GT2024-128885: Flow Reconstruction in a Transonic Turbine Cascade using Physics-Informed Neural Networks (PINNs)

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Background

- What is a PINN?
  - Executive Summary: A PINN is essentially a machine learning methodology where the predictions obey any governing equations that are provided, along with any data that are provided
  - For an in-depth explanation, see (Raissi et al., 2017)
  - Hybrid approach – solution will match supplied data while adhering to the governing equations
- When is a PINN useful?
  - In theory, a PINN can be used completely in the forward sense. However, this is difficult to achieve in practice and provides little to no benefit over classical numerical methods since a hyperparameter study is required (Markidis et al., 2021)
  - PINNs are most useful in the following scenarios:
    - Infer unknown boundary conditions (Cai et al., 2021)
    - Infer unknown parameters (Raissi et al., 2017)
    - Data-driven solution of PDEs

Motivation

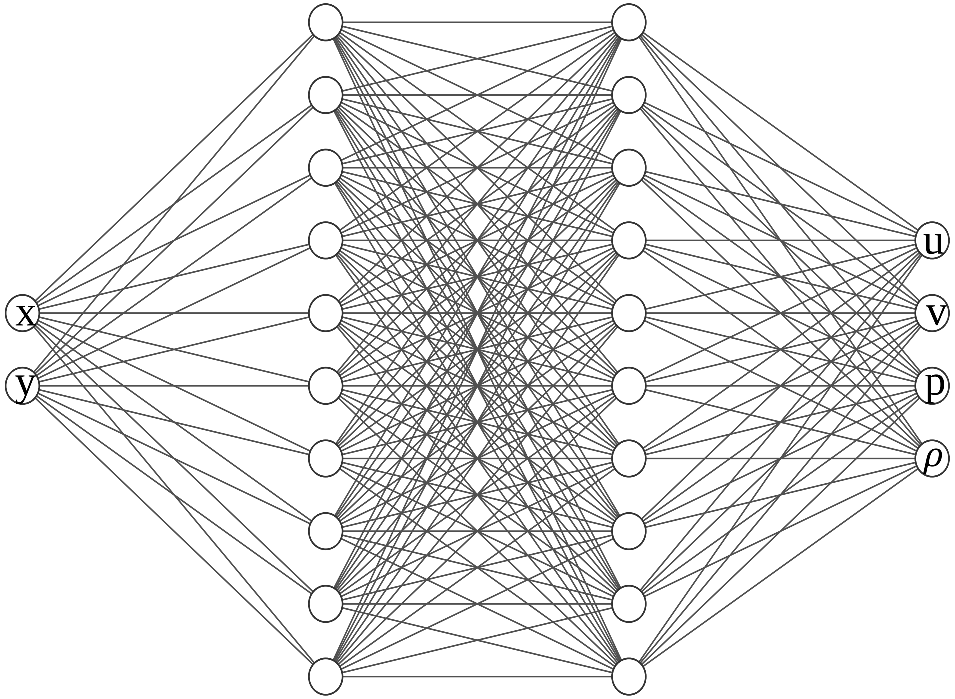
- PINNs have been used for compressible flow analysis (Inviscid and Viscous/RANS) (Hanrahan et al., 2023), (Post et al., 2022), (Mao et al., 2020)
  - In these studies, the PINN was trained on data from a CFD solution
  - CFD solutions yield a ubiquitous amount of data for PINN training

- We want to know how an experimentalist could leverage PINNs to better understand the flow features of interest, with minimal amount of measurements

<u>Questions to be studied:</u>

- Are the data typically acquired in a cascade test enough to train a PINN to reconstruct flow through the passage?
- What effect does the blade loading have on PINN performance?
- What effect does selectively sampled data have on PINN performance?
- Can the PINN infer unknown boundary conditions using sparse data?

PINN Schematic



$$\frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y}$$

$$\frac{\partial^2 u}{\partial x^2} \quad \frac{\partial^2 u}{\partial y^2}$$

Automatic Differentiation

$$\varepsilon_{PDE} = \frac{1}{N_{PDE}} \sum_{i=0}^{N_{PDE}} f_{PDE}$$

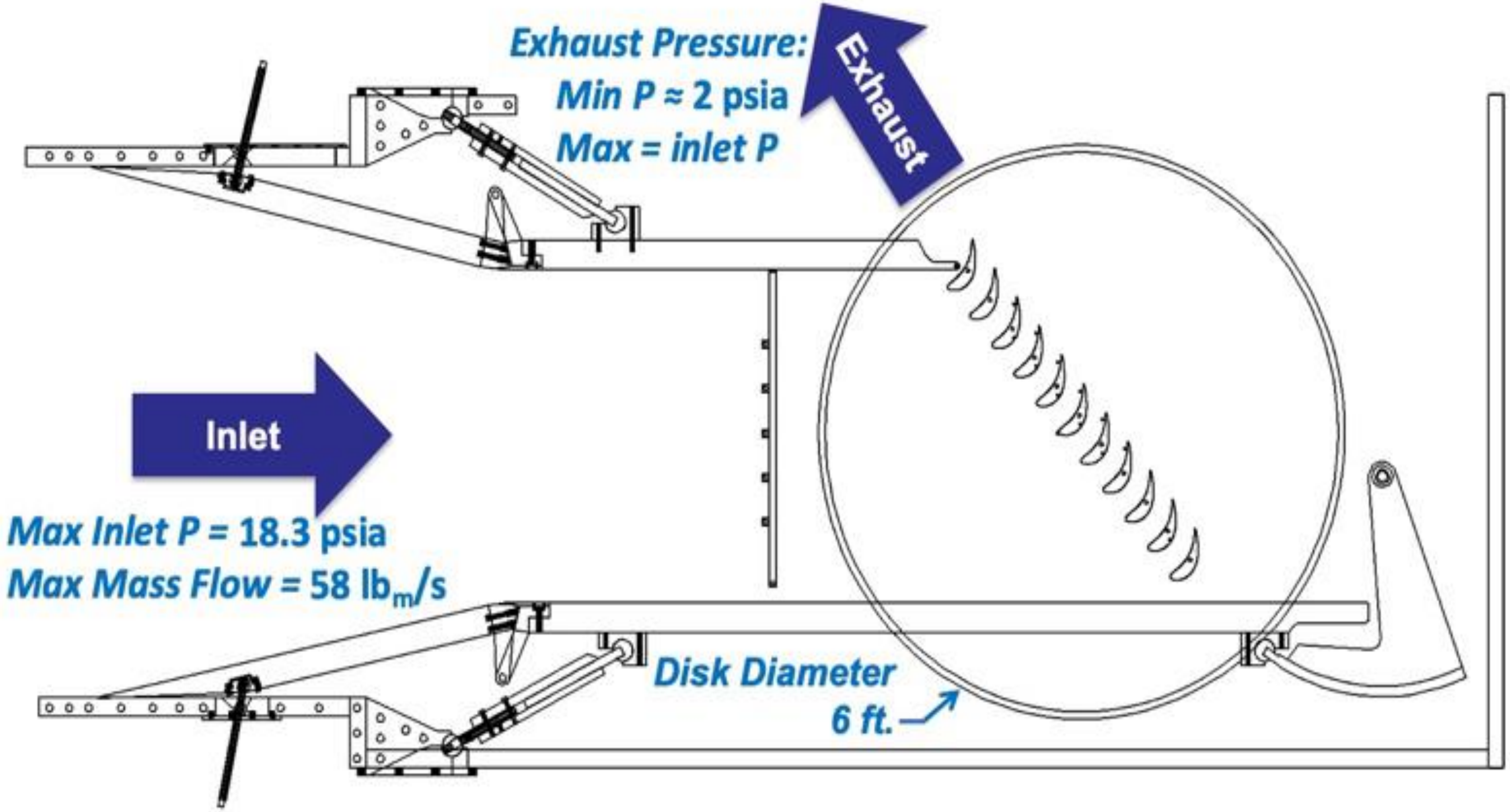
$$\varepsilon_{bc} = \frac{1}{N_{bc}} \sum_{i=0}^{N_{bc}} |u_{bc} - u_{NN}|^2$$

$$\varepsilon_{data} = \frac{1}{N_{data}} \sum_{i=0}^{N_{data}} |u_{data} - u_{NN}|^2$$

Errors

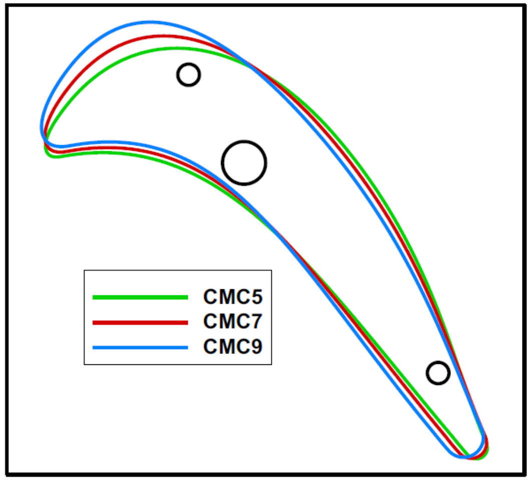
$$Total\ Error = \omega_{PDE}\varepsilon_{PDE} + \omega_{bc}\varepsilon_{bc} + \omega_{data}\varepsilon_{data}$$

Cascade and Geometry Description



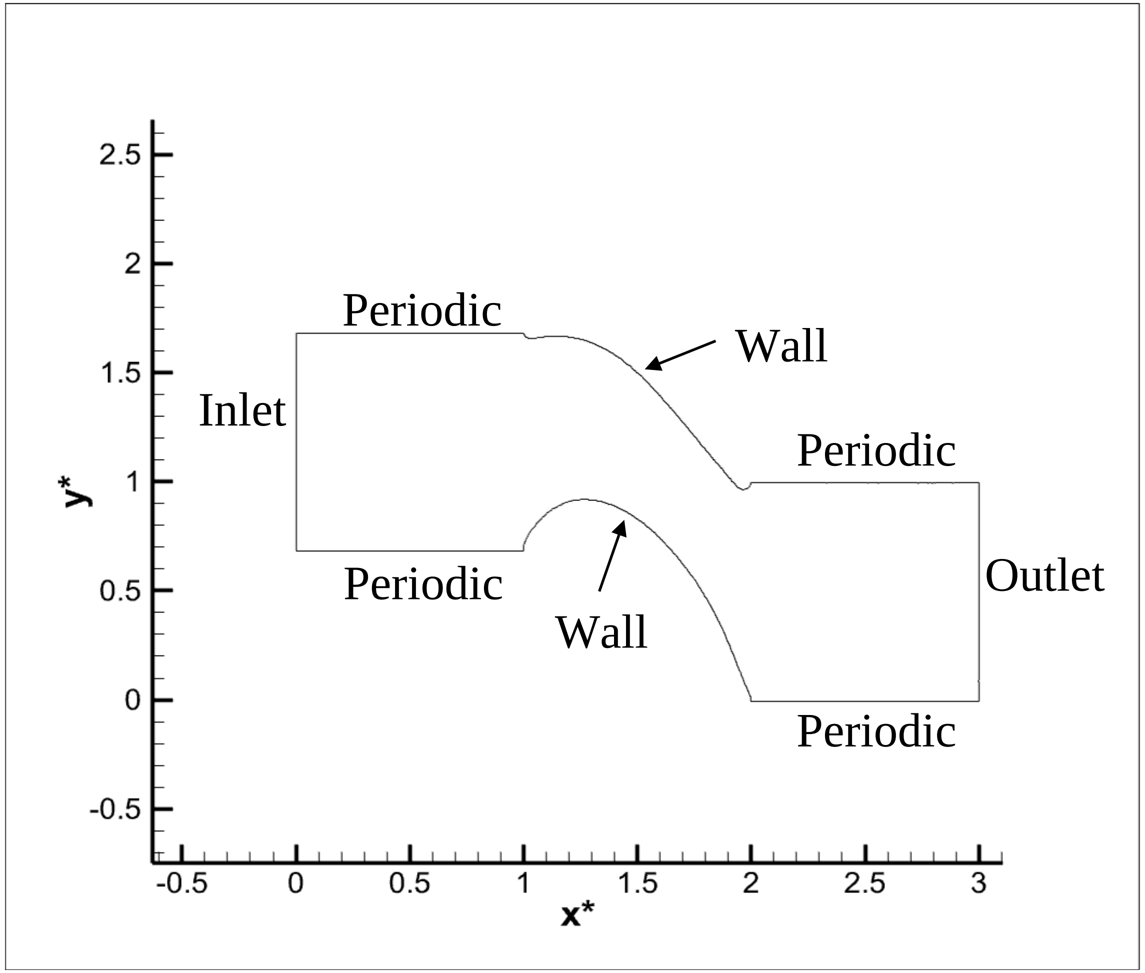
Transonic cascade facility overview (Giel et al., 2020)

Parameter	Value
C_x [in]	5.119
H [in]	6.000
AR (H/C_x)	1.17
Solidity (C_x/Pitch)	1.00
Tu_{inlet} [%]	0.5
$Re_{C_x,inlet}$	310,000
$M_{is,outlet}$	0.74
β_{inlet} [°]	38.8



Simulated CMC blade geometries (Giel et al., 2020)

Governing Equations and Boundary Conditions



Boundary labels and general domain used for all models

2-D, Nondimensional, Inviscid, Compressible Equations

$$0 = u^*\frac{\partial \rho^*}{\partial x^*} + v^*\frac{\partial \rho^*}{\partial y^*} + \rho^*\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right)$$

$$0 = \rho^* u^*\frac{\partial u^*}{\partial x^*} + \rho^* v^*\frac{\partial u^*}{\partial y^*} + \frac{\partial P^*}{\partial x^*}$$

$$0 = \rho^* u^*\frac{\partial v^*}{\partial x^*} + \rho^* v^*\frac{\partial v^*}{\partial y^*} + \frac{\partial P^*}{\partial y^*}$$

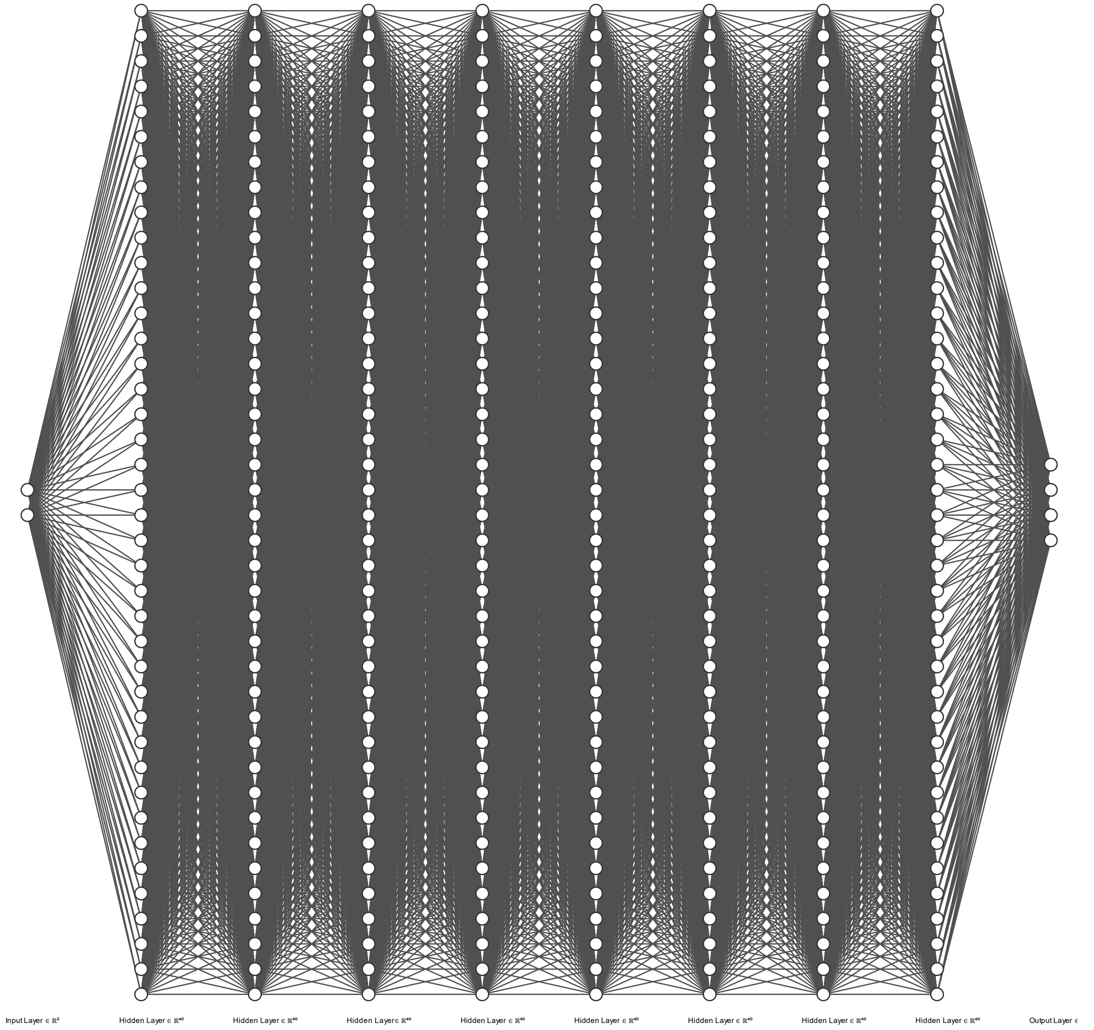
$$0 = u^*\frac{\partial P^*}{\partial x^*} + v^*\frac{\partial P^*}{\partial y^*} + \gamma P^*\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right)$$

Boundary Conditions

1. Inlet
   - u^*, v^*, P^*, ρ^*
2. Outlet
   - P^*
3. Walls
   - $\vec{u} \cdot \hat{n} = 0$
4. Periodic
   - $u^*, v^*, P^*\ \rho^*$ periodic in the y-direction

Neural Network and Training

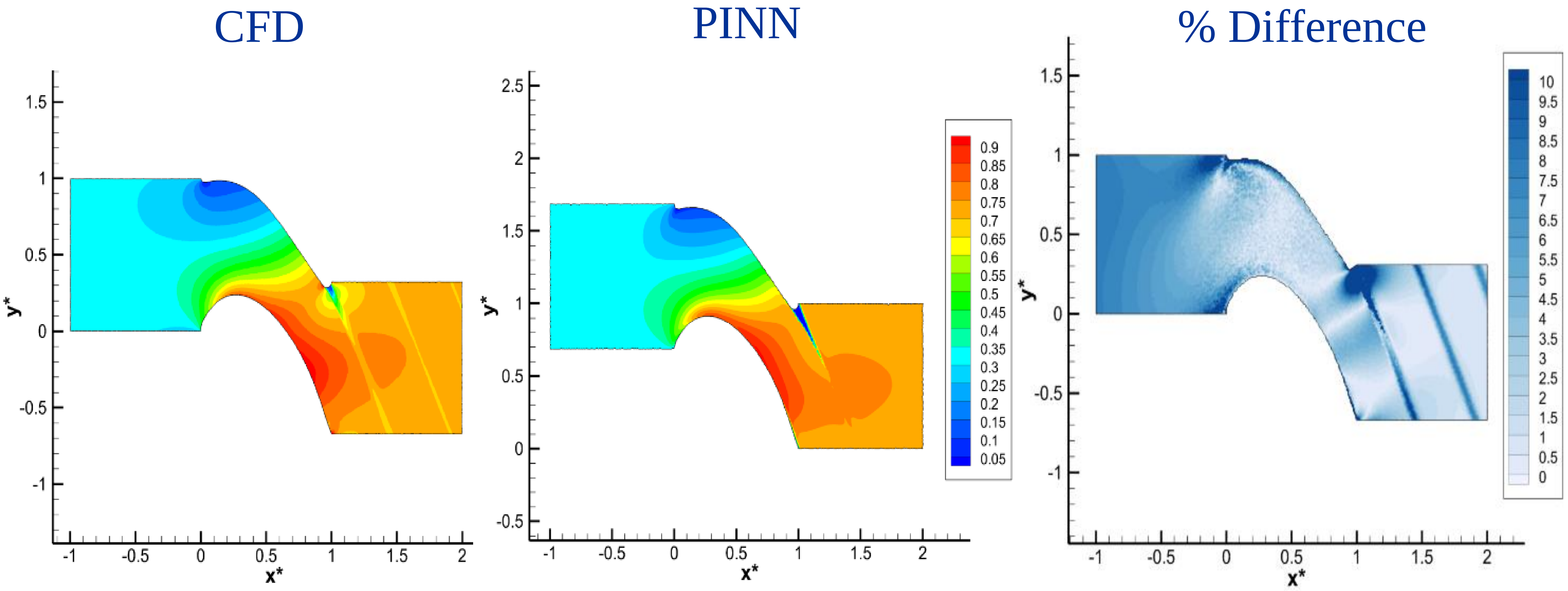
- 7 hidden layers
- 40 neurons per layer
- Activation function
  - tanh
- Output Transformation
  - Pressure and density were transformed using an exponential function to ensure positive values
- Residual Points
  - Domain points were randomly sampled
  - Boundary points were uniformly spaced
  - 5,000 points in domain used for training
  - 5,000 points on boundaries used for training
- Training Process
  - Adam – 1000 iterations, learning rate = 1e-3
  - L-BFGS – 50,000 iterations



Neural network architecture

CFD and PINNs Comparison

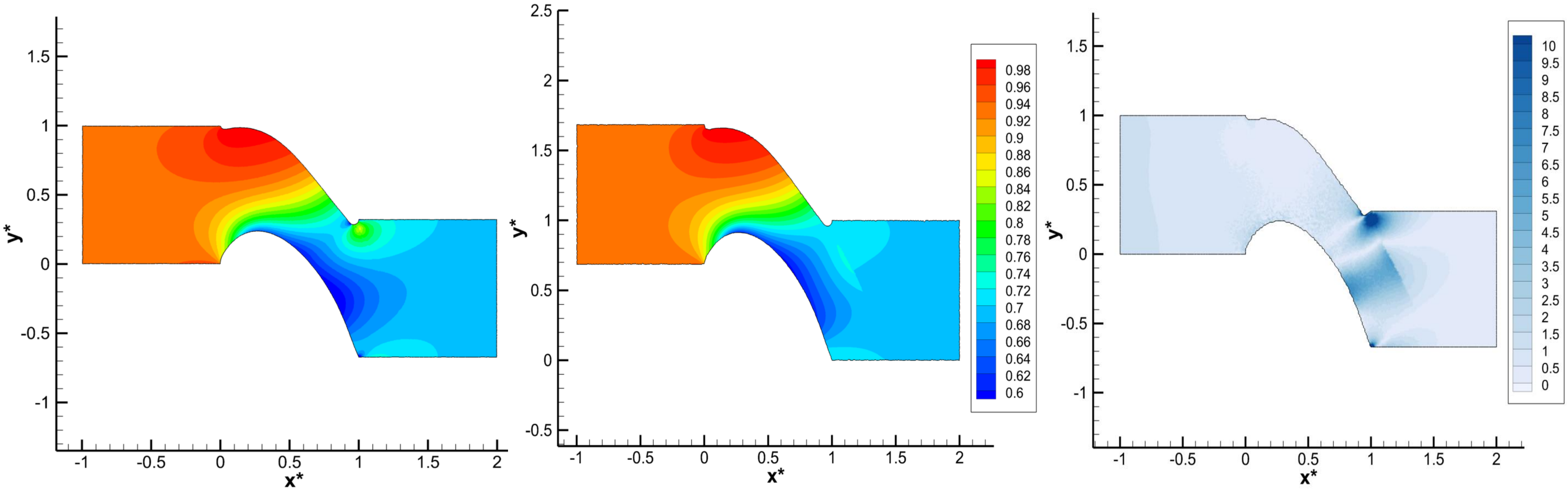
Comparison of Mach Number – CMC7



PINN shows “wedge” behind TE; this is smoothed out in inviscid CFD solution

Comparison of Pressure – CMC7

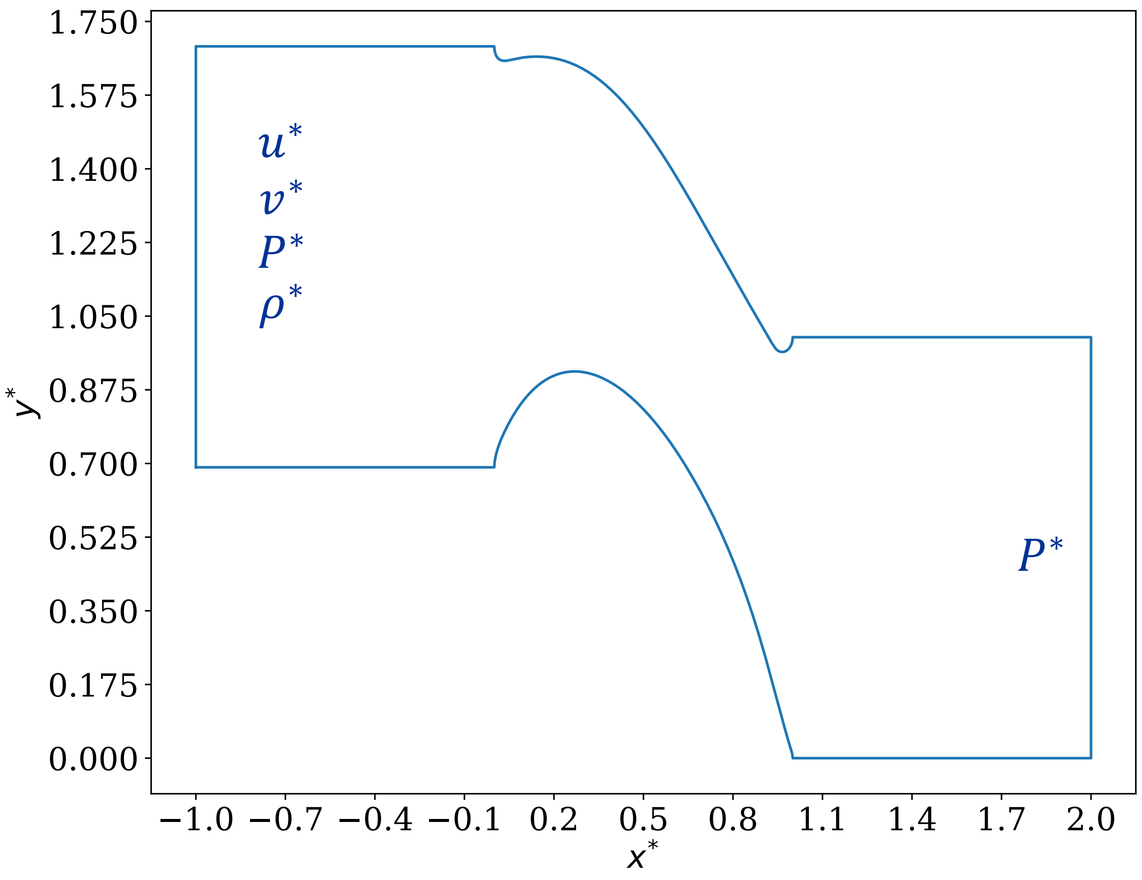
CFD | PINN | % Difference



Only region of significant difference is behind TE

Three Case Studies for Each Blade

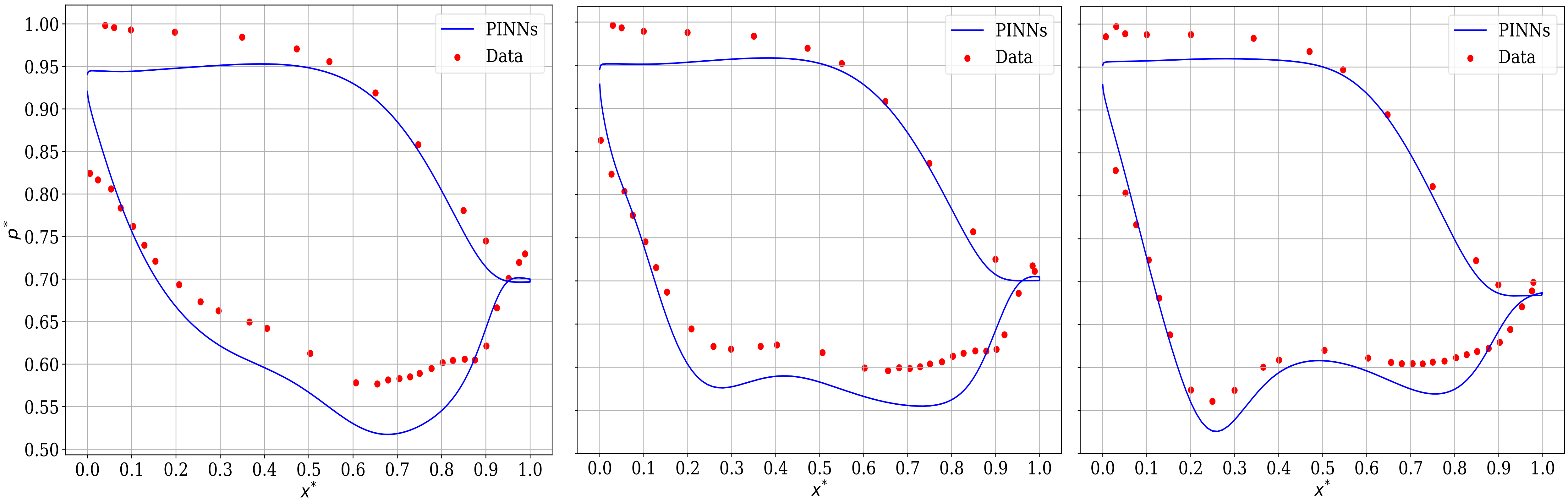
1. Forward Problem
   - Not trained on experimental data
   - Includes all inlet/outlet BCs
2. Assisted Training
   - Trained with experimental data
     - Static pressure data on PS and SS
   - Includes all inlet/outlet BCs
3. Inverse Problem
   - Trained with experimental data
     - Static pressure data on PS and SS
   - Does not include all inlet/outlet BCs
     - Inlet and outlet pressure not prescribed, they are solved for



Case Study 1: Forward Problem

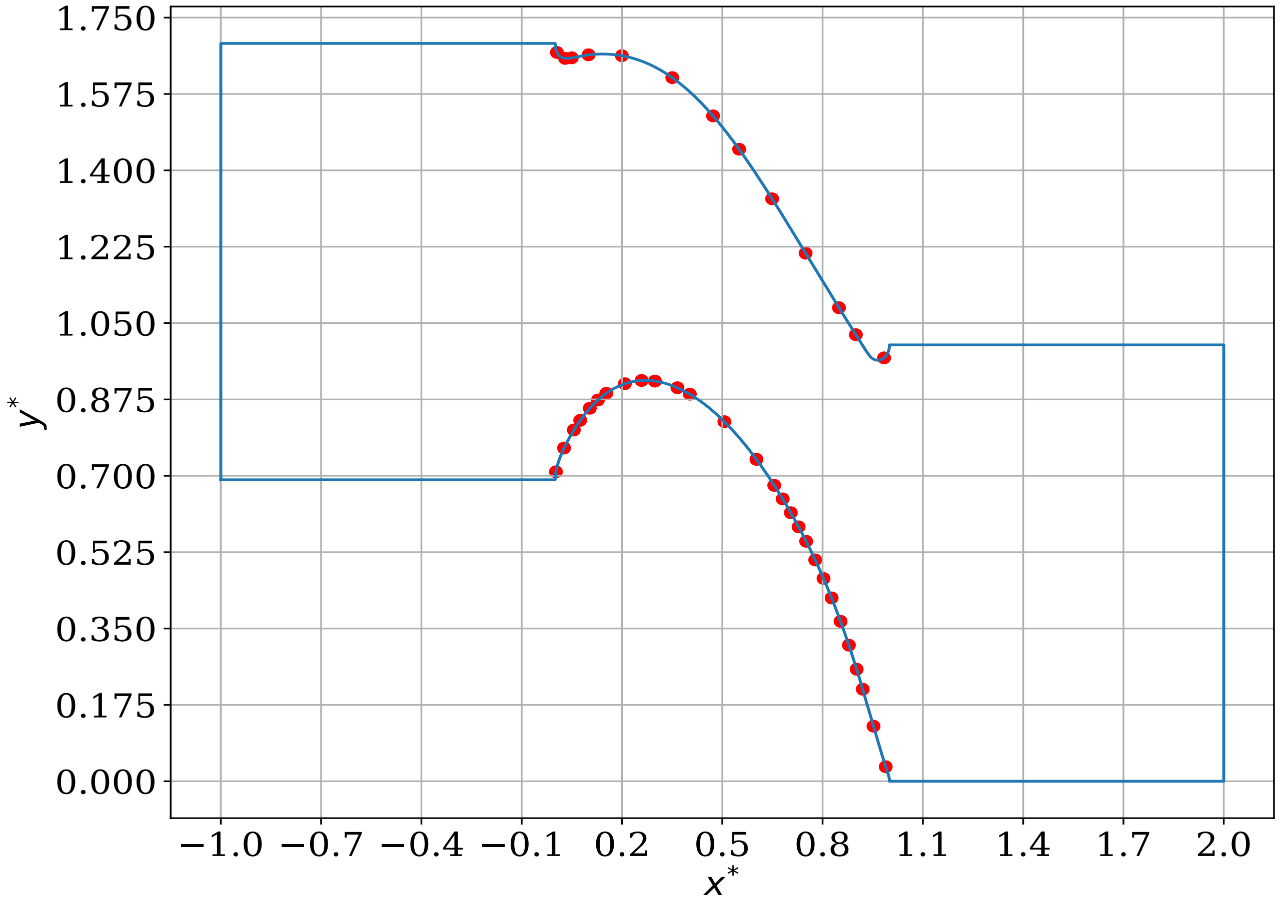
Comparison to Experimental Data

CMC5 CMC7 CMC9

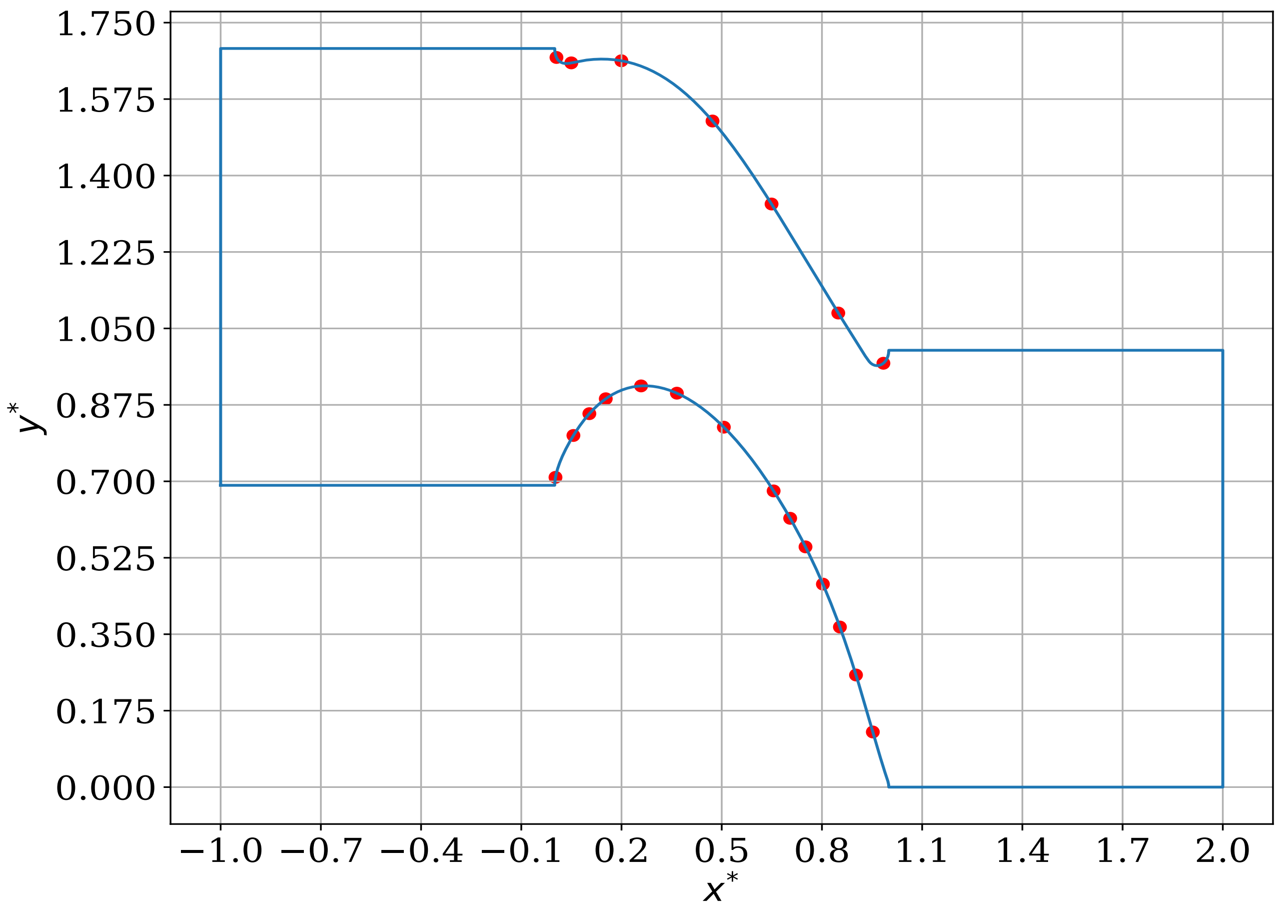


Case Study 2: Assisted Training

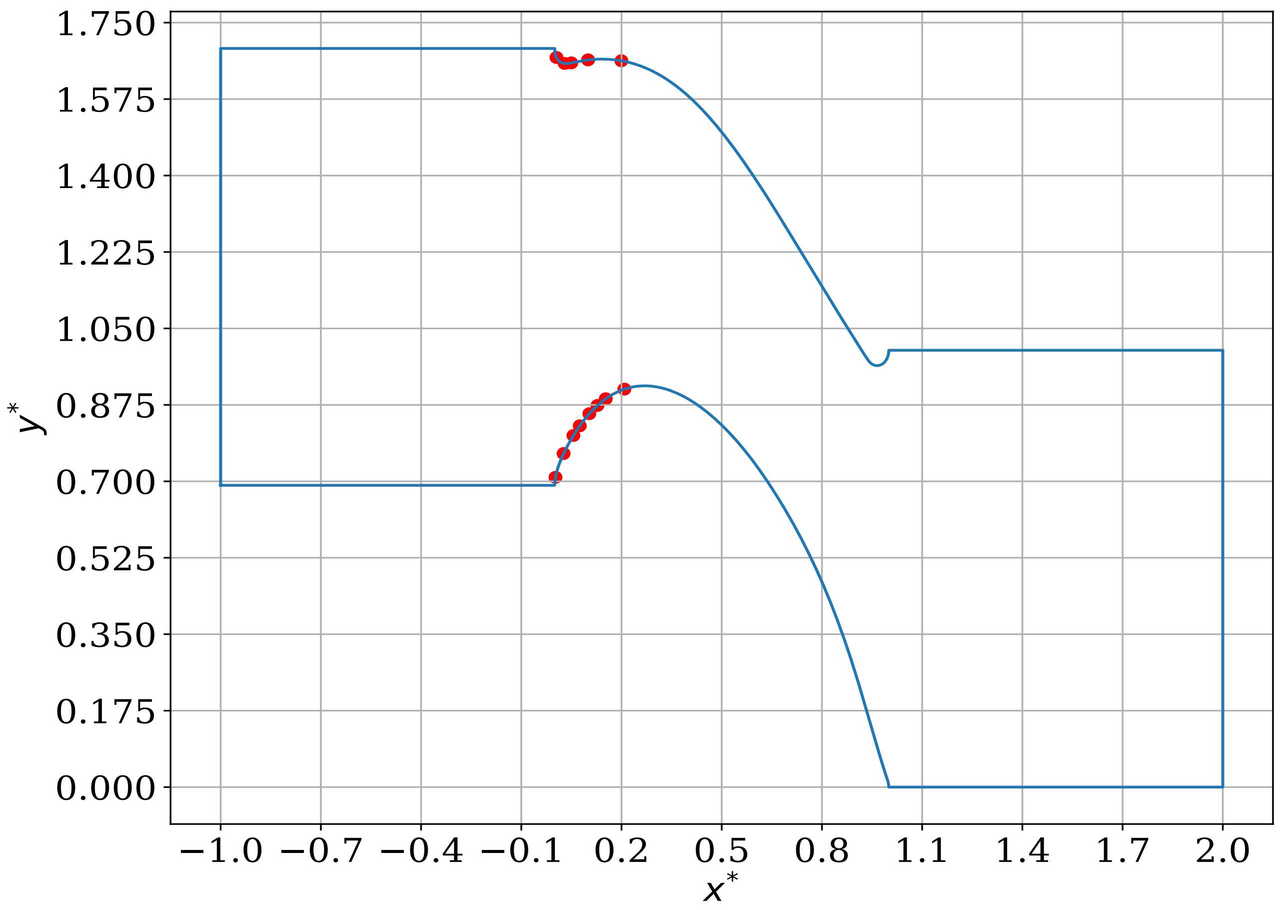
Experimental Data Used for Training – All Data



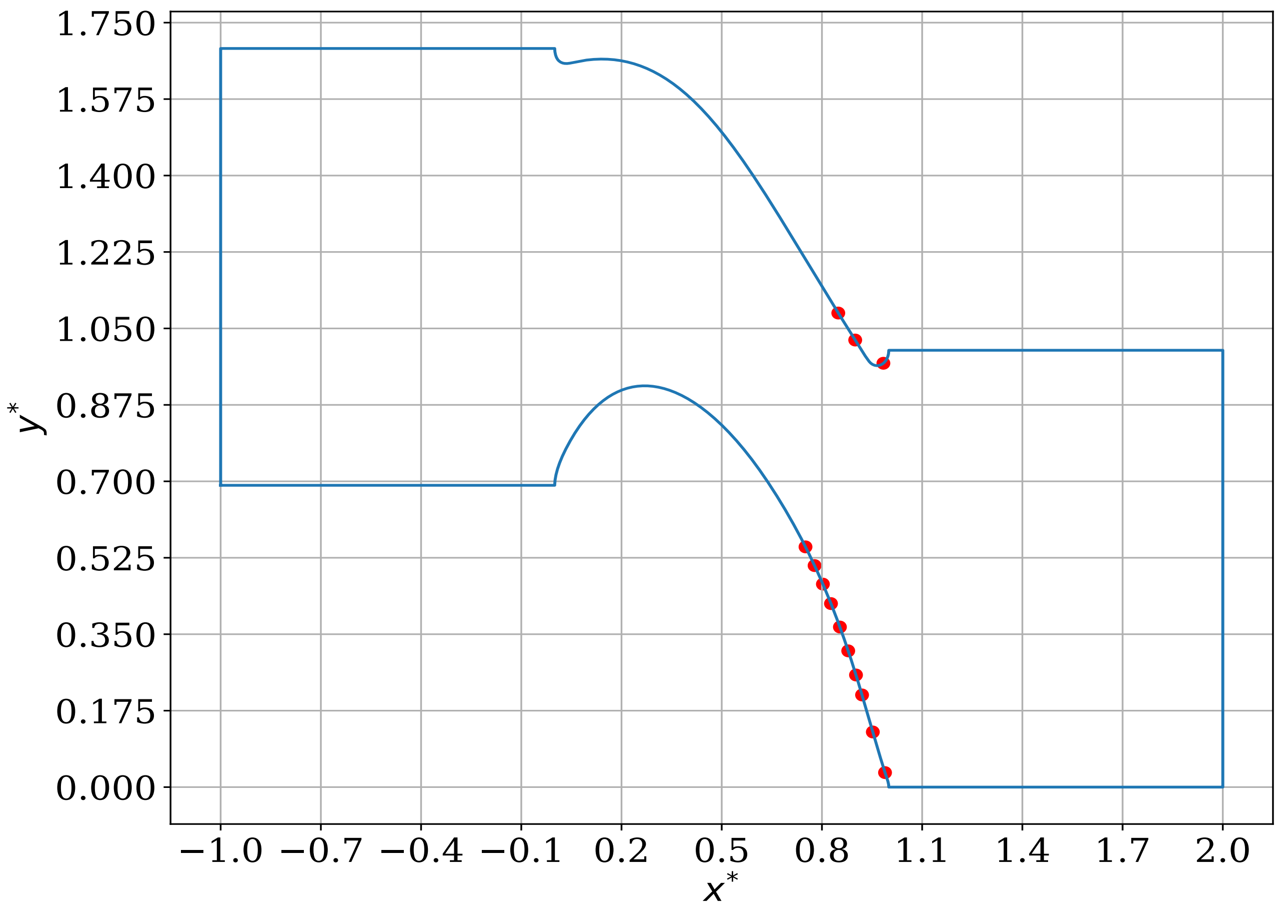
Experimental Data Used for Training – Half Data



Experimental Data Used for Training – LE Data

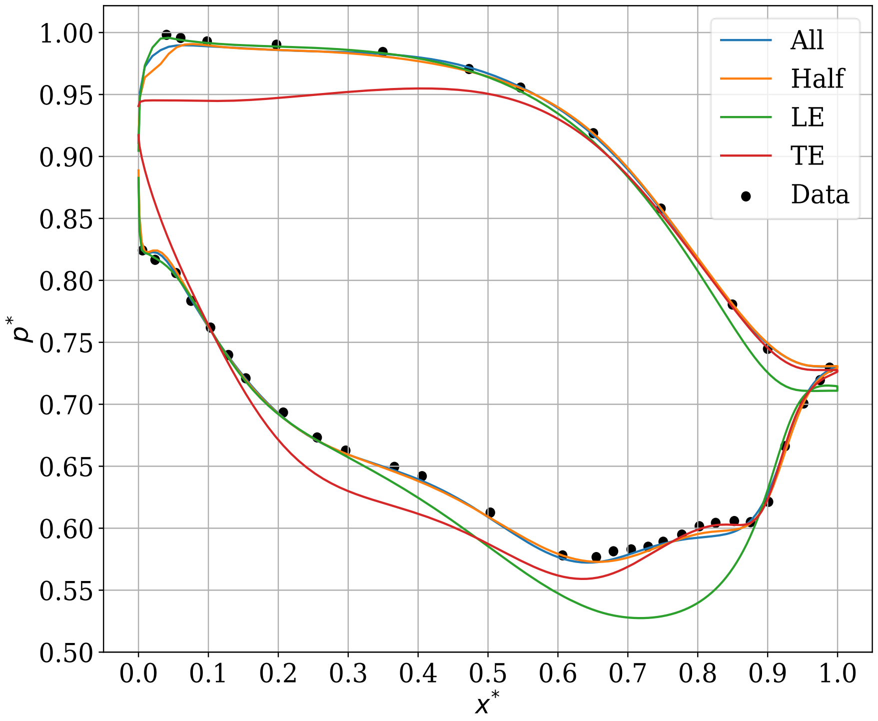


Experimental Data Used for Training – TE Data

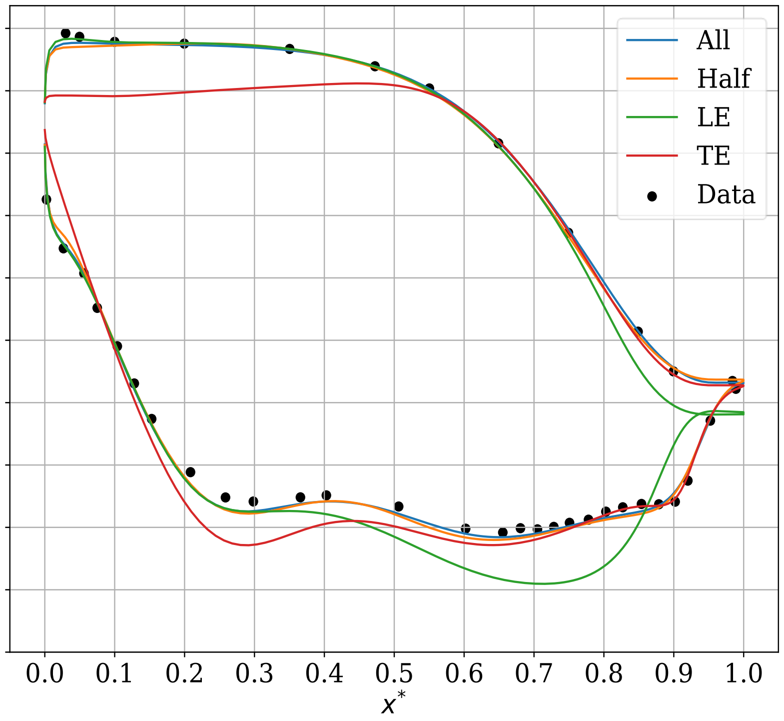


Fit to Experimental Data

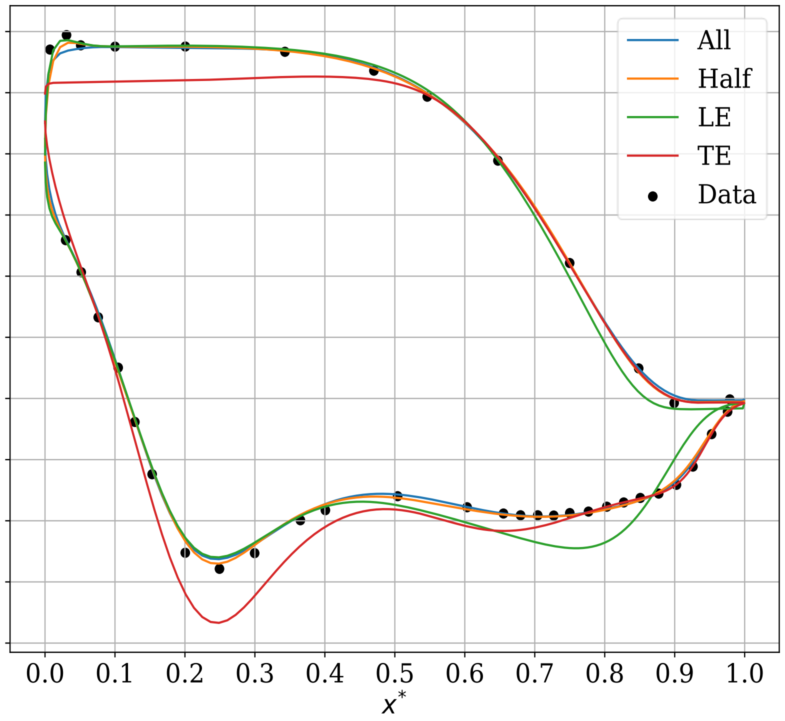
CMC5



CMC7

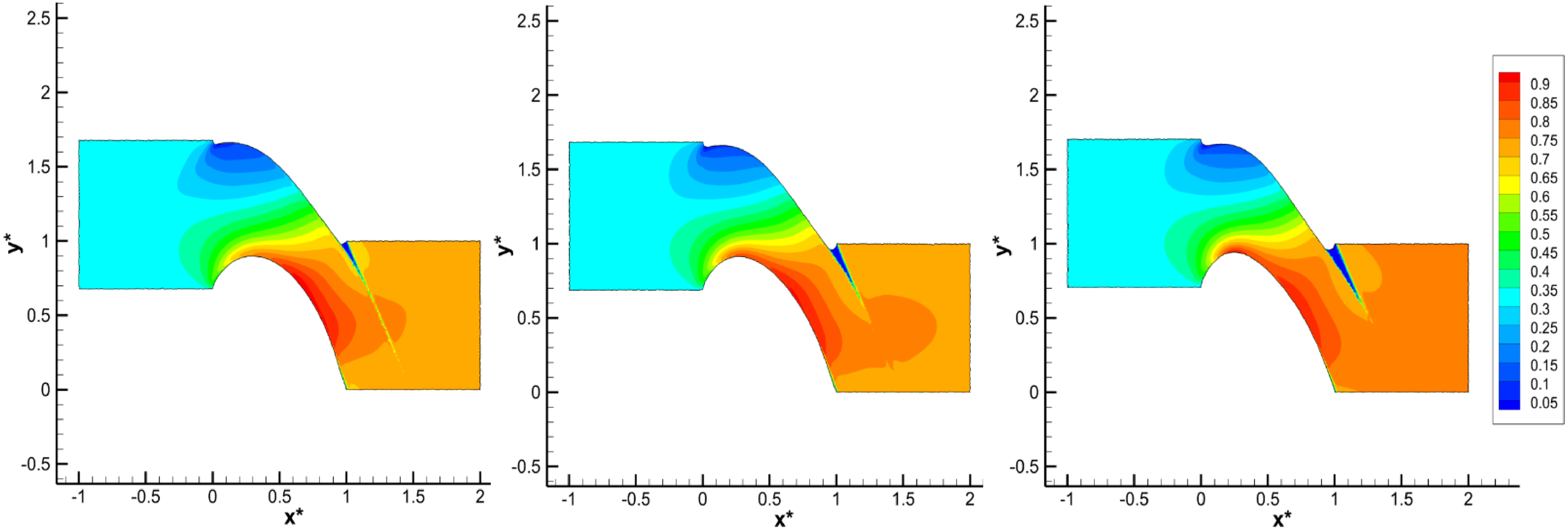


CMC9



Mach Number

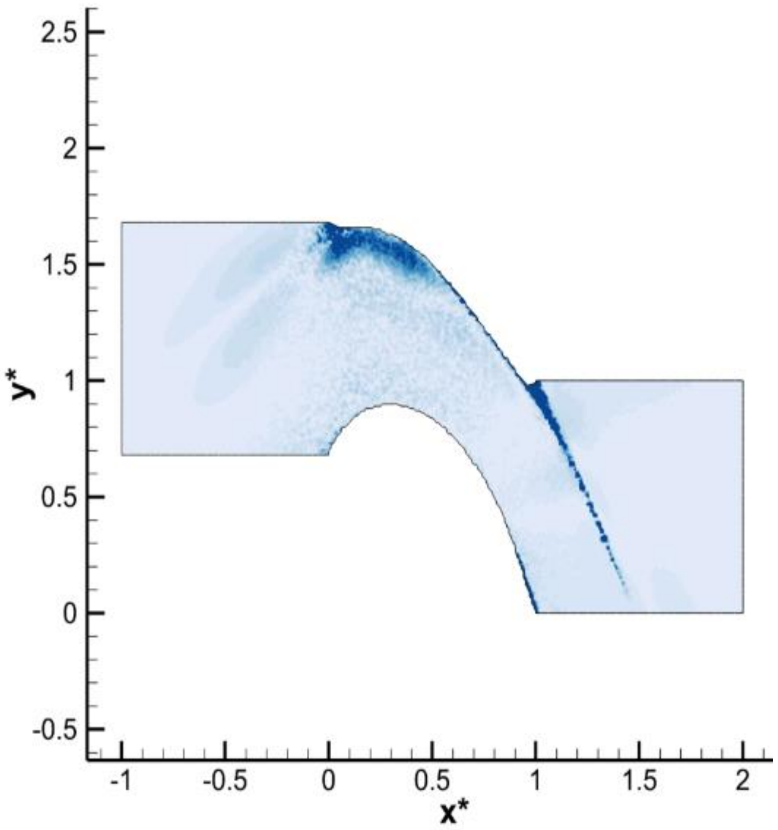
CMC5 CMC7 CMC9



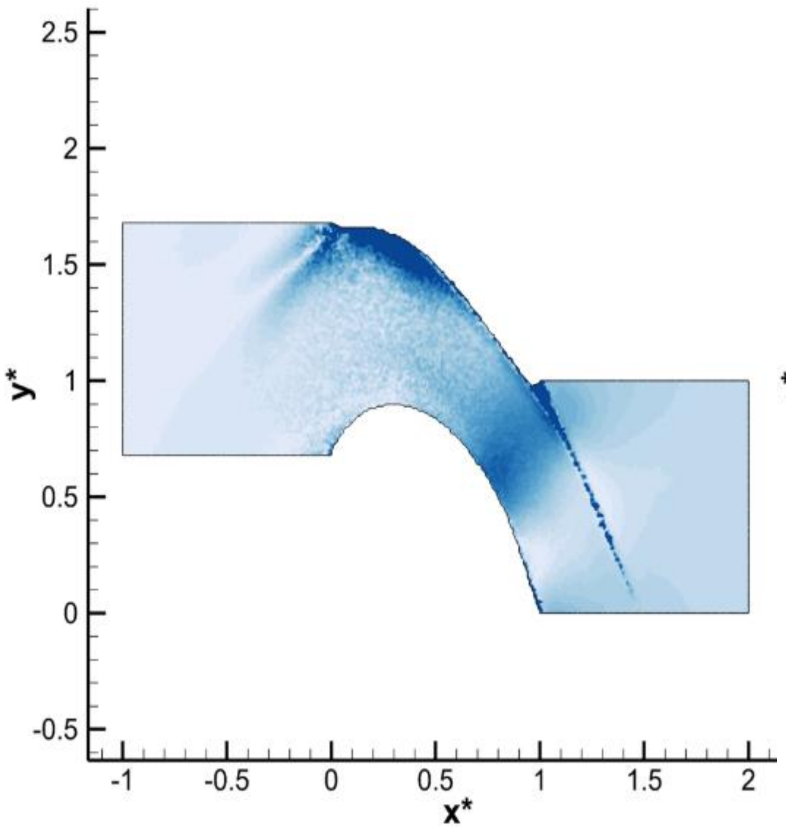
These results are treated as “ground truth” moving forward, due to lowest errors

Comparison of Training Data Effects – CMC5

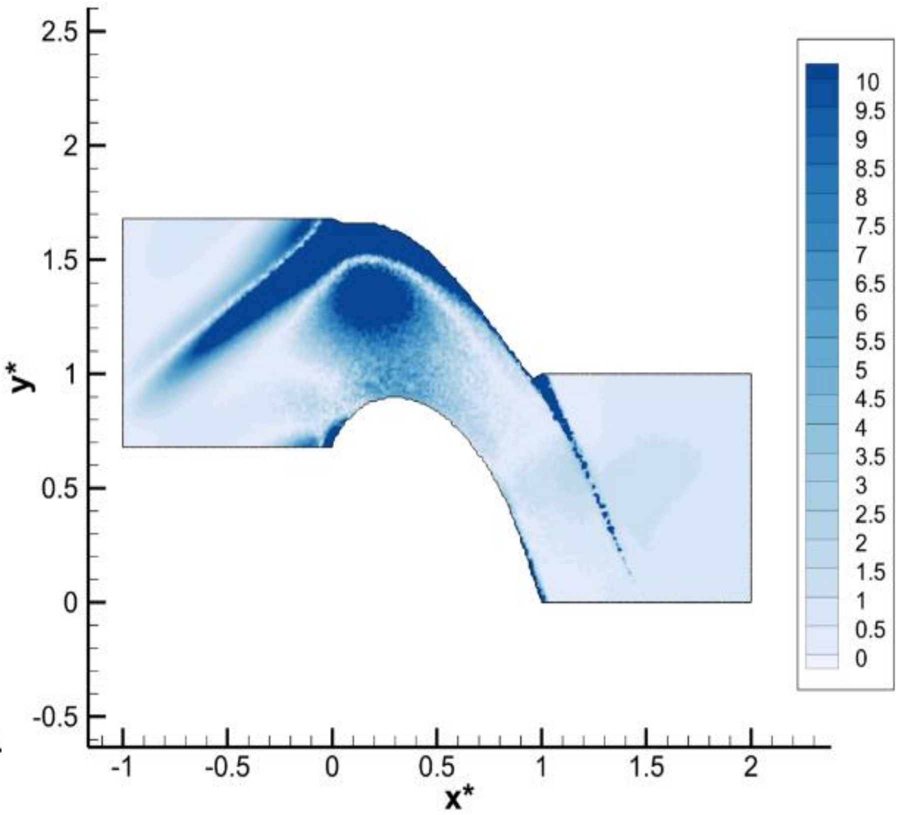
Half



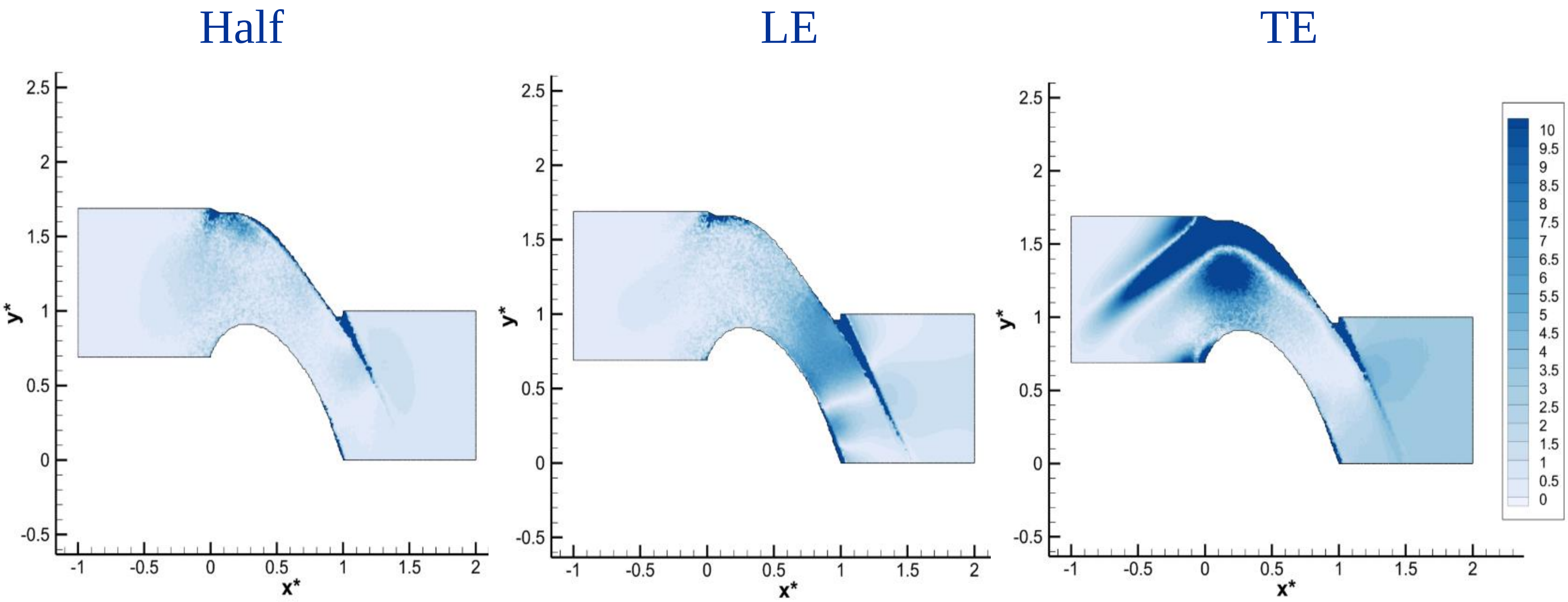
LE



TE

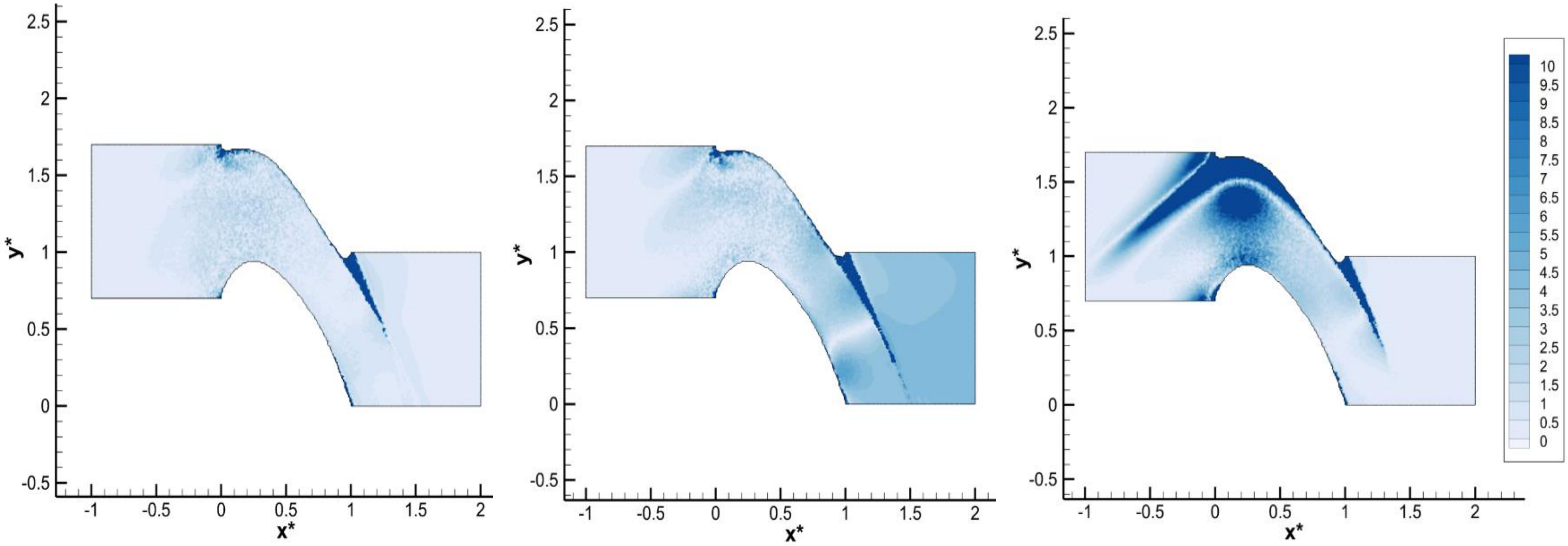


Comparison of Training Data Effects – CMC7



Comparison of Training Data Effects – CMC9

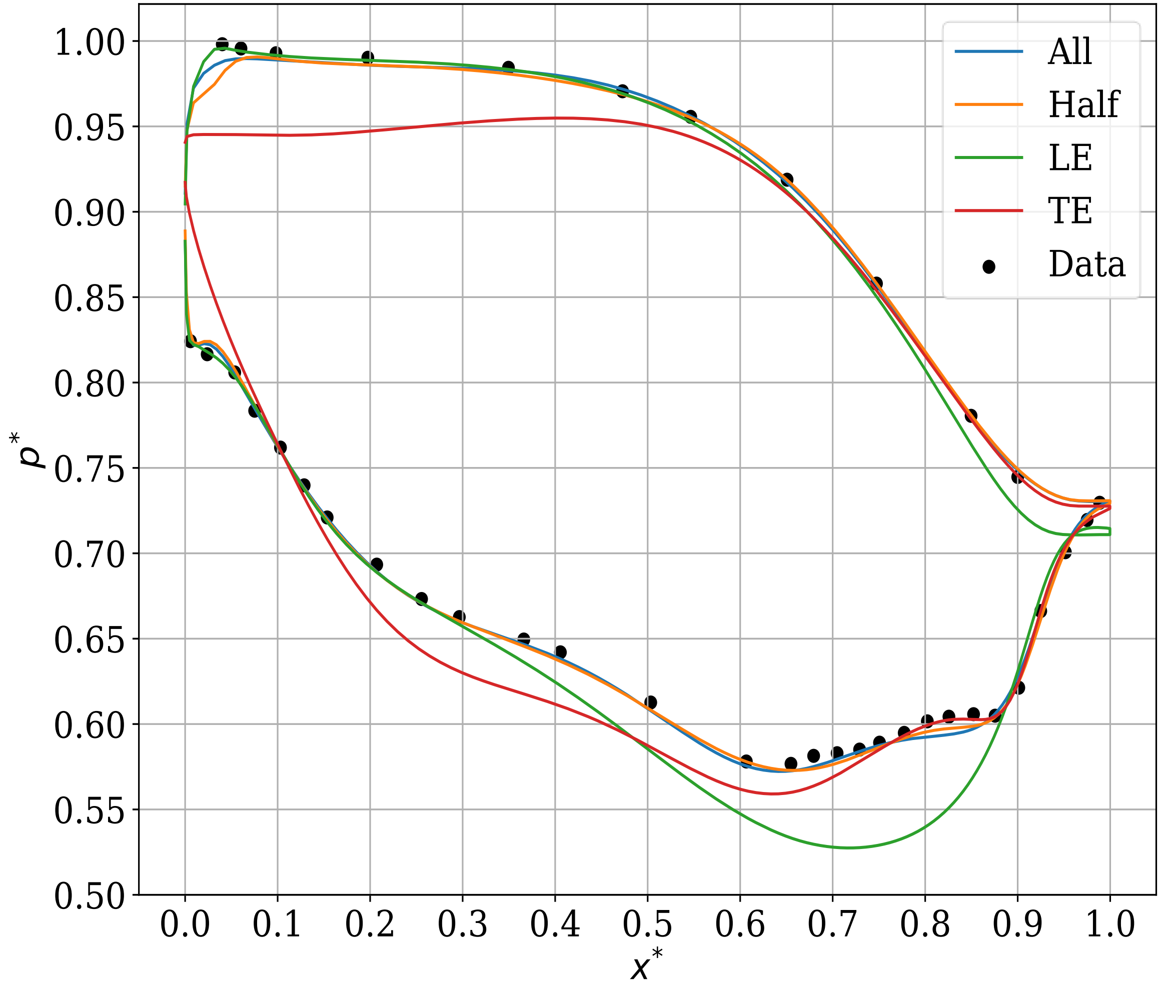
Half LE TE



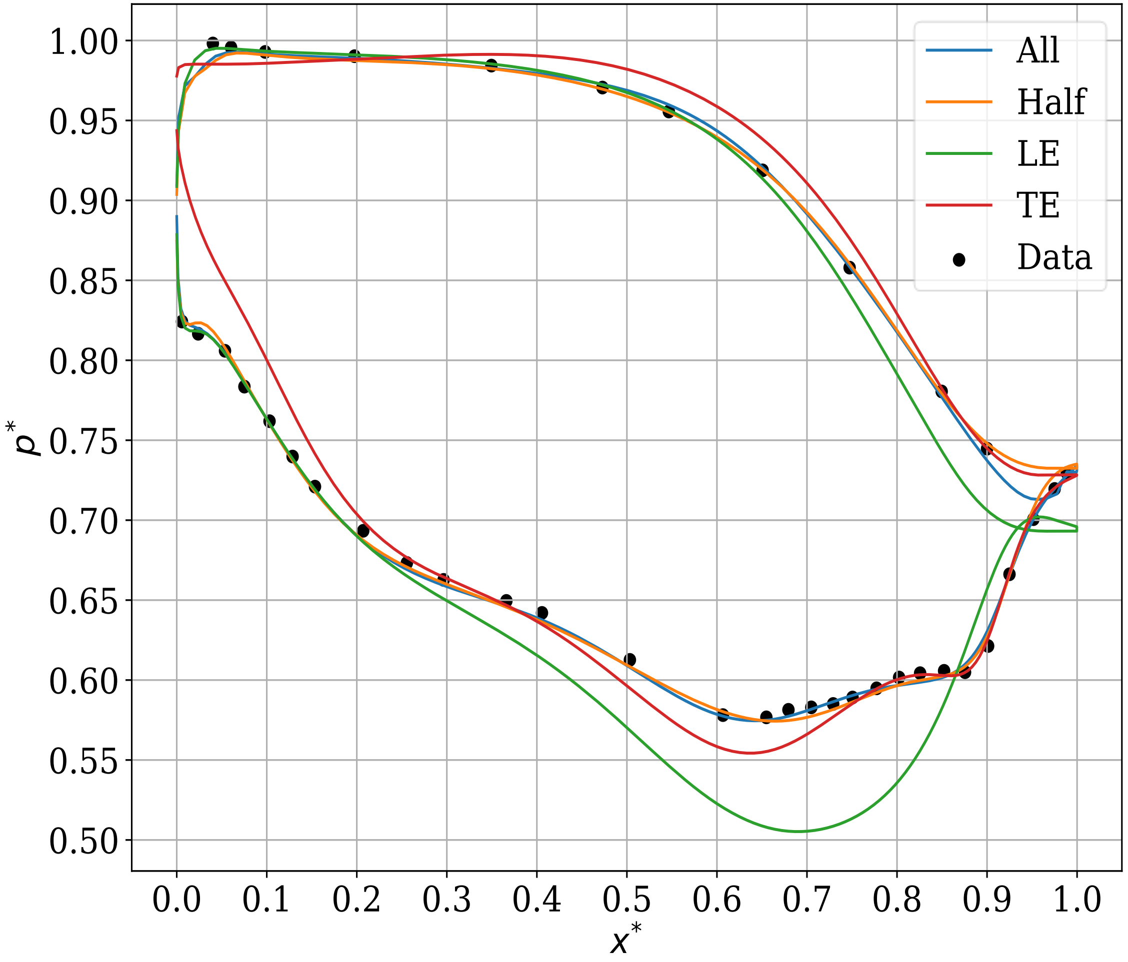
Case Study 3: Inverse Problem

Fit to Experimental Data – CMC5

Assisted-Training

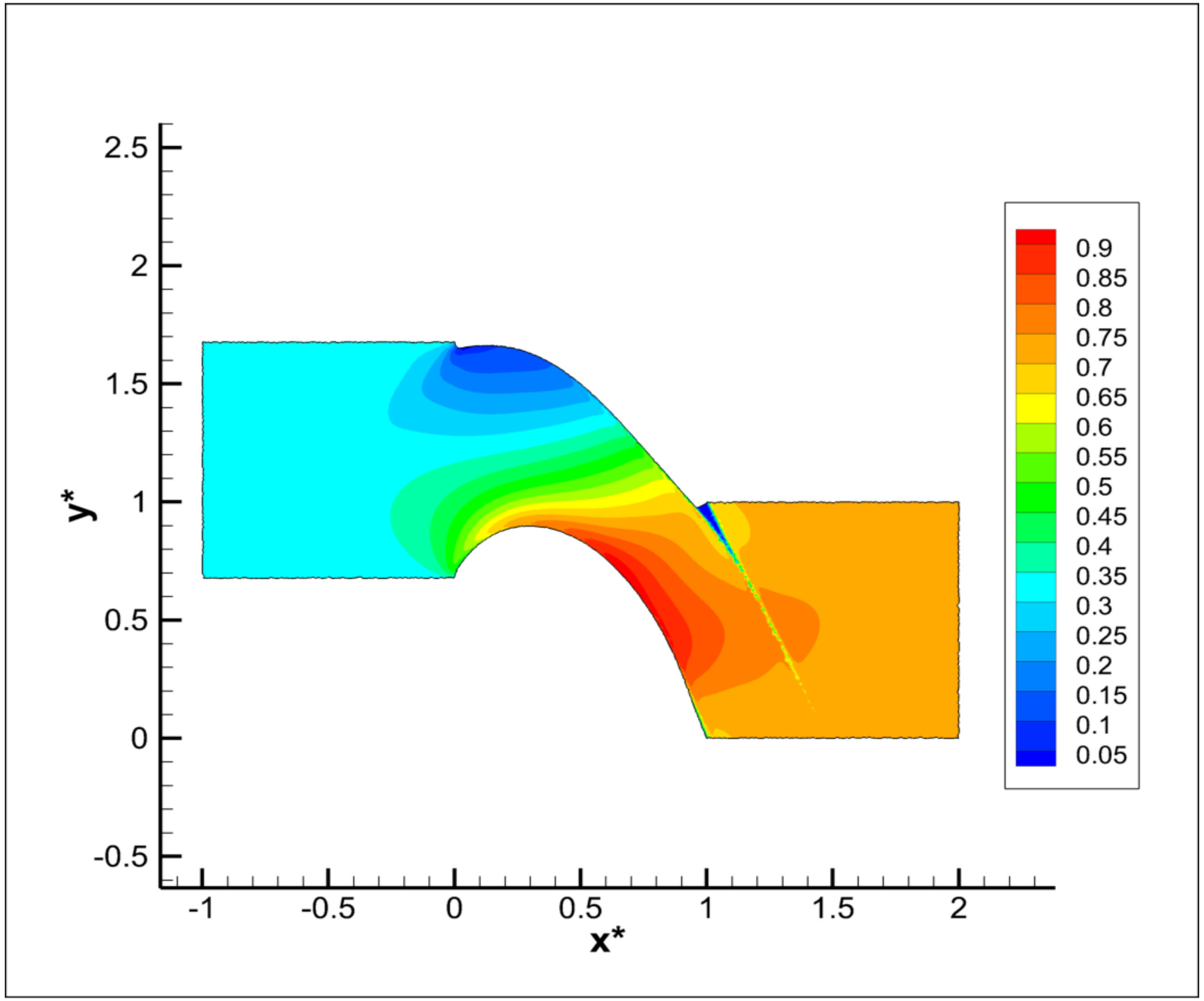


Inverse

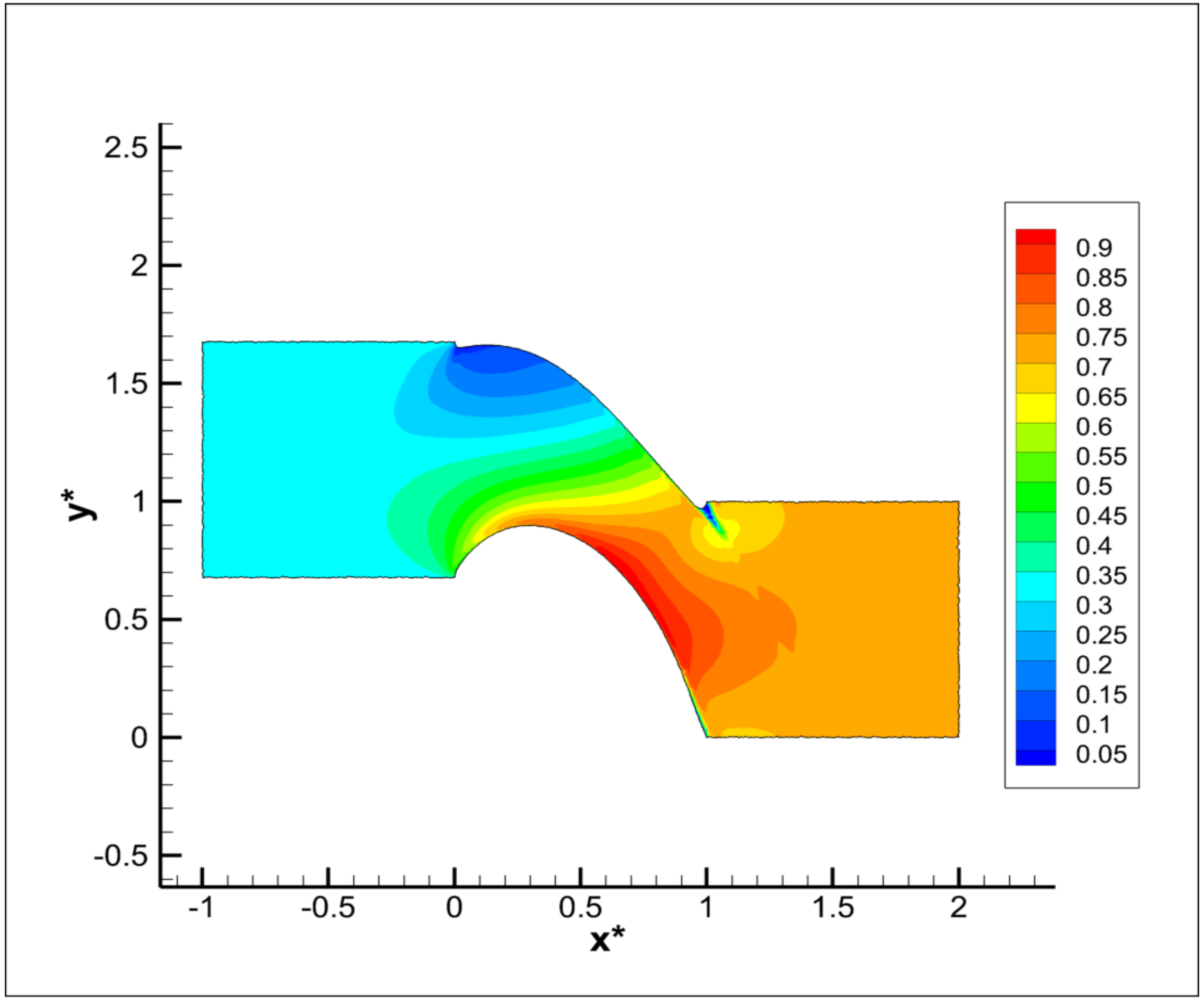


Comparison of Mach Number – CMC5

Assisted-Training

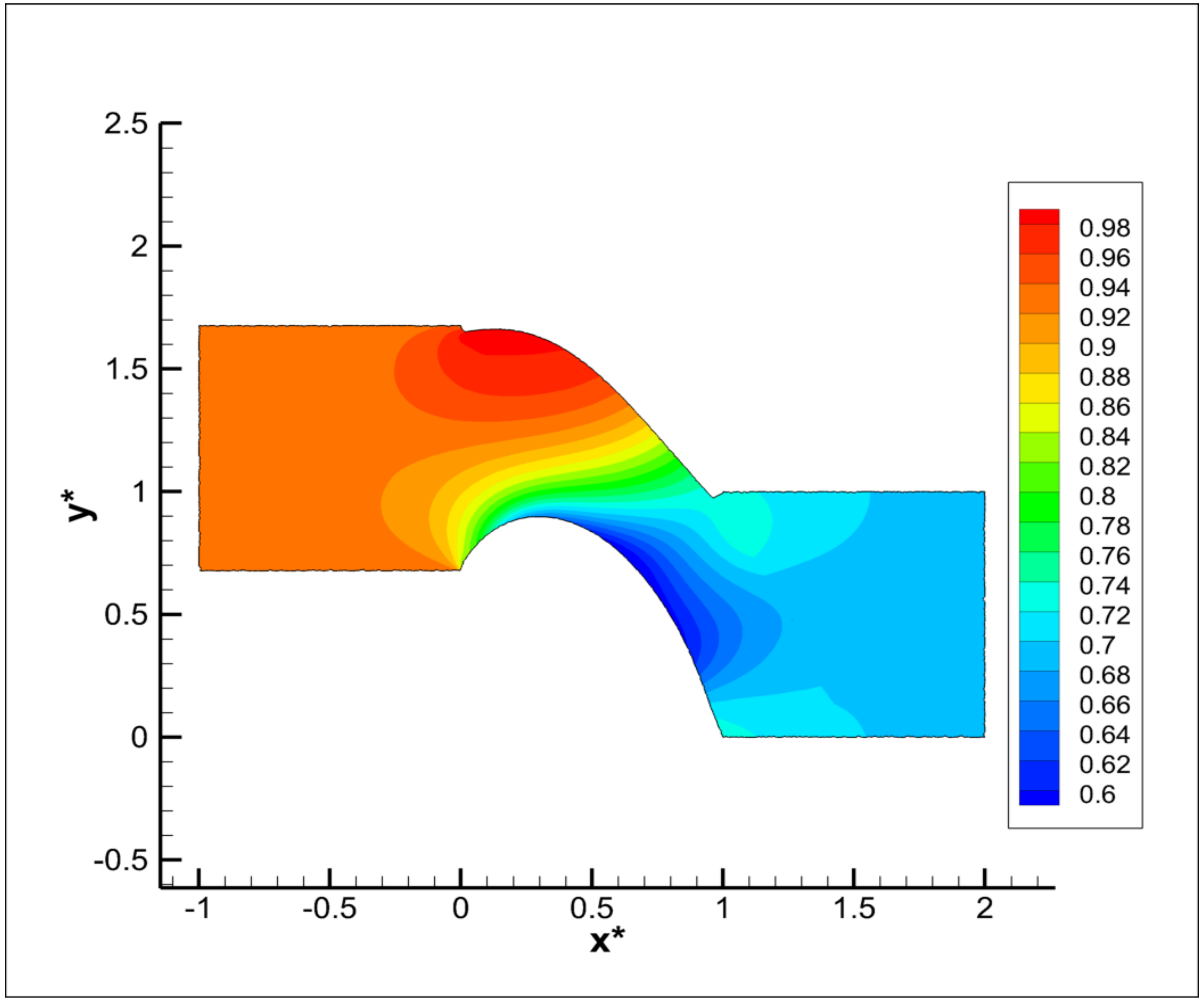


Inverse

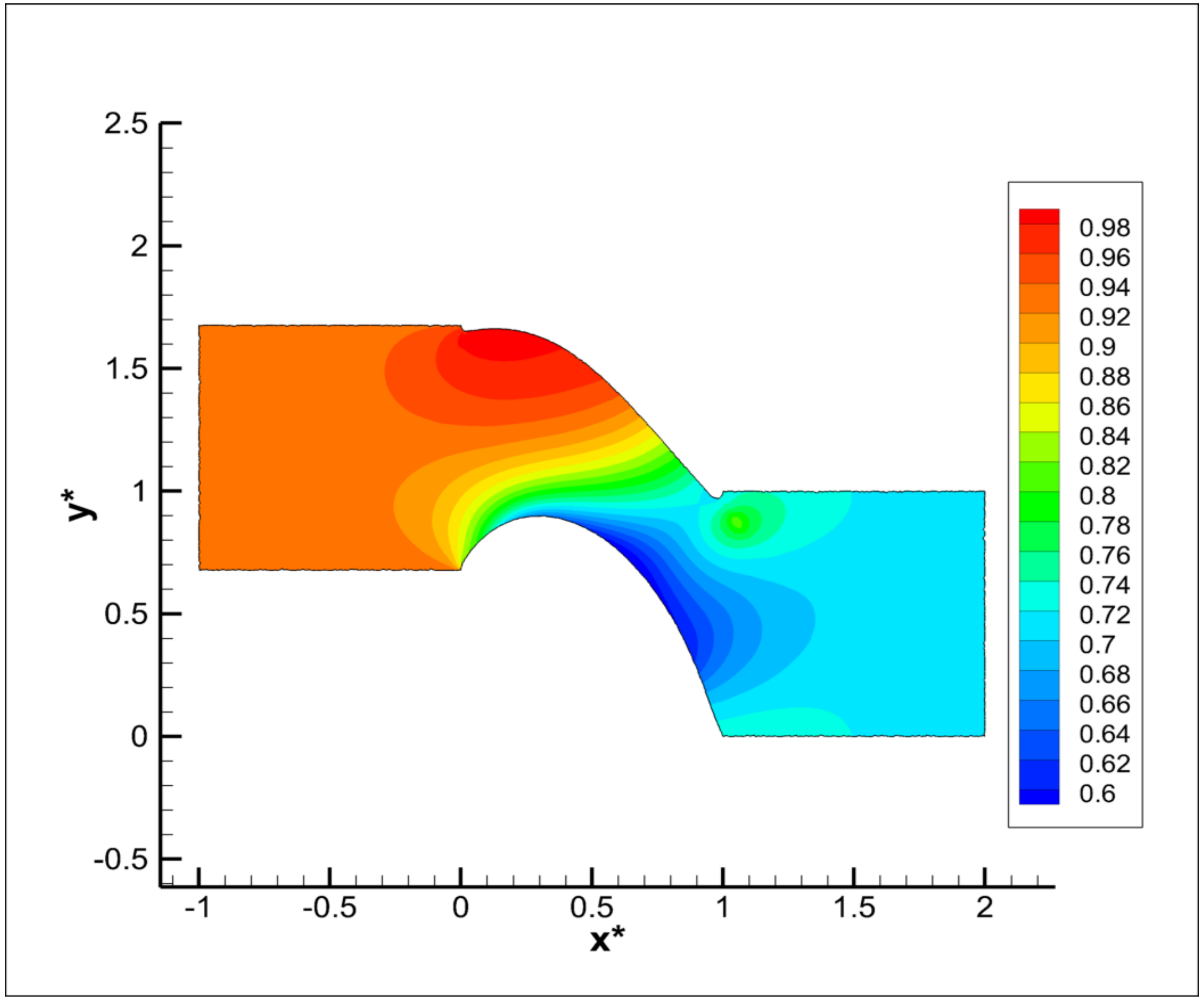


Comparison of Pressure – CMC5

Assisted-Training



Inverse



Percent Errors – Inverse Cases

	$\varepsilon_{Ma,inlet}$ [%]	$\varepsilon_{Ma_{is},outlet}$ [%]
CMC5	0.699	4.215
CMC7	0.984	2.940
CMC9	0.617	0.942

	$\varepsilon_{p,inlet}$ [%]	$\varepsilon_{p,outlet}$ [%]
CMC5	0.33	2.87
CMC7	0.74	2.04
CMC9	0.11	0.67

- Percent error between PINNs-predicted values and the measured values from (Giel et al., 2020).
- Errors for Mach number ranged from 0.62 – 4.22%
- Errors for Pressure ranged from 0.11 – 2.87%
- Errors were consistently higher at the outlet

Conclusions

Questions to be studied:

- Are the data typically acquired in a cascade test enough to train a PINN to reconstruct flow through the passage?
  - Yes, the 41 static taps allowed the PINN to reconstruct the field
  - Even with half the amount of data, the PINN reconstructed the field with low errors
- What effect, if any, does the blade loading have on PINN performance?
  - Using data along the entire profile resulted in the lowest error, regardless of the blade loading
- What effect, if any, does selectively sampled data have on PINN performance?
  - For the more front-loaded blade, CMC9, the errors outside of the LE section were lower than the other blades when trained with only the LE data
  - Generally, the PINN solution resulted in low errors in areas where data were supplied, but high errors outside of those areas
- Can the PINN infer unknown boundary conditions, using sparse data?
  - Yes, but to accurately infer both inlet and outlet conditions at once, data along the entire profile were required

Acknowledgements:
This work was funded by NASA's Advanced Air Transport Technology (AATT) Project

References

[1] Raissi, M., Perdikaris, P., and Karniadakis, G. E., 2017, "Physics Informed Deep Learning (Part I): Data-Driven Solutions of Nonlinear Partial Differential Equations."

[2] Raissi, M., Perdikaris, P., and Karniadakis, G. E., 2017, "Physics Informed Deep Learning (Part II): Data-Driven Discovery of Nonlinear Partial Differential Equations."

[3] Markidis, S., 2021, "The Old and the New: Can Physics-Informed Deep-Learning Replace Traditional Linear Solvers?," Front. Big Data, **4**.

[4] Cai, S., Wang, Z., Wang, S., Perdikaris, P., and Karniadakis, G. E., 2021, "Physics-Informed Neural Networks for Heat Transfer Problems," J. Heat Transf., **143**(060801).

[5] Hanrahan, S. K., Kozul, M., and Sandberg, R. D., 2023, "Predicting Transitional and Turbulent Flow Around a Turbine Blade With a Physics-Informed Neural Network," American Society of Mechanical Engineers Digital Collection.

[6] Post, P., Winhart, B., and di Mare, F., 2022, "Investigation of Physics-Informed Neural Networks Based Solution Techniques for Internal Flows," *GT2022*, Volume 10C: Turbomachinery — Design Methods and CFD Modeling for Turbomachinery; Ducts, Noise, and Component Interactions.

[7] Mao, Z., Jagtap, A. D., and Karniadakis, G. E., 2020, "Physics-Informed Neural Networks for High-Speed Flows," Comput. Methods Appl. Mech. Eng., **360**, p. 112789.

[8] Giel, P., Shyam, V., Juangphanich, P., and Clark, J. P., 2020, "Effects of Trailing Edge Thickness and Blade Loading Distribution on the Aerodynamic Performance of Simulated CMC Turbine Blades," *Proceedings of the ASME Turbo Expo*, American Society of Mechanical Engineers, pp. 1–12.

Thank you for your attention

Appendix

Future Work

- Include viscous terms
  - Finished for CMC9 (primary interest)
- Incorporate measured data from the wake region
  - Finished for CMC9 at same conditions as this study
- Expand into the FANS equations
  - Use measured loading data and wake data from Station 2 (u, v, p, T along pitch at x*=1.203)
  - Assess whether this is enough data to close FANS equations, or if more data is needed.
    - <u>If more data is needed</u>: Use data from LES of CMC9 to assess concept, with a focus on using the minimum amount of data possible
    - Assess which Reynolds stress terms are most important for training
- Apply to aerothermal studies
- Apply to rotating test campaigns

Percent Error – Forward Problem

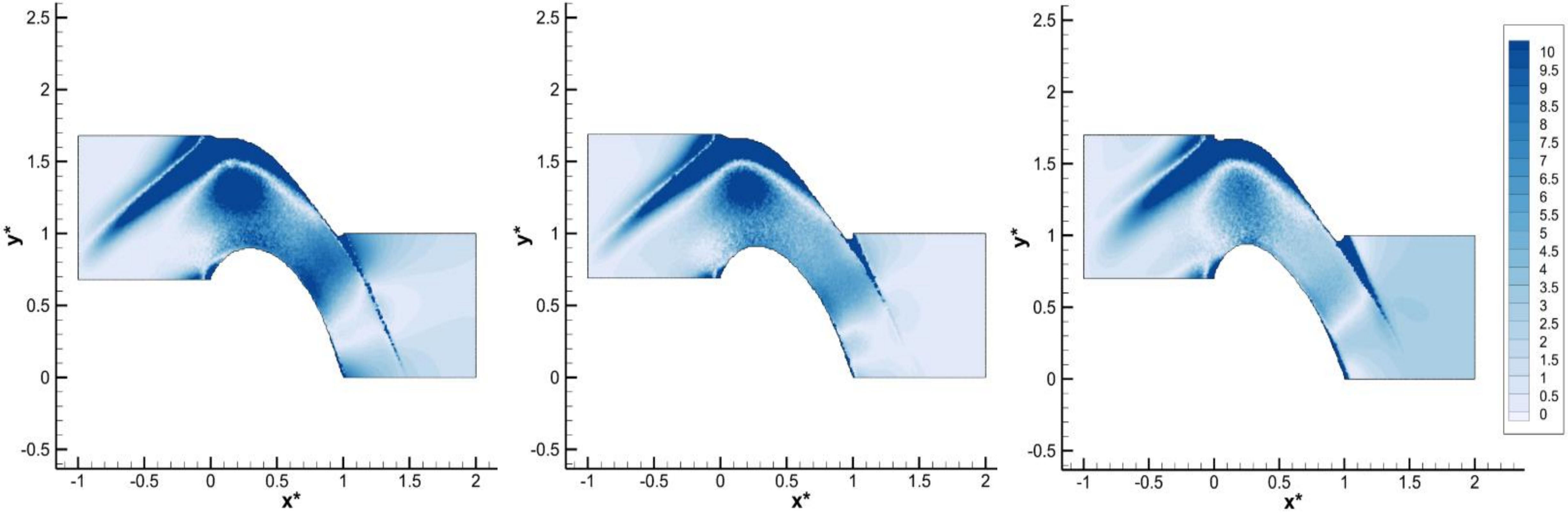


FIGURE A1: Percent error between the calculated Mach number from the PINN solution of the case trained with all available data and the forward problem for the CMC5 (left), CMC7 (center), and CMC9 (right)

Fit to Data – Trained vs Inverse (CMC7)

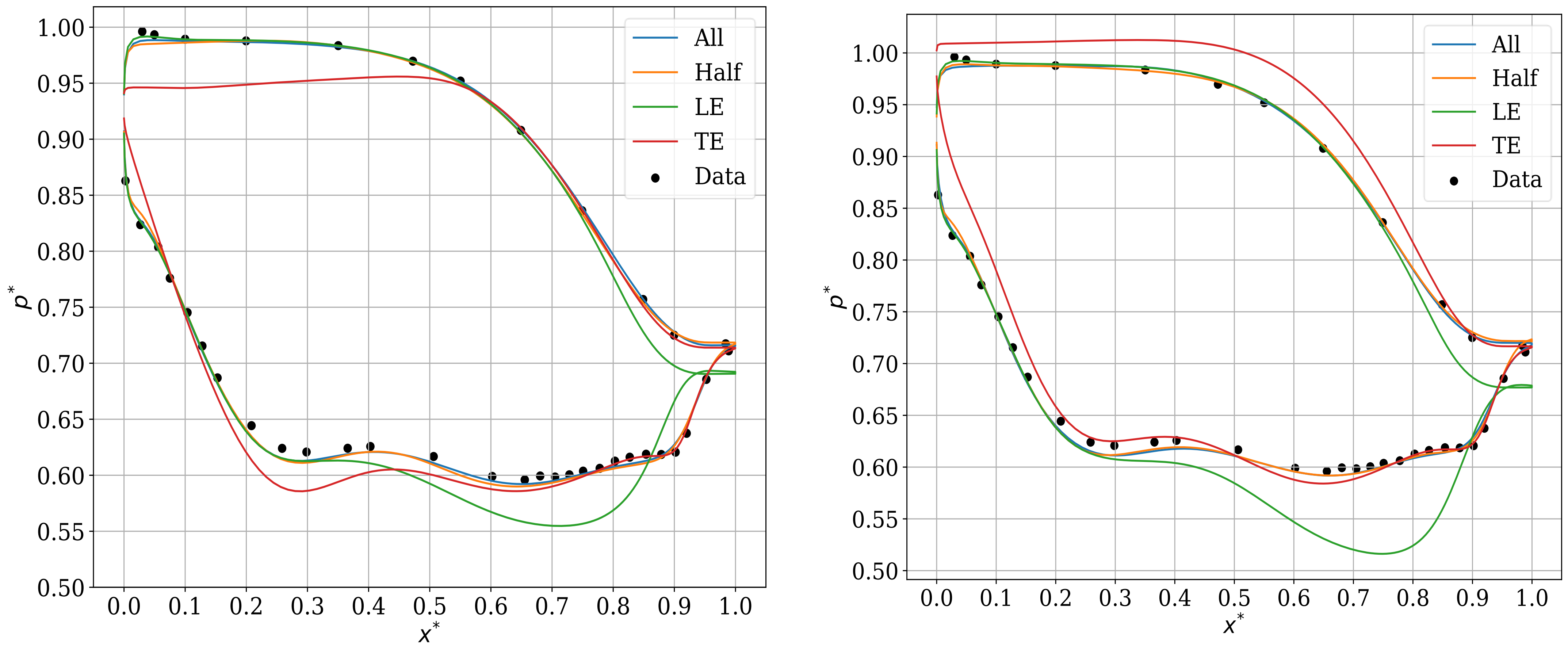


FIGURE A2: Pressure distribution for the assisted training (left) and inverse (right) models

Mach – Trained vs Inverse (CMC7)

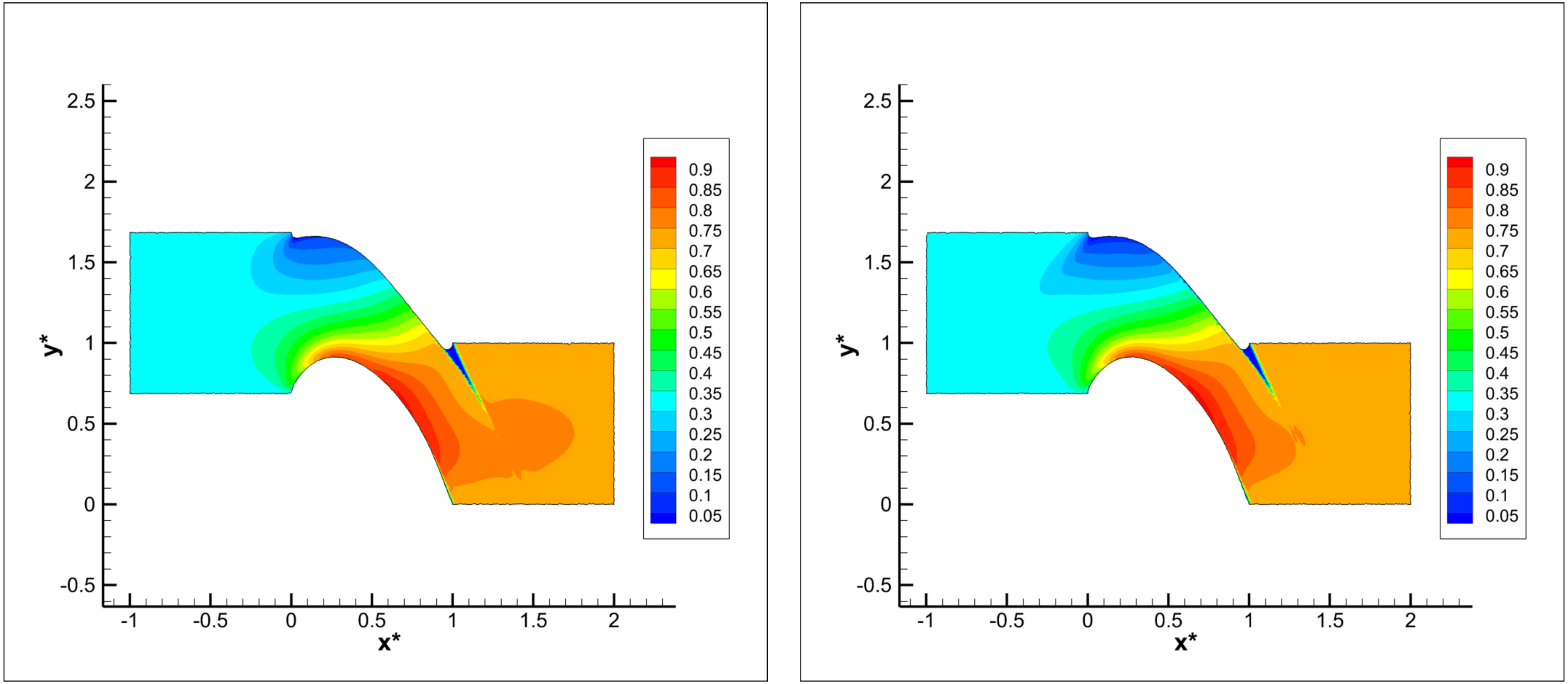


FIGURE A3: Mach number contours for the assisted training (left) and inverse (right) models

Pressure – Trained vs Inverse (CMC7)

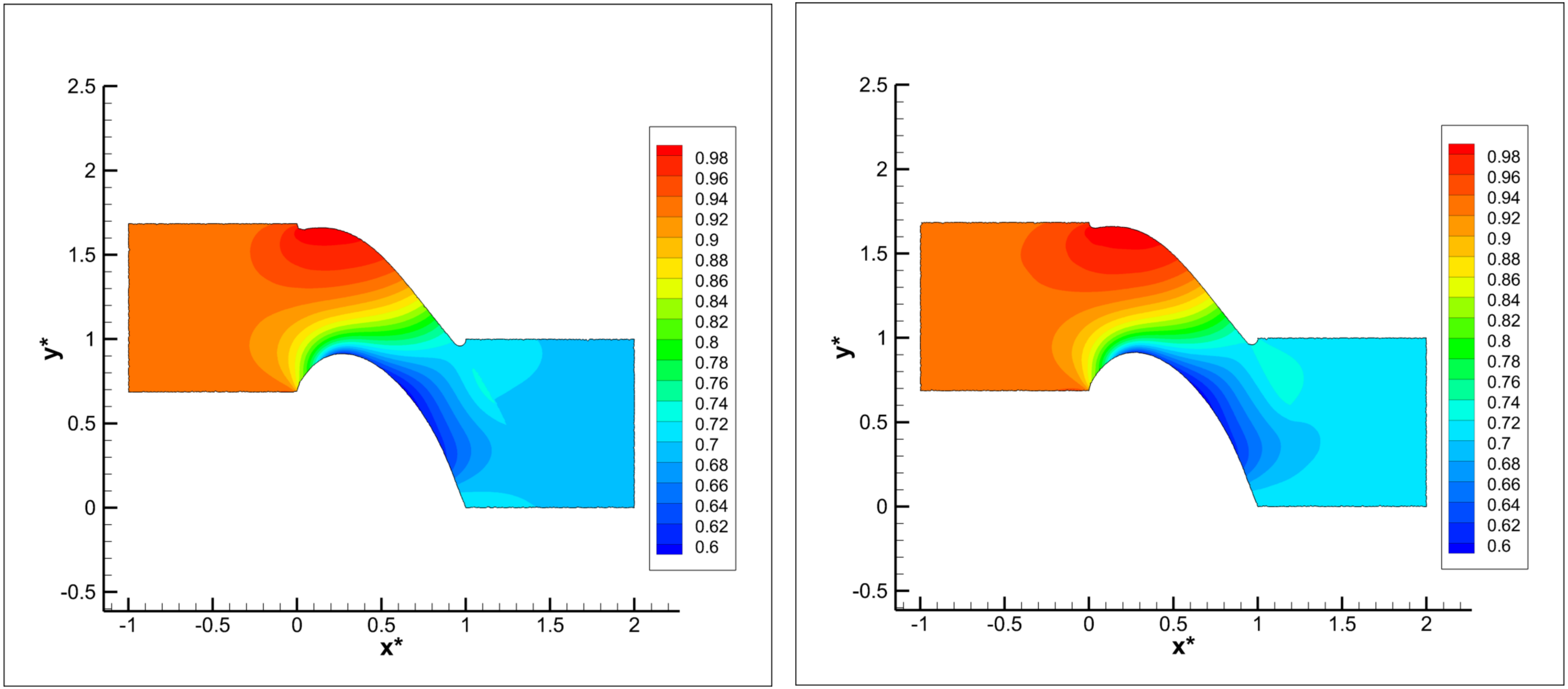


FIGURE A4: Pressure contours for the assisted training (left) and inverse (right) models

Fit to Data – Trained vs Inverse (CMC9)

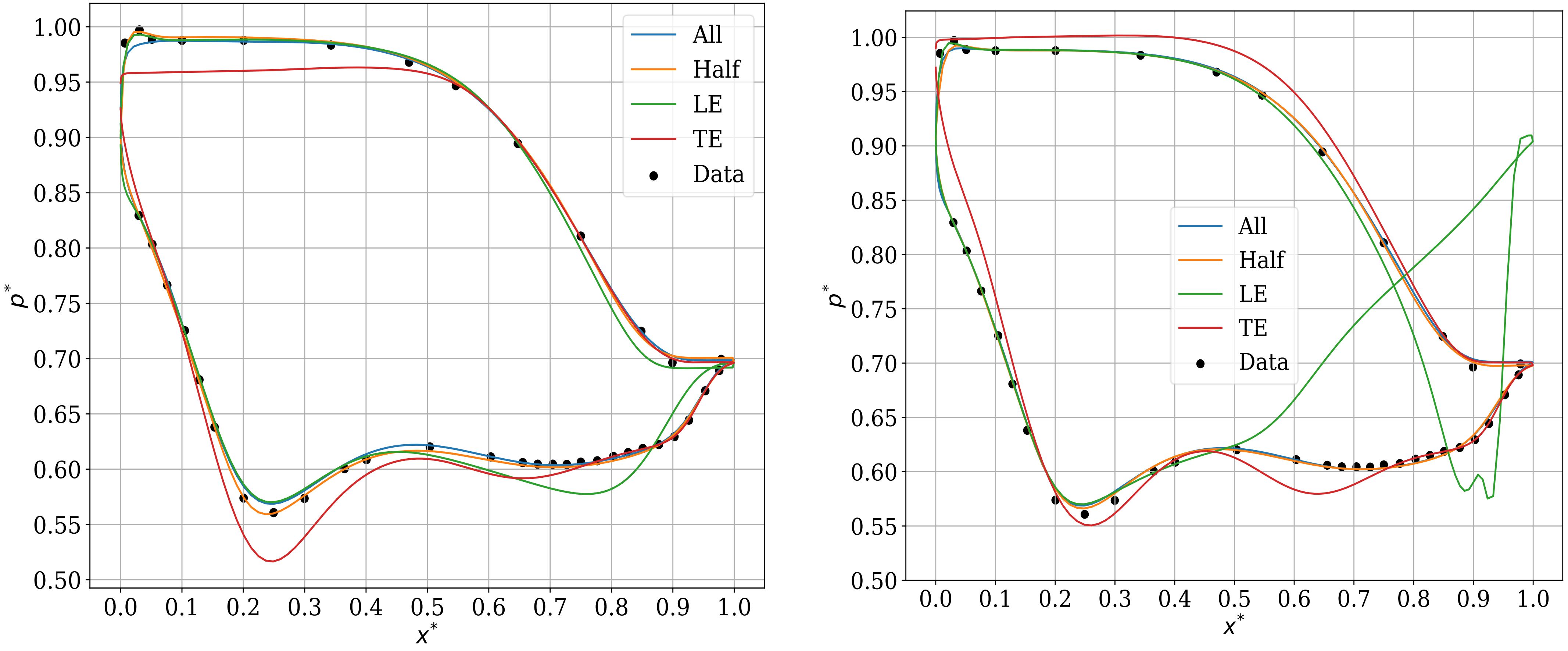


FIGURE A5: Pressure distribution for the assisted training (left) and inverse (right) models

Mach – Trained vs Inverse (CMC9)

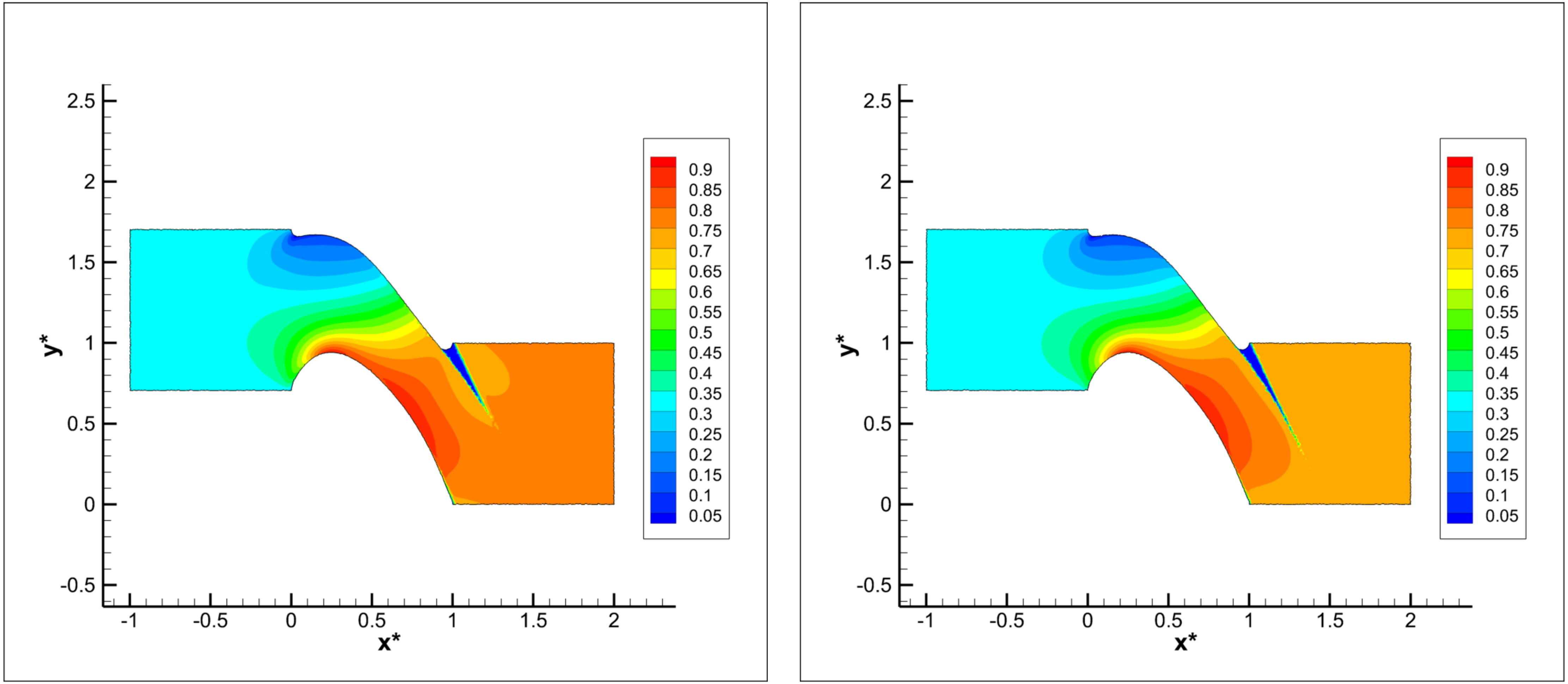


FIGURE A6: Mach number contours for the assisted training (left) and inverse (right) models

Pressure – Trained vs Inverse (CMC9)

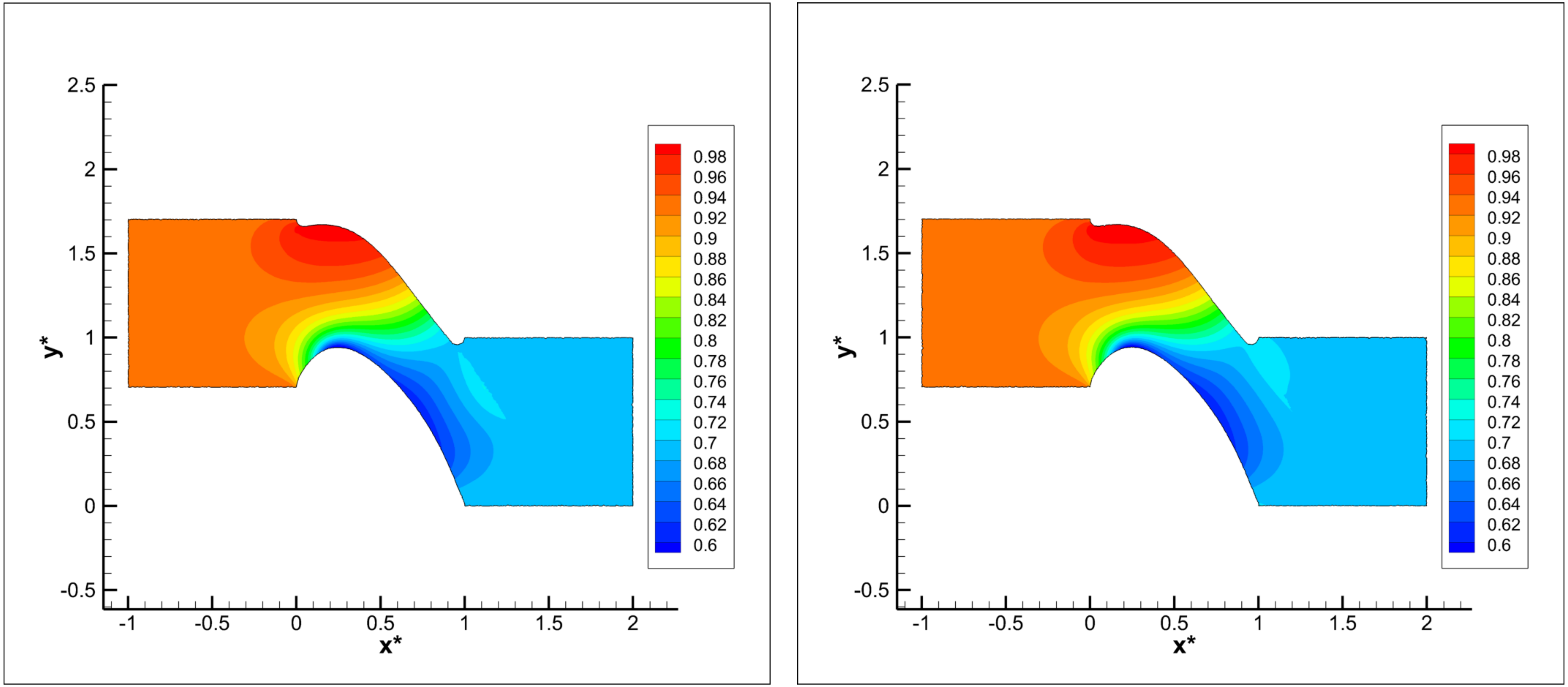


FIGURE A7: Pressure contours for the assisted training (left) and inverse (right) models

Inlet and Outlet Pressure Profiles

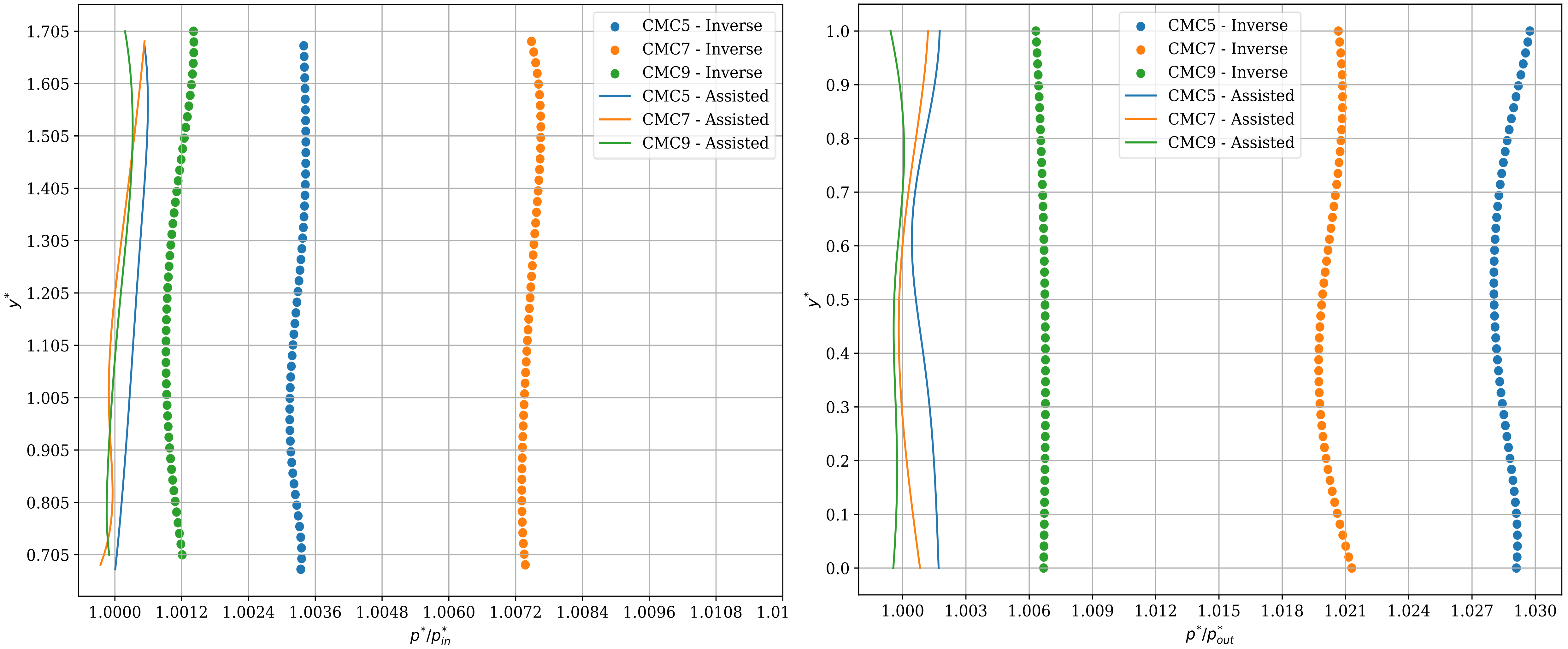


FIGURE A8: Pressure distribution at the inlet (left) and outlet (right) for all-data inverse and assisted training cases