

Discretization Error Estimation and Control for Farfield Acoustic Signatures

Sriram K. Rallabhandi, Marian Nemec , Michael J. Aftosmis

Computational Aerosciences Branch, NASA Ames Research Center



QUEST

Session: SPSN-03, Supersonics

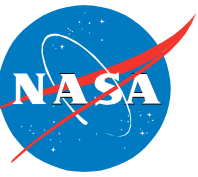
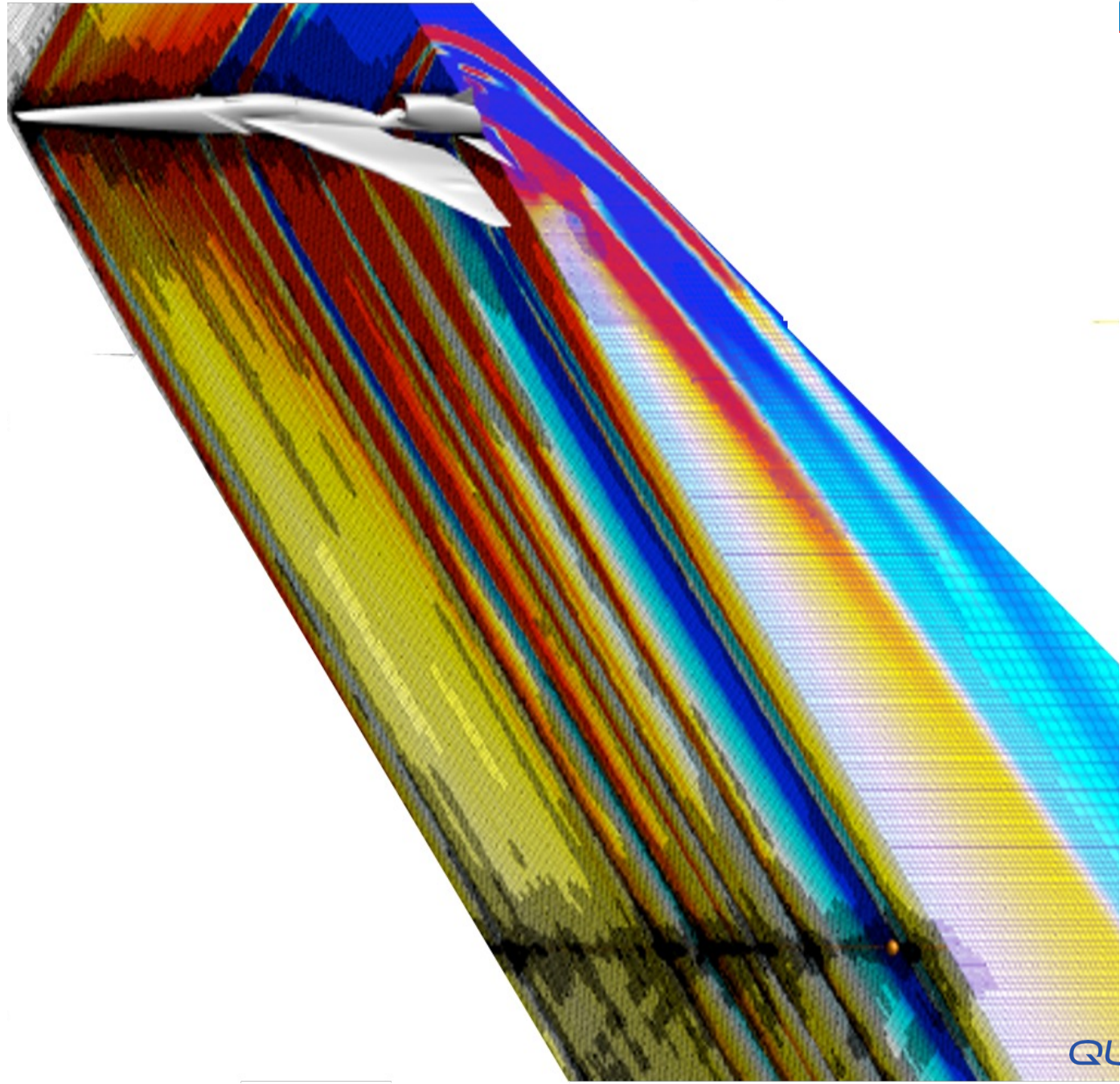
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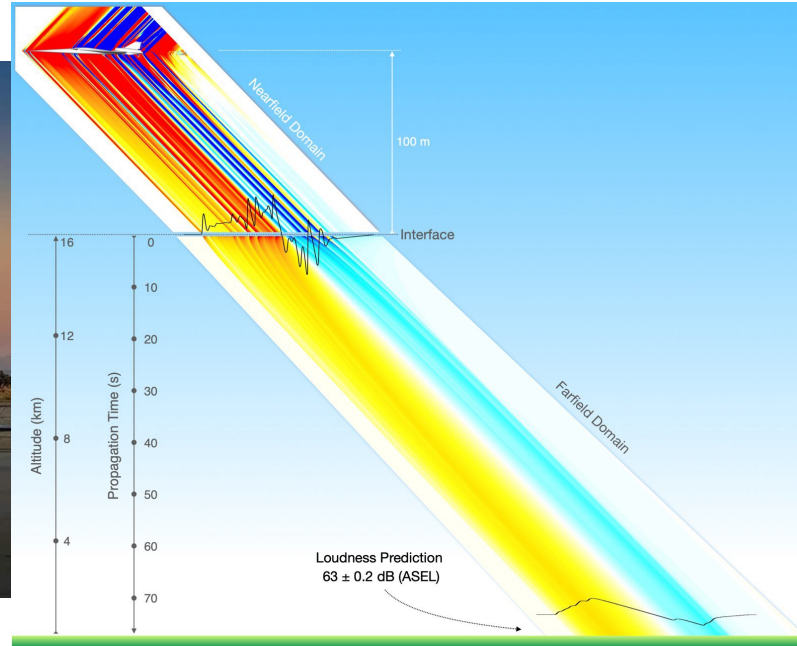
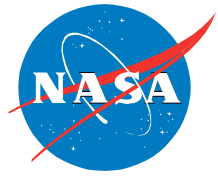
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Outline

- Introduction
 - Quesst Mission
 - Sonic Boom
 - Governing Equations
- Motivation
- sBOOM
 - Primal
 - Dual (Adjoint)
- Discretization Error Estimation
- Results
 - N-wave (Exact)
 - NACA 0003
 - X-59 Loudness Analysis
- Concluding Remarks & Future Work



Quesst Mission



Phase 1—Aircraft Development (X-59) Phase 2—Acoustic Validation

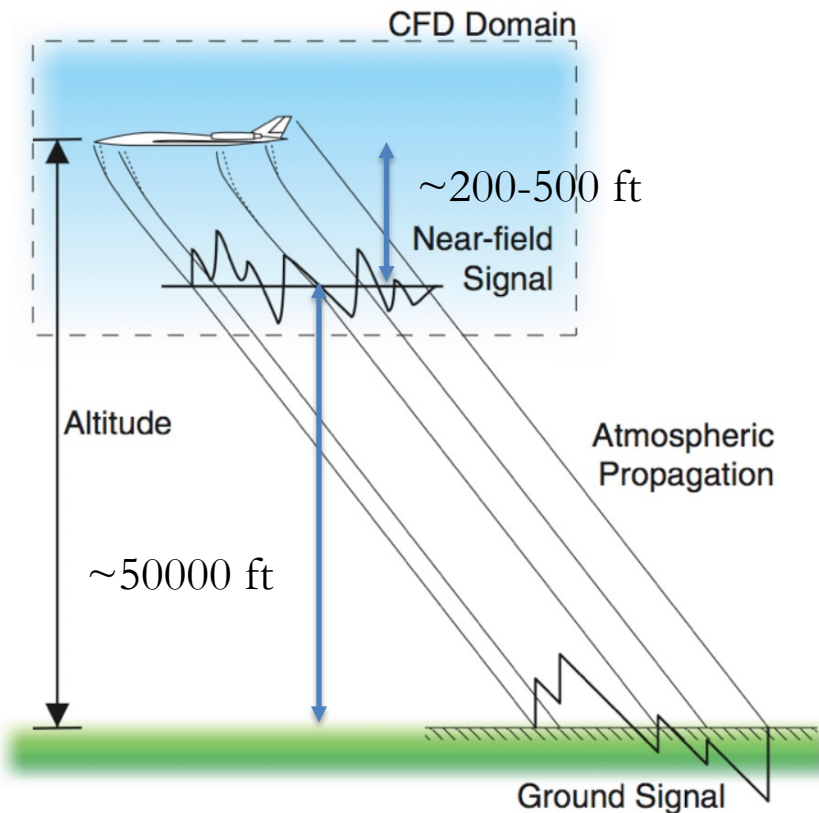
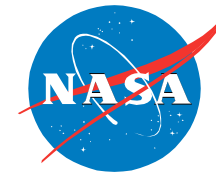
- Detailed Design
- Fabrication, Integration, Ground Test
- Checkout Flights
- Subsonic Envelope Expansion
- Supersonic Envelope Expansion

- Near-, Mid-, Far-field Measurements
- Ground Measurements

Phase 3—Community Response

- Initial community response overflight study
- Multiple campaigns (4 to 6) over representative communities and weather across the U.S.

Sonic Boom Computation and Ray Paths

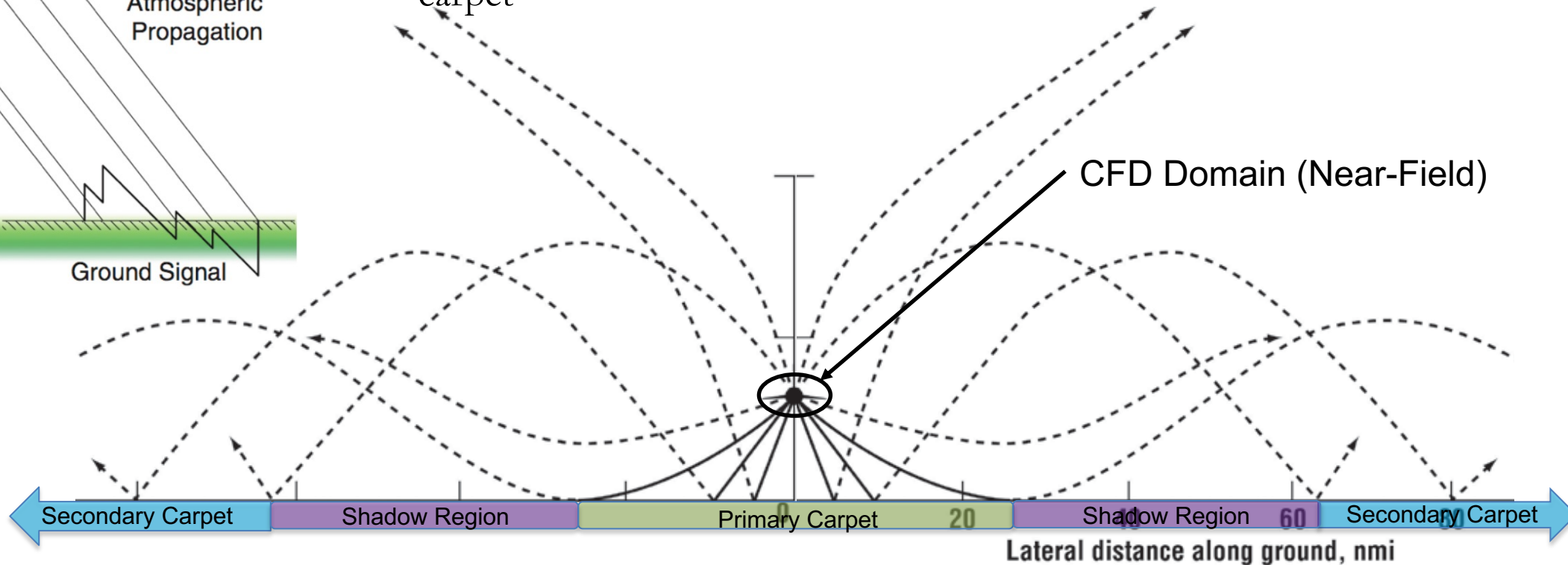


Sonic boom computation involves

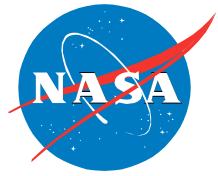
- Near-field pressures from CFD
- Atmospheric propagation to the ground

Aircraft disturbances propagated along ray paths

- Rays go in all directions
- Can separate impact into primary carpet, shadow region and secondary carpet



Boom Modeling: Augmented Burgers' Equation



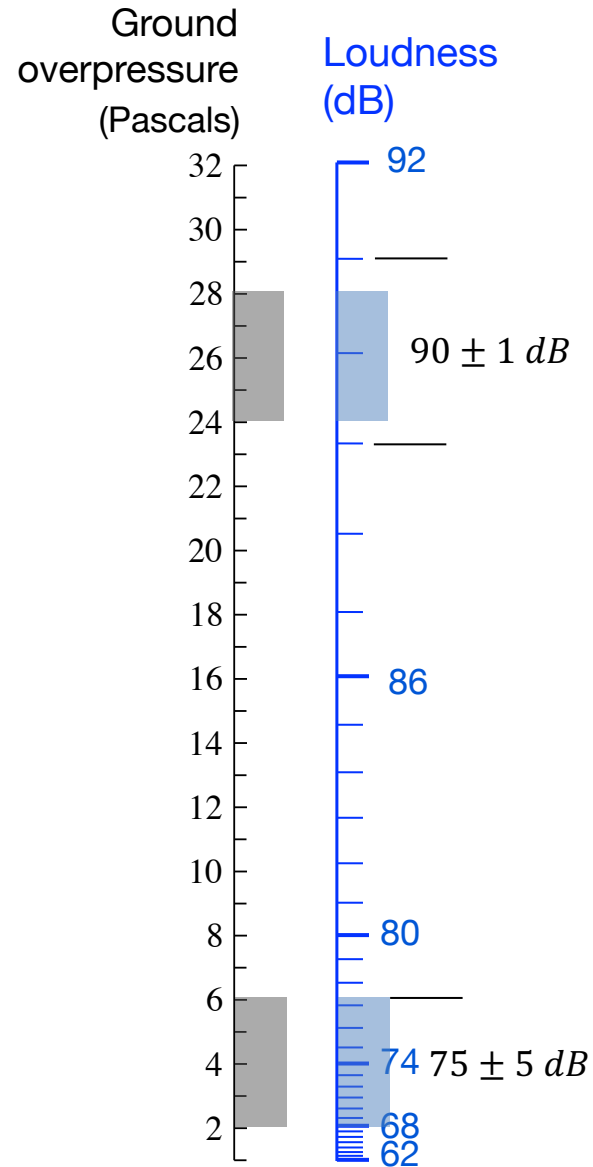
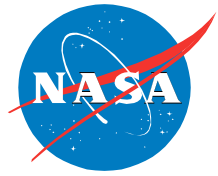
- Sonic boom propagation based on numerical solution of 1-D augmented Burgers' Equation

$$\frac{\partial p}{\partial \sigma} = \frac{\frac{\partial B}{\partial \sigma}}{B} p + \underbrace{\frac{\beta \Omega}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \frac{\partial p^2}{\partial \tau}}_{\text{Nonlinearity}} + \underbrace{\frac{\delta \Omega^2}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \frac{\partial^2 p}{\partial \tau^2}}_{\text{Thermo-viscous absorption}} + \underbrace{\sum_v \frac{m_v}{2 c_0} \frac{t_v \Omega^2}{1 + t_v \Omega \frac{\partial}{\partial \tau}} \left(\frac{c_0}{v_g} \right) \frac{\partial^2 p}{\partial \tau^2}}_{\text{Molecular Relaxation}}$$

$$B = \sqrt{\frac{\rho c S_0 D_0^2}{\rho_0 c_0 S D^2} \left(\frac{\frac{c v_{g0}}{D_0}}{\frac{c_0 v_g}{D}} \right)}, v_g = |c \cdot \bar{n} + \bar{w}|, D = 1 + \frac{\bar{w} \cdot \bar{n}}{c}, \Omega = \frac{1}{D}$$

- D = Doppler wind effects

Sonic Boom Modeling: Numerical Challenges

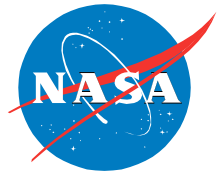


- Impact of an error of ± 2 Pa on a 90 PLdB signal may be less than 1 dB
- The same error on a 75 PLdB signal could be ± 5 dB
- Mesh requirements and analysis uncertainty increase as aircraft designs becomes quieter!

Motivation:

- What is the discretization error in ground signatures?
- What is the discretization error in the predicted loudness metrics?
- Are solutions mesh converged?

Sonic Boom Analysis (Primal Algorithm)



$$\frac{\partial p}{\partial \sigma} = \frac{\frac{\partial B}{\partial \sigma}}{B} p + \frac{\beta \Omega}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \frac{\partial p^2}{\partial \tau} + \frac{\delta \Omega^2}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \frac{\partial^2 p}{\partial \tau^2} + \sum_v \frac{m_v}{2 c_0} \frac{t_v \Omega^2}{1 + t_v \Omega \frac{\partial}{\partial \tau}} \left(\frac{c_0}{v_g} \right) \frac{\partial^2 p}{\partial \tau^2}$$

- Operator splitting to tackle each mechanism separately
- Iterative solution of a set of 7 equations

Relaxation Step 1

Relaxation Step 2

Absorption

Nonlinearity

$$A_{v,n,1} P_{v,n,1} = k_n B_{v,n,1} p_n$$

$$A_{n,1} q_n = k_n p_n + B_{n,1} P_{v,n,1}$$

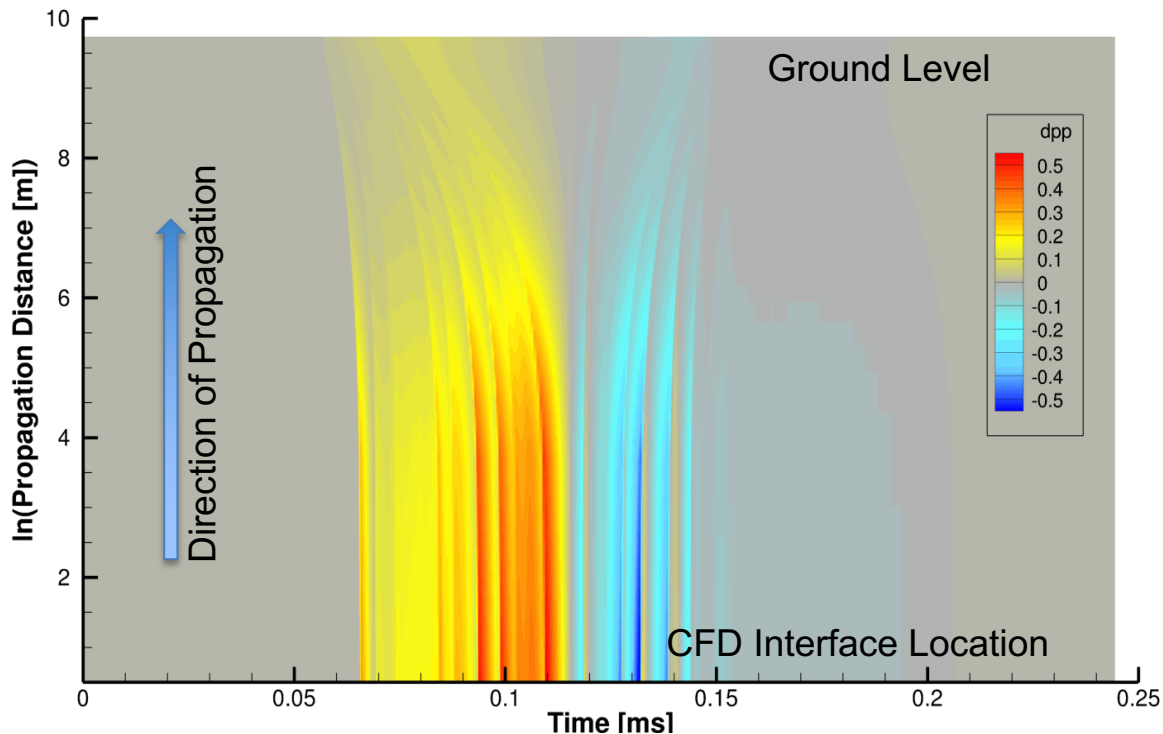
$$A_{v,n,2} P_{v,n,2} = B_{v,n,2} q_n$$

$$A_{n,2} r_n = I q_n + B_{n,2} P_{v,n,2}$$

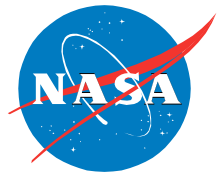
$$A_{n,3} t_n = B_{n,3} r_n$$

$$s_n = f(t_n)$$

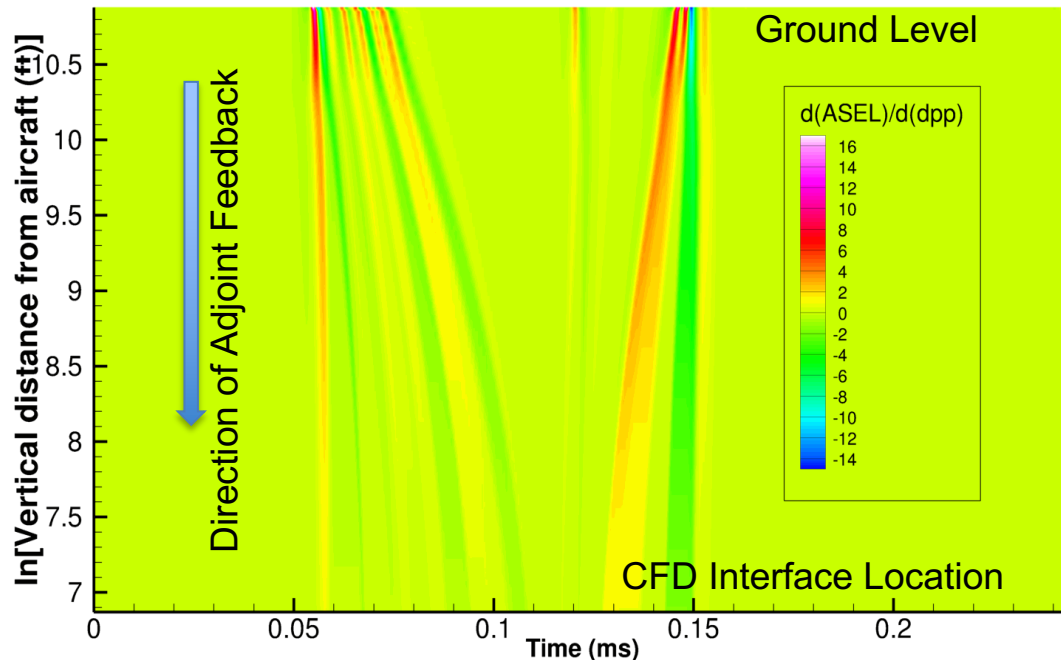
$$p_{n+1} = \frac{1}{2} f(s_n) + \frac{1}{2} t_n$$



Sonic Boom Adjoints (Dual Algorithm)



- Adjoint (dual) solution procedure involves iterative solution of a set of 7 equations
- Solution begins at the ground (end of primal solve) and proceeds backwards



$$\lambda_{n,2}^T = -\frac{\partial J}{\partial p_N} \text{ when } n = N$$

$$\lambda_{n,2}^T = \epsilon_n^T k_n B_{n,v,1} + k_n \gamma_{0,n}^T \text{ Otherwise}$$

$$\lambda_{n,1}^T = \frac{1}{2} \lambda_{n,2}^T \frac{\partial f}{\partial s_n}$$

$$\beta_n^T A_{n,3} = \lambda_{n,1}^T \frac{\partial f}{\partial t_n} + \frac{1}{2} \lambda_{n,2}^T$$

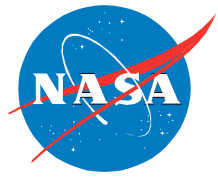
$$\gamma_{1,n}^T A_{n,2} = \beta_n^T B_{n,3}$$

$$\delta_n^T A_{n,v,2} = \gamma_{1,n}^T B_{n,2}$$

$$\gamma_{0,n}^T A_{n,1} = \delta_n^T B_{n,v,2} + \gamma_{1,n}^T$$

$$\epsilon_n^T A_{v,1} = \gamma_{0,n}^T B_{n,1}$$

Farfield Discretization Error Estimation



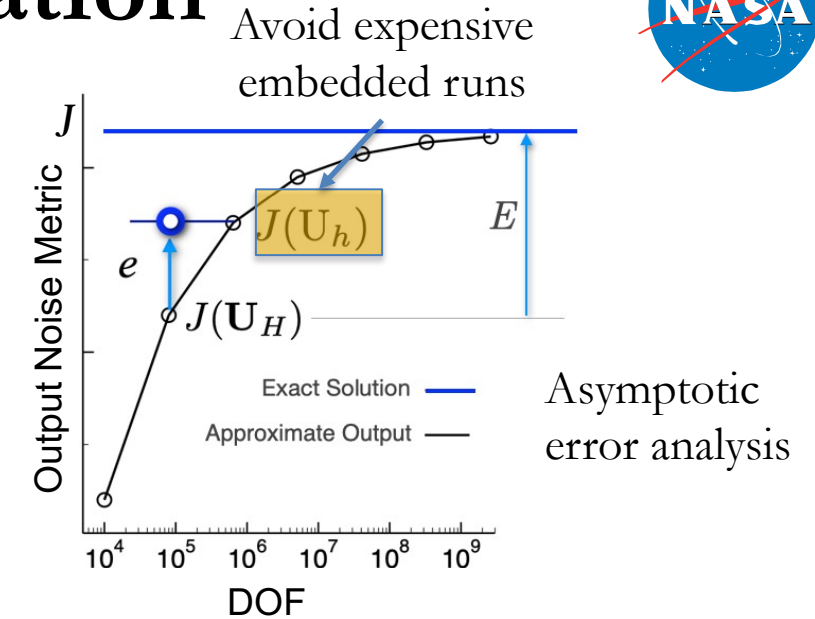
J = Cost functional

H = Current mesh

h = Embedded mesh

E = Error w.r.t. exact solution

e = Relative error w.r.t. embedded grid solution

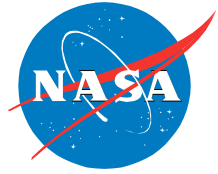


$$J(U_h) \approx J(U_h^H) + \frac{\partial J(U_h^H)}{\partial U_h} (U_h - U_h^H) \qquad J(U_h) \approx J(U_h^H) - \sum_{n=1}^N \psi_{h,n}^T R(U_{h,n}^H)$$

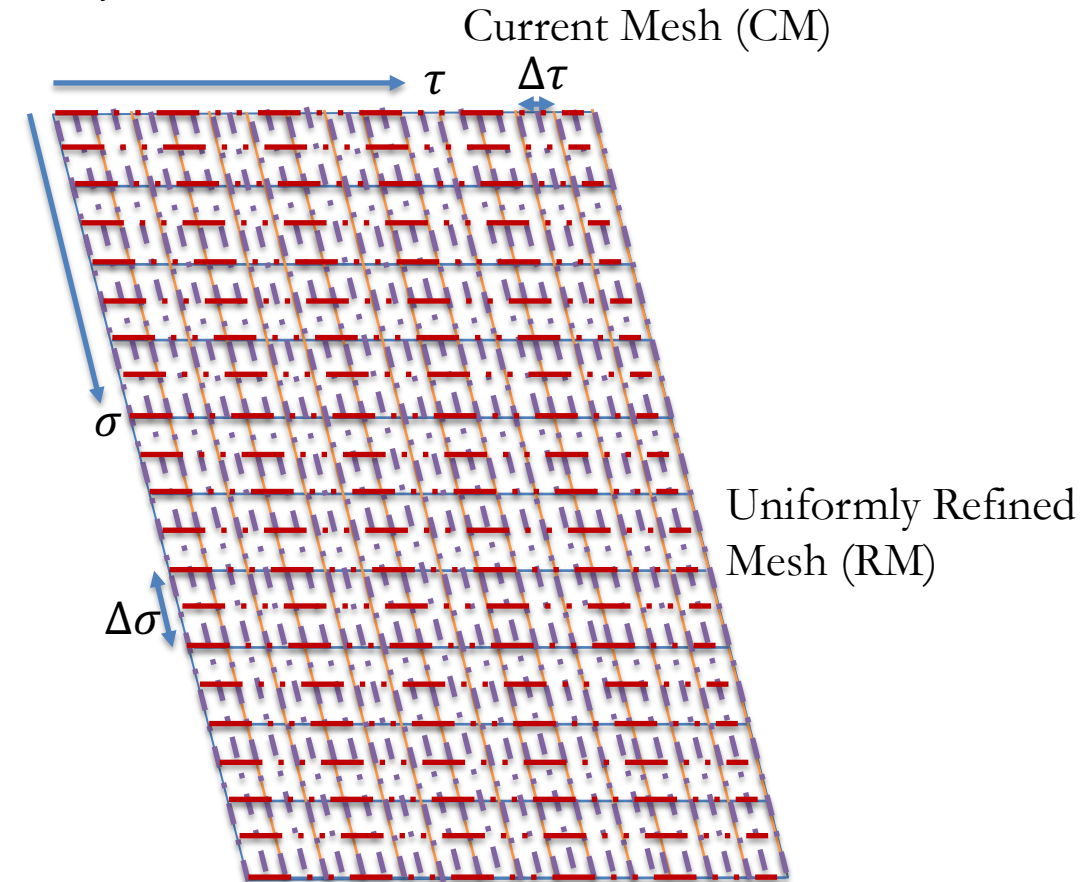
$$J(U_h) \approx J(U_h^H) - \sum_{n=1}^N (\psi_{h,n}^H)^T R(U_{h,n}^H)$$

If nonzero residuals occur in regions where the adjoint quantities are large, then these residuals will contribute a relatively large amount to the total output error

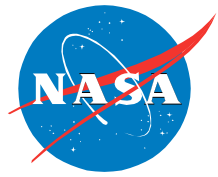
Error Estimation Setup



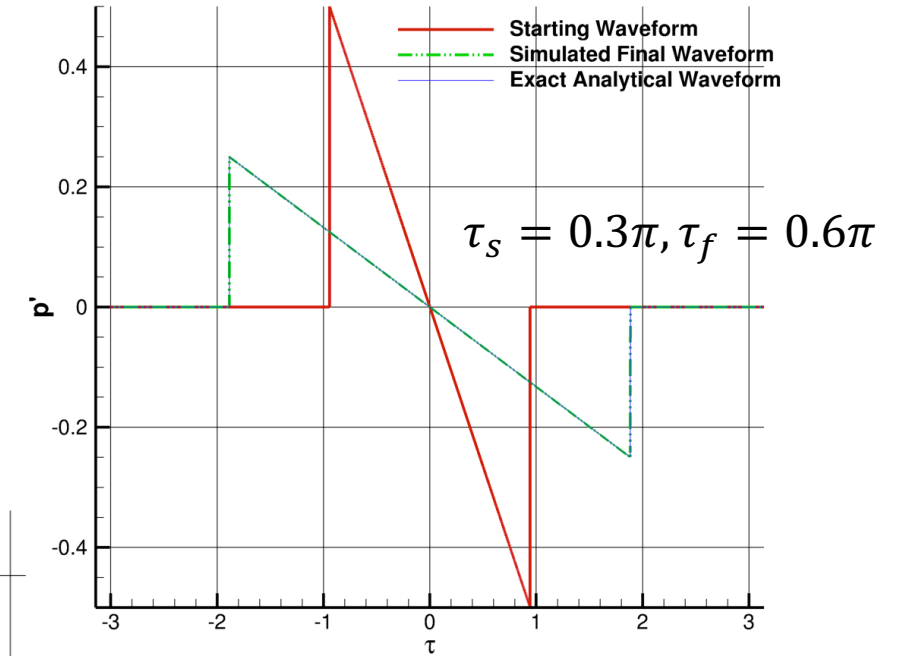
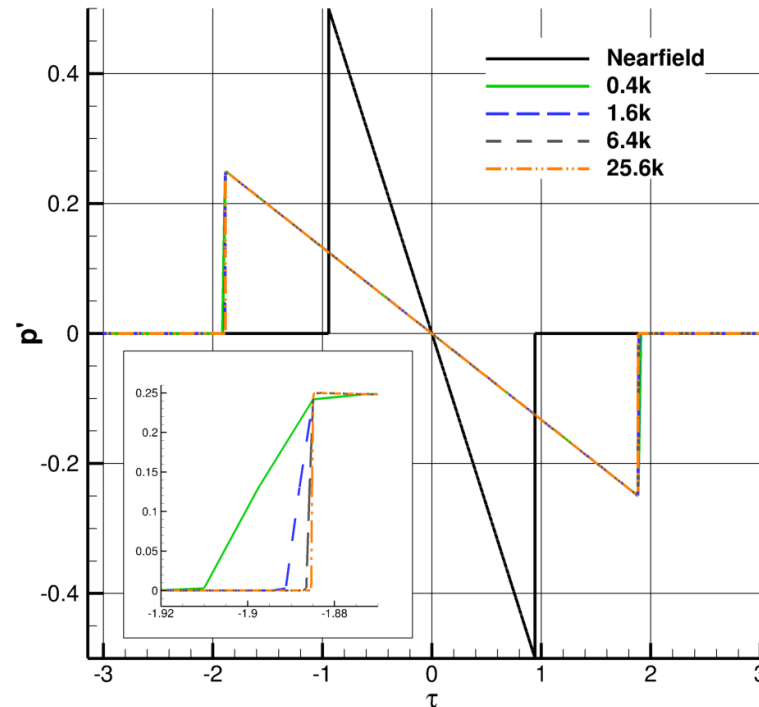
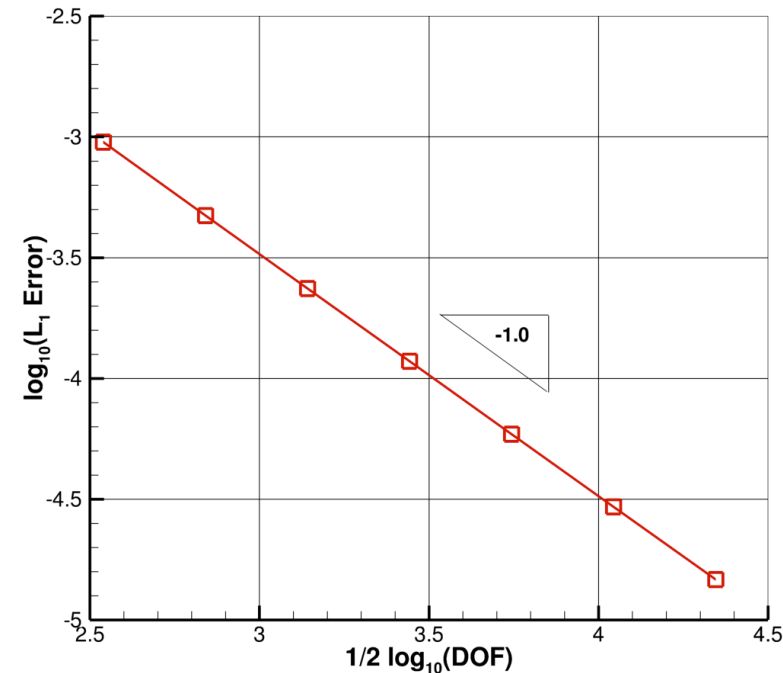
- Investigate errors as the space(τ)-time(σ) mesh is uniformly refined
 - Spacing in τ and σ related through CFL
 - $\Delta\sigma = \frac{(CFL)\Delta\tau}{\max(P)}$
- Refined Mesh
 - $\Delta\sigma_{RM} = \frac{1}{2}\Delta\sigma_{CM}$
 - $\Delta\tau_{RM} = \frac{1}{2}\Delta\tau_{CM}$
 - $DOF_{RM} = 4 \times DOF_{CM}$
- Interpolation
 - Bilinear: Residuals
 - Biquadratic: Adjoint solutions



Results: N-Wave



- N-wave initial pressure waveform
 - Existence of analytical solution
 - Uniform atmosphere, absorption/relaxation ignored
 - $M=1.4$, $\sigma_f = 1.8\pi$
- L_1 error norm of ground pressures as the space(τ)-time(σ) mesh is uniformly refined is $O(h)$
 - Error computed w.r.t. exact analytical solution

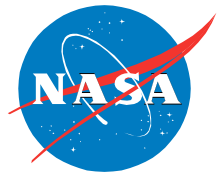


$$p'(\tau, 0) = -\Delta P \begin{cases} \frac{\tau}{\tau_s} & -\tau_s \leq \tau \leq \tau_s \\ 0.0 & \text{otherwise} \end{cases}$$

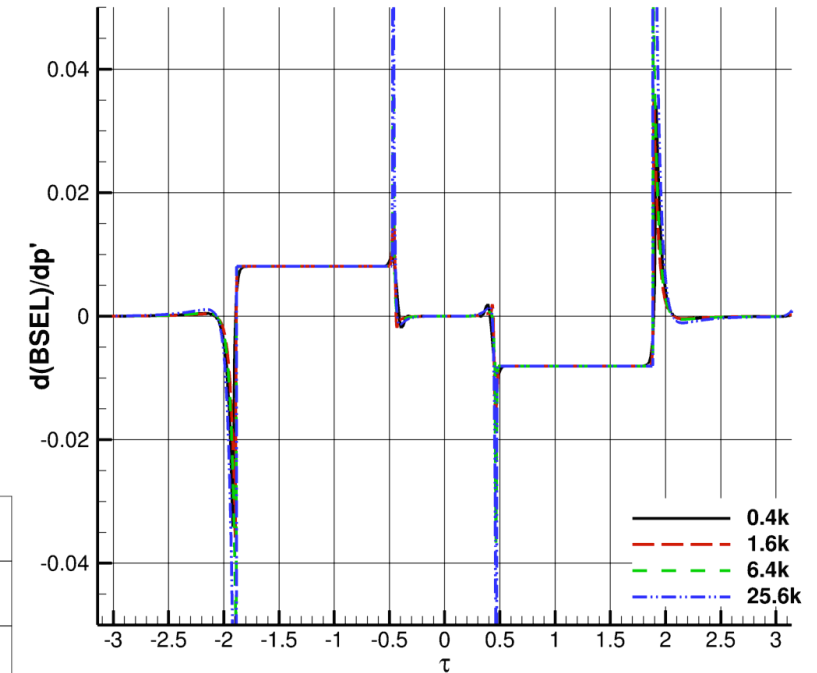
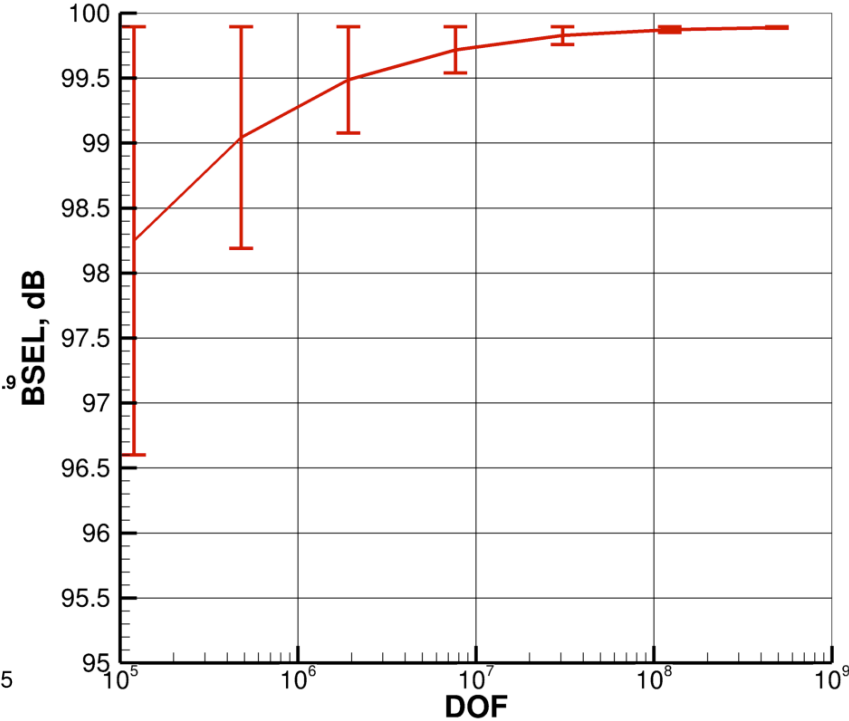
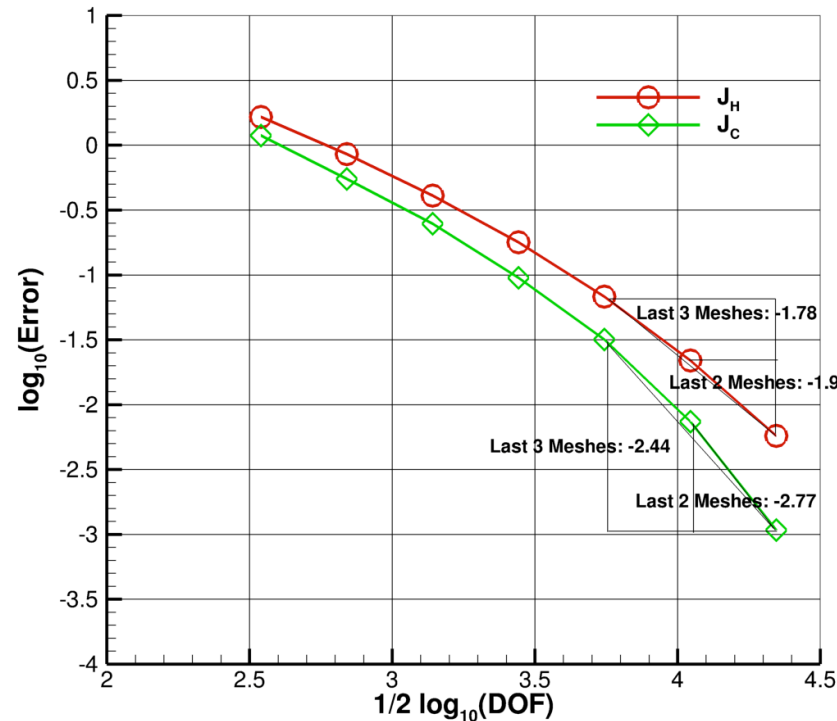
$$p'(\tau, \sigma) = -\Delta P \begin{cases} \frac{\tau}{\tau_s [1 + \frac{\sigma \Delta P}{\tau_s}]} & -\tau_f \leq \tau \leq \tau_f \\ 0.0 & \text{otherwise} \end{cases}$$

$$\tau_f = \tau_s \sqrt{1 + \frac{\Delta P \sigma}{\tau_s}}$$

Results: N-Wave, Errors in B-SEL

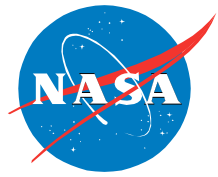


- Investigation of errors in B-weighted Sound Exposure Level (B-SEL)
- Adjoint (dual) solution convergence with uniform refinement
- Adjoint-weighted residual errors in B-SEL
 - Acceleration with increasing DOF
 - ~ 0.001 dB by the largest mesh (~ 175 kHz)

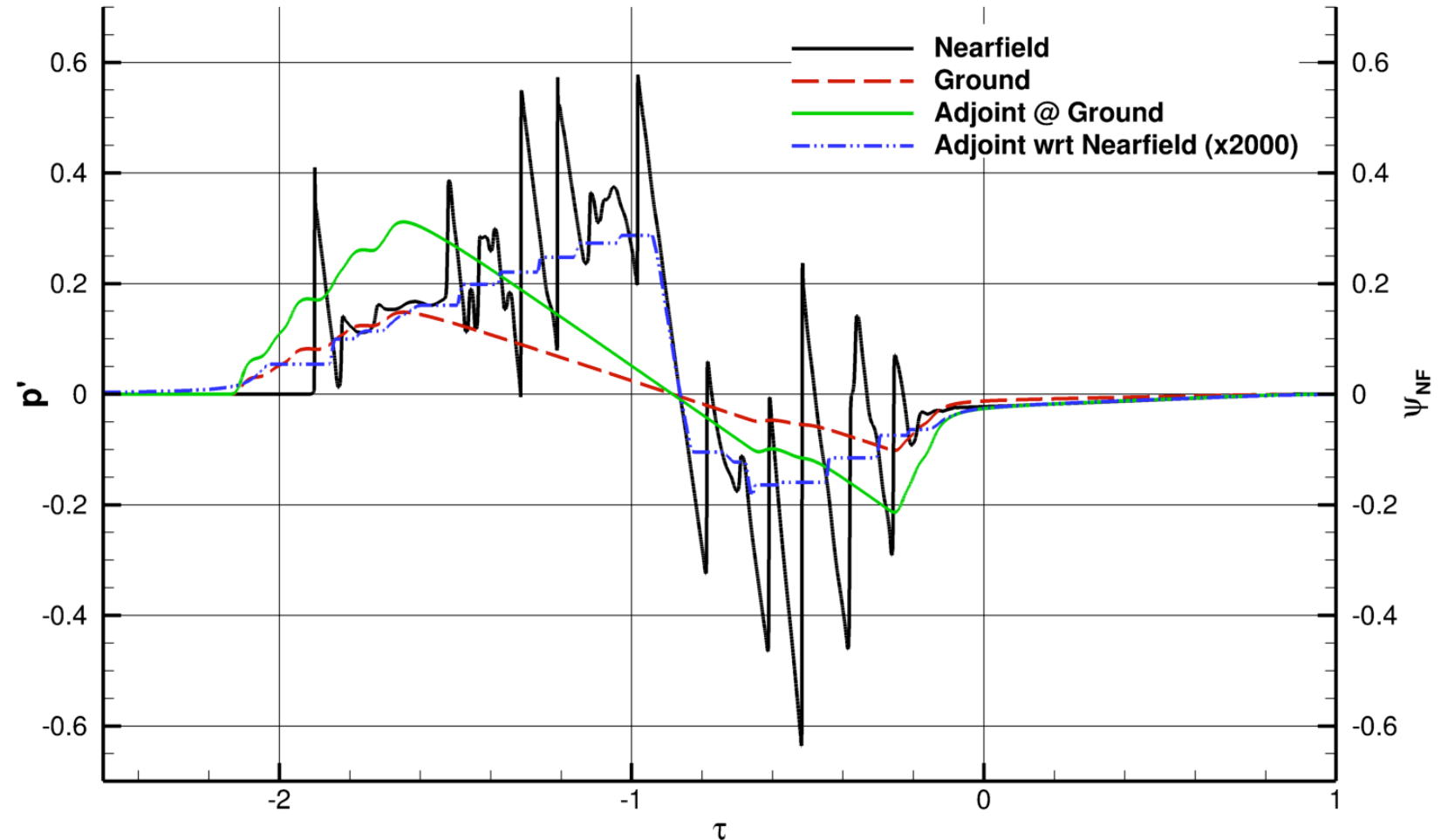


- Convergence of B-SEL
 - Error bars computed using adjoint-weighted residuals
 - Significant and continued reduction in errors

Results: X-59

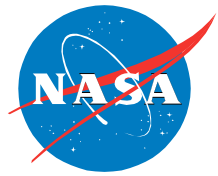


- Discretization error formulation applied to X-59 (C612A)
 - Stratified atmosphere
 - Nonlinearity, absorption, and relaxation mechanisms considered

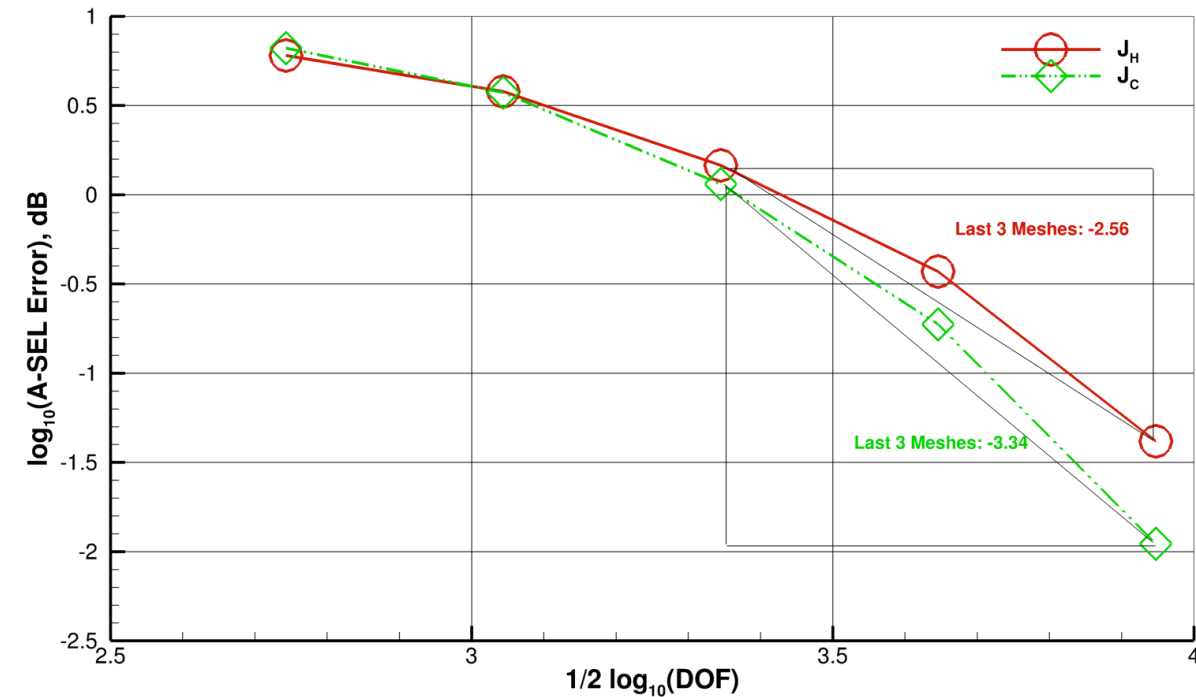
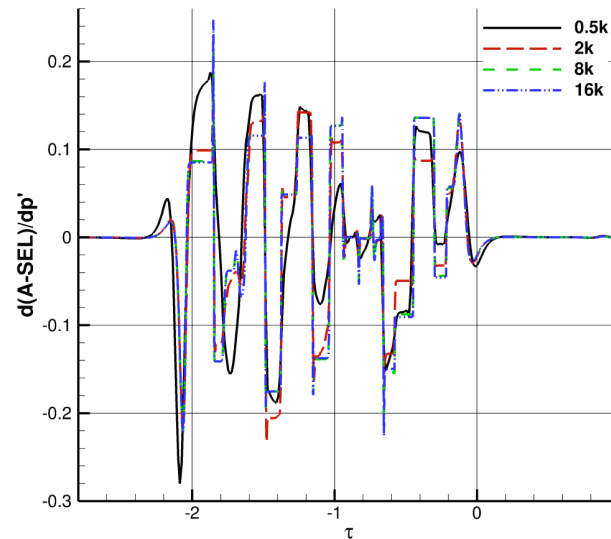
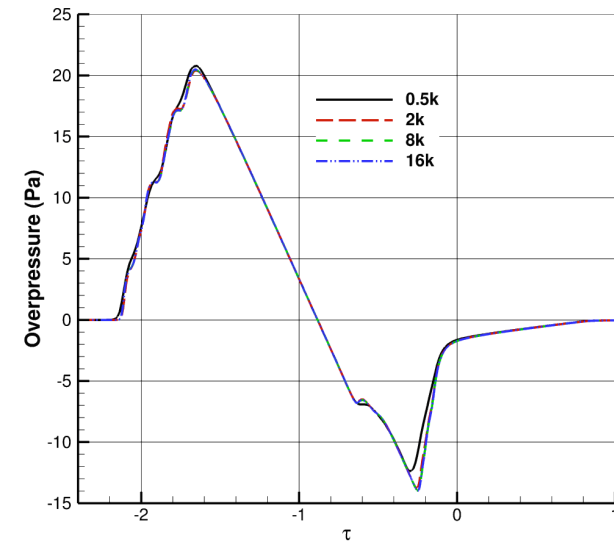
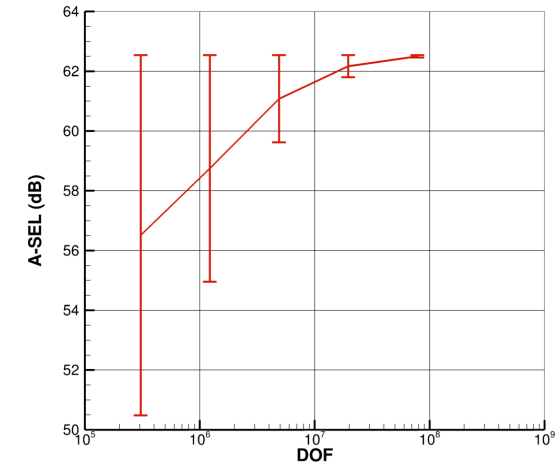


- Primal and adjoint solutions corresponding to J_G
 - Shaped low-boom (76 PLdB) ground signature
 - Expected behavior of the adjoint solutions

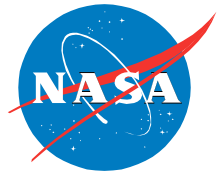
Results: X-59



- Discretization errors corresponding to A-SEL
 - Coarsest mesh 501 nodes (1.5 kHz); Finest mesh 16001 nodes (~ 53 kHz)
 - “Exact” levels computed on 64001 nodes (~ 210 kHz)
 - Primal solutions converge earlier than adjoint solutions
 - Adjoint-weighted residuals predict monotonically reducing errors with uniform space-time mesh refinement
 - At a modest sampling frequency of ~ 53 kHz, A-SEL loudness values are predicted to within 0.01 decibels



Concluding Remarks and Future Work



- Method of adjoint-weighted residuals applied to sonic boom predictions to estimate discretization errors in farfield simulations
 - Leverages asymptotically consistent adjoint discretization
- Predicted level of discretization error in sonic boom farfield metrics offer insight whether additional refinement is needed to reduce them below a desired threshold
- Results indicate loudness errors < 0.01 decibels for realistically complex practical examples (X-59) when analyzed at modest sampling frequencies

Future Work

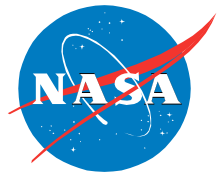
- Explore opportunities for reducing memory footprint and improving computational efficiency of the primal and dual solvers
- Leverage updated farfield formulation and couple with nearfield adjoints to allow reliable and robust aerodynamic shape optimization of low-boom aircraft

Acknowledgements

- NASA Commercial Supersonic Technology (CST) project under Aeronautics Mission Directorate (ARMD)
- NASA High-End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center
- Dave Darmofal and Renato Trono Figueras (MIT)



QUESTIONS?



BACK-UP

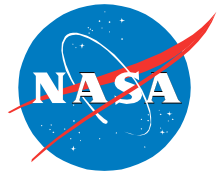
Sonic Boom Introduction



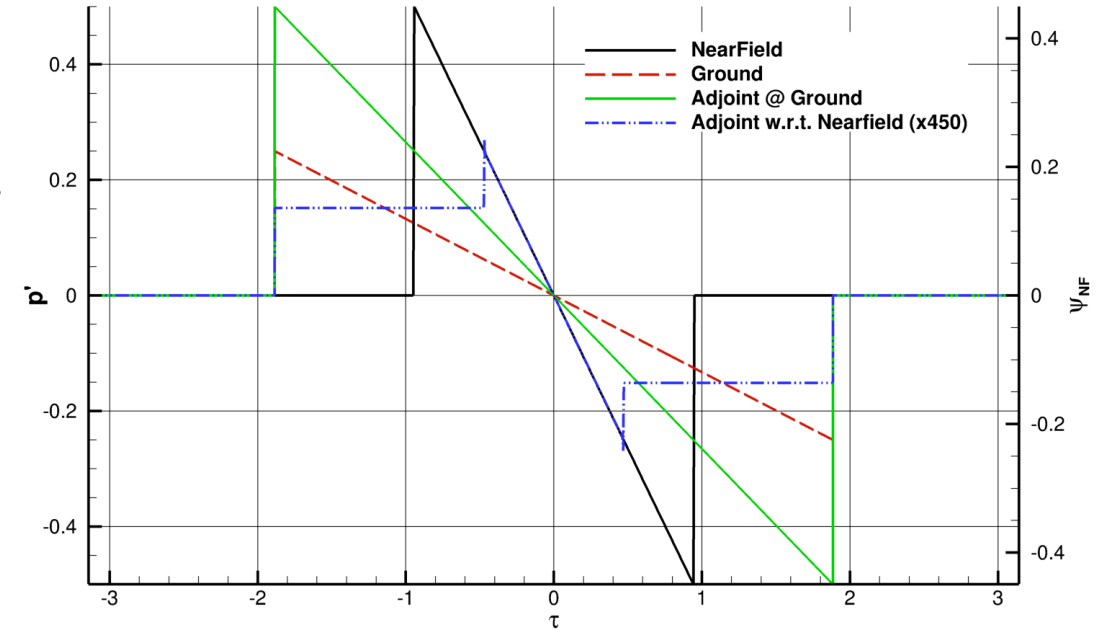
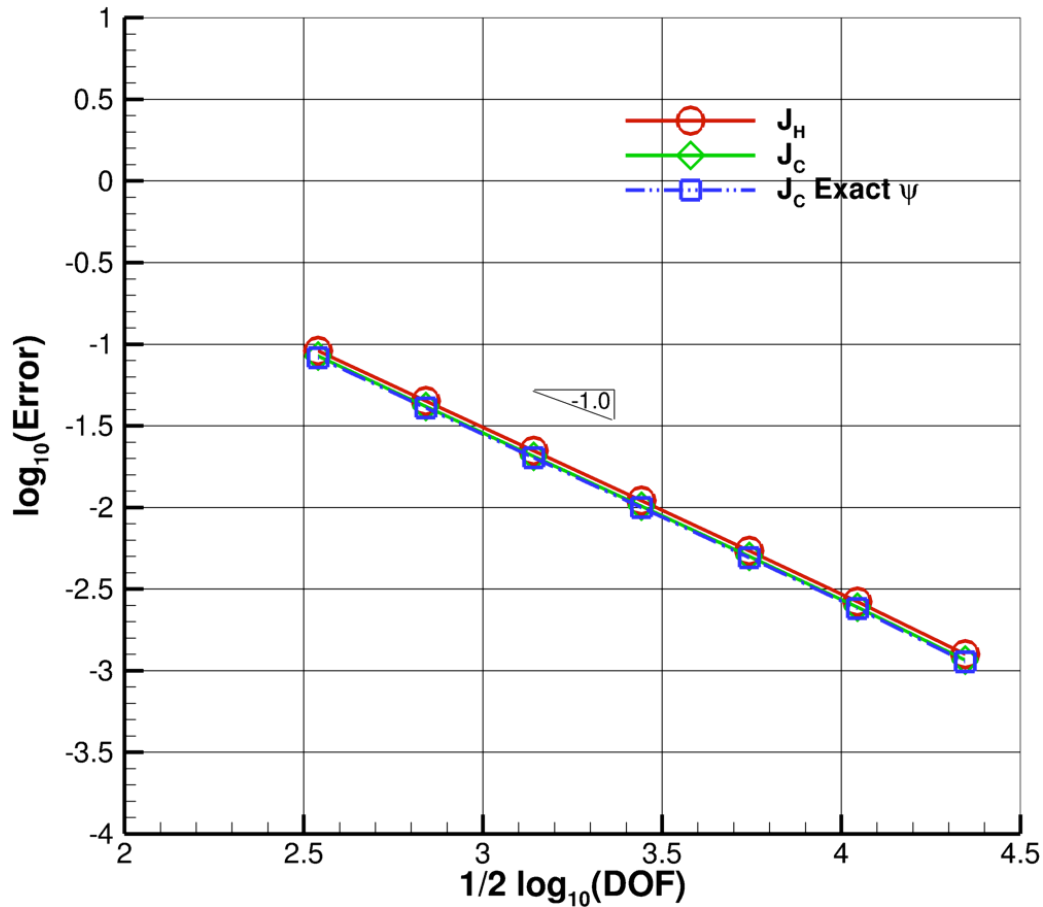
Disturbances from supersonic aircraft propagating to generate a sonic boom on the ground

Primary Sonic Boom Carpet (Sweeping Parabolic Region) on the Ground

Results: N-Wave, Errors in Ground Pressures

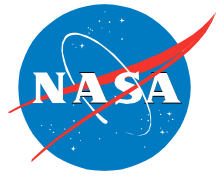


- Investigation of errors in $J_G = [\sum_{i=1}^N P_i^2] \Delta \tau$
- Pressure waveforms (primal) and adjoint (dual) solutions

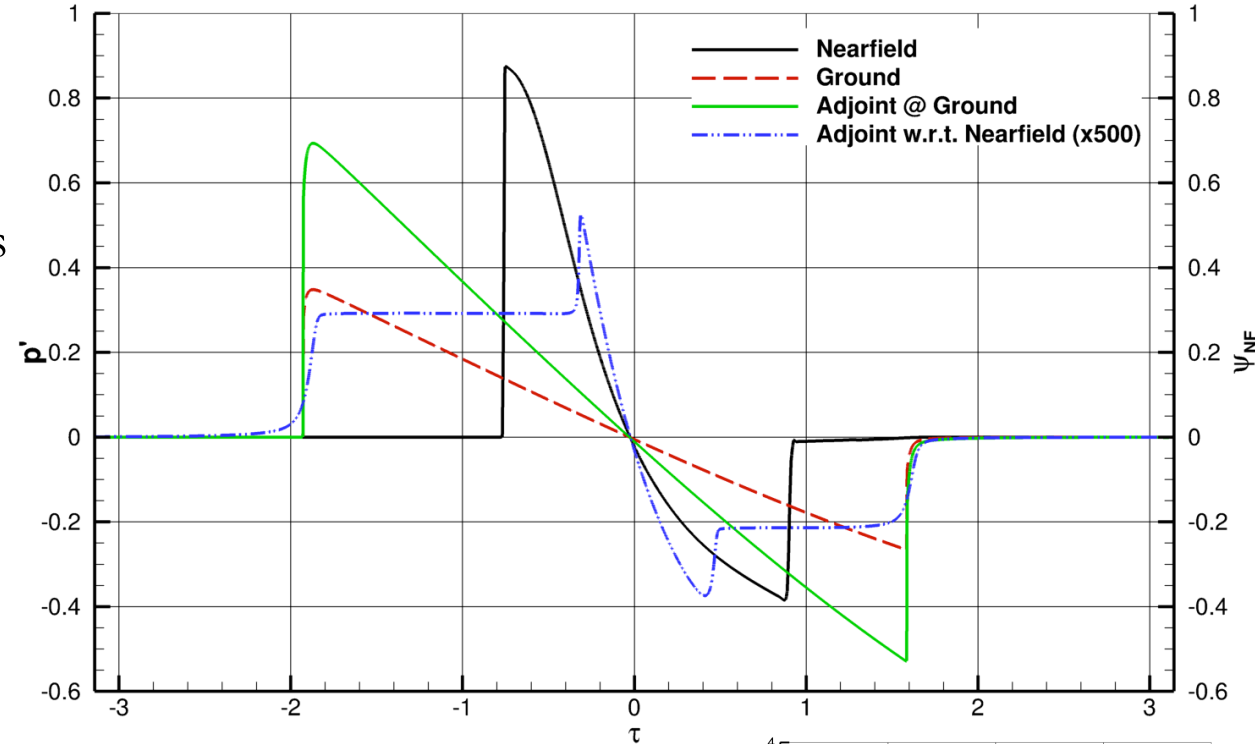
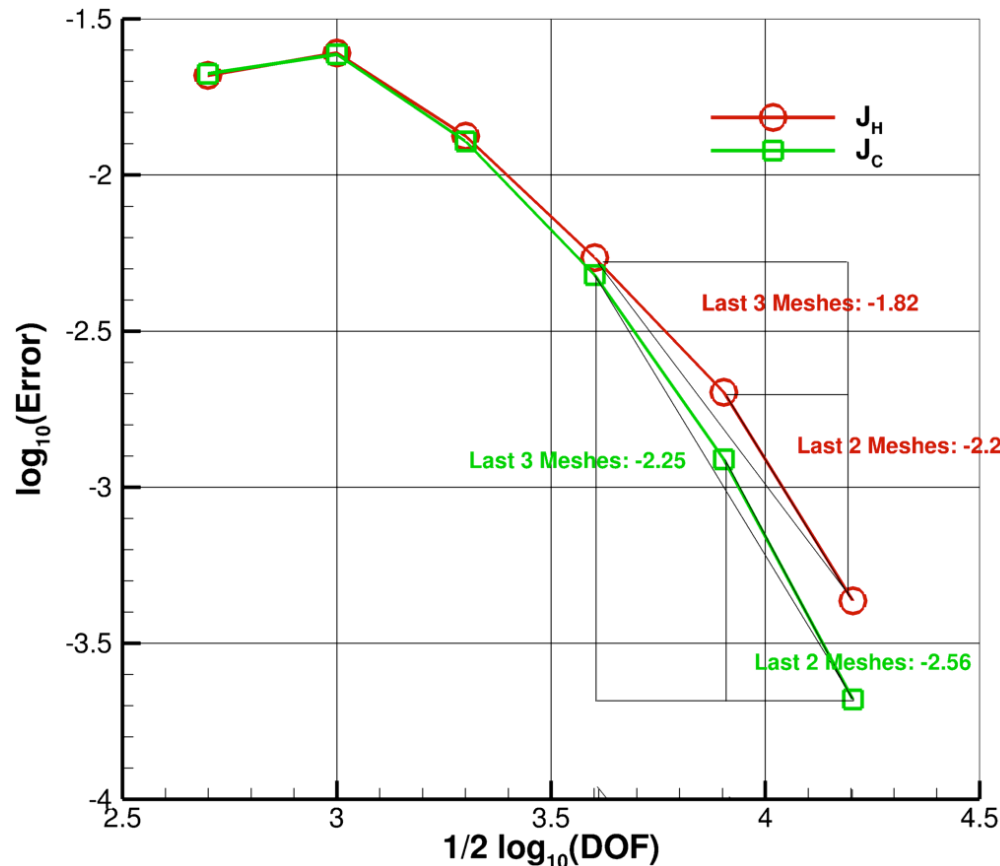


- Adjoint-weighted residual errors in J_G
 - Linear drop in errors with uniform refinement

Results: NACA 0003 Airfoil



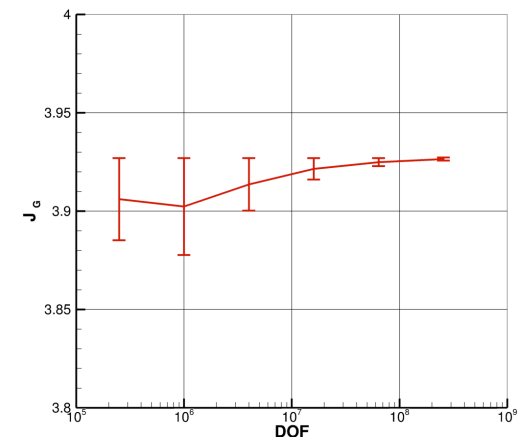
- Pressure waveform from NACA 0003
 - Scaled to 100 ft, off-body distance of 50 ft, $M=1.3$
 - All loss mechanisms modeled under uniform atmosphere
- Pressure waveforms (primal) and adjoint (dual) solutions



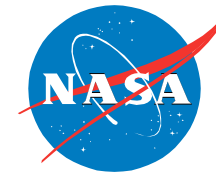
- Adjoint-weighted residual errors in J_G

- Error correction accelerates with increasing refinement
- Errors $\sim O(h^{5/2})$
- Errors monotonically reducing, ~ 0.0001

$$J_G = \left[\sum_{i=1}^N P_i^2 \right] \Delta \tau$$



Quantifying Impact of Sonic Booms



Impulsive Noise Source

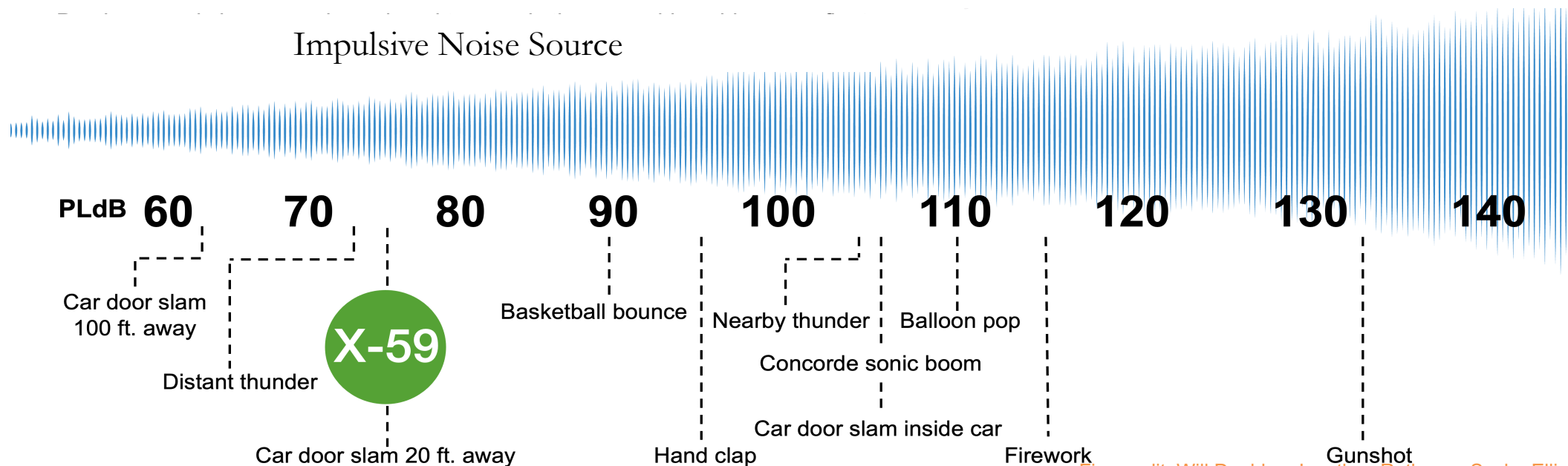
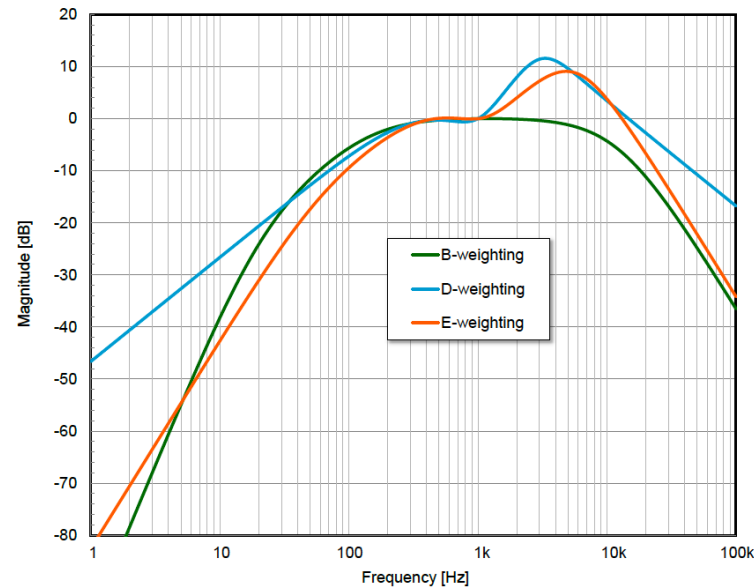
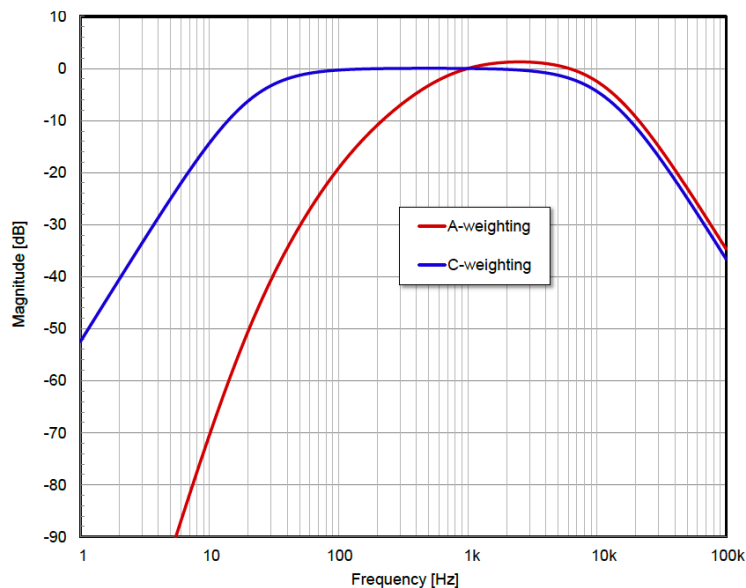
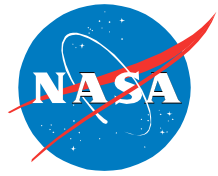


Fig. credit: Will Doebl, Jonathan Rathsam, Sasha Ellis

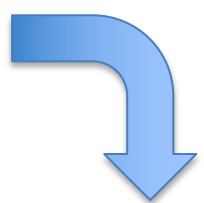
https://www.nasa.gov/sites/default/files/thumbnails/image/x-59_sonic_thump_75_pldb.png



Loudness Metric Sensitivities



- Recall 2nd order cascaded filters

$$W(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=0}^N b_i z^i}{\sum_{i=0}^M a_i z^i}$$

$$W(z) = G \prod_{k=1}^L \frac{\sum_{i=0}^2 b_{i,k} z^i}{\sum_{i=0}^2 a_{i,k} z^i}$$

Loudness Metric Sensitivities:

- Cascaded filters analytically differentiated
- Sensitivities from different frequency weightings show smooth behavior

