Noise Reduction Trajectory Analysis of a Supersonic Business Jet using Novel Optimization Tools

Jeshurun P. Horton*, Gregory R. Busch†

NASA Glenn Research Center, Cleveland, OH, 44135, United States

Proposals to reduce airport noise during takeoff and landing for supersonic aircraft using methods such as variable noise reduction systems add complexity to conceptual flight models. To better optimize these takeoff and landing profiles for noise certification, new modeling methods are explored in this paper with the future goal of more effectively determining the sensitivities of multidisciplinary design variables on airport noise. Takeoff and landing trajectories are modeled for a notional supersonic business jet concept developed by NASA for use in environmental impact studies conducted by the International Civil Aviation Organization. New tools capable of gradient-based optimal control and collocation solving methods are examined and verified against existing methods used in previous studies of the airplane concept. The benefits and limitations of these new methods are discussed. It is found that the new and existing methods match closely for a standard takeoff and landing trajectory, with a cumulative effective perceived noise difference of 0.1 EPNdB. When modeling a variable noise reduction system, the new modeling methods offer alternate, optimized solutions, with noise levels of two trajectories reduced by 0.8 EPNdB and 1.3 EPNdB.

I. Introduction

A critical step to achieving sustainable supersonic commercial aviation is the establishment of new standards and reference procedures that will provide regulatory certainty to manufacturers and other aviation stakeholders. This is part of an ongoing effort by the Committee on Aviation Environmental Protection (CAEP), a technical committee of the International Civil Aviation Organization (ICAO). From 2019-2022, CAEP conducted an exploratory study on the global environmental impact of civil Supersonic Transport (SST) aircraft. This was accomplished in part through the study of a Supersonic Technology Concept Airplane (STCA) model developed by NASA. STCAs are notional research vehicles studied with the intent of providing information to ICAO on the environmental impact of supersonic aviation. The STCA concept investigated by NASA makes use of a Variable Noise Reduction System (VNRS) to help mitigate airport noise. A VNRS, as proposed, would consist of a set of control equipment onboard an airplane that automatically engages procedures to reduce noise in the vicinity of an airport. These systems, recognized previously for use on rotorcraft, would reduce crew workload by controlling the engines and control surfaces automatically. VNRS systems have been proposed as a means to safely reduce noise of future supersonic airplanes to help meet regulatory limits. This innovation was endorsed by the Federal Aviation Administration (FAA) with the intent of revising noise standards for supersonic airplane provisions, leading to a notice of proposed rulemaking in 2020 [1] describing the acceptable use of VNRS systems. The VNRS method is supported by ICAO based on results of the exploratory study and is endorsed in their public proceedings [2]. While it is anticipated that VNRS systems will be used to reduce noise for SSTs, the details of the procedure are not rigorously outlined and will require careful modeling

* Aerospace Engineer, Propulsion Systems Analysis Branch, jeshurun.p.horton@nasa.gov, AIAA Member

† Aerospace Engineer, Acoustics Branch, gregory.r.busch@nasa.gov, AIAA Member

considerations from the designer. In this paper a low-fidelity example of such a model is presented, based on previous work on the STCA vehicle investigated by NASA.

The STCA examined in this paper is a notional eight-passenger supersonic business jet with a takeoff gross weight of 121,000 lb (55,000 kg), designed to cruise at Mach 1.4 over transatlantic distances. It is equipped with three conceptual engines derived from an "off-the-shelf" subsonic turbofan discussed in the next section. Based on relatively near-term technology, it is intended to represent early-market private jet entrants proposed by industry. All components of the airframe and engines are based on information in the public domain, making it ideal for public studies. Previous work by Berton et al. [3] identified the airport noise, emissions, and mission performance of the STCA for the CAEP exploratory study. The STCA has since been used for multiple studies, conducted by NASA [3-7] as well as externally [8-19]. Berton's work included the optimization of an advanced VNRS procedure based on several takeoff design variables [7]. Work by Voet et al. [16] identified an optimal noise trajectory for the STCA, determining design sensitivities and using automatic continuous thrust control schedules. An overview of the STCA design and requirements is discussed in Section II.

While optimizing the takeoff trajectory for a fixed airframe and propulsion design is useful for studying the effects of a VNRS, the ability to consider the impact of other discipline design variables for airport noise certification is important. This prompts the need for novel multidisciplinary modeling methods to predict sensitivities for a tightly-coupled aircraft model. The purpose of this study is twofold: to examine novel techniques for optimizing the STCA takeoff trajectory and estimating noise, and to verify these techniques against existing methods. This is done with the hope of providing NASA and industry leaders with new tools to proactively design supersonic aircraft for airport noise certification.

The previous VNRS study by Berton [7] used the Flight Optimization System (FLOPS) [20] for weight analysis and Landing and Take-Off (LTO) performance, paired with the Aircraft Noise Prediction Program (ANOPP) [21] to estimate the noise certification measurements of the generated trajectories. These tools have been used by NASA for decades to evaluate aircraft model performance. They can be linked together through an administrative command program to evaluate multidisciplinary design problems, such as optimizing the takeoff trajectory for noise certification. However, complex optimization problems can be challenging to implement using these tools because they do not perform automatic differentiation at the present time. Techniques such as finite difference approximation are often used with these tools, or a design space exploration is used in place of optimization for more manageable design spaces. While these are perfectly appropriate methods for many design problems, an increasing push towards tightly-coupled models with many design variables prompts new optimization methods to be explored that can evaluate analytic gradients.

In the place of FLOPS, as in the study by Berton [7], the Aviary tool is adopted for this study. Aviary is an open-source, Python-based aircraft design framework in development by NASA with the goal of streamlining multidisciplinary design and optimization of aircraft systems [22, 23]. Aviary recreates the methods used in FLOPS to model transport-category aircraft [24] and methods from the General Aviation Synthesis Program [25] for normal-category aircraft [26]. For non-conventional aircraft designs, user-built external model components can be linked to the core Aviary components, making Aviary compatible with nearly any type of aircraft design problem. Aviary implements gradient-based optimization techniques, which can enable analysis of larger-scale design spaces and optimal control problems. Verifications of the tool against existing data have been conducted for single-aisle transonic airplanes [23] and are conducted against previous FLOPS analyses as part of the present study. Aviary interfaces with OpenMDAO, an open-source Python environment designed for multidisciplinary optimization [27]. OpenMDAO allows systems in separate disciplines to be analyzed and optimized collectively, making it well suited for multidisciplinary aircraft design problems. Aviary also incorporates Dymos [28] components to enhance trajectory modeling capability. Dymos is an open-source optimal control library for OpenMDAO designed for trajectory optimization problems. A recent study by Voet et al. [16] has created a VNRS optimal control model for the STCA using OpenMDAO and Dymos. This study aims to apply similar modeling methods to that study, but examine the advantages and drawbacks of Aviary as an open-source, Python-compatible option in place of the FLOPS system. An in-depth discussion of Aviary, its development, and its applications can be found in Ref. [23], and demonstration aircraft studies using Aviary can be found in Refs. [29, 30]. A comparison of FLOPS and Aviary takeoff modeling methods is discussed in Section III.

To easily link the flight trajectories generated by Aviary with noise prediction, the open-source Python Noise Assessment (pyNA) tool developed by Voet, et al. [17, 19] was selected in place of ANOPP. PyNA is a tool to assess the noise footprint of aircraft and is largely based on the ANOPP theoretical manual [21]. Voet previously used pyNA to conduct a noise sensitivity analysis with respect to design variables for the STCA engine [17] and takeoff trajectory [16, 19]. This study incorporates pyNA in a similar way to estimate airport noise, although engine and airframe design sensitivities are not yet considered. A comparison of the ANOPP and pyNA noise estimation tools is discussed in

Section IV. Comparisons between the newly proposed toolset and existing toolset for different takeoff trajectories are discussed in Section V, with conclusions drawn in Section VI.

This paper contains several statements on design needs for supersonic airplanes and the impact of noise reduction procedures. NASA does not wish to presume or impose any standards and/or best practices for the regulation of supersonic aircraft. Instead, these remarks should be viewed as impartial observations on the STCA model represented in this study. Indeed, the optimal implementation of VNRS systems for noise reduction along a takeoff trajectory will be highly dependent on the particular aircraft design and configuration.

II. Airplane Design

A. Airframe

Trajectory noise analysis of the STCA first requires the assembly of a comprehensive vehicle model. The STCA model examined in this study is a collaboration between NASA Glenn and Langley Research Centers. The airframe and corresponding aerodynamics tables were designed and computed by NASA Langley Research Center. The design uses three engines, one mounted over the aft portion of each wing using short pylons connected to the fuselage, and one integrated with the vertical stabilizer. The Open Vehicle Sketch Pad software [31] was used to define airplane geometry and compute internal volume. Aerodynamics analysis codes based on modified linear theory [32] were used to compute drag polars and flap performance. The codes were organized together to design the airframe using a multidisciplinary frameworking tool [33]. FLOPS methods were used originally to estimate weights [34], leading to a fixed weight of 121 klb. Profile and planform views of the concept airplane are shown in Fig. 1. Additional information on the airframe design can be found in Ref. [3].

While NASA has ongoing interest in low-boom supersonic technology [35], this concept does not contain any features to reduce sonic boom levels. It is intended to represent a near-term market entrant for which such technologies may not be implemented. Thus, supersonic flight over land would be restricted to permitted areas for this design.

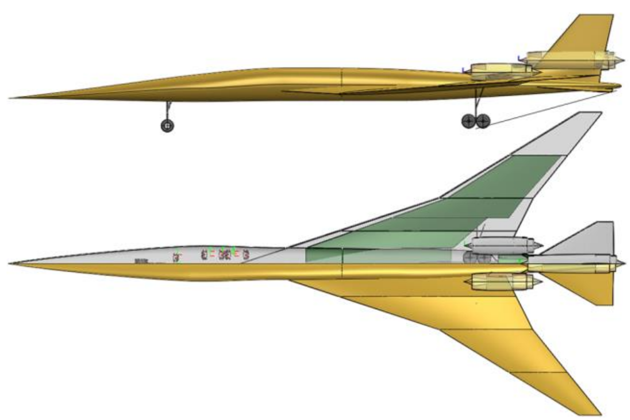

Fig. 1 Profile (above) and planform (below) views of the STCA airframe [3].

B. Propulsion

The propulsion system was designed by NASA Glenn Research Center, using the Numerical Propulsion System Simulation (NPSS) code [36]. For the near-term timeframe of this vehicle, it is deemed unlikely that a completely new engine could be developed and marketed for supersonic use due to economic viability. It is more likely that an off-the-shelf turbofan would be repurposed for supersonic applications by redesigning the low-pressure spool for higher pressures. Such a nominal supersonic engine is derived from a model of the CFM56-7B27 subsonic turbofan engine from CFM International. The engine model is adapted from work by the FAA Environmental Design Space Initiative [37, 38]. The fan is redesigned with lower bypass and higher pressure for supersonic cruise. The low-pressure compressor is discarded to prevent overheating effects due to supersonic ram effects. The bypass and core streams pass through a lobed mixer to improve thrust efficiency. A plug nozzle is used to accelerate the mixed flow to supersonic speeds. The turbomachinery comparison between the CFM56 model and the derived supersonic engine is shown in Fig. 2 (the inlet and nozzle sections are not shown). More information on the derived engine can be found in Ref. [3].

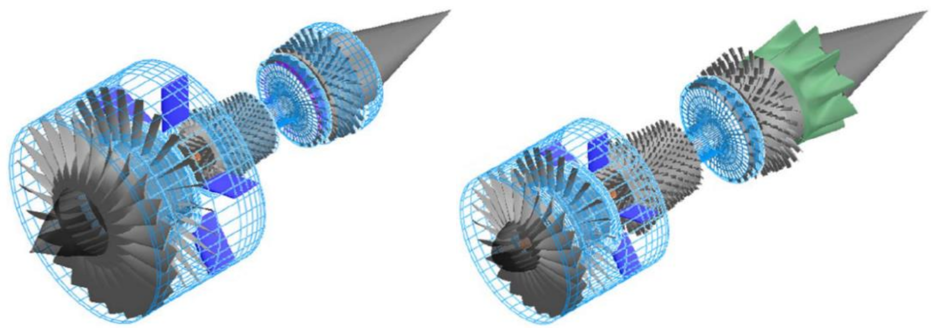

Fig. 2 CFM56 engine model (left) and derived supersonic engine model for the STCA (right) [3].

C. Mission

Based on the known aerodynamics and engine performance, a mission analysis of the airplane concept was performed in a previous study [3]. The design mission is at maximum takeoff weight, with a single transoceanic cruise segment at Mach 1.4. The resulting design range (for maximum supersonic cruise) was found to be 4243 nautical miles. As a prerequisite to the study in Ref. [6], an airworthiness takeoff study was performed to determine the takeoff safety speed (V_2), flap deflections, and required field distance required to meet Code of Federal Regulations (CFR) Part 25 requirements. In addition to a minimum field length takeoff profile, a delayed rotation takeoff profile with higher ground speed was assessed. Where possible, a delayed rotation takeoff may be preferred for supersonic airplanes to reach better takeoff climb rates and less required thrust during second segment climb, leading to lower airport noise. A more detailed analysis of the design mission and airworthiness study can be found in Ref. [3].

The noise certification takeoff trajectory modeled in this study consists of a delayed rotation takeoff at maximum weight. While the ICAO Annex 16 on Environmental Protection [39] describes a maximum-weight takeoff for noise certification, this is not ideal for field testing since multiple landings and refuelings would be required. The ICAO Environmental Technical Manual [40] describes equivalent procedures for noise certification, which permit an aircraft being certified to intercept the flight path described by Annex 16, thus meeting the required speed, altitude, and climb criteria before noise measurements are taken. The Environmental Technical Manual also permits measurements to be taken individually for different noise criteria. In this study, a singular takeoff trajectory is modeled as described in Annex 16.

III. Trajectory Model

A. Requirements

In initial studies conducted by NASA [3], the STCA used just one unoptimized VNRS procedure called a programmed lapse rate (PLR). This procedure would automatically reduce or "lapse" engine throttle shortly after liftoff to decrease noise from the engine. In addition to a PLR procedure, a more recent study by Berton [7] examines a programmed flap retraction as part of the VNRS. The flap deflection, among other things, determines how much the engine thrust can be reduced by the pilot when performing the throttle cutback procedure prescribed in Ref. [41]. By automatically retracting the flaps to a clean configuration, the airplane benefits from an improved lift-drag ratio allowing better climb rates before the throttle cutback and less required thrust after the throttle cutback, effects which both lead to reduced noise as measured from the ground. Typically, flaps would not be permitted to retract during takeoff for noise certification, but it is anticipated this would be permissible if the flaps could be controlled by an automated VNRS system. The flap retraction schedule was previously optimized for the delayed rotation takeoff of the STCA. Taking these procedures together, it was found that the combined VNRS could significantly reduce takeoff noise levels relative to a takeoff trajectory with a PLR procedure alone [3, 7].

A conceptual model of such a system aimed at minimizing noise is constrained by takeoff and landing regulations, noise monitor locations, and flight physics of the airplane. A reference diagram of the noise monitor locations to be considered for takeoff and landing certification is shown in Fig. 3. The airplane system-level noise assessments are conducted relative to the LTO noise certification metric regulated by ICAO: the effective perceived noise level (EPNL). The cumulative EPNL is an aggregate metric of separate EPNL measurements occurring at three noise monitor locations: the approach measurement located directly behind the runway (approach EPNL), the flyover measurement located over the takeoff flight path (flyover EPNL), and the peak lateral measurement along the flight path sideline (sideline EPNL).

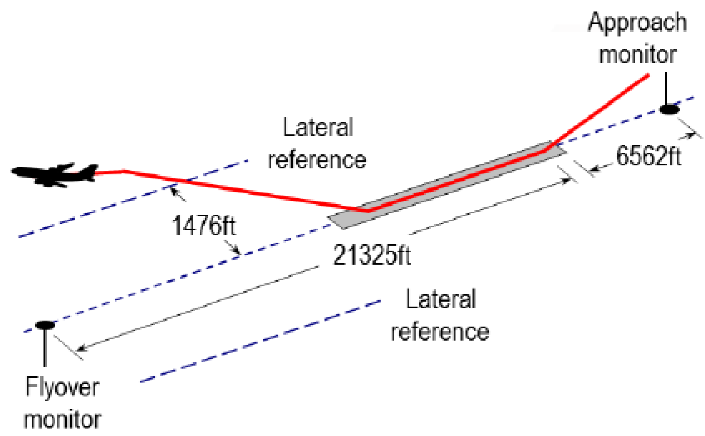

Fig. 3 Monitor arrangement for takeoff and landing noise certification [3].

The envisioned takeoff procedure is as follows: the takeoff begins as usual, with 10% derated engine thrust from brake release to rotation and liftoff. The delayed rotation occurs at a speed of approximately 195 knots, and the liftoff speed is limited to 210 knots due to estimated tire ratings. The takeoff procedure aims to take advantage of ground attenuation effects of noise for the lateral observer caused by refraction and scattering of sound from the ground. For this reason, full derated thrust is applied until the ground attenuation effects begin to diminish with increasing altitude, where the PLR procedure will begin automatically reducing throttle at a linear rate. The trajectory is constrained by one-engine-inoperative (OEI) minimum available climb gradients for transport-category trijets per 14 CFR §25.111 and §25.121 [24], which may control the limits of the PLR procedure. The airplane will continue at a reduced thrust output until a desired climbout reference airspeed is attained. Section 3.6.2(d)(1) of Annex 16 [39] states that in the case of certification, the reference airspeed should be reached "as soon as practicable," and that it must be stabilized and maintained at no greater than the airplane's takeoff safety speed plus 20 knots. However, it is proposed that this requirement could be extended to higher speeds for supersonic aircraft if an approved VNRS is used. In this model the allowable reference airspeed is extended, so long as it falls within the takeoff speed limit of 250 knots established in the United States. Shortly after the climbout reference airspeed is reached, the programmed flap retraction procedure will initiate, allowing a greater lift-to-drag benefit and higher climb rate. The pilot then initiates a manual thrust cutback before reaching the flyover noise monitor, as is common practice for current subsonic aircraft for noise certification. A labeled example diagram of the altitude, airspeed, and per-engine thrust for this procedure is shown in Fig. 4, based on results from the current model.

The design variables for optimizing the VNRS are: 1) the magnitude of the PLR thrust lapse, 2) the starting altitude of the PLR segment, 3) the PLR thrust lapse rate, 4) the increment X such that $V_2 + X$ is the stabilized climbout reference airspeed, 5) the programmed flap retraction altitude, and 6) the start of the pilot-initiated cutback. To simplify the problem, it is assumed that the optimal value for variable 5 will occur right after the stabilized airspeed is reached, as the reduced drag from a clean flap configuration offers an improved climb rate which will benefit flyover noise measurements in most cases. The flap retraction is further simplified as an instantaneous process rather than a slow retraction. In the previous study, variable 6 was determined post-process to balance the engine noise before and after the thrust cutback measured on the lateral sideline. To compare trajectories equally between the toolsets, a fixed distance of 17,000 ft is used as a reasonable approximation for this study. As such, a summary of the remaining four design variables to be optimized and their ranges is shown in Table 1. The PLR and flap retraction altitudes are defined as height above field elevation (AFE). The climbout reference airspeed is defined as calibrated airspeed in knots (KCAS). The pilot-initiated cutback distance is defined as distance from brake release (DFBR). It should be again noted that the optimal values and ranges of the design variables as shown in Table 1 are for this STCA configuration only, and may not be appropriate for all supersonic aircraft designs. Different classes and configurations of supersonic aircraft may have different LTO performance capabilities and constraints, and variables should be based on the particular design being investigated. Future airworthiness requirements for supersonic aircraft may also affect the validity of the VNRS design values used in this study.

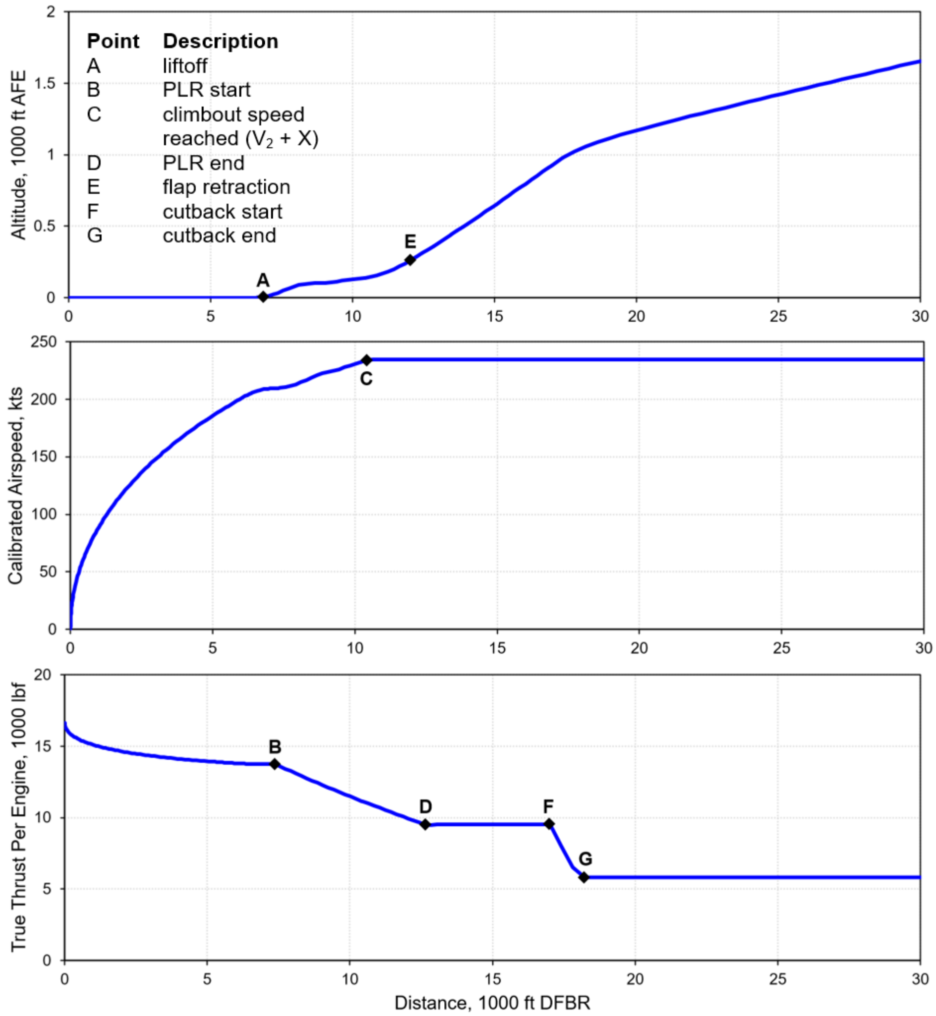

Fig. 4 Notional VNRS controls superimposed on example altitude, airspeed, and thrust plots.

Table 1 Design Variable Summary [7].

Design Variable	Minimum	Maximum
PLR thrust lapse, %	10	30
PLR start altitude, ft AFE	35	150
PLR thrust lapse rate, %/s	1.0	4.0
Climbout airspeed (increment above V2), KCAS	35	55

The objective function is the cumulative EPNL, measured in EPNdB. Because an approach VNRS is not used, the approach noise is unaffected by the design variables. A single value for approach EPNL is used when calculating the cumulative EPNL for each of the results. The focus of the work presented in this study and future follow-on studies for the STCA are concentrated on minimizing the lateral and flyover EPNL values.

B. Previous Approach

The approach used in the previous trajectory optimization study [7] applies a Design Space Exploration (DSE) method to find the set of design variables associated with the optimal trajectory. A series of trajectories are generated in FLOPS, covering a full set of permutations of the design variables with pre-set increments. Each trajectory dataset is passed to ANOPP to estimate the corresponding noise output. The set of noise outputs are then queried to find the design variables associated with the optimal case or cases.

The trajectories in FLOPS are fully solved by providing a deterministic set of control values for the points in each flight segment. After the 35 ft runway obstacle, the problem is constrained by providing either a target climb gradient or target airspeed. By constraining the flight segments by starting distance or altitude, the solver quickly reaches a single solution for the given set of design variables. This method is desirable whenever a trajectory can be "solved" for, but it is not immediately applicable to optimal control problems. Since the climb gradient of the airplane must be determined for the trajectory, the overall trajectory solution is iterated at different target climb gradients until the climbout airspeed (V_2 + X) is reached at the desired point. This effectively overcomes the need for optimal control, at the cost of running multiple iterations of the model.

The equations of motion (EOM) of the airplane are solved using an explicit shooting method. With explicit shooting, the initial state of the model is propagated point-by-point with a given time step until the constraints on the problem are satisfied. For a fixed control problem, this method is highly efficient. However, the method is dependent on the length of the time steps and can be subject to rapid changes in state values when control inputs are changed. In many cases, this method is not efficient for optimal control problems since many iterations are required to propagate states along an entire flight trajectory to determine their time-dependent values. A multiple shooting method that allows separate propagation of flight segments can increase efficiency, but this method is not available in FLOPS. Using this approach in the previous study, the minimum noise trajectory was found to have a 30% programmed throttle lapse beginning at the 35 ft obstacle with a 2% lapse rate, and a stabilized climbout speed of V_2 + 55 KCAS [7].

C. New Approach

The eventual goal of the new modeling approach is to use Aviary and pyNA to create an integrated optimal control problem that can measure sensitivities of the individual design variables to noise, but further development is needed to dynamically connect variables between the tools. As a first step in examining the utility of the new approach, individual trajectory cases are compared to verify the results of the new toolset and highlight any key differences or issues to address in future development. Individual cases are run in Aviary, and then trajectory data is passed to pyNA to estimate the corresponding noise.

Aviary treats the model as an optimal control problem and calculates the trajectory using a nonlinear optimizer, in this case the Sparse Nonlinear Optimizer (SNOPT) [42]. Instead of solving the trajectory using a given climb gradient or airspeed, the current Aviary implementation finds an altitude and Mach speed that drive residual defects of the problem to zero. This allows the optimal state outputs (e.g., climb gradient and airspeed) to be determined for each point through a single iteration of the model. Due to the mostly constrained nature of the VNRS trajectory, many of the state values must be fixed inputs which makes determining optimal control values that meet all the constraints time-consuming for the optimizer. This is especially true since large changes in the design variables make small and often redundant changes to the overall altitude or airspeed of the airplane, requiring considerable time for the optimizer to sift through local minima. While this approach is beneficial for large-scale optimal control problems, for this example it is less desirable than the previous approach in terms of computational efficiency due to the number of invalid solutions that exist. Initial guesses and bounds of the trajectory can be tightened to reduce the number of invalid solutions, but this must either be done in such a way that they scale with the design variables automatically, or that the trajectory is valid for any combination of design variables. In a future implimentation of Aviary, the climb gradient and airspeed could ideally be exposed as direct controls for the optimizer, which would greatly reduce runtime by allowing altitude and Mach speed to "fall out" from the EOM.

In place of explicit shooting, an implicit collocation method is used. In this case, a Radau pseudospectral method [43] mimics the trajectory using a series of 3rd-order polynomial segments. These polynomials can be interpolated at any point along the trajectory to determine the approximate state values. By modeling the trajectory as a continuous set of functions, gradient-based optimization techniques can be applied to determine the optimal control values. Collocation methods can help smooth out rapid state changes observed in explicit shooting methods, as well as reduce the overall number of solved points required to define a trajectory. However, more constraints must be introduced to the model to ensure polynomial segments are connected and continuous. More complexity is added to the model if higher-order polynomial segments are needed to accurately capture trajectory behavior, but in this case 3rd-order segments are sufficient.

Aviary has the benefit of having openly available source code, making it simple to develop desired features for nonconventional models such as the VNRS trajectory. The Python codebase allows Aviary to be compatible with a wide range of external optimization and modeling tools.

Additional limitations include added runtime for the Python codebase of Aviary compared with the monolithic, FORTRAN-based code of FLOPS. Also, some continuity constraints and airworthiness requirements [24] are not included in Aviary at the time of writing, requiring them to be added manually.

The first verification of a standard noise certification trajectory in Aviary is conducted to ensure the results closely match that of FLOPS. The trajectory consists of a takeoff with a 10% derated throttle, and a thrust cutback at 17000 ft. No PLR or flap retraction procedures associated with the VNRS are performed. A comparison of the FLOPS and Aviary trajectories is shown in Fig. 5. The trajectories are generally very consistent, with small differences due to modeling of the rotation segment.

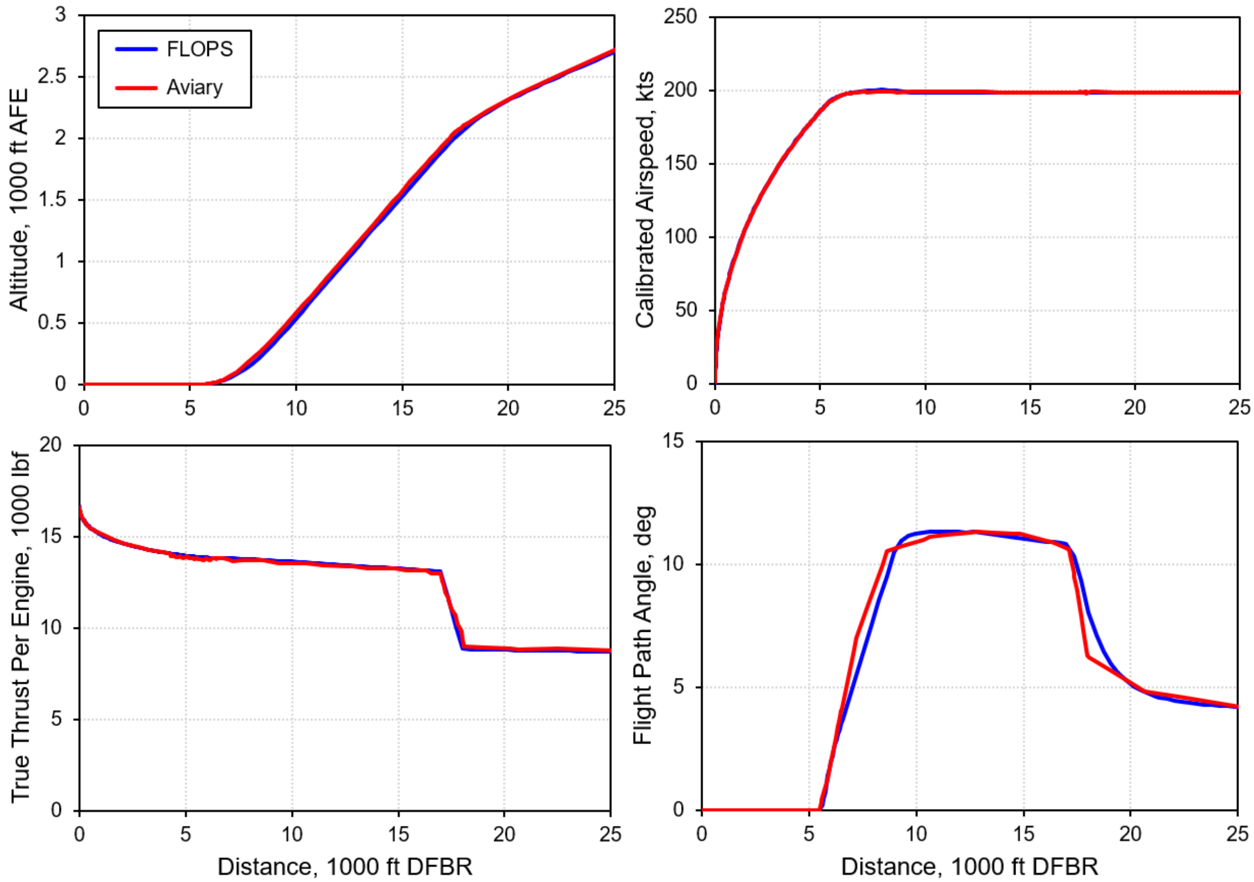

Fig. 5 FLOPS and Aviary comparison of standard noise certification trajectory.

A component-level flowchart in the style of an XDSM diagram [44] listing the components of the new modeling approach is shown in Fig. 6. Vertical lines represent component inputs, and horizontal lines represent outputs. The red octagon represents an implicit group that contains the optimization process for the takeoff trajectory.

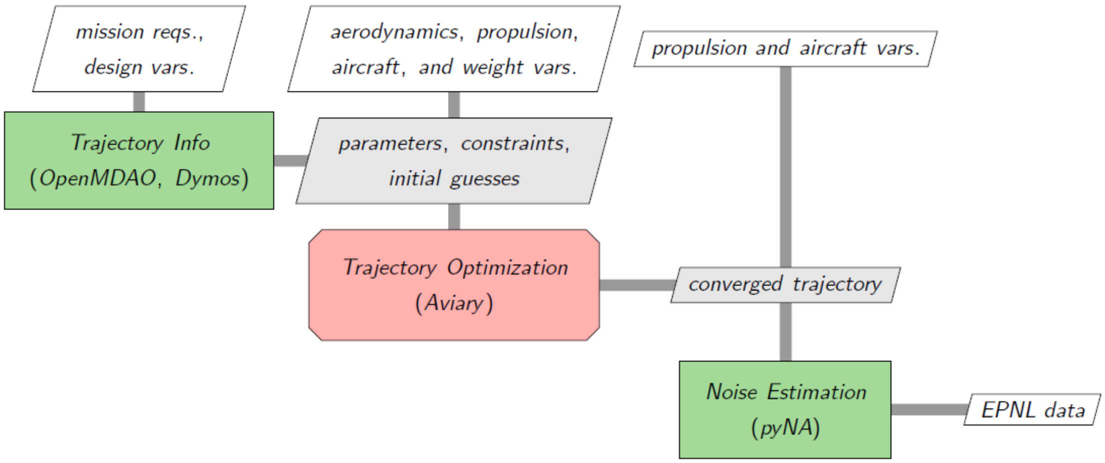

Fig. 6 Flowchart of new modeling approach for a single trajectory.

IV. Noise Assessment Model

A. Requirements

Noise prediction tools for aircraft takeoff and landing noise assessments come in a variety of fidelities and complexities, from empirical-based models to higher-fidelity computational aeroacoustics analyses. The appropriate tool to use for a particular noise assessment is primarily dependent on the aircraft concept's stage of development and computational resources available. Since the STCA is an exploratory conceptual design, a mid-fidelity, computationally inexpensive tool which requires less detailed information about the design is appropriate for use.

The airplane system-level noise assessments are conducted in terms of EPNL. Instantaneous sound pressure levels (SPLs) are determined throughout the operational trajectory and converted to perceived noise levels (PNLs) using a Noy table to account for human annoyance. The PNLs are then corrected for tonal content and differences to tone-corrected perceived noise levels (PNLTs). The maximum PNLT is determined and the PNLTs within a 10dB difference of that maximum (commonly called the 10dB down period) are then used for the EPNL calculation, accounting for the duration of the 10dB-down period [39]. The minimization of the cumulative EPNL noise metric is commonly the goal when investigating VNRS takeoff trajectories, but maintaining an acceptable or desirable margin to the individual observer requirements is also important to monitor.

B. Previous Approach

For previous noise assessments of the STCA conducted by NASA, the existing internal source-noise modules of ANOPP were used without modification aside from the fan noise source and the airframe noise source. The fan source-noise predictions include acoustic treatment suppression based on empirical methods [45] for both the inlet-radiated and discharge-radiated fan noise. The airframe noise predictions are initially calculated using ANOPP's Fink airframe noise prediction method [46] and then adjusted based on acoustic testing of the high-speed civil transport airframe (HSCT) in 1996 [47]. Another characteristic of the STCA is its over-wing mounted engines, which result in a large amount of the fan noise being shielded by the wing and body of the airplane. This was accounted for in ANOPP using its wing shielding module, which is based on methods described in Ref. [48]. Atmospheric absorption, ground reflections and lateral attenuation effects are included in the LTO noise assessments. Because of the importance of the PLR procedure in reducing the lateral observer noise, it becomes crucial to capture the lateral attenuation and ground reflection effects accurately for optimization of the VNRS trajectory. Depending on the supersonic aircraft configuration and performance capabilities, the error produced by using ANOPP's accounting methods could be significant. However, due to the dominance of the broadband jet noise and shielding of the fan tones for the STCA model, the error for lateral noise predictions due to attenuation and ground reflections was assumed to be minimal as discussed in Refs. [7, 49].

C. New Approach

Although the pyNA tool includes a native takeoff trajectory calculation and optimization framework, the goal of this work is to utilize Aviary's trajectory calculations with pyNA's noise predictions to assess the viability of integrating pyNA with Aviary in the future. The pyNA tool allows for programming in both Julia and Python languages and is built in the OpenMDAO / Dymos environment. This allows for much easier connection and integration with Aviary than other noise prediction tools and will facilitate future development of Aviary's noise prediction capabilities. PyNA's openly available code is another benefit of selecting this tool, as it can enable updates and releases faster than controlled software and can be accessed by a larger audience. Currently, not all of ANOPP's noise source modules are available in pyNA, but the critical modules relevant to the STCA noise assessment are present. PyNA will need further development in the future to be able to replicate all of ANOPP's capabilities. The ultimate goal in selecting pyNA is to eventually create an integrated optimal control problem to rapidly find the VNRS profile that results in the minimum cumulative EPNL for supersonic aircraft. It is the vision of the authors to bring low-noise objectives into the aircraft design optimization loop rather than be relegated to a post-processing step as a check against requirements. This would hopefully reduce the need for costly DSE analyses and speed up the process of determining a potential aircraft design's LTO noise metrics. The work presented in this paper is the first step toward reaching that goal.

A verification of pyNA's aircraft noise models was performed against the STCA ANOPP noise predictions and presented in Ref. [17], resulting in a maximum difference in the total EPNL levels of 0.1 EPNdB using a standard noise certification trajectory (i.e., using no VNRS). To ensure that pyNA generates similar results to ANOPP before conducting a VNRS trajectory comparison, a similar verification case is run using a standard noise certification trajectory produced from FLOPS. This comparison is similar to what is presented in Ref. [17] and isolates the noise source prediction differences between pyNA and ANOPP. The results of this effort are shown in Table 2 and show that the codes have generally very good agreement and result in a maximum difference in total EPNL of 0.1 EPNdB for each of the measurement locations.

Table 2 STCA standard takeoff trajectory noise prediction comparison.

	Lateral EPNL (EPNdB)			Flyover EPNL (EPNdB)			Approach EPNL (EPNdB)		
	ANOPP/FLOPS	pyNA/FLOPS	Delta	ANOPP/FLOPS	pyNA/FLOPS	Delta	ANOPP/FLOPS	pyNA/FLOPS	Delta
Jet	93.6	93.6	0	87.8	87.8	0	90.1	90	-0.1
Fan Inlet	50.1	49.6	-0.5	38.4	37.8	-0.6	71.1	71.3	0.2
Fan Discharge	75.8	75.8	0	71.8	71.9	0.1	91.6	91.4	-0.2
Core	76.7	76.9	0.2	73.6	73.6	0	80.4	80.3	-0.1
Airframe	66.2	66.3	0.1	64.8	64.7	-0.1	85.2	84.9	-0.3
Total	94.1	94.1	0	88.5	88.6	0.1	96.2	96.1	-0.1

V. Results

The FLOPS and Aviary trajectory models are first compared for a default set of design variables. Displayed in Fig. 7 are the takeoff trajectories for a VNRS procedure with a 10% programmed throttle lapse at a rate of 1%/s, initiated at the 35 ft obstacle. The climbout airspeed is V2 + 35 kts (212 kts).

For this case, the trajectories match relatively closely between the two models. However, there are differences in the models' determination of the climb gradient, which results in a slight difference in altitude initiated during the programmed lapse segment. The FLOPS model iterates a "target" gradient until the climbout airspeed is reached at or before the end of the programmed lapse. The Aviary model does not contain a notion of a target gradient, and rather varies the gradient as a control for the optimal rate of climb at any given point. This results in a climb gradient that increases with velocity, leading to increased overall altitude and reduced overall noise. The trajectory modeled by Aviary describes an upper limit to the solution space that assumes the flight path of the aircraft can be controlled dynamically, whether by a pilot or through extended control of the VNRS. These findings suggest that dynamic control of the airplane's climb gradient during acceleration could potentially reduce noise during certification significantly, especially at high stabilized airspeeds (V_2 + 35 or greater). Realistically, an aircraft designer may wish to constrain the solution space to represent a fixed climb gradient for more standardized noise predictions and flight procedures. Further development of the Aviary tool should allow a fixed gradient to be specified similarly to FLOPS or determined using an optimizer, but for now there is a discrepancy between the models' solutions.

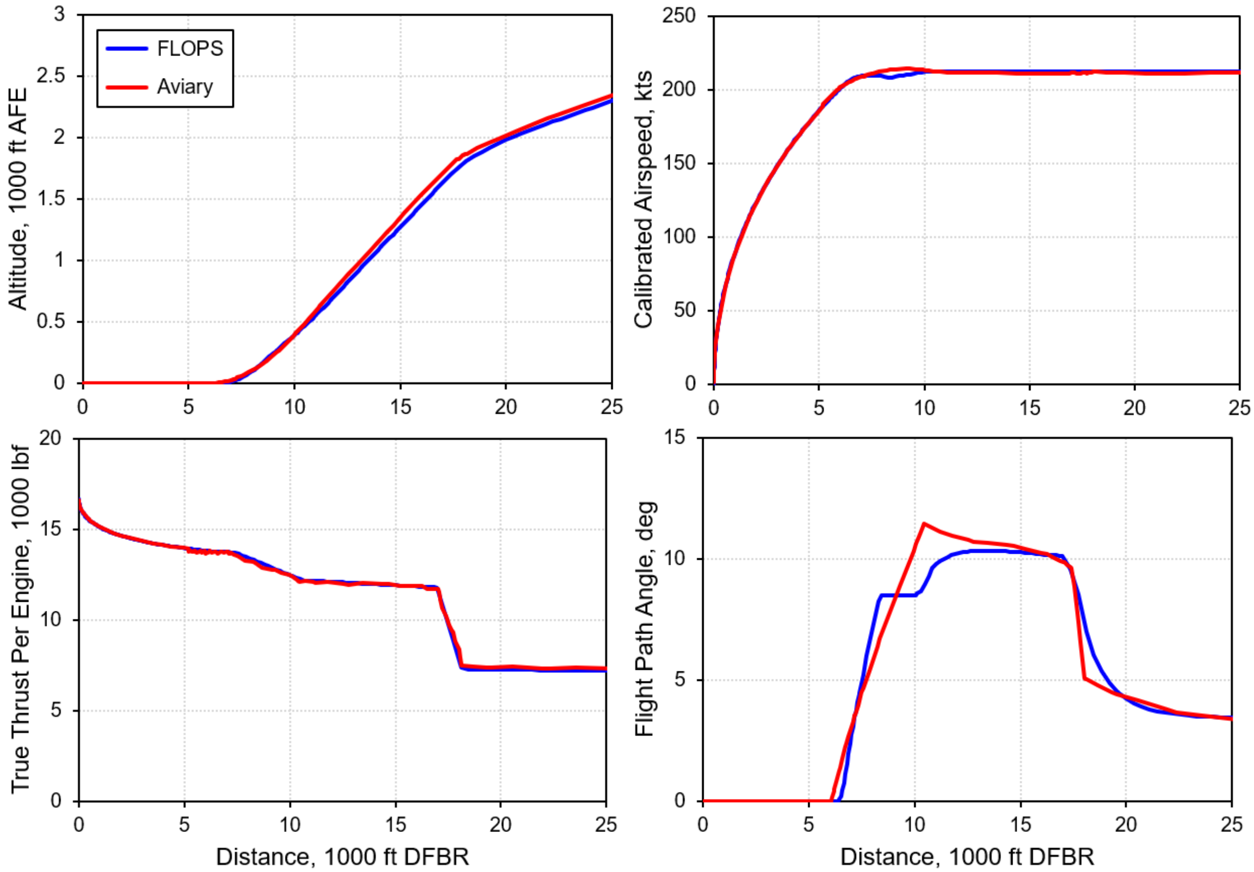

Fig. 7 FLOPS and Aviary comparison of low throttle lapse trajectories.

An example of the ANOPP STCA noise prediction results for a nominal VNRS trajectory are shown in Fig. 8, represented by the blue data bars. These results are broken down into the different noise source components that contribute to the overall noise levels at the lateral and flyover observers. The trajectory used for the ANOPP predictions was determined using FLOPS to be consistent with previous approaches. The results show a strong dominance of the jet mixing noise at both the lateral and flyover observers. It is also shown that the fan inlet noise is low relative to other sources resulting from the over-wing nacelles of the STCA and resulting noise shielding effects. The results of the pyNA noise prediction using the Aviary trajectory are represented by the green data bars. The lateral observer total EPNL is identical between both ANOPP/FLOPS and pyNA/Aviary even though some of the component noise levels are slightly more misaligned than the standard takeoff trajectory case. The largest discrepancies between the two toolsets occurs at the flyover observer, which has a total EPNL difference of 0.8 EPNdB. This is likely due to the differences in the trajectory calculations, since the source noise verification for the standard takeoff trajectory shows minimal discrepancies at the same observer. As can be seen in Fig. 7, the largest difference between the FLOPS trajectory and the Aviary trajectory occurs around the pilot-initiated cutback point and results in the airplane being at a higher altitude over the flyover observer for the Aviary trajectory. This explains the consistently lower noise levels predicted by pyNA for all the noise sources at the flyover observer. The largest discrepancy between the two approaches at a source-noise level is the fan inlet source. Since the fan inlet noise is shielded and does not contribute significantly to the total noise levels, this is deemed acceptable for the STCA assessments. However, these results do warrant further investigation for supersonic configurations different than the STCA, especially those with under-wing mounted engines.

It should be noted that in the study by Berton [7], post-process ANOPP methods were implemented to modify the results to account for the noise-reducing effects of refraction and ground attenuation. These methods are not included in ANOPP or pyNA by default, and are not implemented in this study so that the toolsets can be compared equally. Thus, the total EPNL values shown are higher than previously estimated and should be used only in the context of comparing tools and noise prediction methods.

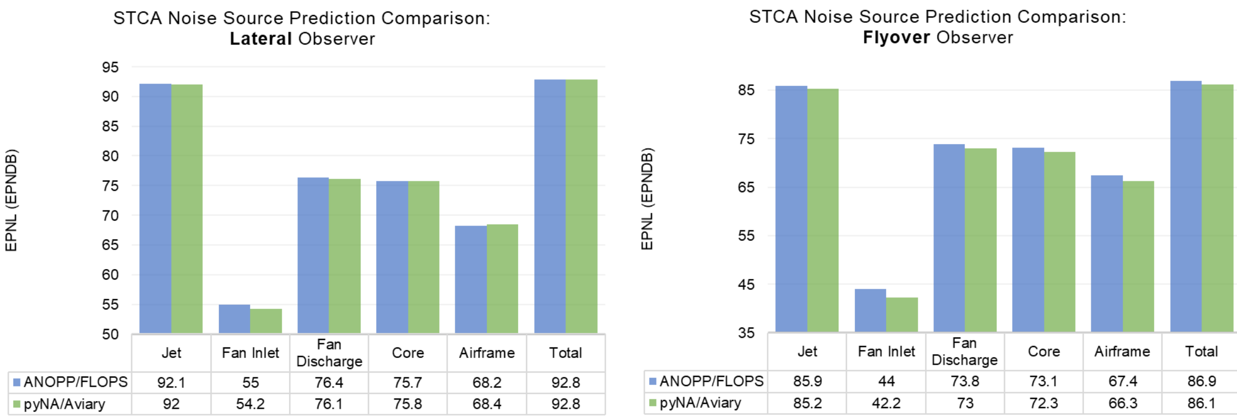

Fig. 8 ANOPP and pyNA comparison of STCA noise prediction at lateral (left) and flyover (right) for low throttle lapse trajectories.

The FLOPS and Aviary trajectory models are next compared for the optimal set of design variables determined by Berton [7]. Displayed in Fig. 9 are the takeoff trajectories for a VNRS procedure with a 30% programmed throttle lapse at a rate of 2%/s, initiated at the 35 ft obstacle. The climbout airspeed is V2 + 55 kts (232 kts).

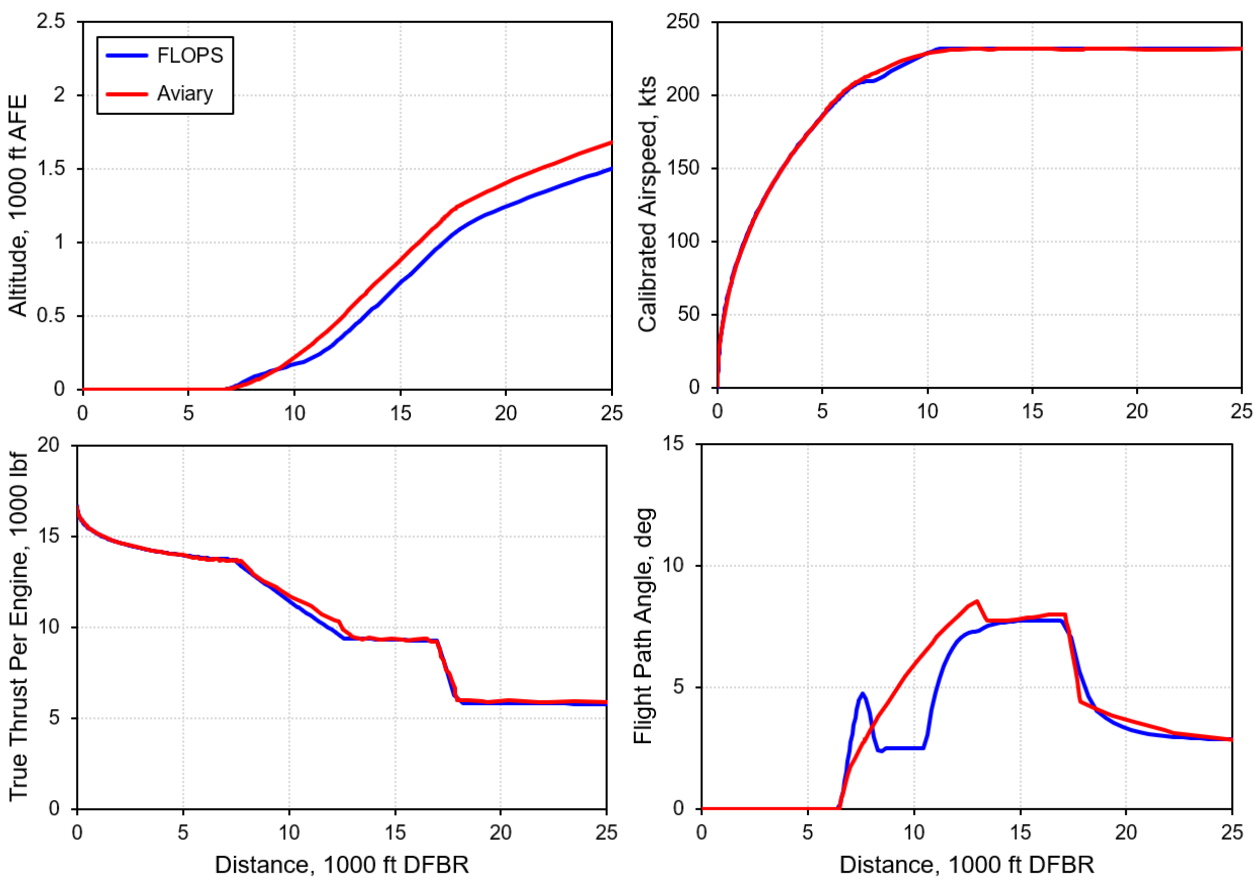

Fig. 9 FLOPS and Aviary comparison of previously optimized VNRS trajectory [7].

Similar results are observed between the two models, but with a greatly amplified difference between the climb gradient calculations. The FLOPS model's iterated climb gradient ends up being lower than the climb gradient during the liftoff segment, leading to a drop in the climb gradient at the onset of the programmed lapse. Since the FLOPS model outputs the first trajectory that is valid for the throttle and climbout speed constraints, it is possible that better results could be achieved with a higher fixed climb gradient. As discussed above, the Aviary model optimizes the climb gradient as the airplane accelerates, leading to a greater altitude difference (and larger noise reduction) than that shown in Fig. 7 due to the greater throttle lapse and higher stabilized airspeed.

A noise prediction comparison between ANOPP/FLOPS and pyNA/Aviary was also performed for this optimized VNRS takeoff trajectory. The results of these assessments are shown in Fig. 10, with blue representing ANOPP/FLOPS predictions and green representing pyNA/Aviary predictions.

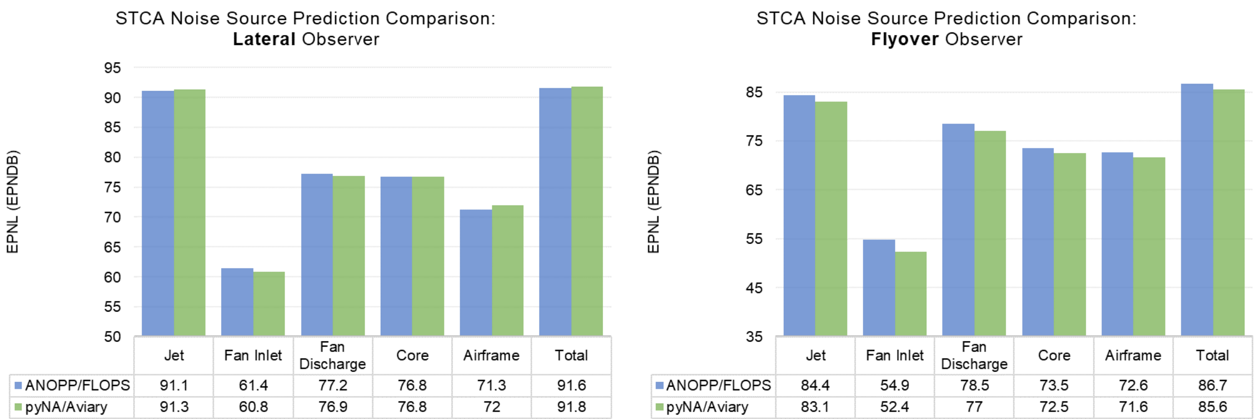

Fig. 10 ANOPP and pyNA comparison of STCA noise prediction at lateral (left) and flyover (right) for previously optimized VNRS trajectory.

For this case, the differences between the noise prediction methods are much more pronounced, primarily due to the differences in the takeoff trajectory calculations. At the lateral observer the total EPNL difference is 0.2 EPNdB, with the largest component differences being the airframe and fan inlet noise at 0.7 EPNdB and 0.6 EPNdB respectively. These discrepancies are primarily driven by the differences in the trajectories generated by Aviary and FLOPS. The maximum lateral EPNL number using the FLOPS trajectory occurs at around 13,550 ft from brake release, where the airplane is at an altitude of 520 ft. This is slightly different for the Aviary generated trajectory where the maximum lateral EPNL occurs at around 12,570 ft from brake release where the airplane is at an altitude of 530 ft. Although the distance from brake release at which the maximum lateral noise occurs is different for each trajectory, the airplane altitudes are very similar between the two. The flyover observer total EPNL difference between the toolsets is 1.1 EPNdB. This difference is driven by the large discrepancy in airplane altitude at the flyover observer location, 21,325 ft from brake release. The FLOPS trajectory predicts the altitude of the STCA to be 1,320 ft, whereas the Aviary trajectory predicts the airplane altitude to be 1,500 ft. This results in consistently lower noise levels for all source components due to the longer propagation distance from the airplane to the observer.

VI. Conclusions

An initial investigation of the prospects of using Aviary and pyNA to address optimization and compatibility challenges for LTO prediction of supersonic aircraft performance and noise shows great promise. A verification study demonstrates similar results between the new and existing toolsets, with a maximum EPNL difference of 0.1 EPNdB. Results for a low throttle lapse example case showed a cumulative EPNL noise prediction difference of 0.8 EPNdB, and results for the previously optimized VNRS trajectory showed a cumulative EPNL noise prediction difference of 1.3 EPNdB. The optimal control approach of the new toolset results in lower noise predictions using a VNRS, suggesting that dynamically controlling the flight path of a supersonic aircraft during takeoff could be beneficial for noise reduction. However, further development is needed to standardize the flight path in the new toolset for an equal comparison with the existing toolset. Additional developments are needed to address concerns with computational speed. The next step in this ongoing study is to test the robustness of the new approach by conducting a DSE similar to the study conducted by Berton [7], which will double as a verification exercise. Future work will also involve connecting Aviary and pyNA to pass variables dynamically, leveraging OpenMDAO and Dymos optimization tools to conduct an integrated optimization of the trajectory with design sensitivities. This capability can be expanded in the future to conduct engine and aerodynamics design optimizations for noise, and to examine other supersonic aircraft configurations and designs that vary significantly from the STCA. This will further increase understanding of the supersonic aircraft design space and the benefits, limitations, and requirements of a VNRS system.

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