

Evaluation of finite impulse response filters for turbulence effects on sonic booms

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Abstract

Turbulence effects on sonic booms lead to random variability of sonic boom waveforms measured on the ground, complicating the prediction of such waveforms using sonic boom propagation codes that do not account for turbulence effects. The NASA PCBoom software is one such code shown to accurately predict sonic booms above the atmospheric boundary layer but not those on the ground. Analyses of measured sonic booms show that, on average, the turbulence effects result in sonic boom loudness reduction that increases with propagation distance and turbulence strength. To efficiently account for such effects, a signal processing-based approach has been developed at NASA using finite impulse response filters derived from predicted waveforms by solving a nonlinear parabolic equation. Analyses of the cases used here indicate that the mean loudness reduction obtained using such filters does not increase with propagation distance or with increasing turbulence strength, in disagreement with flight test data. An alternative approach to the filters is currently under development employing the multiple scattering theory (MST) of wave propagation. Available results from the physics-based MST approach are used to evaluate the predictive capability of the filters, suggesting that the filters can severely underestimate the loudness reduction prediction of the MST approach.

I. Introduction

The development of a low-amplitude sonic boom (low boom) technology to enable commercial supersonic travel overland has been pursued¹ by the National Aeronautics and Space Administration (NASA) in collaboration with the aerospace industry.^{2,3} One component of this goal is accurate prediction of sonic booms generated by supersonic aircraft that propagate to the ground. To help achieve this goal, an efficient numerical solver of the governing nonlinear equation for sonic boom propagation, called a Burgers equation, has been developed⁴ and integrated into the NASA PCBoom prediction software.⁵ The solver is anticipated for use in a low boom signature validation of the NASA X-59 aircraft currently being built and for estimating the loudness of its low booms in subsequent community overflight tests. Due to the large number of predictions required to support

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these tests, the solver must be both efficient and accurate, accounting for dominant propagation effects. Although the solver already accounts for nonlinearity, absorption, geometrical spreading, and wind effects, it does not currently account for turbulence effects in the atmospheric boundary layer (ABL). Consequently, it can accurately predict sonic booms measured above the ABL⁴ but overpredicts, on average, sonic booms measured on the ground^{6–8} in flight tests.^{9,10}

To predict turbulence effects on sonic boom waveforms, finite impulse response filters were originally developed for N-waves⁹ and, subsequently,^{11,12} for shaped booms using waveforms obtained from a nonlinear parabolic equation called the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation. Such an equation takes into account diffraction effects, nonlinearity, and atmospheric absorption from thermoviscosity and molecular relaxation, but not backscattering¹³ caused by turbulence. The filters, aptly called KZK filters, can be applied efficiently as a signal processing-based surrogate model of the physics-based KZK equation, which requires large computational runtimes. When they are applied to a clean waveform or waveform not having gone through a turbulence field, the waveform becomes turbulized, resulting in varying shapes from a spiky waveform to a rounded one. One important property of the filters is that they recover the waveforms generated by the KZK equation when applied to the same clean waveform used to formulate them.^{11,12}

For shaped booms, there are two different implementations of the KZK filters in the latest PCBoom version, giving rise to two different sets of filters. One is called the regular KZK filters consisting of three filters, for a given turbulence condition, from which the resulting waveforms yield the mean loudness level and the levels corresponding to the mean plus/minus 1 standard deviation. Briefly discussed in this paper, such a set of filters can yield two inconsistent standard deviation values that necessitated the development of the second set of filters. Called the enhanced KZK filters, the second set is obtained from over 1,000 turbulized waveforms that were randomly chosen from over 50,000 waveforms generated by a KZK equation solver. Each of the filters is then applied to an input waveform to account for the turbulence effects. The mean and the standard deviation values of the loudness level are directly calculated from the chosen turbulized waveforms.

A physics-based approach to account for the effects of a turbulence field is currently being developed and uses the multiple scattering theory (MST) of wave propagation. It obtains the effective or mean scattering effects of the turbulence field^{14–16} on an acoustic wavefield. Initial results from this approach include the calculation of the mean waveform using the correlation function of the turbulence field. The mean waveform is obtained without calculating the individual waveforms that would result from an ensemble of random realizations,¹⁷ requiring subsecond computational runtimes. The calculation of the mean waveform implies the inclusion of random focusing, defocusing, and backscattering effects of the turbulence field. The physics-based MST approach obtains an acousto-turbulence interaction energy representing the interaction of the acoustic wavefield with the turbulence field. The interaction energy gives rise to a scattering wavenumber, a complex-valued correction to the wavenumber without turbulence. The scattering wavenumber leads to dispersion and attenuation of the mean waveform due to backscattering.

Predictions from the KZK filters are compared with predictions from the MST-based approach and with data from a flight test. The scattering predicted by the MST approach leads to a loudness reduction of the mean waveform¹⁷ in sonic booms that has been observed in data^{6,8} from a flight test.⁹ Such a loudness reduction is shown here to be underpredicted by the filters. Furthermore, the flight test data indicate that the mean loudness is reduced by turbulence effects and that such a reduction increases with propagation distance and with increasing turbulence strength. However, results from the filters for the atmospheric profiles and waveforms used here indicate that the loudness reduction does not increase with propagation distance or with increasing turbulence

strength. This discrepancy can be attributed to the one-way propagation approximation that the KZK equation uses and, thus, it cannot take into account energy losses due to backscattering.¹³ Additionally, the loudness reduction using the filters does not account for the numerical losses in the KZK equation solver that have been discussed elsewhere⁷ and compensated for in a subsequent work.¹⁸

The remainder of this paper is structured as follows. Section II presents the theoretical background in the development and implementation of the KZK filters. In Sec. III, the evaluation of the filters is made using physics-based solutions. Section IV discusses the predictive power of the filters. Finally, Sec. V provides the conclusion.

II. Theoretical background

A. The KZK equation

The KZK equation¹⁹ with sound speed and velocity fluctuations is used to simulate the turbulence effects on sonic booms propagating through the atmospheric boundary layer (ABL). This equation is augmented in Ref. [7] to account for the molecular relaxation and can be written as

$$\frac{\partial p}{\partial x} = (\mathcal{D} + \mathcal{T} + \mathcal{N} + \mathcal{A})p, \quad (1)$$

where

$$\mathcal{D} = \frac{\bar{c}}{2} \int_{-\infty}^{\tau} d\tau' \frac{\partial^2}{\partial \mathbf{r}_{\perp}^2}$$

is a mathematical operator associated with the diffraction effect,

$$\mathcal{T} = \frac{\tilde{n} + \tilde{m}_x}{\bar{c}} \frac{\partial}{\partial \tau} - \tilde{\mathbf{m}}_{\perp} \cdot \frac{\partial}{\partial \mathbf{r}_{\perp}}, \quad (2)$$

is the turbulence effect operator,

$$\mathcal{N} = \frac{\beta p}{\bar{\rho} \bar{c}^3} \frac{\partial}{\partial \tau} \quad (3)$$

is the nonlinear effect operator, and

$$\mathcal{A} = \frac{\delta_d}{2\bar{c}^3} \frac{\partial^2}{\partial \tau^2} + \sum_{\nu} \frac{c'_{\nu}}{\bar{c}^2} \int_{-\infty}^{\tau} d\tau' e^{-\frac{(\tau-\tau')}{t_{\nu}}} \frac{\partial^2}{\partial \tau'^2} \quad (4)$$

is the absorption effect operator consisting of the thermoviscous dissipation in the first term and the molecular relaxation in the second term. In Eqs. (1) and (3), $p = p(\mathbf{r}, \tau)$ is the acoustic pressure, τ is the retarded time, and the spatial coordinate $\mathbf{r} = (x, \mathbf{r}_{\perp})$ is decomposed into the axis of the primary propagation direction x and the transverse coordinates $\mathbf{r}_{\perp}$. In the current application, $p(\mathbf{r}, \tau)$ is initialized by the overpressure near the aircraft typically obtained using computational fluid dynamics (CFD) codes. In addition, $\bar{c}$, $\bar{\rho}$, β , and δ_d are the mean sound speed, mean density, coefficient of nonlinearity, and sound diffusivity, respectively; c'_{ν} and t_{ν} are the phase speed change and relaxation time associated with a single relaxation process ν ; and $\tilde{n}$, $\tilde{m}_x$, and $\tilde{\mathbf{m}}_{\perp}$ are the nondimensionalized quantities denoting the sound speed fluctuation, velocity fluctuation along x , and velocity fluctuation transverse to the x -axis, respectively.

Equation (1) is a three-dimensional (3D) equation and can be solved numerically but with typically prohibitively large computational runtimes. Consequently, the results presented here are obtained using a 2D approximation⁷ to reduce the computational runtimes to a practical level. Such an approximation assumes that the turbulence field only varies along a plane, leading to an acoustic pressure field that also varies only within this plane. The solver, called the KZKFourier engine code⁹ uses a split-step approach or operator-splitting approximation,⁷ as is commonly done in sonic boom propagation through turbulence fields (e.g., Refs. [19–22]). In the split-step approach, each of the four operators is applied independently at each propagation step. The split-step approach can yield a sufficiently accurate solution if the step size is small enough such that the nonlinear effect is negligible.

In acoustic propagation modeling, a turbulence field is usually treated as a random field^{7,9,16,21–24} due to the inherent difficulty in accurately measuring the fluctuation in a given volume of space. The effects of turbulence on an acoustic waveform can be studied using Monte Carlo-type simulations, where the waveform is propagated through multiple random realizations (N) of the turbulence field by solving the KZK equation. The realizations are created such that an ensemble of realizations obeys certain spatial correlation. The numerical simulation produces N different waveforms at a specific point on the ground. The energy of each waveform can be expressed in a logarithmic scale using an acoustic metric. Since the turbulence field is treated as a random field, the metric level will also be a random variable with a mean and variance.

B. Implementation of the KZK filters

Because the KZKFourier code is computationally expensive and inefficient, the effects of turbulence on a waveform are studied using a set of surrogate models. These models are implemented as finite impulse response filters called KZK filters,^{9,11,12} where a single KZK filter represents the effect of a single realization of a turbulence condition on a waveform. The filters are convolved with the waveform to obtain filtered waveforms, which are aptly called turbulized waveforms. These filters recover the waveforms generated by the KZKFourier code when applied to the same input waveform used to formulate them. An ensemble of filters representing the effects of many realizations of a turbulence condition can then be used to develop statistics describing the turbulence effects.

The KZK filter coefficients are calculated using an input waveform $p_i(\tau)$ and an output waveform $p_o(\tau)$. The input waveform is generated by the KZKFourier code without turbulence for which the terms containing the operators $\mathcal{D}$ and $\mathcal{T}$ in Eq. (1) vanish. On the other hand, $p_o(\tau)$ is generated by the KZKFourier code with turbulence. Note that the numerical losses in the KZKFourier code reported in Ref. [7] have not been compensated for in the calculation of $p_o(\tau)$ for the filters in the latest PCBoom version. The filter coefficients are then obtained using the process represented by $p_o(\tau) = h(\tau) * p_i(\tau)$, where the symbol $*$ represents the convolution operator and $h(\tau)$ is the filter coefficients that act as the impulse response of the ABL. Numerically, this equation can be rewritten as $\hat{p}_o = \hat{H}\hat{p}_i$ where $\hat{p}_o$ and $\hat{p}_i$ are vectors representing the discretized $p_o(\tau)$ and $p_i(\tau)$, respectively, and $\hat{H}$ is a matrix operator from the discretized $h(\tau)$. From this equation, $\hat{H}$ can be calculated and the filter coefficients $h(\tau)$ reconstructed if both $\hat{p}_o$ and $\hat{p}_i$ are known. This process is inherently linear; thus, the applicability of the filtering approach to waveforms of arbitrary spectral level is accurate only if the nonlinearity operator $\mathcal{N}$ in Eq. (1) is small.

The filter coefficients $h(\tau)$ only accounts for the diffraction and turbulence effects, by construction, and can thus be evaluated for accuracy using existing physics-based solutions. Since Eq. (1) is

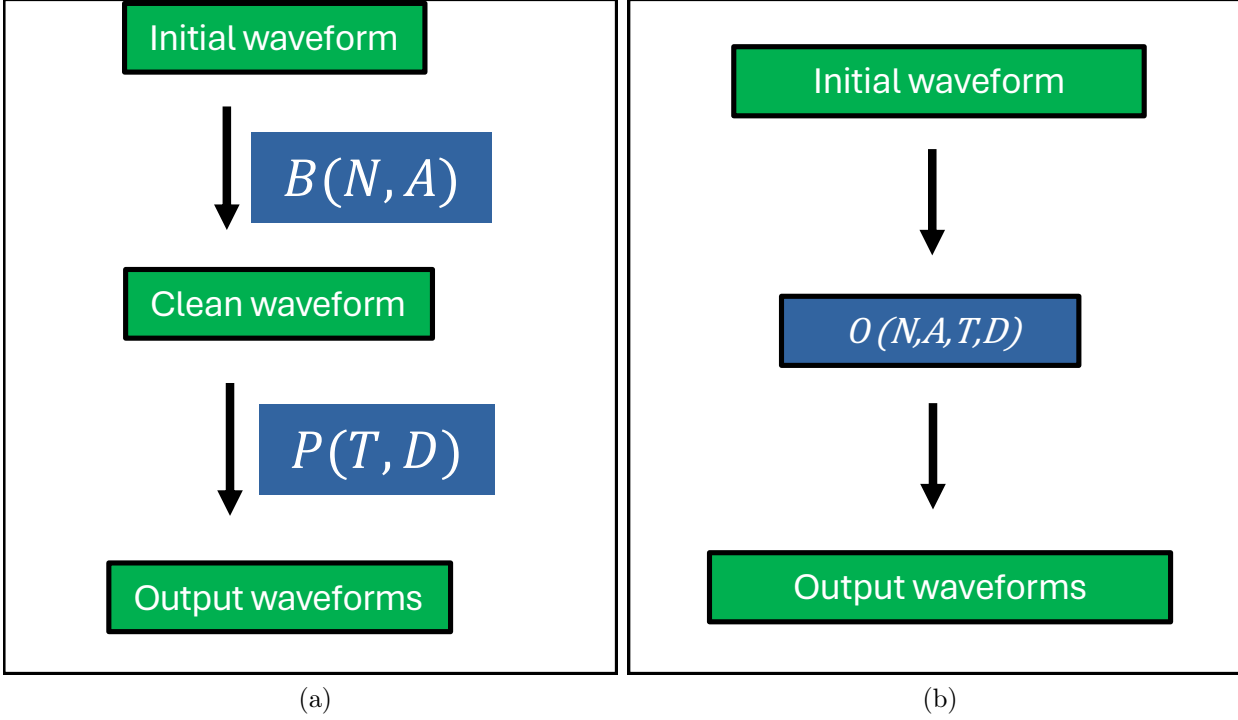


Figure 1: The implementation of the KZK filters in PCBoom. In (a), the initial waveform is the input waveform to the operator B representing the Burgers equation consisting of the nonlinearity N and atmospheric absorption A operators. The clean waveform is then acted upon by the operator P denoting the parabolic equation consisting of the turbulence T and diffraction D operators. This process is an approximation of the whole process shown in (b) in which the operator O consists of the operators N , A , T , and D simultaneously acting on the initial waveform yielding the output waveforms.

solved numerically⁷ using an operator-splitting approximation, then it can be split into an equation consisting of the operators $\mathcal{N}$ and $\mathcal{A}$ and another consisting of the operators $\mathcal{D}$ and $\mathcal{T}$. The first equation is known as a Burgers equation and the second a parabolic equation, both of which have known solutions (e.g., see Refs. [4, 17, 25, 26]). In the frequency domain, the parabolic equation can be written as

$$\frac{\partial P}{\partial x} = \left(\frac{i}{2\kappa_o} \frac{\partial^2}{\partial \mathbf{r}_\perp^2} - i\kappa_o \tilde{n}_e - \tilde{\mathbf{m}}_\perp \cdot \frac{\partial}{\partial \mathbf{r}_\perp} \right) P, \quad (5)$$

where $P(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} p(\mathbf{r}, \tau)$ is the Fourier transform of the acoustic pressure, $\kappa_o = \omega/\bar{c}$ is the wavenumber without turbulence, and $\tilde{n}_e(\mathbf{r}) = \tilde{n}(\mathbf{r}) + \tilde{m}_x(\mathbf{r})$ is the effective sound speed fluctuation along the propagation direction.^{16, 17, 23} Essentially, the KZK filters model the solution of Eq. (5).

The KZK filters are implemented in PCBoom following the procedure described in Ref. [9]. In this implementation, a sonic boom waveform is propagated all the way to the ground using a Burgers equation^{4, 5} taking into account the nonlinearity and atmospheric absorption only ($\mathcal{N}$ and $\mathcal{A}$). Such a ground sonic boom waveform constitutes the input (clean) waveform to the KZK filters. The turbulence effects are then taken into account by applying an ensemble of filters corresponding to the turbulence condition and propagation distance, generating an ensemble of waveforms. As shown in Fig. 1(a), the implementation can be illustrated by a successive application of two operations: the first operation is the use of the Burgers equation, represented by the operator B , propagating the

waveform from the aircraft to the ground. It is then followed by the operation using the parabolic equation, denoted by the operator P , to apply the turbulence effects through the atmospheric boundary layer. This process is an approximation of the whole process shown in Fig. 1(b), and is discussed in more detail in Ref. [9].

III. Evaluation of the KZK filters using physics-based solutions

A. Physics-based solutions

Various numerical solutions of the one-way propagation model given by Eq. (5) and the more accurate two-way propagation model, accounting for backscattering effects, already exist for acoustic propagation in the atmosphere (e.g., see Ref. [16] and the references cited therein). For propagation through random turbulence fields in the atmosphere, the ensemble mean of some physical quantities can be computed. One such quantity is the mean acoustic pressure field $\langle p(\mathbf{r}, \tau) \rangle$ that has already been derived and presented in Eq. (7.150) of Ref. [16] using the on-shell approximation. Such an approximation requires a weak turbulence field and is only accurate at low frequency.¹⁷ For an initial plane wave, the mean acoustic pressure field can be expressed by

$$\langle p(\mathbf{r}, \tau) \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A(\omega) e^{-i\omega\tau} e^{-\int_0^x \alpha_{vk}(x'; \omega) dx'}, \quad (6)$$

which depends on the propagation distance x , but not on the transverse coordinates $\mathbf{r}_\perp$. In Eq. (6), $A(\omega)$ is the spectrum of the acoustic pressure without turbulence and α_{vk} is the attenuation coefficient given by Eq. (7.184) of Ref. [16], which can be given by

$$\alpha_{vk} = \frac{\sqrt{\pi}\Gamma(5/6)}{\Gamma(1/3)} \kappa_o^2 (\sigma_c^2 \ell_c + \sigma_v^2 \ell_v) \quad (7)$$

for a turbulence field modeled by the von Kármán correlation function. In Eq. (7), Γ is the gamma function, σ_c^2 and ℓ_c are the nondimensionalized variance and correlation of the sound speed fluctuation, respectively, while σ_v^2 and ℓ_v are the nondimensionalized variance and correlation of the velocity fluctuation. In using Eq. (7.184) of Ref. [16], we use $\beta_c = 1$ and $\sigma_T^2/T_o^2 = 4\sigma_c^2$, as well as replace σ_v^2/c_o^2 in Ref. [16] with σ_v^2 here. In deriving Eq. (7), Eq. (5) was solved that takes backscattering into account.

A solution to the two-way propagation model, of which Eq. (5) is a one-way approximation, has also been obtained in Ref. [17] without using the on-shell approximation. The solution is also given by Eq. (6) but with α_{vk} replaced by the complex-valued scattering wavenumber $k_s = k_e - \kappa_o$, where k_e is the root of the dispersion relation given by Eq. (19) in Ref. [17] that can be rewritten as

$$k_e^2 - \kappa_o^2 - \tilde{\Sigma}(k_e) = 0. \quad (8)$$

In Eq. (8), $\tilde{\Sigma}$ is the acousto-turbulence interaction given by

$$\tilde{\Sigma}(k_e) = \sum_{\nu} \frac{4\kappa_o^4 \ell_\nu^2 \sigma_\nu^2}{(k_e^2 - \kappa_o^2) \ell_\nu^2 - 2i\kappa_o \ell_\nu + 1}$$

for a turbulence field modeled by the exponential correlation function. For the von Kármán correlation function, additional work is needed to obtain an expression for $\tilde{\Sigma}$. However, the exponential and the von Kármán correlation functions yield nearly the same physical results.¹⁷

The turbulence parameters (i.e., σ_c^2 , ℓ_c , σ_ν^2 , and ℓ_ν) can depend on x due to a weak anisotropy of the turbulence field in the ABL caused by the atmospheric stratification. The most commonly used anisotropic model of this turbulence field is the Monin-Obukov Similarity Theory (MOST).¹⁶ Since MOST is the turbulence anisotropy model used in the KZKFourier code, this theory is used in the current paper to estimate the turbulence parameters as a function of height in the ABL through the propagation distance x .

B. Comparisons with the enhanced KZK filters

Except in Sec. IVC discussing the regular KZK filters, the KZK filters from hereon refer to the enhanced KZK filters, which are an ensemble of over 1,000 filters for a given turbulence condition formulated in Ref. [11,12]. These filters are obtained from a random subset of waveforms generated by the KZKFourier code at over M (where $M=1,000$) ground points using over N (where $N=25$) random realizations of a turbulence field, similar to the process discussed in Ref. [9]. The ground points are equally-spaced, forming a straight line transverse to the propagation direction. The filters are applied by convolving each of them with an input waveform $p_i(\tau)$ to obtain as many output waveforms $p_o(\tau)$ as there are KZK filters. Using the ergodic assumption (e.g., Ref. [7]), the statistics of these turbulized waveforms are considered to be representative of the statistics of the turbulized waveforms that would be obtained at a given location on the ground using the more than $M \times N$ filters.

The KZK filters are evaluated for a specific turbulence condition characterized by the turbulence parameters such as the height of the ABL H , friction velocity scale u_* , mixed-layer velocity scale w_* , surface layer temperature scale T_* , surface pressure P_s , and surface temperature T_s . For the comparison made here, these parameters take the following values: $H = 800$ m, $u^* = 0.8$ m/s, $w^* = 0.8$ m/s, $T^* = -1.54$ K, $P_s = 0.87$ atm, and $T_s = 269.35$ K. The waveforms with turbulence (or turbulized waveforms) obtained using the KZKFourier code^{7,9,18} are shown in Fig. 2(a), while the input (clean) waveform obtained without turbulence using PCBoom^{4,5} is shown in Fig. 2(b). Some of the turbulized waveforms have slightly larger peak overpressures and others have slightly smaller peak overpressures. Turbulence effects increase the overpressure due to focusing of energy at locations with negative effective sound speed fluctuation, and decrease it due to defocusing at locations with positive effective sound speed fluctuation. The numerical losses in the KZKFourier code that were reported elsewhere⁷ and compensated for in a subsequent work¹⁸ using the code have not been compensated for in the results presented in Fig. 2(a).

The ensemble mean of the turbulized waveforms (also called the mean turbulized waveform) in Fig. 2(a) is calculated and shown in the dash-dotted red curve in Fig. 3(a), along with the clean waveform shown in the solid black curve. The loudness level of such waveforms can be expressed by the Perceived Level (PL), a loudness metric developed specifically for N-waves and shaped booms²⁷ employing the method developed in Ref. [28]. Using this loudness metric, the clean waveform has a PL of 78.7 dB, while the mean turbulized waveform has a PL of 76.1 dB. The figure also shows the predicted mean waveform (or exact mean waveform) in the dotted blue curve, obtained using the exact solution given by Eqs. (6) and (8) for the exponential correlation function. This waveform has a PL of 70.1 dB. Furthermore, the figure also shows the predicted mean waveform (on-shell

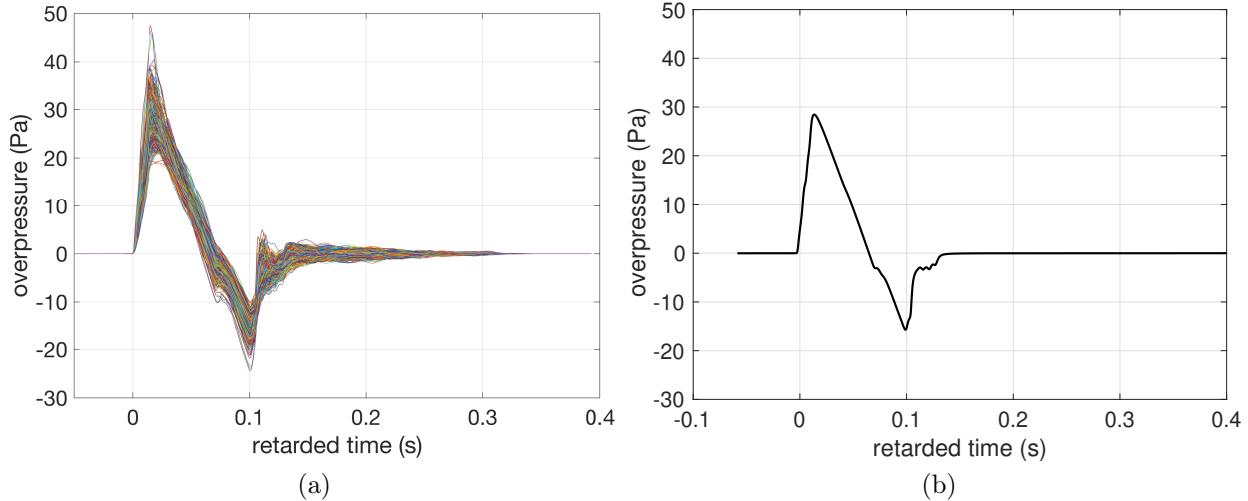


Figure 2: An ensemble of turbulized waveforms (a) calculated using the KZK filters (with turbulence) and generated from a clean (initial) waveform (b).

mean waveform) in the dashed green curve, obtained using Eq. (6) and the on-shell approximation given by Eq. (7). This waveform has a PL of 35.1 dB.

All the mean waveforms propagated through turbulence and calculated using various approaches have reduced loudness values relative to the clean waveform. The mean turbulized waveform has a loudness reduced by about 2.6 dB. Although this waveform shown by the dash-dotted red curve in Fig. 3(a) appears to be nearly identical with the clean waveform, its energy spectral density shown in the dash-dotted red curve in Fig. 3(b) is significantly different at high frequencies from that of the clean waveform shown in the solid black curve. Since the parabolic equation given by Eq. (5) is a one-way propagation model, there is no energy loss due to backscattering.¹³ The loudness reduction could be partially due to the reported numerical losses⁷ in the KZKFourier code that were compensated for in a subsequent work.¹⁸ Unlike the mean turbulized waveform, both the exact mean waveform and the on-shell mean waveform take backscattering into account and are, thus, expected to suffer energy losses due to this effect. Such energy losses are, indeed, manifested in the loudness reduction of about 8.6 dB for the exact mean waveform and 43.6 dB for the on-shell mean waveform. These are much larger than the loudness reduction for the mean turbulized waveform that does not take backscattering into account. These losses can also be seen in the reduction of their energy spectral densities shown in Fig. 3(b).

The extremely large loudness reduction for the on-shell mean waveform is most likely an overprediction. This waveform uses the on-shell approximation that has been shown to be accurate only for weak turbulence and low frequency.¹⁷ With the on-shell approximation, the extinction coefficients (attenuation coefficients caused by backscattering due to turbulence) are increasingly overestimated with increasing frequency above a transition frequency¹⁷ given by $f_t = \bar{c}/4\pi\ell\sigma$. Such an overestimation of the extinction coefficients would then result in an excessive signal attenuation. For the case considered here and for the buoyancy-produced velocity fluctuation where $\bar{c} \approx 330$ m/s, $\ell = 0.23H = 184$ m, and $\sigma = \sqrt{0.35}w_*/\bar{c} = 0.0014$, the transition frequency is 99.5 Hz. Well above this frequency, the extinction coefficient for the exact mean waveform approaches $1/2\ell = 0.0027$ m⁻¹, which is independent of frequency f . On the other hand, the on-shell extinction coefficient increases with f^2 as shown by Eq. (7), severely overestimating signal attenuation at high frequencies. Although less accurate, the on-shell solution given by Eqs. (6) and (7) demonstrates our

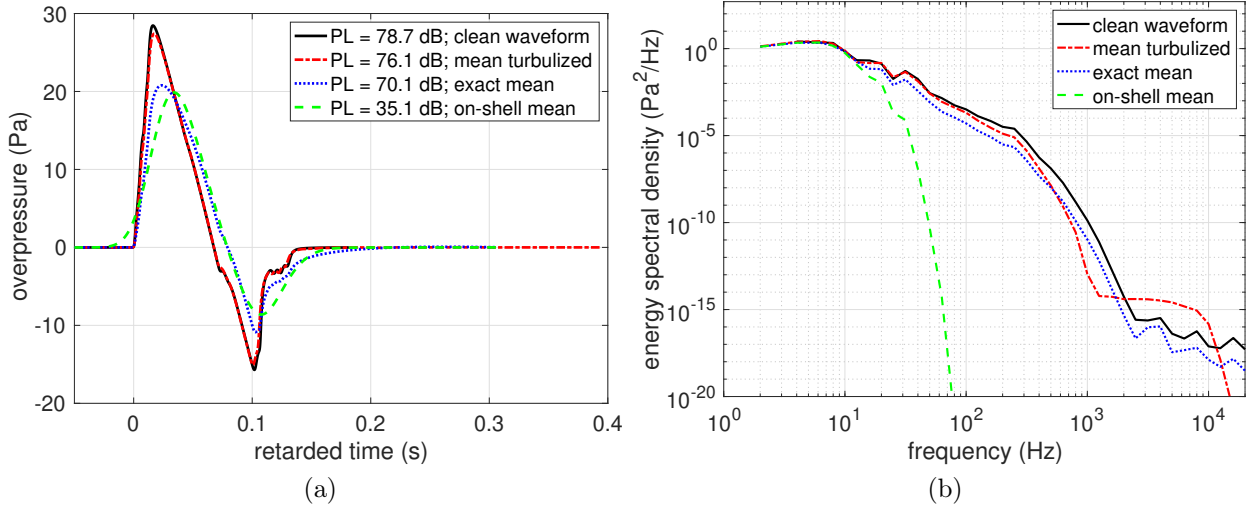


Figure 3: Clean acoustic field and mean acoustic fields obtained using various approaches. Panel (a) shows four different waveforms: clean waveform obtained without turbulence, mean turbulized waveform from the ensemble of the KZK turbulized waveforms shown in Fig. 2(a), exact mean waveform obtained using the exact solution given by Eq. (8), and on-shell mean waveform using the on-shell approximation given by Eq. (7). Panel (b) shows their corresponding energy spectral densities averaged within each one-third octave band.

understanding of the energy losses associated with backscattering in wave propagation that must not be ignored.

IV. Discussion of the prediction of the KZK filters

In the previous section, the loudness of the mean turbulized waveform is compared with the clean waveform, indicating that the turbulized waveforms undergo loudness reduction while propagating through turbulence. Since the turbulized waveforms are obtained using the parabolic equation, which does not take backscattering into account, there are no loss mechanisms. Such a loudness reduction could therefore be attributed to the reported numerical losses^{7,18} in the KZKFourier code. Furthermore, this loudness reduction is much smaller than the loudness reduction obtained by the multiple scattering theory by at least 8 dB (PL). Consequently, the KZK filters and MST-based approach appear to yield results that are in disagreement.

In this section, we present some results obtained from the KZK filters by varying certain turbulence parameters. Unlike in the previous section in which the loudness of the mean waveform is the quantity of interest, here the results are presented in terms of the mean loudness, defined as the mean of an ensemble of loudness values obtained from different turbulized waveforms for a given turbulence condition. Such turbulence parameters include the ABL height H and the strength of each of the temperature, shear-produced velocity, and buoyancy-produced velocity fluctuations controlled by T_* , u_* , and w_* .

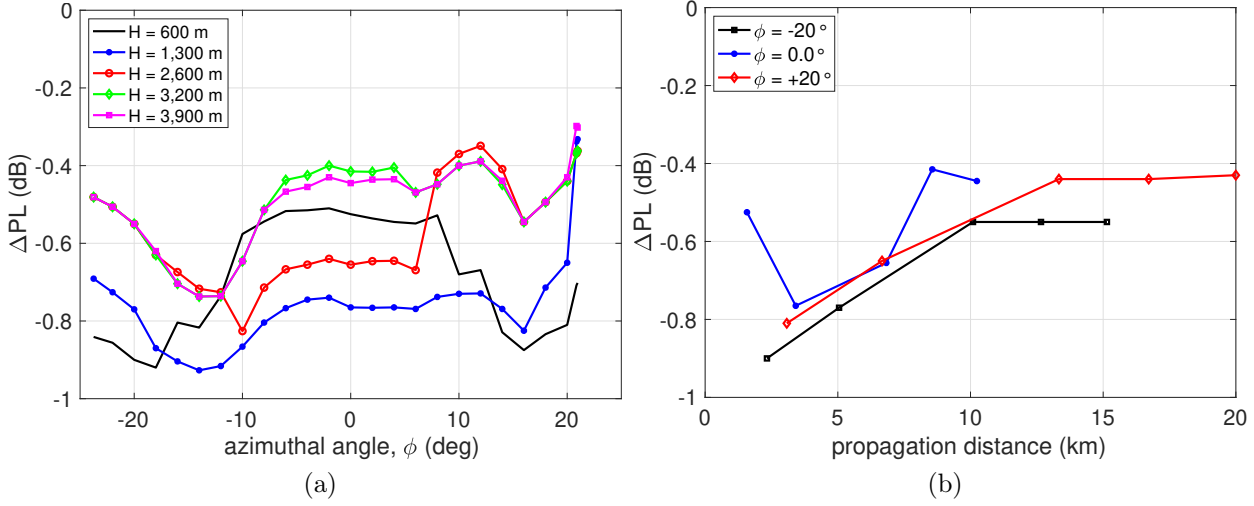


Figure 4: Dependence of the loudness reduction (ΔPL) obtained from the KZK filters on the ABL height H . The azimuthal angle in the horizontal axis of panel (a) is the angle around the aircraft, from which acoustic rays (sonic booms) are emitted. The ΔPL in the vertical axis is defined as the mean PL with turbulence minus the PL without. Panel (b) shows the mean PL reduction as a function of the propagation distance through the ABL for azimuthal angles ϕ of -20, 0, and 20 deg.

A. Dependence on the ABL height

A dependence of the loudness reduction obtained from the KZK filters on the ABL height H and propagation distance x in a numerical simulation is shown in Fig. 4. In this simulation, four different heights are used: 600 m shown in black curve, 1,300 m in blue, 2,600 m in red, 3,200 m in green, and 3,900 m in magenta. The following turbulence parameters are used: $u_* = 0.4$ m/s, $w_* = 2.21$ m/s, $T_* = -0.4$ K, $T_s = 307.15$ K, and $P_s = 0.93$ atm. In Fig. 4(a), the azimuthal angle in the horizontal axis is the angle around the aircraft, from which acoustic rays or sonic booms are emitted. The 0-deg azimuthal angle refers to the ray emitted directly below the aircraft. The ΔPL in the vertical axis is defined as the mean PL with turbulence minus the PL without. By definition, negative ΔPL indicates a reduction in the loudness when turbulence is introduced. The simulation uses the atmospheric profiles from the NASA Propagation Dry Run Case 4 as well as waveforms from the X-59 aircraft obtained using a CFD code.

From Fig. 4(a), the loudness reduction does not conclusively depend on the ABL height H , although it does appear that the loudness reduction generally decreases with H if the data for $H = 600$ m are ignored. This result is consistent with the loudness reduction that appears to decrease with propagation distance x through the ABL as shown in Fig. 4(b) for three different azimuthal angles of -20°, 0°, and +20°. Furthermore, the loudness reduction is only limited to within a dB (PL) and varies within a narrow range of about 0.6 dB. Based on this plot, the loudness reduction obtained using the KZK filters for the case used here does not increase with x . Depending on the robustness of the filters, this finding may not hold for certain atmospheric profiles. Nevertheless, this finding is in disagreement with the data presented in Ref. [7] obtained in a flight test⁹ where the loudness reduction was found to increase with x , on average. In this data, however, the variance of the velocity fluctuation was reported to also increase with x . It is also possible that the other turbulence parameters could have changed between supersonic passes.

To ensure that all turbulence parameters are fixed except for the propagation distance, an analysis

using the three microphone arrays in SonicBat 1 flight test⁹ is instructive. The first array was near the flight undertrack, the second was 2.4 km away, and the third was 4.8 km away. The mean PL for each pass at each array is calculated, from which the loudness reduction ΔPL can also be computed given the PL without turbulence obtained using PCBoom. Figures 5(a) - 5(c) show the PL reduction distribution (probability density function) for the first, second, and third array, respectively, using all the passes during the flight test. In these figures, μ is the mean of the ensemble of PL reduction values from all the supersonic passes and σ is the corresponding standard deviation. The mean of the ensemble of PL reduction is plotted in Fig. 5(d) as a function of the distance of the arrays from the flight undertrack, which is a proxy for the propagation distance through the ABL. This plot has eliminated the effects of the variation of the turbulence conditions between passes since the arrays were exposed to the same conditions, assuming turbulence conditions are nearly independent of the horizontal distance, which is likely true for the short horizontal distance involved. It is clear from Fig. 5(d) that the mean of the PL reduction increases with propagation distance, on average.

B. Dependence on the velocity scales

In addition to the ABL height, we would like to know the dependence of the loudness reduction on other turbulence parameters. In this section, this dependence on the mixed-layer velocity scale w_* and the friction velocity scale u_* is investigated.

For the mixed-layer velocity scale w_* , the different values used are 0.5, 1.0, 2.0, 3.0, and 3.8, all in units of m/s. The values of the other turbulence parameters are $H = 1,300$ m, $u_* = 0.4$ m/s, $T_* = -0.4$ K, $T_s = 307.15$ K, and $P_s = 0.93$ atm. The loudness reduction ΔPL is shown in Fig. 6(a) as a function of the azimuthal angle around the aircraft for the five different values of w_* . Based on this figure, the ΔPL is only limited to within a dB (PL) and does not appear to increase with w_* . This finding is confirmed in Fig. 6(b) showing the ΔPL as a function of w_* for azimuthal angles of -20, 0, and 20 deg. Based on this figure, the loudness reduction appears to slowly increase with increasing w_* for the 0-deg azimuthal angle. However, it appears to decrease with increasing w_* near the lateral edge (-20 and +20 deg). This result is inconsistent with the flight test data reported in Ref. [7], where the loudness reduction, on average, was found to increase with increasing variance of the velocity fluctuation, which depends on w_* .

For the friction velocity scale u_* , the different values used are 0.2, 0.4, 0.6, 0.8, and 1.0, all in units of m/s. The values of the other turbulence parameters are $H = 1,300$ m, $w_* = 2.21$ m/s, $T_* = -0.4$ K, $T_s = 307.15$ K, and $P_s = 0.93$ atm. The loudness reduction ΔPL is shown in Fig. 6(c) as a function of the azimuthal angle around the aircraft for the five different values of u_* . As with the dependence on w_* , the ΔPL is only limited to a dB and does not appear to depend on u_* . This finding is confirmed in Fig. 6(d) showing the ΔPL as a function of u_* for azimuthal angles of -20, 0, and 20 deg. Based on this figure, the ΔPL only vary within a narrow range of about 0.5 dB. As with the mixed-layer velocity scale, this result is inconsistent with the flight test data reported in Ref. [7].

C. Regular KZK filters

Before the enhanced KZK filters discussed above were developed, the regular filters were first chosen by NASA to represent the mean loudness level μ and the associated standard deviation σ of the turbulence effects on sonic booms. For a given turbulence condition, three filters were formulated

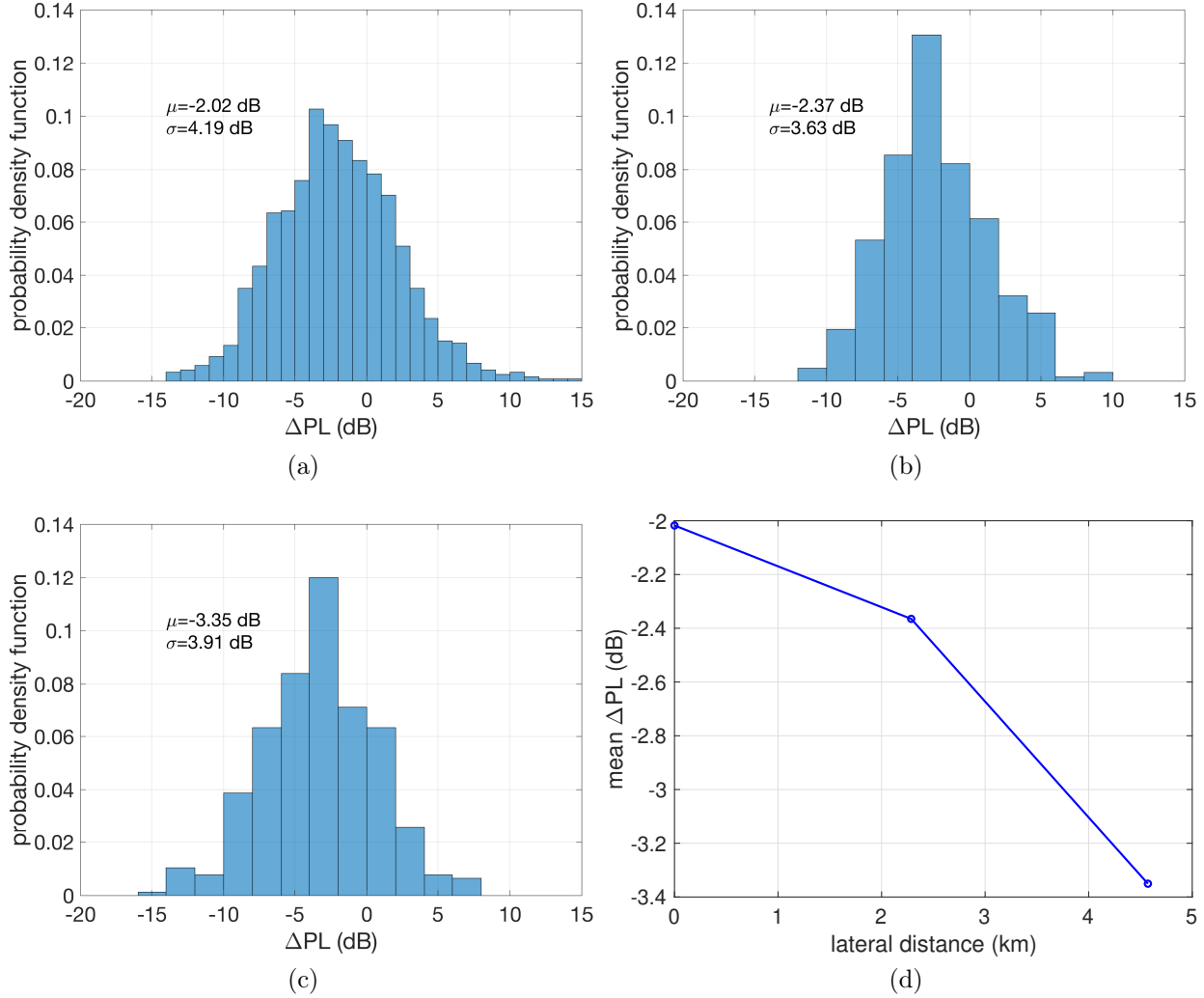


Figure 5: Loudness reduction as a function of lateral distance from flight undertrack. Panels (a)–(c) show the distributions of the loudness reduction in 3 microphone arrays for all the passes in SonicBAT flight test. The first array was near the flight undertrack, the second was 2.4 km away, and the third was 4.8 km away. Panel (d) shows the loudness reduction as a function of the distance of the arrays from the flight undertrack.

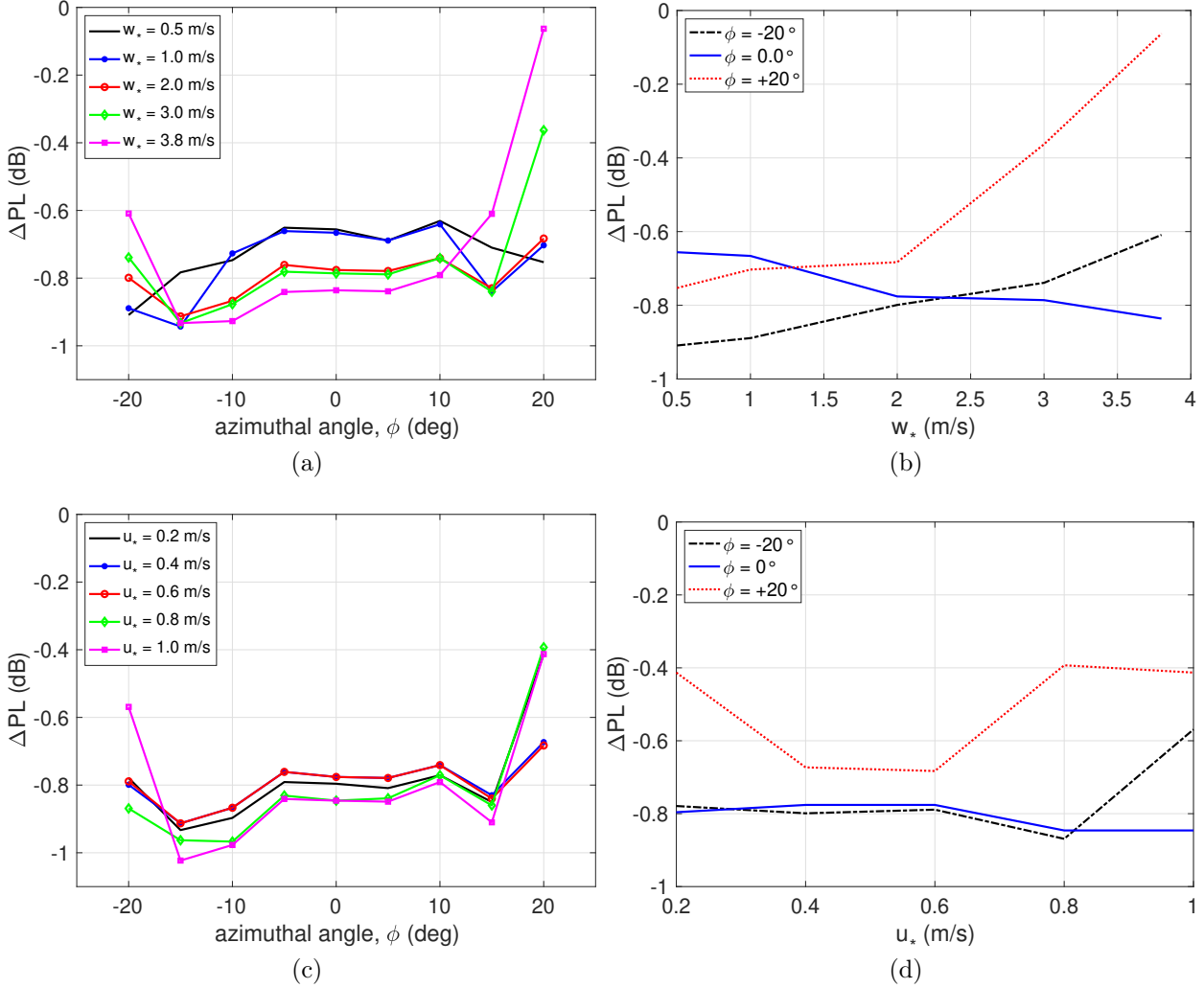


Figure 6: Dependence of the loudness reduction ΔPL on the turbulence velocity scales. Panel (a) shows the ΔPL as a function of the azimuthal angle around the aircraft for different mixed-layer velocity scale w_* values, and panel (b) plots the reduction as a function of w_* for azimuthal angles ϕ of -20, 0, and +20 deg. Panel (c) shows the ΔPL for different friction velocity u_* values, while panel (d) plots the reduction as a function of u_* for azimuthal angles ϕ of -20, 0, and +20 deg.

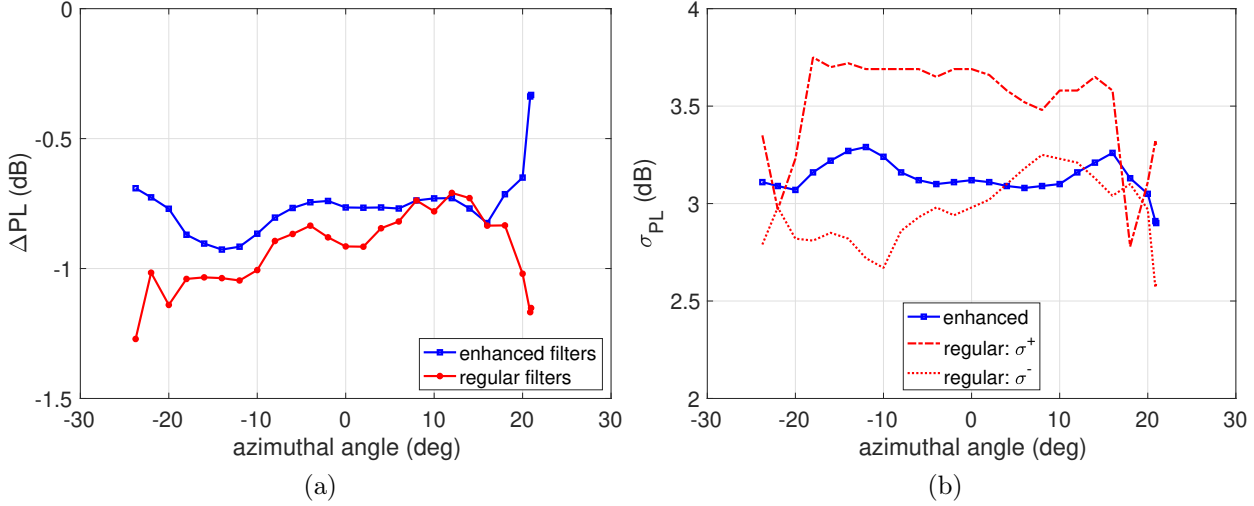


Figure 7: A loudness reduction (a) and the corresponding standard deviations σ^+ and σ^- (b) obtained using the regular KZK filters, shown along with those obtained using the enhanced KZK filters.

using three turbulized waveforms, one of which yields μ and $\mu \pm \sigma$ values of a loudness metric. Such turbulized waveforms were taken from an ensemble of turbulized waveforms generated by the KZKFourier code.

To evaluate the regular KZK filters, we use the same turbulence condition used in obtaining the blue curve shown in Fig. 4(a), for which the turbulence parameters are $H = 1,300$ m, $u_* = 0.4$ m/s, $w_* = 2.21$ m/s, $T_* = -0.4$ K, $T_s = 307.15$ K, and $P_s = 0.93$ atm. The resulting loudness reduction is shown by the red curve in Fig. 7(a), along with the loudness reduction using the enhanced KZK filters (blue curve). These two approaches of calculating the loudness reduction appear to yield comparable results, where neither one shows a discernible relationship between the loudness reduction and the azimuthal angle or, by extension, the propagation distance through the ABL.

The standard deviation corresponding to the turbulence condition in the previous paragraph is shown by the red curves in Fig. 7(b), along with the standard deviation calculated using the enhanced KZK filters that appears to be nearly bracketed by the two σ values of the regular KZK filters. Note that the regular KZK filters were constructed such that two σ values can be obtained relative to μ : one is σ^+ from the filter for $\mu + \sigma$ level and the other is σ^- from the filter for the $\mu - \sigma$ level. Regardless of the choice, the two values should be nearly the same for self-consistency, since a Gaussian distribution of the loudness level is assumed in the formulation of the KZK filters. However, these two values appear to significantly differ from each other by as much as 1 dB for the case investigated here. Due to this lack of self-consistency in the regular KZK filters, NASA had decided to further investigate the utility of the enhanced KZK filters as an alternative to the regular KZK filters.

V. Conclusion

Effects of turbulence fields on sonic booms can significantly modify and deform sonic boom waveforms from supersonic aircraft. Data from flight tests previously reported (e.g., see Ref. [9]) show

that these effects include random formation of spiky waveforms as well as rounded ones. The data reported here and elsewhere⁷ also indicate that the sonic booms, on average, undergo loudness reduction and that such a reduction increases with propagation distance and with increasing turbulence strength. The reduction is potentially due to backscattering induced by the turbulence fields.

To predict turbulence effects on the loudness of sonic booms generated by supersonic aircraft, a set of finite impulse response filters were formulated using a KZK equation solver. These filters are signal processing-based surrogate models of the solution of the physics-based KZK equation typically solved numerically, requiring large computational runtimes. For the cases used in this paper, analyses show that the filters yield a loudness reduction that appears to be limited to within a dB (PL) and that does not increase with propagation distance or with increasing turbulence strength, in disagreement with the flight test data. This disagreement could be attributed to several factors including potential issues on the accuracy of the filters, the 2D approximation used in the KZKFourier code, and the one-way approximation of the KZK equation in which it does not take into account energy losses due to backscattering. The loudness reduction obtained using the filters could be partially attributed to known numerical losses in the KZK equation solver.

The filters were compared with a physics-based approach, currently under development, using the multiple scattering theory (MST). The mean turbulized waveform, obtained from an ensemble of turbulized waveforms using the filters, is found to have a much smaller loudness reduction than that of the mean waveform obtained using the MST-based approach for the turbulence condition investigated here. The difference could be caused by the backscattering effects that are taken into account by the MST approach but not by the filters.

Predictions using the MST approach are given here in terms of the mean waveforms and the corresponding loudness values, without calculating an ensemble of turbulized waveforms. This approach is several orders of magnitude faster than the use of the KZK filters. Work is underway to extend the approach to predict the mean loudness of sonic booms, without calculating the ensemble of loudness values.

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