



Mirror Surface Contamination Specification Derived from Coronagraphy Scatter Error Budget Allocation

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Executive Summary



Near-Angle Scatter (NAS) of Host Star's Light may define the Dark-Hole's Noise Floor.

- NAS from each optical surface produces an E-field across the Dark Hole that is coherent.
- NAS E-fields sum – they do not RSS – to add a positive bias onto the WF instability E-field.

There are multiple sources of NAS

- Long-Spatial Structure (2023 O&P paper) which can be corrected by DM
- Microstructure (2024 Astro paper)
- Contamination (topic of this paper)
- Micrometeoroids (topic of this paper)
- Coating Structure and Defects

2024 Astro paper

- Derived a relationship between BRDF and Coronagraph Contrast
- Developed process for defining Microstructure Specifications

This paper applies the BRDF/Contrast relationship for defining Contamination Specification

This analysis does not include:

- Diffraction or scatter from segment edges
- Polarization – **important:** different polarization states scatter differently.



Motivation & Definitions

Near Angle Scatter

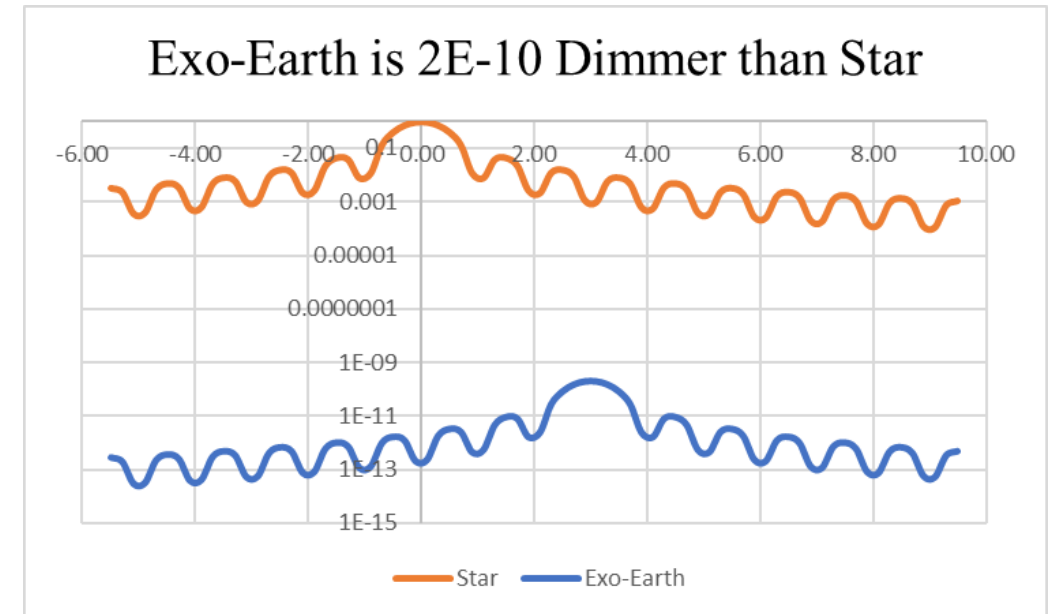
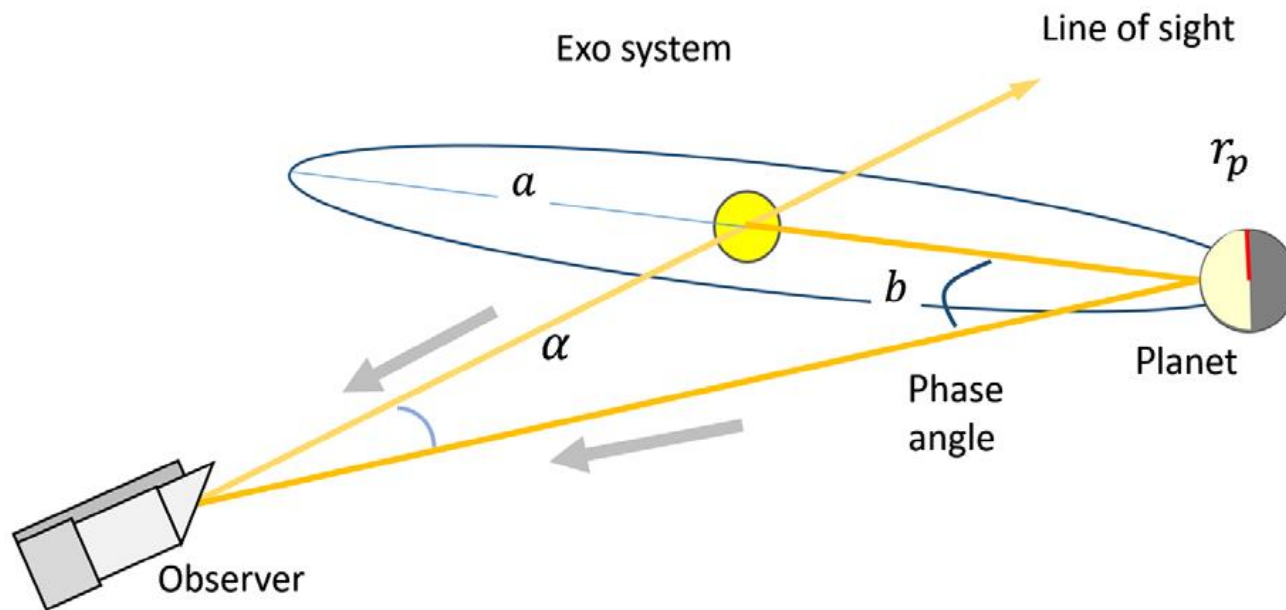
Per Pfisterer, Harvey and Breckinridge (2018), near angle scatter from mirror surface microroughness and particulate contamination “may hopelessly overwhelm” the ability to the ability to detect and characterize an Earth class exoplanet.



Science Measurement

To characterize an exo-Earth, its PSF needs to be detectable above starlight halo.

Flux ratio for an exo-Earth at 10 pc is 210 parts per trillion (ppt). (2×10^{-10})

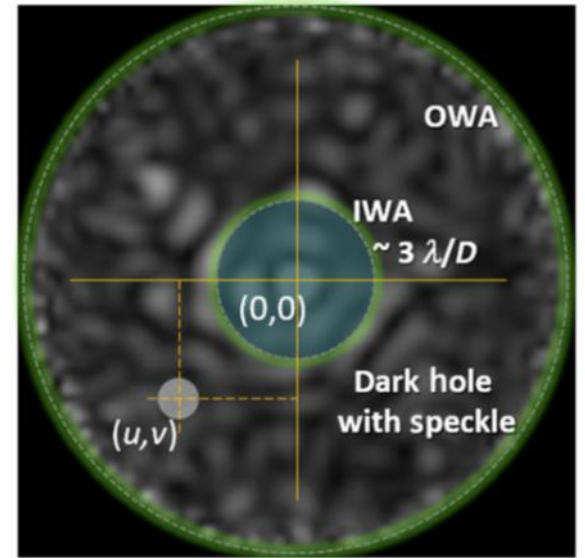
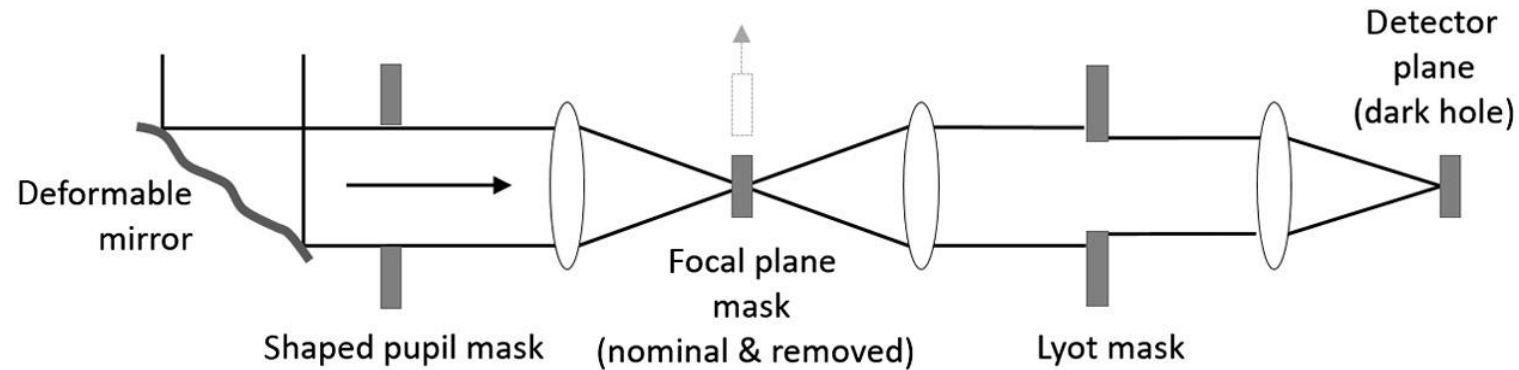


Flux ratio is different for planets that are larger or smaller, closer to or further from their star.

For the purpose of this analysis, the exo-Earth is located $2\lambda/D$ from the host star.

Dark Hole

Coronagraph images planet by reimaging Telescope Prime Focus onto occulting mask where star light is blocked.



Planet light pass occulter and is reimaged into the detector focal plane.

The ability of a coronagraph ability to detect an exo-Earth is quantified by 'Raw Contrast'.

Raw Contrast (C)

Raw Contrast, $C(u,v)$, is defined as the ratio of two throughputs:

$$C(u, v) = \frac{\tau(u, v; 0, 0)}{\tau(u, v; u, v)}$$

where:

$\tau(u, v; 0, 0)$ = throughput at (u,v) for a point source located on-axis $(0,0)$ (i.e. Host Star)

$\tau(u, v; u, v)$ = throughput for a point source located at (u,v) (i.e. Exoplanet)

Throughput is ratio of power at (u,v) to power incident on entrance pupil (i.e. primary mirror)

$$\tau(u, v; 0, 0) = \frac{\Phi(u, v)}{\Phi_i(0, 0)}$$

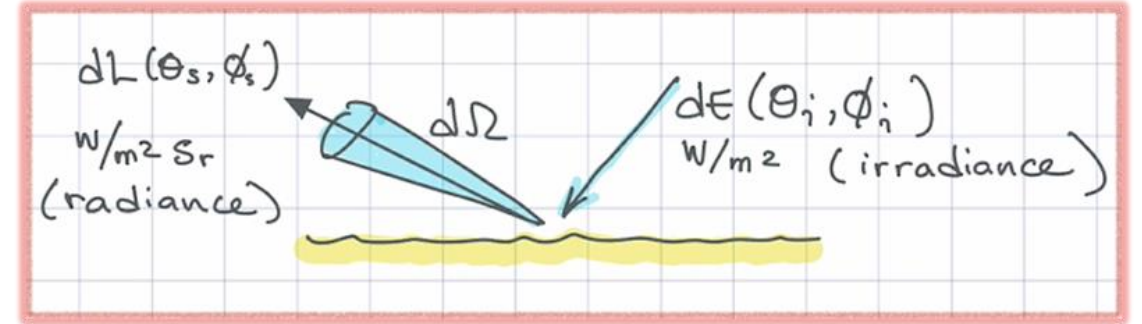
$$\tau(u, v; 0, 0) = \tau_{\text{Coronagraph Leakage}} + \tau_{\text{Wavefront Stability}} + \tau_{\text{NAS}} + \dots$$

Bidirectional Reflectance Distribution Function (BRDF)



BRDF (Nicodemus, 1977) is defined as the reflected radiance, L , (power per unit solid angle per unit projected area) divided by the incident irradiance, E , (power per unit area):

$$\begin{aligned} BRDF(\theta_s, \varphi_s; \theta_i, \varphi_i) &= \frac{dL(\theta_s, \varphi_s; \theta_i, \varphi_i)}{dE(\theta_s, \varphi_s; \theta_i, \varphi_i)} \\ &= \frac{\Phi_s / \Omega}{\Phi_i \cos(\theta_s, \varphi_s)} = \frac{\Phi_s / \Omega}{\Phi_i} \end{aligned}$$



For the purpose of modeling near-angle scatter of a telescope/coronagraph system:

- Φ_i = Total Power Incident on Entrance Pupil, i.e. Primary Mirror Area, $= E_i \times A_{PM}$
- Φ_s / Ω = Power per unit Solid Angle Scattered from Entrance Pupil, i.e. PM Area
- $(\theta_s, \varphi_s; \theta_i, \varphi_i)$ = $(N\lambda/D, N\lambda/D; 0, 0)$ where $N = 1$ to 30 so that (θ_s, φ_s) are very small
- Note: BRDF is frequently plotted in $(\beta - \beta_0)$ space where $\beta = \sin(\theta_s)$, $\beta_0 = \sin(\theta_i)$

Thus, Power scattered off Primary Mirror and imaged directly into the Dark Hole is

$$\Phi_s(u, v)_{Core} = BRDF(\lambda/D < (\beta - \beta_0) < N\lambda/D) \cdot \Phi_i \cdot \Omega_{Core}$$

Nicodemus, F E, et al "Geometric Considerations and Nomenclature for Reflectance", US Dept of Commerce, Washington DC, NBS Monograph 160, 1977

Stover, John C, Optical Scattering Measurement and Analysis, McGraw-Hill, New York, NY, 1990

Harvey, James E, Understanding Surface Scatter Phenomena a Linear Systems Formulation, SPIE, Bellingham, WA, 2019

Near-Angle Scatter Throughput = $BRDF \times \Omega_{Core}$



Using the definitions for Throughput and BRDF, we now define a ‘new’ Throughput.

Near-Angle Scatter Throughput is the ratio of power scattered off an optical surface (e.g. primary mirror) and imaged into Dark Hole relative to power incident on optical surface.

$$\text{NAS Throughput} = \tau_{NAS} = \frac{\Phi_{NAS}(u, v)_{Dark\ Hole}}{\Phi_i} = BRDF(2\lambda/D) \cdot \Omega_{Core}$$

$$\tau_{NAS} = BRDF(0.2 \mu rad) \cdot 10^{-14} \text{ Sr}$$

where:

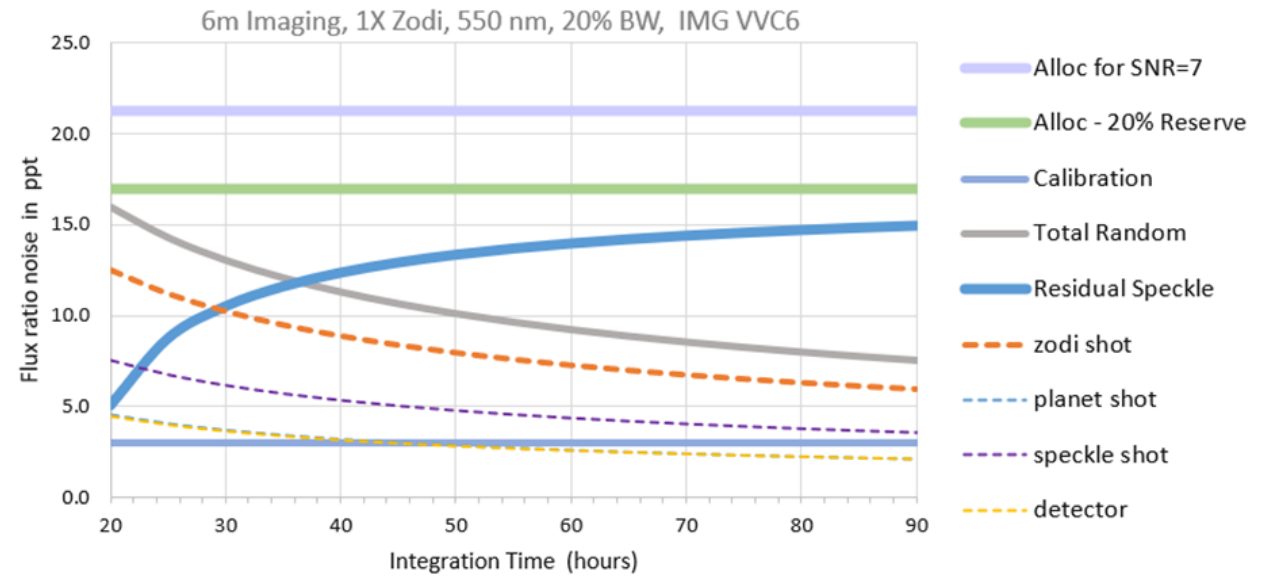
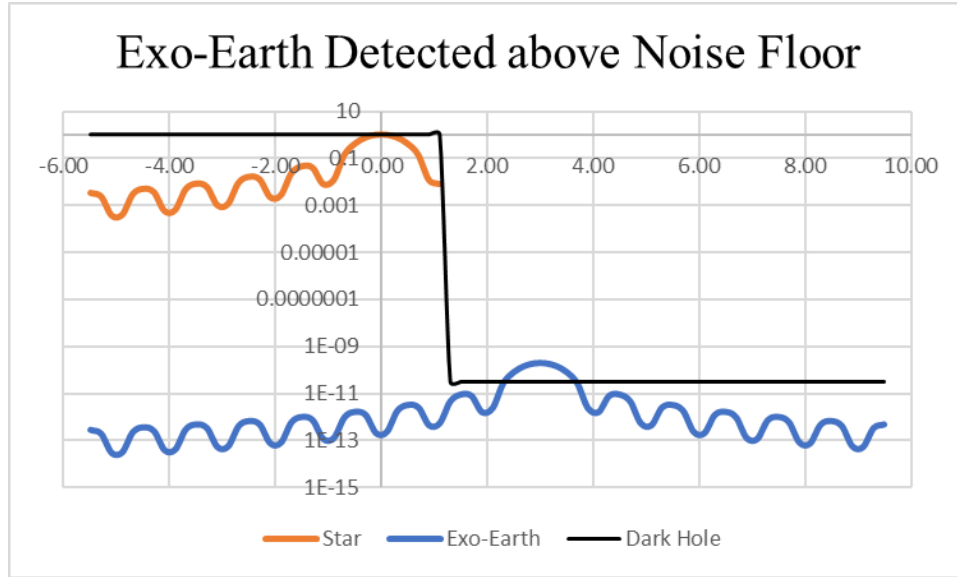
- $\lambda/D = 0.1 \mu rad$ (for $\lambda = 600 \text{ nm}$, $D = 6\text{m}$)
- $\Omega_{Core} = \pi \theta_{Core}^2 = \pi (\lambda/D)^2 = (0.0\pi) \text{ pico-Steradians}$

Note: **BRDF** is essentially flat for all angles inside the dark-hole

$$\tau_{NAS} = BRDF_0 \cdot \Omega_{Core}$$

Error Budget

But an Exo-Earth is only observable if its signal is above the Noise Floor.

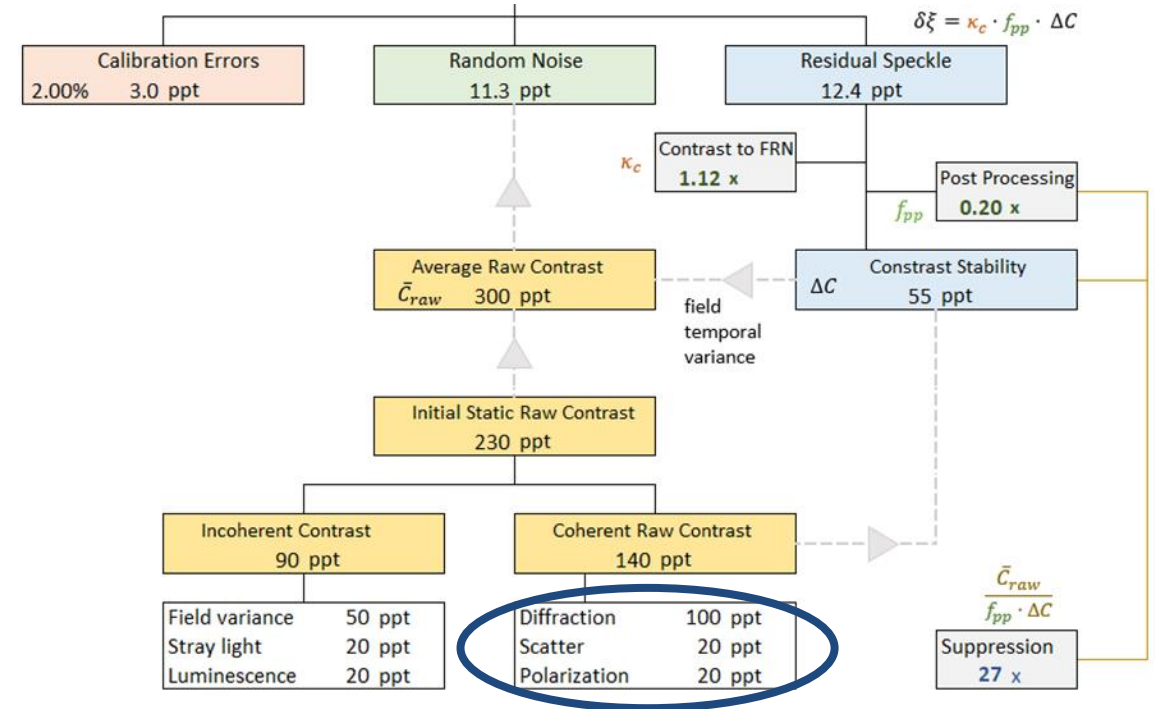
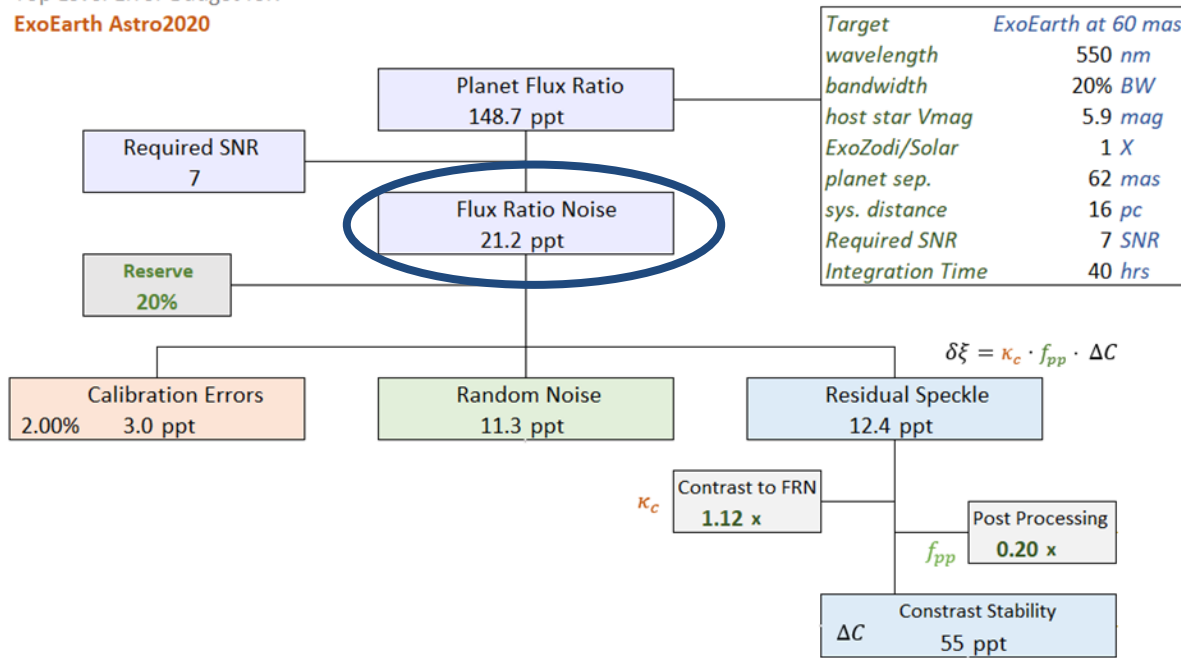


For SNR= 7, noise from all sources must contribute less than ~20 ppt (2×10^{-11} flux ratio).

Error Budget

~ 20 ppt of Flux Noise flows-down to 20 ppt ‘post-correction’ scattered light allocation.

Top Level Error Budget for:
ExoEarth Astro2020



Because scattered light sums (does not RSS), we sub-allocate 5 ppt each to Microstructure, Contamination, Micrometeoroids, RESERVE.



Microstructure

Executive Summary (corrected)



An expression was developed that specifies mirror statistical surface rough for a given specified near-angle scatter throughput (τ_{NAS})

If we assume the surface roughness is Gaussian:

$$\sigma[\text{\AA}] = \frac{1}{4\pi} \left[\frac{\lambda^2}{\ell} \cdot 10^4 \right] \left[\frac{\tau_{NAS}}{\pi \Omega_{\text{core}}} \right]^{1/2}$$

10^4 is a wavelength scale factor from μm to $\text{\AA}$, because λ and ℓ are in units of μm .

where:

- λ and ℓ are in units of μm
- $\lambda/D = 0.1 \mu\text{rad}$ (for $\lambda = 600 \text{ nm}$, $D = 6\text{m}$)
- $\Omega_{\text{Core}} = \pi \theta_{\text{Core}}^2 = \pi (\lambda/D)^2 \sim (0.0\pi) \text{ pico-Steradians}$

For this analysis, we assume that the specification for $\tau_{NAS} = 0.5 \text{ ppt per surface}$.

RMS Surface Microstructure Specification from NAS Throughput



For EAC-1 and EAC-2 there are 14 optical surfaces before the focal plane occulting mask.

If we allocated $\tau_{NAS} = 0.5\text{E-}12$ to each surface, then the surface specification becomes

Surface RMS Microstructure [$\text{\AA}$] to have NAS Throughput $< 0.5\text{E-}12$				
	Wavelength (λ) [μm]			
Correlation Length (ℓ) [μm]	0.300	0.680	1.000	1.800
10	16.2	83.2	180.0	583.2
100	1.6	8.3	18.0	58.3
1000	0.2	0.8	1.8	5.8
10000	0.0	0.1	0.2	0.6

Note: $\sigma \sim \sqrt{\tau_{NAS}}$. Thus, if τ_{NAS} changes by 10X, RMS roughness changes by $\sim 3\text{X}$

Specification is driven by shortest coronagraph wavelength.

This analysis assumes that Roughness is Gaussian.

RMS Slope

Another potential Key Performance Metric is RMS Slope

$$RMS\ Slope = \sqrt{2} \frac{\sigma}{\ell}$$

Slope is potentially a more intuitive parameter, providing an angular spread.

Table 2: Surface RMS Slope [μ rad] to have NAS Throughput $< 0.5E-12$				
	Wavelength (λ) [μ m]			
Correlation Length (ℓ) [μ m]	0.300	0.680	1.000	1.800
10	229.1	1177.1	2545.6	8247.7
100	2.3	11.8	25.5	82.5
1000	0.0	0.1	0.3	0.8
10000	0.0	0.0	0.0	0.0



Contamination Scattering

Surface Defect Scatter

Particle Mie Scatter, described by the Henyey-Greenstein function, is directional based on defect size.

The larger the particle or defect, the narrower and more directional its scatter cone

Scattering from Particulate Contaminants

79

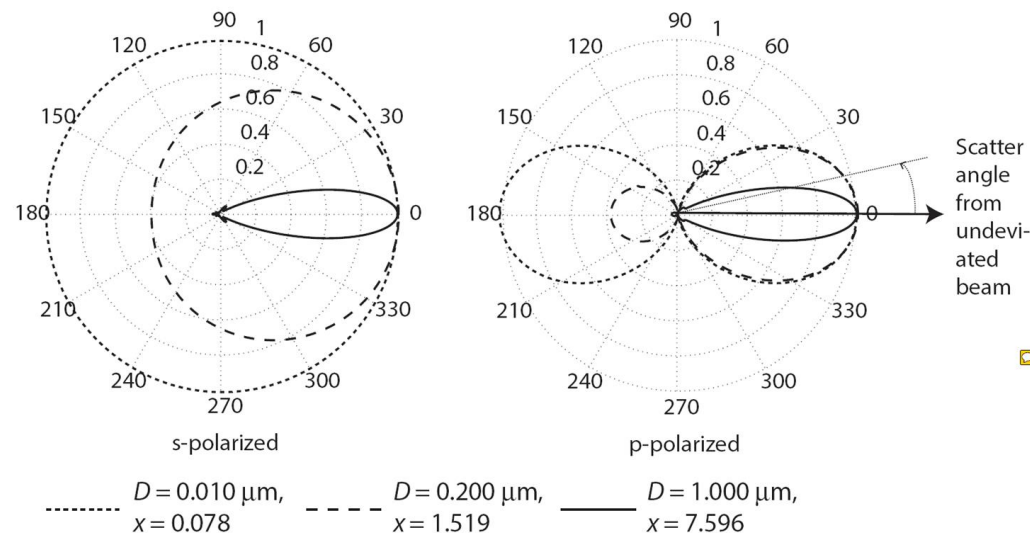


Figure 5.2 Normalized intensity I versus scatter angle from an undeviated beam θ for an s-polarized (left) and p-polarized (right) incident beam. D is equal to the sphere diameter, and x is the size factor [$\lambda = 0.6328 \mu\text{m}$, and $N = m = 1.53 + 0.0005i$ (typical dust)].

Fest (2013) *Stray Light Analysis & Control*

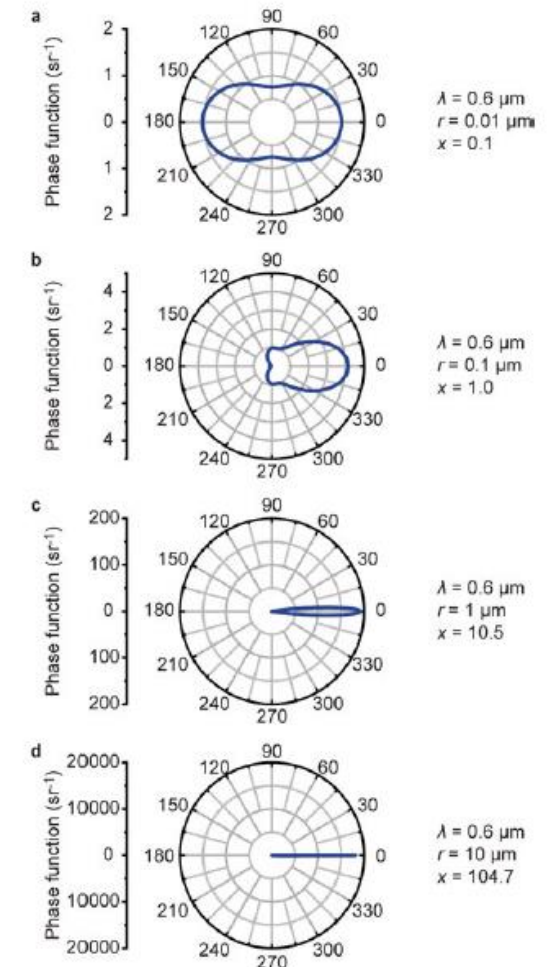


Figure 2.3. Scattering phase function of Au nanosphere in water. The wavelength of the incident light is 600 nm. The radius of the Au nanospheres are (a) 0.01 μm , (b) 0.1 μm , (c) 1 μm and (d) 10 μm .

Angle dependent light scattering of functional nanoparticle composites, Wang, 2018

Contamination Statistics (1)

All surfaces have particulate contamination, but (because each contamination population is unique) it is impossible to deterministically model contamination scatter.

Thus, contamination is described statistically

The most common statistical parameter for process control (from witness samples, etc.) is Cleanliness Level (CL):

- CL corresponds to the particle size whose average density is one (1) particle per 0.1-m²
- Thus, a CL-200 Surface has average of one 'equivalent' 200-μm particle per 0.1-m²

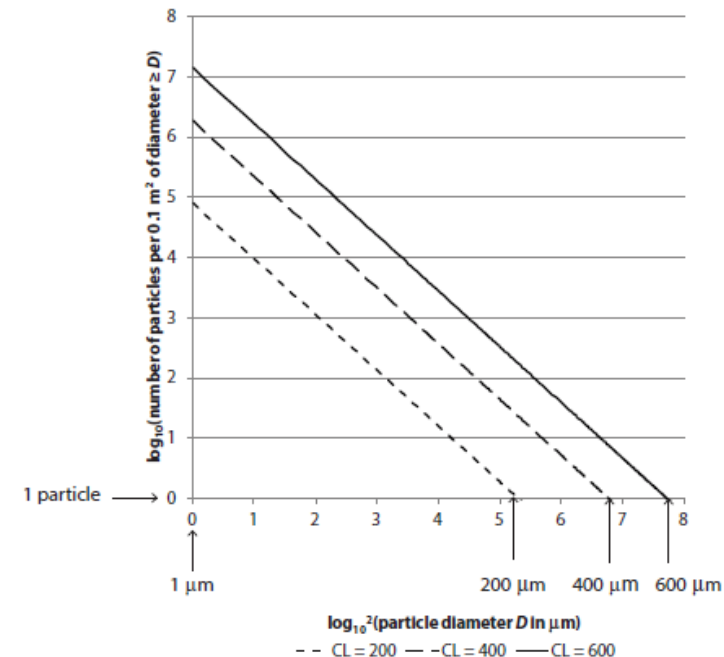


Figure 5.4 Number of particles of diameter $\geq D$ versus D given by IEST-STD-CC1246D (slope = -0.926) for CLs 200, 400, and 600. The x intercept of each line occurs at the particle whose diameter is equal to CL μm .

Eric C. Fest, Straylight Analysis and Control, SPIE Press, 2023

Richard N. Pfisterer, James E. Harvey, James B. Breckinridge, "The role of narrow-angle forward surface scatter and particulate scatter in exoplanet exploration," Proc. SPIE 10698, Space Telescopes and Instrumentation 2018: Optical, Infrared, and Millimeter Wave, 106985F (6 July 2018); <https://doi.org/10.1117/12.2314229>

Contamination Scatter (2)

For Straylight Analysis, BRDF/BSDF models have been developed for each CL.

$$\text{BRDF} = \frac{\text{Mie Intensity}}{\left(\frac{2\pi}{\lambda}\right)^2}$$

where λ = wavelength of incident light.

Peak BRDF is proportional to CL

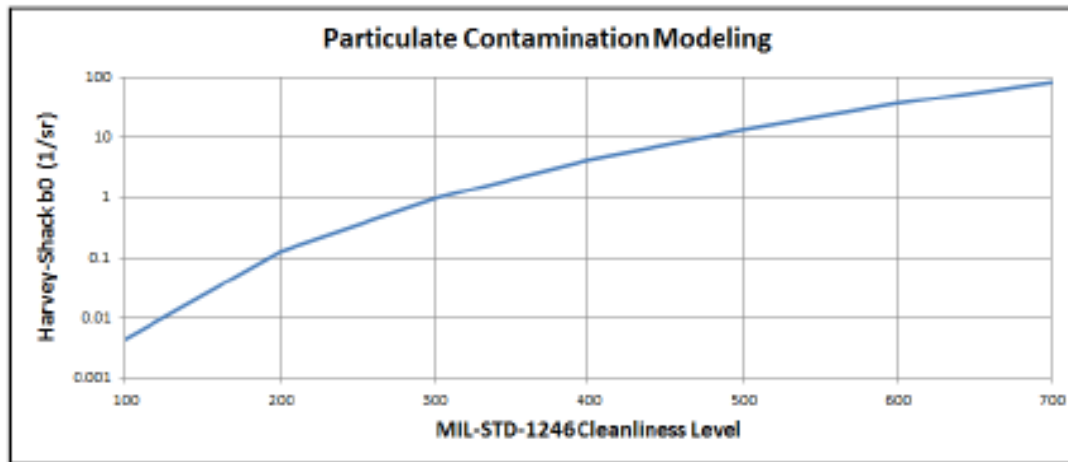


Figure 4 Derived relationship between Harvey-Shack b_0 parameter and MIL-STD-1246 cleanliness level.

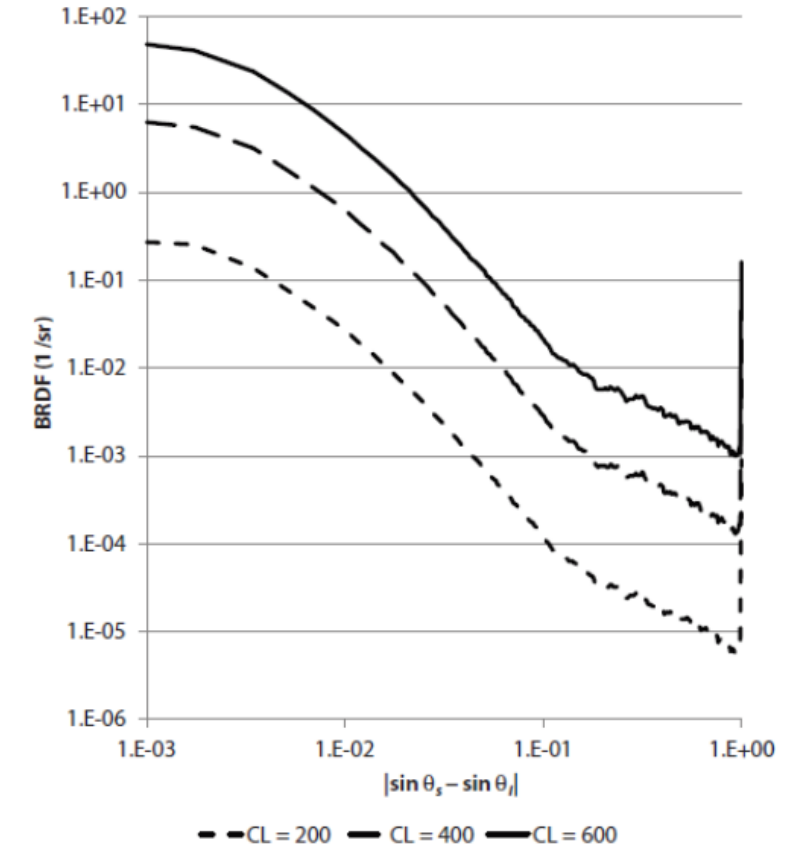


Figure 5.6 BRDF versus $|\sin \theta_s - \sin \theta_i|$ of a perfect mirror (reflectance = 1) due to contamination scatter (0.926 slope assumed, $\lambda = 0.6328 \mu\text{m}$). Calculations performed in FRED.

Richard N. Pfisterer, James E. Harvey, James B. Breckinridge, "The role of narrow-angle forward surface scatter and particulate scatter in exoplanet exploration," Proc. SPIE 10698, Space Telescopes and Instrumentation 2018, 106985F (6 July 2018); <https://doi.org/10.1117/12.2314229>

Fest, Eric, Stray Light Analysis and Control, Chapter 5 Scattering from Particulate Contaminants, SPIE Press, 2013, <https://doi.org/10.1117/3.1000980>

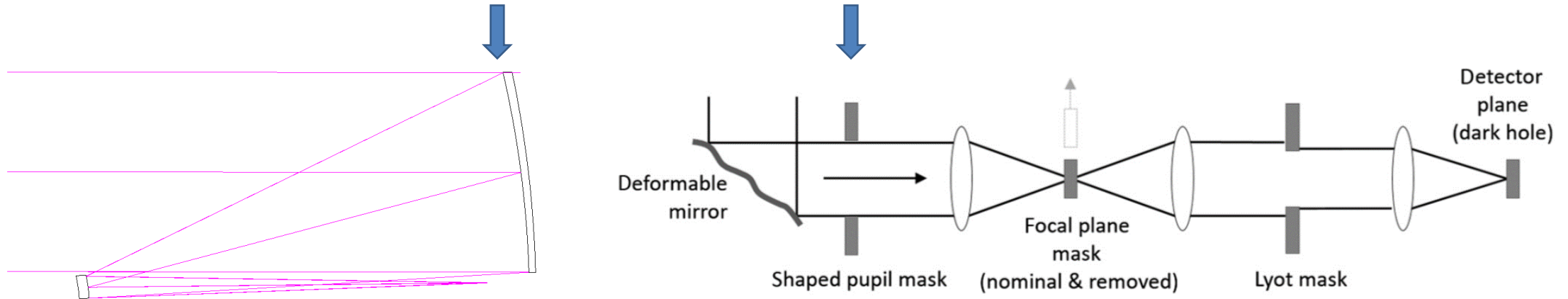
Point Scatterers on PM = Coherent Planar E-Field in Dark Hole



A Point Scatterer on the Primary Mirror will produce a Spherical Wavefront.

SM and Auxiliary Mirrors (M3 & M4) image that wavefront into a focus at PM Conjugate Plane (where the Shaped Pupil Mask is located).

This point focus is then collimated, passing the Focal Plane Mask and reimaged into Dark Hole.



Scatter is Coherent: Because the Coronagraph filters the light to a very narrow spectral band, and because the point scatter is located on the PM, the Planar E-Field is **coherent and in phase** with the host star's light – thus, it increases the heterodyne amplification of any wavefront instability.

$$|E + \Delta E|^2 = |E|^2 + |\Delta E|^2 + 2R\{E^* \cdot \Delta E\}$$

Leakage → Static Speckle Contrast Noise



In the Dark-Hole, after focal plane mask star-light suppression, some residual error remains – leading to a ‘Leakage’ field $E(u,v)$.

$$E(u,v) = E_{Leakage} + E_{NAS} + \dots$$

Which produces a Speckle Intensity Pattern $= |E(u,v)|^2$

Theoretically this pattern can be corrected by wavefront sensing and control – but the level of correction is unknown as well as the effect of physical issues such as polarization.

If corrected pattern is constant or static, it can be removed via Differential Imaging.

The maximum allowed amplitude of this static error (including Near-Angle Scatter) is defined by Shot Noise.

Leakage → Contrast Instability or Speckle Noise



‘Temporal’ disturbance or drift errors in the telescope or coronagraph optomechanical configuration adds a small amount $\Delta E(u,v,t)$ which is coherently added to $E(u,v)$.

$$|E + \Delta E|^2 = |E|^2 + |\Delta E|^2 + 2R\{E^* \cdot \Delta E\}$$

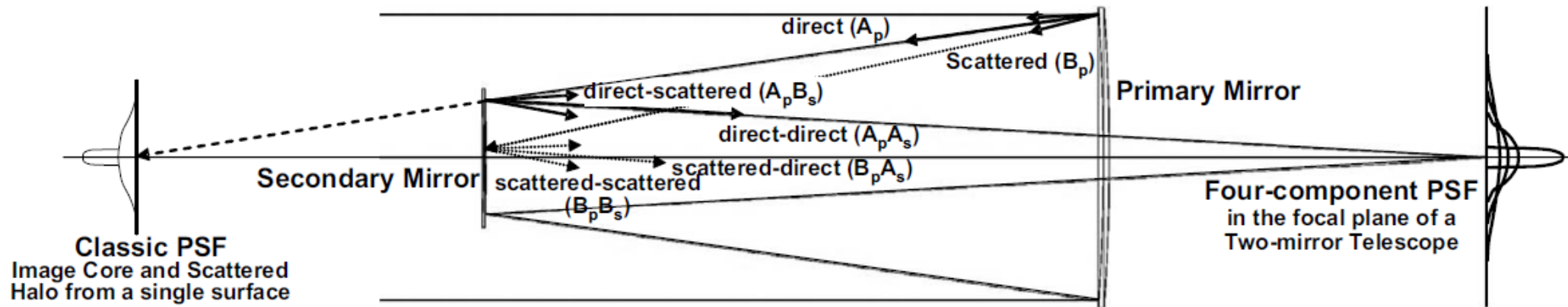
The coherent mixing or cross term = Contrast Instability or Speckle Noise.

- **Wavefront Instability (ΔE) causes ‘speckle’ noise that is not correctable by a DM.**
- **If Near-Angle Scatter increases the ‘post-correction’ static field $E(u,v)$ it causes an increase of the heterodyne amplification of the ΔE instability.**
- **If there is a maximum allowed amplitude for the Heterodyne Term, then any increase of the ‘static’ E term due to Near-Angle Scatter ‘requires’ a decrease in the allowed ‘temporal’ ΔE term.**

For example, if 5 pm wavefront instability is the limit to keep speckle noise below a threshold and the static E-field term increases by 20%, then instability allocation must be reduced to 4 pm.

Scattered Light Sums

Each optical surface before the coronagraph mask illuminated by host star light produces a scattered light wavefront that propagates to the Dark Hole and **Sums**.



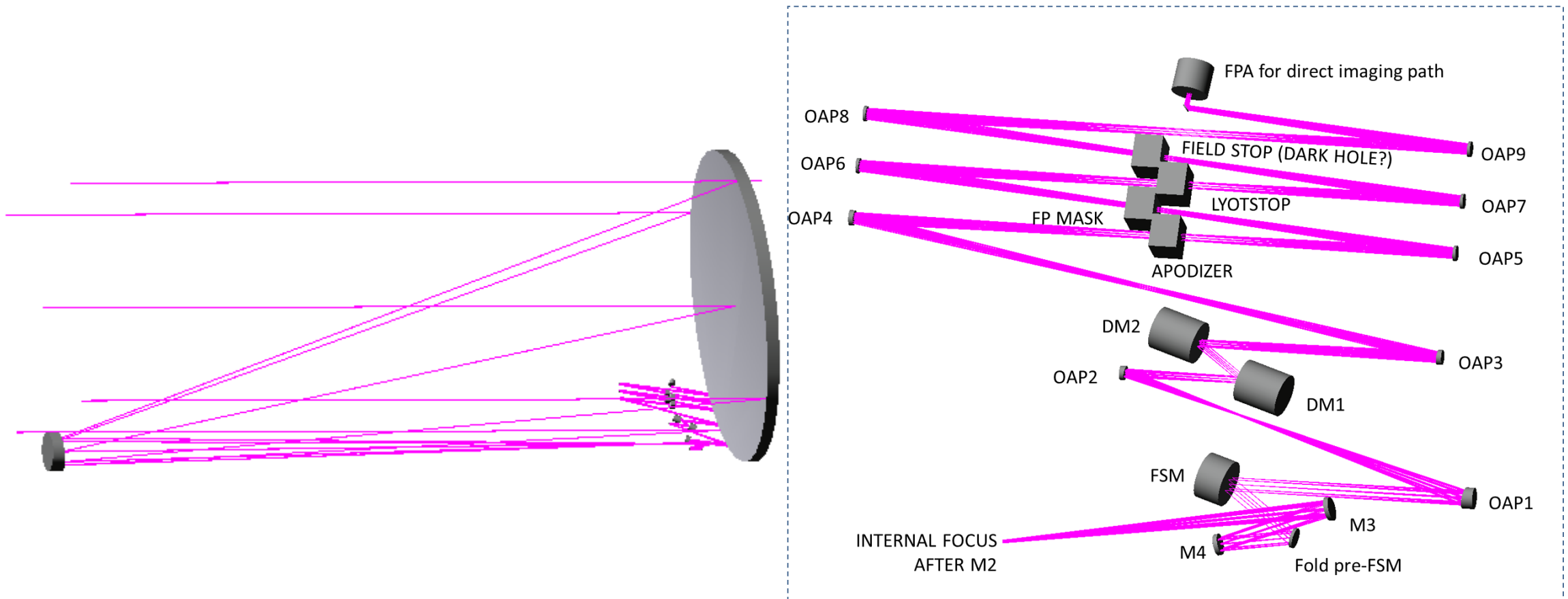
As the host star's wavefront propagates through the optical system, each time it reflects off an optical surface, a small amount is scattered by contamination producing an E-Field that is in-phase (coherent) with the wavefront.

NOTE: Every Surface before Focal Plane Mask scatters Light to Dark Hole.

EAC Cases have Many Surfaces

EAC1 and EAC2 have 14 optical surfaces before the Focal Plan Mask – each contributing coherent scattered light into the dark hole.

In the coronagraph, the FSM and DM1 are conjugate with the PM.



1/7/2024

Contamination Specification

Using the CL to BRDF (figure) and BRDF to Near-Angle-Scatter Throughput (equation), a surface contamination specification can be defined for a given Raw Contrast allocation.

Assuming a TOTAL Raw Contrast allocation for Contamination of 5 ppt AND that there are 10 optical surfaces before the Coronagraph Focal Plane Mask, then the Raw Contrast allocation per surface is 0.5 ppt.

For a 6-m diameter telescope, the BRDF per surface that contributes 0.5 ppt:

$$\text{BRDF}_0 = \frac{\tau_{NAS}}{\Omega_{\text{core}}} = \frac{0.5E-12}{\pi E-14} = 16 [1/\text{sr}]$$

Per Pfisterer, Harvey, & Breckinridge Fig 4: this defines a per surface spec of < CL-500.

Webb was specified to be < CL-600.

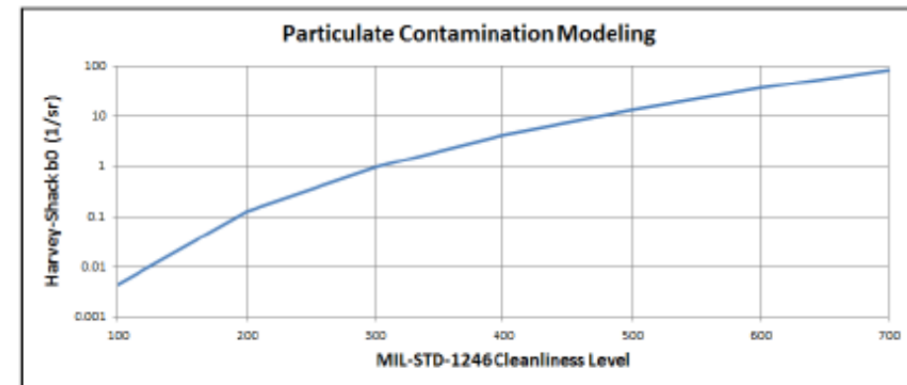


Figure 4 Derived relationship between Harvey-Shack b₀ parameter and MIL-STD-1246 cleanliness level.

Independent Cross-Check

Kunjithapatham Balasubramanian, Stuart Shaklan, Amir Give'on, "Stellar coronagraph performance impact due to particulate contamination and scatter," Proc. SPIE 7440, Techniques and Instrumentation for Detection of Exoplanets IV, 74400T (25 August 2009); <https://doi.org/10.1117/12.825182>

Scatter Throughput vs Uncorrected Contrast

We developed a relationship between Scatter Throughput and BRDF

$$\tau_{NAS} = BRDF_0 \cdot \Omega_{Core}$$

Balasubramanian, Shaklan and Give'on (2009) called this Uncorrected Contrast.

Large Particles are More Important than Small Particles

Scatter increased with particle diameter to the 4th power.

A 100 μm particle scatters 10^4 more than a 10 μm particle

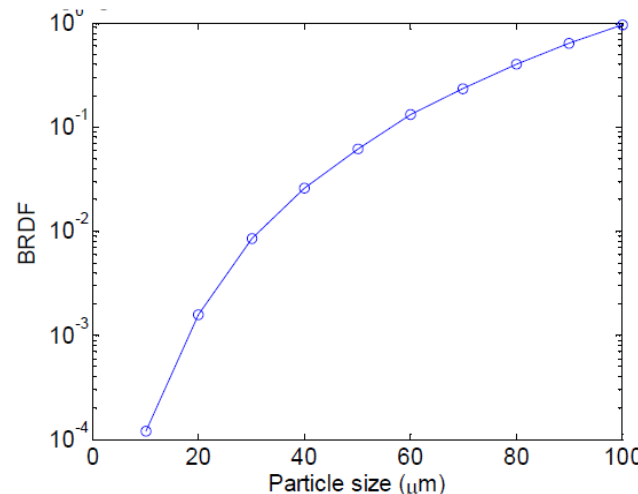


Figure 7. BRDF at <0.1 deg vs particle size showing a 4th order dependence; particle density = 1/sq cm. $\lambda = 800$ nm; Uncorrected contrast $UC = BRDF \cdot \pi \cdot (\lambda/D)^2 = 2.2 \cdot BRDF \cdot 10^{-9}$ for $D = 30$ mm.

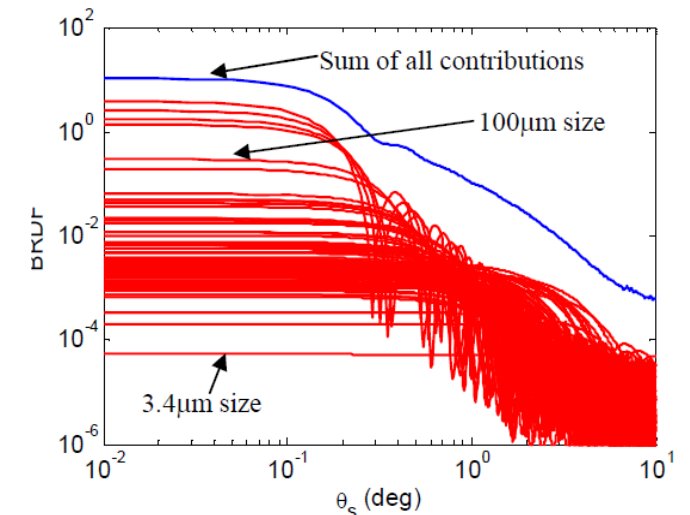


Figure 6. BRDF vs scatter angle for the distribution of particles on sample gold mirror

Correctability of Contamination Scatter

Because near angle scatter is coherent, it may be partially correctable.

The top row of this simulation shows that a single 100 μm particle on a 30-mm mirror increase the Dark Hole Contrast.

The bottom row shows that a DM can correct for that scatter and create a one-sided Dark Hole (moving the scattered light to the opposite side).

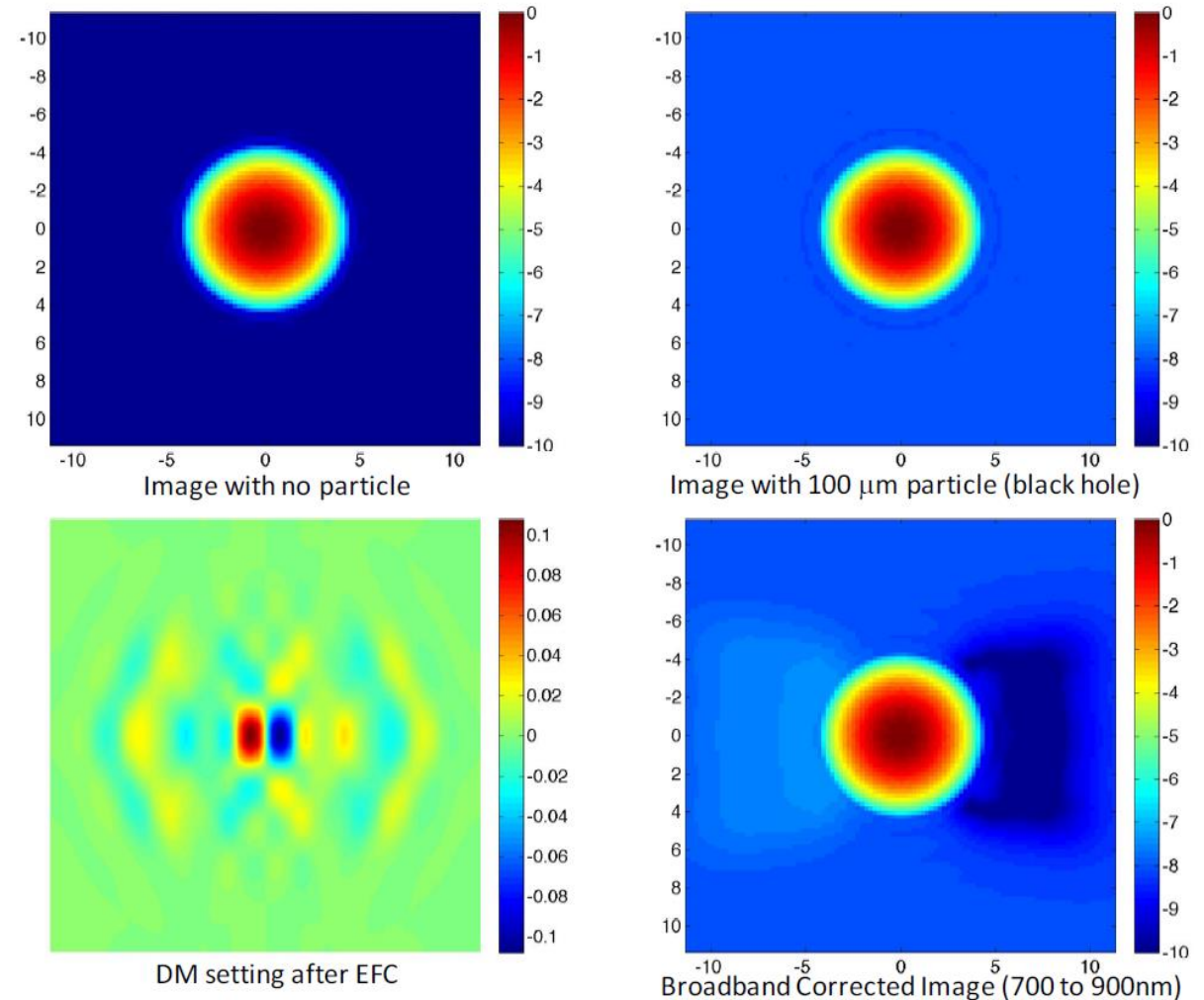
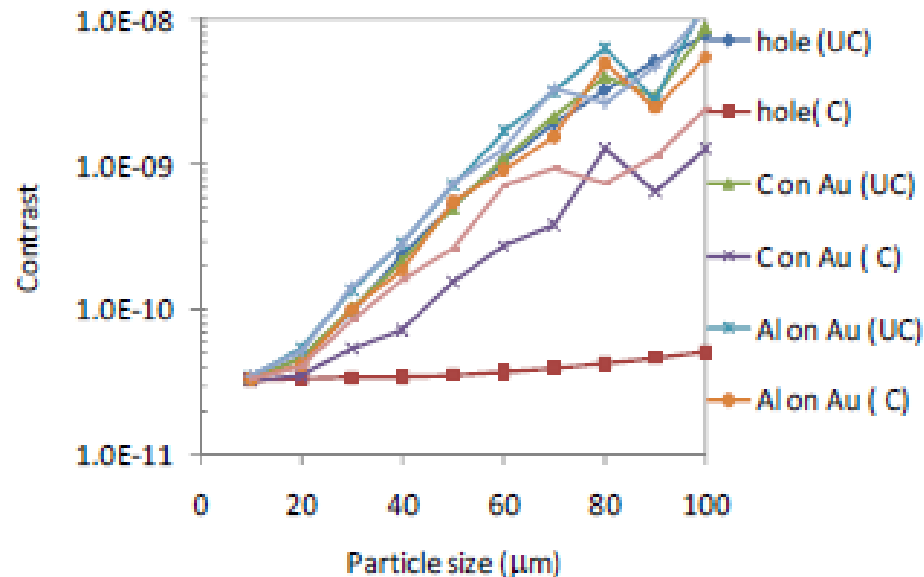


Figure 10. DM correction of particle induced aberrations. Top left: Image from an apodized pupil coronagraph (APC) with 30 mm pupil. Top right: Image from the same APC with a 100 μm particle (a perfect absorber) placed in the center of the pupil. Background level rises by ~ 3 orders. Bottom left: Setting of the DM surface after EFC corrects the aberrated field. Bottom right: Image after DM correction showing the recovered dark hole contrast. $\lambda = 700$ to 900 nm for this simulation.

Correctability of Contamination Scatter

DM correction is limited to the amount of absorption in the particle. ‘Corrected contrast roughly equals uncorrected contrast scaled by average particle reflectance.’ Thus, a shiny particle produces more uncorrectable contrast than a black particle.

Paper states ‘For typical dust, the correct limit is $\sim 0.4 \times$ Uncorrected’



- ‘H’ = ideal ‘black’ hole
- ‘C’ = carbonaceous ‘black’ particle.
- ‘Al’ = metallic ‘shiny’ particle

Figure 11. Uncorrected (UC) and corrected contrast (C) with various particle types. One particle is placed on a 30 mm pupil of the Apodized Pupil Coronagraph in this broadband simulation with EFC.



Allocation Between Surfaces

This paper arbitrarily allocates an equal amount of scatter to each of 10 surfaces.

Bala et. al. (2009) allocated different scatter based on mirror size.

Table 1. Requirements on Particulate Contamination Limits on Coronagraph Optics

Optic Dia (m)	Typical # of optics in system	Contrast budget per optic for 25% BW	CL requirement on each optic	PAC requirement on each optic	CL requirement with 4x margin due to model assumptions	PAC requirement with 4x margin due to model assumptions
0.03	5	6×10^{-12}	<75	<0.00009	<50	<0.00002
0.1	5	6×10^{-12}	<150	<0.001	<100	<0.0002
1.0	2	1.5×10^{-11}	<400	<0.09	<300	<0.02
4.0	1	1×10^{-11}	<700	<1.5	<500	<0.25



Micrometeoroid Scattering



Micrometeoroid Impacts

Similar to contamination, Micrometeoroid Impacts produce near-angle scatter that can reach Dark Hole.

One difference, Micrometeoroid are likely to only impact the Primary Mirror.

Scatter increases with exposure.

- Webb is seeing ~ 1 impact per day.
- Most are small, some are not.
- For HWO, a tube should reduce impacts to maybe 1 per 100 days.

So, how does one define a tolerance: CL? BRDF? Fourier Transform?

Micrometeoroid Impacts

High-Velocity Impacts (HVI) of glass surface result in spalling $\sim 11X$ larger in diameter than the central pit diameter and $\sim 10X$ larger in diameter than the pit depth.

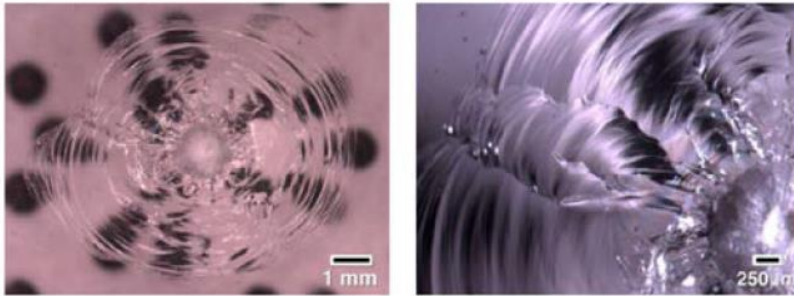


Figure 2 – Top views of surfaces that have undergone surface spalling (conchoidal fracture) in an approximately circular region surrounding the impact crater on a glass substrate impacted at ~ 5.6 km/s by a $400\text{ }\mu\text{m}$ -dia. Aluminum alloy sphere. As seen from the picture on the left, the diameter of the spalled region is ~ 5 times the diameter of the central pit or crater.

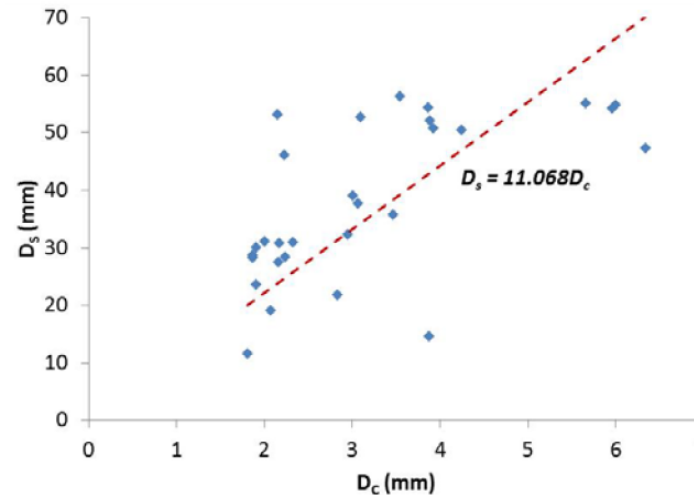


Figure 6 - Values of surface spall diameter, D_s , in relation to HVI crater diameter, D_c , obtained from HVI tests up to 7 km/s on spacecraft-grade glass window material [6].

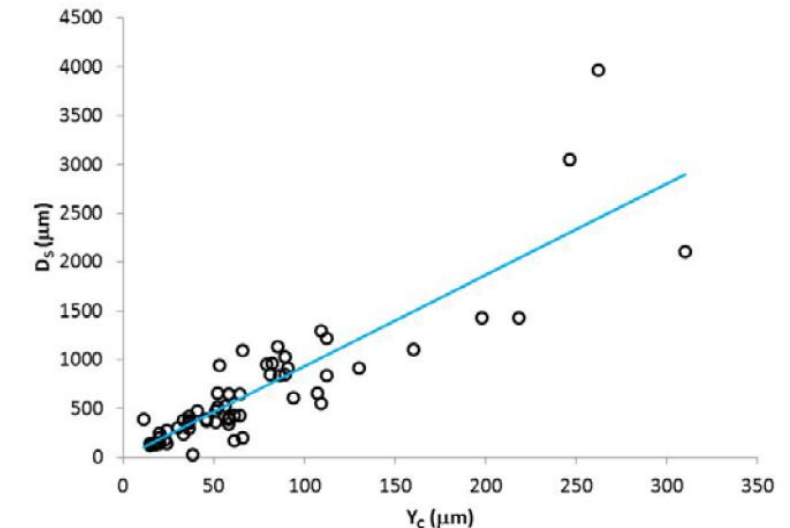


Figure 10 – Surface spall diameter values in relation to crater depth values for pits found on NASA STS glass window tiles returned from space [9].

Micrometeoroid Impacts

The more energetic the HVI, the larger the spall diameter.

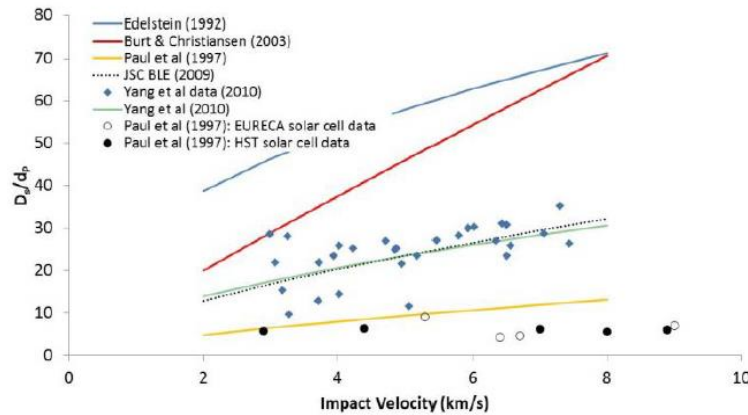


Figure 9 – Published values of surface spall diameter normalized by impacting particle diameter, D_s/d_p , in relation to impact velocity. Regressions obtained by previous authors [4-8] are also shown for comparison. The spall diameter developed on solar cell glass covers due to microparticle HVI [8] follows a different relationship than that developed on unsupported fused silica tiles due to milliparticle HVI [6].

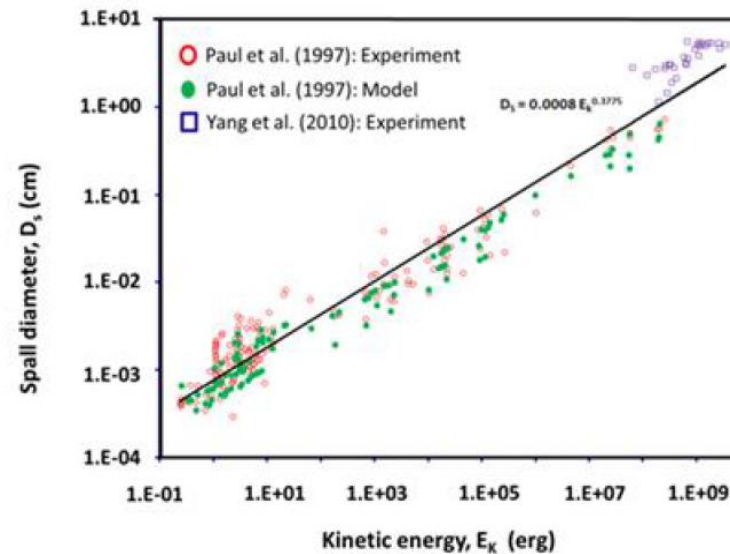
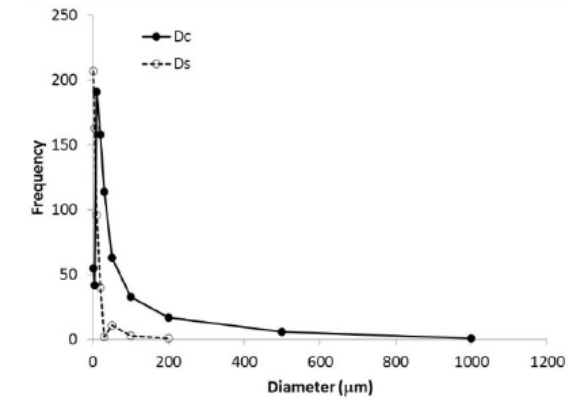
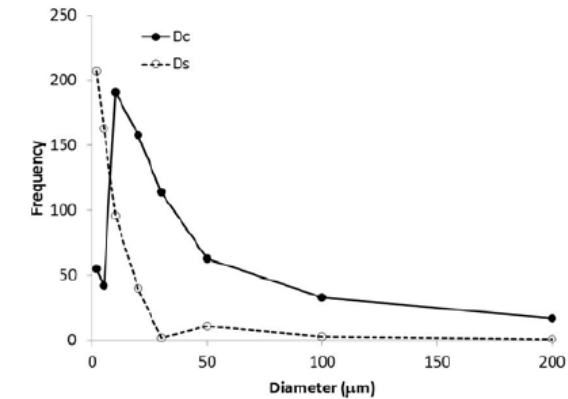


Figure 8 – Published values of surface spall diameter, D_s , in relation to particle kinetic energy from HVI tests on a cerium-doped glass solar cell cover with microparticles [8] and an unsupported fused silica monolith with milliparticles.



(a)



(b)

Figure 11 – Measured values of spall and pit diameters and frequency of occurrence on HST and EURECA solar array glass surfaces exposed to micrometeoroid HVI: (a) 0-1000 μm range, and (b) 0-200 μm range. Spall diameter values in the 10-30 μm range show the highest frequencies. Crater pit diameters in the 2-5 μm range show the highest frequencies.

For this analysis, we assume 3 sizes: 6, 30 and 60-mm.



Effective Cleanliness Level

CL defined by the average particle size per 0.1 m^2

HWO is specified to have a 6-m mirror with an area $\sim 28 \text{ m}^2$.

Thus, for any given CL, the PM will have a total of 280 particles of that size.

QUESTION: Is a surface with one 1.67-mm impact (which has same area as 280 100- μm particles) considered CL-100?

‘Effective’ CL per single Micrometeoroid Impact

Impact Size	‘Effective’ CL
6-mm	CL-360
30-mm	CL-1800
60-mm	CL-3600

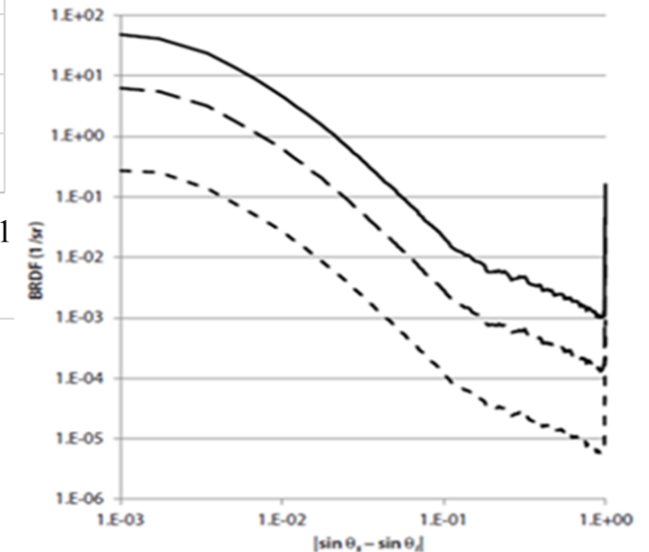
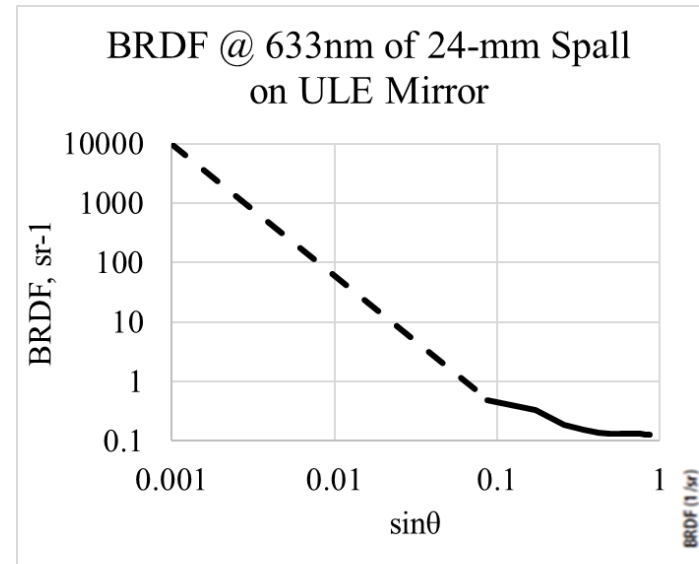
Don’t have a good path forward for defining specification using CL.

BRDF

The Webb Program Office has impacted glass samples with hypervelocity particles to study micrometeoroid impacts.

BRDF was measured for a sample with a ~ 5-mm central crater and ~ 24-mm spall with a scatterometer from 5 to 60 deg.

‘Extrapolating’ measured data with slope similar to CL BRDF curves (from Fest) gives a ‘peak’ BRDF estimate of ~1E+3.



Multiplying by area ratio of 24-mm spall to 6-m PM gives:

$$\text{BRDF} = \sim 2\text{E-}2$$

$$\tau_{NAS} = \sim (5\text{E-}10) \text{ BRDF} = \sim 10 \text{ ppt}$$

One 24-mm spall may contribute the entire Raw Contrast micrometeoroid allocation.

Georgi Georgiev, Private Communication, 22 July 24



Fourier Transform Analysis

Assuming that a micrometeoroid impact has a ‘rect(x)’ profile with diameter (d), then its scattered wavefront has a ‘sinc()’ amplitude with angular spread λ/d .

$$\text{rect}(x/d) \xrightarrow{\mathcal{F}} d \cdot \text{sinc}(d \xi) = \frac{d \sin(d \xi)}{\pi \xi}$$

To calculate Dark Hole power at $2\lambda/D = 0.2 \mu\text{rad}$ (for $\lambda = 600 \text{ nm}$, $D = 6\text{m}$)

- Ratio Area of Impact to Primary Mirror
- Multiple Area Ratio times ‘peak’ $\sim d^2/\pi$ (because small angle $\text{Sin}(d \xi) = (d \xi)$)

Relative Power in Dark hole from Micrometeoroid Impact					
Impact Diameter [mm]	Input Power Ratio	Scatter Angular Spread [μrad]	Scatter Angular Spread [λ/D]	Peak Scatter Amplitude	Relative Power at $2\lambda/D$
24	1.6E-05	25.00	125	2E-04	3E-09
12	4.0E-06	50.00	250	5E-05	2E-10
6	1.0E-06	100.00	500	1E-05	1E-11
3	2.5E-07	200.00	1000	3E-06	7E-13

One 6-mm impact may contribute the entire Raw Contrast micrometeoroid allocation.



Future

Coating Columnar Structure

Summary



Near-Angle Scatter (NAS) of Host Star's Light may define the Dark-Hole's Noise Floor.

- NAS from each optical surface produces an E-field across the Dark Hole that is coherent.
- NAS E-fields sum – they do not RSS – to add a positive bias onto the WF instability E-field.

There are multiple sources of NAS

- Long-Spatial Structure (2023 O&P paper) which can be corrected by DM
- Microstructure (2024 Astro paper)
- Contamination (topic of this paper)
- Micrometeoroids (topic of this paper)
- Coating Structure and Defects

Using relationship (from 2024 Astro paper) between BRDF and Coronagraph Contrast:

- Derived a Specification of Surface Contamination
- Derived a Tolerance for Micrometeoroid Impacts.

Next is Scatter from Coating Structure.

This analysis does not include:

- Diffraction or scatter from segment edges
- Polarization – **important**: different polarization states scatter differently.



Questions & Discussion



APPENDIX A:

Long-Spatial Scale Structure

Point Spread Function (PSF) & Power Spectral Density (PSD)

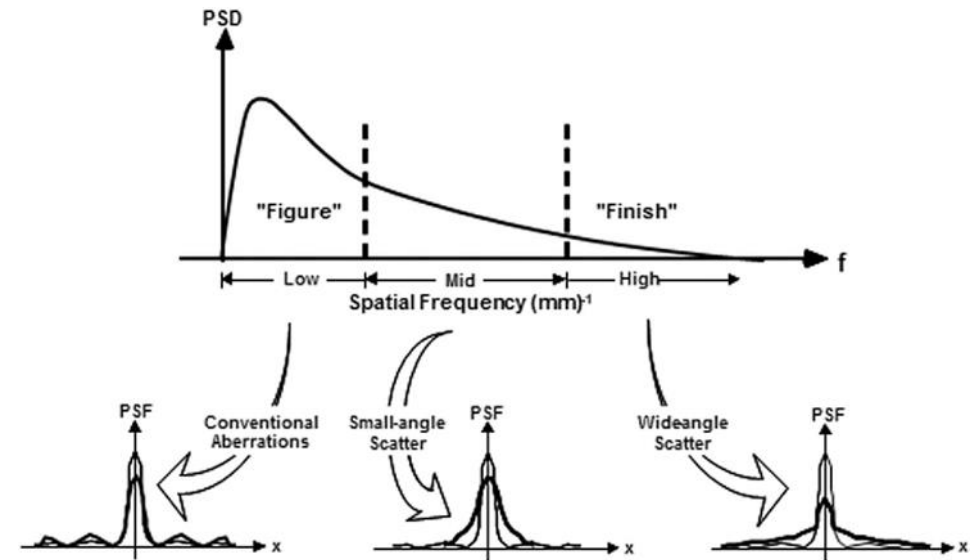


Coronagraphs operate on a telescope's Point Spread Function (PSF).

PSF is driven by telescope's mirror's (or wavefront) Power Spectral Density (PSD).

PSD is typically divided into 3 spatial frequency regimes: Low (Figure), Mid and High

- Low-spatial frequency (figure) structure determines the shape of the core PSF.
- Mid-spatial frequency structure produces small-angle scatter which broadens the PSF
- High-spatial frequency (micro-roughness) structure produces wide angle scatter.



Harvey & Kotha, Optical Engineering (1995)

Distorted PSF or Energy Removed from Core is not blocked by the occulting mask.

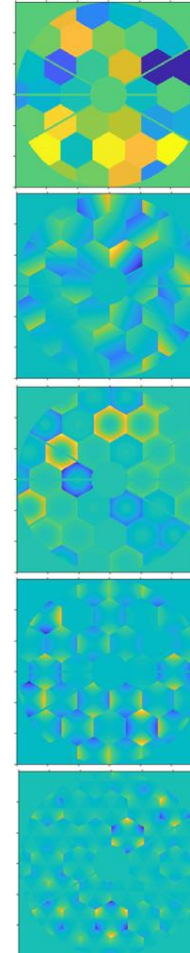
Multiple Researchers have investigated Contrast Leakage as a function of Wavefront/Surface Instability

- Stahl, Mark T., H. Philip Stahl, and Stuart B. Shaklan “Preliminary analysis of effect of random segment errors on coronagraph performance”, Proc. SPIE. 9605, Techniques and Instrumentation for Detection of Exoplanets VII, 96050P. (September 24, 2015) doi: 10.1117/12.2190160
- Coyle, et. al., “Ultra-Stable Telescope Research and Analysis (ULTRA) Program, Phase 1 Report, (30 April 2019)
- Nemati, Bijan, H. Philip Stahl, Mark T. Stahl, Garreth J. Ruane, Leah J. Sheldon, "Method for deriving optical telescope performance specifications for Earth-detecting coronagraphs," J. Astron. Telesc. Instrum. Syst. 6(3), 039002 (2020), doi: 10.1117/1.JATIS.6.3.039002.
- Sahoo, et. al., "Segment-level thermal sensitivity analysis for exo-Earth imaging," Proc. SPIE 12180, (27 August 2022); doi: 10.1117/12.2630592
- Pueyo, et. al., "Coronagraphic detection of Earth-like planets with large, actively controlled space telescopes," J. Astron. Telesc. Instrum. Syst. 8(4) 049002 (25 October 2022)
<https://doi.org/10.1117/1.JATIS.8.4.049002>

Wavefront Stability Requirement (1)

Independent analysis of two different coronagraphs indicate that required wavefront stability needs to be < 2 pm rms (< 1 pm rms surface) for all Zernike modes.

6-m off-axis segmented with VVC-6			
Perturbation mode	Sensitivities (ppt/pm)	Allocations (ppt)	Tolerances (pm)
Bend	3.21	5.53	1.72
Power (Seidel)	4.64	7.99	1.72
Spherical (Seidel)	4.51	7.76	1.72
Coma (Seidel)	3.78	6.51	1.72
Coma (Zernike)	5.19	8.93	1.72
Trefoil (Zernike)	5.82	10.0	1.72
Hexafoil (Zernike)	10.6	18.3	1.72
Segment piston	8.84	15.2	1.72
Segment tip/tilt	9.09	15.6	1.72
Segment power (Seidel)	3.68	6.34	1.72
Segment astigmatism (Zernike)	9.33	16.1	1.72
Segment trefoil (Zernike)	15.0	10.6	1.72
Segment hexafoil (Zernike)	0.745	1.28	1.72
Quadrature sum roll-up	--	40	6.21



6-m segmented with an aperture and APLC as per Ref. 1			
Column 1	Columns 2, 3, 4		
Optical perturbation mode	Sensitivity (ppt/pm)	Allocation (ppt)	Tolerance (pm)
Bend	1.65	2.70	1.64
Power	1.96	3.21	1.64
Spherical	1.26	2.07	1.64
Coma	3.26	5.34	1.64
Coma	3.31	5.43	1.64
Trefoil	4.66	7.64	1.64
Hexafoil	1.95	3.20	1.64
Segment piston	6.74	11.05	1.64
Segment tip/tilt	11.10	18.20	1.64
Segment power	3.71	6.08	1.64
Segment astigmatism	18.30	30.00	1.64
Segment trefoil	4.55	7.46	1.64
Segment hexafoil	1.48	2.43	1.64
Quadrature sum roll up	—	40.00	5.91

Nemati, et. al., "Method for deriving optical telescope performance specifications for Earth-detecting coronagraphs," J. Astron. Telesc. Instrum. Syst. 6(3), 039002 (2020), doi: 10.1117/1.JATIS.6.3.039002.

Pueyo, et. al., "Coronagraphic detection of Earth-like planets with large, actively controlled space telescopes," J. Astron. Telesc. Instrum. Syst. 8(4) 049002 (25 October 2022)
<https://doi.org/10.1117/1.JATIS.8.4.049002>

Wavefront Stability Requirement (3)

Coronagraph's sensitivity also depends on the Segment's position in the Aperture.
Lower sensitivity to Segment errors at edge of Aperture.

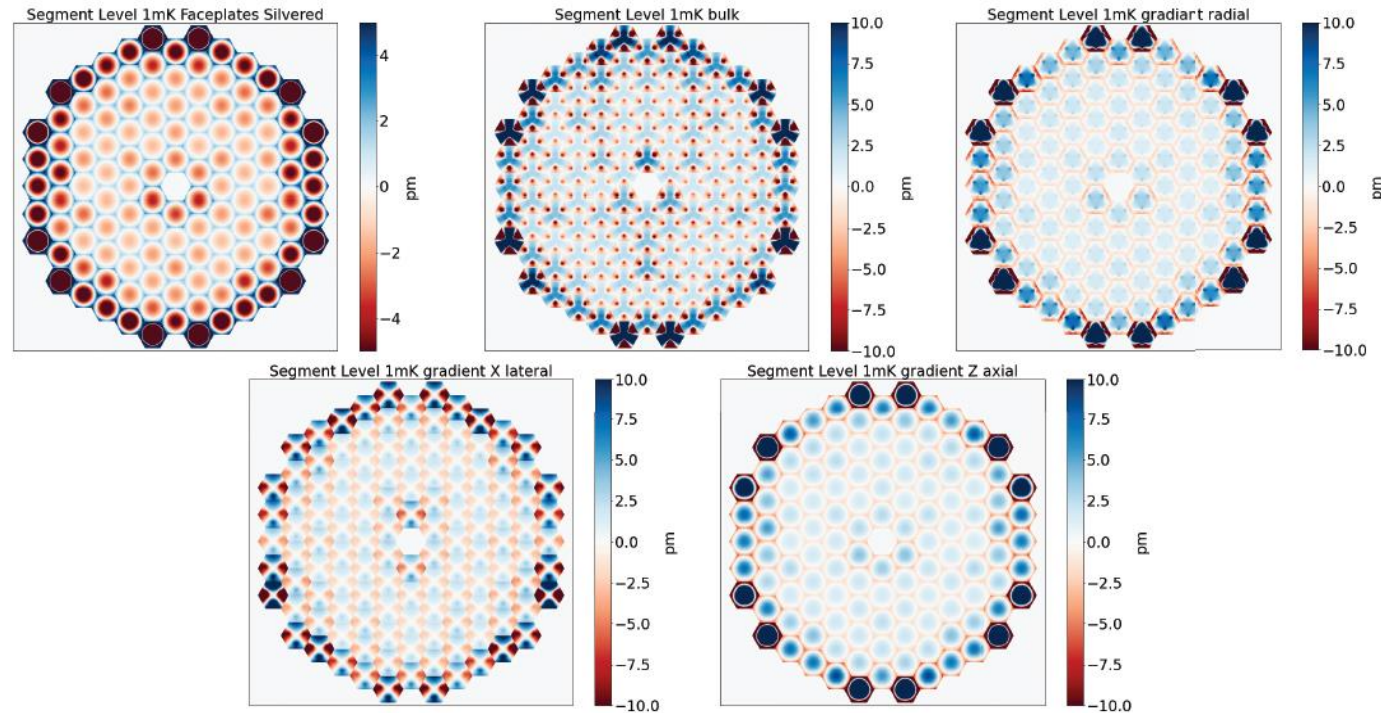
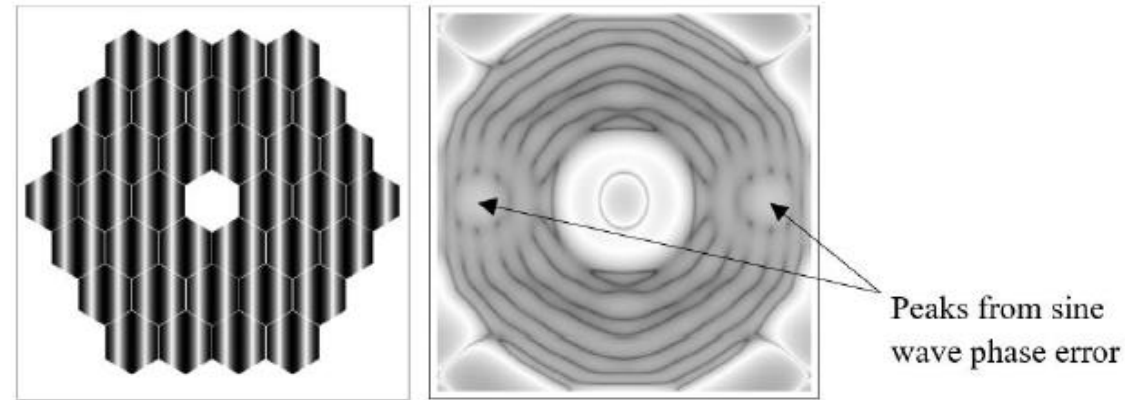


Figure 4: Segment-level wavefront tolerance maps required to maintain a mean dark hole contrast of 10^{-11} for the narrow-angle APLC at a wavelength of 500 nm. Each segment shows the limit of its surface deformation it can withstand while maintaining the desired contrast. Outer-ring segments are covered more by the APLC apodizer than segments on the inner rings, therefore they have looser tolerance requirements.

Sahoo, et. al., "Segment-level thermal sensitivity analysis for exo-Earth imaging," Proc. SPIE 12180, (27 August 2022); doi: 10.1117/12.2630592

Wavefront Stability Requirement (4)

Coronagraphs are also sensitive to mid-spatial frequency errors up to 100 cycles/diameter. The best way to develop an error budget for these terms is via PSD analysis.



A sine wave in the pupil with 1 pm amplitude in wavefront and 10 cycles/dia produces two distinct peaks in the coronagraph dark hole that cause a contrast degradation of 10^{-10} . Calculating the PSD of the error and the corresponding peak amplitude provides a normalization factor for the allowable energy density in the dark hole.

Notes:

Coyle, et. al., “Ultra-Stable Telescope Research and Analysis (ULTRA) Program, Phase 1 Report, (30 April 2019)

- **1 pm PV wavefront amplitude at 10 cycles cause contrast degradation of 10^{-10}**
- An error budget allocation may be as small as 10^{-11} contrast.
- This result is consistent with Rayleigh-Rice Vector Scatter Theory.

Rayleigh-Rice (R-R) Scatter Theory (1)

R-R is a ‘smooth’ surface theory. Where ‘smooth’ is defined to be a surface whose rms roughness is $< \lambda/100$ ($< \sim 5$ nm rms) to $< \lambda/1000$ ($< \sim 0.5$ nm rms).

R-R is the grating equation. PSD spatial frequency components ‘scatter’ flux into an angle.

$$\sin(\theta) = \lambda/d = \lambda \cdot f_x, \text{ where } d = \text{spatial period and } f_x = \text{spatial frequency}$$

Scattered flux ratio as function of angle is proportional to PSD and scales with wavelength

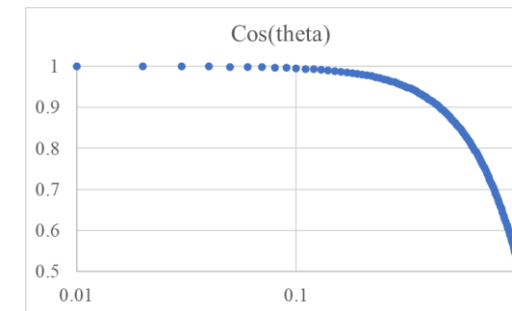
$$\frac{P_{\pm 1}}{P_i} = \left(\frac{2 \pi a}{\lambda} \right)^2 \cos \theta_i \cos \theta_{\pm 1}$$

where a = **peak-valley** amplitude of the sinusoidal PSD spatial frequency.

Note: the two cosine terms are essentially 1.0

Incident Angle is 0, so $\cos(\theta_i) = 1.0$

For Scatter Angles < 0.1 rad $\cos$ is essentially = 1



Thus, R-R predicts that the scatter flux ratio for near-angle scatter for a given wavelength is essentially constant and proportional to the PV Surface Roughness.

Rayleigh-Rice Scatter Theory (2)

Because diffraction is a physical optics phenomena, I refer to RR as ‘coherent’ scatter.

And because PSD spatial frequencies that cause near-angle scatter are so long, RR near-angle scatter is same phenomena as coherent speckle noise from surface figure errors.

Table 1: Low-Spatial Frequency Errors that Scatter into Dark Hole			
Scatter Angle [mas]	PSD Spatial Frequency [1/μm] for $\lambda = 600$ nm	Spatial Period [m]	Cycles per 6-m Aperture
30	2.5×10^{-7}	4	1.5
60	5×10^{-7}	2	3
200	1.7×10^{-6}	.6	10
600	5×10^{-6}	0.2	30

If the speckle noise is static, it can be subtracted – except for its shot noise

What is important is temporally changing scatter caused by surface PSD components that are changing with time – due to thermal drift or mechanical motion.

Rayleigh-Rice Scatter Theory (3)

Solving the grating equation for maximum allowed change in PV surface PSD amplitude to have a scattered light flux ratio of 10^{-11}

$$a = \sim \left(\frac{\lambda}{2} \right) \times 10^{-6}$$

PV Surface Amplitude Stability as function of Wavelength for Temporal Scatter from Star to be 10% of exo-Earth Flux (i.e. 10^{-11})			
Wavelength	300 nm	600 nm	1800 nm
PV Surface Amplitude	0.15 pm	0.3pm	0.9 pm
Surface Spatial Period at 60 mas	1 m	2 m	6 m

This result is consistent with published wavefront stability analysis

‘Coherent’ error is correctable via a DM, but the ability of a DM to correct wavefront instability to sub-picometer precision is limited by sense and control resolution.



APPENDIX B:

Microstructure Derivation

Point Spread Function (PSF) & Power Spectral Density (PSD)

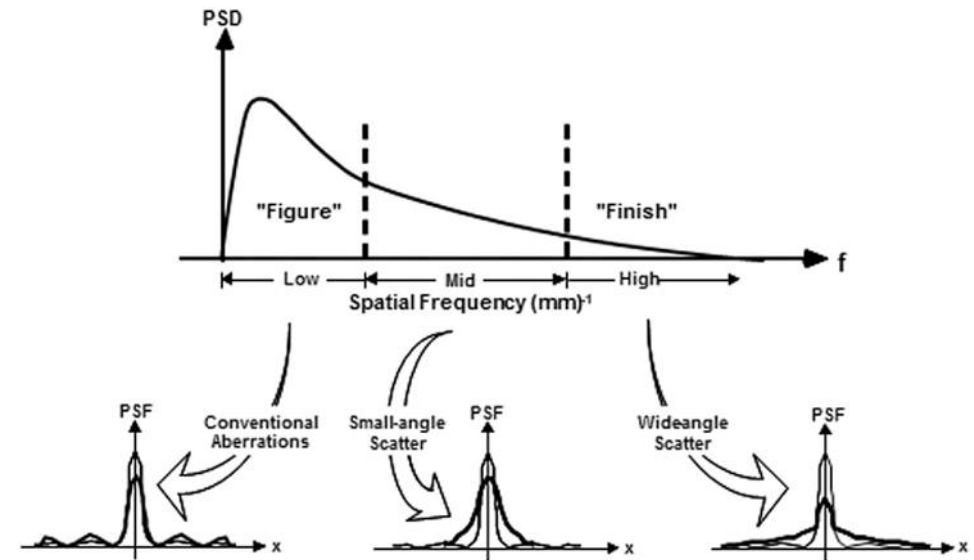


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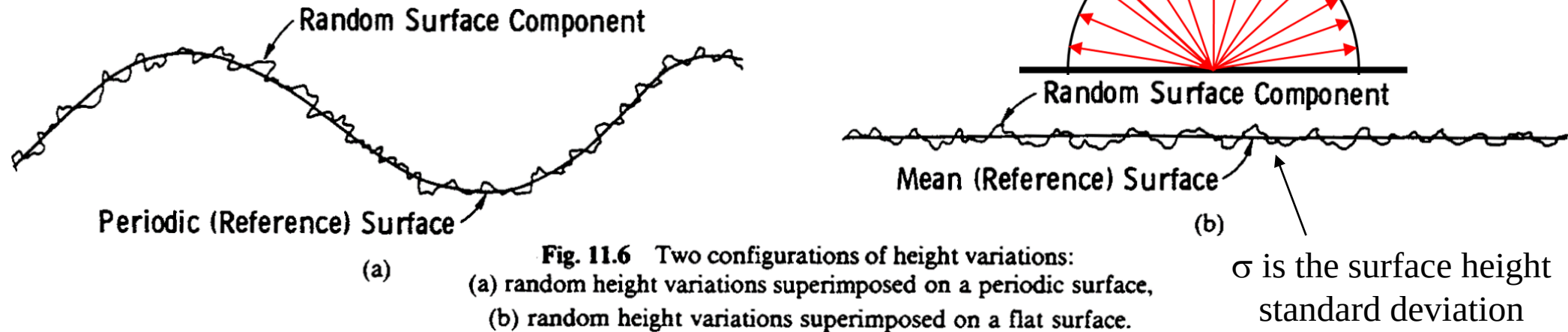


Harvey & Kotha, Optical Engineering (1995)

Distorted PSF or Energy Removed from Core is not blocked by the occulting mask.

Surface Structure Function

Surface structure can be decomposed into Correlated and Uncorrelated Components



Correlated components can be decomposed (via Fourier Transform) into Periodic Structure and for long period structure manifest themselves as the specular point spread function (PSF)

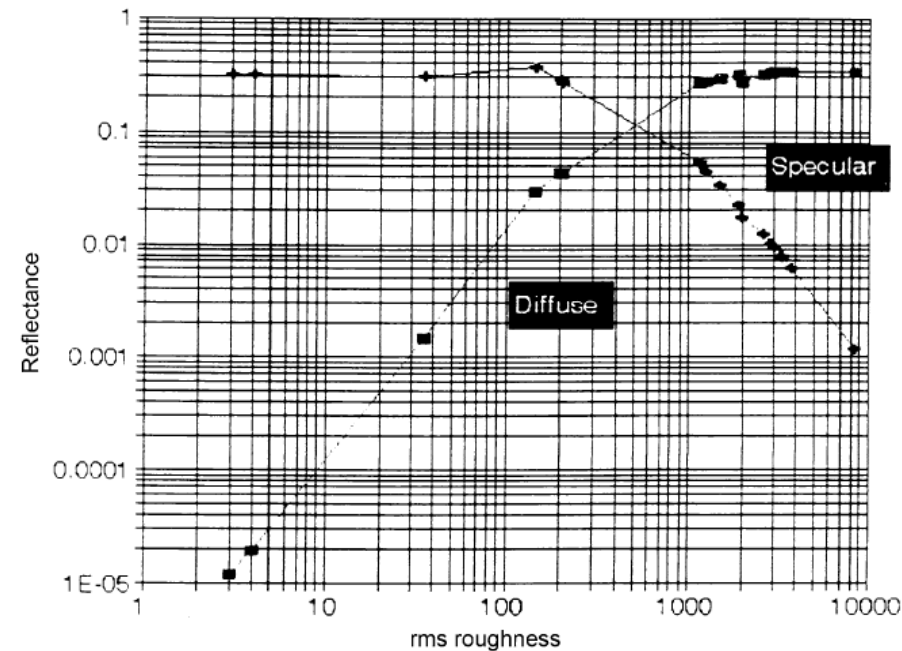
Removing correlated components, via a deformable mirror, leaves uncorrelated Gaussian Random Structure which scatters light in all directions, i.e. Lambertian Scatter.

If partially random (beyond correlation length) the uniform scatter is around paraxial ray

Conservation of Energy

Assuming no absorption or transmittance, conservation of energy requires:

Host Star Incident Power = Specular Reflection + Correlated Scatter + Diffuse Scatter

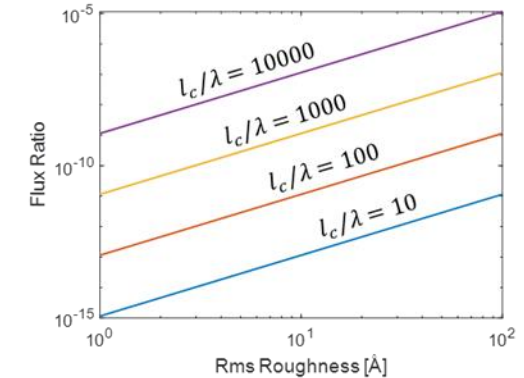
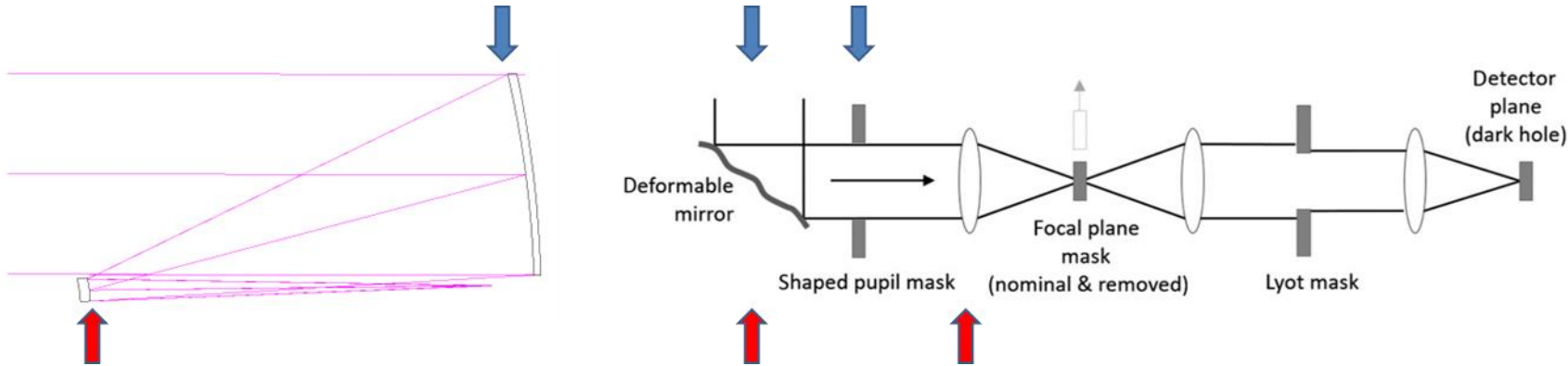


All surfaces will have some amount of micro-roughness and thus diffuse scatter.

Surface Structure Scatters E-Field into Dark Hole

Micro-Structure on Primary Mirror produces speckle in Conjugate Planes.

Speckle in Conjugate Planes produce planar E-fields in Dark Hole.



Scatterers located on other surfaces produce 'defocused' E-field in Dark Hole.

For the EAC1 & EAC2 optical designs:

- PM scatter produces a collimated wavefront at the dark hole
- SM scatter produces a spherical wavefront with a radius of 12.8 meters

Only surfaces before the occulting mask are significant.

Amplitude depends on roughness and correlation length.

Scattered Light is Coherent

Scattered Light is coherent because

- Optical Surfaces are $\ll \lambda$ and WFE $\ll \lambda$.
- Coronagraph is operating on 'narrow' 20% band.

$$\text{Coherence Length} = \frac{\lambda^2}{\Delta\lambda}$$

Coherence Length for 20% Band					units
Wavelength	300	680	1000	1800	nm
Coherence Length	1500	3400	5000	9000	nm

Light Scattered from a single point will have 'zero' optical path difference (OPD) from the input wavefront that illuminates that scattering point.

As the host star's wavefront propagates through the optical system, each time it reflects off an optical surface, a small amount becomes a diffuse E-Field, scattered by uncorrelated microstructure, that is in-phase (coherent) with the wavefront.

Leakage → Static Speckle Contrast Noise



In the Dark-Hole, after focal plane mask star-light suppression, some residual error remains – leading to a ‘Leakage’ field $E(u,v)$.

$$E(u,v) = E_{Leakage} + E_{NAS} + \dots$$

Which produces a Speckle Intensity Pattern = $|E(u,v)|^2$

Theoretically this pattern can be corrected by wavefront sensing and control – but the level of correction is unknown as well as the effect of physical issues such as polarization.

If corrected pattern is constant or static, it can be removed via Differential Imaging.

The maximum allowed amplitude of this static error (including Near-Angle Scatter) is defined by Shot Noise.

Leakage → Contrast Instability or Speckle Noise



‘Temporal’ disturbance or drift errors in the telescope or coronagraph optomechanical configuration adds a small amount $\Delta E(u,v,t)$ which is coherently added to $E(u,v)$.

$$|E + \Delta E|^2 = |E|^2 + |\Delta E|^2 + 2R\{E^* \cdot \Delta E\}$$

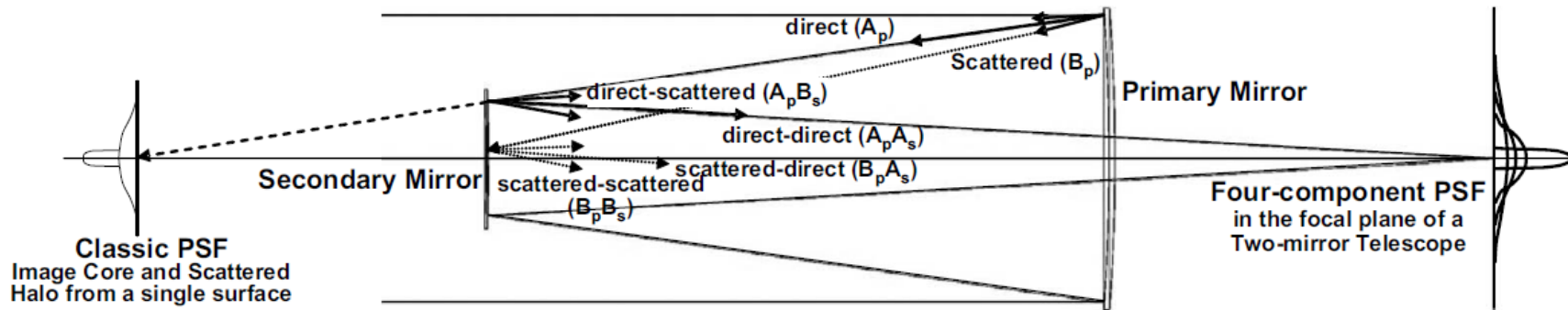
The coherent mixing or cross term = Contrast Instability or Speckle Noise.

- **Wavefront Instability (ΔE) causes ‘speckle’ noise that is not correctable by a DM.**
- **If Near-Angle Scatter increases the ‘post-correction’ static field $E(u,v)$ it causes an increase of the heterodyne amplification of the ΔE instability.**
- **If there is a maximum allowed amplitude for the Heterodyne Term, then any increase of the ‘static’ E term due to Near-Angle Scatter ‘requires’ a decrease in the allowed ‘temporal’ ΔE term.**

For example, if 5 pm wavefront instability is the limit to keep speckle noise below a threshold and the static E-field term increases by 20%, then instability allocation must be reduced to 4 pm.

Scattered Light Sums

Each optical surface before the coronagraph mask illuminated by host star light produces a scattered light wavefront that propagates to the Dark Hole and **Sums**.



The below plot shows the relative power of scatter-direct paths from PM & SM

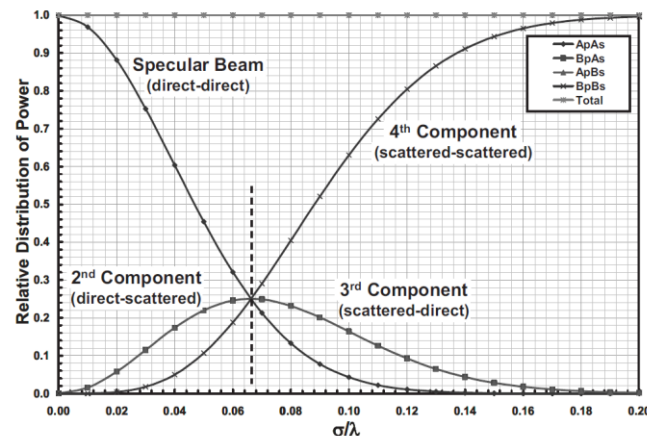
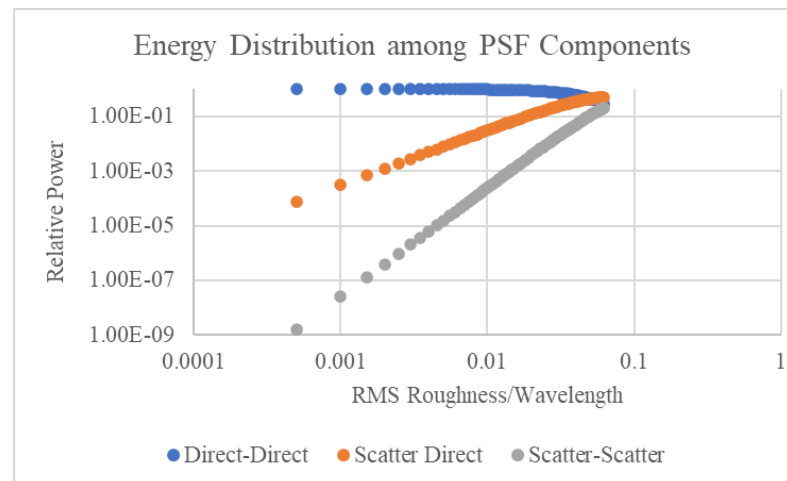


Fig. 3 Energy distribution among the four PSF components.

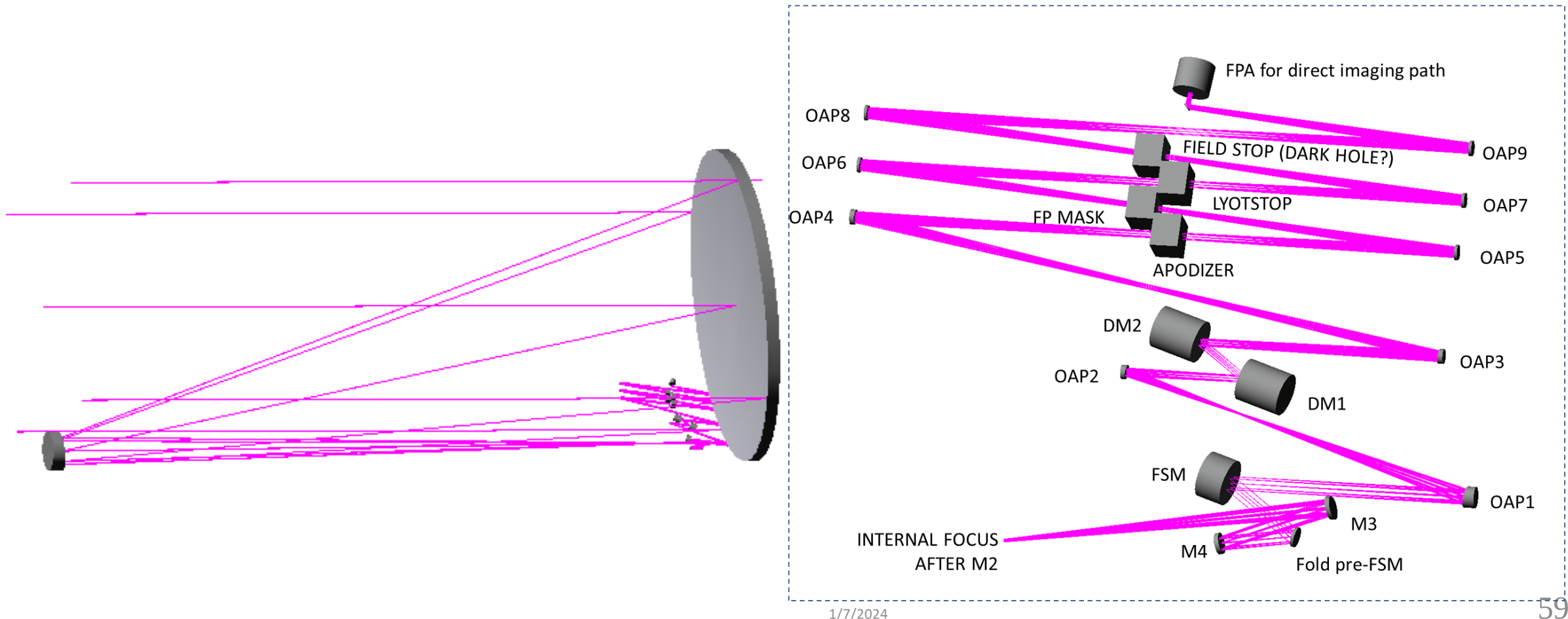


While small, single digit Angstrom surface roughness from 2 surfaces is significant,

EAC Cases have Many Surfaces

EAC1 and EAC2 have 14 optical surfaces before the Focal Plan Mask – each contributing coherent scattered light into the dark hole.

In the coronagraph, the FSM and DM1 are conjugate with the PM.



Power Spectral Density (PSD) Analysis

$$\text{PSD} \rightarrow \text{BRDF} \rightarrow \tau_{NAS}$$

Power Spectral Density (PSD)

PSD decomposes any data set into contributions per frequency (spatial or temporal)

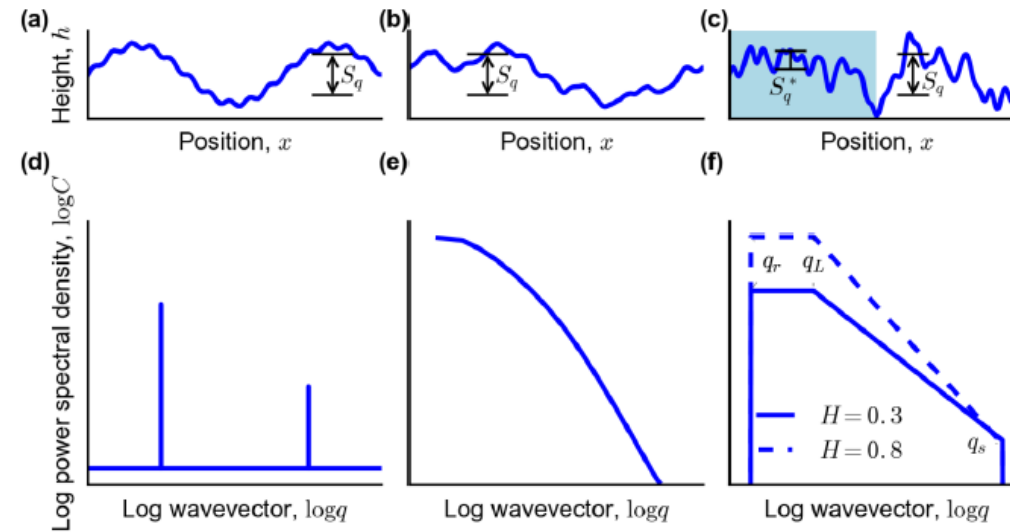


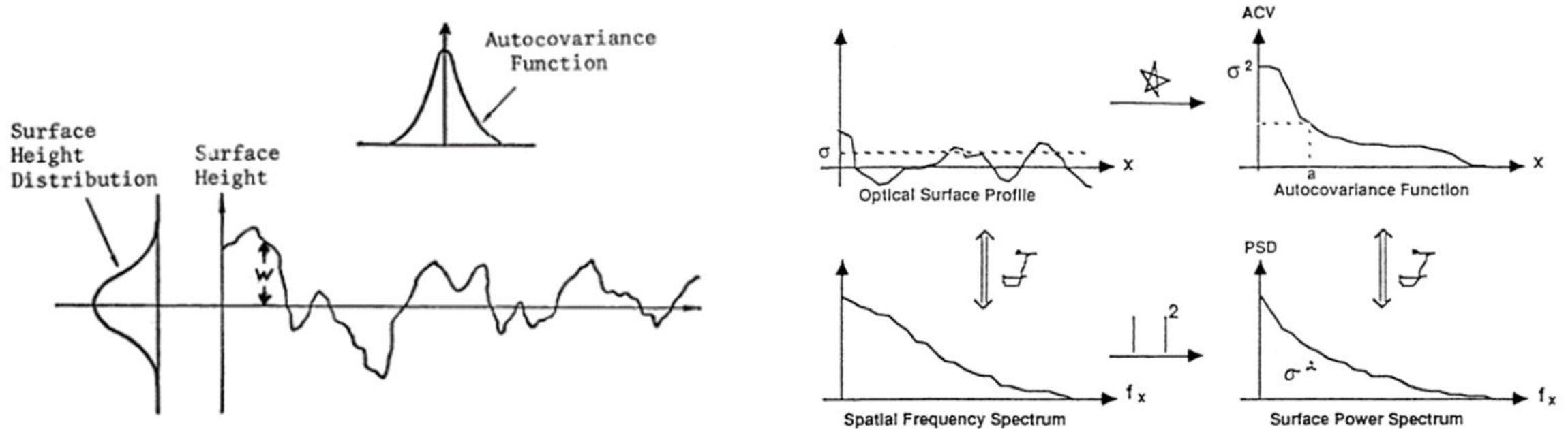
Figure 1: Examples for regular and random surfaces. Row (a)-(c) shows a line scan of a surface and (d)-(f) illustrates the corresponding power spectral density (PSD). Panel (a) shows a superposition of two sine waves of different wavelengths (wavevectors), panel (b) a frozen capillary wave and panel (c) a self-affine, randomly rough surface. (d) For the sine waves, the PSD has a single peak at the respective wavevector. The PSDs of randomly rough surfaces, as shown here for a non-self-affine frozen-capillary-wave surface (e) and two self-affine surfaces (f), have contributions from many wavevectors. The scalar measure of root mean square height, S_q , is identical for these three surfaces, although their topography is clearly different. The value S_q^* shown for the randomly rough surface is measured over half the domain length indicated by the blue rectangle. This illustrates that S_q depends on the domain over which the measurement takes place. Panel (f) shows the PSD for two surfaces with different Hurst exponents (solid and dashed line) but identical RMS slope $h'_{rms} = 0.1$. q_r , q_L and q_s indicate roll-off, large and small wavelength cutoffs.

PSD has been used to describe fractal statistics of data as diverse as lithographic line widths, surfaces (optical, roads, fields, oceans, etc) to flow rates in the Nile River (Hurst exponent)

Power Spectral Density (PSD)



PSD is the Fourier transform of the autocovariance function (ACF) of the data set.



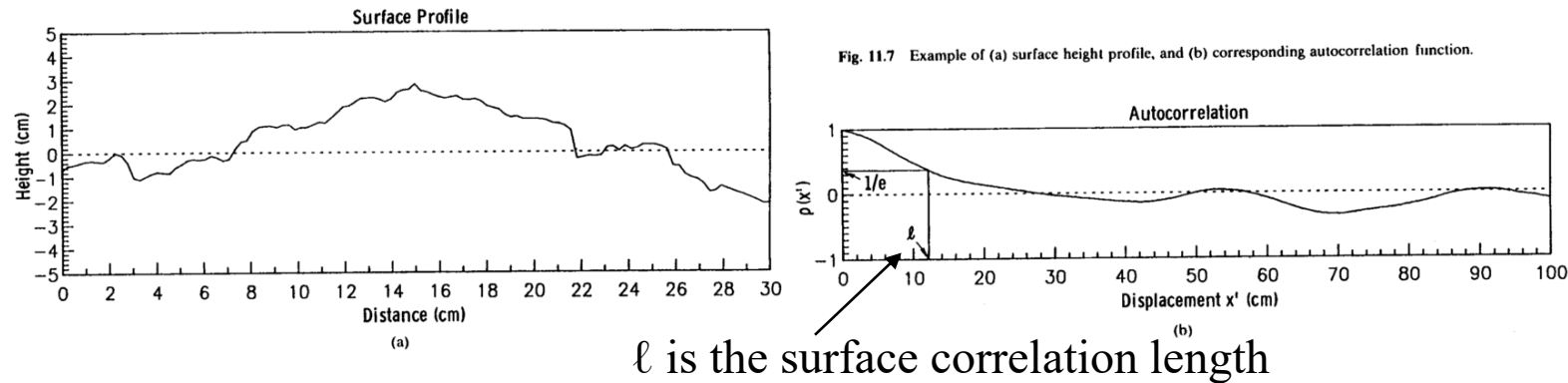
James E. Harvey, "Light-Scattering Characteristics Of Optical Surfaces," Proc. SPIE 0107, Stray Light Problems in Optical Systems, (26 September 1977); <https://doi.org/10.1117/12.964594>

James E. Harvey, Kristin L. Lewotsky, "Scattering from multilayer coatings: a linear systems model," Proc. SPIE 1530, Optical Scatter: Applications, Measurement, and Theory, (1 December 1991); <https://doi.org/10.1117/12.50494>

Autocovariance Surface Structure Function



Autocovariance Function (ACF) is calculated by convolving the Surface Structure against itself, i.e. shear the dataset against itself.



Correlation Length (ℓ) is maximum shear for which the surface spatial structure is correlated (i.e. longest spatial wavelength Fourier decomposed periodic component).

- Some define ℓ to be where autocovariance function = 0; others define it as $1/e$.

For very smooth surfaces, ℓ is L to $L/2$ where L = sample length. (depending on definition)

Shears beyond the Correlation Length are uncorrelated.

Correlation Length

Correlation Length is very important.

- PSD beyond the Correlation Length cut-off is Correlated and falls off as a statistical function with negative slope.
 - Typical Slope for polished optical surfaces is -2.5.
 - Correlated structure scatters light into Angles proportional to their Spatial Frequency
 - Dark Hole Angles are important for Coronagraphy
- PSD below the Correlation Length cut-off is Flat ('white noise')
 - Uncorrelated structure scatters light into all angles.
 - **NOTE that PSD(0) is the 'flat' Amplitude**
- Surface structure separated by a distance larger than correlation length cut-off are statistically independent i.e uncorrelated.
Typically, such structure has Gaussian Random statistics.

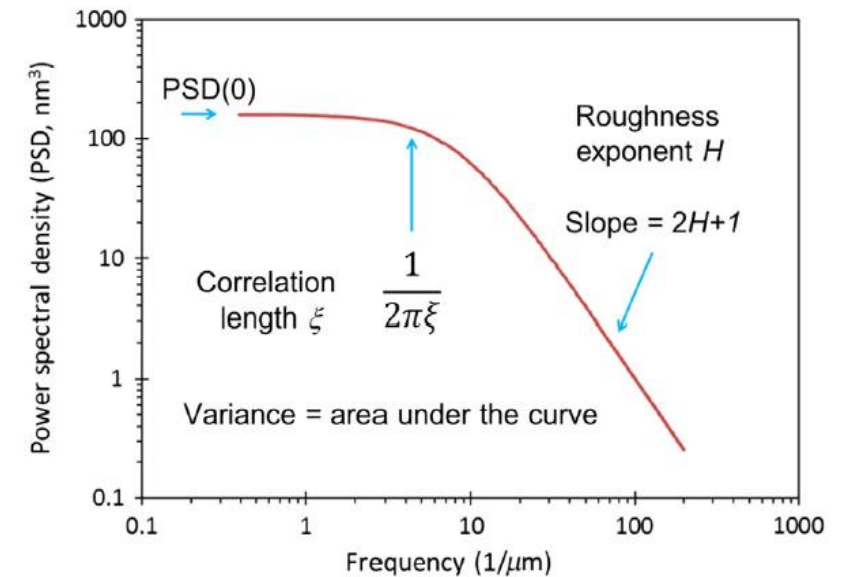
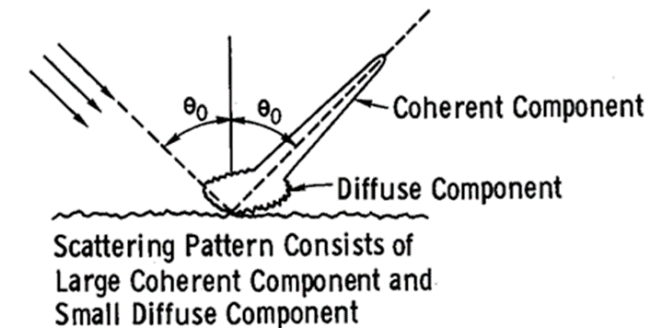


Fig. 3 A typical PSD can be described by three parameters: PSD(0), the low-frequency value of the PSD, the correlation length ξ , and the roughness exponent H . The variance of the roughness is the area under the PSD curve.



Chris A. Mack, "Reducing roughness in extreme ultraviolet lithography," J. Micro/Nanolith. MEMS MOEMS 17(4) 041006 (7 August 2018)

<https://doi.org/10.1117/1.JMM.17.4.041006>

Allen, Chris, "Surface Scattering", Course website URL people.eecs.ku.edu/~callen/823/EECS823.htm

PSD is Band Limited



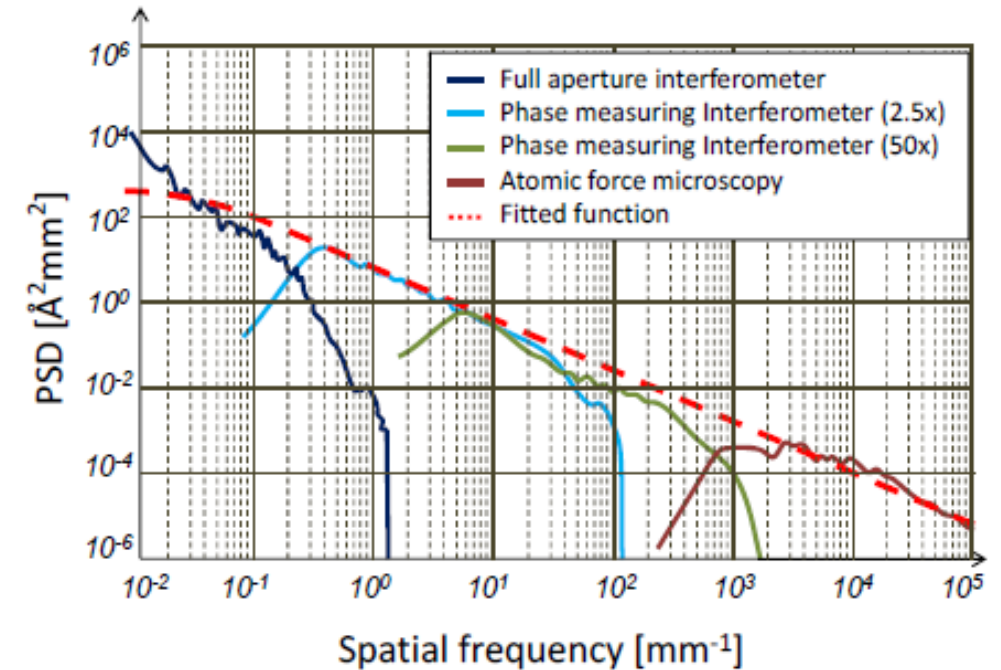
Problem with calculating ACF and PSD from surface height data is that no single instrument can sample the entire spatial band

- Full aperture interferometer
- Sub-aperture interferometer
- Microprofiler

and the data are band limited.

- Longest Spatial Period (Lowest Spatial Frequency) is limited to Sample Length.
- Shortest Spatial Period (Highest Spatial Frequency) is limited by Sample Size & Spacing.

Also, full-aperture interferometry is blind to microstructure.



Measured Surface RMS Roughness over a spatial band:

$$\sigma = \left[\int_{f_1}^{f_2} PSD(f) df \right]^{1/2}$$

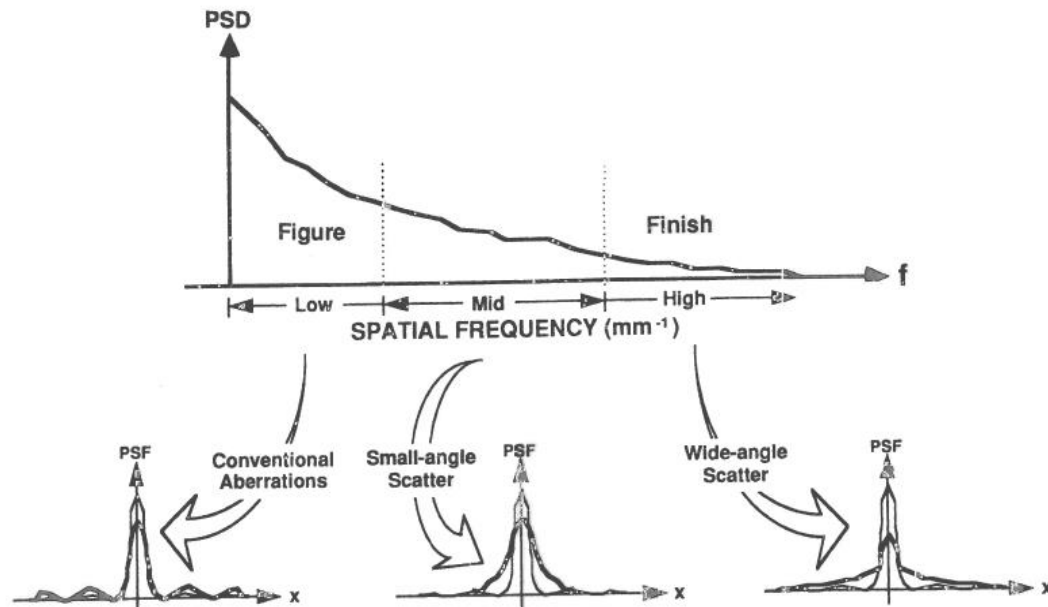
Narak Choi, James E. Harvey, "Linear systems formulation of image analysis in the presence of both aberrations and surface scatter," Proc. SPIE 8128, Current Developments in Lens Design and Optical Engineering XII; and Advances in Thin Film Coatings VII, 81280B (7 September 2011);

<https://doi.org/10.1117/12.910930>

PSD and PSF

PSD is important because it informs the Optical System's Point Spread Function (PSF)

- Low Spatial Frequency Structure shape the Core
- Mid-Spatial Frequency Structure produce Small-Angle Scatter
- High Spatial Frequency Structure produce Wide-Angle Scatter



Surface total variance under the PSD is sum of Bank-Limited variances

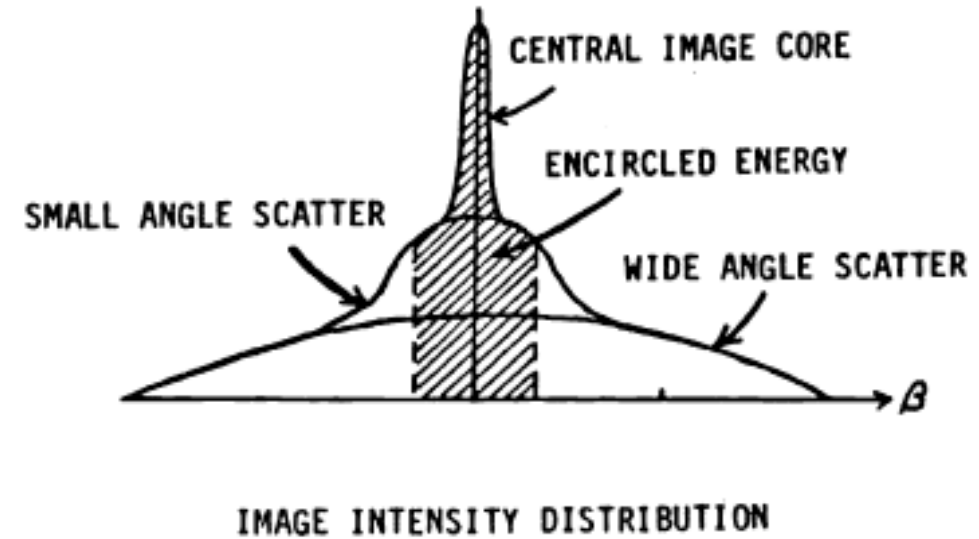
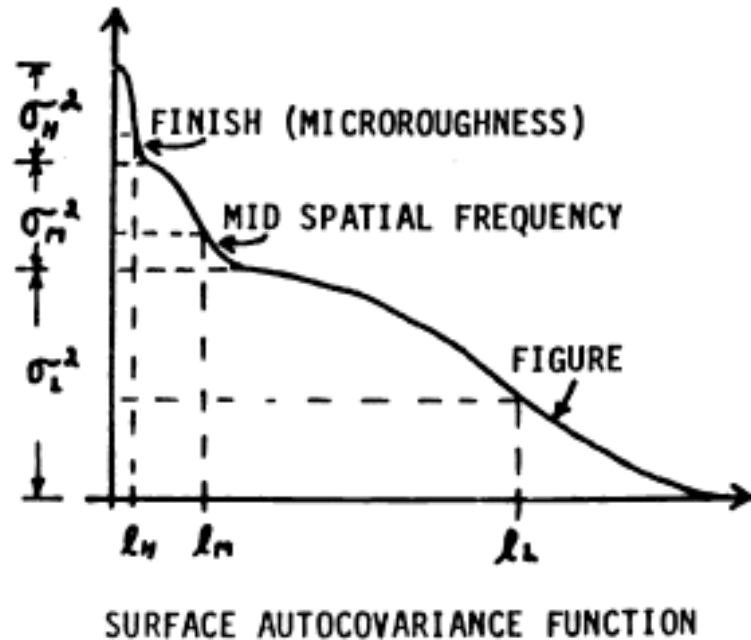
$$\sigma_{\text{Total}}^2 = \sigma_{\text{Low}}^2 + \sigma_{\text{Mid}}^2 + \sigma_{\text{High}}^2$$

and Surface total RMS

$$\sigma_{\text{RMS}} = \sqrt{\sigma_{\text{Total}}^2}$$

Band-Limited Autocovariance Function (ACF)

Total Autocovariance Function (ACF) for a Surface is the sum of individual band-limited ACFs.



Band-Limited surface roughness variances sum to a total surface variance:

$$\sigma_{\text{Total}}^2 = \sigma_{\text{Low}}^2 + \sigma_{\text{Mid}}^2 + \sigma_{\text{High}}^2$$

NOTE: Each Band-Limited ACF has its own Correlation Length

Autocovariance Function and PSF

Also important is that **PSDs add**. Total PSD is **stack** of individual PSDs.

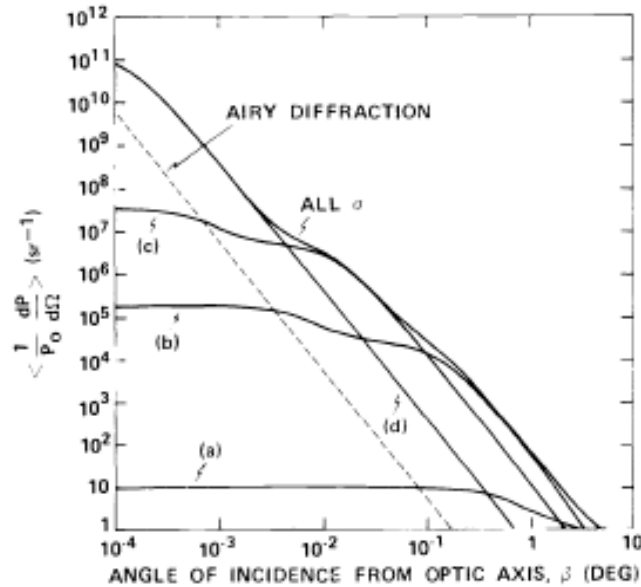


Figure 5. Angle-resolved scattering (ARS) versus angle of incidence β for an exponential autocovariance function. The Airy diffraction envelope is shown in the dotted line. The lines designated (a)-(d) are for (a) $\sigma = 10 \mu\text{m}$, (b) $\sigma = 1 \text{ mm}$, (c) $\sigma = 1 \text{ cm}$, and (d) $\sigma = 1 \text{ m}$, where σ is the correlation length. The $\sigma = 0.1 \mu\text{m}$ line is $\sim 10^{-4}$ and is off scale. The all- σ line is the net result of all σ values. It is seen that the ARS considerably exceeds the Airy limit. The incident light is unpolarized at wavelength $\lambda = 5000 \text{ \AA}$.

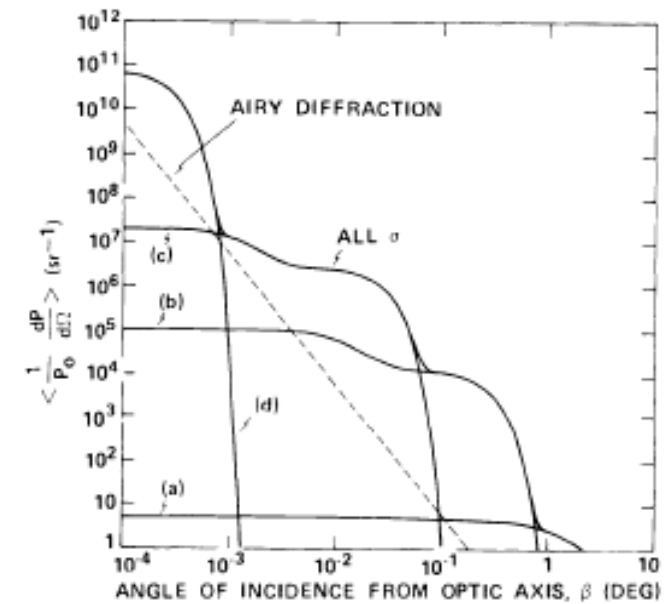


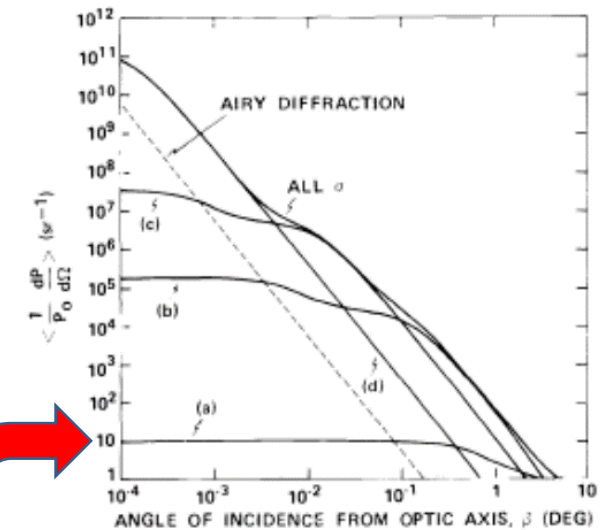
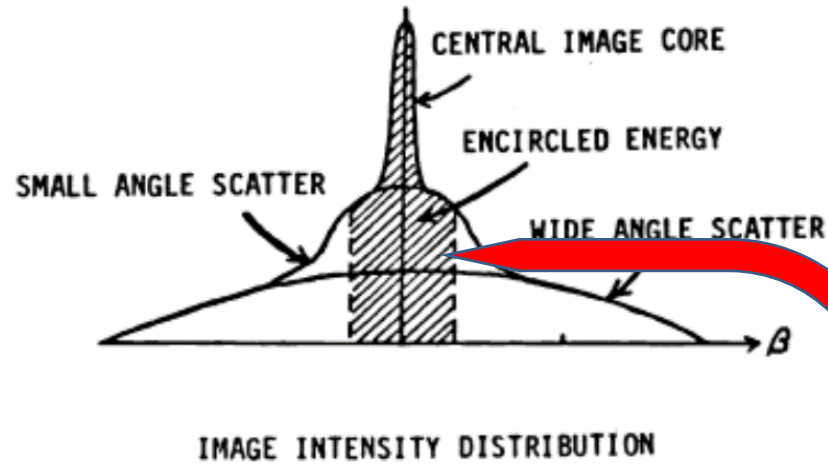
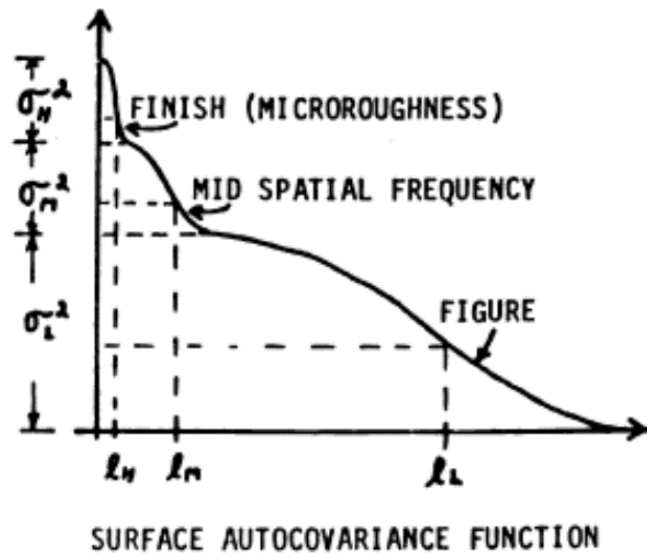
Figure 6. This figure parallels Figure 5 except that a Gaussian autocovariance function is used.

i	rms Roughness, μm	Autocorrelation length, μm
1	0.0015	0.1
2	0.0025	10
3	0.0035	1000
4	0.0050	10,000
5	0.0086	1,000,000

ACF, PSD and PSF

Low- and some Mid-Frequency (to 30 or 40 cycles) Structure scatters light into the Dark Hole, this Scatter can be corrected by Deformable Mirrors

Correlated Mid- and High-Frequency Structure scatters light beyond the Dark Hole



Uncorrelated High-Frequency Gaussian Random Structure scatters light over all Angles

William P. Zmek, Edward C. Moran, James E. Harvey, "The Effects Of Surface Quality Upon The Performance Of Normal Incidence X-Ray/XUV Imaging Systems," Proc. SPIE 0984, X-Ray Multilayers in Diffractometers, Monochromators, and Spectrometers, (16 December 1988); <https://doi.org/10.1117/12.948788>

J. M. Elson, H. E. Bennett, "Image Degradation Caused By Direct Scatter From Optical Components Into The Image Plane," Proc. SPIE 0511, Stray Radiation IV, (17 January 1985); doi: 10.1117/12.945028

PSD and Surface Roughness (Mack)

Because the total area under the PSD curve is equal to the Surface Variance.

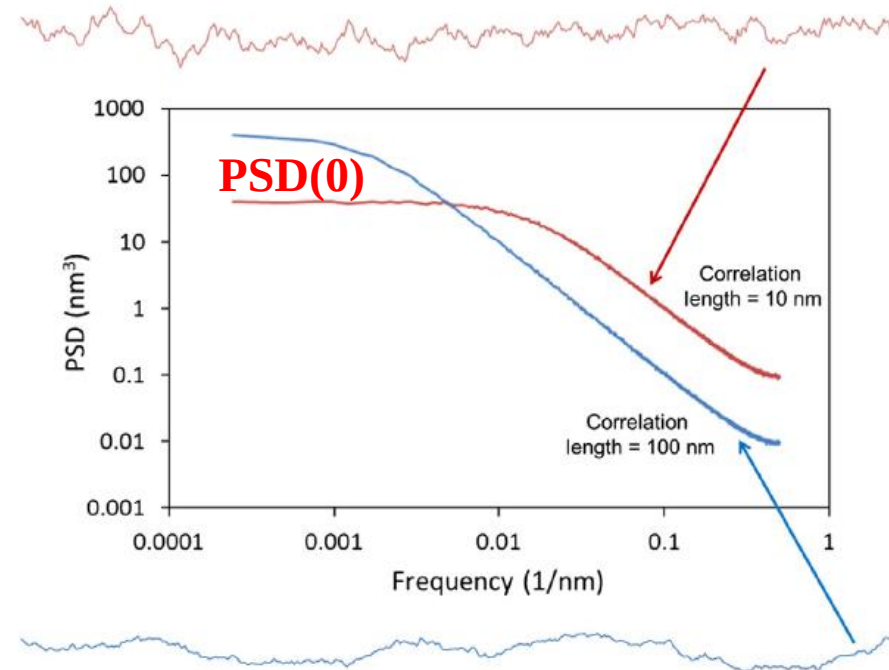
The value of $\text{PSD}_{2D}(0)$ depends on Surface Roughness and Correlation Length.

$$\text{PSD}_{2D}(0) \sim \ell^2 \cdot \sigma^2$$

For Isotropic Surfaces (i.e. Polished), PSD can be integrated azimuthally into a 1D radial function.

$$\text{PSD}_{1D}(0) \sim \ell \cdot \sigma^2$$

For a given surface roughness, longer correlation length = larger $\text{PSD}(0)$



PSD and Surface Roughness (Stover)



Beckmann-Kirchhoff scatter from isotropic randomly rough surface with normal (Gaussian) height distribution and Gaussian autocorrelation function

$$\text{PSD}_{2D}(0) = \pi \ell^2 \sigma^2 \exp(-(\pi f \cdot \ell)^2) \quad (\text{Stover Eq 3.52})$$

$$f = \frac{(\beta - \beta_0)}{\lambda}$$

Assuming that $f \cdot \ell$ is very small:

$$\text{PSD}_{2D}(0) = \pi \ell^2 \sigma^2$$

Because PSD(0) is flat across the entire dark hole, we can derive optical surface roughness specifications as a function of NAS Throughput.

2D PSD $\rightarrow$ BRDF



Rayleigh-Rice theory relates PSD of an isotropic (i.e. polished) surface to BRDF:

$$\text{PSD}(f_x, f_y) = \left[\frac{10^8 \lambda^4}{16\pi^2} \right] \cdot \text{BRDF}(\beta - \beta_0) [\text{\AA}^2 \mu\text{m}^2] \quad (\text{Stover Eq 4.3})$$

This equation allows one to calculate the surface statistics to achieve a Near-Angle Scatter Throughput (τ_{NAS}) at any physical location in the dark hole.

Remember that units of BRDF is $[\text{Sr}^{-1}]$

The equation simplifies to PSD_0 and BRDF_0 , because they are constant across the dark hole.

$$\text{PSD}_0 = \left[\frac{10^8 \lambda^4}{16\pi^2} \right] \cdot \text{BRDF}_0 [\text{\AA}^2 \mu\text{m}^2]$$

$$\text{PSD} \rightarrow \text{BRDF} \rightarrow \tau_{NAS}$$

Starting with:

$$\text{PSD}_0 = \left[\frac{10^8 \lambda^4}{16\pi^2} \right] \cdot \text{BRDF}_0$$

Using:

$$\tau_{NAS} = \text{BRDF}_0 \cdot \Omega_{\text{core}}$$

$$\text{BRDF}_0 = \frac{\tau_{NAS}}{\Omega_{\text{core}}}$$

$$\text{2D PSD}_0 = \pi \ell^2 \sigma^2$$

Near-Angle Throughput can be connected to Surface Statistics

$$\text{PSD}_0 [\text{\AA}^2 \mu\text{m}^2] = \left[\frac{10^8 \lambda^4}{16\pi^2} \right] \cdot \frac{\tau_{NAS}}{\Omega_{\text{core}}} = \pi \ell^2 \sigma^2$$

PSD $\rightarrow \tau_{NAS} \rightarrow$ Surface Statistics

One can solving for the surface statistics needed to achieve a τ_{NAS} specification:

$$\sigma[\text{\AA}] = \left[\left[\frac{10^8}{16\pi^3} \right] \cdot \left[\frac{\lambda^4}{\ell^2} \right] \cdot \left[\frac{\tau_{NAS}}{\Omega_{\text{core}}} \right] \right]^{1/2} = \frac{1}{4\pi} \left[\frac{\lambda^2}{\ell} \cdot 10^4 \right] \left[\frac{\tau_{NAS}}{\pi \Omega_{\text{core}}} \right]^{1/2}$$

10^4 is a wavelength scale factor from μm to $\text{\AA}$, because λ and ℓ are in units of μm .

If $\tau_{NAS} = 0.5\text{E-}12$ and $\Omega_{\text{core}} = \pi\text{E-}14$

Then $\frac{\tau_{NAS}}{\Omega_{\text{core}}} = 16$

$$\sigma[\text{\AA}] = \frac{1}{4\pi} \left[\frac{\lambda^2}{\ell} \cdot 10^4 \right] \left[\frac{16}{\pi} \right]^{1/2} = \frac{4}{4\pi^{3/2}} \left[\frac{\lambda^2}{\ell} 10^4 \right] = 0.18 \left[\frac{\lambda^2}{\ell} \cdot 10^4 \right]$$

where λ and ℓ are in units of μm

PSD and Surface Roughness (Church)



BUT ... MAYBE NOT

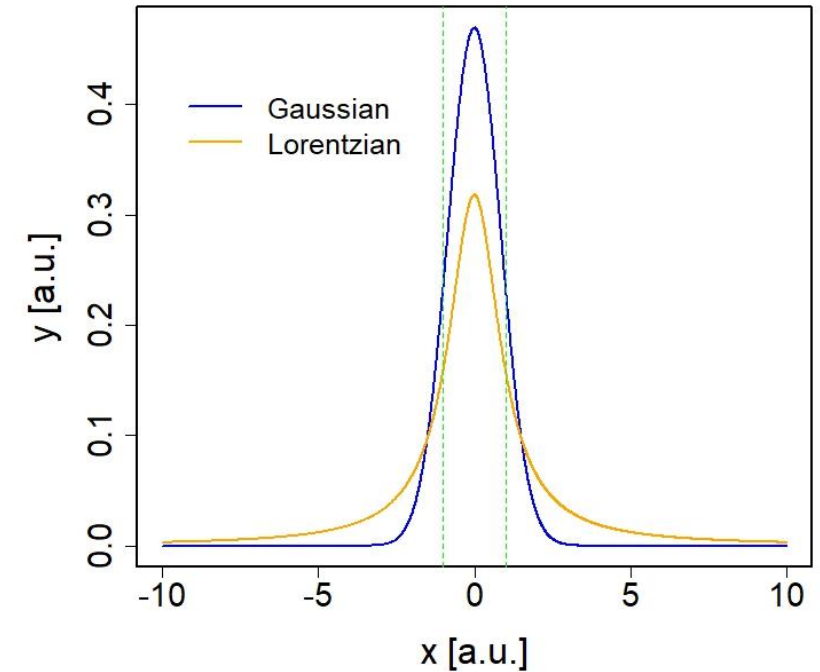
If the surface micro-roughness is Lorentzian instead of Gaussian, the surface specification is different:

$$\text{PSD}_{2D}(0) = \frac{\ell^2 \sigma^2}{2\pi}$$

And the Surface Specification is:

$$\sigma[\text{\AA}] = \left[\frac{\lambda^2}{\ell} \cdot 10^4 \right] \left[\frac{\tau_{NAS}}{8\pi \Omega_{\text{core}}} \right]^{1/2}$$

where λ and ℓ are in units of μm



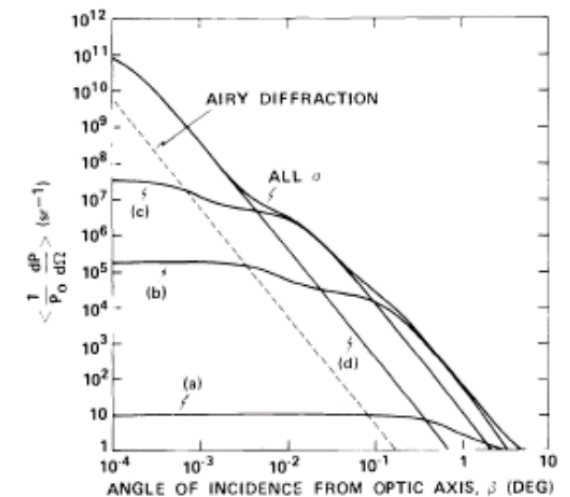
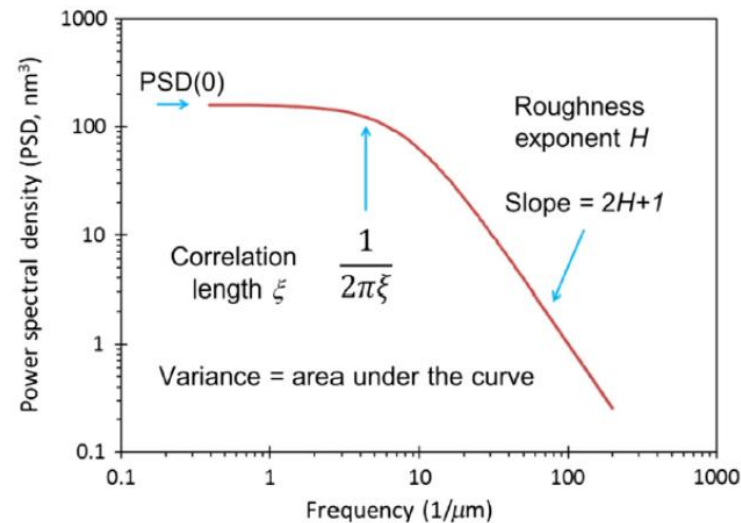
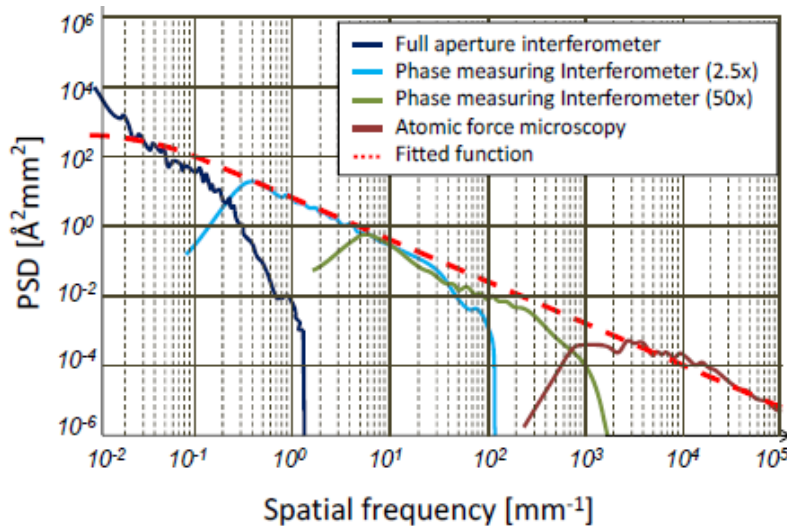
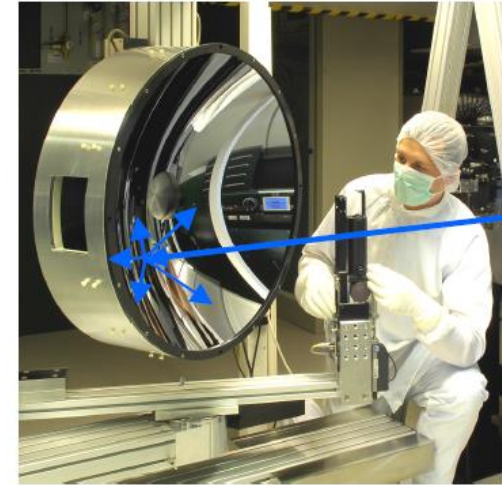
But, correlation length for the two distributions is also different.

For same rms, correlation length of Gaussian will be about 4 times smaller than that of the Lorentzian.

Microstructure Specification

Until this analysis, I never noticed that, for this super-polished EUV mirror, the highest spatial frequency PSD clearly shows a roll-over at ~ 900 [1/mm] for a Correlation Length $\sim 7 \mu\text{m}$ before it hits its measurement band limit.

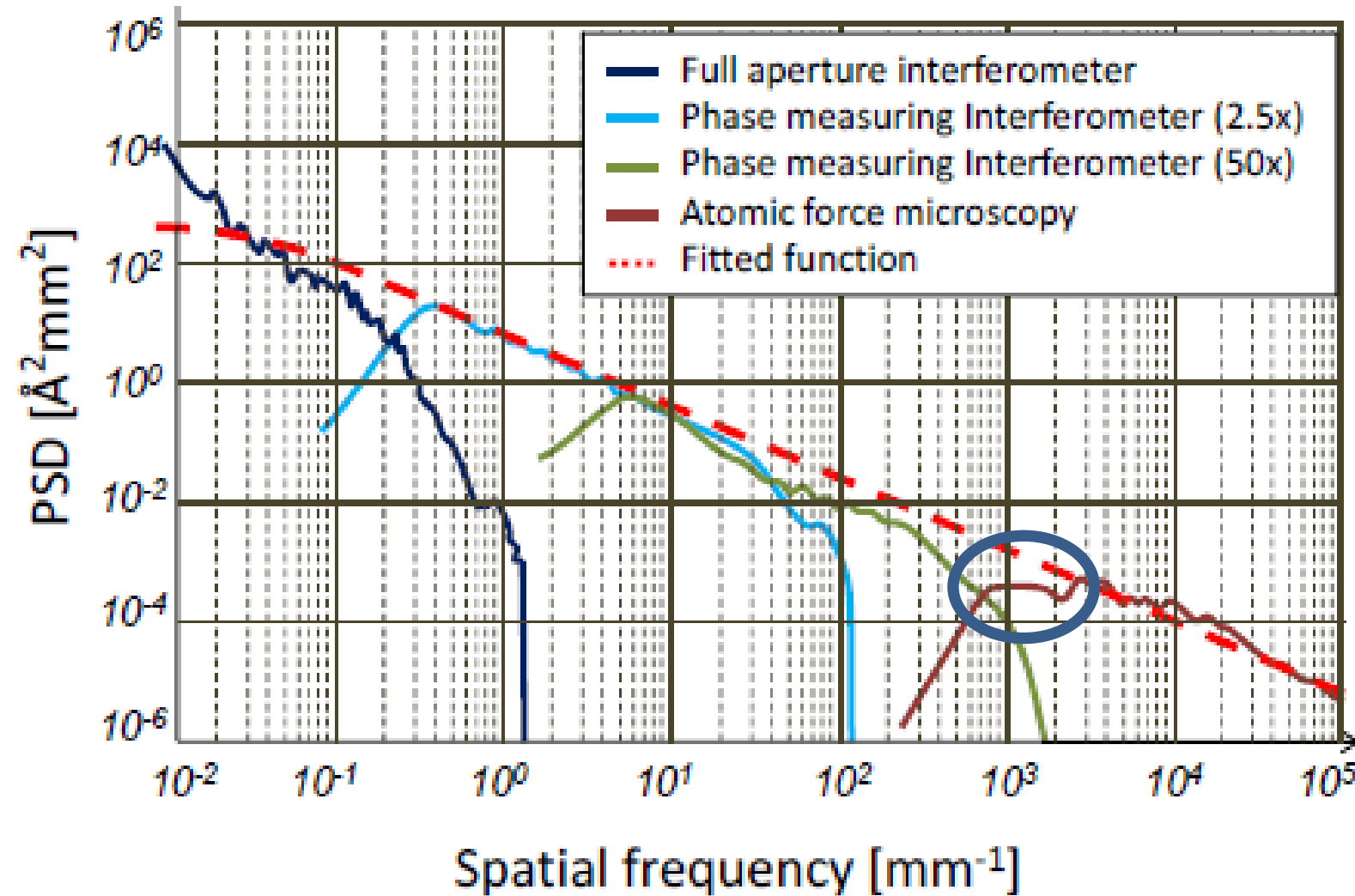
The other band-limited measurements do not have a flat uncorrelated PSD component.



Narak Choi, James E. Harvey, "Linear systems formulation of image analysis in the presence of both aberrations and surface scatter," Proc. SPIE 8128, 81280B (7 September 2011); <https://doi.org/10.1117/12.910930>

Sven Schröder, Marcus Trost, Torsten Feigl, James E. Harvey, Angela Duparré, "Efficient specification and characterization of surface roughness for extreme ultraviolet optics," Proc. SPIE 7969, Extreme Ultraviolet (EUV) Lithography II, 79692C (8 April 2011); doi: 10.1117/12.891280

Microstructure Specification



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<https://doi.org/10.1117/12.910930>