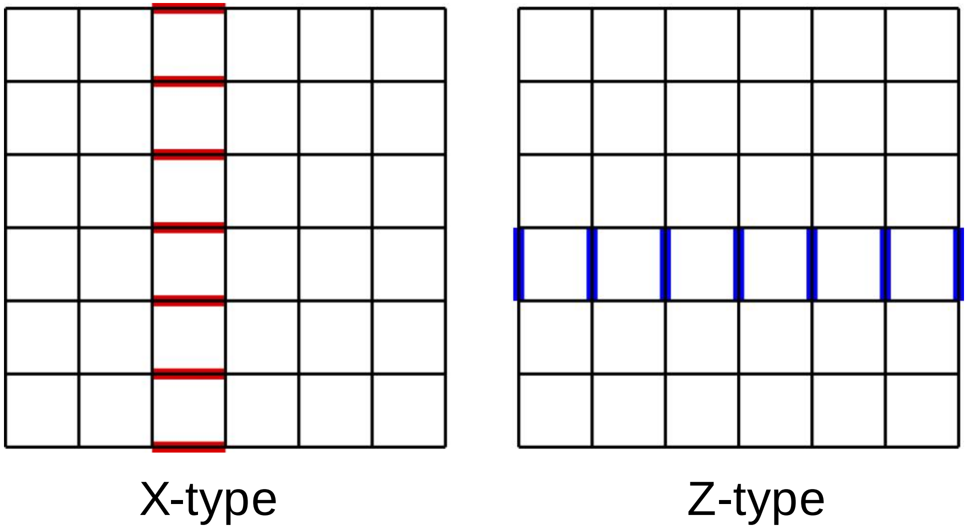


Z-type

Stabilizers

[1] Peter W. Shor, Scheme for reducing decoherence in quantum computer memory, Physical Review A **52** (4): R2493-R2496 (1995)

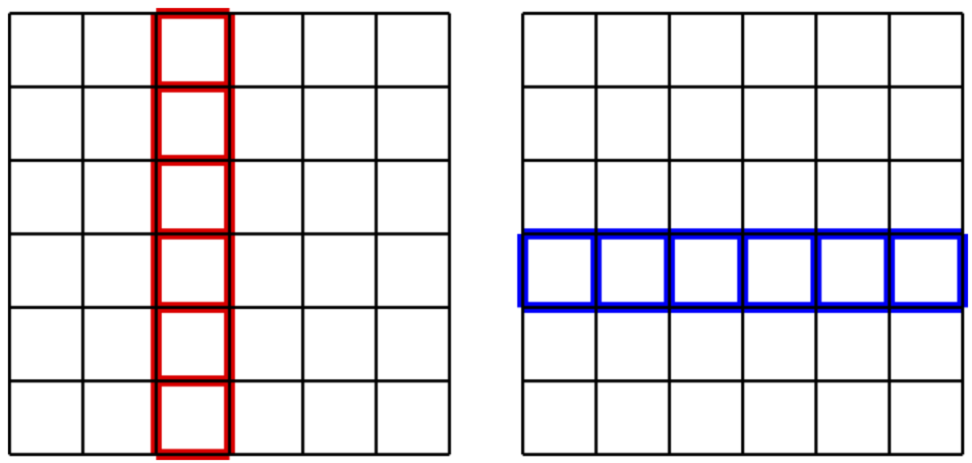
Bacon-Shor code



Gauge checks

[1] D. Bacon, Operator quantum error-correcting subsystems for self-correcting quantum memories, Phys. Rev. A 73, 012340 (2006)

Bacon-Shor code



X-type Z-type

Stabilizers

[1] D. Bacon, Operator quantum error-correcting subsystems for self-correcting quantum memories, Phys. Rev. A 73, 012340 (2006)

Subspace (stabilizer) codes

Stabilizer group

$$\mathcal{S} = \langle S_1, \ldots, S_r \rangle = \langle Z'_1, \ldots, Z'_r \rangle$$

Code space

$$C = \{|\psi\rangle : S_j|\psi\rangle = |\psi\rangle\},\ \dim(C) = 2^{n-r}$$

$$|\psi\rangle = |\underbrace{0\ldots0}_{\text{stabilizers}}\overbrace{z'_{r+1}\ldots z'_n}^{\text{logical states}}\rangle \qquad \mathcal{H} = C \oplus C^{\perp}$$

Subsystem codes

Stabilizer: center of the gauge group

$$\mathcal{Z}(\mathcal{G}) = \mathcal{G} \cap C(\mathcal{G}) = \langle i, \mathcal{S} \rangle$$

$$\mathcal{G} = \langle i, \underbrace{Z'_1, \ldots, Z'_r}_{\text{stabilizers}}, \overbrace{X'_{r+1}, Z'_{r+1}, \ldots, X'_{r+k}, Z'_{r+k}}^{\text{gauge degrees of freedom}} \rangle$$

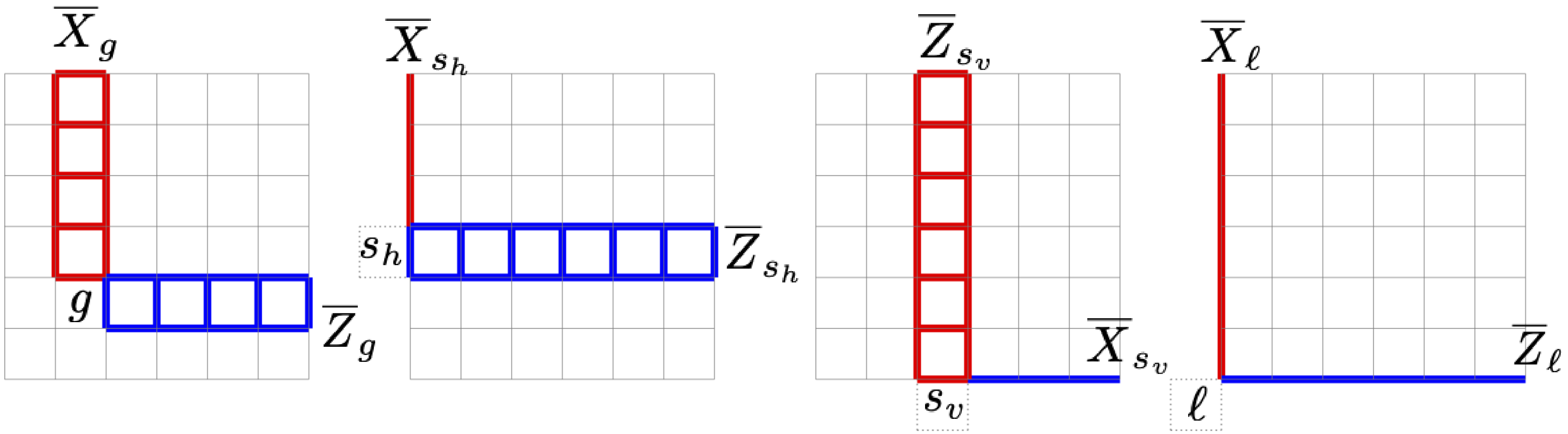
$n - (r+k)$ logical qubits

$$\mathcal{L} \simeq \langle X'_{r+k+1}, Z'_{r+k+1}, \ldots, X'_n, Z'_n \rangle$$

$$\mathcal{L} = \mathcal{N}(\mathcal{S})/\mathcal{G}, \quad [\mathcal{L}, \mathcal{G}] = 0$$

Bacon-Shor code

Virtual qubit operators



Gauge

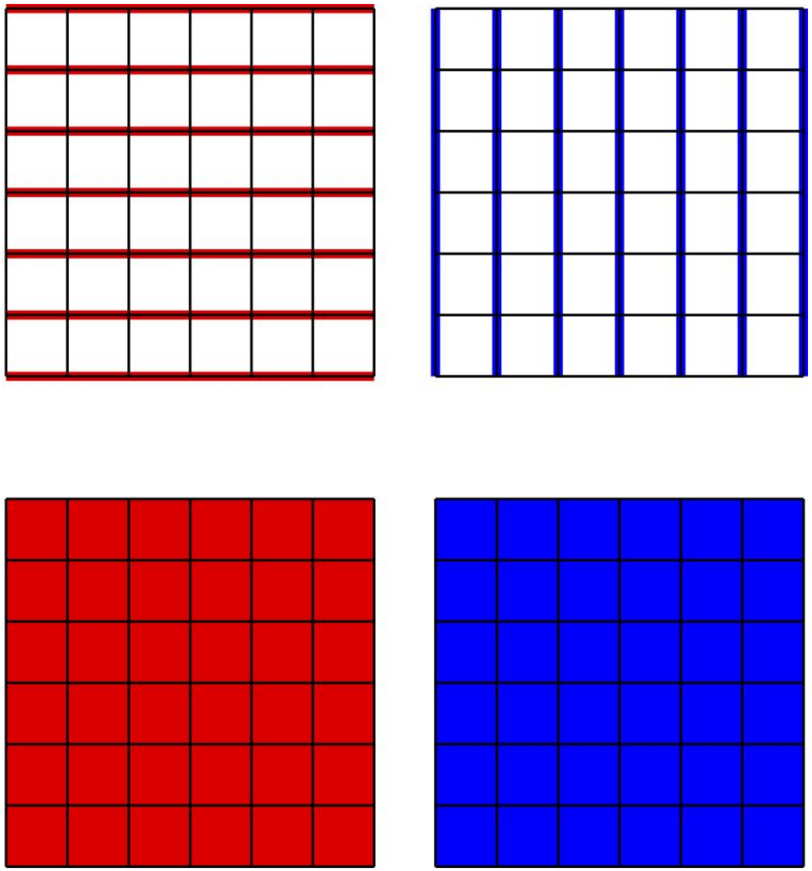
Stabilizer (horizontal)

Stabilizer (vertical)

Logical

Bacon-Shor code

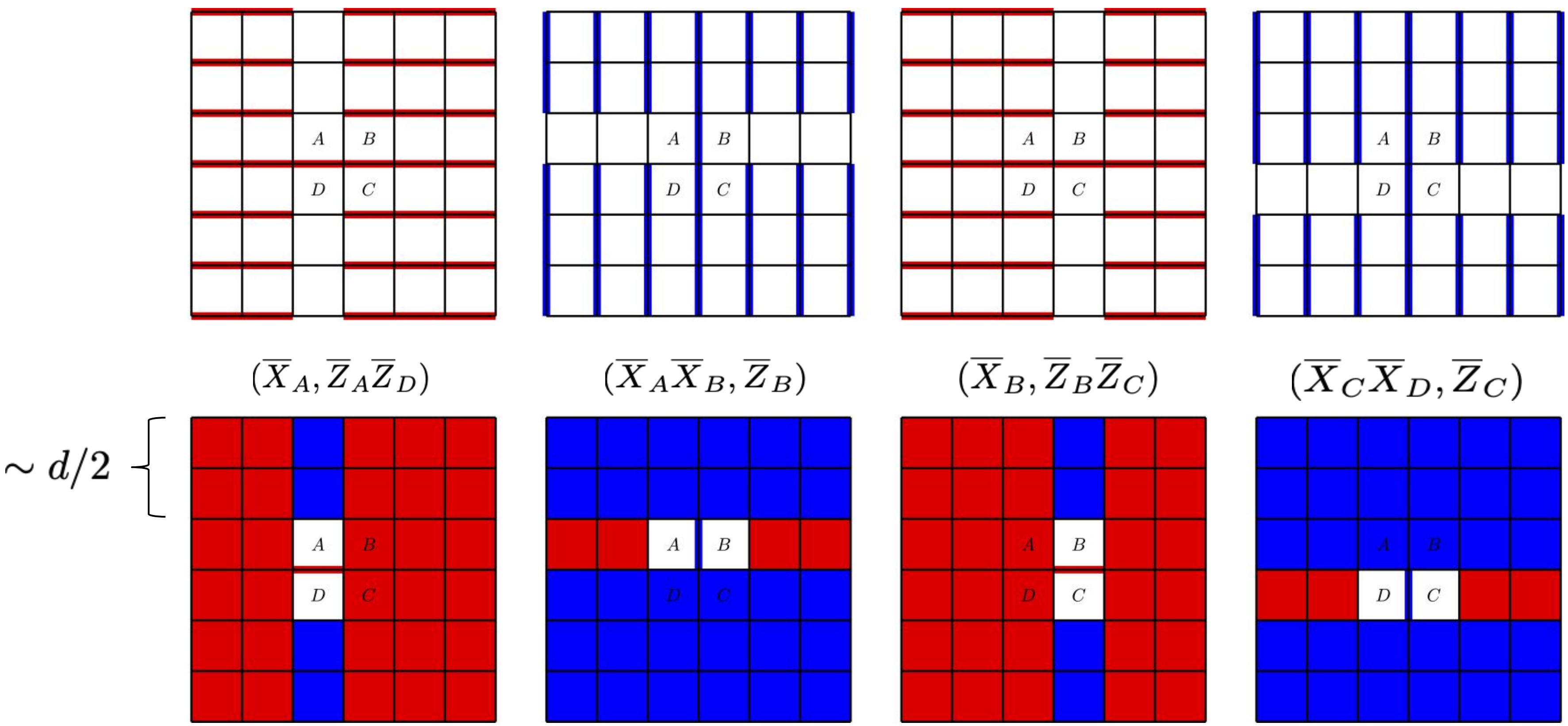
Measurement schedule



Instantaneous Stabilizer Groups (ISGs)

Floquet-Bacon-Shor code

Measurement schedule



Instantaneous Stabilizer Groups (ISGs)

Floquet-Bacon-Shor code

Preservation of logical information

$$s^{(r)} O^{(r)} = s^{(r+1)} O^{(r+1)}$$

$$(s \in \mathcal{S}, O \in \mathcal{N}(\mathcal{S}) \backslash \mathcal{S})$$

ensures

$$|\psi^{(r)}\rangle = \alpha |0^{(r)}\rangle + \beta |1^{(r)}\rangle$$

$$\rightarrow |\psi^{(r+1)}\rangle = \alpha |0^{(r+1)}\rangle + \beta |1^{(r+1)}\rangle$$

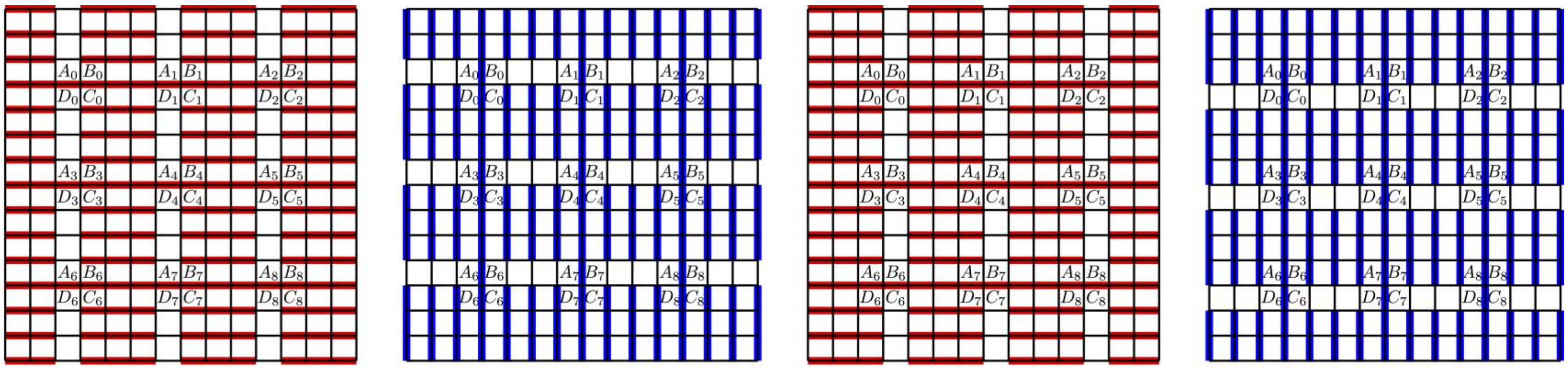
Floquet-Bacon-Shor code

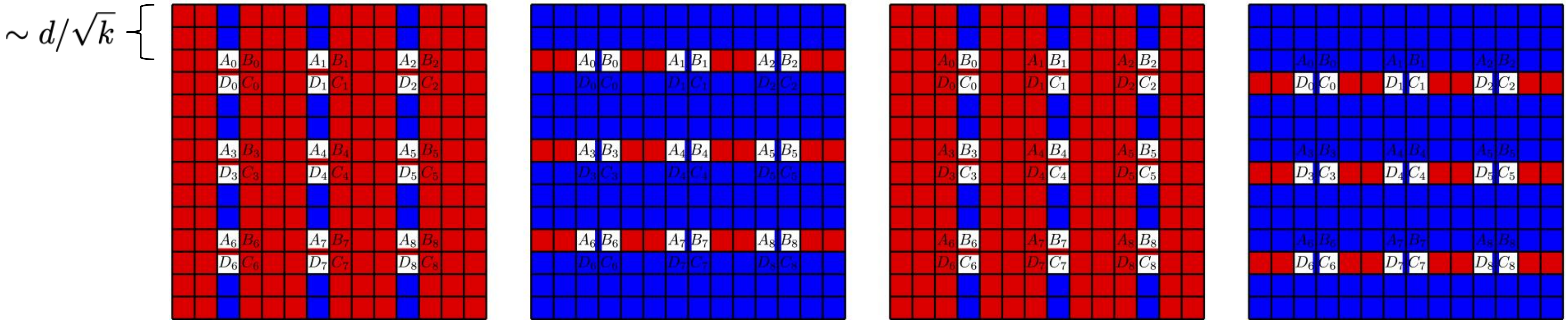
Preservation of logical information

Round	0	1	2	3
$\overline{X}^{(r)}$	$\overline{X}_A$	$\overline{X}_A\overline{X}_B$	$\overline{X}_B$	$\overline{X}_C\overline{X}_D$
$\overline{Z}^{(r)}$	$\overline{Z}_A\overline{Z}_D$	$\overline{Z}_B$	$\overline{Z}_B\overline{Z}_C$	$\overline{Z}_C$
$s_x^{(r)}$	$\overline{X}_A\overline{X}_D \cdot \overline{X}_C$	$\mathbb{I}$	$\overline{X}_A$	$\mathbb{I}$
$s_x^{(r-1)}$	$\mathbb{I}$	$\overline{X}_B$	$\mathbb{I}$	$\overline{X}_B\overline{X}_C \cdot \overline{X}_D$
$s_z^{(r)}$	$\mathbb{I}$	$\overline{Z}_D \cdot \overline{Z}_A\overline{Z}_B$	$\mathbb{I}$	$\overline{Z}_B$
$s_z^{(r-1)}$	$\overline{Z}_C\overline{Z}_D \cdot \overline{Z}_A$	$\mathbb{I}$	$\overline{Z}_C$	$\mathbb{I}$

Floquet-Bacon-Shor code

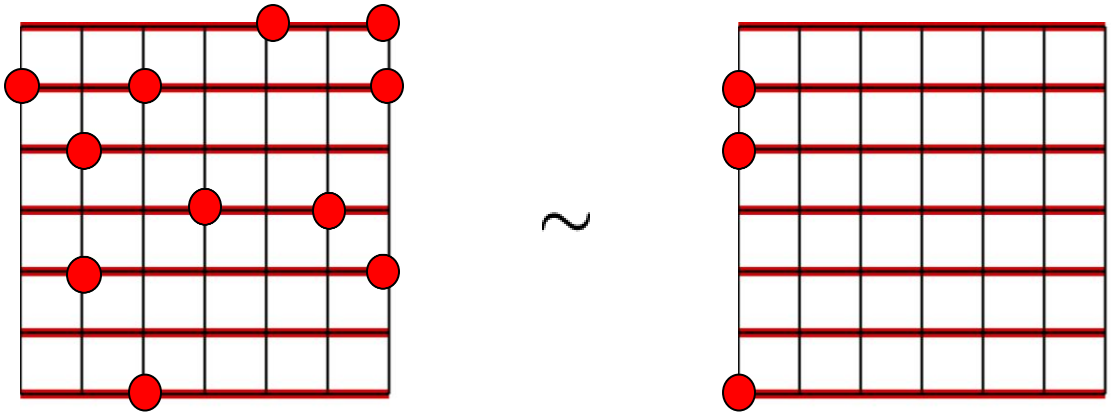
Measurement schedule





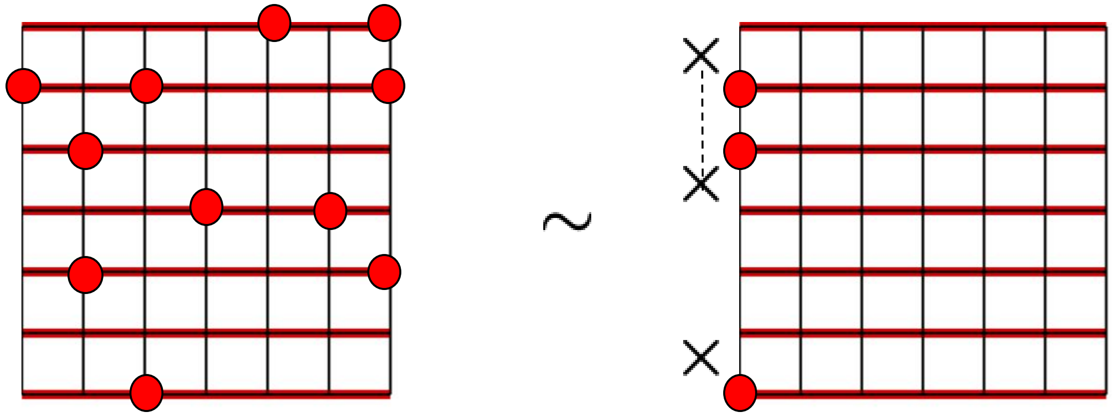
Instantaneous Stabilizer Groups (ISGs)

Bacon-Shor code



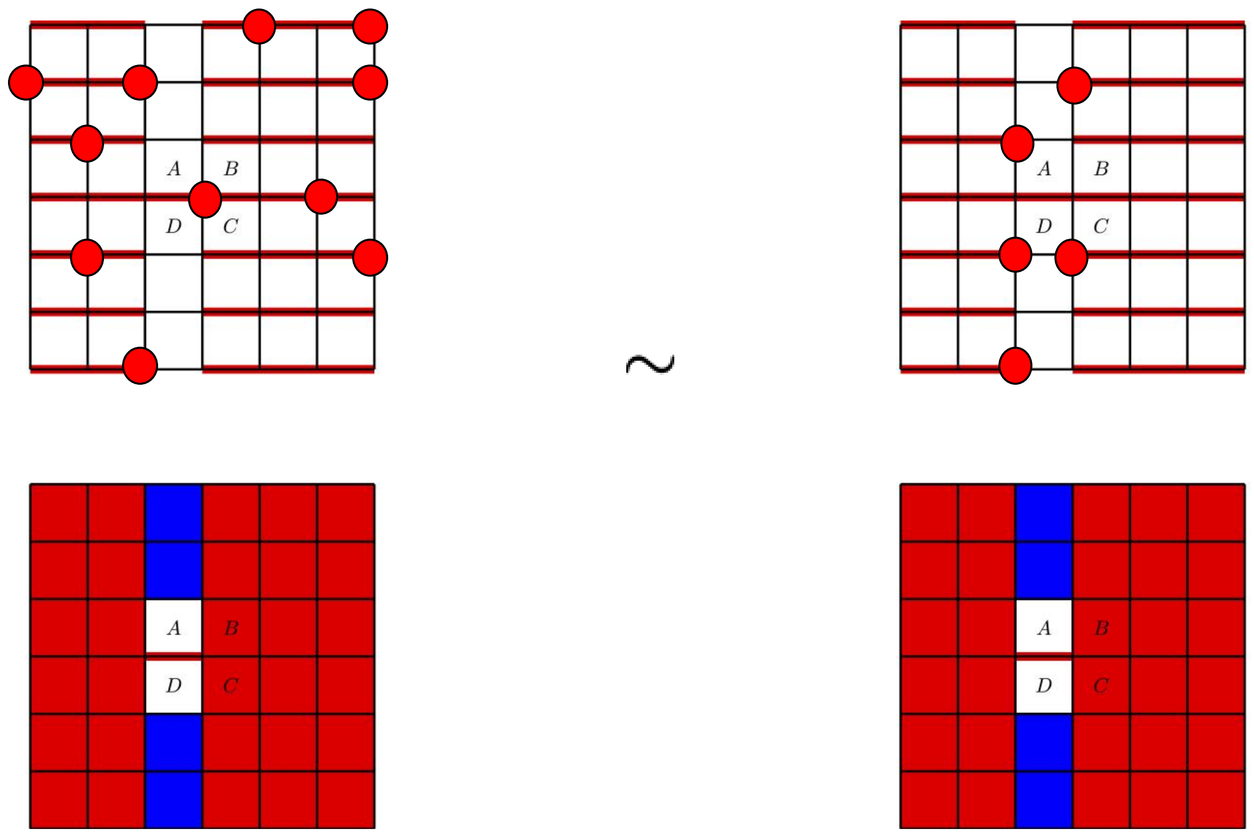
Error equivalence

Bacon-Shor code



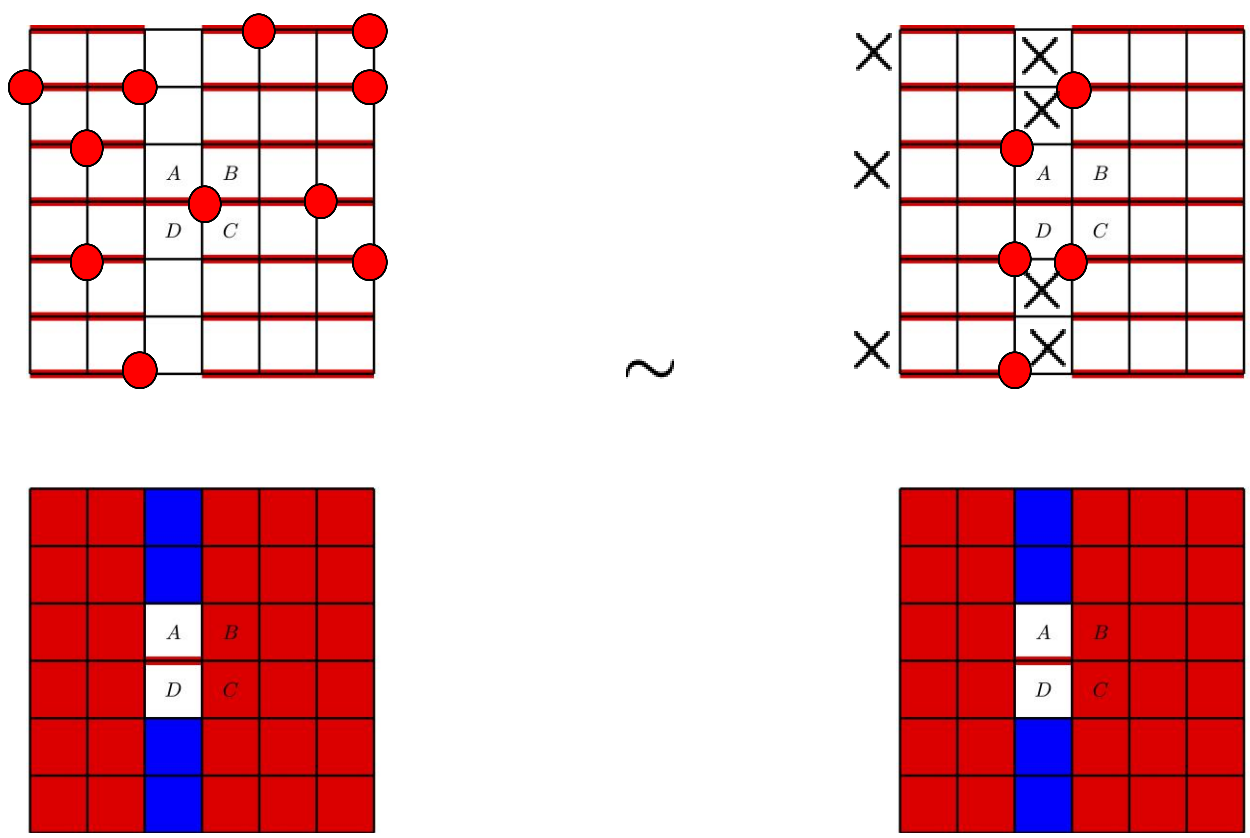
Error detection

Floquet-Bacon-Shor code



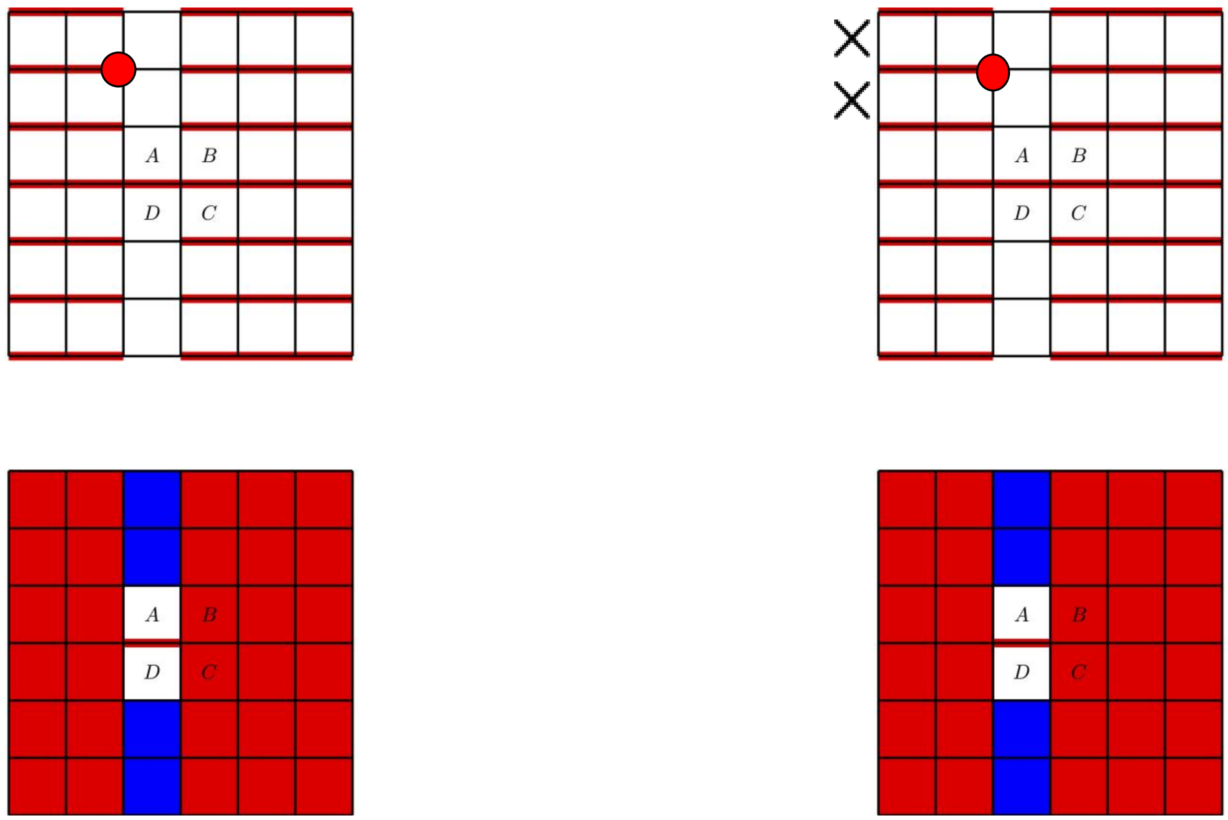
Error equivalence

Floquet-Bacon-Shor code



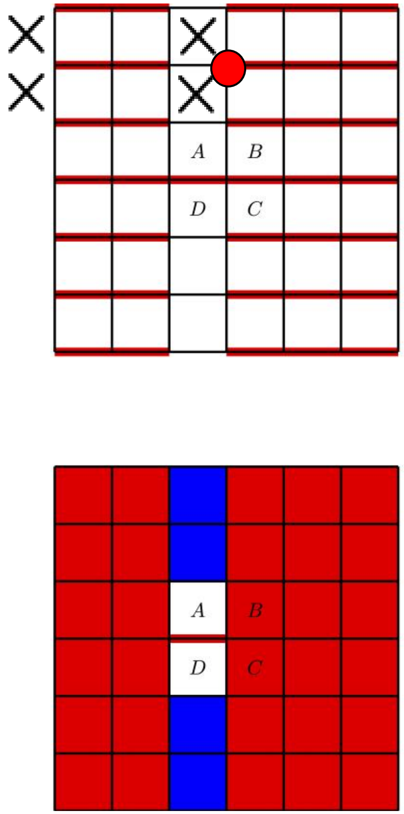
Error detection

Floquet-Bacon-Shor code



Error detection

Floquet-Bacon-Shor code

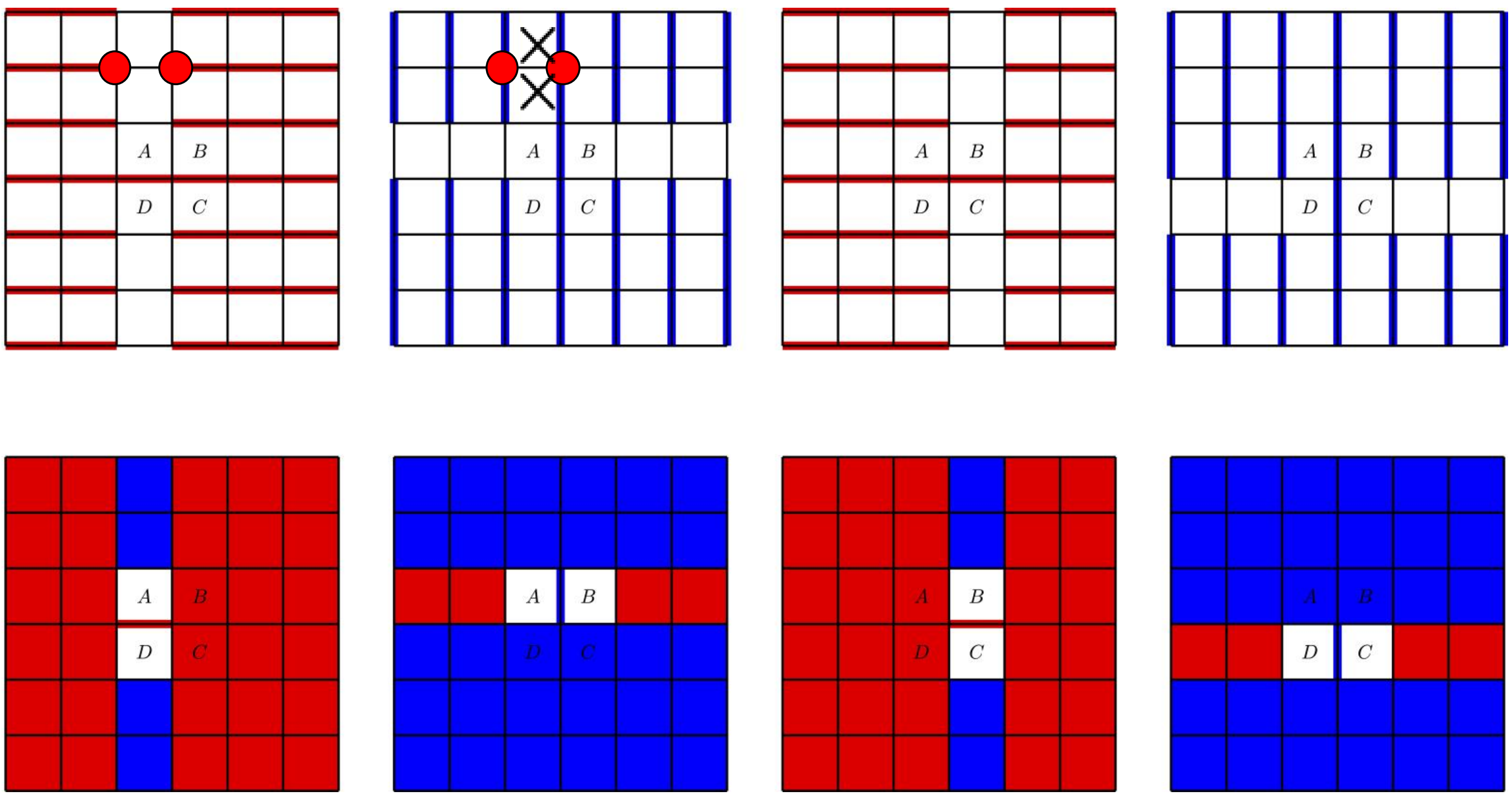


Error detection

National Aeronautics and Space Administration

Ames
Discovery Innovations Solutions

Floquet-Bacon-Shor code



Self correction

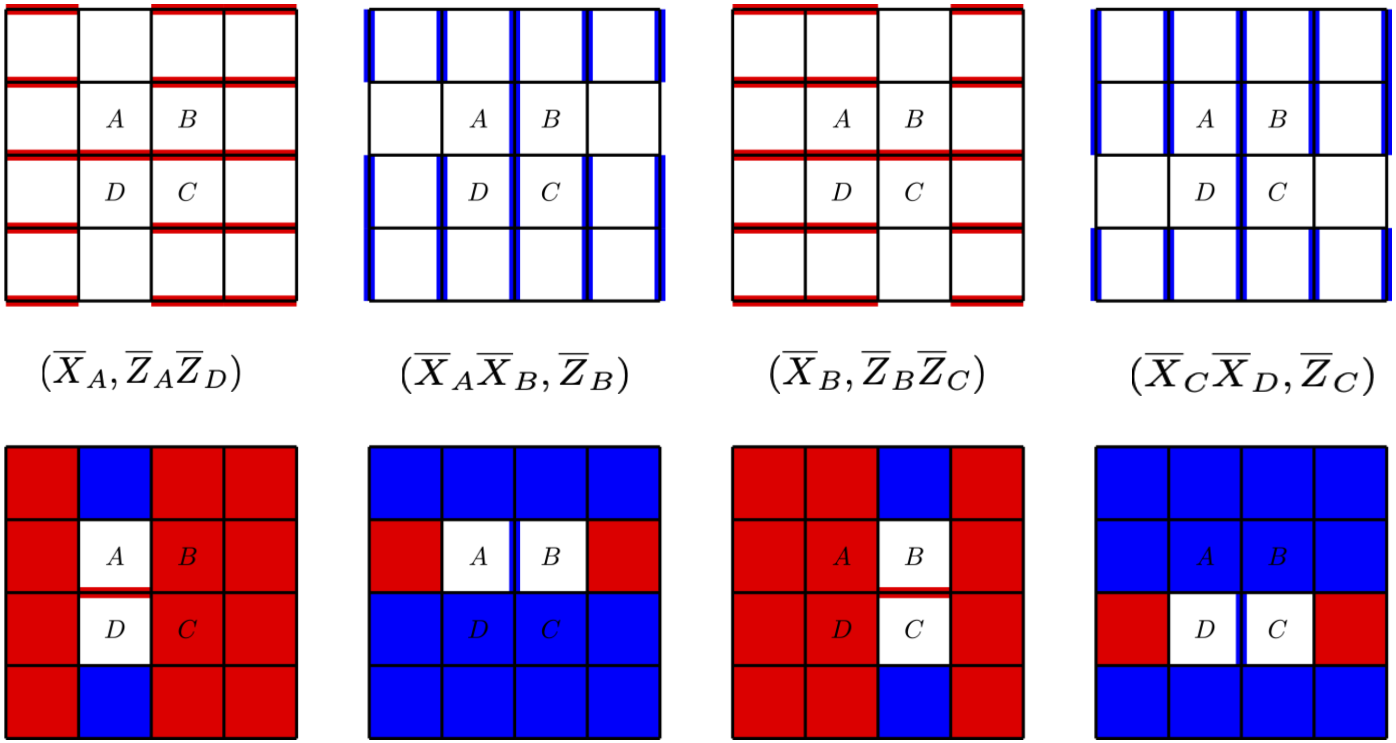
Dynamical quantum memory experiment?

Schematically,

- Prepare logical $|0\rangle$ state of dynamical operator
- Perform several rounds of syndrome extraction
- Measure logical operator at the end
- Identify discrepancy between measured and decoded value as a logical error

[1] State preservation by repetitive error detection in a superconducting quantum circuit, J. Kelly et al., Nature **519**, 66-69 (2015)
[2] Suppressing quantum errors by scaling a surface code logical qubit, Google Quantum AI, Nature **614**, 676-681 (2023)

Dynamical quantum memory experiment?



$\#$ of data qubits: 25

$\#$ of edges: 40

Thank you!

Email: malam@usra.edu; sohaib.alam@nasa.gov

Funding support:

Universities Space Research Association

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