# A Green's Function Sensor Fusion Approach for Evaluating Spacecraft Entry Heating from On-Board Thermal Instrumentation

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*University of Maryland, College Park, MD*

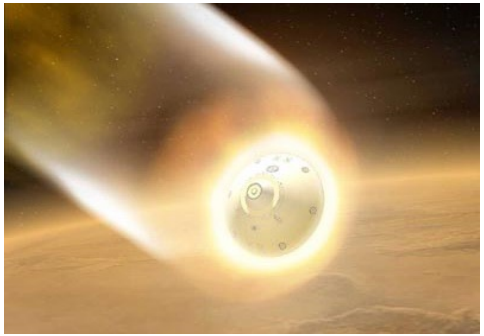
UNIVERSITY OF MARYLAND

**Hannah S. Alpert**
*NASA Ames Research Center, Moffett Field, CA*

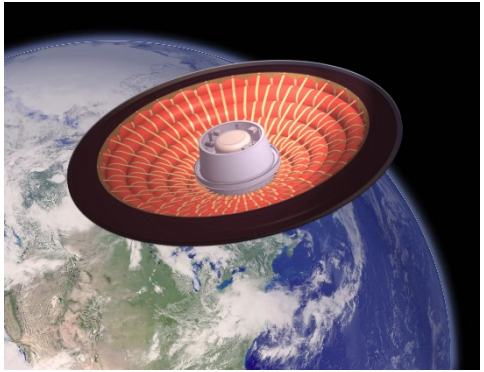
NASA

AIAA Aviation Forum, July 31st, 2024

# Motivation

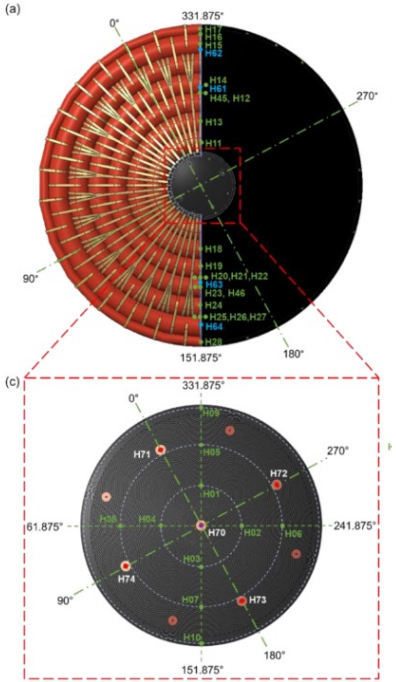


https://mars.nasa.gov/mer/gallery/artwork/entry_br.htm



https://www.nasa.gov/general/low-earth-orbit-flight-test-of-an-inflatable-decelerator-loftid-overview/

## LOFTID (2022)

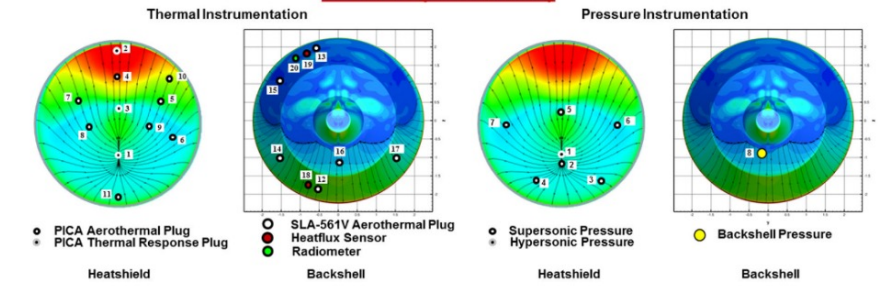



Swanson, G., et al. AIAA SciTech (2024)

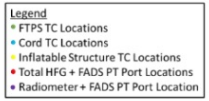
## MEDLI (MSL 2012)



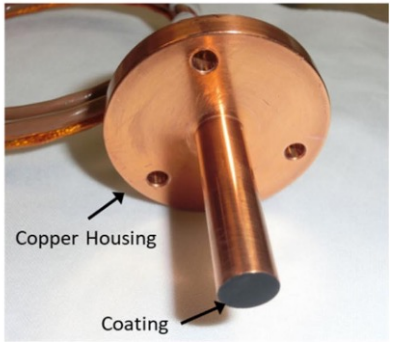

## MEDLI2 (Mars 2020)

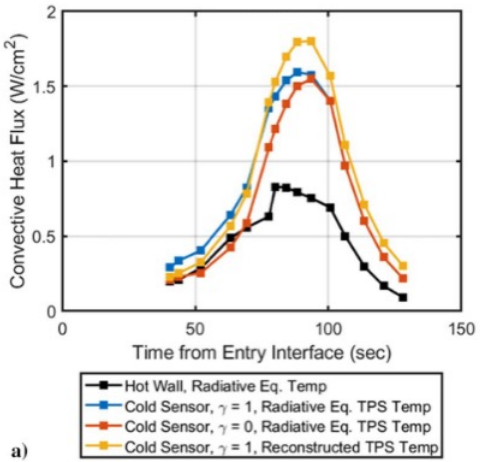



Hwang, H., et al. AIAA Aviation (2016)

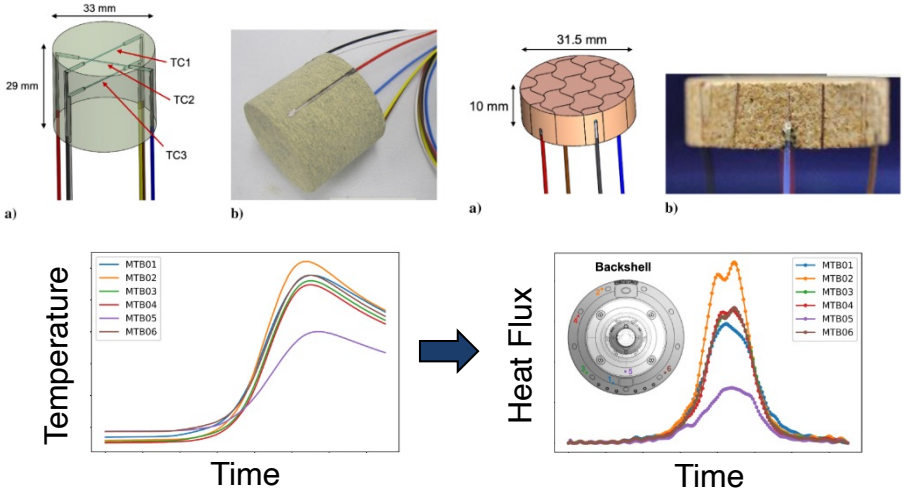


# Recovering TPS Surface Heat Flux




Miller, R., et al. J Spacecr Rockets (2024)



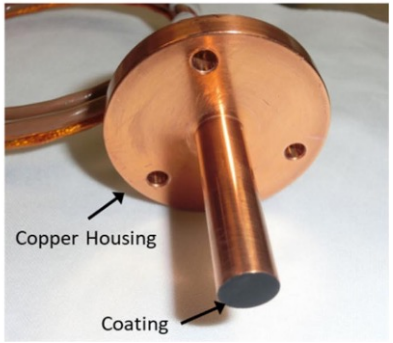

Alpert, H., et al. J Spacecr Rockets (2023)

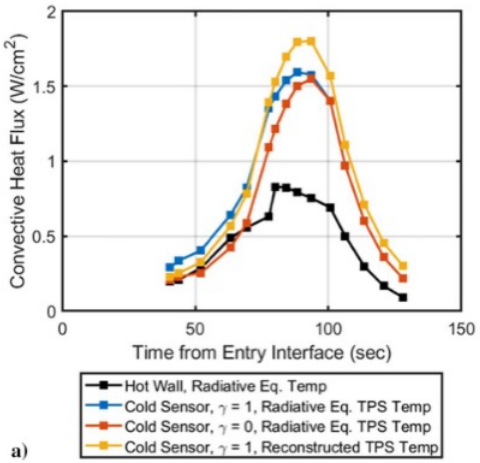
+ Direct measurement (stable)
– Cold-wall conditions

+ Hot wall conditions
– Indirect reconstruction (unstable)

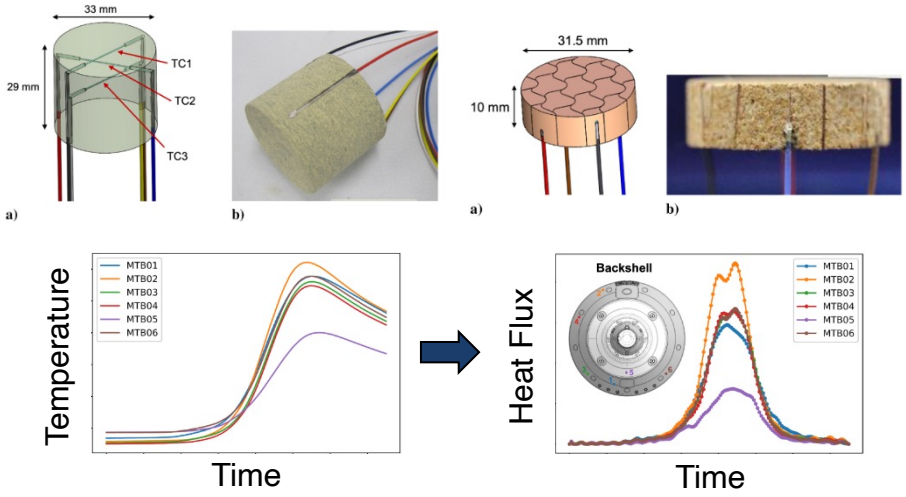
– Time-marching based methods

# Recovering TPS Surface Heat Flux




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+ Direct measurement (stable)
– Cold-wall conditions

+ Hot wall conditions
– Indirect reconstruction (unstable)

– Time-marching based methods

# Hot-Wall Heat Flux Reconstruction Algorithm

- Objective:
  - *Develop an algorithm to reconstruct the local hot-wall TPS heat flux solely from collocated temperature and heat flux sensor instrumentation.*
- Two Components
  1. Efficient thermal model of the TPS-Sensor system
     - $q \rightarrow T$ and $T \rightarrow q$ in one (or few) step(s).
  2. Mechanism to stabilize hot-wall heat flux recovery

# Thermal Model

| | | | |
|---|---|---|---|
| $T$ | Temperature | $G$ | Green's Function |
| $\alpha$ | Thermal Diffusivity | $\delta$ | Dirac-Delta Function |
| $k$ | Thermal Conductivity | $g$ | Volu. Energy Generation |

- Green's function framework

Cole, K., et al. CRC Press (2010)

Massa, L., and Schetz, J.,
J Thermophys Heat Trans (2020)

Whalen, T., et al.
Meas Sci Technol (2021)

Nguyen, N., et al.
J Thermophys Heat Trans (2021)

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} - \nabla^2 T = \frac{1}{k} g(\boldsymbol{x}, t)$$

Heat Conduction Equation

$$\frac{1}{\alpha}\frac{\partial G}{\partial t} - \nabla^2 G = \frac{1}{\alpha}\delta(\boldsymbol{x} - \boldsymbol{x_0}, t - \tau)$$

Green's Function Analogue

$$T(\boldsymbol{x}, t) = \underbrace{\int_V ICs \; dV_0}_{\text{Initial conditions}} + \underbrace{\int_0^t \int_S BCs \; dS_0 \, d\tau}_{\text{Boundary conditions}} + \underbrace{\int_0^t \int_V Energy \; gen \; dV_0 \, d\tau}_{\text{Volumetric energy generation}}$$

# Thermal Model

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$T_0$

Initial conditions

Boundary conditions

Volumetric energy generation

# Thermal Model

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Green's Function Analogue

$$T(\boldsymbol{x}, t) - T_0 = \int_0^t \int_S BCs \; dS_0 \; d\tau$$

# Thermal Model – Applied

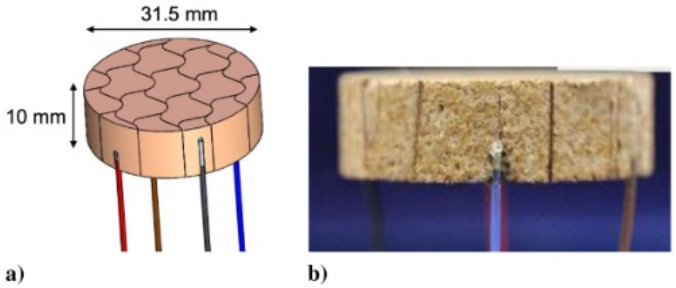
- Applied to the TPS

TPS properties: $\frac{\alpha_{TPS}}{k_{TPS}}$

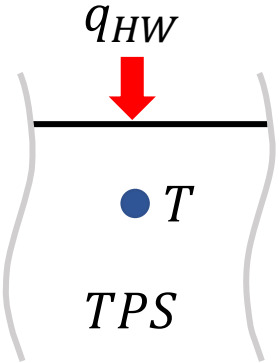
$TPS \rightarrow TPS$ Green's function: $G_{TPS,TPS}(\boldsymbol{x}, t, \boldsymbol{x_0}, \tau)$

Hot-wall heat flux: $q_{HW}(\tau)$

$$T(\boldsymbol{x}, t) - T_0 = -\int_0^t \int_{S_{TPS}} \frac{\alpha_{TPS}}{k_{TPS}} G_{TPS,TPS}(\boldsymbol{x}, t, \boldsymbol{x_0}, \tau) q_{HW}(\tau) dS_0 \, d\tau$$




Alpert, H., et al. J Spacecr Rockets (2023)



# Thermal Model – TPS-Sensor System

- TPS-sensor system

TPS properties: $\frac{\alpha_{TPS}}{k_{TPS}}$

$TPS \rightarrow TPS$ Green's function: $G_{TPS,TPS}(\boldsymbol{x}, t, \boldsymbol{x_0}, \tau)$

Hot-wall heat flux: $q_{HW}(\tau)$

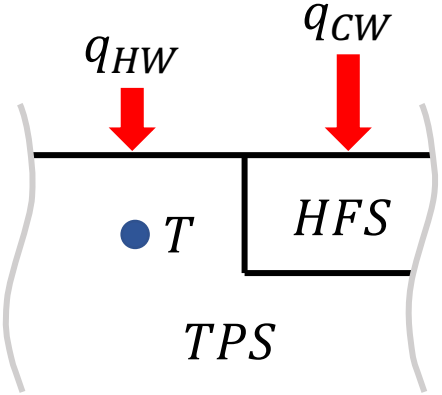
$$T(\boldsymbol{x}, t) - T_0 = -\int_0^t \int_{S_{TPS}} \frac{\alpha_{TPS}}{k_{TPS}} G_{TPS,TPS}(\boldsymbol{x}, t, \boldsymbol{x_0}, \tau) q_{HW}(\tau) dS_0 \, d\tau$$

$$-\int_0^t \int_{S_{HFS}} \frac{\alpha_{HFS}}{k_{HFS}} G_{TPS,HFS}(\boldsymbol{x}, t, \boldsymbol{x_0}, \tau) q_{CW}(\tau) dS_0 \, d\tau$$

HFS properties: $\frac{\alpha_{HFS}}{k_{HFS}}$

$HFS \rightarrow TPS$ Green's function: $G_{TPS,HFS}(\boldsymbol{x}, t, \boldsymbol{x_0}, \tau)$

Cold-wall heat flux: $q_{CW}(\tau)$



# Thermal Model – TPS-Sensor System

- TPS-sensor system

$$T(\boldsymbol{x},t) - T_0 = -\int_0^t \int_{S_{TPS}} \frac{\alpha_{TPS}}{k_{TPS}} G_{TPS,TPS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau) q_{HW}(\tau) dS_0\, d\tau$$

$$-\int_0^t \int_{S_{HFS}} \frac{\alpha_{HFS}}{k_{HFS}} G_{TPS,HFS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau) q_{CW}(\tau) dS_0\, d\tau$$

$$\{\Delta T\} + \frac{\alpha_{HFS}}{k_{HFS}} [G_{TPS,HFS}]\{q_{CW}\} = -\frac{\alpha_{TPS}}{k_{TPS}} [G_{TPS,TPS}]\{q_{HW}\}$$

Whalen, T., et al. Meas Sci Technol (2021)

# Thermal Model – TPS-Sensor System

- TPS-sensor system

$$T(\boldsymbol{x},t) - T_0 = -\int_0^t \int_{S_{TPS}} \frac{\alpha_{TPS}}{k_{TPS}} G_{TPS,TPS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau) q_{HW}(\tau) dS_0\, d\tau$$

$$-\int_0^t \int_{S_{HFS}} \frac{\alpha_{HFS}}{k_{HFS}} G_{TPS,HFS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau) q_{CW}(\tau) dS_0\, d\tau$$

Measurables $= \{\gamma\}$ ⇩ Unknown

$$\{\Delta T\} + \frac{\alpha_{HFS}}{k_{HFS}} [G_{TPS,HFS}]\{q_{CW}\} = -\frac{\alpha_{TPS}}{k_{TPS}} [G_{TPS,TPS}]\{q_{HW}\}$$

Whalen, T., et al. Meas Sci Technol (2021)

# Recovering $q_{HW}$

- Regularization used to stabilize inversion
- E.g., Tikhonov regularization

User defined

$$\min\left\{\left\|\{\gamma\}+\frac{\alpha_{TPS}}{k_{TPS}}\left[G_{TPS,TPS}\right]\{q_{HW}\}\right\|_2^2+\|\lambda\{q_{HW}\}\|_2^2\right\}$$

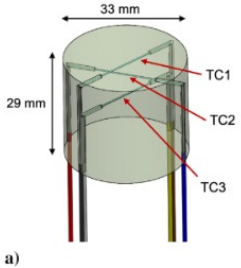
Residual

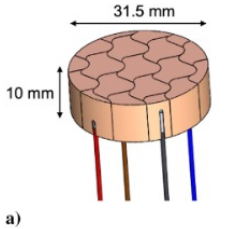
Penalty Function

Hansen, P. C., Numer Algorithms (2002)

# Recovering $q_{HW}$ – TPS Challenges

- System condition is worsened due to
  - TPS is significantly more insulating than most materials.
    - E.g., SLA-561 (heritage): $k \approx 0.05\ W/m \cdot K$, $\alpha \approx 2{\times}10^{-7}\ m^2/s$
  - Temperature sensors are embedded under the TPS surface
- Large values of $\lambda$ are needed to damp instabilities
  - loss of sharp solution features.




Alpert, H., et al. J Spacecr Rockets (2023)

Smyly, E. and Pears, C., NAS1-7732-1 (1971)

# Recovering $q_{HW}$ – Regularization Surrogate

- Achieve stability by coupling $q_{HW}$ and $q_{CW}$

$$q_{HW} = C(t)q_{CW}$$

where $C(t)$ is an arbitrary, but ***smooth***, function

- Linear system becomes

$$\{\gamma\} = -\frac{\alpha_{TPS}}{k_{TPS}}\left[G_{TPS,TPS}\right]\left[diag(q_{CW})\right]\{C\}$$

# Recovering $q_{HW}$ – Regularization Surrogate

- Solution of $C$ is stabilized by penalizing the 2nd derivative

$$\min\left\{\left\|\{\gamma\}+\frac{\alpha_{TPS}}{k_{TPS}}[G]\,[diag(q_{CW})\,]\{C\}\right\|_2^2+\|\lambda_2[L_2]\{C\}\|_2^2\right\}$$

User defined ($\lambda_2$)

Residual

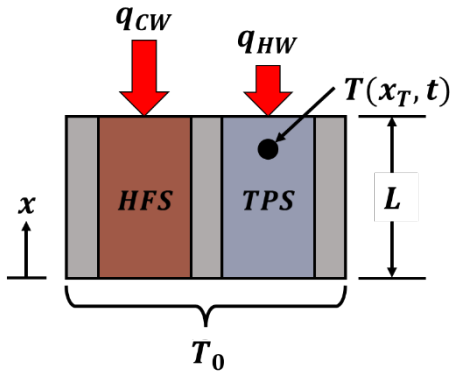
2nd derivative penalty term

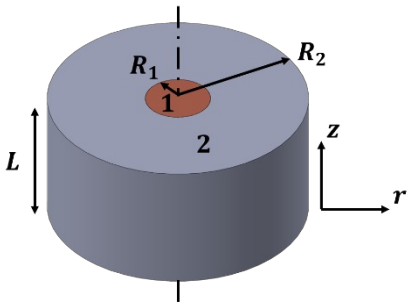
- $q_{HW}$ is recovered via

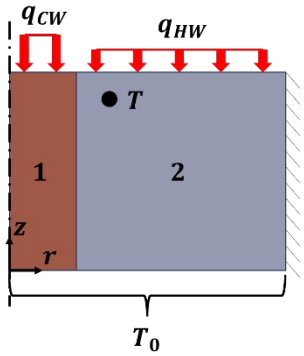
$$\{q_{HW}\}=[diag(q_{CW})]\{C\}$$

# Results

- 1D Model System
  - Algorithm verification
  - Convergence criteria
  - Analysis of algorithm robustness
- Axisymmetric Model System
  - Model verification
  - Analysis of algorithm robustness

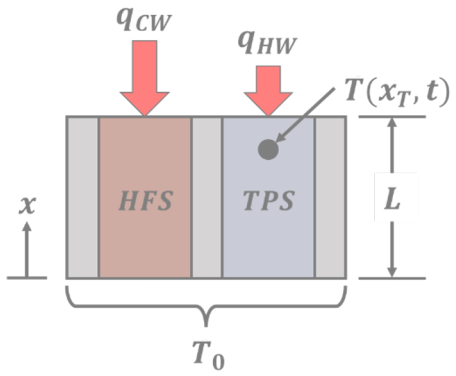


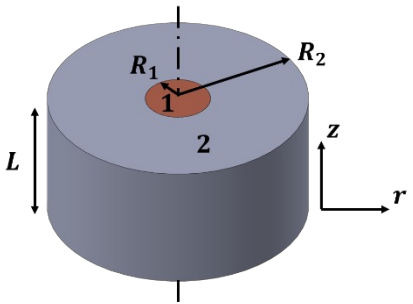



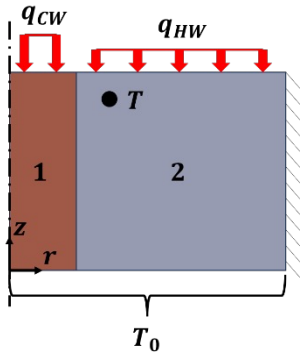



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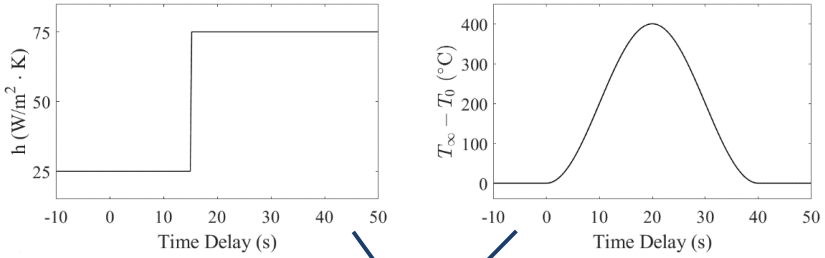


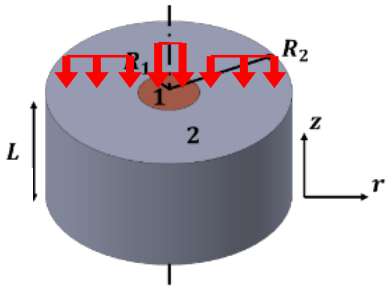



# Results – Axisymmetric Model System

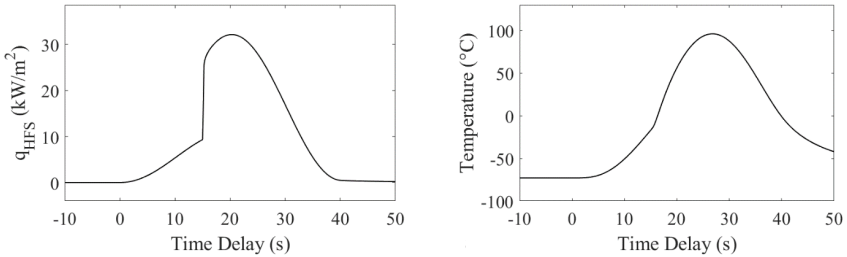
- Axisymmetric model with convective boundary conditions




$$q = h(T_\infty - T_{surf})$$



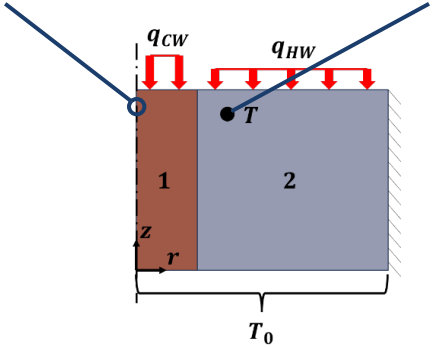

$1/\Delta t = 8Hz$



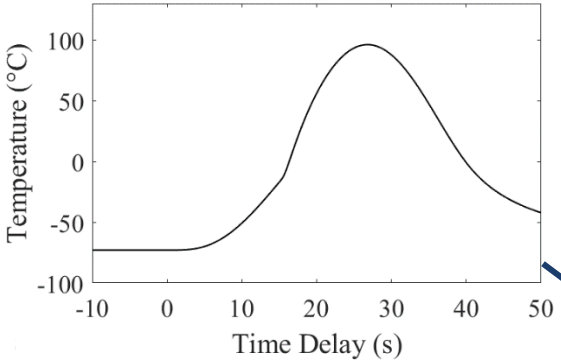

1. Copper
2. SLA-561

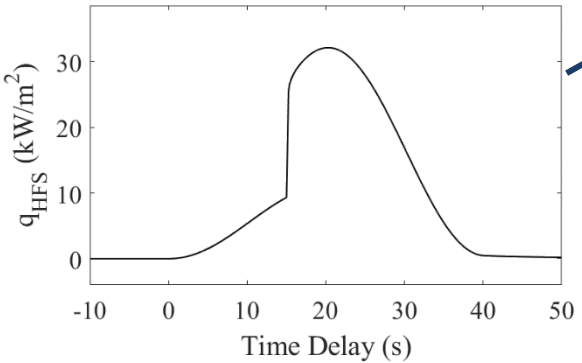
Smyly, E. and Pears, C., NAS1-7732-1 (1971)

Finite Element Analysis (FEA)



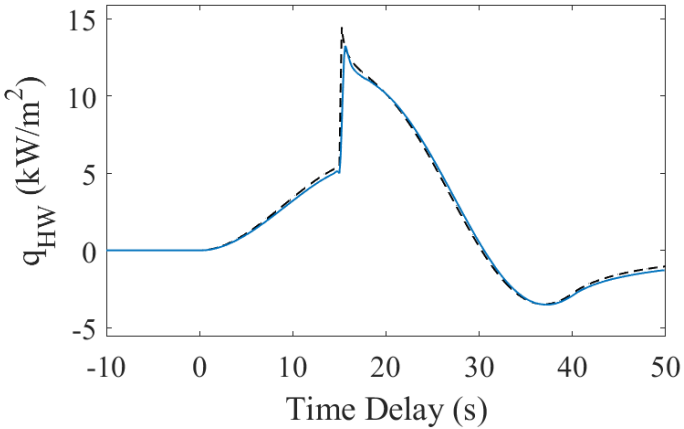
# Algorithm Verification



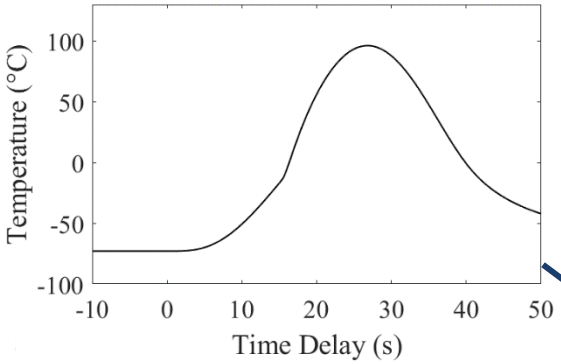



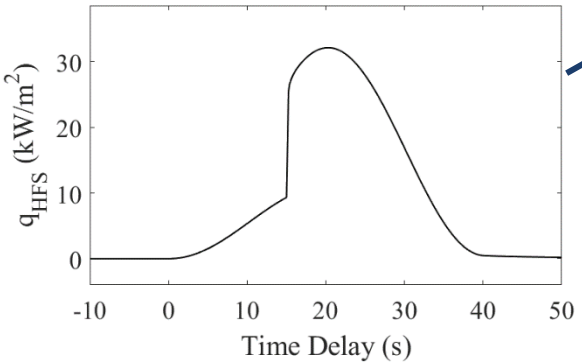

Reconstruction Algorithm

Solution time $\sim\mathcal{O}(20s)$



# Algorithm Verification



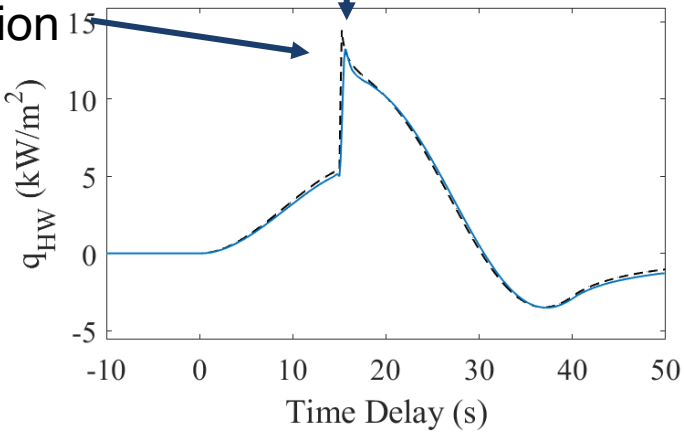




Peak heat flux predicted to within 9% FEA solution

Reconstruction Algorithm

Solution time $\sim\mathcal{O}(20s)$

Peak heat time predicted to within 0.375s FEA solution



# Algorithm Robustness

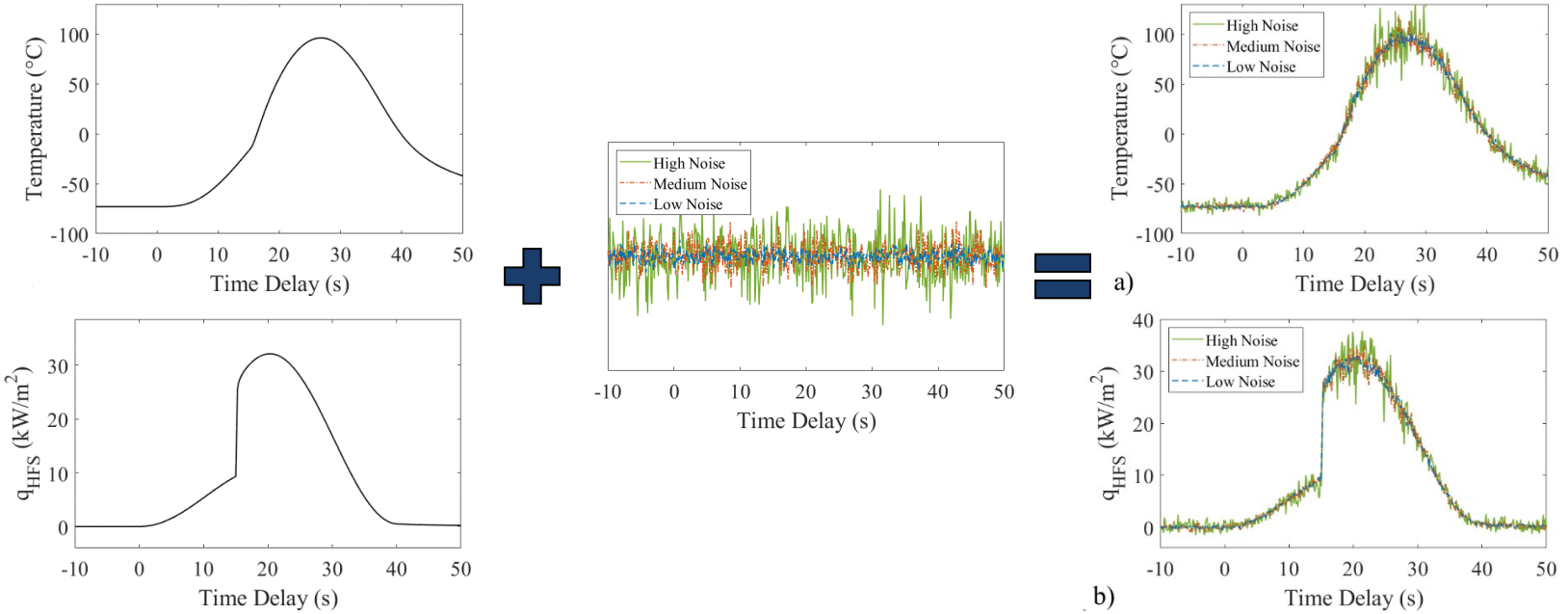
- Real measurements will contain noise or errors.
- Goal: Analyze the impact of varying $\lambda_2$ subject to different input noise levels.

$$\min\left\{\left\|\{\gamma\}+\frac{\alpha_{TPS}}{k_{TPS}}[G]\,[diag(q_{CW})\,]\{C\}\right\|_2^2+\|\lambda_2[L_2]\{C\}\|_2^2\right\}$$

User defined

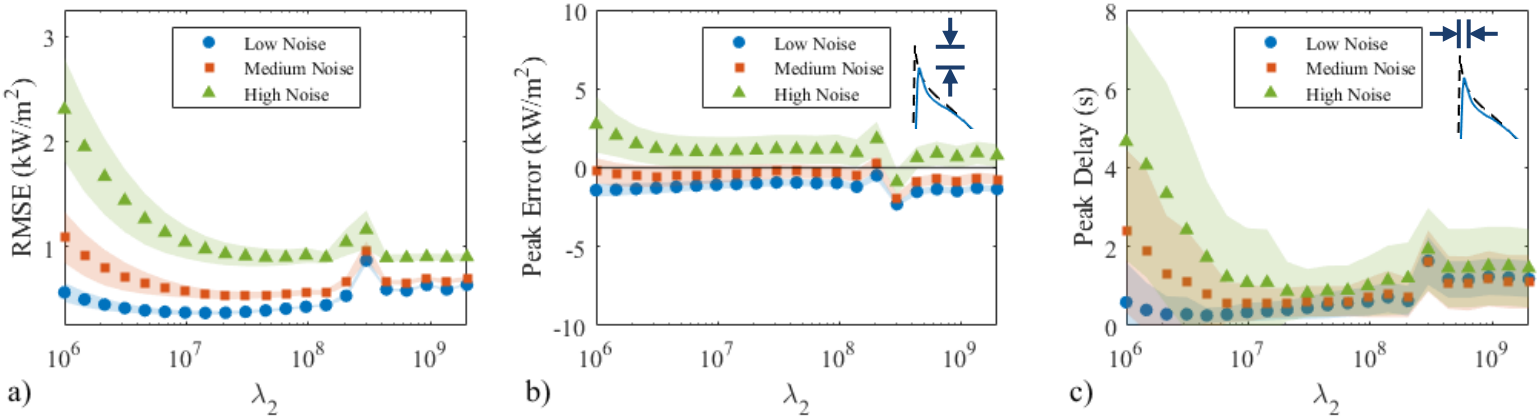
# Algorithm Robustness – Measurement Noise

- FEA-generated measurements perturbed with noise.



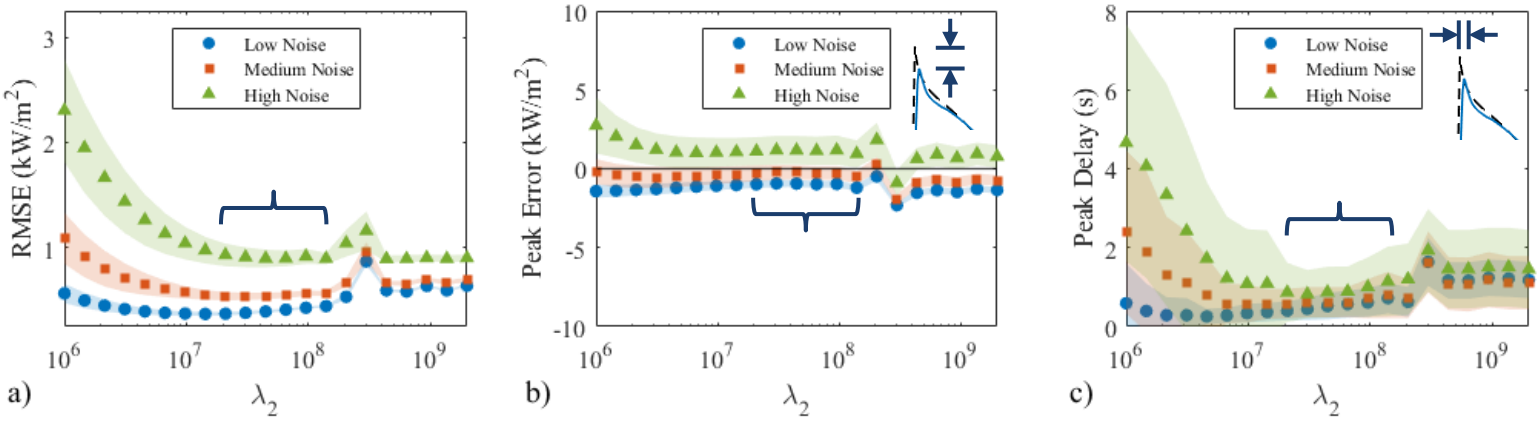
# Varying $\lambda_2$

- Sweep of $\lambda_2$ performed for 3 levels of noise
- Sweeps repeated 50 times to generate distribution of results



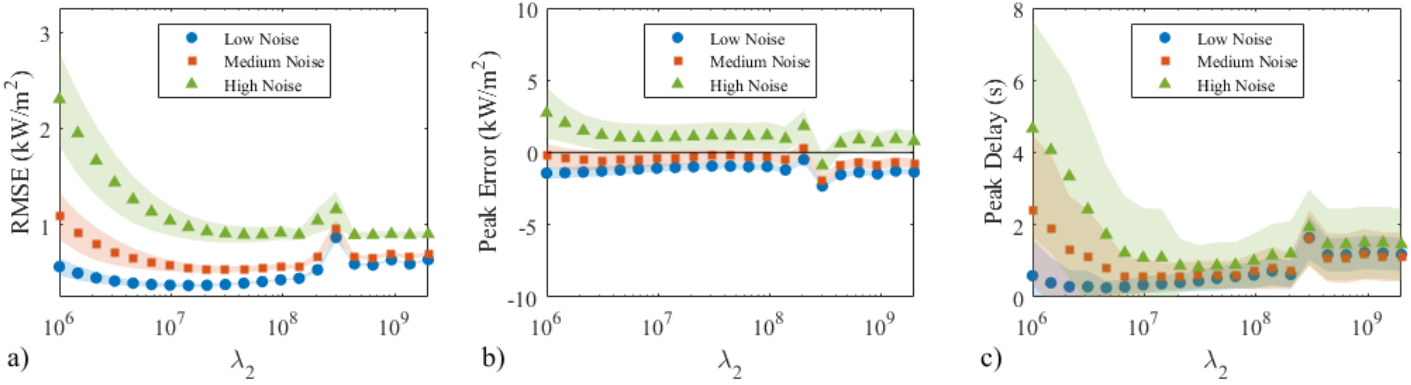
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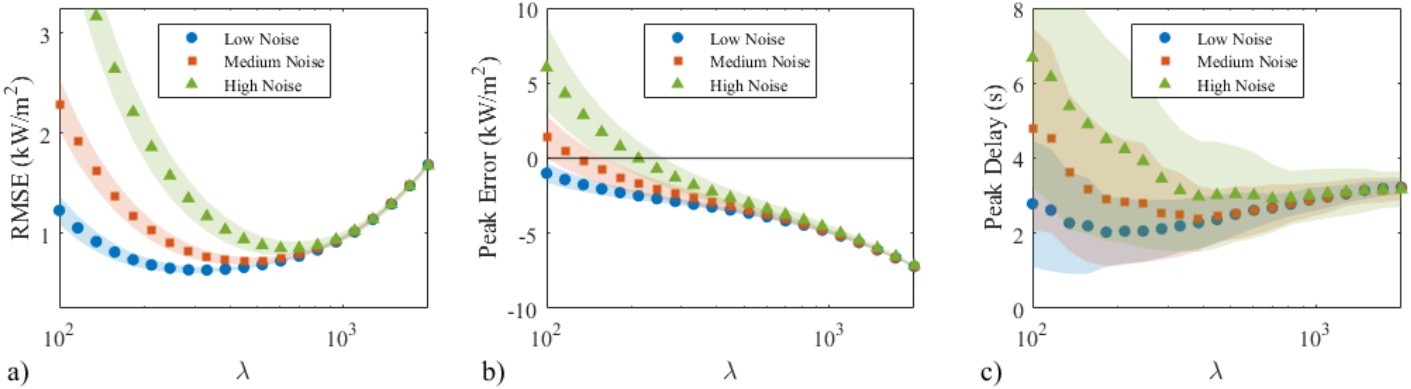


# Comparison vs. Tikhonov Regularization

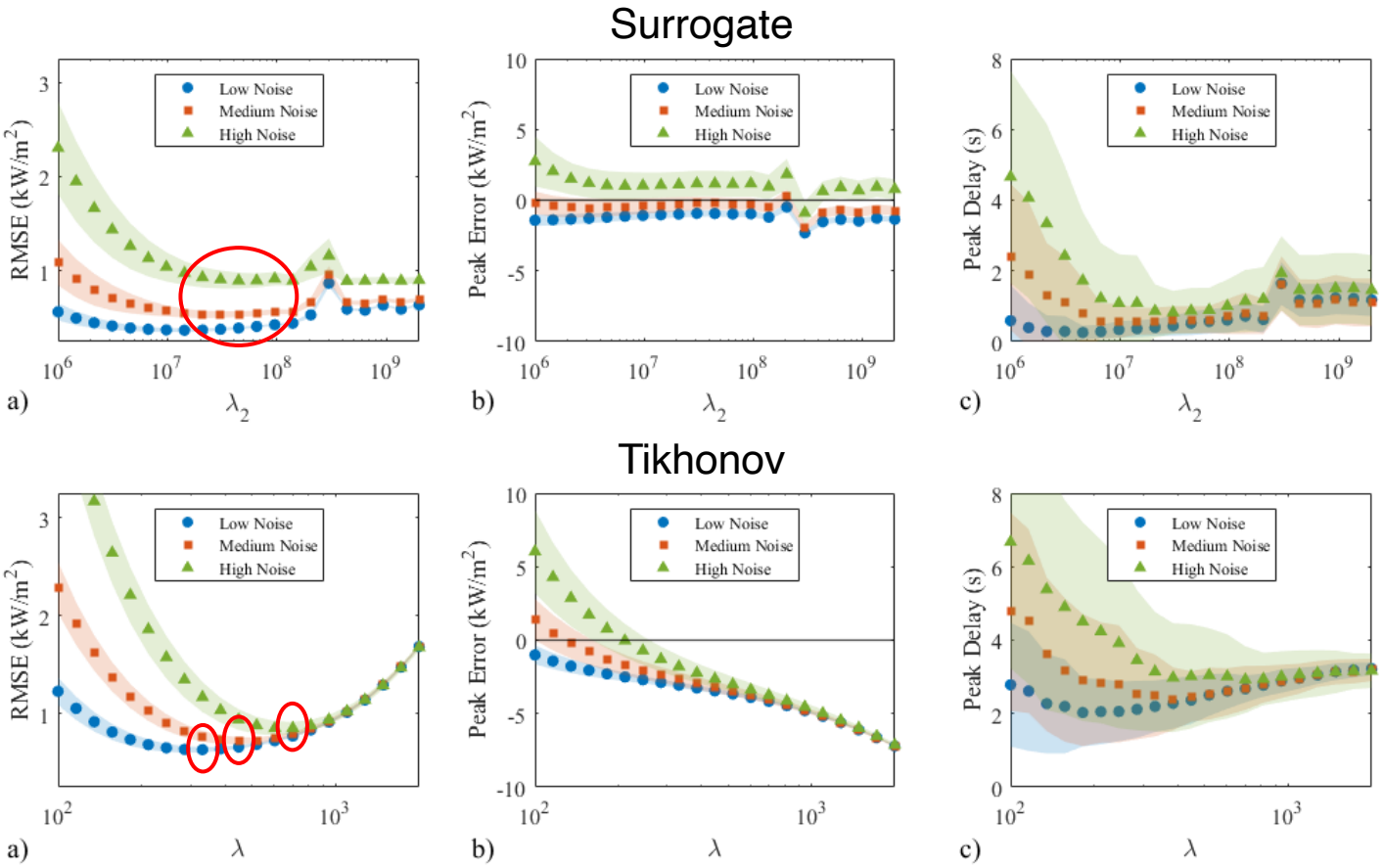
Surrogate



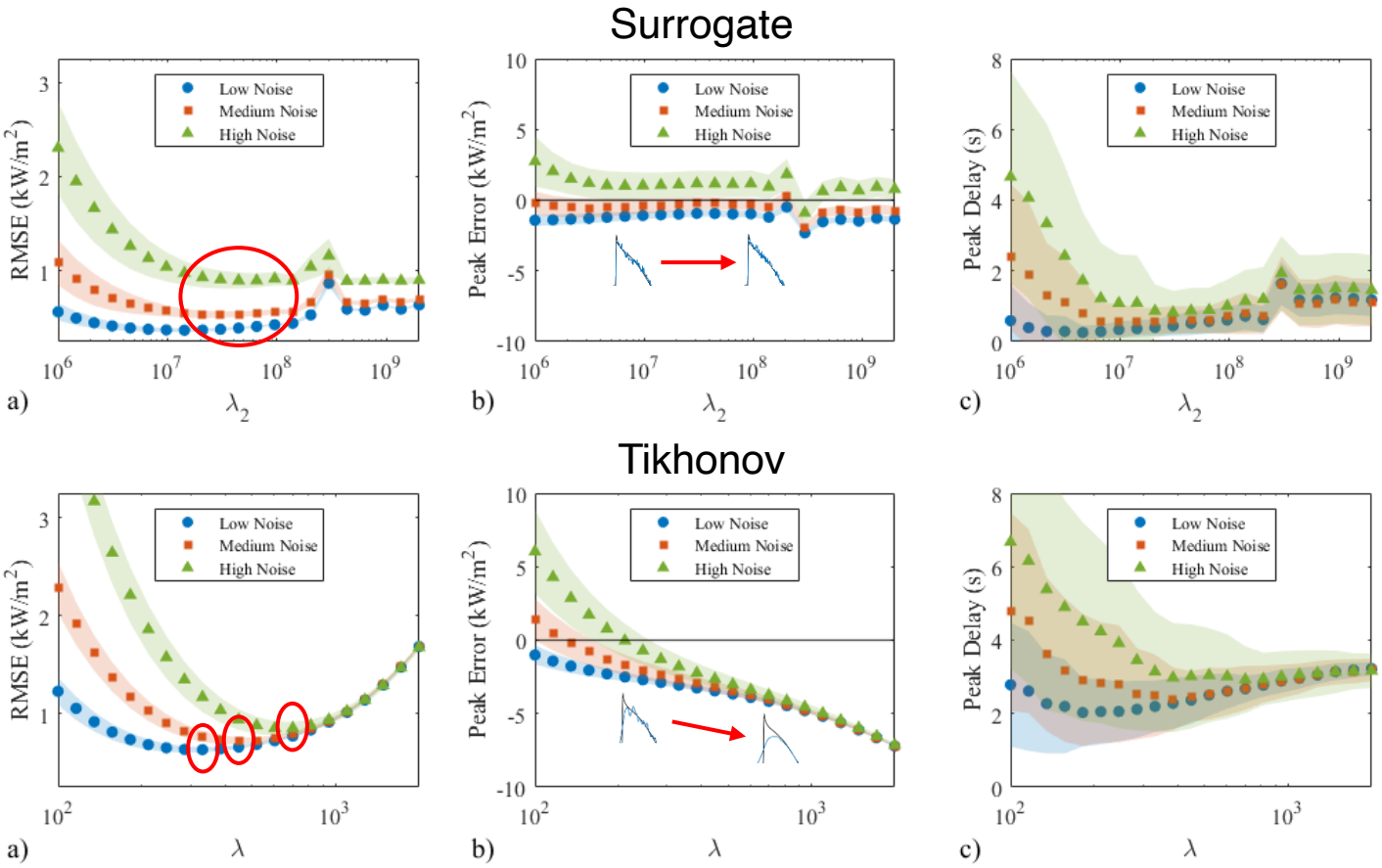

Tikhonov



# Comparison vs. Tikhonov Regularization

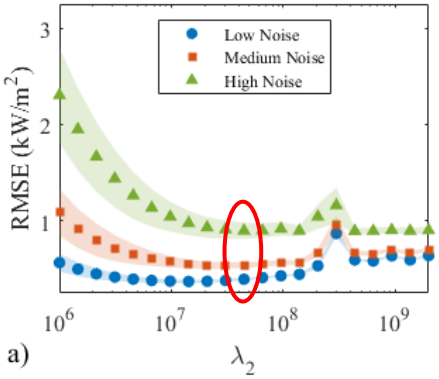


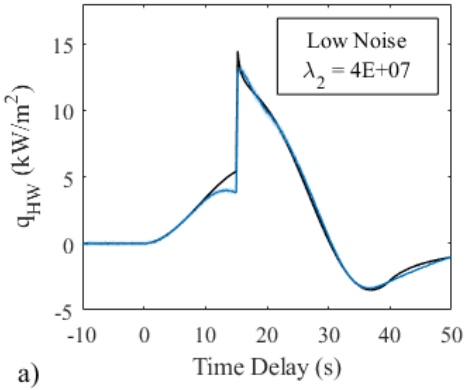
# Comparison vs. Tikhonov Regularization



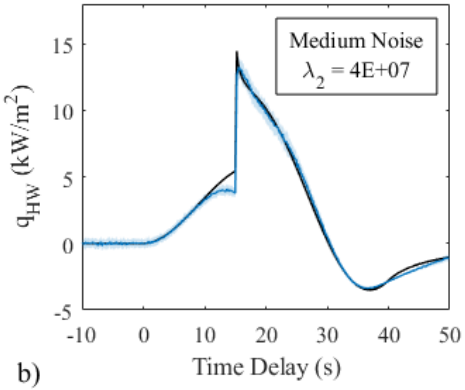
# RMSE-Optimized Solutions

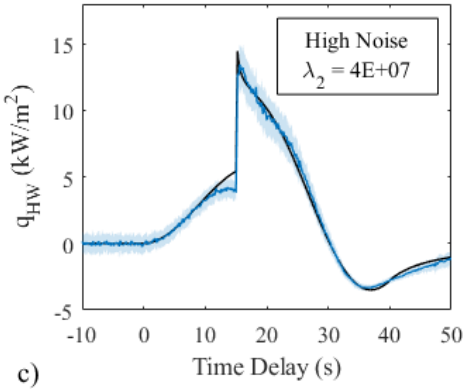
Surrogate



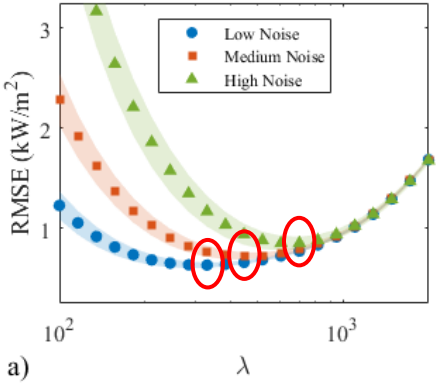



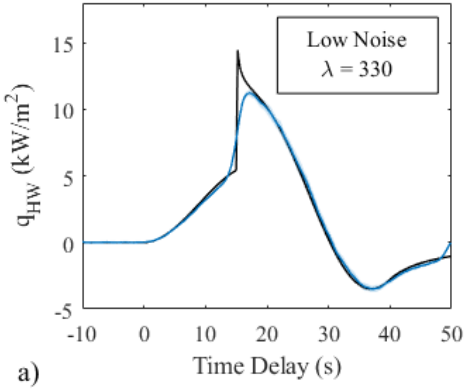

Increasing noise →



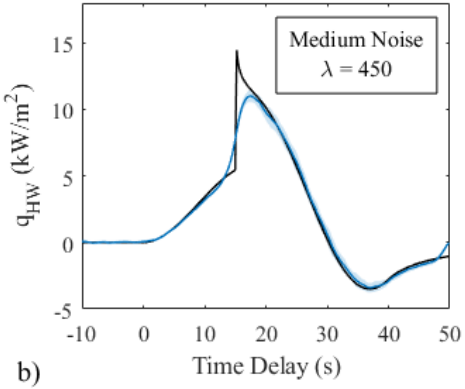



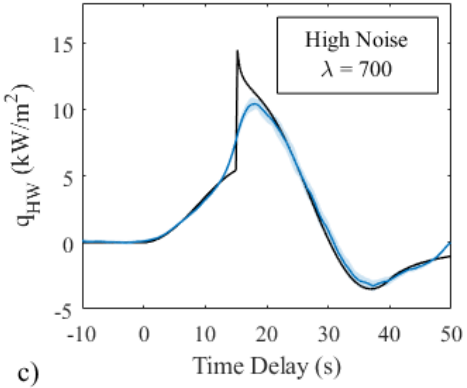

Tikhonov




Increasing noise →



# Conclusions

- We have developed an algorithm to recover the TPS surface heat flux from on-board thermal instrumentation.
- The algorithm can resolve sharp solution features in an ill-conditioned TPS-sensor system.
- The algorithm demonstrates strong robustness to varying input measurement characteristics.

# Acknowledgments

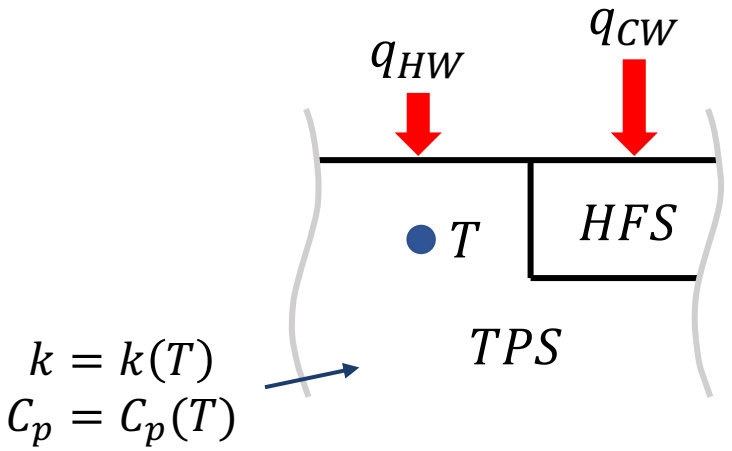

- This work was supported by a NASA Space Technology Graduate Research Opportunity (grant 80NSSC23K1201).

# Questions?

# Backup Slides

# Temperature-Dependent Thermal Properties

- TPS-sensor system (Temp-dependent properties)



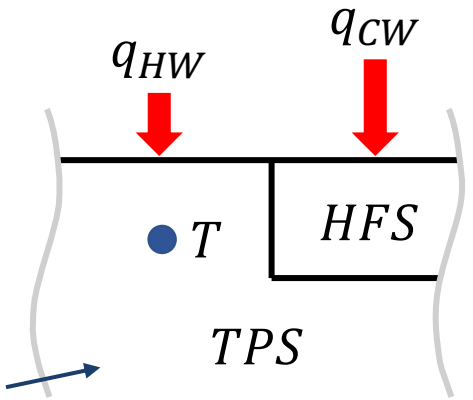
# Temperature-Dependent Thermal Properties

- TPS-sensor system (Temp-dependent properties)
  - Cole-Hopf transformation → $T = f(\theta)$

$$\rho C_p(T)\frac{\partial T}{\partial t} - \nabla \cdot (k(T)\nabla T) = 0$$

$$\frac{1}{\alpha(T)}\frac{\partial \theta}{\partial t} - \nabla^2 \theta = 0$$

$$k = k(T)$$
$$C_p = C_p(T)$$



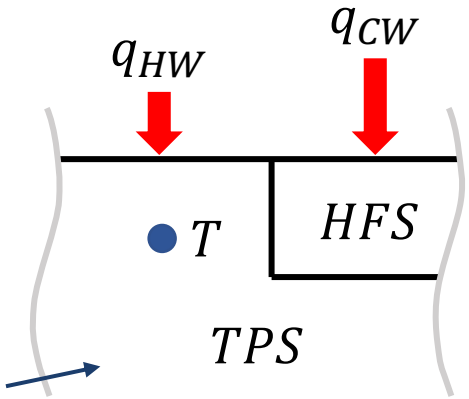
# Temperature-Dependent Thermal Properties

- TPS-sensor system (Temp-dependent properties)
  - Cole-Hopf transformation → $T = f(\theta)$
  - Isolation of temp-dependence → $\alpha(T) = \alpha_0 + \alpha_T(T)$

$$\rho C_p(T)\frac{\partial T}{\partial t} - \nabla \cdot (k(T)\nabla T) = 0$$

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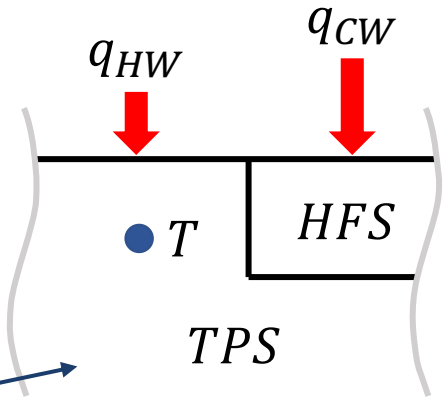
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  - Inhomogeneous source term → $\frac{g'}{A} = \frac{\alpha_T(T)}{\alpha_0}\nabla^2\theta$

$$\rho C_p(T)\frac{\partial T}{\partial t} - \nabla \cdot (k(T)\nabla T) = 0$$

$$\frac{1}{\alpha_0}\frac{\partial \theta}{\partial t} - \nabla^2\theta = \frac{g'(\boldsymbol{x}, t)}{A}$$

$$k = k(T)$$
$$C_p = C_p(T)$$



# Temperature-Dependent Thermal Properties

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  - Cole-Hopf transformation → $T = f(\theta)$
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$$\theta(\boldsymbol{x},t) = -\int_0^t \int_{S_{TPS}} \frac{\alpha_{0,TPS}}{A_{TPS}} q_{HW}(\tau) G_{TPS,TPS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau)\, dS_0\, d\tau - \int_0^t \int_{S_{HFS}} \frac{\alpha_{0,HFS}}{A_{HFS}} q_{CW}(\tau) G_{TPS,HFS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau)\, dS_0\, d\tau$$
$$+ \int_0^t \int_{V_{TPS}} \frac{\alpha_{0,TPS}}{A_{TPS}} g'_{TPS}(\boldsymbol{x_0},\tau) G_{TPS,TPS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau)\, dV_0\, d\tau + \int_0^t \int_{V_{HFS}} \frac{\alpha_{0,HFS}}{A_{HFS}} g'_{HFS}(\boldsymbol{x_0},\tau) G_{TPS,HFS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau)\, dV_0\, d\tau$$

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Effects of heat flux boundary conditions

$$\theta(\boldsymbol{x},t) = -\int_0^t \int_{S_{TPS}} \frac{\alpha_{0,TPS}}{A_{TPS}} q_{HW}(\tau) G_{TPS,TPS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau)\, dS_0\, d\tau - \int_0^t \int_{S_{HFS}} \frac{\alpha_{0,HFS}}{A_{HFS}} q_{CW}(\tau) G_{TPS,HFS}(\boldsymbol{x},t,\boldsymbol{x_0},\tau)\, dS_0\, d\tau$$
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Effects of temp-dependent properties

# Green's Function

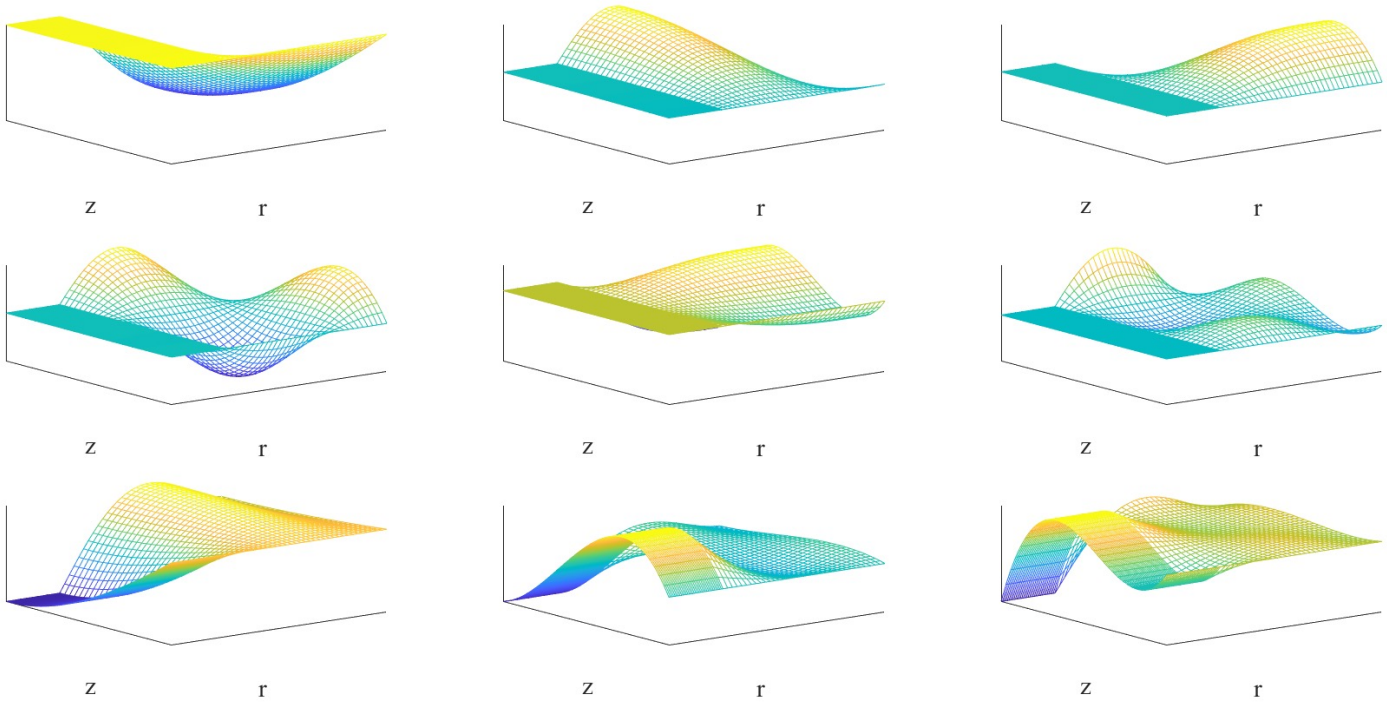
- General Green's function formulation

$$G_{l,m}(\boldsymbol{x}, t, \boldsymbol{x_0}, \tau) = \sum_{n=1}^{N} \frac{k_{0,m}}{\alpha_{0,m}} \phi_n^{(l)}(\boldsymbol{x}) \phi_n^{(m)}(\boldsymbol{x_0}) e^{-\Lambda_{n,n}(t-\tau)} \qquad l, m = TPS \; or \; HFS$$

- Galerkin Eigenvalue Approximation

$$\phi_n^{(l)} = \sum_{j=1}^{N} \psi_{j,n} \chi_j^{(l)} \qquad l = TPS \; or \; HFS$$

# Eigenfunctions (First 3 Modes)





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