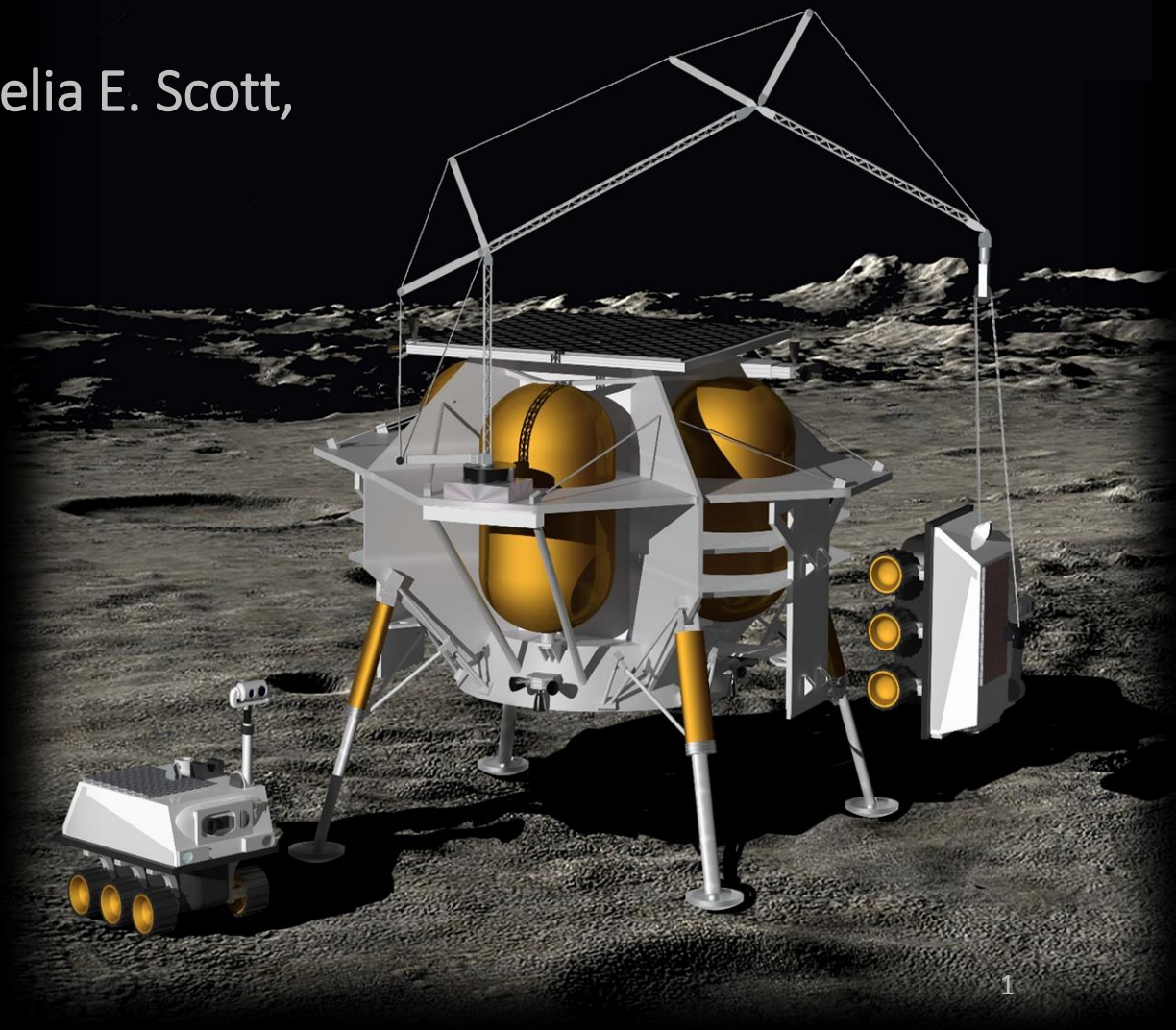
# Safe Operational Envelope for the LSMS Family of Cable-Driven Cranes

Javier Puig-Navarro, Dominic R. Bisio, Amelia E. Scott,
Julia Cline, B. Danette Allen

*NASA Langley Research Center*



# Outline

- Introduction
- Geometric Model
- Dynamic Model
- Safe Operational Constraints
- Envelope Visualization
- Conclusions

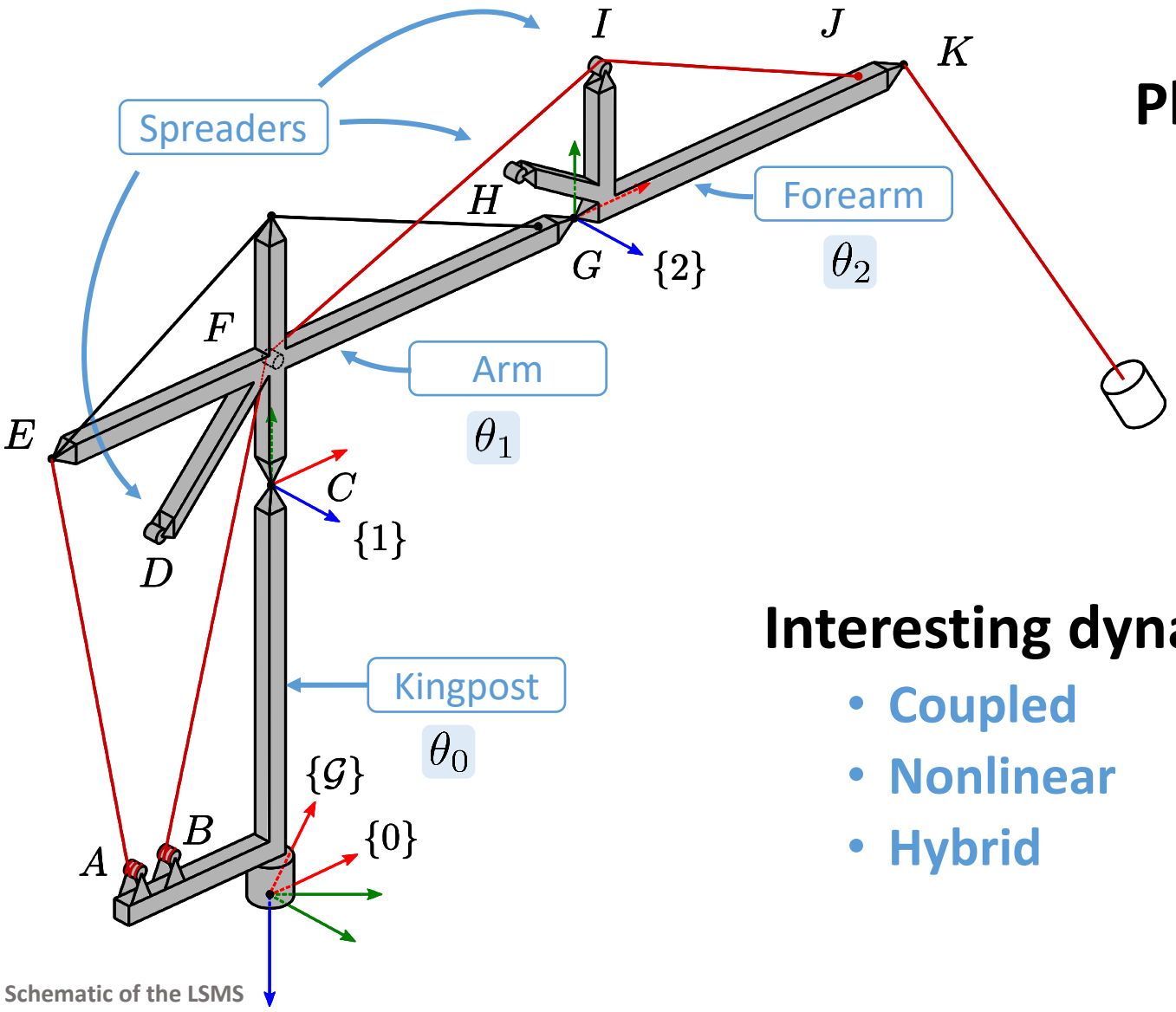
# Introduction

# A Trajectory Tracking Algorithm
## for the LSMS Family of Cable-Driven Cranes




NASA Langley Research Center

# Interesting Dynamics of the LSMS



Schematic of the LSMS

## Planetary multi-purpose robotic crane

- Cable **actuated** and **stabilized**

| 3 links | 3R joints |
|---|---|
| Kingpost | Waist |
| Arm | Shoulder |
| Forearm | Elbow |

## Interesting dynamics

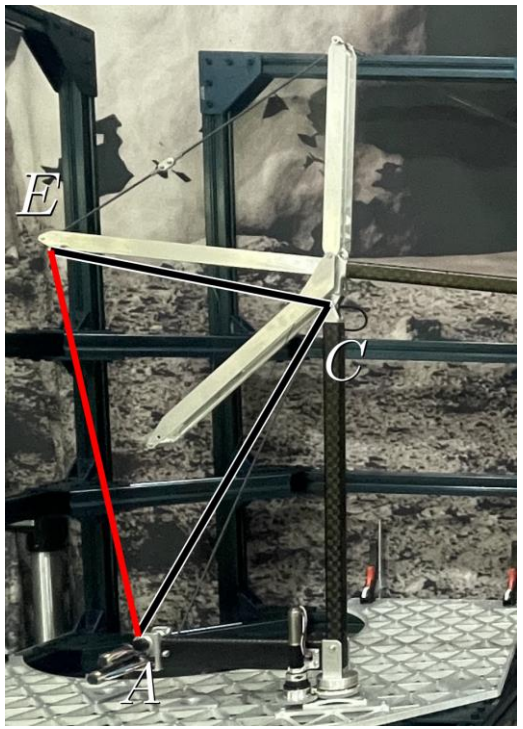
- **Coupled** → shoulder and elbow angles
- **Nonlinear** → different from a serial manipulator
- **Hybrid** → 6 spreader configurations

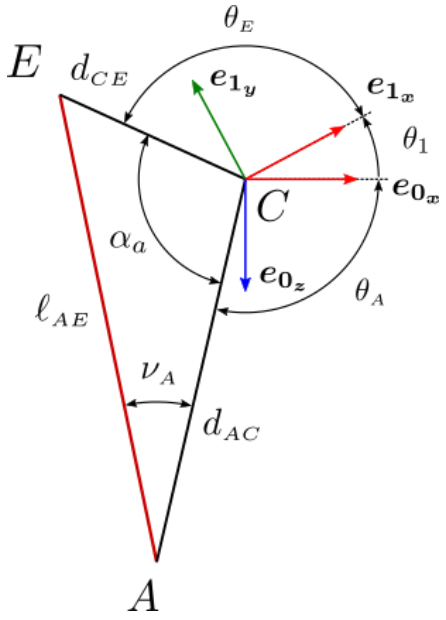
# Geometric Model

# Geometry of Cable A – Switched Dynamics

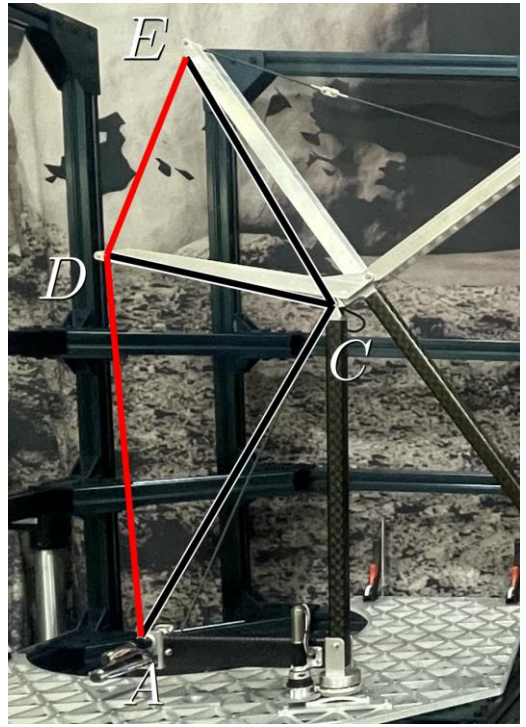
- As the arm lowers → spreader D becomes load bearing

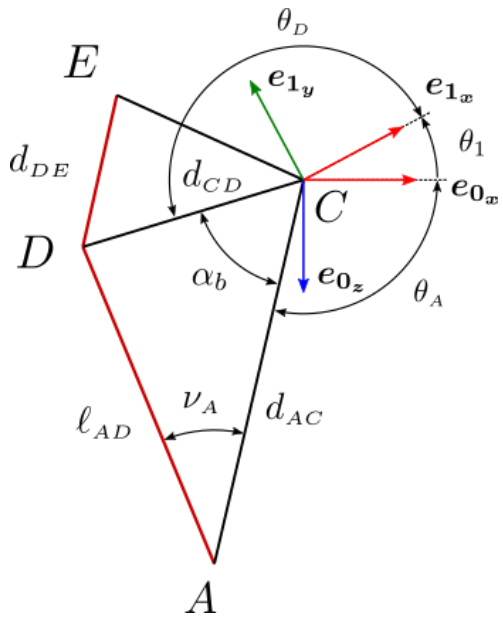
**Configuration (a)** $\theta_1 \geq \theta_1^*$




**Configuration (b)** $\theta_1 < \theta_1^*$



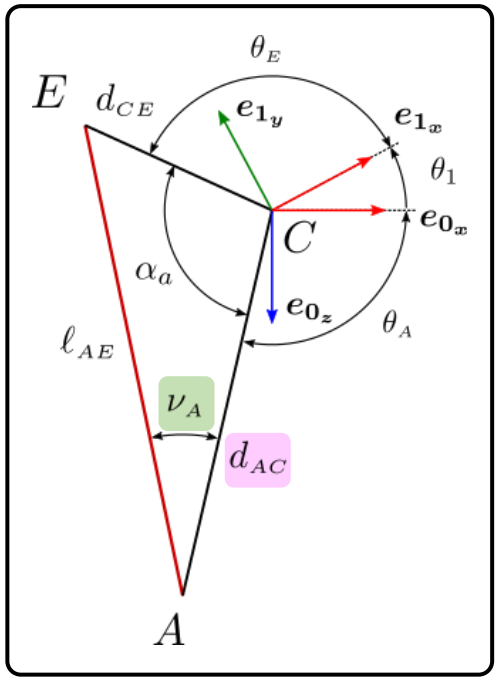
# Geometry of Cable A – Switched Dynamics

gear ratio and drum radius

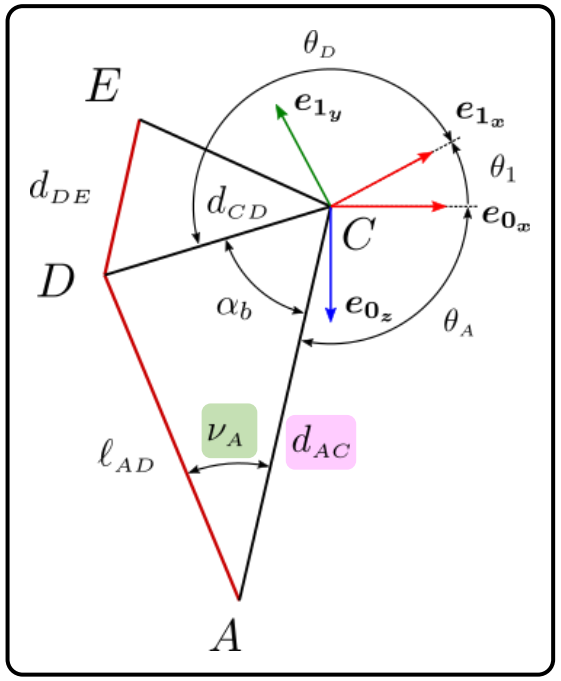
$$\dot{\theta}_1 = \frac{i_A r_A}{d_{AC} \sin \nu_A} \omega_A$$

$$\sin \nu_A := \begin{cases} \dfrac{d_{CE} \sin \alpha_a}{\ell_{AE}}, & \text{if } \theta_1 \geq \theta_1^*, \\ \dfrac{d_{CD} \sin \alpha_b}{\ell_{AD}}, & \text{if } \theta_1 < \theta_1^*, \end{cases}$$

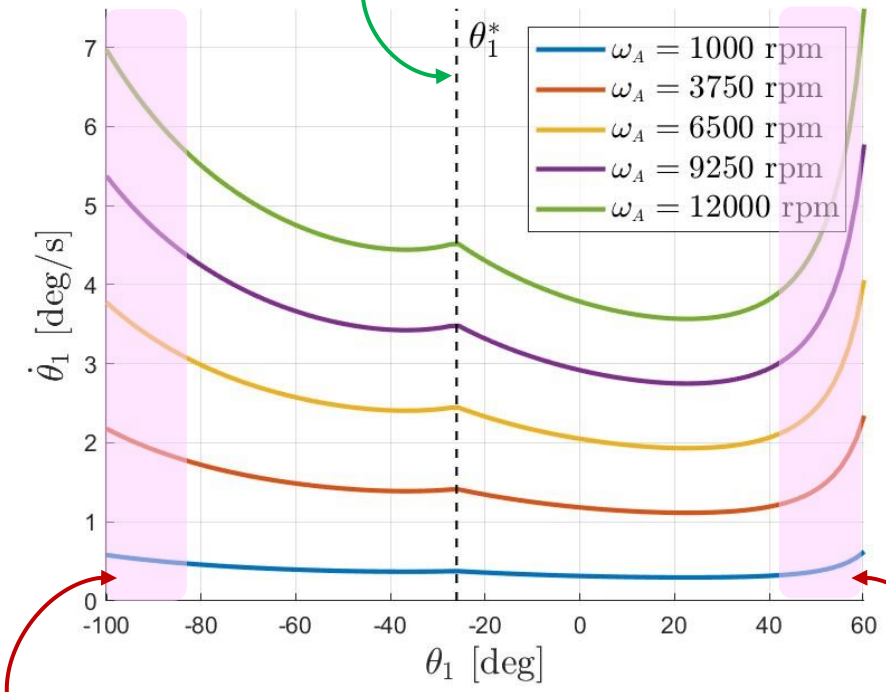
Configuration (a) $\theta_1 \geq \theta_1^*$




Configuration (b) $\theta_1 < \theta_1^*$



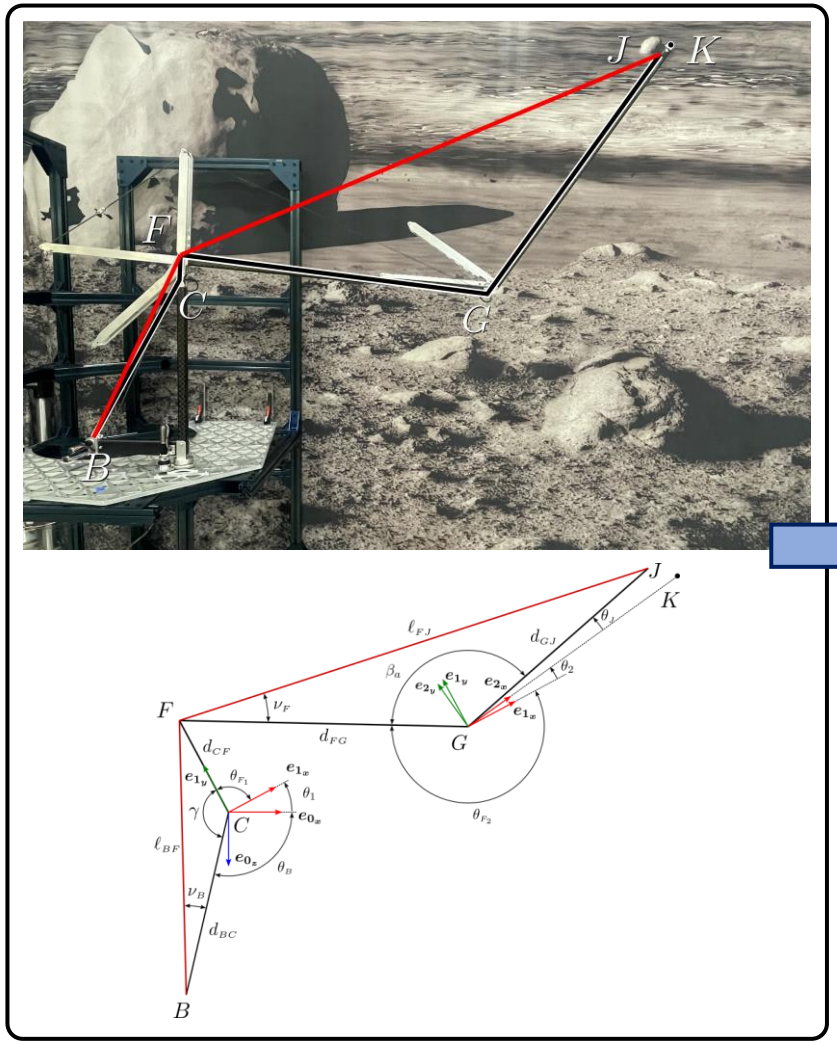

configuration switch



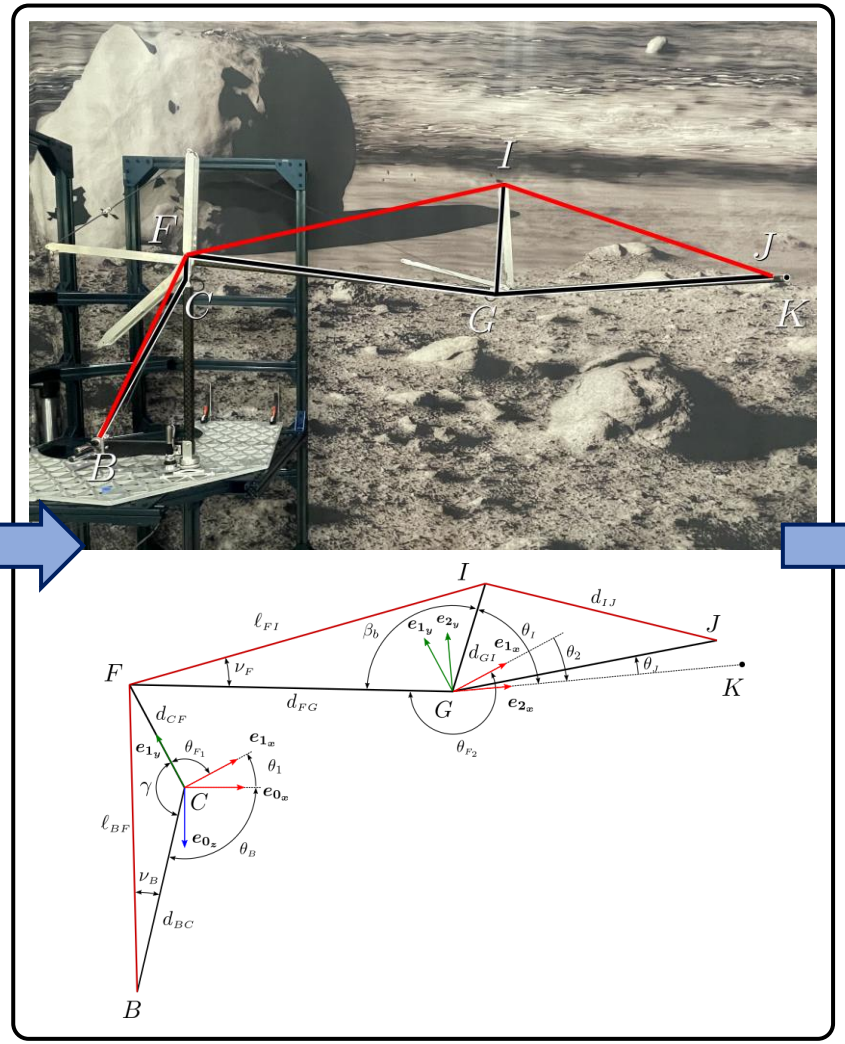

for extreme values of $\theta_1$ slope grows unbounded

# Geometry of Cable B – Coupled Dynamics

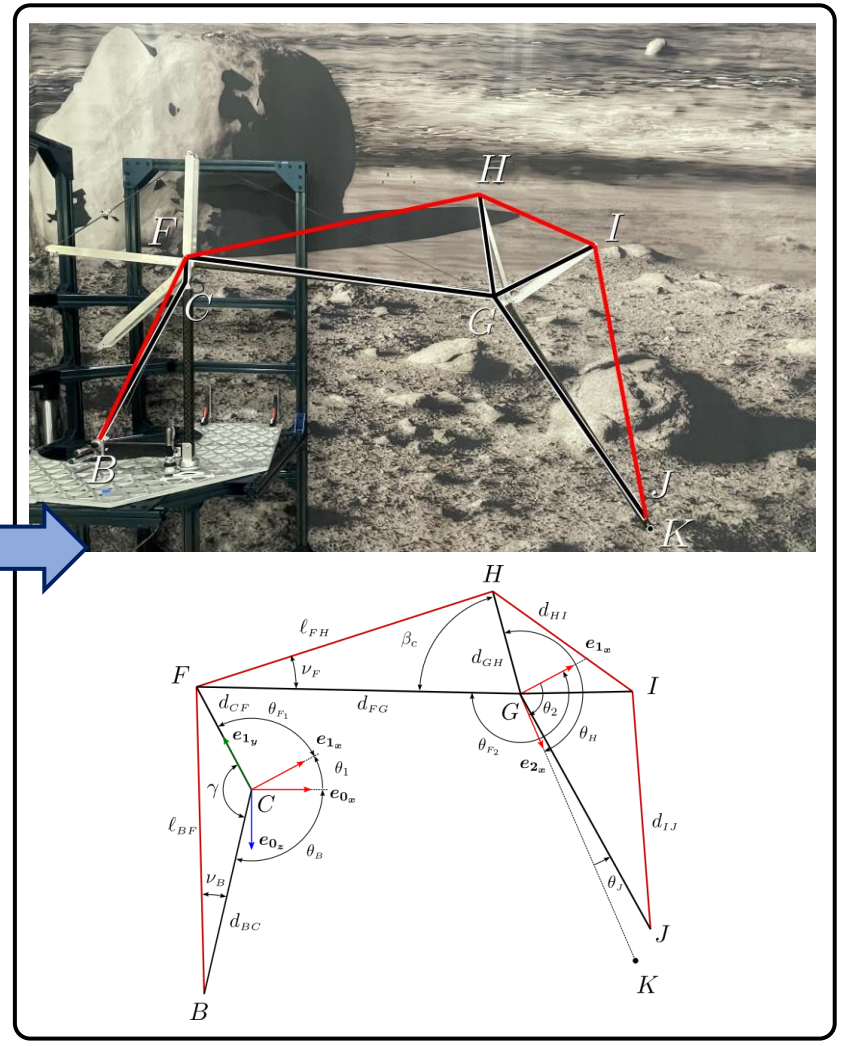
**Configuration (a)** $\theta_2 \geq \theta_2^*$




**Configuration (b)** $\theta_2^{**} \leq \theta_2 < \theta_2^*$




**Configuration (c)** $\theta_2 < \theta_2^{**}$



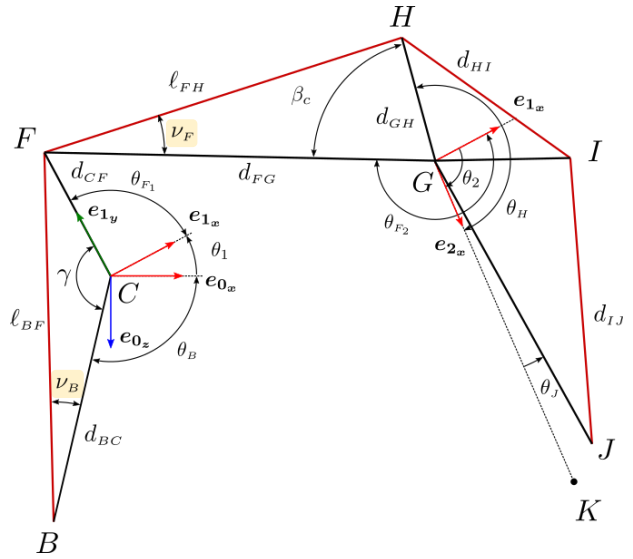
# Geometry of Cable B – Coupled Dynamics

$$\dot{\theta}_2 = -\frac{i_A r_A}{d_{FG}\sin\nu_F}\frac{d_{BC}\sin\nu_B}{d_{AC}\sin\nu_A}\omega_A + \frac{i_B r_B}{d_{FG}\sin\nu_F}\omega_B$$

gear ratio and drum radius

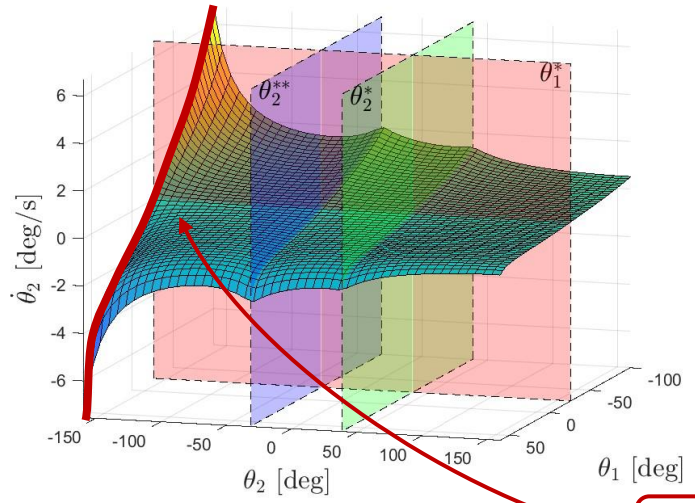
Shoulder- elbow coupling

$$\sin\nu_A := \begin{cases} \frac{d_{CE}\sin\alpha_a}{\ell_{AE}}, & \text{if } \theta_1 \geq \theta_1^*, \\ \frac{d_{CD}\sin\alpha_b}{\ell_{AD}}, & \text{if } \theta_1 < \theta_1^*, \end{cases} \qquad \sin\nu_B := \frac{d_{CF}\sin\gamma}{\ell_{BF}}, \qquad \sin\nu_F := \begin{cases} \frac{d_{GJ}\sin\beta_a}{\ell_{FJ}}, & \text{if } \theta_2 \geq \theta_2^*, \\ \frac{d_{GI}\sin\beta_b}{\ell_{FI}}, & \text{if } \theta_2^{**} \leq \theta_2 < \theta_2^*, \\ \frac{d_{GH}\sin\beta_c}{\ell_{FH}}, & \text{if } \theta_2 < \theta_2^{**}, \end{cases}$$

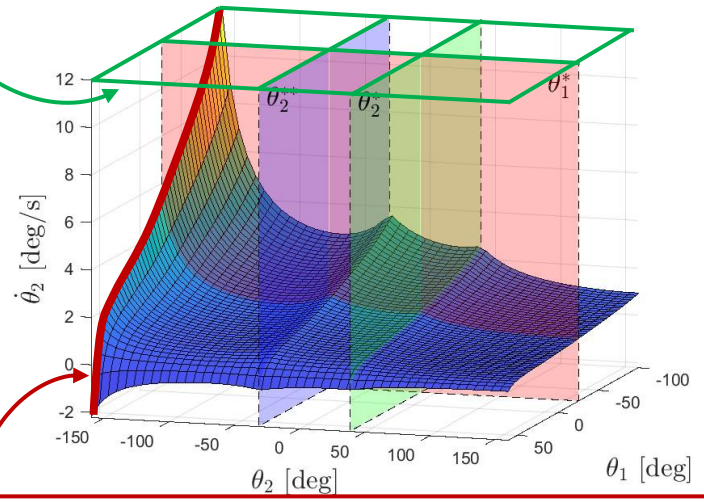



**configuration switches**

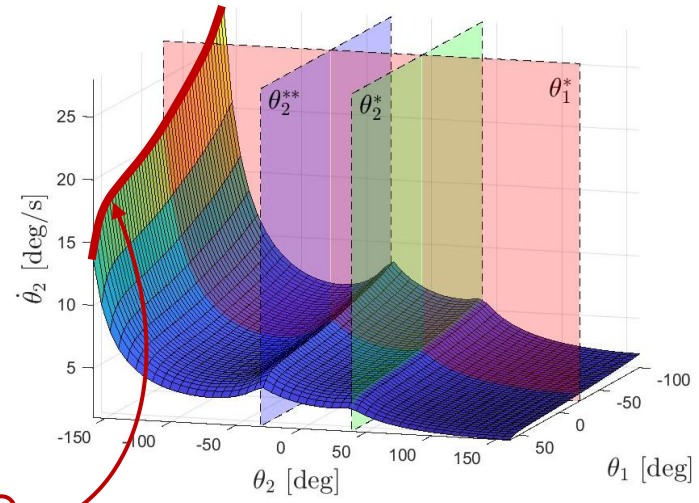
**Elbow rate,**
$\omega_A = 12,000$ rpm, $\omega_B = 0$ rpm.




**Elbow rate,**
$\omega_A = 12,000$ rpm, $\omega_B = 3,000$ rpm.



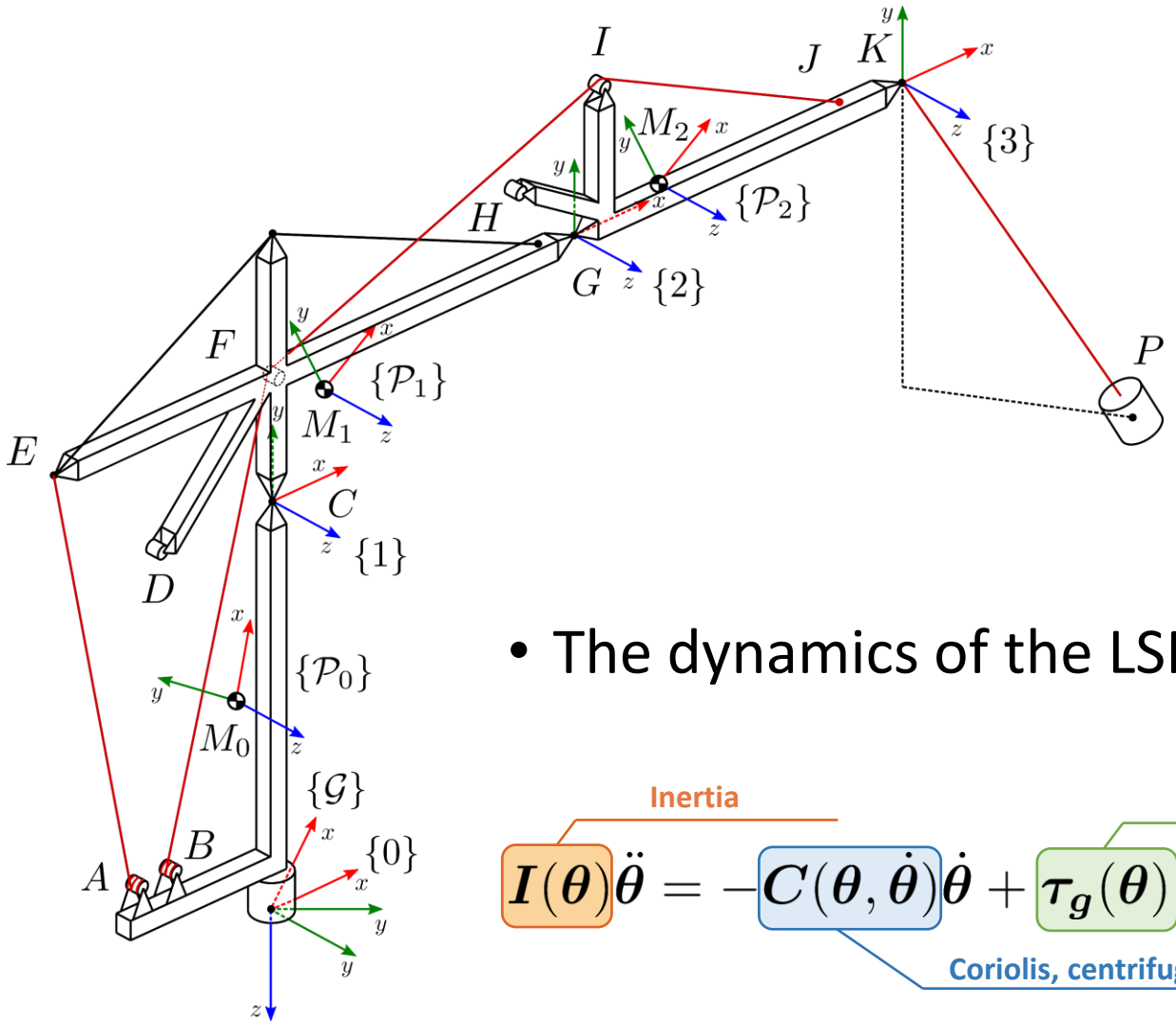

**Elbow rate,**
$\omega_A = 12,000$ rpm, $\omega_B = 12,000$ rpm.




**for extreme values of $\theta_2$ slope grows unbounded**

# Dynamic Model

# Dynamic Model of the LSMS




Schematic of the LSMS

- Assume rigid, massless cables
- Joint angles as the generalized coordinates

$$\boldsymbol{\theta} := [\theta_0, \theta_1, \theta_2]^\top$$

- The dynamics of the LSMS derived using the Lagrange's equation

$$\underbrace{\boldsymbol{I}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}}_{\text{Inertia}} = -\underbrace{\boldsymbol{C}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}}_{\text{Coriolis, centrifugal}} + \underbrace{\boldsymbol{\tau_g}(\boldsymbol{\theta})}_{\text{Gravity}} + \underbrace{\boldsymbol{\tau_d}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})}_{\text{Friction}} + \underbrace{\boldsymbol{J}_G^\top(\boldsymbol{\theta})^r\boldsymbol{w}_G + \boldsymbol{J}_K^\top(\boldsymbol{\theta})^s\boldsymbol{w}_K}_{\text{Payload wrench}} + \underbrace{\boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{\tau_m}}_{\text{Control}}$$

# Safe Operational Constraints

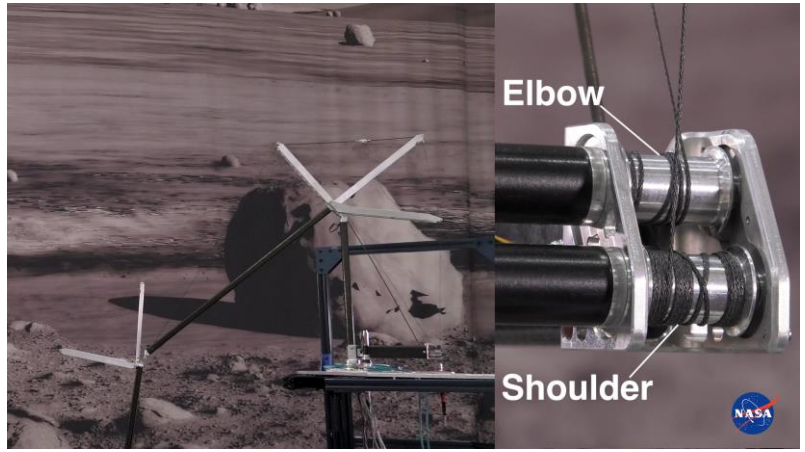
# Safe Operational Constraints

- Motor constraints
  - Maximum speed

$$|\omega_0| \leq \bar{\omega}_0, \quad |\omega_A| \leq \bar{\omega}_A, \quad |\omega_B| \leq \bar{\omega}_B$$

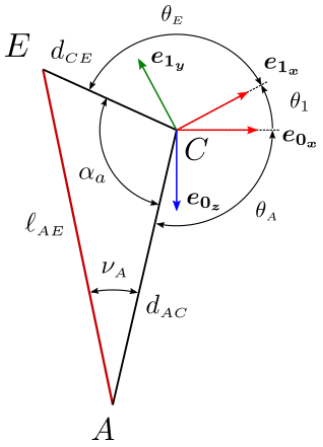
  - Maximum torque

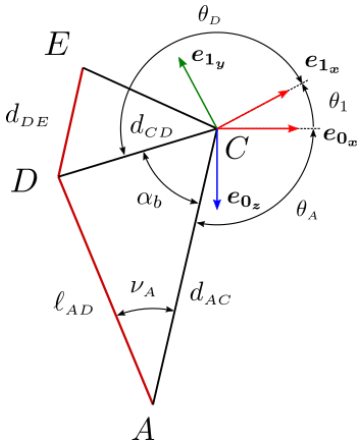
$$|\tau_{m_0}| \leq \bar{\tau}_{m_0}, \quad |\tau_{m_A}| \leq \bar{\tau}_{m_A}, \quad |\tau_{m_B}| \leq \bar{\tau}_{m_B}$$



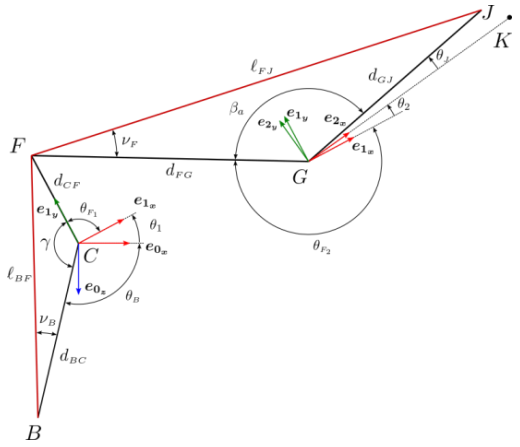

- Geometric constraints
  - Avoid elongated triangles in the pattern

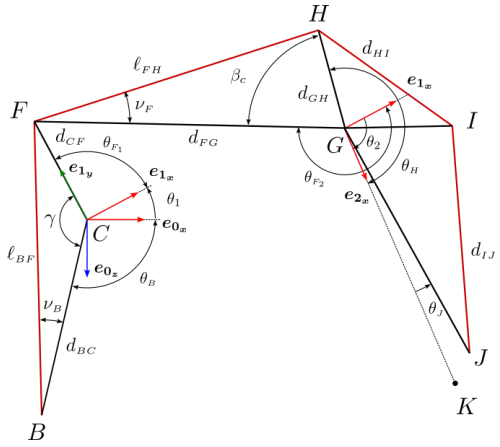
$$\sin \nu_A \geq \zeta_A > 0 \implies \underline{\theta}_1 \leq \theta_1 \leq \bar{\theta}_1$$




$$\sin \nu_F \geq \zeta_F > 0 \implies \underline{\theta}_2 \leq \theta_2 \leq \bar{\theta}_2$$



# Safe Operational Constraints

- Motor constraints
  - Maximum speed

$$|\omega_0| \leq \bar{\omega}_0, \quad |\omega_A| \leq \bar{\omega}_A, \quad |\omega_B| \leq \bar{\omega}_B$$

  - Maximum torque

$$|\tau_{m_0}| \leq \bar{\tau}_{m_0}, \quad |\tau_{m_A}| \leq \bar{\tau}_{m_A}, \quad |\tau_{m_B}| \leq \bar{\tau}_{m_B}$$

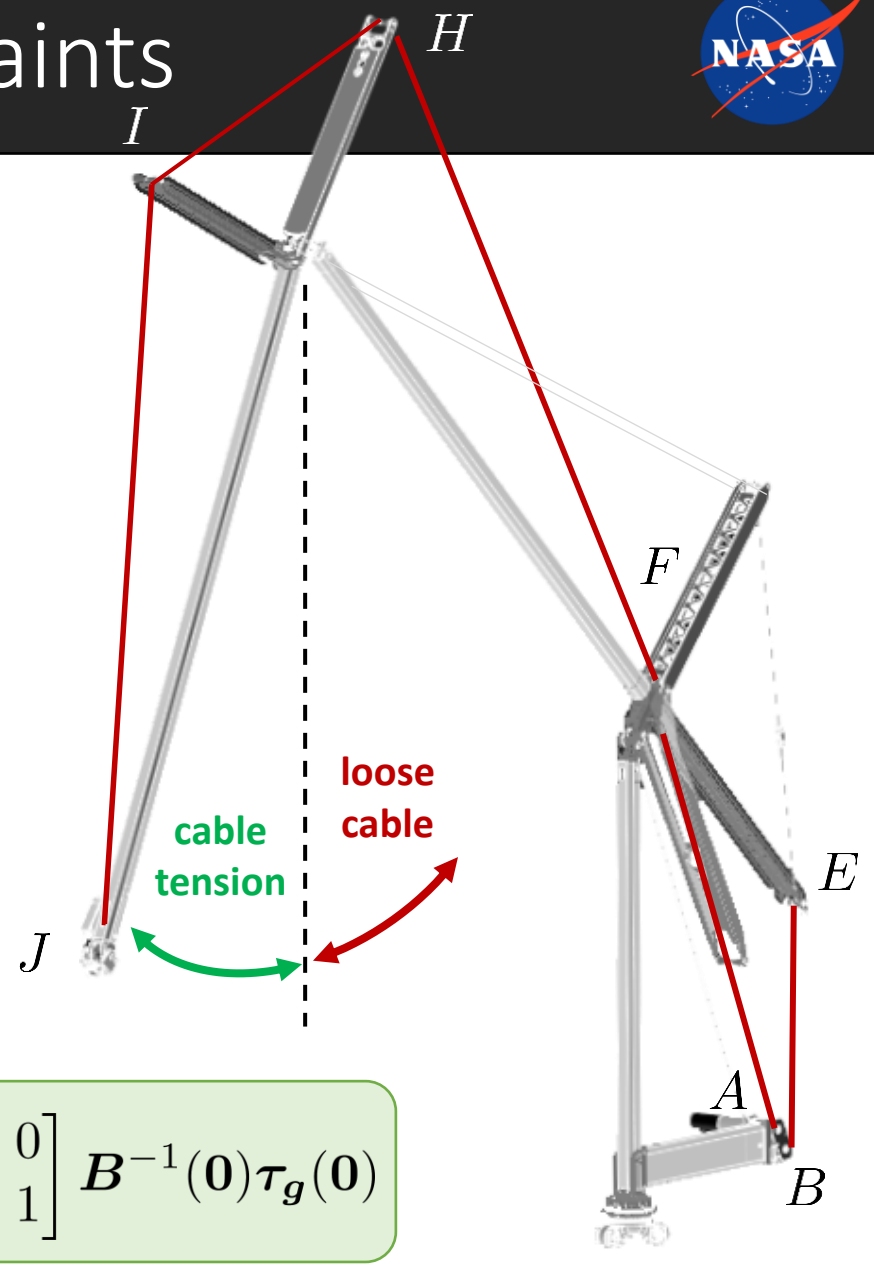
- Geometric constraints
  - Avoid elongated triangles in the pattern

$$\sin \nu_A \geq \zeta_A > 0 \implies \underline{\theta}_1 \leq \theta_1 \leq \bar{\theta}_1$$

- Cable tension constraints

$$\tau_{m_A} \geq \underline{\tau}_{m_A}, \quad \tau_{m_B} \geq \underline{\tau}_{m_B}$$

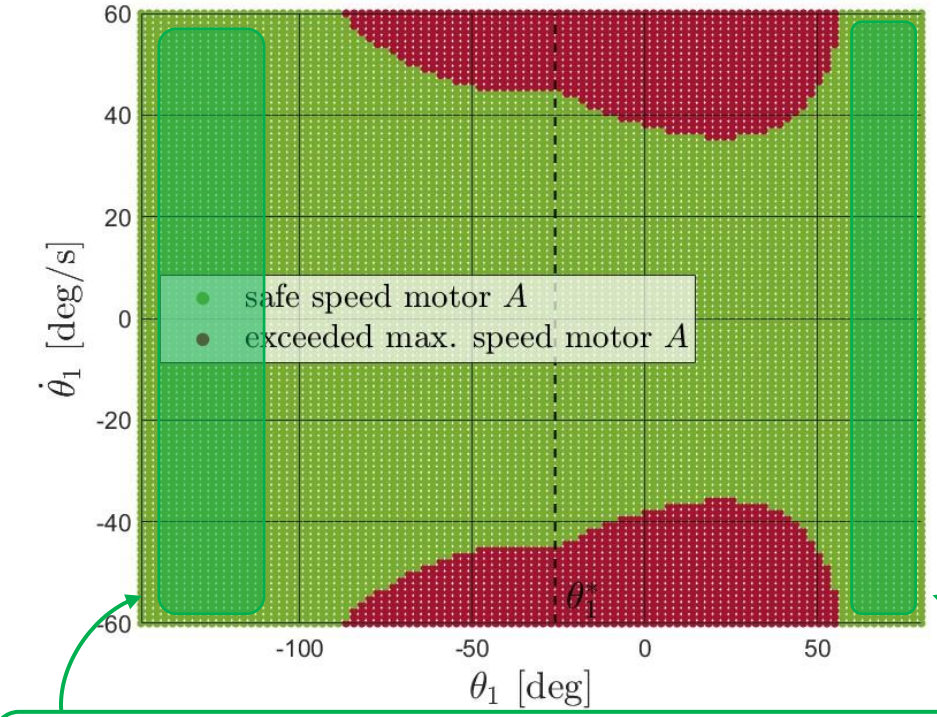
$$\begin{bmatrix} \underline{\tau}_{m_A} \\ \underline{\tau}_{m_B} \end{bmatrix} = -\epsilon_m \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{B}^{-1}(\mathbf{0})\boldsymbol{\tau_g}(\mathbf{0})$$



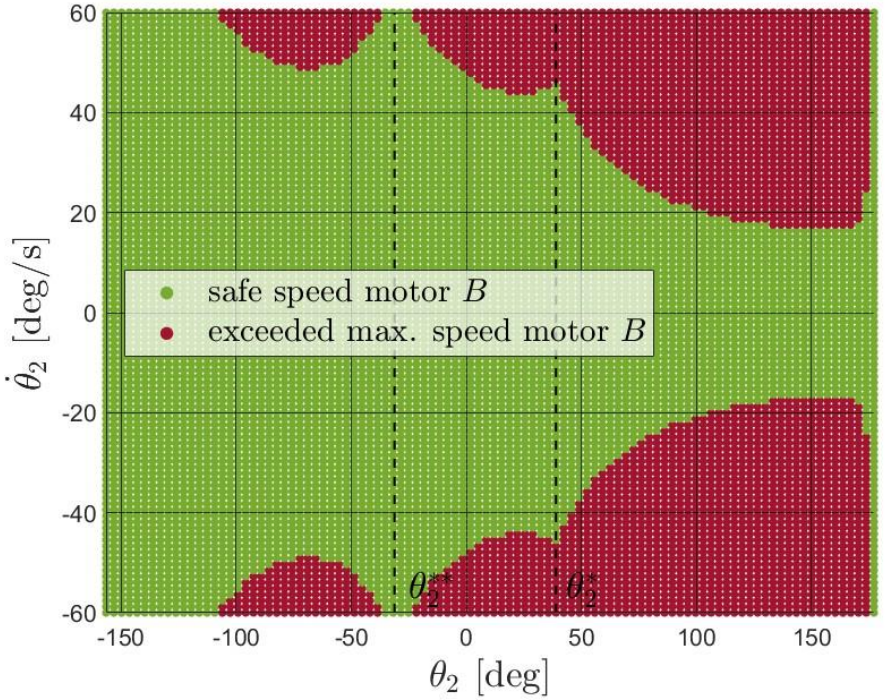
# Envelope Visualization

# Motor Constraints

**Motor A – speed constraints**




**Motor B – speed constraints**




**slower motor speeds achieve greater joint rates at extreme values of $\theta_1$**

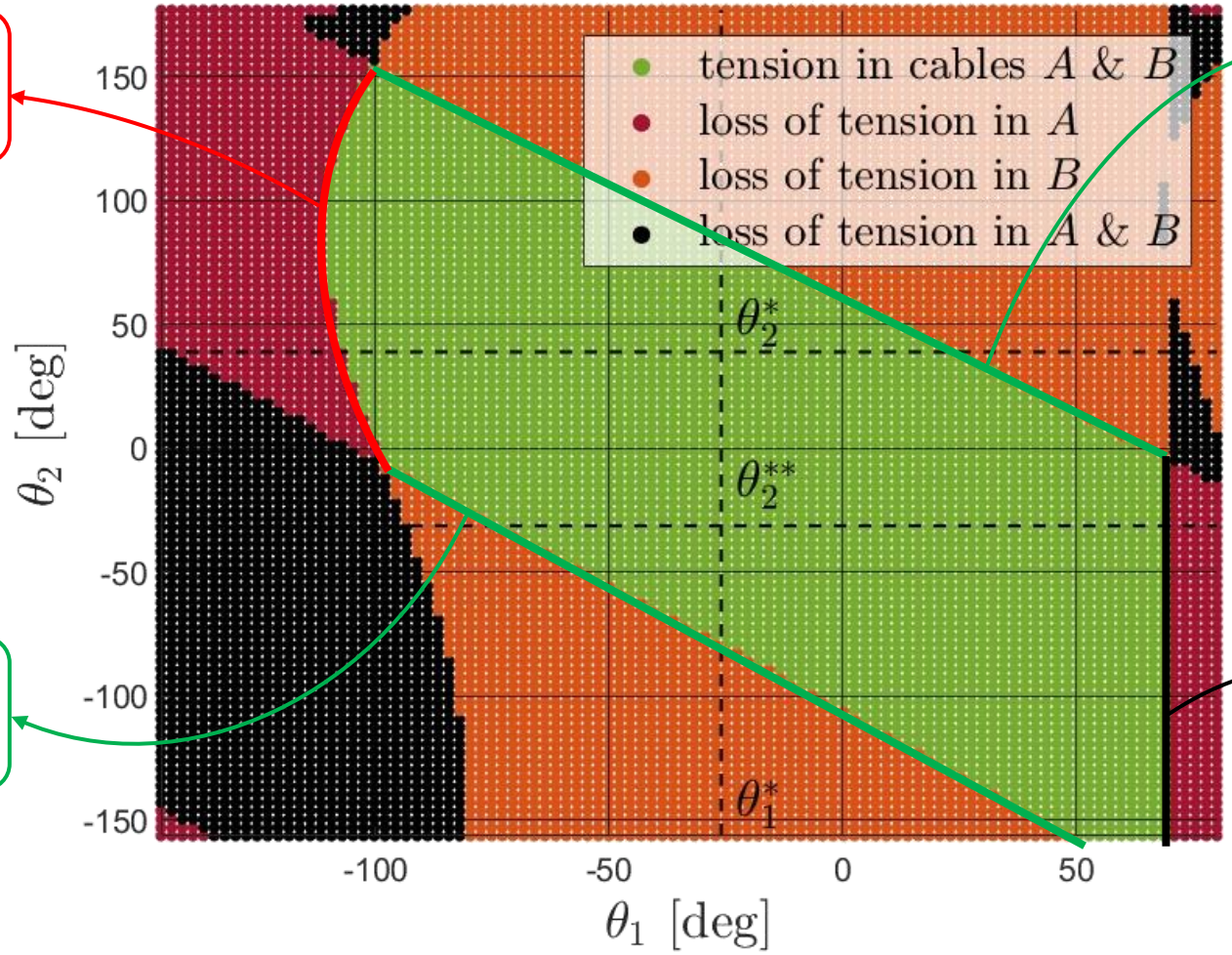
# Loss of Tension - No Payload

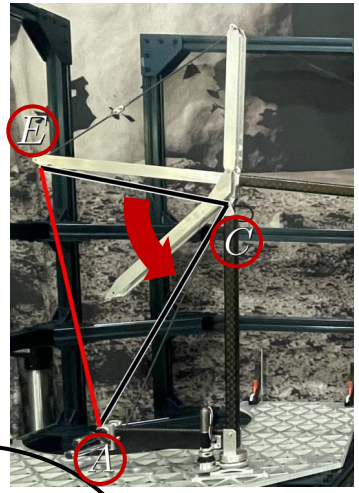
- Identical for the Earth, the Moon, and Mars

**excessive reel out of cable A**

**excessive cable B reel out**

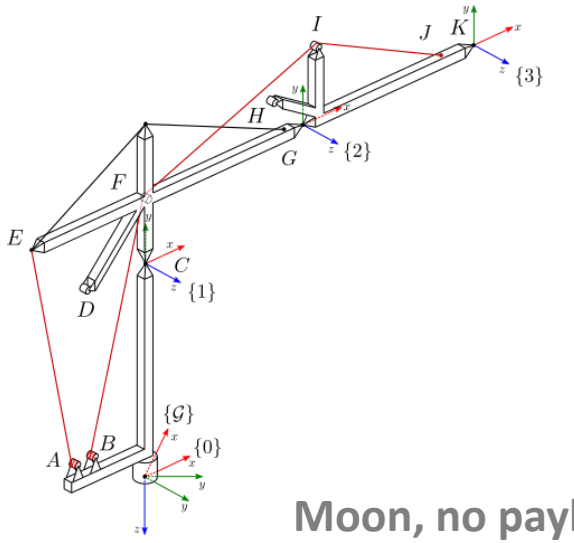
**elbow tip-over**



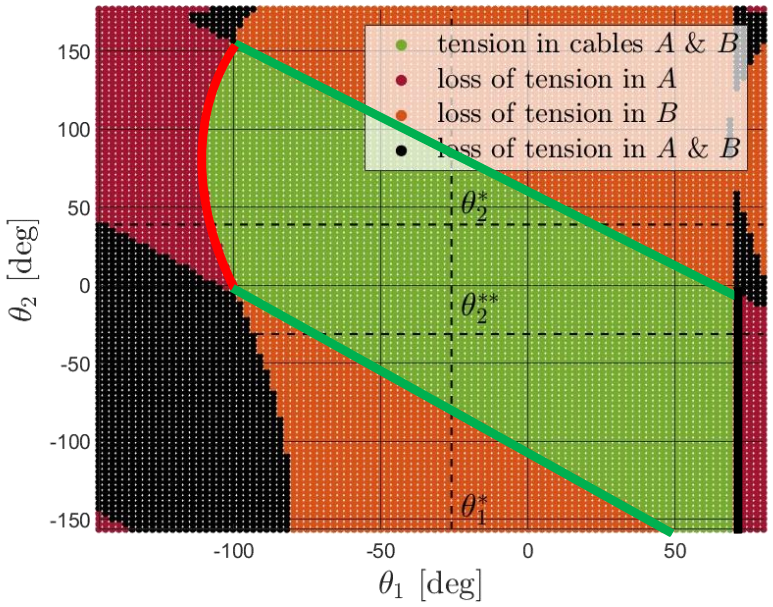




**Points A, E, and C aligned**

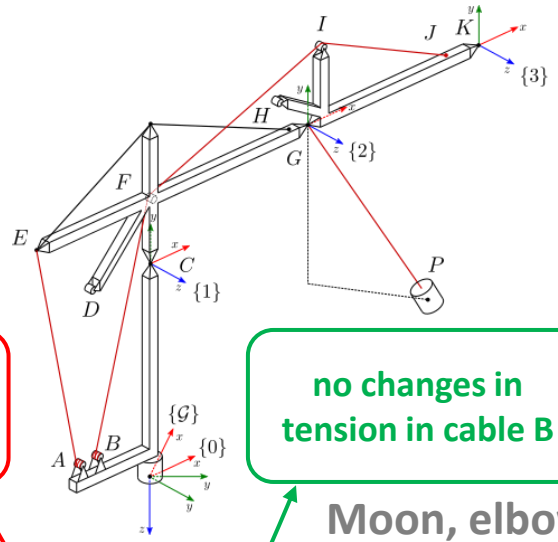
# Loss of Tension – Loading Configurations



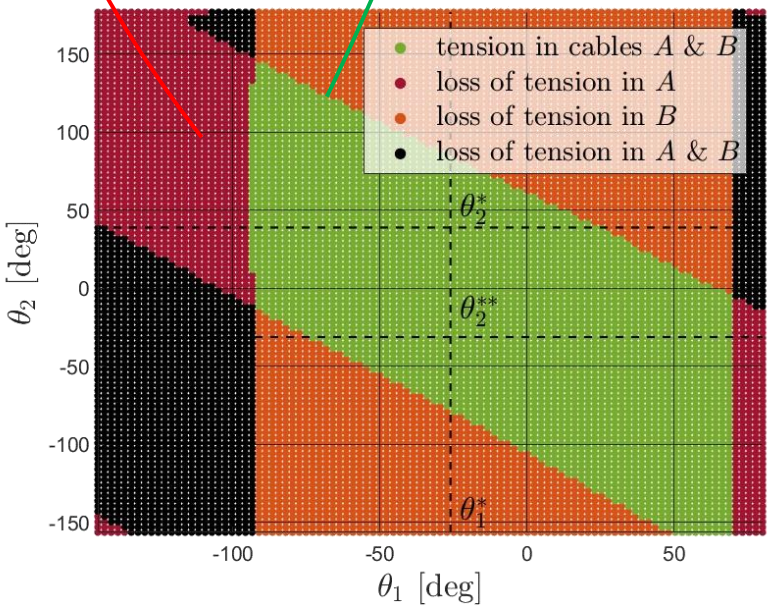

Moon, no payload




earlier LOT at low $\theta_1$

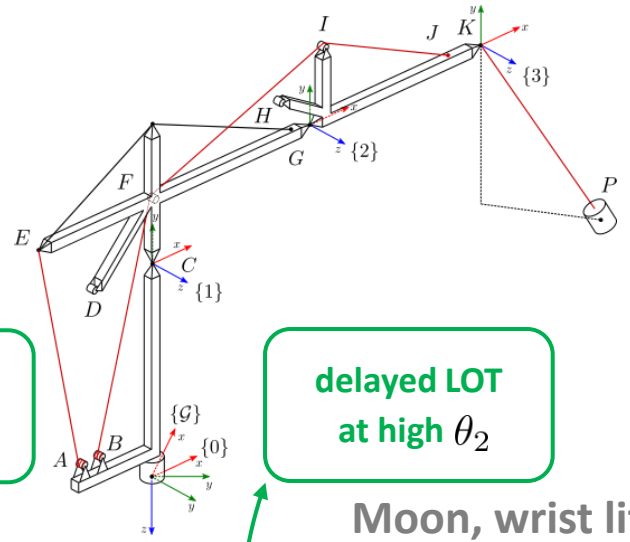



Moon, elbow lift

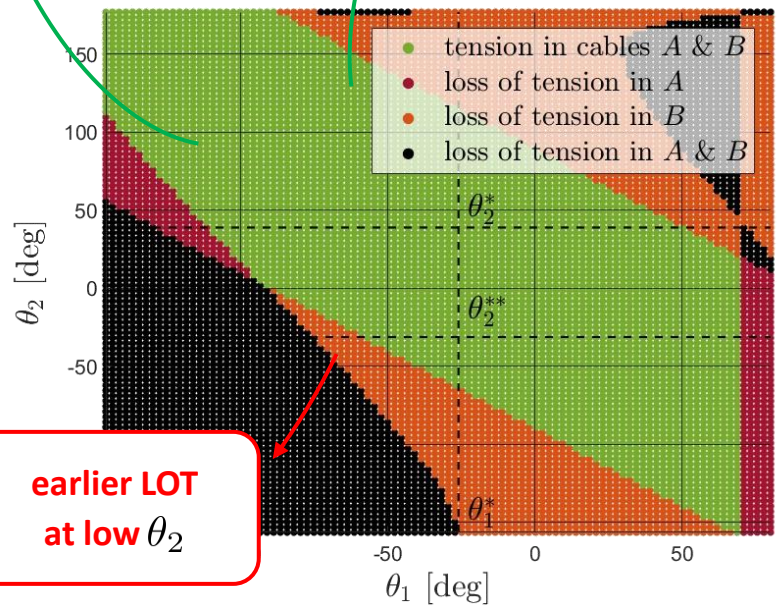



no changes in tension in cable B

delayed LOT at low $\theta_1$




Moon, wrist lift



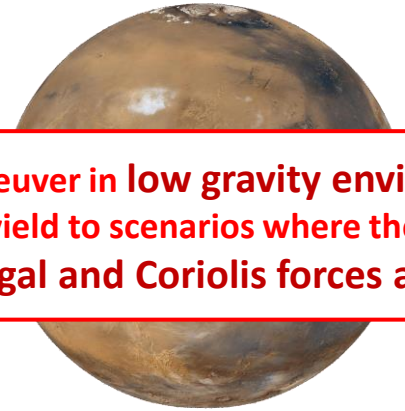

delayed LOT at high $\theta_2$

earlier LOT at low $\theta_2$


javier.puignavarro@nasa.gov

# Loss of Tension – Constant Speed Maneuver

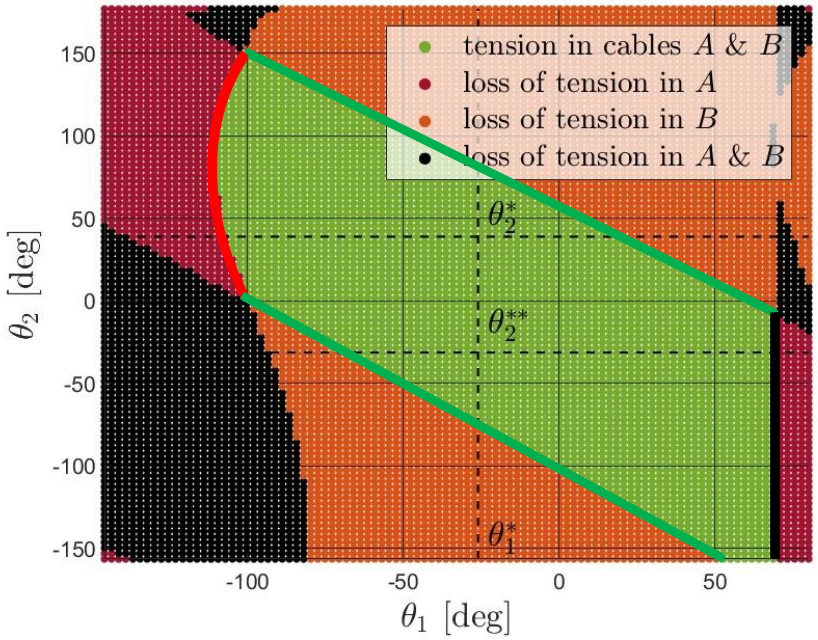




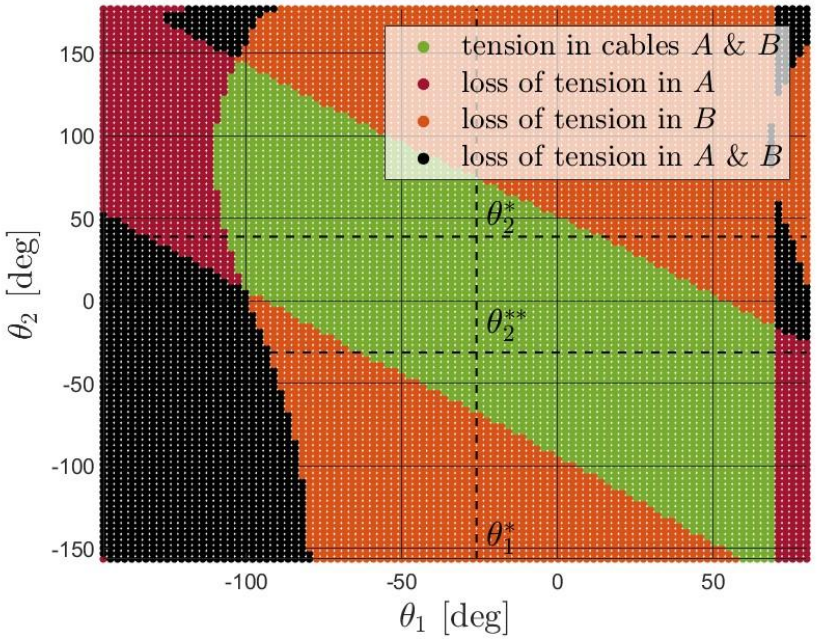


**The same maneuver in low gravity environments can yield to scenarios where the inertial, centrifugal and Coriolis forces are dominant**

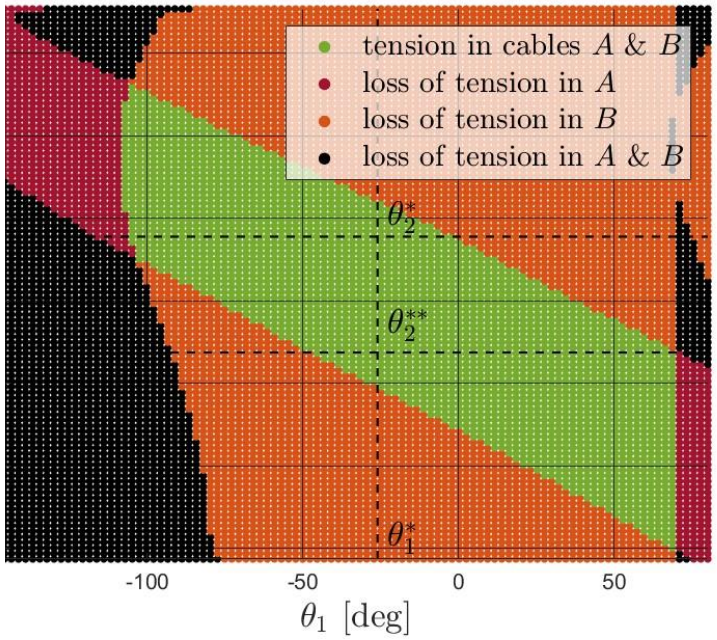
**Earth,** $\dot{\theta}_2 = -20$ deg/s




**Mars,** $\dot{\theta}_2 = -20$ deg/s




**Moon,** $\dot{\theta}_2 = -20$ deg/s



# Conclusion

- Models leveraged to define the **safe operational envelope**
  - Geometric
  - Dynamic
- Constraints considered
  - Motor → speed and torque limits
  - Geometric → avoid degenerate triangles
  - Cable tension → avoid compression
- Constraints **not** considered in this paper
  - Structural limits
  - Self collision constraints
- Scenarios considered
  - Loading → no lift, elbow and wrist lifts
  - Planetary surfaces → Earth, Moon, Mars

