

Uncertainty Quantification Methodology for Sonic Boom Loudness of a Low-Boom Supersonic Concept

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This paper presents a comprehensive analysis of uncertainty quantification for sonic boom loudness metrics associated with the X-59 aircraft during its acoustic validation phase. The study focuses on employing advanced methodologies to characterize uncertainties in key sonic boom parameters predicted from a database derived from a computational fluid dynamics (CFD) solver. These methodologies include creating a polynomial chaos expansion (PCE) surrogate model, which is then used to run a Monte Carlo analysis to obtain 95% uncertainty intervals for sonic boom loudness parameters across the full carpet. Atmospheric uncertainties are rigorously considered, leveraging weather models based on historical data near Edwards Air Force Base to simulate potential flight conditions. The findings include preliminary uncertainty results for the full carpet during the validation phase of the Quesst mission. Additionally, a sensitivity study reveals that the primary sources of uncertainty are humidity and cruise weight.

Nomenclature

A_e	equivalent area	p	pressure
a_i	coefficient for i th basis function	q	order of polynomial
$\mathbf{D}$	deterministic variables	ST_j	total Sobol parameter for metric j
m	number of uncertain variables	$\mathbf{X}_j$	uncertainty parameter j
N_{MC}	number of Monte Carlo samples	$\mathbf{Y}$	uncertain output parameter
N_t	number of samples needed	Ψ_i	basis function representing i th mode
n	number of PCE models	ξ	standard random variable

I. Introduction

The Quesst mission seeks to advance supersonic travel by demonstrating advance aerodynamic shaping techniques aimed at dramatically reducing sonic boom loudness. Because the goals of the mission are to achieve a nominal Mach of 1.4, an altitude of 52,000 feet, and a sonic boom perceived level (PL) [1, 2] of less than 75 dB during cruise, the accurate assessment of uncertainties becomes paramount during X-59's acoustic validation phase. This work presents an exploration of both epistemic and aleatory uncertainties inherent to the mission, outlining various sources and their potential effects on noise metrics. The methodologies that will be used for uncertainty assessment during the acoustic validation phase are detailed.

The computation of aircraft sonic boom transmission to the ground is initiated from a pressure waveform in the near-field, located three to five body lengths below the aircraft. This waveform must sufficiently resolve the three-dimensional effects around the aircraft body. This near-field signature is then propagated through the atmosphere using a quasi-1D augmented Burgers' equation to simulate waveform propagation through the atmosphere while taking energy losses into consideration. Figure 1 provides a schematic representation of the sonic boom computation and propagation process.

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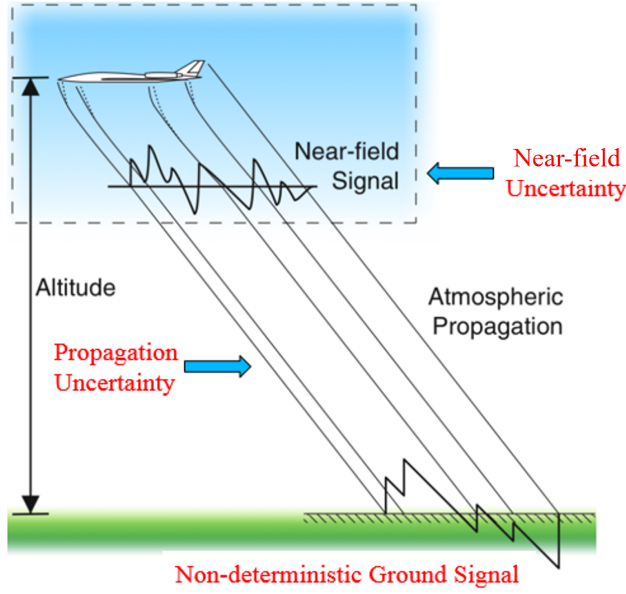


Fig. 1 Propagation of a pressure signature to the ground. [3, 4]

Although rigorous techniques and simulations have been applied to study the X-59's mission, uncertainties persist, especially concerning sonic boom loudness during the acoustic validation phase. This work addresses uncertainties related to the near-field, influenced by piloted trajectory uncertainties, and ground signal, driven by the atmosphere uncertainties. The methodology outlined in this work assesses potential uncertainties that may arise during the X-59's validation flights.

Previous research has been dedicated to the investigation of uncertainty in the prediction of low-boom aircraft signatures for implementation in certification procedures. West et al. [5, 6] have demonstrated efficient methods for certification prediction of low-boom aircraft. That work used a non-intrusive polynomial chaos approach in conjunction with novel dispersion methods outlined by Pinier [7] to place uncertainty bounds on both the near-field pressure signatures and overall perceived levels on the ground. The methodology employed by West et al. is applicable to both experimentally and numerically generated signatures. Phillips and West [8] also studied the impact of structural and aeroelastic uncertainties, which was an expansion on previous work by Nikbay et al. [9].

Building upon previous research focused on low-boom aircraft signature prediction for certification procedures, this work employed an uncertainty quantification (UQ) approach for the X-59 Quesst mission. The uncertainty method included in this work builds localized surrogate models on the equivalent area in the near-field, then transforms this equivalent area into a pressure signature by differentiating it along the longitudinal axis. Details on this methodology are provided in Section II. Section III delves into sonic boom uncertainty results for the X-59 Quesst mission during the acoustic validation phase, showcasing the developed tools and methods, and Section IV outlines the conclusions and future work.

II. Methodology

This section is divided into two subsections: the first focuses on near-field pressure signature generation, and the second introduces the uncertainty modeling methodology.

A. Near-field Pressure Signature Generation

This work used the Community Low-boom Exposure, Operations, Piloting, And TRim Analyzer (CLEOPATRA) program that is currently under development by NASA's Commercial Supersonic Technology (CST) project. CLEOPATRA is being developed as a waypoint planning and post-flight analysis tool for the community response phase of

the Quesst mission*. For mission waypoint planning, CLEOPATRA employs Bayesian optimization to determine the Mach number and altitude the aircraft must fly (at a given cruise weight) to yield a desired sonic boom loudness level. The starting coordinates of the vehicle’s trajectory are determined by comparing the locations of the predicted sonic boom carpet and the desired location, then appropriately translating the trajectory so that they overlap as desired by the mission planner. Two separate Gaussian process models are used by the optimizer to estimate the aircraft’s trim condition and the near-field pressure cylinder that is representative of the selected trim conditions. CLEOPATRA is written in Python and employs the Gaussian process functions found in the Scikit-learn machine learning package [10]. The kernel used in this work was defined as the sum of the Matern 3/2 and white noise kernels [11].

The computational fluid dynamic (CFD) database for CLEOPATRA comes from the Launch Ascent and Vehicle Aerodynamics (LAVA) CFD solver [12] for the as-designed outer mold line of the X-59 vehicle. LAVA is a second-order finite-volume solver, which can use both structured and unstructured meshes, designed for complex aerodynamic flows. For the database, LAVA best practices were employed for mesh generation and analysis setup, which were derived from the Sonic Boom Prediction Workshops hosted by the AIAA [13]. The refactored curvilinear solver and the Spalart–Allmaras (SA) turbulence model were used. A structured overset mesh with 340 million nodes and a Koren limiter were employed in the solver [14]. Convergence was assessed by monitoring the L2 norm of the flow equation residuals, and a fixed number of iterations were run, ensuring at least a four order of magnitude reduction in residuals.

To construct the database implemented in CLEOPATRA, a Latin hypercube sampling (LHS) design was created for Mach, weight, and altitude conditions. The variable bounds are presented in Table 1. These bounds were determined by using data from the X-59 simulator, currently located at NASA Armstrong Flight Research Center. The simulator was used to replicate the aircraft’s cruise nominal trajectory for pass 1 in the validation phase of the Quesst mission. While flying the simulator, pilots introduced fluctuations in cruise Mach number, altitude, and cruise weight due to minor adjustments in control settings during the climb phase. Subsequently, LAVA CFD mesh and boundary conditions are defined using a Gaussian process surrogate model based off Lockheed Martin’s 6-DOF trim controller to establish trimmed aircraft conditions, including angle of attack, control surface deflections, and power settings.

Table 1 CLEOPATRA database bounds and epistemic uncertainties

Parameter	Bounds
Mach	[1.38, 1.42]
Weight, lb	[20,705.83, 22,682.29]
Altitude, ft	[50,039.37, 52,959.18]

B. Uncertainty Quantification

Uncertainty quantification of computational models involves the examination of the impact of uncertainties in input parameters and modeling simplifications on the outcomes of a process or simulation. The scope of UQ can vary, ranging from accounting for uncertainty in a single model, multiple models, or experimental data. The objective of uncertainty quantification is to enhance the robustness of the design process by identifying uncertainty drivers and mitigating the potential impact of uncertainties through informed, targeted resource investments.

A crucial aspect of UQ is the proper characterization and treatment of simulation input uncertainties [3, 15], which can be categorized into two types: aleatory and epistemic. Epistemic uncertainty is due to a lack of knowledge or incomplete information, whereas aleatory uncertainty refers to uncertainty arising from inherent variability in a system. An example of aleatory uncertainty is the atmospheric variability over an entire month. An example of epistemic uncertainty in the analysis of sonic boom signatures is piloted trajectory uncertainty. To accommodate mixed uncertainties, this work employs a second-order probability analysis approach developed by Eldred and Swiler [16] to obtain 95% uncertainty intervals from the probability boxes (p-boxes) for the loudness metrics, as depicted in Fig. 2. For the p-boxes, the spread in the min and max of the 95% intervals is due to epistemic uncertainty, and the distribution of the points on each curve is due to the aleatory samples, as depicted in Fig. 3. To handle the computational burden of this type of approach, a non-intrusive polynomial chaos method is used as an efficient and accurate means of propagating uncertainty through the computational model.

*The community response phase of the Quesst mission consists of flying the X-59 aircraft in various locations throughout the U.S. to gain an understanding of how the sonic boom loudness affects various populations.

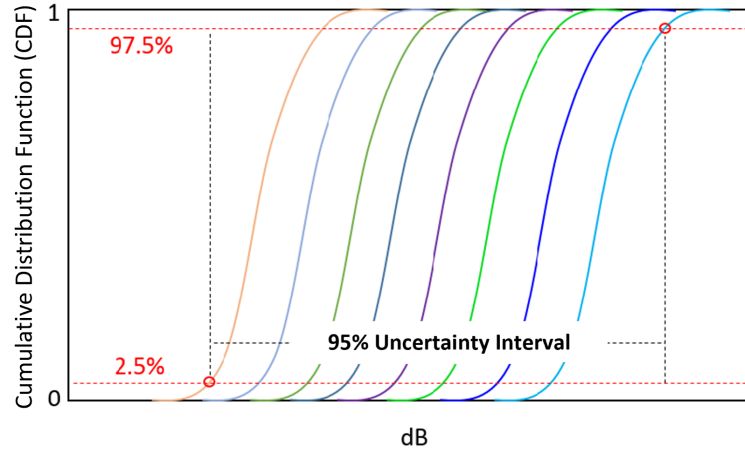


Fig. 2 Example of a p-box plot from a second-order probability approach used to obtain the 95% uncertainty interval. [17]

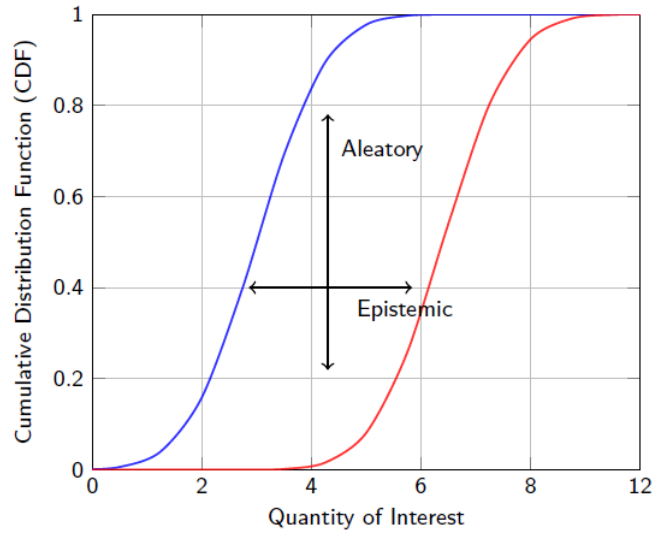


Fig. 3 Pictorial of how the epistemic and aleatory samples relate to p-boxes.

1. Uncertainty Sources

As previously discussed, this work addressed two types of uncertainties: epistemic and aleatory. Epistemic uncertainties encompassed factors originating from the piloted trajectory such as Mach number, cruise weight of the X-59 aircraft at the start of the cruise, and cruise altitude. On the other hand, aleatory uncertainties arose from atmospheric conditions. Both types of uncertainties could have an impact on the loudness metric. This study analyzed the sensitivity of the loudness metric to these uncertainties and established final loudness bounds based on the input uncertainties.

The atmospheric uncertainty was propagated through NASA's far-field propagation code, PCBoom version 7.2.0 [18]. Given that the validation phase for the X-59's Quesst mission will occur at Edwards Air Force Base, the atmospheric uncertainty was assessed using modeled atmospheric data near Edwards Air Force Base. Data from a weather model were taken from the month of August over two decades to simulate a potential flight during the Quesst mission's validation phase. The atmospheric modeled data were sourced from the European Centre for Medium-Range Weather

Forecasts (ECMWF) reanalysis data (ERA5) [19]. Figure 4 shows the samples gathered from the weather model for each quantity of interest.

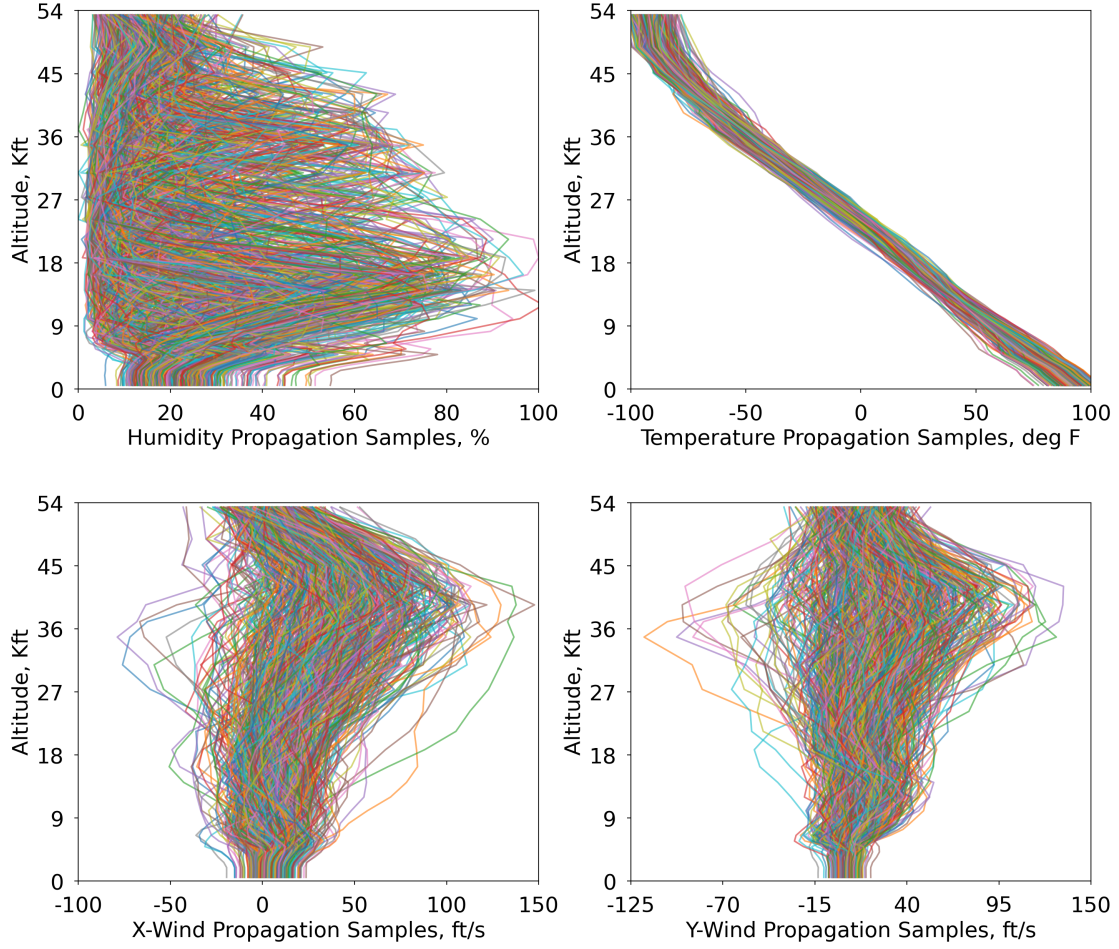


Fig. 4 Twenty years of ERA5 atmospheric model data for the month of August over Edwards Air Force Base.

From the historical atmospheric weather model data, a multivariate kernel density estimation probability distribution was formulated across 27 altitude levels. A multivariate distribution was chosen because it can capture the dependency of each atmospheric variable on altitude, rather than treating each altitude level as a separate, independent distribution. Given that the multivariate probability distribution has an infinite range, precautions were taken to exclude any samples where humidity dropped below 0% or exceeded 100%. Additionally, measures were implemented to ensure that samples drawn from the multivariate distributions fell within the minimum and maximum bounds of the ERA5 sample data at each altitude level. The 2600 samples used for propagation from the multivariate probability distributions can be seen in Fig. 5. Observing Fig. 5, the average of the samples closely aligns with the ERA5 weather model data averages at each altitude level. Table 2 shows the maximum average difference between the sample propagation data and the ERA5 weather model data across the 27 altitudes. Table 2 reveals the largest difference occurs in humidity, with a 6% difference. When propagating this difference through PCBoom, the 6% difference translates to a max of 0.67 dB greater loudness (specifically in PL, dB at -30 degree off-track angle) in the propagation samples average than the original samples average. Future work, will include investigating a better way to align the humidity propagation samples with the ERA5 samples. Generally, the propagation samples demonstrate alignment with the ERA5 data.

Table 2 The maximum value of the differences between the average of the atmospheric propagation samples and ERA5 weather model data across the 27 altitudes, i.e. $\max(\text{samples avg}-\text{ERA5 data avg})$

Parameter	Max Avg Difference
Humidity, %	5.99
Temperature, deg F	0.05
X-Wind, ft/s	0.63
Y-Wind, ft/s	1.89

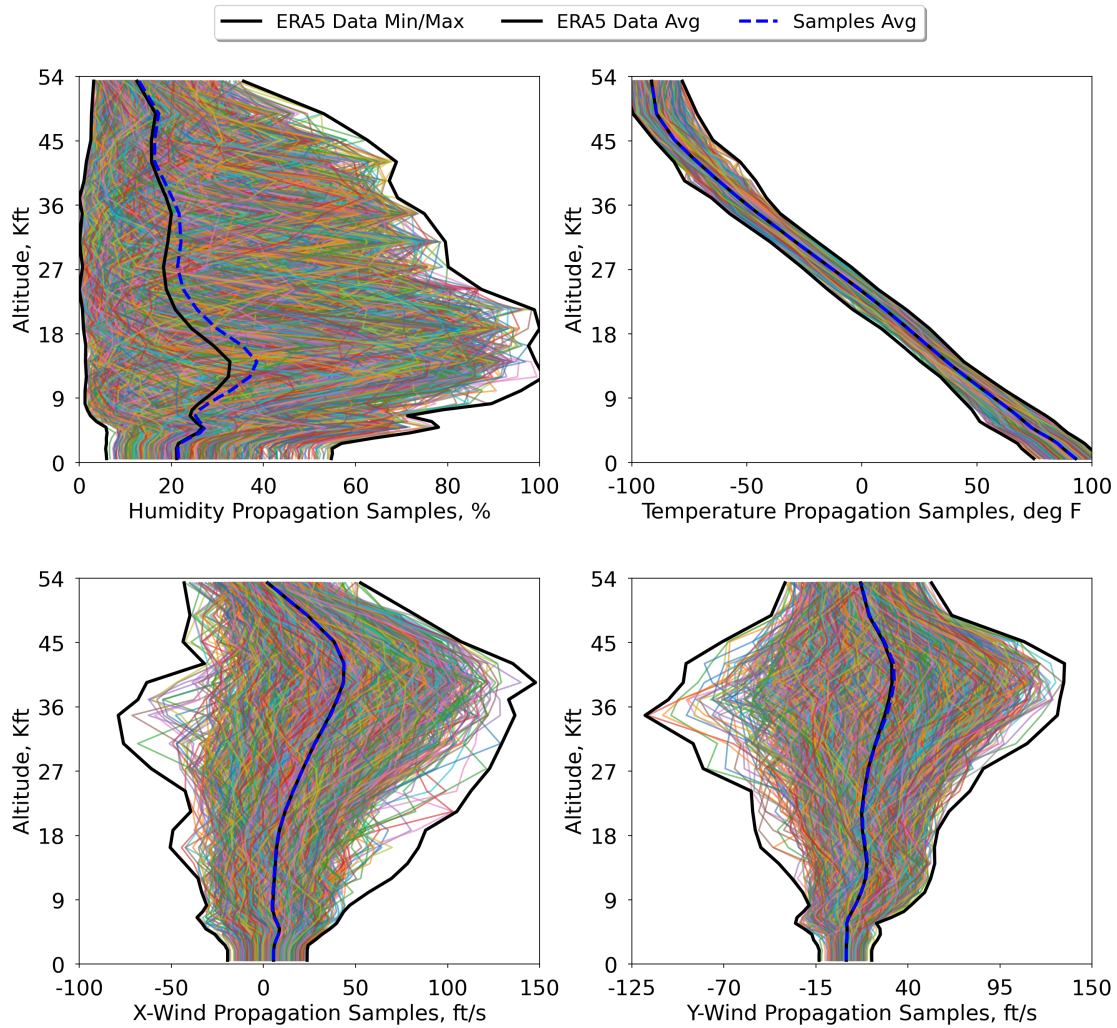


Fig. 5 The 2600 aleatory samples drawn from the multivariate distribution and used for propagation. The black outer lines represent the min and max bounds for each altitude on the ERA5 weather model data that was used to create the multivariate probability distributions.

Because the CLEOPATRA database bounds were established by using data from the X-59 simulator, and reflect piloted trajectory uncertainty, the UQ bounds are set to be equal to the CLEOPATRA database. These uncertainties fall into the epistemic category because with additional simulated flights, their bounds could be quantified more accurately. The bounds for the piloted trajectory uncertainties can be found in Table 1.

2. Non-intrusive Polynomial Chaos

Polynomial chaos expansion (PCE) is a way of representing a quantity of interest as a function of random variables using a polynomial expansion [20]. The polynomial expansion gives a spectral representation of the uncertainty that can represent the response value as a linear combination of orthogonal stochastic or basis functions, Ψ_i , and suitable coefficients, a_i , as seen in Eq. (1). Further details on polynomial chaos theory can be found throughout the literature [21–24].

$$Y(\mathbf{D}, \xi^m) = \sum_{i=0}^{\infty} a_i(\mathbf{D}) \Psi_i(\xi^m) \quad (1)$$

Here, $\mathbf{D}$ represents the deterministic variables, and ξ^m consists of m independent, standard normal variables (also known as the uncertainty variables). The basis functions of each random variable are represented by an orthogonal polynomial of order q that is determined using the Askey scheme [20] and is dependent on the distribution of each variable. For practical applications, Eq. (1) must be truncated to a limited number of basis functions, N_t , which leads to Eq. (2).

$$Y(\mathbf{D}, \xi^m) \approx \sum_{i=0}^{N_t-1} a_i(\mathbf{D}) \Psi_i(\xi^m) \quad (2)$$

To form a complete basis, the number of terms, N_t , can be determined by Eq. (3).

$$N_t = \frac{(m+q)!}{m!q!} \quad (3)$$

To obtain a PCE model of the form of Eq. (2), the coefficients a_i must be computed. Here, a point-collocation non-intrusive polynomial chaos (NIPC) method was used due to the ease of implementation with complex models. This method has also been used extensively in aerospace applications for improved computational efficiency over other spectral-based approaches [25–30]. The point-collocation NIPC method starts by computing coefficients using a least squares problem. Given N_t samples of the random variable ξ_k^m for $k = 1, \dots, N_t$, the corresponding basis functions, $\Psi_i(\xi_k^m)$, and the stochastic process, $Y(\mathbf{D}, \xi_k^m)$, given by Eq. (2), was used to find a suitable linear system as shown in Eq. (4).

$$\begin{pmatrix} Y(\mathbf{D}, \xi_0^m) \\ \vdots \\ Y(\mathbf{D}, \xi_{N_t}^m) \end{pmatrix} = \begin{pmatrix} \Psi_0(\xi_0^m) & \cdots & \Psi_{N_t}(\xi_0^m) \\ \vdots & \ddots & \vdots \\ \Psi_0(\xi_{N_t}^m) & \cdots & \Psi_{N_t}(\xi_{N_t}^m) \end{pmatrix} \begin{pmatrix} a_0 \\ \vdots \\ a_{N_t} \end{pmatrix} \quad (4)$$

For this system, N_t is the minimum number of required samples. If there are more samples, the system in Eq. (4) is over-determined and can be solved as a least squares problem [31]. The number of samples required beyond the minimum is known as the over sampling ratio (OSR). The number of collocation points is determined by multiplying Eq. (3) by the OSR.

3. Quasi-Monte Carlo Variance Decomposition

Given that this work employs a Monte Carlo simulation for propagation using PCBoom to obtain the final loudness metrics, the Sobol sensitivity index method [32] was used to assess the sensitivity of these metrics to the uncertain input parameters. The Sobol method, a variance-based approach for global sensitivity analysis, quantifies the influence of input parameters on overall output variation. This method was chosen due to its compatibility with Monte Carlo techniques and its capability to provide a quantifiable measurement of the impact of input parameters on output metrics.

The total-effect index, S_{T_j} , given in Eq. (5), measures the contribution of the output variance of parameter X_j including all the variance caused by interactions with other parameters.

$$S_{T_j} = \frac{\mathbb{E}_{\sim j}[\mathbb{V}_j[\mathbf{Y}|\mathbf{X}_{\sim j}]]}{\mathbb{V}[\mathbf{Y}]} \quad (5)$$

In this equation, $\mathbb{E}_{\sim j}[\mathbb{V}_j[\mathbf{Y}|\mathbf{X}_{\sim j}]]$ represents the expected value of the remaining variation in the output $\mathbf{Y}$ when all parameters are fixed except $\mathbf{X}_j$. For this work, a quasi-Monte Carlo method is applied, following the approach outlined by Saltelli et al. [33]. This approach replaces random sequences with low-discrepancy sequences, allowing the sensitivities to be determined using less samples than full Monte Carlo.

4. Uncertainty Methodology

The simulation to obtain the ground loudness metrics involves two phases. In the first phase, a CFD solver is used to compute the near-field pressure signature, which is located three body lengths below the aircraft. The second phase entails solving an augmented Burgers' equation to propagate the near-field signature through the atmosphere, resulting in the ground loudness metrics and a far-field waveform.

The near-field signature represents the pressure disturbance resulting from the complex system which causes shock interactions around a supersonic aircraft. Consequently, the near-field pressure signature exhibits various sharp changes in pressure caused by shock waves emanating from the aircraft's body. When employing the PCE surrogate for analysis, modeling these sharp pressure changes across different Mach numbers, weights, and altitudes poses a significant challenge. This difficulty arises because a PCE model struggles to fit the discontinuities that result from these abrupt pressure variations.

Previous research [5, 6, 9, 17] has used a Fourier dispersion method developed by Pinier [7]. This method generates near-field signatures within a range of lower and upper bounds. Although it is efficient and produces realistic near-field signatures, one limitation that this method brings is that it does not establish a direct correlation between the generated signature and specific Mach, weight, and altitude values. Since Mach and altitude are used for far-field propagation, having a way to associate a near-field signature with these values can impact uncertainty statistics.

In contrast to the Fourier dispersion method, this work employed the construction of PCE models of the equivalent area to connect the Mach, weight, and altitude for each near-field prediction. The equivalent area (A_e) distribution is chosen to build the PCE models on because it is the integration of the pressure, which creates a smooth monotonic function. This characteristic facilitates PCE models with better fits. Changing from A_e to near-field pressure has been shown to have negligible difference for the boom loudness metrics [34]. After the models are constructed and an A_e curve has been predicted for a given set of flight conditions, the A_e can then be converted back to a near-field pressure signature. For details on how the conversion between equivalent area and pressure along the near-field is constructed, refer to Rallabhandi [34].

The process of generating near-field pressure signatures from the PCE surrogate model is depicted in Fig. 6. Step 1 uses the uncertainties as inputs into the simulation to generate $N_t = 21$ near-field pressure signatures. The number of training samples was determined by employing Eq. (3) with an oversampling ratio (OSR) of two [29]. Step 2 converts those near-field signatures into 21 different A_e curves. In Step 3, $n = 2888$ unique second-order PCE models are built on A_e based on the 21 samples at each of the 2888 x locations. The number of PCE models was determined by the resolution of the CFD mesh grid. A second-order PCE model was verified to be sufficient for the smooth monotonic A_e distribution. Step 4 uses the uncertainty variables (Mach, weight, and altitude) to determine N_{MC} Monte Carlo samples, which are used as inputs into the PCE models to generate A_e . In Step 5, differentiation is used to convert the PCE-generated A_e distributions into near-field pressure signatures. The result yields $N_{MC} = 10.4$ million near-field pressure signatures to estimate the uncertainty intervals. These Monte Carlo samples along with the atmospheric uncertainty are propagated through the atmosphere using the PCBoom propagation code to obtain loudness p-boxes from the cumulative distribution functions (CDF) curves, as seen in step 6.

The loudness metrics used in this work included perceived level (PL), the A, B, D, and E-weighted sound exposure levels (ASEL, BSEL, DSEL, ESEL), and the indoor sonic boom annoyance predictor (ISBAP) [35]. To account for all uncertainties, a Latin Hypercube Sampling (LHS) design of Mach, weight, and altitude samples was created, and random samples were drawn from the atmospheric multivariate probability distributions. Using randomly drawn samples over an LHS design for the multivariate distributions is necessary due to the way the multivariate distribution is constructed with altitude dependency. For LHS to be applicable, accurate spacing between all atmospheric variables at each altitude must be maintained while preserving the altitude dependency for each atmospheric variable, a method not yet studied in the literature.

To ensure an adequate number of random samples from the multivariate probability distributions, convergence was assessed by tracking the 95% bounds for the aleatory variables (i.e., atmospheric variables) until minimal changes were observed, as shown in Fig. 7. Similarly, convergence for the LHS sampling (epistemic samples) was deemed sufficient by ensuring minimal changes in the 95% bounds, as seen in Fig. 8. Note that in Fig. 8, there was a very small change in the upper bound of BSEL and ESEL, but the change is less than 0.02 dB for BSEL and 0.04 dB for ESEL after 3500 samples. This convergence resulted in 2600 aleatory samples and 4000 epistemic samples, creating $N_{MC} = 10.4$ million PCE-generated Monte Carlo near-field pressure signatures. The resulting near-field signatures from the converted PCE models and the atmospheric data were propagated through the atmosphere via PCBoom to obtain the loudness metrics.

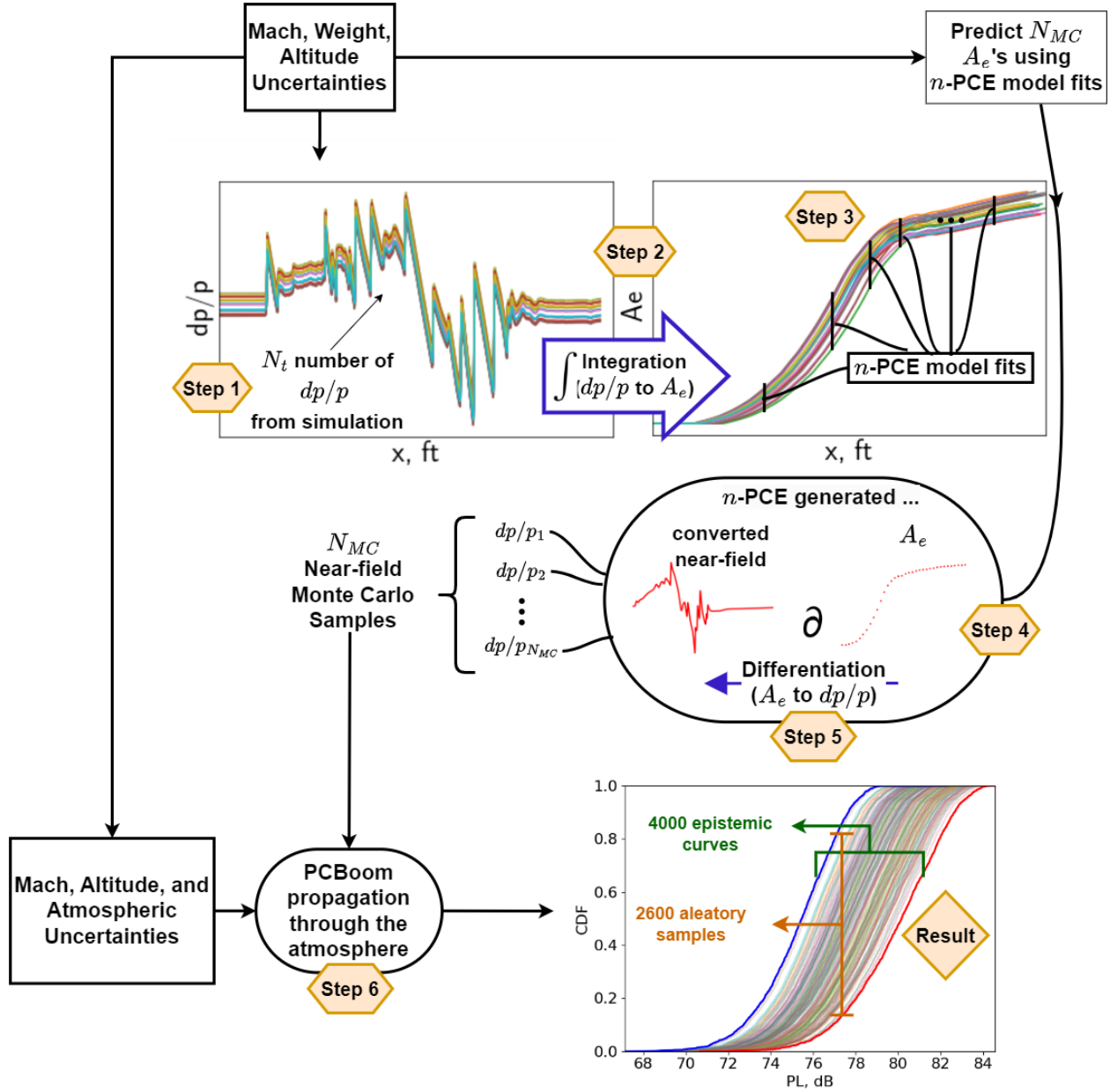


Fig. 6 Process for obtaining the p-boxes for this work.

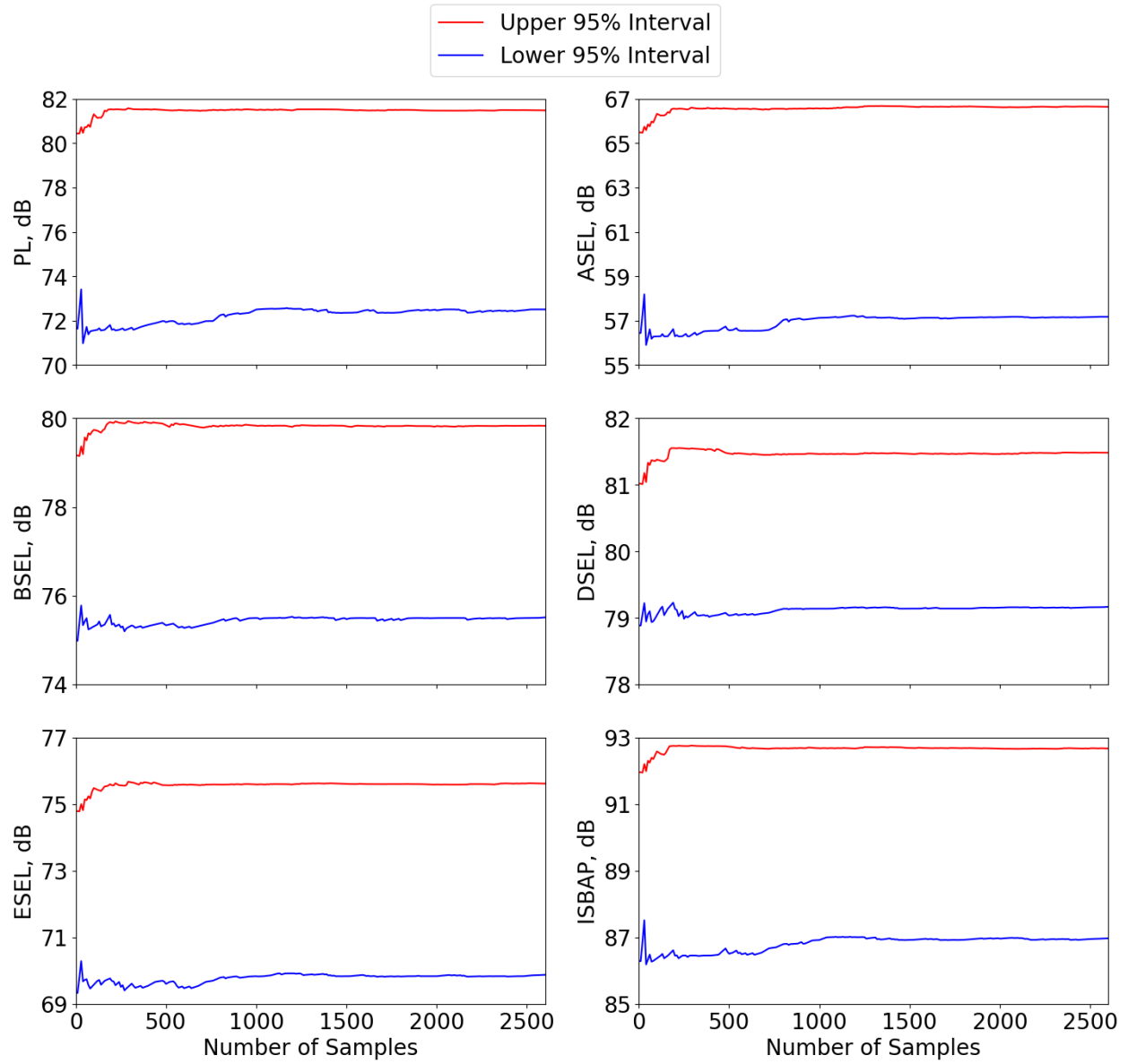


Fig. 7 Aleatory convergence of the 95% uncertainty intervals for the loudness metrics for the undertrack.

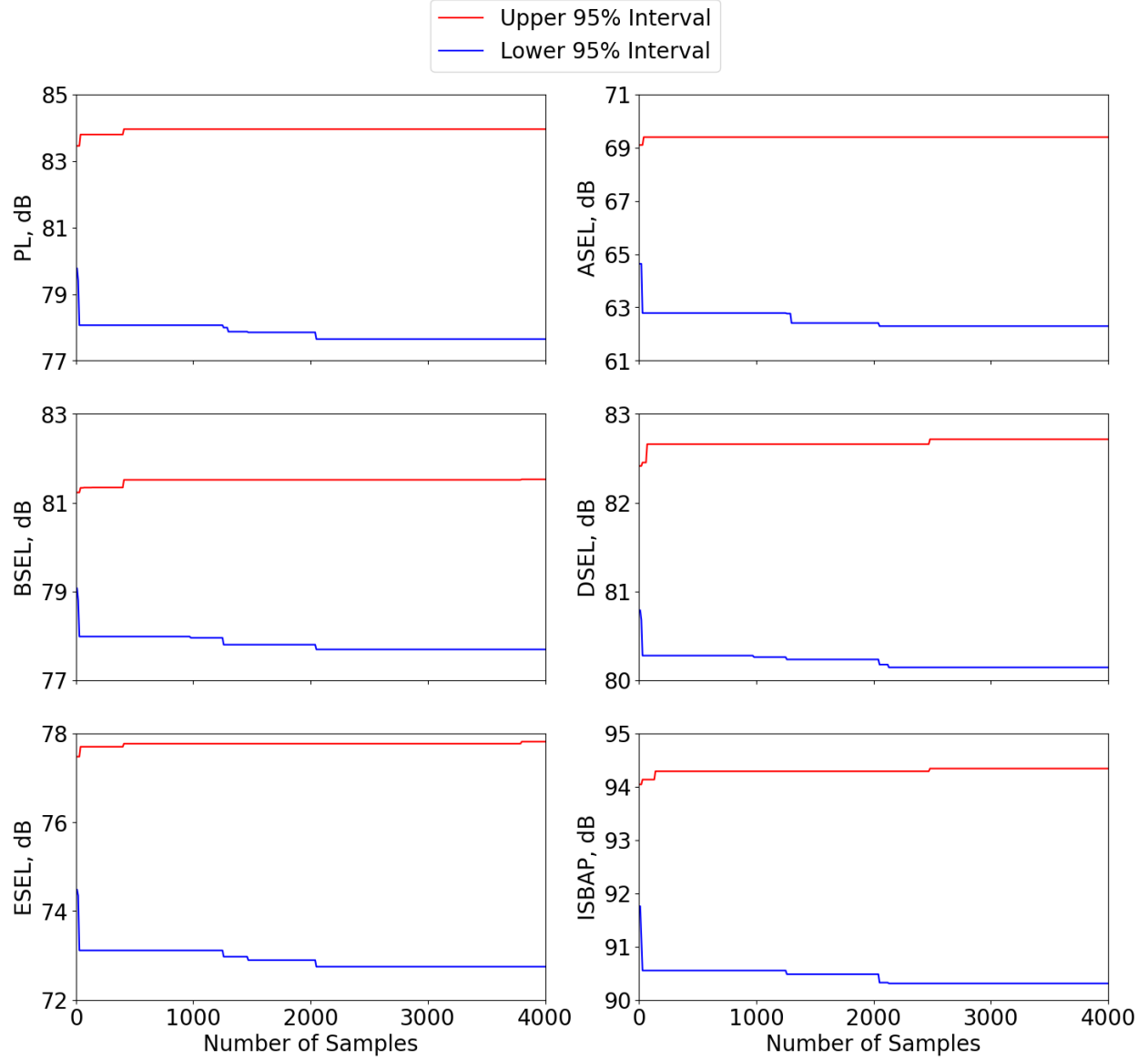


Fig. 8 Epistemic convergence of the 95% uncertainty intervals for the loudness metrics for the undertrack.

5. PCE Model Verification

For the method described in Section II.B.4, 67 verification cases were used to assess the model's fit. Residuals between the PCE-predicted model and the CLEOPATRA LAVA CFD database were calculated using Eq. (6). The full carpet average residuals and the maximum residuals are presented in Table 3. The mean residual error does not exceed 0.38 dB for any metric, indicating that the PCE model provides a good fit for the near-field signature. For the maximum residual error case (undertrack), the PCE-predicted A_e and near-field signature are compared with CLEOPATRA LAVA CFD data in Figs. 9 and 10, respectively. The predicted near-field signature closely matches the LAVA case, except around 300 and 350 feet. It is worth noting that the loudness metrics are particularly sensitive to overpredictions of shock peaks (caused by model fit error), which explains the largest error observed in the verification case. Future work will include incorporating the PCE model error into the overall uncertainty framework.

$$\text{residual} = |\text{PCE-Predicted} - \text{CLEOPATRA LAVA CFD}| \quad (6)$$

Table 3 The min, max, and average residuals of the 67 verification cases for each loudness metric across the full carpet.

Metric, dB	Residual Type	Off-track Angle, deg												
		-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30
PL	Average	0.298	0.298	0.348	0.329	0.379	0.344	0.329	0.340	0.368	0.340	0.346	0.305	0.308
	Min	0.000	0.015	0.018	0.001	0.026	0.002	0.003	0.011	0.000	0.015	0.001	0.008	0.008
	Max	1.058	0.922	1.205	1.066	1.137	1.185	1.253	1.366	1.179	1.059	1.171	0.964	1.028
ASEL	Average	0.279	0.293	0.368	0.359	0.402	0.355	0.355	0.355	0.397	0.368	0.364	0.302	0.290
	Min	0.001	0.009	0.003	0.008	0.003	0.017	0.006	0.008	0.004	0.012	0.004	0.012	0.027
	Max	0.989	0.955	1.318	1.098	1.211	1.266	1.320	1.323	1.278	1.077	1.258	0.856	0.938
BSEL	Average	0.214	0.191	0.205	0.206	0.229	0.225	0.213	0.220	0.227	0.213	0.204	0.179	0.204
	Min	0.003	0.000	0.002	0.006	0.005	0.009	0.003	0.005	0.003	0.010	0.004	0.001	0.005
	Max	0.622	0.586	0.646	0.627	0.637	0.695	0.747	0.784	0.673	0.621	0.631	0.554	0.655
DSEL	Average	0.129	0.123	0.132	0.131	0.140	0.141	0.137	0.138	0.139	0.132	0.131	0.116	0.126
	Min	0.001	0.005	0.003	0.004	0.005	0.002	0.004	0.000	0.002	0.003	0.000	0.000	0.001
	Max	0.376	0.347	0.415	0.391	0.406	0.435	0.458	0.452	0.408	0.394	0.397	0.344	0.394
ESEL	Average	0.222	0.206	0.237	0.238	0.270	0.246	0.233	0.241	0.260	0.246	0.234	0.205	0.216
	Min	0.007	0.000	0.006	0.000	0.012	0.002	0.001	0.000	0.001	0.012	0.005	0.004	0.001
	Max	0.679	0.620	0.822	0.754	0.784	0.813	0.911	0.930	0.841	0.738	0.797	0.598	0.721
ISBAP	Average	0.232	0.221	0.244	0.235	0.274	0.270	0.269	0.266	0.277	0.244	0.241	0.221	0.233
	Min	0.009	0.007	0.002	0.000	0.009	0.001	0.011	0.007	0.001	0.007	0.007	0.003	0.001
	Max	0.776	0.713	0.873	0.775	0.914	0.881	0.883	0.996	0.926	0.739	0.837	0.772	0.786

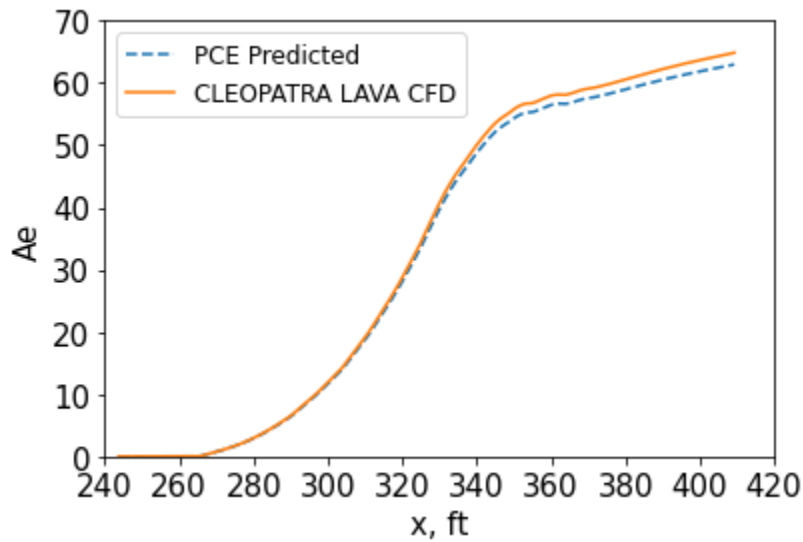


Fig. 9 A_e comparison between the PCE-predicted model and CLEOPATRA LAVA CFD data for maximum residual error verification case for the undertrack.

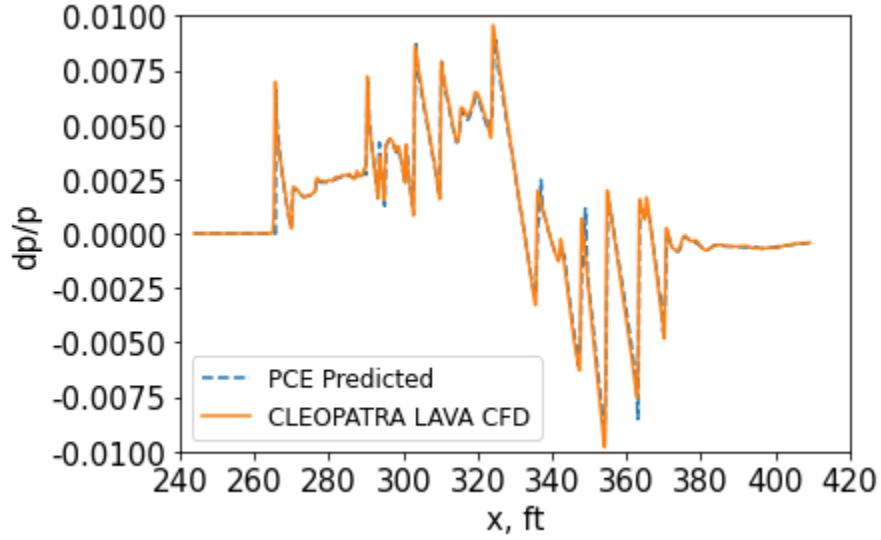


Fig. 10 Near-field pressure comparison between the PCE-predicted model and CLEOPATRA LAVA CFD data for maximum residual error verification case for the undertrack.

III. Results

This section presents the preliminary results for the X-59's undertrack loudness and full carpet metrics, including uncertainty intervals and sensitivities. These results are subject to change as the predictions are based on the as-designed outer mold line, which may have insufficient mesh resolutions for the LAVA CFD. Future updates to the predictions will be made when the as-built outer mold line becomes available and data from the acoustic validation phase of the Quesst mission is gathered.

A. Sensitivities

The Sobol sensitivities for each metric are illustrated in Fig. 11. Recall that for the second-order probability approach, the spread in the uncertainty intervals is due to the epistemic uncertainty, whereas the shape of the CDFs inside the p-boxes is due to aleatory uncertainty, as illustrated in Fig. 3. Humidity emerges as the primary contributor to the 95% intervals, driven by its aleatory nature, shaping the CDFs within the probability boxes. Weight, the second largest contributor, is epistemic, indicating its role in widening the spread of the 95% intervals. Altitude, contributing almost as much as weight, may also influence the spread. Gaining more understanding of cruise weight and altitude could potentially narrow the spread in the intervals.

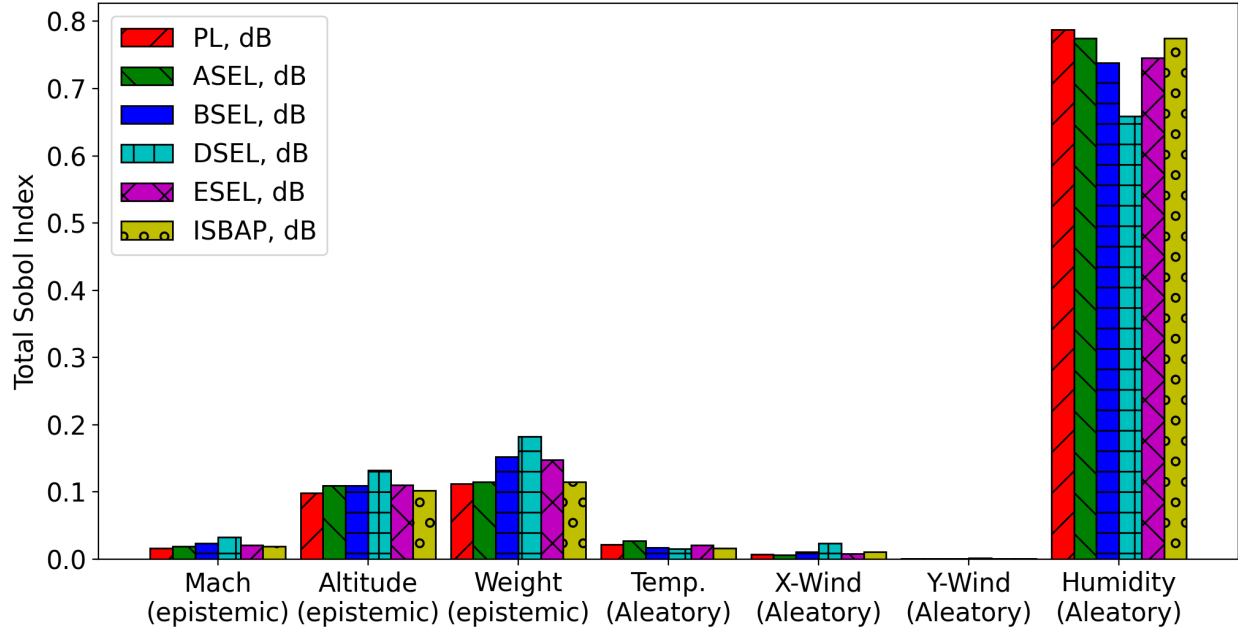


Fig. 11 Total Sobol index for each loudness metric on the undertrack.

B. Undertrack Loudness Metrics

To assess the final uncertainty in loudness metrics for the X-59's cruise mission, 10.4 million propagation samples were run through PCBoom. For propagation, a sampling frequency of 50 kHz with a step size of 600 ft along the ray was used. These settings were sufficient to achieve the desired accuracy and convergence behavior while balancing computational run times. The final probability boxes for the undertrack can be seen in Fig. 12. The ground altitude was set to the Edwards Air Force Base altitude of 2303 ft with an assumed heading of 78 deg and longitude and latitude of -118 deg and 35 deg, respectively. The 95% uncertainty intervals for the undertrack of each loudness metric are provided in Table 4.

Table 4 Undertrack 95% uncertainty intervals for each loudness metric

Loudness Metric	95% Uncertainty Interval
PL, dB	[70.39, 83.90]
ASEL, dB	[55.06, 69.33]
BSEL, dB	[74.34, 81.86]
DSEL, dB	[78.18, 83.06]
ESEL, dB	[68.51, 77.84]
ISBAP, dB	[85.60, 94.69]

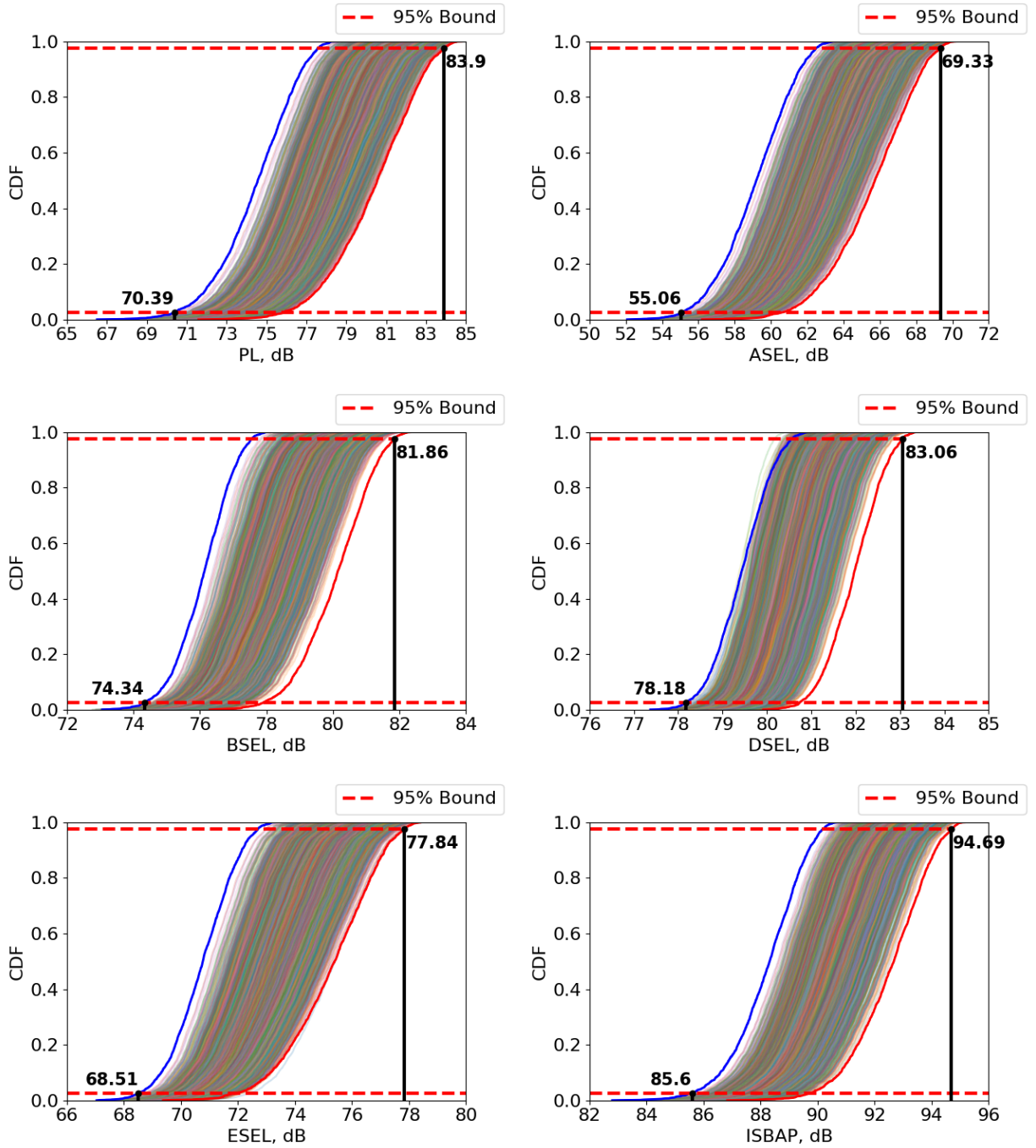


Fig. 12 Undertrack probability boxes for each loudness metric. The horizontal dashed lines represent the 95% uncertainty intervals and the black vertical lines are the decibels values associated with those probability levels.

In terms of decibel spread, the undertrack ranges from 4.88 dB to 14.27 dB within the 95% uncertainty intervals. The largest spread for the undertrack is observed in ASEL, at 14.27 dB, with PL following closely at 13.51 dB. Traditionally, ASEL and PL converge less reliably than the other metrics, likely contributing to the spread. Conversely, the smallest spread is observed in the DSEL metric, at 4.88 dB, as DSEL typically converges more reliably. Future investigations will explore how weight correlates with these metrics.

The given methodology showed that for PL, there is at least a 1.00% and up to a 58.66% probability of reaching a perceived level of less than 75 dB for the undertrack at Edwards Air Force Base on an arbitrary day in the month

of August, as shown in Fig. 13. The large spread in the chances of obtaining the 75 dB perceived level is due to the epistemic uncertainty, which is driven by the cruise weight of the aircraft, as depicted in Fig. 11. Due to the nature of epistemic uncertainty, as more knowledge is gained regarding the cruise weight of the aircraft, the spread in the probability bounds at 75 PLdB will lessen. However, the upper bound will not exceed a 58.66% probability level. The precise probability bound end points (1.00% and 58.66%) are due to the aleatory uncertainty whose largest contributor is the humidity, as seen in Fig. 11. Given that humidity controls the absorption rate of sound and that it has substantial variations throughout the altitude, as depicted in Fig. 4, it contributes more to the uncertainty than other factors. Weight is the second-highest contributing factor, which can significantly affect noise.

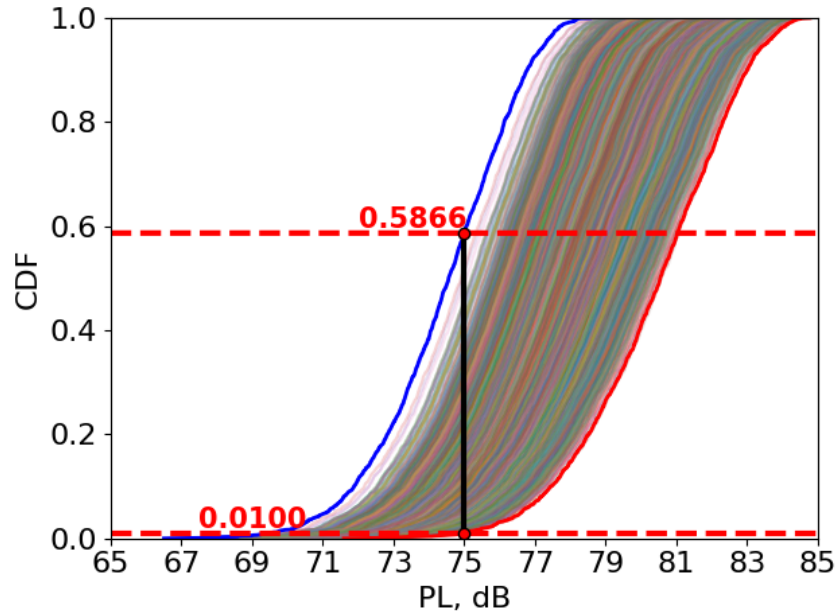


Fig. 13 Probability levels for reaching a PL of 75 dB for the undertrack on an arbitrary day in August.

C. Full Carpet Loudness Metrics

For the full carpet, the 95% uncertainty intervals can be seen in the violin plots of Fig. 14 for each metric. The figure has the upper uncertainty CDFs shaded red and the lower uncertainty CDF shaded in blue. The means for each CDF curve can be seen in the black horizontal line. Notice that the sound metrics are the loudest at the undertrack and then lessen towards the off-track.

Observing Fig. 14, the minimum change in the upper uncertainty interval across the full carpet is 2.27 dB, occurring in DSEL, whereas the maximum change is 3.59 dB, occurring in ASEL. The minimum change in the lower uncertainty interval is 2.72 dB in DSEL, with a maximum change of 6.34 dB in PL. Therefore, the boom loudness shows significant dissipation—up to 3.59 dB for the upper uncertainty interval and up to 6.34 dB for the lower uncertainty interval—as it expands across the full carpet. Similar to the undertrack, the uncertainty interval spread from upper to lower intervals changes from 4.85 dB to 16.89 dB across the entire carpet. The smallest spread occurs in the DSEL metric at an off-track angle of 15 degrees, and the largest spread occurs in PL at an off-track angle of 30 degrees. DSEL typically converges better than other loudness metrics, so it is expected to show the least fluctuations across the carpet width, whereas PL and ASEL usually converge less reliably and are subject to more fluctuations. The large spread and specific bounds of the uncertainty are primarily due to humidity and cruise weight, respectively, which are the most significant contributors to the uncertainty estimates.

It is important to note that this work uses data based on the as-designed outer mold line of the X-59 aircraft, rather than the as-built aircraft. As new data become available, the presented methodology will be re-applied, likely leading to changes in the results. Additionally, this work does not account for atmospheric turbulence effects, which tend to reduce mean loudness levels. Furthermore, grid error from the LAVA CFD database, a potential significant factor for

uncertainty in computational models, is not incorporated in this analysis. Finally, this work indicates that the humidity propagation samples need further investigation to ensure alignment with the historical weather data.

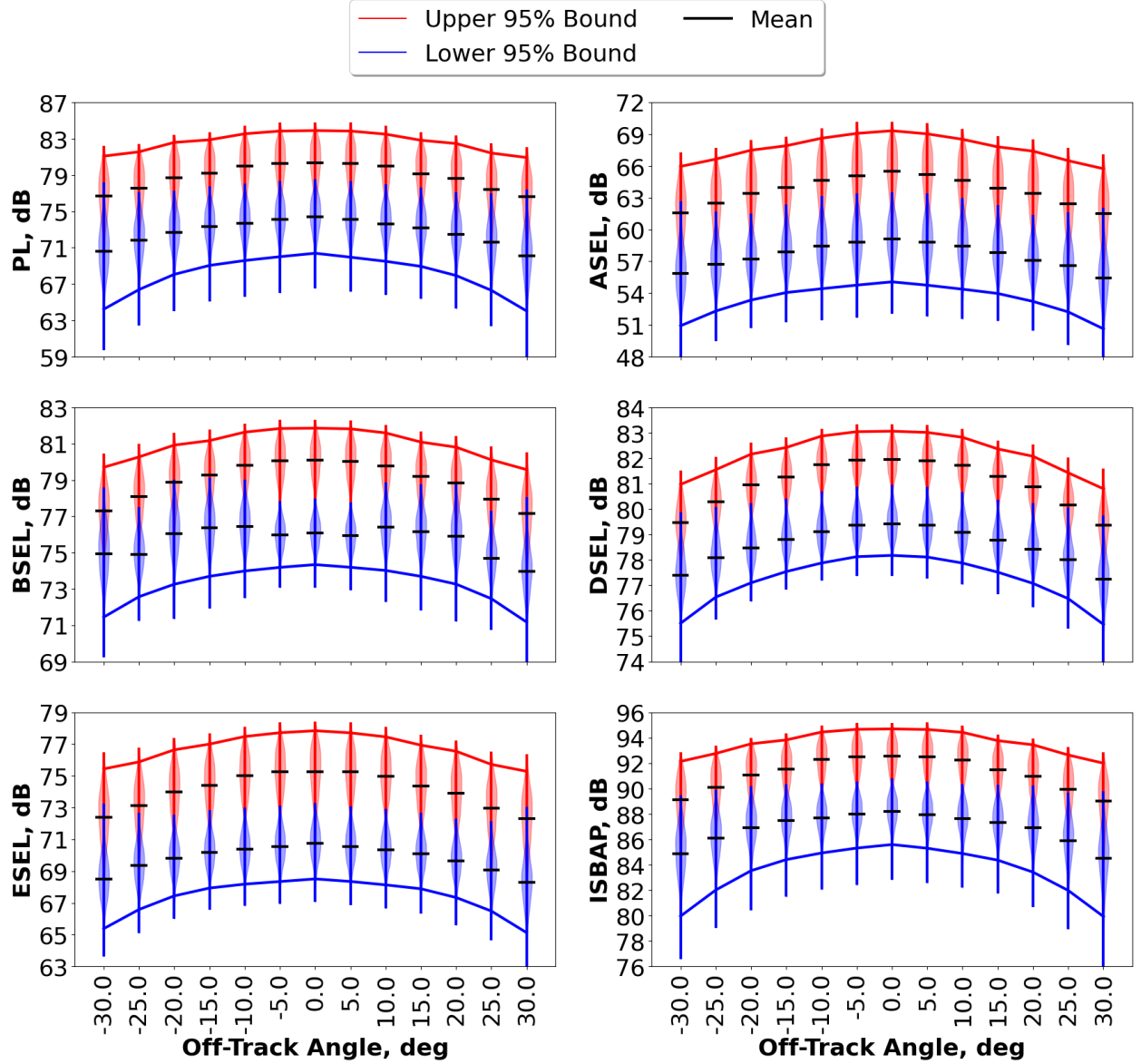


Fig. 14 Violin plots of the full carpet upper (red) and lower (blue) uncertainty CDF bounds for each metric. The black horizontal lines represent the mean of the uncertainty intervals CDFs. The red line represents the upper 95% bound and the blue line represents the lower 95% bound.

IV. Conclusions

In conclusion, this paper has shown a UQ methodology which gathered preliminary results for the uncertainty associated with the X-59 Quesst mission during the acoustic validation phase. Input uncertainties for the atmosphere were derived using atmospheric weather models based on historical data for Edwards Air Force Base, while piloted trajectory uncertainties were obtained from the X-59 simulator. These uncertainties were propagated through derived PCE models for the near-field and NASA's PCBoom propagation code, which established final uncertainty bounds for the sonic boom loudness on an arbitrary day in August for a fixed heading. This approach provided a comprehensive

understanding of the potential variability in sonic boom loudness, highlighting the significant impact of atmospheric conditions, particularly humidity, on the uncertainty.

This work used preliminary data to describe the chances of having PL of 75 dB loudness for the undertrack of a low-boom supersonic design for an arbitrary day in August, given the current CLEOPATRA LAVA CFD database. Uncertainty bounds for six loudness metrics were established using the CLEOPATRA LAVA CFD database, revealing variability ranging from 4.85 dB to 16.89 dB across the full carpet.

The limitations of this work include the absence of accounting for surrogate model error in the final analysis and the reliance solely on the as-designed geometry rather than the as-built geometry. Another limitation includes experiencing a higher PL due to the way the humidity samples are determined from the multivariate probability distribution. Future endeavors will involve updating uncertainty bounds as the CLEOPATRA database evolves, developing a robust method to incorporate the PCE surrogate model into the final probability bounds, and ensuring better representation of the humidity propagation samples with the historical weather data. Furthermore, consideration will be given to atmospheric turbulence and its potential impact on current uncertainty estimates.

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